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Date \_\_\_\_\_  
Algebra II

## Solving Linear Systems in Three Variables

1. Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ :

- 1  $x + 3y + 5z = 45$
- 2  $6x - 3y + 2z = -10$
- 3  $-2x + 3y + 8z = 72$

1 and 2

$$\begin{array}{r} x + 3y + 5z = 45 \\ + 6x - 3y + 2z = -10 \\ \hline 7x + 7z = 35 \end{array}$$

1 and 3

$$\begin{array}{r} x + 3y + 5z = 45 \\ -(-2x + 3y + 8z = 72) \\ \hline 3x - 3z = -27 \end{array}$$

4 and 5

$$\begin{array}{r} 3(7x + 7z = 35) \\ 7(3x - 3z = -27) \\ \hline 21x + 21z = 105 \\ -21x + 21z = -189 \\ \hline 42z = -84 \\ \frac{42z}{42} = \frac{-84}{42} \\ z = -2 \end{array}$$

$$\begin{array}{r} x + 3y + 5z = 45 \\ -2 + 3y + 5(-2) = 45 \\ -2 + 3y - 10 = 45 \\ 3y - 12 = 45 \\ 3y = 57 \\ y = 19 \end{array}$$

$$\begin{array}{r} 7x + 7z = 35 \\ 7(-2) + 7z = 35 \\ -14 + 7z = 35 \\ 7z = 49 \\ z = 7 \end{array}$$

2. Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ :

- A  $x + 2y - 3z = -2$
- B  $2x - 2y + z = 7$
- C  $x + y + 2z = -4$

A and B

$$\begin{array}{r} x + 2y - 3z = -2 \\ + 2x - 2y + z = 7 \\ \hline 3x - 2z = 5 \end{array}$$

B and C

$$\begin{array}{r} 2x - 2y + z = 7 \\ 2(x + y + 2z = -4) \\ \hline 2x - 2y + z = 7 \\ 2x + 2y + 4z = -8 \\ \hline 4x + 5z = -1 \end{array}$$

D and E

$$\begin{array}{r} 5(3x - 2z = 5) \\ 2(4x + 5z = -1) \\ \hline 15x - 10z = 25 \\ + 8x + 10z = -2 \\ \hline 23x = 23 \\ \frac{23x}{23} = \frac{23}{23} \\ x = 1 \end{array}$$

$$x + 2y - 3z = -2$$

$$1 + 2y - 3(-1) = -2$$

$$\begin{array}{r} 1 + 2y + 3 = -2 \\ 2y + 4 = -2 \\ 2y = -6 \\ y = -3 \end{array}$$

$$\begin{array}{r} 4x + 5z = -1 \\ 4(1) + 5z = -1 \\ 4 + 5z = -1 \\ 5z = -5 \\ z = -1 \end{array}$$

3. Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ :

A  $-x + y + 2z = 7$   
 B  $2x + 3y + z = 1$   
 C  $-3x - 4y + z = 4$

B and C  
 $2x + 3y + z = 1$   
 $-1(-3x - 4y + z = 4)$

A and B  
 $+1(-x + y + 2z = 7)$   
 $-2(2x + 3y + z = 1)$   
 $-x + y + 2z = 7$   
 $-4x - 6y - 2z = -2$

D and E  
 $+5x + 7y = -3$   
 $-5x - 5y = 5$

$2x + 3y + z = 1$   
 $+3x + 4y + z = -4$

$\frac{2y}{2} = \frac{2}{2}$

$y = 1$

D  $5x + 7y = -3$

E  $-5x - 5y = 5$

$5x + 7y = -3$   
 $5x + 7(1) = -3$   
 $5x + 7 = -3$   
 $-7 -7$

$\frac{5x}{5} = \frac{-10}{5}$   
 $x = -2$

$-x + y + 2z = 7$   
 $-(-2) + 1 + 2z = 7$   
 $2 + 1 + 2z = 7$   
 $\frac{2z + 3}{2} = \frac{4}{2}$   
 $z = 2$

4. Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ :

A  $2x - y + z = 7$   
 B  $x + 2y - 5z = -1$   
 C  $x - y = 6$

A and B  
 $5(2x - y + z = 7)$   
 $1(x + 2y - 5z = -1)$

C and D  
 $-3(x - y = 6)$   
 $1(11x - 3y = 34)$

$2 - y = 6$   
 $-2 -2$

$10x - 5y + 5z = 35$   
 $+x + 2y - 5z = -1$

$-3x + 3y = -18$   
 $+11x - 3y = 34$

$-y = 4$   
 $-1 -1$

D  $11x - 3y = 34$

$\frac{8x}{8} = \frac{16}{8}$   
 $x = 2$

$y = -4$

$2x - y + z = 7$   
 $2(2) - (-4) + z = 7$   
 $4 + 4 + z = 7$

$z + z = 7$   
 $z - 8$   
 $z = -1$

5. Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ :

A  $-2x + y + 3z = 20$   
 B  $-3x + 2y + z = 21$   
 C  $3x - 2y + 3z = -9$

B and C  

$$\begin{array}{r} -3x + 2y + z = 21 \\ + 3x - 2y + 3z = -9 \\ \hline 4z = 12 \\ \frac{4}{4} \quad \frac{12}{4} \\ \hline z = 3 \end{array}$$

A  

$$\begin{array}{r} -2x + y + 3(3) = 20 \\ -2x + y + 9 = 20 \\ \quad \quad \quad -4 \quad -9 \\ \hline -2x + y = 11 \end{array}$$

B  

$$\begin{array}{r} -3x + 2y + 3(3) = 21 \\ \quad \quad \quad -3 \quad -3 \\ \hline -3x + 2y = 18 \end{array}$$

D and E  

$$\begin{array}{r} -2(-2x + y = 11) \\ 1(-3x + 2y = 18) \\ \hline 4x - 2y = -22 \\ + -3x + 2y = 18 \\ \hline x = -4 \end{array}$$

$$\begin{array}{r} -2x + y = 11 \\ -2(-4) + y = 11 \\ 8 + y = 11 \\ \quad \quad -8 \\ \hline y = 3 \end{array}$$

6. Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ :

$2x + 3y = 5 - z$   
 $x - 2z = 2y - 4$   
 $-3x - y + 3z = -7$

A  $2x + 3y + z = 5$   
 B  $x - 2y - 2z = -4$   
 C  $-3x - y + 3z = -7$

A and B  

$$\begin{array}{r} 2(2x + 3y + z = 5) \\ 1(x - 2y - 2z = -4) \\ \hline 4x + 6y + 2z = 10 \\ + x - 2y - 2z = -4 \\ \hline 5x + 4y = 6 \end{array}$$

A and C  

$$\begin{array}{r} 3(2x + 3y + z = 5) \\ 1(-3x - y + 3z = -7) \\ \hline -6x - 9y - 3z = -15 \\ + -3x - y + 3z = -7 \\ \hline -9x - 10y = -22 \end{array}$$

D and E  

$$\begin{array}{r} 10(5x + 4y = 6) \\ 4(-9x - 10y = -22) \\ \hline 50x + 40y = 60 \\ -36x - 40y = -88 \\ \hline 14x = -28 \\ \frac{14}{14} \quad \frac{-28}{14} \\ \hline x = -2 \end{array}$$

$$\begin{array}{r} 5x + 4y = 6 \\ 5(-2) + 4y = 6 \\ -10 + 4y = 6 \\ +10 \quad +10 \\ \hline 4y = 16 \\ \frac{4}{4} \quad \frac{16}{4} \\ \hline y = 4 \end{array}$$

$$\begin{array}{r} 2x + 3y + z = 5 \\ 2(-2) + 3(4) + z = 5 \\ -4 + 12 + z = 5 \\ 8 + z = 5 \\ \quad \quad -8 \\ \hline z = -3 \end{array}$$

$$\begin{array}{r} + 50x + 40y = 60 \\ -36x - 40y = -88 \\ \hline 14x = -28 \\ \frac{14}{14} \quad \frac{-28}{14} \\ \hline x = -2 \end{array}$$

7. Solve the following system of equations algebraically for all values of  $a$ ,  $b$ , and  $c$ .

$$a + 4b + 6c = 23$$

$$a + 2b + c = 2$$

$$6b + 2c = a + 14$$

$$\begin{array}{r} -a \\ -a \end{array}$$

A  $a + 4b + 6c = 23$

B  $a + 2b + c = 2$

C  $-a + 6b + 2c = 14$

A and C

$$\begin{array}{r} + a + 4b + 6c = 23 \\ - a + 6b + 2c = 14 \\ \hline \end{array}$$

D  $10b + 8c = 37$

B and C

$$\begin{array}{r} + a + 2b + c = 2 \\ - a + 6b + 2c = 14 \\ \hline \end{array}$$

E  $8b + 3c = 16$

D and E

$$\begin{array}{r} -3(10b + 8c = 37) \\ 8(8b + 3c = 16) \\ \hline \end{array}$$

$$\begin{array}{r} -30b - 24c = -111 \\ 64b - 24c = 128 \\ \hline \end{array}$$

$$\begin{array}{r} + 94b = 17 \\ + 34 \\ \hline \end{array}$$

$b = .5$

$$10b + 8c = 37$$

$$10(.5) + 8c = 37$$

$$\begin{array}{r} 5 + 8c = 37 \\ -5 \quad -5 \\ \hline \end{array}$$

$$8c = 32$$

$c = 4$

$$a + 2b + c = 2$$

$$a + 2(.5) + 4 = 2$$

$$a + 1 + 4 = 2$$

$$\begin{array}{r} a + 5 = 2 \\ -5 \quad -5 \\ \hline \end{array}$$

$a = 3$

8. Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ :

$$y = -2x + 14$$

$$3x - 4z = 2$$

$$3x - y = 16$$

A  $2x + y = 14$

B  $3x - 4z = 2$

C  $3x - y = 16$

A and C

$$\begin{array}{r} 2x + y = 14 \\ + 3x - y = 16 \\ \hline \end{array}$$

$$\begin{array}{r} 5x = 30 \\ \hline \end{array}$$

$x = 6$

(2x2 system already)

$$2x + y = 14$$

$$2(6) + y = 14$$

$$12 + y = 14$$

$$\begin{array}{r} -12 \quad -12 \\ \hline \end{array}$$

$y = 2$

$$3x - 4z = 2$$

$$3(6) - 4z = 2$$

$$\begin{array}{r} 18 - 4z = 2 \\ -18 \quad -18 \\ \hline \end{array}$$

$$\begin{array}{r} -4z = -16 \\ \hline \end{array}$$

$z = 4$