$\qquad$

Date
Algebra II

## Algebra II Guide to 65 Test

## Part I: (2 Points Each)

1. What is the inverse of $f(x)=\frac{x}{x+2}$, where $x \neq-2$ ?
1) $f^{-1}(x)=\frac{2 x}{x-1}$
2) $f^{-1}(x)=\frac{-2 x}{x-1}$
3) $f^{-1}(x)=\frac{x}{x-2}$
4) $f^{-1}(x)=\frac{-x}{x-2}$
2. Which value is not contained in the solution of the system shown below?
$4 x-5 y+2 z=130$
$3 x+2 y-7 z=-99$
$10 x-6 y-4 z=112$
1) -8
2) -12
3) 10
4) 15
3. Given the parent function $p(x)=\cos x$, which phrase best describes the transformation used to obtain the graph of $g(x)=\cos (x+a)-b$, if $a$ and $b$ are positive constants?
1) right $a$ units, up $b$ units
2) left $a$ units, up $b$ units
3) right $a$ units, down $b$ units
4) left $a$ units, down $b$ units
4. The expression $\left(\frac{m^{2}}{m^{\frac{1}{3}}}\right)^{-\frac{1}{2}}$ is equivalent to
1) $-\sqrt[6]{m^{5}}$
2) $\frac{1}{\sqrt[6]{m^{5}}}$
3) $-m \sqrt[5]{m}$
4) $\frac{1}{m^{5} \sqrt{m}}$
5. Where $i$ is the imaginary unit, the expression $(x+3 i)^{2}-(2 x-3 i)^{2}$ is equivalent to
1) $-3 x^{2}$
2) $-3 x^{2}-18$
3) $-3 x^{2}+18 x i$
4) $-3 x^{2}-6 x i-18$
6. Which factorizations are correct?
I. $a^{3}+27 b^{3}=(a+3 b)\left(a^{2}-3 a b+9 b^{2}\right)$
II. $c^{3}-6 c^{2}+8 c+5 c^{2}-30 c+40=(c-2)(c-4)(c+5)$
III. $1-x^{4}=(1+x)^{2}(1-x)^{2}$
1) I, only
2) II and III only
3) I and II only
4) I, II, and III
7. For which values of $x$, rounded to the nearest hundredth, will $\left|x^{2}-9\right|-3=\log _{3} x$ ?
1) 2.29 and 3.63
2) 2.37 and 3.54
3) 2.84 and 3.17
4) 2.92 and 3.06
8. The solution set of the equation $\sqrt{2 x-4}=x-2$ is
1) $\{-2,-4\}$
2) $\{2,4\}$
3) $\{4\}$
4) $\}$
9. Which scenario is best described as a controlled experiment?
1) For a class project, students in Health class ask every tenth student entering the school if they eat breakfast in the morning.
2) A social researcher wants to learn whether or not there is a link between attendance and grades. She gathers data from 15 school districts.
3) A researcher wants to learn whether or not there is a link between children's daily amount of physical activity and their overall energy level. During lunch at the local high school, she distributed a short questionnaire to students in the cafeteria.
4) Sixty seniors taking a course in Advanced Algebra Concepts are randomly divided into two classes. One class uses a graphing calculator all the time, and the other class never uses graphing calculators. A guidance counselor wants to determine whether there is a link between graphing calculator use and students' final exam grades.
10. The yearbook staff has designed a survey to learn about student opinions on how the yearbook could be improved for this year. If they want to distribute this survey to 100 students and obtain the most reliable data, they should survey
1) Every third student sent to the office
2) Every third student to enter the library
3) Every third student to enter the gym for the basketball game
4) Every third student arriving at school in the morning

## Part II: (2 Points Each)

11. Determine if $x+3$ is a factor of $p(x)=x^{4}+7 x^{3}+9 x^{2}-21 x-36$. Explain your answer.
12. The parabola $y=-\frac{1}{4}(x+3)^{2}-6$ has a directrix at $y=-5$. What is the focus?

13. On the grid below, sketch a quartic polynomial whose factors are $x+5, x+2, x-1$, and $x-4$.

14. Graph $c(x)=-9(3)^{x-4}+2$ on the axes below.

15. Write the equation of the parabola with a focus of $(1,3)$ and a directrix of $y=-5$

16. Kina earns a $\$ 27,000$ salary for the first year of work at her job. She earns annual increases of $2.5 \%$. What is the total amount, to the nearest cent, that Kina will earn for the first eight years at this job?
17. Is $f(x)=\frac{10}{x}$ even, odd, or neither? Justify your answer.
18. For the function $f(x)=3^{x}$, find the average rate of change over the interval -5 to -1 rounded to the nearest thousandth.
19. A middle school conducted a survey of students to determine if they spent more of their time playing games or watching videos on their tablets. The results are shown in the table below. Of the students who spent more time playing games on their tablets, to the nearest percent, what percent were boys?

|  | Playing <br> Games | Watching <br> Videos | Total |
| :--- | :---: | :---: | :---: |
| Boys | 138 | 46 | 184 |
| Girls | 54 | 142 | 196 |
| Total | 192 | 188 | 380 |

20. The weights of students on the boys cross country team is normally distributed with a mean of 135.3 pounds and a standard deviation of 2.8 pounds. If the team has 32 members, how many of them, rounded to the nearest person, would be expected to weigh less than 130 pounds?
21. The results of a survey of the student body at Central High School about television viewing preferences are shown below.

|  | Comedy Series | Drama Series | Reality Series | Total |
| :--- | :---: | :---: | :---: | :---: |
| Males | 95 | 65 | 70 | 230 |
| Females | 80 | 70 | 110 | 260 |
| Total | 175 | 135 | 180 | 490 |

Are the events "student is a male" and "student prefers reality series" independent of each other? Justify your answer.
22. A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, $t$, in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation $t=2 \pi \sqrt{\frac{L}{g}}$ where $L$ is the length of the pendulum in meters and $g$ is a constant of $9.81 \mathrm{~m} / \mathrm{s}^{2}$. The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m . Determine, to the nearest tenth of a second, the time it takes this pendulum to complete one swing.

## Part III: (4 Points Each)

23. A group of students was trying to determine the proportion of candies in a bag that are blue. The company claims that $24 \%$ of candies in bags are blue. A simulation was run 100 times with a sample size of 50 , based on the premise that $24 \%$ of the candies are blue. The approximately normal results of the simulation are shown in the dot plot below.


The simulation results in a mean of 0.254 and a standard deviation of 0.060 . Based on this simulation, what is a plausible interval containing the middle $95 \%$ of the data? A student found that 18 out of 50 of the candies were blue. Use statistical evidence to explain why this is an expected value.
24. The accompanying table shows the number of bacteria present in a certain culture over a 5-hour period, where $x$ is the time, in hours, and $y$ is the number of bacteria.

Write an exponential regression equation for this set of data, rounding all values to four decimal places. Using this equation, determine the number of whole bacteria

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 0 | 1,000 |
| 1 | 1,049 |
| 2 | 1,100 |
| 3 | 1,157 |
| 4 | 1,212 |
| 5 | 1,271 | present after 6.5 hours.

25. Monthly mortgage payments can be found using the formula below:

$$
M=\frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{n}}{\left(1+\frac{r}{12}\right)^{n}-1}
$$

$$
\begin{gathered}
M=\text { monthly payment } \\
P=\text { amount borrowed } \\
r=\text { annual interest rate } \\
n=\text { number of monthly payments }
\end{gathered}
$$

The Banks family would like to purchase a home for $\$ 220,000$. They qualified for an annual interest rate of $4.8 \%$. If they put make a down payment of $\$ 100,000$ and plan to spend 15 years to repay the loan, what will be the monthly payment rounded to the nearest cent?

If they want their monthly payment to be $\$ 1500$, what would their down payment have to be?

## Part IV: (6 Points Each)

26. The value of a certain small passenger car based on its use in years is modeled by $V(t)=28482.698(0.684)^{t}$, where $V(t)$ is the value in dollars and $t$ is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t)=22151.327(0.778)^{t}$, where $Z(t)$ is measured in dollars, and $t$ is the time in years, models the unpaid amount of Zach's loan over time. Graph $V(t)$ and $Z(t)$ over the interval $0 \leq t \leq 5$, on the set of axes below.


State when $V(t)=Z(t)$, to the nearest hundredth, and interpret its meaning in the context of the problem. Zach takes out an insurance policy that requires him to pay a $\$ 3000$ deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the nearest year, how long will it take Zach to cancel this policy? Justify your answer.

