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Common Core Algebra II Regents Review Packet!

Mr. Schlansky

2024

Multiple Choice Strategy with Variables

If variables in the problems and answers:
10 STO $\rightarrow$ X, 15 STO $\rightarrow$ Y
Type in original problem, write down the value.
Type in each choice, write down the value.
If they match up, they are equivalent.
Check all four choices as more than one may be equivalent!

$\frac{7802}{23}$

1. The expression $\frac{6x^3+17x^2+10x+2}{2x+3}$ equals

1) $3x^2+4x-1+\frac{5}{2x+3}$ $\frac{7802}{23}$
2) $6x^2+8x-2+\frac{5}{2x+3}$
3) $6x^2-x+13-\frac{37}{2x+3}$
4) $3x^2+13x+\frac{49}{2}+\frac{151}{2x+3}$

$\frac{4060}{23}$

2. The expression $\frac{4x^3+5x+10}{2x+3}$ is equivalent to

1) $2x^2+3x-7+\frac{31}{2x+3}$
2) $2x^2-3x+7-\frac{11}{2x+3}$ $\frac{4060}{23}$
3) $2x^2+2.5x+5+\frac{15}{2x+3}$
4) $2x^2-2.5x-5-\frac{20}{2x+3}$

18432

3. What is the completely factored form of $k^4-4k^2+8k^3-32k+12k^2-48$?

1) $(k-2)(k-2)(k+3)(k+4)$
2) $(k-2)(k-2)(k+6)(k+2)$
3) $(k+2)(k-2)(k+3)(k+4)$
4) $(k+2)(k-2)(k+6)(k+2)$ 18432

2100

4. When factored completely, the expression $3x^3-5x^2-48x+80$ is equivalent to

1) $(x^2-16)(3x-5)$ 2100
2) $(x^2+16)(3x-5)(3x+5)$
3) $(x+4)(x-4)(3x-5)$ 2100
4) $(x+4)(x-4)(3x-5)(3x-5)$

not factored completely

$-96-40i$

5. Given i is the imaginary unit, $(2-yi)^2$ in simplest form is

1) $y^2-4yi+4$
2) $-y^2-4yi+4$
3) $-y^2+4$
4) y^2+4

$-96-40i$

$40i$

6. The expression $(x+i)^2-(x-i)^2$ is equivalent to

1) 0
2) $-2+4xi$
3) -2
4) $4xi$ $40i$

$-2400-300i$

7. The expression $6xi^3(-4xi+5)$ is equivalent to

1) $2x-5i$
2) $-24x^2-30xi$
3) $-24x^2+30x-i$
4) $26x-24x^2i-5i$

$-2400-300i$

8. The expression $\frac{a^2b^{-3}}{a^{-4}b^2}$ is equivalent to $\frac{320}{243}$

1) $\frac{a^6}{b^5}$ $\frac{320}{243}$

2) $\frac{b^5}{a^6}$

3) $\frac{a^2}{b}$

4) $a^{-2}b^{-1}$

9. Which expression is equivalent to $\frac{x^{-1}y^2}{x^2y^{-4}}$? $\frac{91125}{8}$

1) $\frac{x}{y^2}$

2) $\frac{x^3}{y^6}$

3) $\frac{y^2}{x}$

4) $\frac{y^6}{x^3}$ $\frac{91125}{8}$

10. What is the product of $\sqrt[3]{4a^2b^4}$ and $\sqrt[3]{16a^3b^2}$? 41774.. math: 4

1) $4ab^2\sqrt[3]{a^2}$ 41774..

2) $4a^2b^3\sqrt[3]{a}$

3) $8ab^2\sqrt[3]{a^2}$

4) $8a^2b^3\sqrt[3]{a}$

11. The expression $\sqrt[4]{16x^2y^7}$ is equivalent to 723.. → 4 math 5

1) $2x^{\frac{1}{2}}y^{\frac{7}{4}}$ 723...

2) $2x^8y^{28}$

3) $4x^{\frac{1}{2}}y^{\frac{7}{4}}$

4) $4x^8y^{28}$

12. For positive values of x, which expression is equivalent to $\sqrt{16x^2}\bullet x^{\frac{2}{3}}+\sqrt[3]{8x^5}$ 278..

1) $6\sqrt[3]{x^5}$ 278..

2) $6\sqrt[5]{x^3}$

3) $4\sqrt[3]{x^2}+2\sqrt[3]{x^5}$

4) $4\sqrt{x^3}+2\sqrt[5]{x^3}$

13. Written in simplest form, $\frac{c^2-d^2}{d^2+cd-2c^2}$ where $c \neq d$, is equivalent to $-\frac{5}{7}$

1) $\frac{c+d}{d+2c}$

2) $\frac{c-d}{d+2c}$

3) $\frac{-c-d}{d+2c}$ $-\frac{5}{7}$

4) $\frac{-c+d}{d+2c}$

14. The expression $\frac{-3x^2-5x+2}{x^3+2x^2}$ can be rewritten as $\frac{-29}{100}$

1) $\frac{-3x-3}{x^2+2x}$

2) $\frac{-3x-1}{x^2}$

3) $-3x^{-1}+1$

4) $-3x^{-1}+x^{-2}$ $\frac{-29}{100}$

Comparing Expressions
Use Multiple Choice Strategy with Variables for each option

1. Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?

I $(m+p)^2 = m^2 + 2mp + p^2$ 625=625 ✓

II $(x+y)^3 = x^3 + 3xy + y^3$ 15625 ≠ 4825 ✗

III $(a^2+b^2)^2 = (a^2-b^2)^2 + (2ab)^2$ 105625=105625 ✓

1) I, only 3) II and III
2) I and II 4) I and III

2. Which expression(s) are equivalent to $\frac{x^2-4x}{2x}$, where $x \neq 0$? 3 3 3

I. $\frac{x}{2} - 2$ 3 II. $\frac{x-4}{2}$ 3 III. $\frac{x-1}{2} - \frac{3}{2}$ 3

1) II, only 3) II and III
2) I and II 4) I, II, and III

3. Which of the following identities hold true for all real numbers?

$I.\ (x^2+2y)^2 = x^4 + 4x^2y + 4y^2$ 16900 = 16900 ✓

$II.\ (x^2-4z^3)(x^2+4z^3) = x^4 - 16z^6$ -182240000 = -182240000 ✓

$III.\ (x+y)(x^2-xy-y^2) = x^3 - y^3$ -6875 ≠ -2375 ✗

1) I, only 3) II and III only
2) I and II only 4) I and III only

4. Which factorizations are correct?

$I.\ a^3 + 27b^3 = (a+3b)(a^2-3ab+9b^2)$ 92125=92125 ✓

$II.\ c^3 - 6c^2 + 8c + 5c^2 - 30c + 40 = (c-2)(c-4)(c+5)$ 720=720 ✓

$III.\ 1 - x^4 = (1+x)^2(1-x)^2$ -9999 ≠ 9801 ✗

1) I, only 3) II and III only
2) I and II only 4) I, II, and III

5. Which factorization is *incorrect*?

1) $4k^2 - 49 = (2k+7)(2k-7)$ 351=351 ✓
2) $a^3 - 8b^3 = (a-2b)(a^2+2ab+4b^2)$ -26000 = -26000 ✓
3) $m^3 + 3m^2 - 4m + 12 = (m-2)^2(m+3)$ 1272 ≠ 6656 ✗
4) $t^3 + 5t^2 + 6t + t^2 + 5t + 6 = (t+1)(t+2)(t+3)$ 1716=1716 ✓

6. Which expression has been rewritten correctly to form a true statement?

1) $(x+2)^2 + 2(x+2) - 8 = (x+6)x$ 160=160 ✓
3) $x^3 + 3x^2 - 4xy^2 - 12y^2 = (x-2y)(x+3)^2$ -10400 ≠ -3380
2) $x^4 + 4x^2 + 9x^2y^2 - 36y^2 = (x+3y)^2(x-2)^2$ 204800 ≠ 193600
4) $(x^2-4)^2 - 5(x^2-4) - 6 = (x^2-7)(x^2-6)$ 8130 ≠ 8742

Multiple Choice Strategy with Equations
-Store each potential answer (_____ STO → X)
-Type in equation
-1 is correct, 0 is incorrect
*Be sure to check all potential answers as most equations have multiple answers

1. The solution set of the equation $\sqrt{x+3} = 3-x$ is
1) {1} 1 STO→X ~~1~~ 2=2 ✓
2) {0}
3) {1,6} 6 STO→X 0 3=-3 ×
4) {2,3}

2. What is the solution set for the equation $\sqrt{5x+29} = x+3$?
1) {4} 4 STO→X ~~1~~ 7=7 ✓
2) {−5}
3) {4,5} 5 STO→X 0 $\sqrt{54} \neq 8$ ×
4) {−5,4} −5 STO→X 0 $2 \neq -2$ ×

3. The solution set of $\sqrt{3x+16} = x+2$ is
1) {−3,4}
2) {−4,3} −4 STO→ 0 $2 \neq -2$ ×
3) {3} 3 STO→X ~~1~~ 5=5 ✓
4) {−4}

4. The solution set of the equation $\sqrt{2x-4} = x-2$ is
1) {−2,−4}
2) {2,4} 2 STO→X ~~1~~ 0=0 ✓
4 STO→X ~~1~~ 2=2 ✓
3) {4}
4) { }

5. What is the solution set of the equation $\frac{30}{x^2-9} + 1 = \frac{5}{x-3}$?
1) {2,3} 3 STO→X ERR
2) {2} 2 STO→X ~~1~~ −5=−5 ✓
3) {3}
4) { }

6. What is the solution set of the equation $\frac{3x+25}{x+7} - 5 = \frac{3}{x}$?

1) $\left\{\frac{3}{2}, 7\right\}$

2) $\left\{\frac{7}{2}, -3\right\}$

3) $\left\{-\frac{3}{2}, 7\right\}$

4) $\left\{-\frac{7}{2}, -3\right\}$ $-\frac{7}{2}$ STO→X $-\frac{6}{7} = -\frac{6}{7}$ ✓
-3 STO→X $-1 = -1$ ✓

7. The solution set for the equation $\sqrt{56-x} = x$ is

1) $\{-8, 7\}$ -8 STO→X $8 = -8$ X

2) $\{-7, 8\}$

3) $\{7\}$ 7 STO→X $7 = 7$ ✓

4) $\{\ \}$

8. The zeros for $f(x) = x^4 - 4x^3 - 9x^2 + 36x$ are 0

1) $\{0, \pm 3, 4\}$ 0 STO→X $0=0$ ✓

2) $\{0, 3, 4\}$ 3 STO→X $0=0$ ✓

3) $\{0, \pm 3, -4\}$ -3 STO→X $0=0$ ✓

4) $\{0, 3, -4\}$ 4 STO→X $0=0$ ✓

9. Which values of the following is a solution of the following system of equations?

$$y = 3x - 6$$
$$y = x^2 - x - 6$$

1) (0,4)

2) (2,0)

3) (4,6) 4 STO→X $6=6$ $6=6$
6 STO→Y

4) (2,−1)

10. Which ordered pair is a solution of the system of equations shown below?

$$x + y = 5$$
$$(x+3)^2 + (y-3)^2 = 53$$

1) (2,3)

2) (5,0)

3) (−5,10) -5 STO→X $5=5$ ✓ $53=53$ ✓
10 STO→Y

4) (−4,9)

11. Which ordered pair is in the solution set of the system of equations shown below?

$$y^2 - x^2 + 32 = 0$$
$$3y - x = 0$$

1) (2,6)

2) (3,1)

3) (−1,−3)

4) (−6,−2) -6 STO→X $0=0$ $0=0$
-2 STO→Y

Open Response Equations

1) Type in left hand side into Y1
2) Type in right hand side into Y2
3) Adjust window (if necessary)
4) 2nd Trace (Calc), 5: Intersect
5) The solution is the x value of the intersection

*You may want to divide both sides at the beginning to make the values smaller

1. Solve for all values of x: $\sqrt{x-5}+x=7$

window's good — Y1 | Y2 — Intersect $x=6$

2. What is the solution set for the equation $\sqrt{56-x}=x$?

window's good — Y1 | Y2 — Intersect $x=7$

3. What is the solution set for the equation $\sqrt{5x+29}=x+3$?

window's good — Y1 | Y2 — Intersect $x=4$

4. Solve algebraically for x: $\sqrt{x^2+x-1}+11x=7x+3$

window's good — Y1 | Y2 — Intersect $x=.\overline{6}$

5. What is the solution set of the equation $\frac{30}{x^2-9}+1=\frac{5}{x-3}$?

window's good — Y1 | Y2 — Intersect $x=2$

6. What is the solution set of the equation $\frac{3x+25}{x+7}-5=\frac{3}{x}$?

window's good — Y1 | Y2 — Intersect $x=-3$

7. What is the solution, if any, of the equation $\frac{2}{x+3}-\frac{3}{4-x}=\frac{2x-2}{x^2-x-12}$?

window's good — Y1 | Y2 — Intersect $x=-1$

8. Solve for x: $\frac{1}{x}-\frac{1}{3}=-\frac{1}{3x}$

Y1 | Y2 — Intersect

window's good but hard to see. I would ZBOX

$x=4$

9. Solve the equation $2x^3 - x^2 - 8x = 4$ for all values of x.

windows good $\quad$ Y1 | Y2 $\quad$ Intersect $\quad x = -2$, $x = .5$, $x = 2$

10. Solve for x: $x^3 + x^2 = 4x + 4$

windows good $\quad$ Y1 | Y2 $\quad$ Intersect $\quad x = -2$, $x = -1$, $x = 2$

11. Solve for x: $x^3 - 2x^2 = x - 2$

windows good $\quad$ Y1 | Y2 $\quad$ Intersect $\quad x = -1$, $x = 1$, $x = 2$

12. Solve for x and round your answer to the nearest thousandth: $\frac{1}{2}(1.8)^x = 7.5$

windows good $\quad$ Y1 | Y2 $\quad$ Intersect $\quad x = 4.607$

13. Solve for x and round your answer to the nearest thousandth: $2\left(\frac{1}{3}\right)^x = 4$

windows good $\quad$ Y1 | Y2 $\quad$ Intersect $\quad x = -.631$

14. Solve for x and round your answer to the nearest thousandth: $1 - 2(3)^{2x} = -5$

windows good $\quad$ Y1 | Y2 $\quad$ Intersect $\quad x = .5$

15. Solve $x^3 + 5x^2 = 4x + 20$.

adjust y max $\quad$ Y1 | Y2 $\quad$ Intersect $\quad x = -5$, $x = -2$, $x = 2$

16. Solve for all values of x: $x^4 + 4x^3 + 4x^2 = -16x$

adjust y max $\quad$ Y1 | Y2 $\quad$ Intersect $\quad x = 0$, $x = -4$

17. Solve for x and round your answer to the nearest hundredth: $4^x - 5 = 12$

adjust y max $\quad$ Y1 | Y2 $\quad$ Intersect $\quad x = 2.04$

18. Solve for x and round your answer to the nearest hundredth: $8 + 2(4)^{x-5} = 14$

adjust y max $\quad$ Y1 | Y2 $\quad$ Intersect $\quad x = 5.79$

Profit

Profit = revenue – cost, $p(x) = r(x) - c(x)$

Net worth = value of accounts - debt

*Keep, change, change when subtracting polynomials

*you can use mc strategy once it's set up

1. Mr. Schlansky's tutoring revenue can be represented by $r(x) = 25x^2 - 90x + 14$ and his costs can be represented by $c(x) = 12x^2 + 21x + 10$. If his profit can be determined using $p(x) = r(x) - c(x)$, write a polynomial function what would represent $p(x)$.

$p(x) = (25x^2 - 90x + 14) - (12x^2 + 21x + 10)$

$25x^2 - 90x + 14$

$-12x^2 - 21x - 10$

$p(x) = 13x^2 - 111x + 4$

2. Stone Manufacturing has developed a cost model, $C(x) = 0.18x^3 + 0.02x^2 + 4x + 180$, where x is the number of sprockets sold, in thousands. The sales price can be modeled by $S(x) = 95.4 - 6x$ and the company's revenue by $R(x) = x \bullet S(x)$. The company's profits, $R(x) - C(x)$, could be modeled by

$x(95.4 - 6x)$

1) $0.18x^3 + 6.02x^2 + 91.4x + 180$
2) $0.18x^3 - 5.98x^2 - 91.4x + 180$
3) $-0.18x^3 - 6.02x^2 + 91.4x - 180$
4) $0.18x^3 + 5.98x^2 + 99.4x + 180$

$x(95.4 - 6x) - (.18x^3 + .02x^2 + 4x + 180)$ $\quad -.18x^3 - 6.02x^2 + 91.4x - 180$

$95.4x - 6x^2 - .18x^3 - .02x^2 - 4x - 180$

3. Chet has \$1200 invested in a bank account modeled by the function $P(n) = 1200(1.002)^n$, where $P(n)$ is the value of his account, in dollars, after n months. Chet's debt is modeled by the function $Q(n) = 100n$, where $Q(n)$ is the value of debt, in dollars, after n months. After n months, which function represents Chet's net worth, $R(n)$?

1) $R(n) = 1200(1.002)^n + 100n$
2) $R(n) = 1200(1.002)^{12n} + 100n$
3) $R(n) = 1200(1.002)^n - 100n$
4) $R(n) = 1200(1.002)^{12n} - 100n$

$R(n) = P(n) - Q(n)$

$R(n) = 1200(1.002)^n - 100n$

4. A manufacturing company has developed a cost model, $C(x) = 0.15x^3 + 0.01x^2 + 2x + 120$, where x is the number of items sold, in thousands. The sales price can be modeled by $S(x) = 30 - 0.01x$. Therefore, revenue is modeled by $R(x) = x \bullet S(x)$. The company's profit, $P(x) = R(x) - C(x)$, could be modeled by

$x(30-.01x)$

1) $0.15x^3 + 0.02x^2 - 28x + 120$
2) $-0.15x^3 - 0.02x^2 + 28x - 120$
3) $-0.15x^3 + 0.01x^2 - 2.01x - 120$
4) $-0.15x^3 + 32x + 120$

$x(30-.01x) - (.15x^3 + .01x^2 + 2x + 120)$

$30x - .01x^2 - .15x^3 - .01x^2 - 2x - 120$

$-.15x^3 - .02x^2 + 28x - 120$

5. A major car company analyzes its revenue, $R(x)$, and costs $C(x)$, in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years, x, using the given functions.

$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$
$$C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$$

The company's profits can be represented as the difference between its revenue and costs. Write the profit function, $P(x)$, as a polynomial in standard form.

$(550x^3 - 12,000x^2 + 83,000x + 7000) - (880x^3 - 21,000x^2 + 150,000x - 160,000)$

$550x^3 - 12,000x^2 + 83,000x + 7000$

$-880x^3 + 21,000x^2 - 150,000x + 160,000$

$-330x^3 + 9,000x^2 - 67,000x + 167,000$

6. The profit function, $p(x)$, for a company is the cost function, $c(x)$, subtracted from the revenue function, $r(x)$. The profit function for the Acme Corporation is $p(x) = -0.5x^2 + 250x - 300$ and the revenue function is $r(x) = -0.3x^2 + 150x$. The cost function for the Acme Corporation is

1) $c(x) = 0.2x^2 - 100x + 300$
2) $c(x) = 0.2x^2 + 100x + 300$
3) $c(x) = -0.2x^2 + 100x - 300$
4) $c(x) = -0.8x^2 + 400x - 300$

$p(x) = r(x) - c(x)$

$c(x) = r(x) - p(x)$

$= (-0.3x^2 + 150x) - (-.5x^2 + 250x - 300)$

$-.3x^2 + 150x$

$.5x^2 - 250x + 300$

$+.2x^2 - 100x + 300$

Dividing Polynomials: (Synthetic Division)
Negative the value of what you are dividing by and put it outside
Bring the first number down
Multiply, Add, Multiply, Add, etc.
Decrease the first terms exponent by 1, the last number is the remainder. The remainder goes over the divisor.
(Put 0 as a placeholder if necessary)

Divide each of the following polynomials

1. $\dfrac{2x^3+5x^2-31x-84}{x+3}$

$$\begin{array}{r|rrrr} -3 & 2 & 5 & -31 & -84 \\ & & -6 & 3 & 84 \\ \hline & 2 & -1 & -28 & 0 \end{array}$$

$2x^2-x-28$

2. $\dfrac{x^4-2x^2-7x+12}{x+6}$

$$\begin{array}{r|rrrrr} -6 & 1 & 0 & -2 & -7 & 12 \\ & & -6 & 36 & 204 & -1182 \\ \hline & 1 & -6 & 34 & 197 & -1170 \end{array}$$

$x^3-6x^2+34x+197-\frac{1170}{x+6}$

3. $\dfrac{x^3+5x^2-1}{x+2}$

$$\begin{array}{r|rrrr} -2 & 1 & 5 & 0 & -1 \\ & & -2 & -6 & 12 \\ \hline & 1 & 3 & -6 & 11 \end{array}$$

$x^2+3x-6+\frac{11}{x+2}$

4. $\dfrac{4x^3+12x^2-5}{x+5}$

$$\begin{array}{r|rrrr} -5 & 4 & 12 & 0 & -5 \\ & & -20 & 40 & -200 \\ \hline & 4 & -8 & 40 & -205 \end{array}$$

$4x^2-8x+40-\frac{205}{x+5}$

5. $\dfrac{6x^3-5x+3}{x-3}$

$$\begin{array}{r|rrrr} 3 & 6 & 0 & -5 & 3 \\ & & 18 & 54 & 147 \\ \hline & 6 & 18 & 49 & 150 \end{array}$$

$6x^2+18x+49+\frac{150}{x-3}$

6. $\dfrac{5x^3-60}{x-2}$

$$\begin{array}{r|rrrr} 2 & 5 & 0 & 0 & -60 \\ & & 10 & 20 & 40 \\ \hline & 5 & 10 & 20 & -20 \end{array}$$

$5x^2+10x+20-\frac{20}{x-2}$

7. $\dfrac{x^2+x-4}{x-3}$

$$\begin{array}{r|rrr} 3 & 1 & 1 & -4 \\ & & 3 & 12 \\ \hline & 1 & 4 & 8 \end{array}$$

$x+4+\frac{8}{x-3}$

8. $\dfrac{-3x^2+10x-6}{x+1}$

$$\begin{array}{r|rrr} -1 & -3 & 10 & -6 \\ & & 3 & -13 \\ \hline & -3 & 13 & -19 \end{array}$$

$-3x+13-\frac{19}{x+1}$

To determine if a binomial is a factor:
Find the remainder! (Use remainder theorem)
If the remainder is 0, it is a factor
If the remainder is not 0, it is not a factor

1. Is $x-6$ a factor of $p(x)=x^3-6x^2+4x-1$? Explain your answer.

$p(6)=(6)^3-6(6)^2+4(6)-1$ No, the remainder is not 0
$p(6)=23$

2. Is $x+2$ a factor of $p(x)=x^3-3x^2-8x+4$? Explain your answer.

$p(-2)=(-2)^3-3(-2)^2-8(-2)+4$ Yes, the remainder is 0
$p(-2)=0$

3. Is $2x+1$ a factor of $p(x)=2x^2+5x+2$? Explain your answer.

$2x+\cancel{1}=0$
$\cancel{-1}\ -1$
$\frac{2x}{2}=\frac{-1}{2}$
$x=\frac{-1}{2}$
$p(-\frac{1}{2})=2(-\frac{1}{2})^2+5(-\frac{1}{2})+2$ Yes, the remainder is 0
$p(-\frac{1}{2})=0$

4. Is $3x-2$ a factor of $p(x)=3x^3-2x^2-27x+18$? Explain your answer.

$3x-\cancel{2}=0$
$+2\ +2$
$\frac{3x}{3}=\frac{2}{3}$
$x=\frac{2}{3}$
$p(\frac{2}{3})=3(\frac{2}{3})^3-2(\frac{2}{3})^2-27(\frac{2}{3})+18$
$p(\frac{2}{3})=0$ Yes, the remainder is 0

5. Determine if $x-5$ is a factor of $2x^3-4x^2-7x-10$. Explain your answer.

$p(5)=2(5)^3-4(5)^2-7(5)-10$ No, the remainder is not 0
$p(5)=105$

6. Which binomial is a factor of x^4-4x^2-4x+8?

1) $x-2$ $p(2)=0$
2) $x+2$ $p(-2)=16$
3) $x-4$ $p(4)=184$
4) $x+4$ $p(-4)=216$

7. Which binomial is *not* a factor of the expression $x^3-11x^2+16x+84$?

1) $x+2$ $p(-2)=0$
2) $x+4$ $p(-4)=-220$
3) $x-6$ $p(6)=0$
4) $x-7$ $p(7)=0$

8. Which binomial is *not* a factor of the expression $x^3 - 6x^2 - 49x - 66$?

1) $x-11$ $p(11)=0$
2) $x+2$ $p(-2)=0$
3) $x+6$ $p(-6)=-204$
4) $x+3$ $p(-3)=0$

9. Which binomial is a factor of the expression $x^3 - 7x - 6$?

1) $x+3$ $p(-3)=-12$
2) $x-1$ $p(1)=-12$
3) $x-2$ $p(2)=-12$
4) $x+2$ $p(-2)=0$

10. Which binomial is *not* a factor of the expression $x^3 - 4x^2 - 25x + 28$?

1) $x+6$ $p(-6)=-182$
2) $x-7$ $p(7)=0$
3) $x-1$ $p(1)=0$
4) $x+4$ $p(-4)=0$

11. Which binomial is not a factor of $p(x) = 2x^3 + 7x^2 - 5x - 4$?

1) $x+4$ $p(-4)=0$
2) $x+1$ $p(-1)=6$
3) $x-1$ $p(1)=0$
4) $2x+1=0$ $p(-\frac{1}{2})=0$

$-1 \quad -1$

$\frac{2x}{2}=\frac{-1}{2}$

12. Which binomial is not a factor of $p(x) = 2x^3 - 5x^2 + 6x - 2$?

1) $x-1$ $p(1)=1$
2) $x-2$ $p(2)=6$
3) $2x-1=0$ $p(\frac{1}{2})=0$
4) $2x+1=0$ $p(-\frac{1}{2})=-\frac{13}{2}$ → not a factor

$2x-1=0$, $+1 \quad +1$, $\frac{2x}{2}=\frac{1}{2}$, $x=\frac{1}{2}$

$2x+1=0$, $-1 \quad -1$, $\frac{2x}{2}=\frac{-1}{2}$, $x=-\frac{1}{2}$

13. Given $P(x) = x^3 - 3x^2 - 2x + 4$, which statement is true?

1) $(x-1)$ is a factor because $P(-1) = 2$.
2) $(x+1)$ is a factor because $P(-1) = 2$.
3) $(x+1)$ is a factor because $P(1) = 0$.
4) $(x-1)$ is a factor because $P(1) = 0$.

$p(-1)=(-1)^3-3(-1)^2-2(-1)+4$
$p(-1)=2$

$p(1)=(1)^3-3(1)^2-2(1)+4$
$p(1)=0$
↓
$x-1$

14. If $f(x) = 2x^4 - x^3 - 16x + 8$, then $f\left(\frac{1}{2}\right)$ $=0$

1) equals 0 and $2x+1$ is a factor of $f(x)$ ($=0$, $-1 \quad -1$, $\frac{2x}{2}=\frac{-1}{2}$)
2) equals 0 and $2x-1$ is a factor of $f(x)$ ($=0$, $+1 \quad +1$, $\frac{2x}{2}=\frac{1}{2}$)
3) ~~does not equal 0 and $2x+1$ is not a factor of $f(x)$~~
4) ~~does not equal 0 and $2x-1$ is a factor of $f(x)$~~

15. Consider the function $f(x) = 2x^3 + x^2 - 18x - 9$. Which statement is true?

$2x-1=0$, $+1 \quad +1$, $\frac{2x}{2}=\frac{1}{2}$

1) $2x-1$ is a factor of $f(x)$. $x=\frac{1}{2}$ $p(\frac{1}{2})=-\frac{35}{2}$
2) $x-3$ is a factor of $f(x)$. $x=3$ $p(3)=0$
3) $f(3) \neq f\left(-\frac{1}{2}\right)$
4) $f\left(\frac{1}{2}\right) = 0$

Finding k in a Polynomial Equation

Finding k in a Polynomial Equation

If $x+a$ is a factor then a is a zero. Replace $p(x)$ with 0 and x with a.

1. Consider the polynomial $p(x)=x^3+kx^2+x+6$. Find a value of k so that $x+1$ is a factor of P.

$0=(-1)^3+k(-1)^2+(-1)+6$ $\quad p(-1)=0$

$0=-1+k-1+6$

$0=k+4$

$-4 \quad -4$

$-4=k$

2. Consider the polynomial $p(x)=x^3+kx-30$. Find a value of k so that $x+3$ is a factor of P.

$0=(-3)^3+k(-3)-30$ $\quad p(-3)=0$

$0=-27-3k-30$

$0=-3k-57$

$+57 \quad +57$

$\frac{57}{-3}=\frac{-3k}{-3}$

$-19=k$

3. If $x-1$ is a factor of x^3-kx^2+2x, what is the value of k?

$p(1)=0$

$0=(+1)^3-k(1)^2+2(1)$

$0=1-k+2$

$0=-k+3$

$+k \quad +k$

$k=3$

4. The polynomial function $g(x)=x^3+ax^2-5x+6$ has a factor of $(x-3)$. Determine the value of a.

$0=(3)^3+a(3)^2-5(3)+6$ $\quad p(3)=0$

$0=27+9a-15+6$

$0=9a+18$

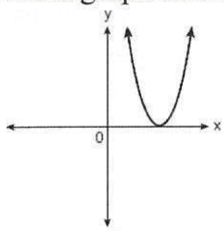
$-18 \quad -18$

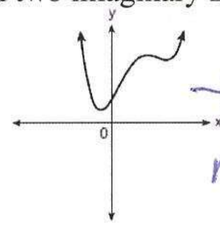
$\frac{-18}{9}=\frac{9a}{9}$

$-2=a$

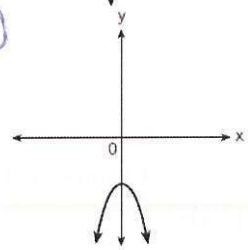
Imaginary Solutions
Imaginary solutions do not touch the x-axis

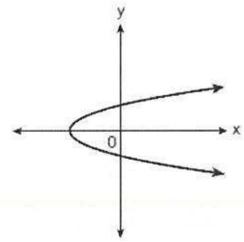
1. Which graph shows a quadratic function with two imaginary zeros?

1)


3)


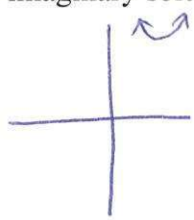
→ imaginary but not quadratic

2)


4)


2. Does the equation $x^2 - 4x + 13 = 0$ have imaginary solutions? Justify your answer.

type into Y=



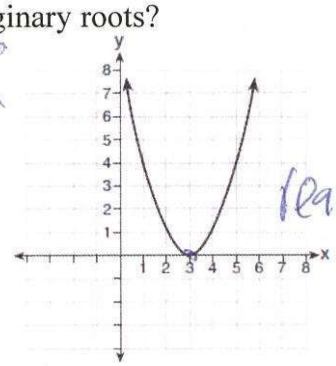
Yes, the graph doesn't touch the x-axis

3. Which representation of a quadratic has imaginary roots?

1)

x	y
−2.5	2
−2.0	0
−1.5	−1
−1.0	−1
−0.5	0
0.0	2

real

3)


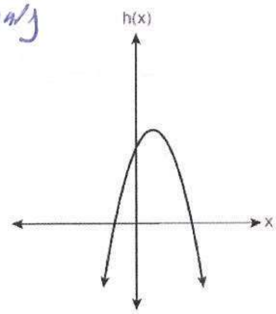
real

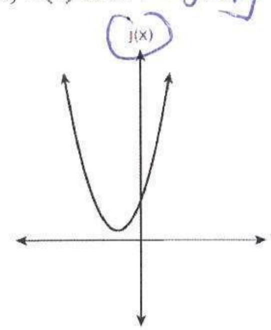
2) $2(x+3)^2 = 64$ $y=2(x+3)^2-64$
$-64 \quad -64$

4) $2x^2 + 32 = 0$
$y = 2x^2+32$

4. In the quadratic formula, $b^2 - 4ac$ is called the discriminant. The function $f(x)$ has a discriminant value of 8, and $g(x)$ has a discriminant value of -16. The quadratic graphs, $h(x)$ and $j(x)$, are shown below.

real imaginary imaginary





Which quadratic functions have imaginary roots?

1) $g(x)$ and $h(x)$
2) $g(x)$ and $j(x)$
3) $f(x)$ and $h(x)$
4) $f(x)$ and $j(x)$

$g(x)$ $j(x)$

Writing Equations of Polynomial Functions

To write the equation of a polynomial function, list the factors. In order to list the factors, list the zeros (where the graph crosses the x-axis). If $x-a$ is a factor, then a is a zero (switch the sign to go back and forth between factors and zeros).

If a is a zero, $p(a)=0$, $x-a$ is a factor, and the polynomial is divisible by $x-a$. Once you have one of the four pieces of information, you have all four.

Single and Double Roots

Single roots pass through the x axis

Double roots bounce off the x axis

Real and Imaginary Roots

Real roots hit the x axis

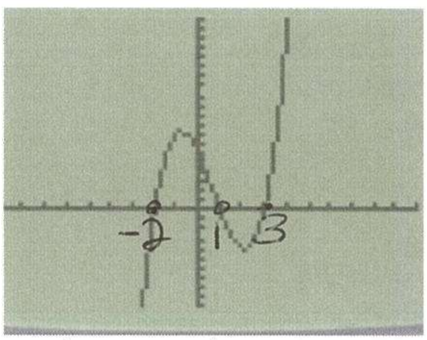
Imaginary roots (roots with an *i*, do not hit the x axis.

Factoring

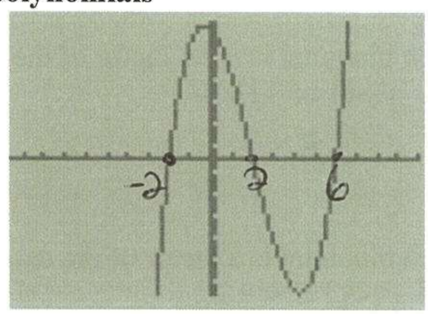
You may need to completely or partially factor to put the equation into factored form

A perfect square trinomial factor leads to a double root

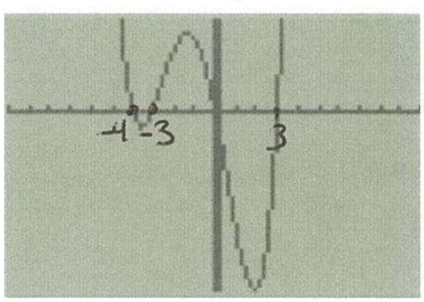
Write a possible equation for each of the following polynomials

1. 

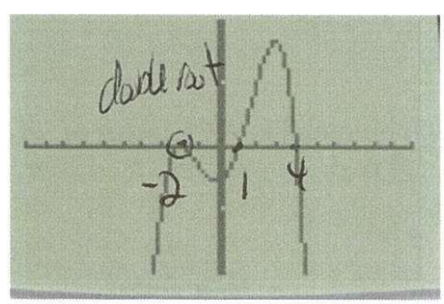
$p(x) = (x+2)(x-1)(x-3)$

2. 

$p(x) = (x+2)(x-2)(x-6)$

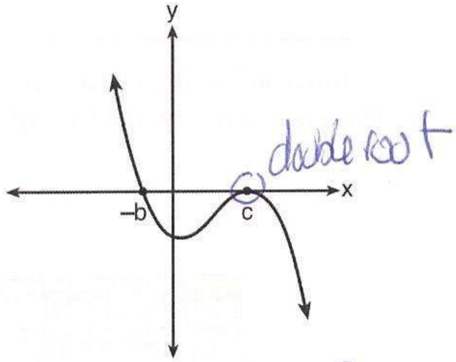
3. 

$p(x) = (x+4)(x+3)(x-3)$

4. 

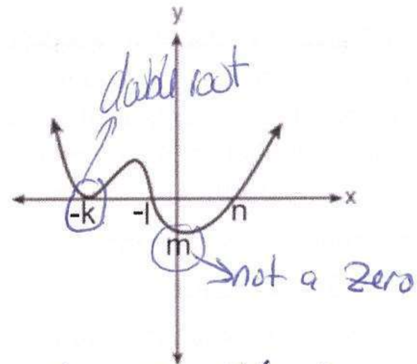
$p(x) = -(x+2)^2(x-1)(x-4)$

5.



$p(x) = -(x+b)(x-c)^2$

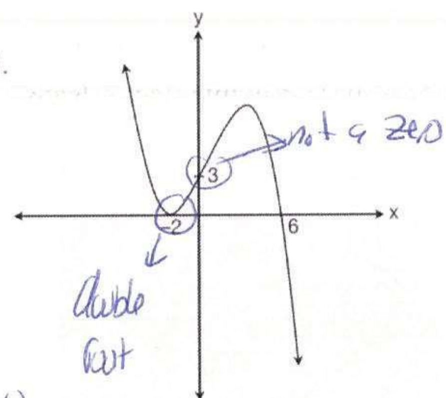
6.



$p(x) = (x+k)^2(x+l)(x-n)$

7\.

The graph below shows the polynomial $y = p(x)$.



The factors of $p(x)$ are

(1) $(x+2)$, $(x-3)$, and $(x+6)$

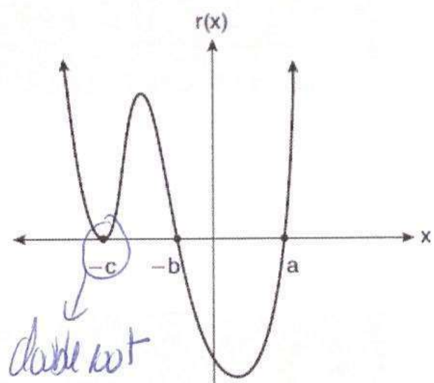
(2) $(x-2)$, $(x+3)$, and $(x+6)$

(3) $(x-2)$, $(x-2)$, and $(x+6)$

(4) $(x+2)$, $(x+2)$, and $(x-6)$

$p(x) = (x+2)^2(x-6)$

8\. A sketch of $r(x)$ is shown below.



An equation for $r(x)$ could be

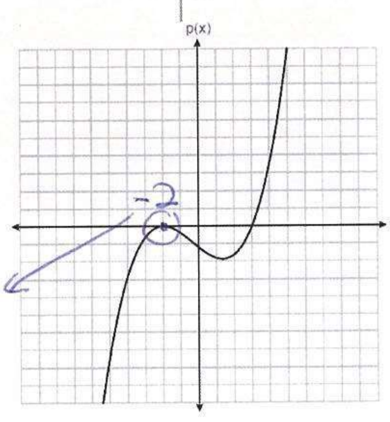
1) $r(x) = (x-a)(x+b)(x+c)$

2) $r(x) = (x+a)(x-b)(x-c)^2$

3) $r(x) = (x+a)(x-b)(x-c)$

4) $r(x) = (x-a)(x+b)(x+c)^2$

9\. The graph of a cubic polynomial function $p(x)$ is shown below.



If $p(x)$ is written as a product of linear factors, which factor would appear twice?

1) $x-2$

2) $x+2$

3) $x-3$

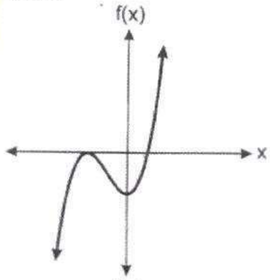
4) $x+3$

$\overset{+\ \to \text{opens up}}{}$

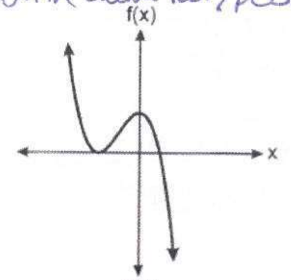
10. Which graph best represents the graph of $f(x) = (x+a)^2(x-b)$, where a and b are positive real numbers?

negative double root, positive single root

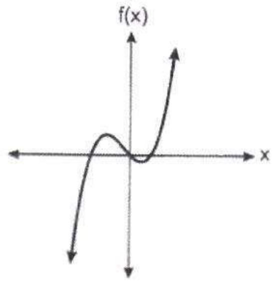
1)



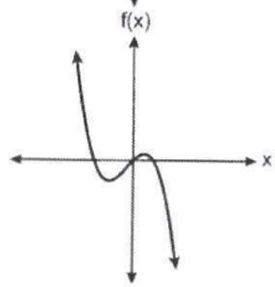
3)



2)



4)

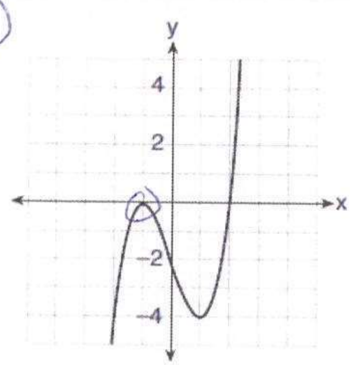


11. Which graph represents a polynomial function that contains $x^2 + 2x + 1$ as a factor?

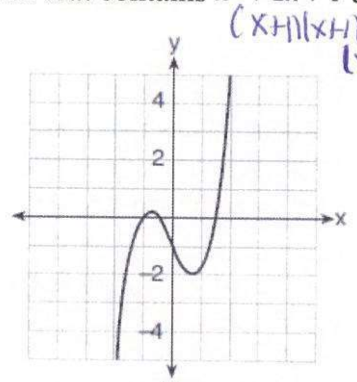
$(x+1)(x+1)$

$(x+1)^2 \to -1$ is a double root

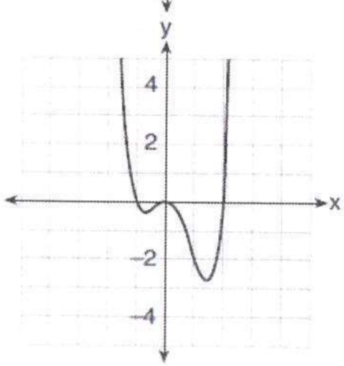
1)



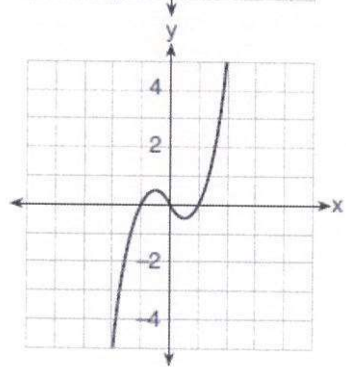
3)



2)



4)



opens down

12. If a, b, and c are all positive real numbers, which graph could represent the sketch of the graph of $p(x) = -a(x+b)\left(x^2 - 2cx + c^2\right)$?

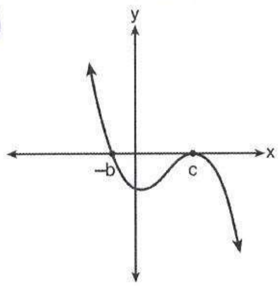
$(x-c)(x-c)$

$(x-c)^2$

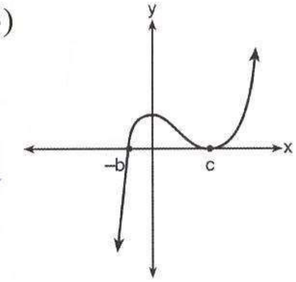
↓

c is a double root

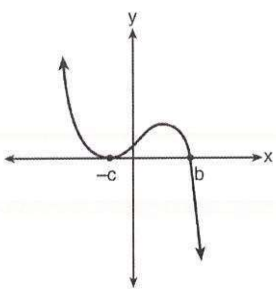
1)



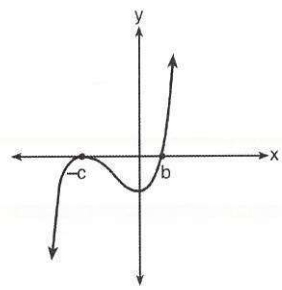
3)



2)



4)



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13. Consider the function $p(x) = 3x^3 + x^2 - 5x$ and the graph of $y = m(x)$ below.

Which statement is true?

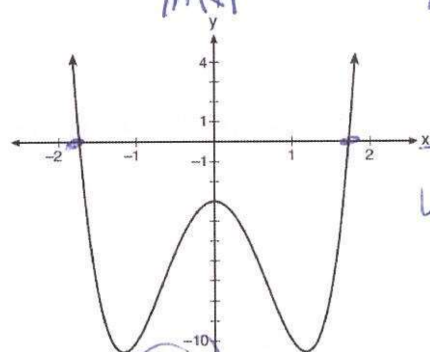
1) $p(x)$ has three real roots and $m(x)$ has two real roots.

2) $p(x)$ has one real root and $m(x)$ has two real roots.

3) $p(x)$ has two real roots and $m(x)$ has three real roots.

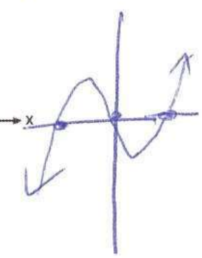
4) $p(x)$ has three real roots and $m(x)$ has four real roots.

m(x)



②

p(x)

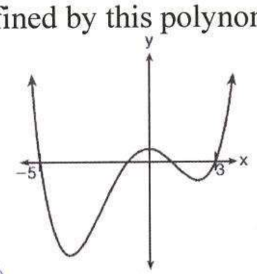


③

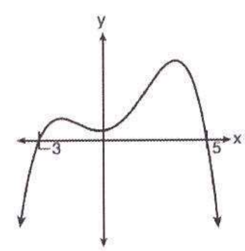
must hit only at –5 and 3

14. A 4th degree polynomial has zeros -5, 3, i, and $-i$. Which graph could represent the function defined by this polynomial?

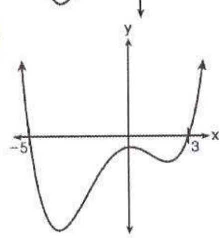
1)



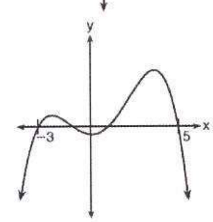
3)



2)



4)



The zeros are where the graph hits the x-axis.
If $x-a$ is a factor, then a is a zero.

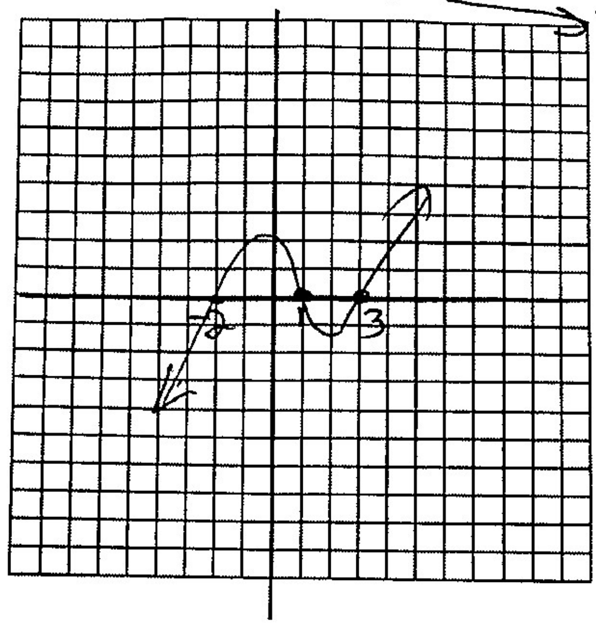
Name Schlansky
Mr. Schlansky

Date ____________
Algebra II

Sketching Polynomial Graphs Regents Practice

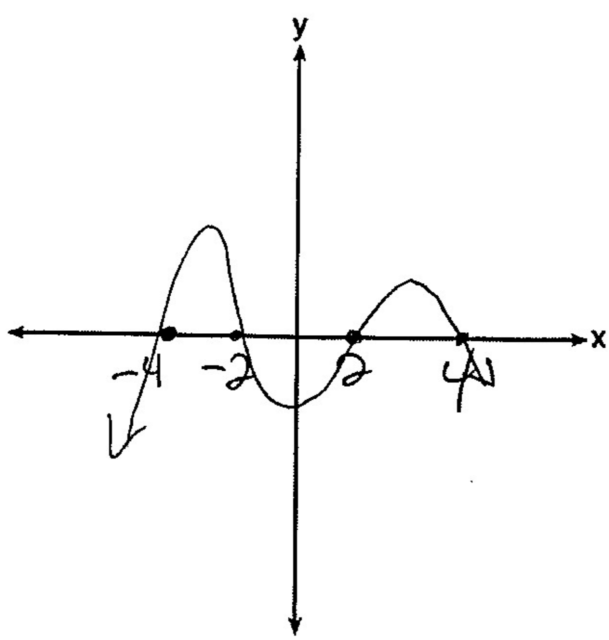
1. On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and -2.

Zeros hit the x-axis. Don't negate!



2. The zeros of a quartic polynomial function are 2, −2, 4, and −4. Use the zeros to construct a possible sketch of the function, on the set of axes below.

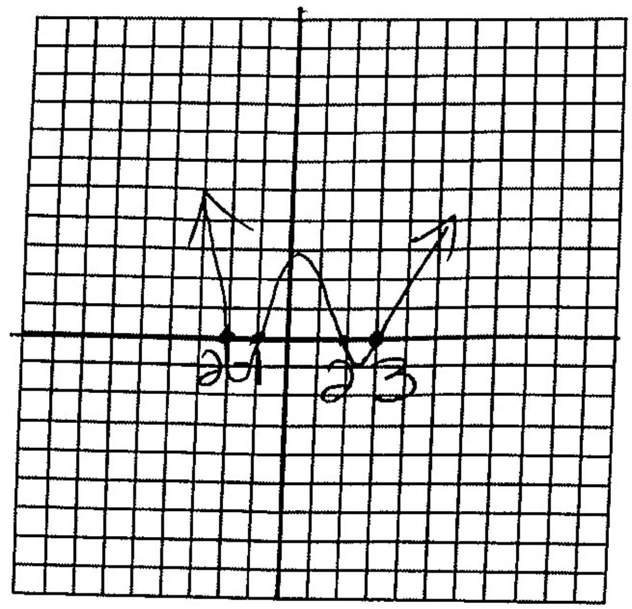
Zeros hit the x-axis
Don't negate



+2 and -2

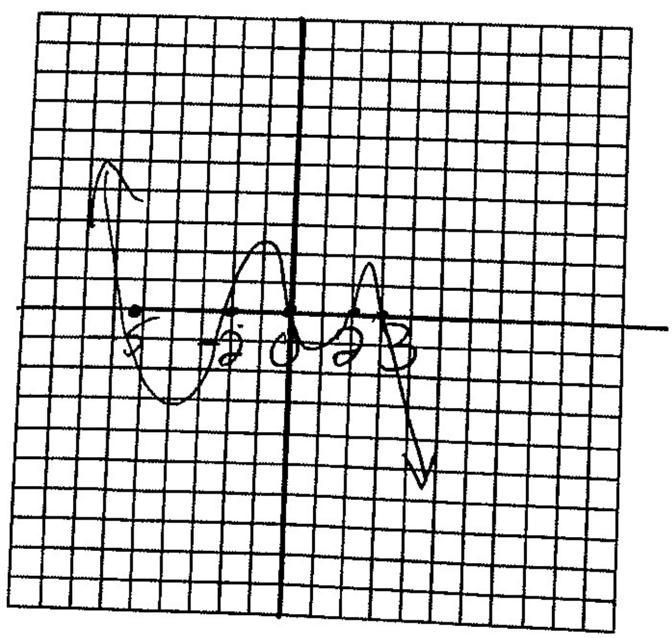
3. The zeros of a quartic polynomial function h are $-1, \pm 2$, and 3. Sketch a graph of $y = h(x)$ on the grid below.

Zeros hit the x-axis. Don't negate



4. The zeros of a polynomial function are $-5, \pm 2, 0,$ *and* 3. Sketch a graph of the polynomial functions on the grid below.

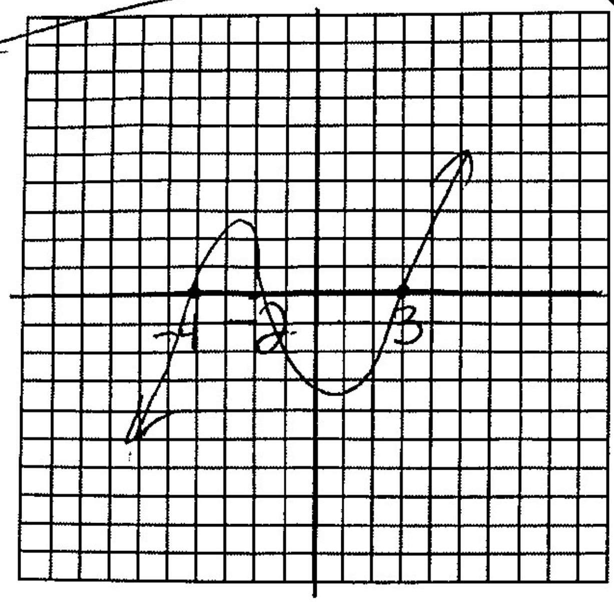
Zeros hit the x-axis. Don't negate



5. On the grid below, sketch a cubic polynomial whose factors are $x-3$, $x+4$, and $x+2$.

3 -4 -2 zeros

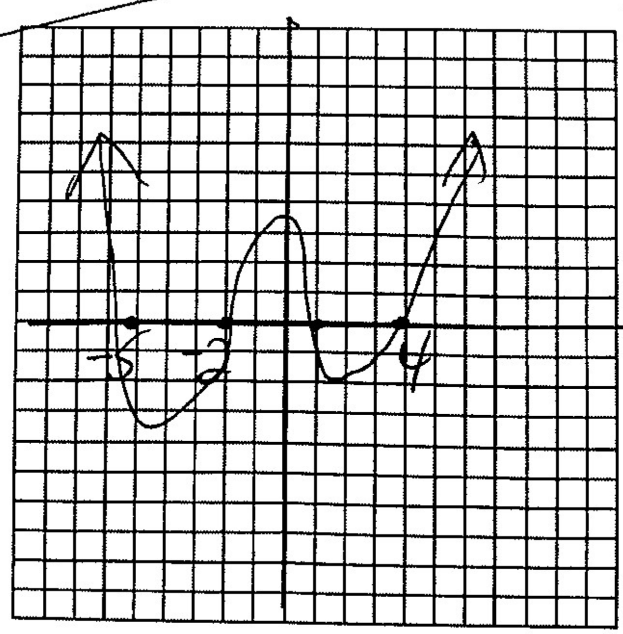
factors don't hit the x-axis, zeros do!



6. On the grid below, sketch a quartic polynomial whose factors are $x+5$, $x+2$, $x-1$, and $x-4$ → 4

-5 -2 1 zeros

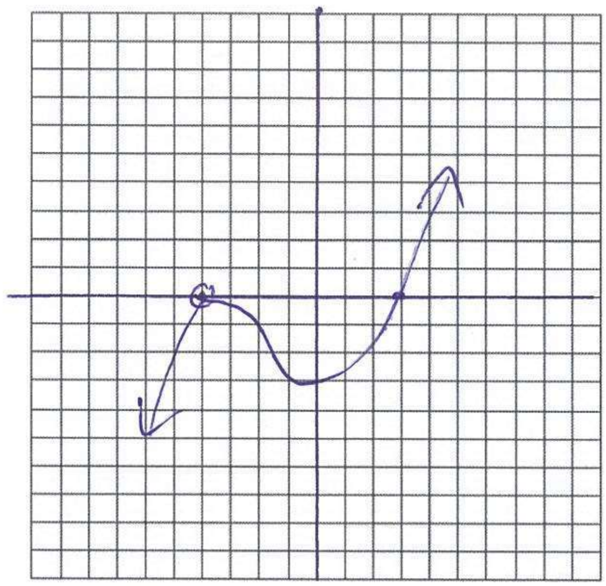
factors don't hit the x-axis, zeros do



7. On the grid below, sketch a cubic polynomial whose factors are $x-3$ and $x^2+8x+16$.

$(x+4)(x+4)$

$(x+4)^2$



Factors

$(x-3)(x+4)^2$

Zeros

3 and -4

double root

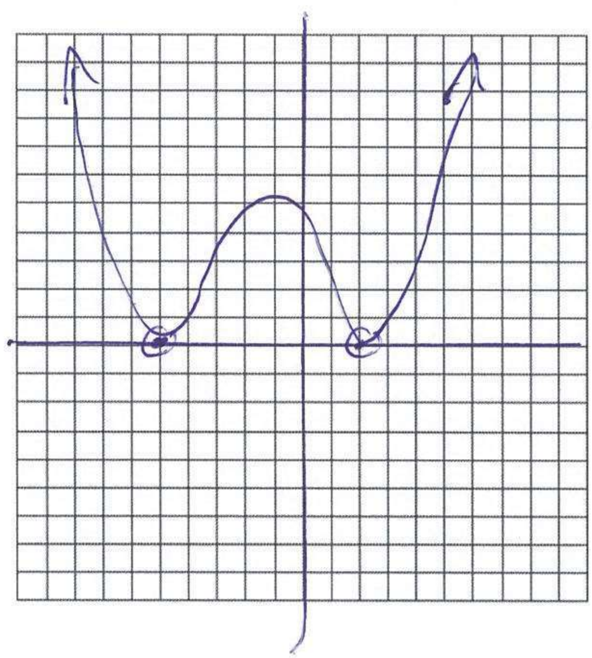
8. On the grid below, sketch a quartic polynomial whose factors are x^2-4x+4 and $x^2+10x+25$.

$(x-2)(x-2)$

$(x-2)^2$

$(x+5)(x+5)$

$(x+5)^2$



Factors

$(x+5)^2(x-2)^2$

Zeros

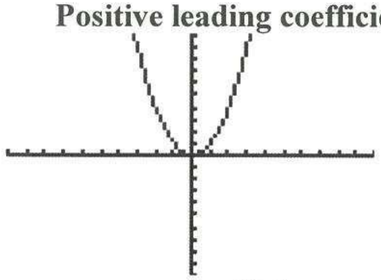
-5 and 2

double root

double root

Sketching Polynomial Graphs (Specific)

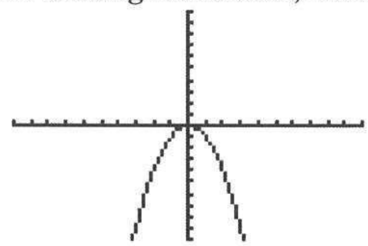
Positive leading coefficient, Even Degree



$x \to -\infty, f(x) \to \infty$

$x \to \infty, f(x) \to \infty$

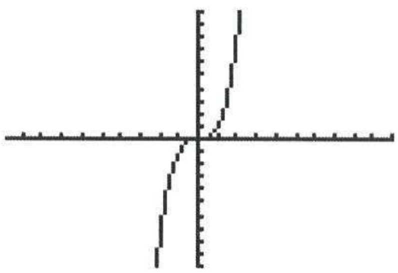
Negative leading coefficient, Even Degree



$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to -\infty$

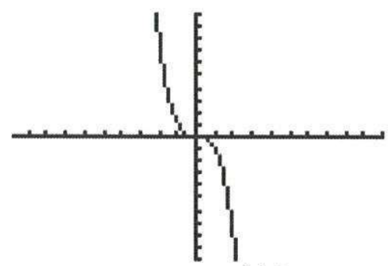
Positive leading coefficient, Odd Degree



$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to \infty$

Negative leading coefficient, Odd Degree



$x \to -\infty, f(x) \to \infty$

$x \to \infty, f(x) \to -\infty$

1. $h(x) = x^6 - 5x^4 + 4x^2$

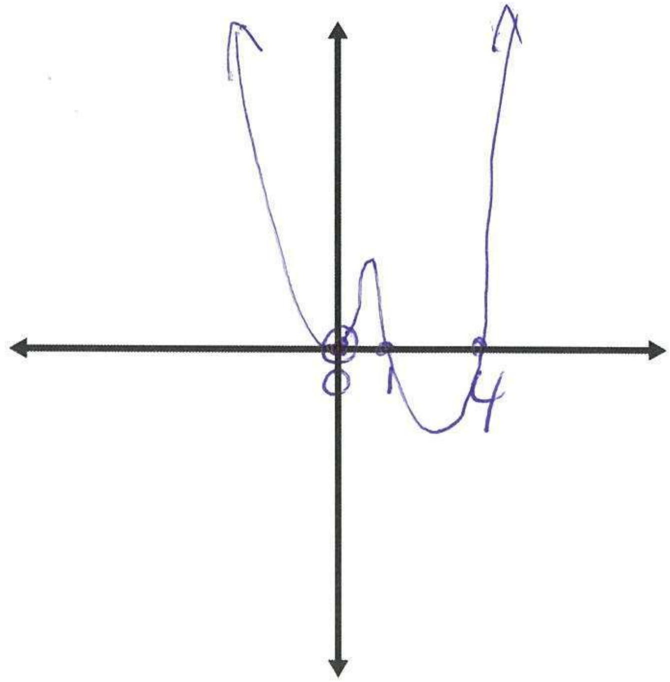
Increasing:
(0, .5)
(2.5, ∞)

Decreasing:
(-∞, 0)
(.5, 2.5)

Positive:
(-∞, 0)
(0, 1)
(4, ∞)

Negative:
(1, 4)

x-intercepts (zeros):
$\{0,0,1,4\}$



End Behavior:

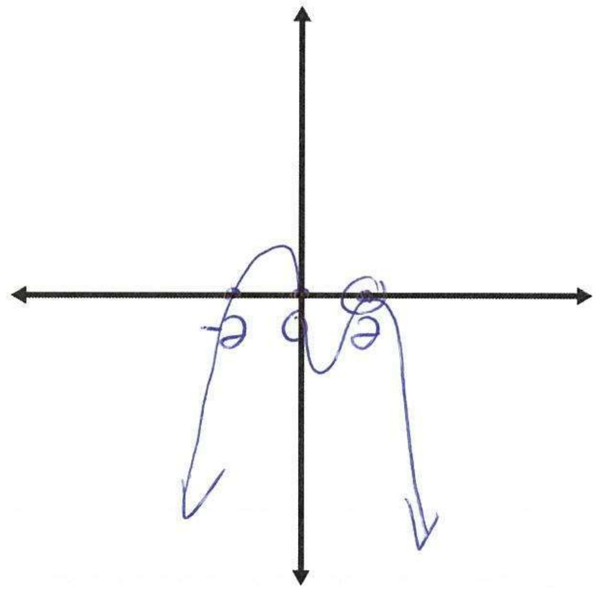
$x \to -\infty,\ f(x) \to \infty$

$x \to \infty,\ f(x) \to \infty$

2. $g(x) = -x^4 + 2x^3 + 4x^2 - 8x$ ∩

Increasing:
$(-\infty, -1)$
$(1, 2)$

Decreasing:
$(-1, 1)$
$(2, \infty)$



Positive:
$(-2, 0)$

Negative:
$(-\infty, -2)$
$(0, 2)$
$(2, \infty)$

x-intercepts (zeros):
{-2,0,2,2}

End Behavior:

$x \to -\infty,\ f(x) \to -\infty$

$x \to \infty,\ f(x) \to -\infty$

3. Consider the end behavior description below.

left / up
- as $x \to -\infty$, $f(x) \to \infty$

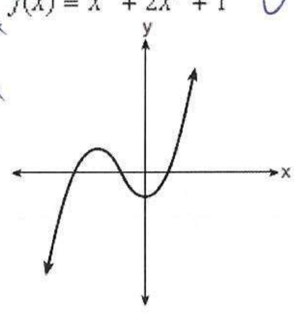
right / down
- as $x \to \infty$, $f(x) \to -\infty$

Which function satisfies the given conditions?

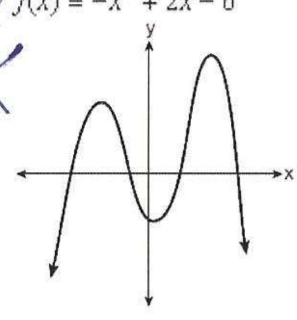
1) $f(x) = x^4 + 2x^2 + 1$ U

3) $f(x) = -x^3 + 2x - 6$

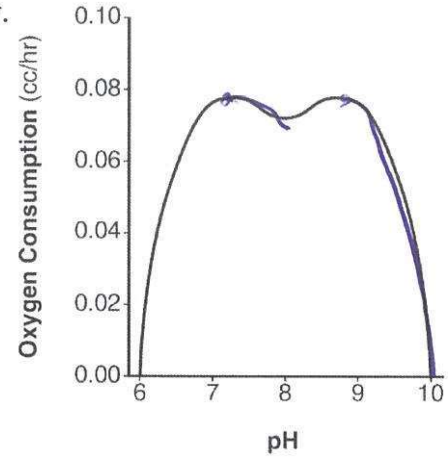
2)



4)



4. There was a study done on oxygen consumption of snails as a function of pH, and the result was a degree 4 polynomial function whose graph is shown below.



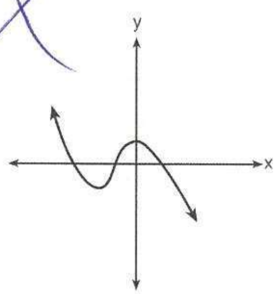
Which statement about this function is *incorrect*?

1) The degree of the polynomial is even. V shaped
2) There is a positive leading coefficient. opens down
3) At two pH values, there is a relative maximum value.
4) There are two intervals where the function is decreasing.

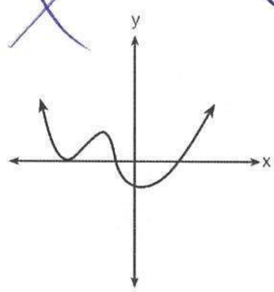
5. Which graph has the following characteristics?

- three real zeros
- as $x \to -\infty$, $f(x) \to -\infty$ left down
- as $x \to \infty$, $f(x) \to \infty$ right up

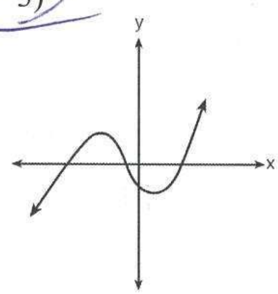
1)



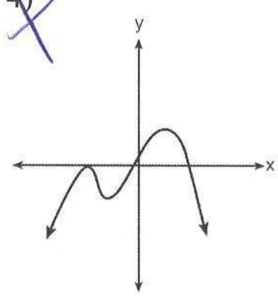
2)



3)



4)



6. Consider a cubic polynomial with the characteristics below.

- exactly one real root
- as $x \to \infty$, $f(x) \to -\infty$ right down — leading coefficient

Given $a > 0$ and $b > 0$, which equation represents a cubic polynomial with these characteristics?

1) $f(x) = (x - a)(x^2 + b)$
2) $f(x) = (a - x)(x^2 + b)$
   $-1(x-a)(x^2+b)$
3) $f(x) = (a - x^2)(x^2 + b)$
4) $f(x) = (x - a)(b - x^2)$

$4x^2(x+a)-1(x+a)$

7. Which description could represent the graph of $f(x) = 4x^2(x + a) - x - a$, if a is an integer?

$(4x^2-1)(x+a)$

3 real roots

positive odd

1) As $x \to -\infty$, $f(x) \to \infty$, as $x \to \infty$, $f(x) \to \infty$, and the graph has 3 x-intercepts.
2) As $x \to -\infty$, $f(x) \to -\infty$, as $x \to \infty$, $f(x) \to \infty$, and the graph has 3 x-intercepts.
3) As $x \to -\infty$, $f(x) \to \infty$, as $x \to \infty$, $f(x) \to -\infty$, and the graph has 4 x-intercepts.
4) As $x \to -\infty$, $f(x) \to -\infty$, as $x \to \infty$, $f(x) \to \infty$, and the graph has 4 x-intercepts.

11. $(2-yi)^2$

$(2-yi)(2-yi)$

	2	$-yi$
2	4	$-2yi$
$-yi$	$-2yi$	$+y^2i^2$

$4-4yi+y^2i^2$

$4-4yi+y^2(-1)$

$4-4yi-y^2$

12. $(3-7i)^2$

$(3-7i)(3-7i)$

	3	$-7i$
3	9	$-21i$
$-7i$	$-21i$	$+49i^2$

$9-42i+49i^2$

$9-42i+49(-1)$

$9-42i-49$

$-40-42i$

1 ~~13~~. $(3k-2i)^2$

$(3k-2i)(3k-2i)$

	$3k$	$-2i$
$3k$	$9k^2$	$-6ki$
$-2i$	$-6ki$	$+4i^2$

$9k^2-12ki+4i^2$

$9k^2-12ki+4(-1)$

$9k^2-12ki-4$

9

2 ~~14~~. $(4x-3yi)^2$

$(4x-3yi)(4x-3yi)$

	$4x$	$-3yi$
$4x$	$16x^2$	$-12xyi$
$-3yi$	$-12xyi$	$+9y^2i^2$

$16x^2-24xyi+9y^2i^2$

$16x^2-24xyi+9y^2(-1)$

$16x^2-24xyi-9y^2$

3 ~~15~~. $3xi(3-2i)$

$9xi-6xi^2$

$9xi-6x(-1)$

$6x+9xi$

11

4 ~~16~~. $5i+4i(2+3i)$

$5i+8i+12i^2$

$13i+12(-1)$

$-12+13i$

5 ~~17~~. $2xi(i-4i^2)$

$2xi^2-8xi^3$

$2x(-1)-8x(-i)$

$-2x+8xi$

13

6 ~~18~~. $6xi^3(-4xi+5)$

$6x(-i)(-4xi+5)$

$-6xi(-4xi+5)$

$24x^2i^2-30xi$

$24x^2(-1)-30xi$

$-24x^2-30xi$

7 ~~19~~. $2i(\sqrt{-4}-4)$

$\sqrt{-4}$

$i\sqrt{4}$

$2i$

$2i(2i-4)$

$4i^2-8i$

$4(-1)-8i$

$-4-8i$

15

8 ~~20~~. $-\frac{1}{2}i^3\left(\sqrt{-9}-4\right)-3i^2$

$\sqrt{-9}$

$i\sqrt{9}$

$3i$

$-\frac{1}{2}(-i)(3i-4)-3(-1)$

$\frac{1}{2}i(3i-4)+3$

$\frac{3}{2}i^2-2i+3$

$\frac{3}{2}(-1)-2i+3$

$-\frac{3}{2}-2i+3$

$\frac{3}{2}-2i$

Writing the Equation of a Parabola

Definition of a Parabola: A parabola is the set of all points equidistant between a point (focus) and a line (directrix).

The vertex is directly in between the focus and the directrix. USE GRAPH PAPER AND COUNT!

$$y=\frac{1}{4p}(x-v)^2+t$$

$(v,t)=vertex$

$p=distance\ from\ vertex\ to\ focus$

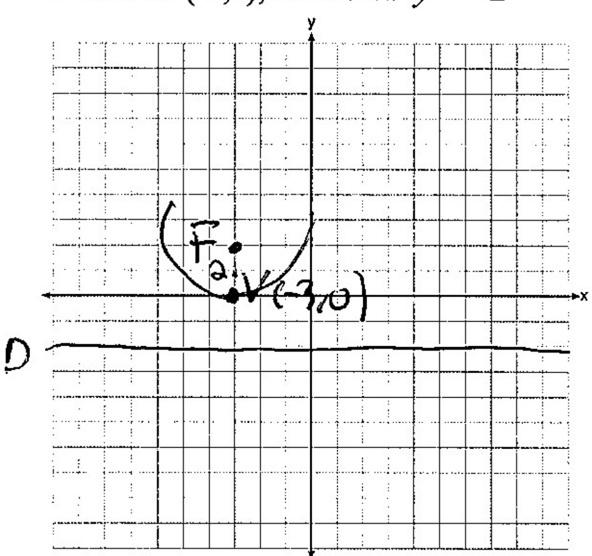
*p is positive when parabola opens up and negative when parabola opens down

You might have to manipulate the equation if it is multiple choice

If given equation, pull the vertex out!

For each of the following problems, state the coordinate of the focus and vertex, the equation of the directrix and the parabola in three different forms.

1. Focus: (-3,2), Directrix: $y=-2$

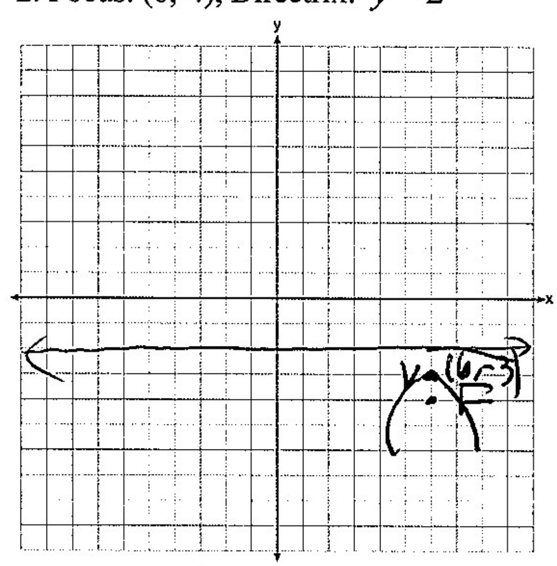


$y=\frac{1}{4p}(x-v)^2+t$ $\quad p=2$, $v=-3$, $t=0$

$y=\frac{1}{4(2)}(x+3)^2+0$

$y=\frac{1}{8}(x+3)^2$

2. Focus: (6,-4), Directrix: $y=2$

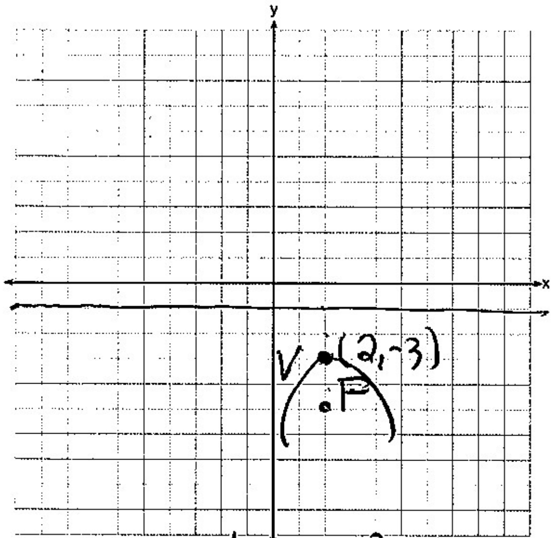


$y=\frac{1}{4p}(x-v)^2+t$ $\quad p=-1$, $v=6$, $t=-3$

$y=\frac{1}{4(-1)}(x-6)^2-3$

$y=-\frac{1}{4}(x-6)^2-3$

3. Focus: (2,-5), Directrix: $y=-1$



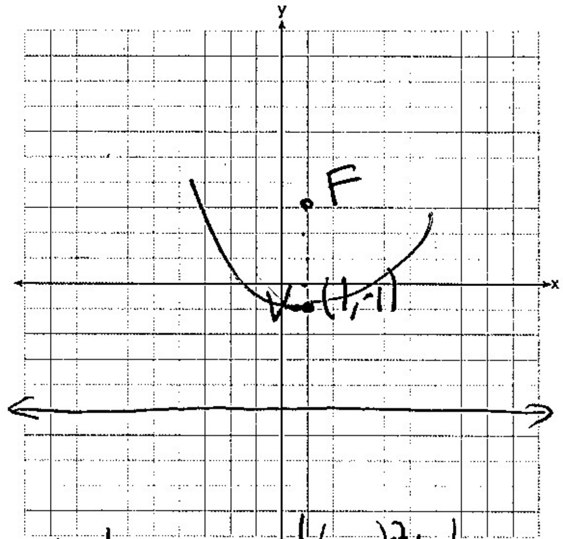
$V=2$
$t=-3$
$p=-2$

$y=\frac{1}{4p}(x-v)^2+t$

$y=\frac{1}{4(-2)}(x-2)^2-3$

$y=-\frac{1}{8}(x-2)^2-3$

4. Focus: (1,3), Directrix: $y=-5$



$V=1$
$t=-1$
$p=4$

$y=\frac{1}{4p}(x-v)^2+t$

$y=\frac{1}{4(4)}(x-1)^2-1$

$y=\frac{1}{16}(x-1)^2-1$

5. Which equation represents the set of points equidistant from line ℓ and point R shown on the graph below?

$y=\frac{1}{4p}(x-v)^2+t$

$V=2$
$t=-1$
$p=-2$

$y=\frac{1}{4(-2)}(x-2)^2-1$

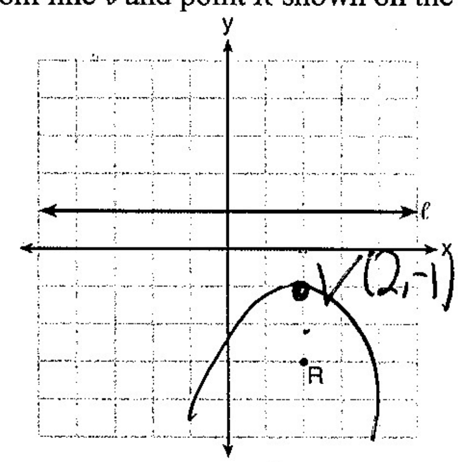
$y=-\frac{1}{8}(x-2)^2-1$

1) $y=-\frac{1}{8}(x+2)^2+1$

2) $y=-\frac{1}{8}(x+2)^2-1$

3) $y=-\frac{1}{8}(x-2)^2+1$

4) $y=-\frac{1}{8}(x-2)^2-1$



6. Which equation represents the equation of the parabola with focus $(-3,3)$ and directrix $y=7$?

1) $y=\frac{1}{8}(x+3)^2-5$

2) $y=\frac{1}{8}(x-3)^2+5$

3) $y=-\frac{1}{8}(x+3)^2+5$

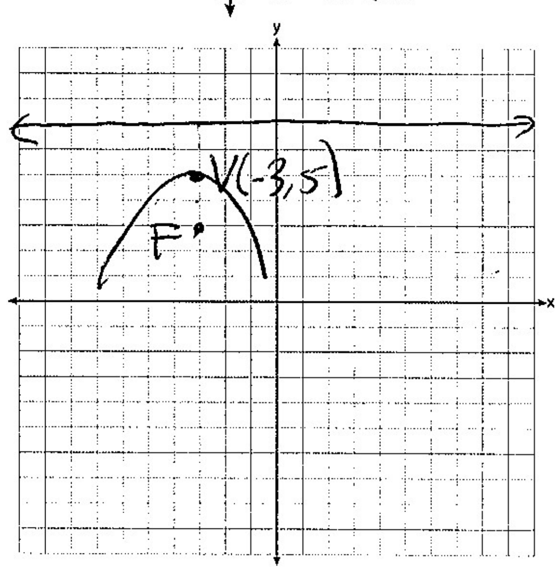
4) $y=-\frac{1}{8}(x-3)^2+5$

$V=-3$
$t=5$
$p=-2$

$y=\frac{1}{4p}(x-v)^2+t$

$y=\frac{1}{4(-2)}(x+3)^2+5$

$y=-\frac{1}{8}(x+3)^2+5$



7. Which equation represents a parabola with a focus of $(0, 4)$ and a directrix of $y = 2$?

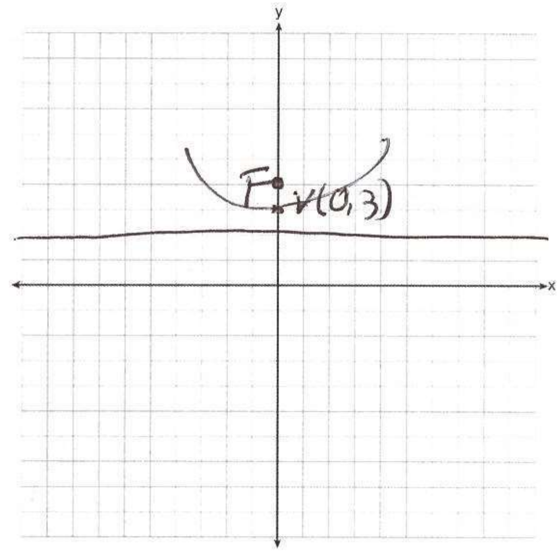
1) $y = x^2 + 3$
2) $y = -x^2 + 1$
3) $y = \frac{x^2}{2} + 3$
4) $y = \frac{x^2}{4} + 3$

$y = \frac{1}{4p}(x - v)^2 + t$ $\quad$ $v = 0$

$t = 3$

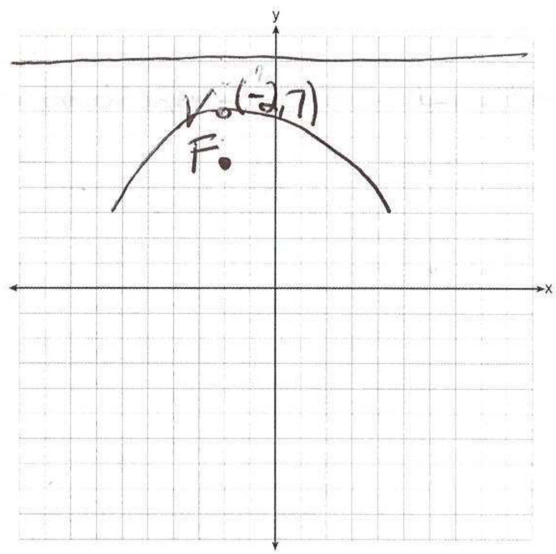
$y = \frac{1}{4(1)}(x - 0)^2 + 3$ $\quad$ $p = 1$

$y = \frac{1}{4}x^2 + 3$



8. Which equation represents a parabola with a focus of $(-2, 5)$ and a directrix of $y = 9$?

1) $(y - 7)^2 = 8(x + 2)$
2) $(y - 7)^2 = -8(x + 2)$
3) $(x + 2)^2 = 8(y - 7)$
4) $(x + 2)^2 = -8(y - 7)$



$y = \frac{1}{4p}(x - v)^2 + t$ $\quad$ $v = -2$

$t = 7$

$y = \frac{1}{4(-2)}(x + 2)^2 + 7$ $\quad$ $p = -2$

$y = -\frac{1}{8}(x + 2)^2 + 7$

$-7 \qquad -7$

$-8(y - 7) = -8\left(-\frac{1}{8}(x + 2)^2\right)$

$-8(y - 7) = (x + 2)^2$

9. A parabola has its focus at $(1, 2)$ and its directrix is $y = -2$. The equation of this parabola could be

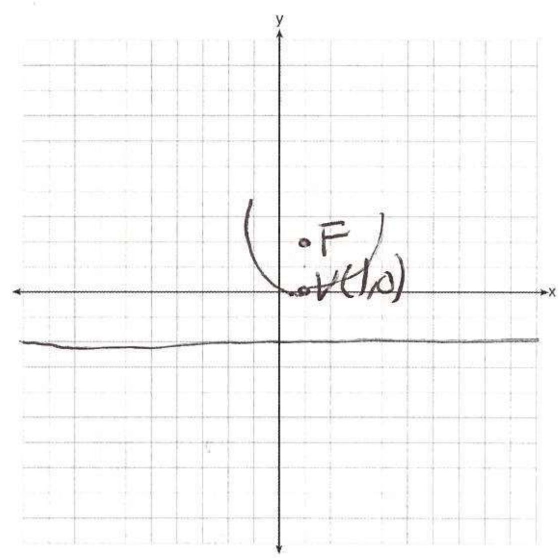
1) $y = 8(x + 1)^2$
2) $y = \frac{1}{8}(x + 1)^2$
3) $y = 8(x - 1)^2$
4) $y = \frac{1}{8}(x - 1)^2$

$y = \frac{1}{4p}(x - v)^2 + t$ $\quad$ $p = 2$

$v = 1$

$y = \frac{1}{4(2)}(x - 1)^2 + 0$ $\quad$ $t = 0$

$y = \frac{1}{8}(x - 1)^2$



Given the Equation of a Parabola

If given equation, pull the vertex and p value out!

To find vertex, negate the x coordinate but not the y (unless it is in cross multiplied form).
To find p, divide the value by 4.
The vertex is directly in between the focus and the directrix. USE GRAPH PAPER AND COUNT!

$$y = \frac{1}{4p}(x-v)^2 + t$$

$(v,t) = vertex$

$p = distance\ from\ vertex\ to\ focus$

*p is positive when parabola opens up and negative when parabola opens down

Find the vertex and p value of the parabolas below

1. $y = \frac{1}{12}(x-5)^2 - 1$
   $(5,-1)$
   $p=3$

2. $y = \frac{1}{8}(x+3)^2 - 4$
   $(-3,-4)$
   $p=2$

3. $y = -\frac{1}{16}(x+9)^2 - 8$
   $(-9,-8)$
   $p=-4$

4. $y = \frac{1}{4}(x+9)^2 - 3$
   $(-9,-3)$
   $p=1$

5. $y = -\frac{1}{12}(x-7)^2 + 1$
   $(7,1)$
   $p=-3$

6. $y = \frac{1}{20}x^2 + 5$
   $(0,5)$
   $p=5$

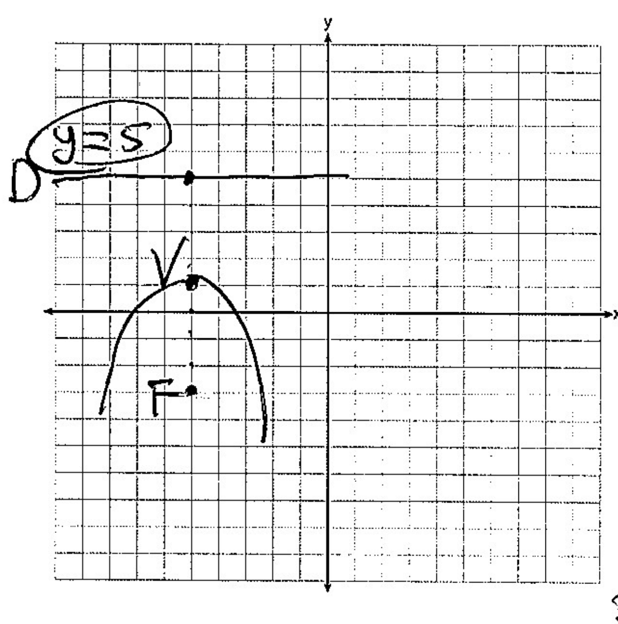
7. $12(y+2) = (x+3)^2$
   $(-3,-2)$
   $p=3$

8. $-4(y+1) = (x-2)^2$
   $(2,-1)$
   $p=-1$

9. $24(y+1) = (x-7)^2$
   $(7,-1)$
   $p=6$

10. The equation of a parabola is $y = -\frac{1}{16}(x+5)^2 + 1$. If the focus is $(-5,-3)$, what is the equation of the directrix?

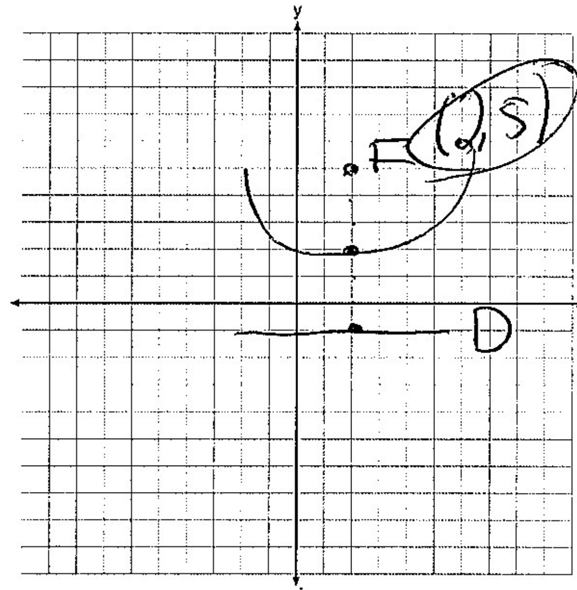
$(-5,1)$
$p=-4$



11. The parabola described by the equation $y = \frac{1}{12}(x-2)^2 + 2$ has the directrix at $y = -1$. What is the focus?

V: (2,2)

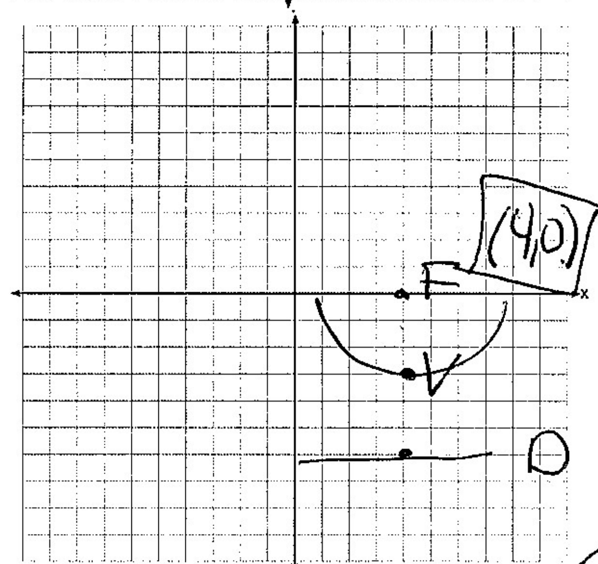
p = 3



12. The directrix of the parabola $12(y+3) = (x-4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

V: (4,-3)

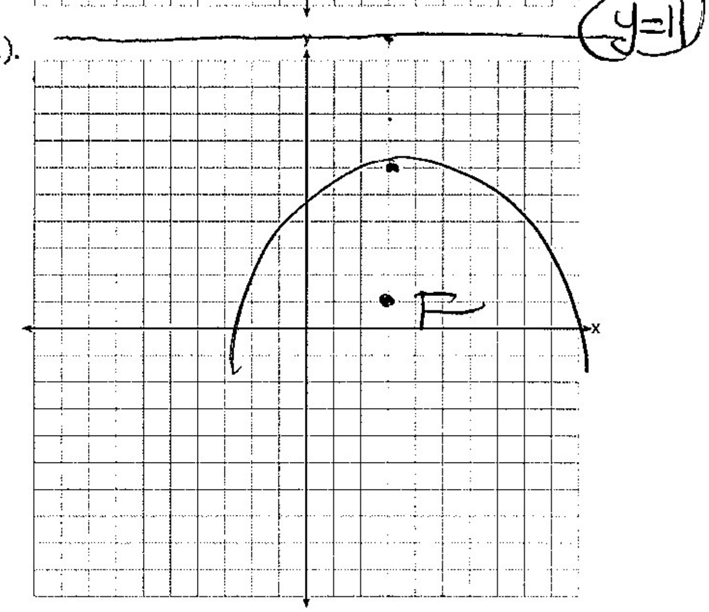
p=3



13. The parabola $y = -\frac{1}{20}(x-3)^2 + 6$ has its focus at (3, 1). Determine and state the equation of the directrix.

V:(3,6)

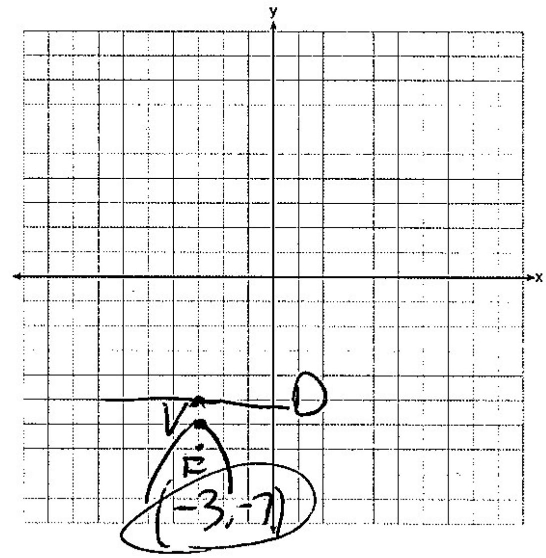
p=-5



14. The parabola $y = -\frac{1}{4}(x+3)^2 - 6$ has a directrix at $y = -5$. What is the focus?

V: (-3, -6)

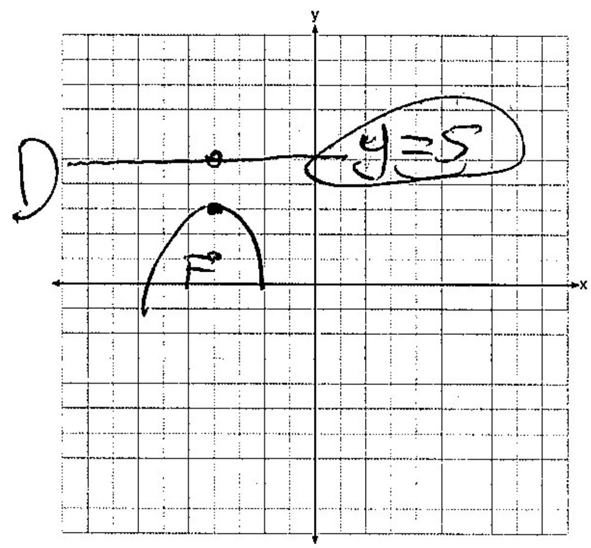
p = -1



15. What is the equation of the directrix for the parabola $-8(y-3) = (x+4)^2$?

V: (-4, 3)

p = 2

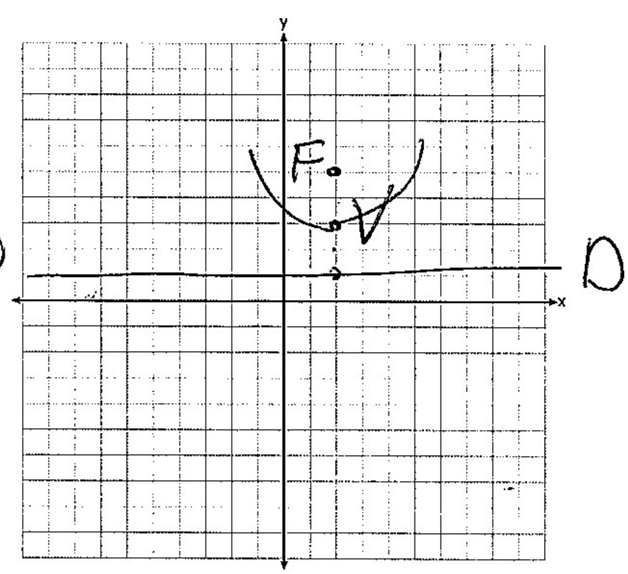


16. The parabola $8(y-3) = (x-2)^2$ has a focus of (2, 5). What is the equation of the directrix?

V: (2, 3)

p = 2

y = 1



Solving Systems of Equations Graphically Using TI-84+ ($f(x) = g(x)$)

1) Type equations into Y_1 and Y_2
2) Zoom 6 (Standard) is your standard window. Adjust window OR try Zoom 0(Fit) if you don't see what you want to see.
3) 2nd Trace (Calc), 5 (Intersect)
4) Place cursor over point of intersection, hit enter, enter, enter. Repeat the process for any other points of intersection.

***The solutions to the system of equations are the x values of the intersections.**

1. To the *nearest tenth*, the value of x that satisfies $2^x = -2x + 11$ is Intersect

Y1 | Y2

1) 2.5
2) 2.6
3) 5.8
4) 5.9

x=2.6

*window's good

2. For which values of x, rounded to the *nearest hundredth*, will $|x^2 - 9| - 3 = \log_3 x$? Intersect

Y1 | Y2

1) 2.29 and 3.63
2) 2.37 and 3.54
3) 2.84 and 3.17
4) 2.92 and 3.06

x=2.29
x=3.63

*window's good

3. For which approximate value(s) of x will $\log(x+5) = |x-1| - 3$?

Y1 | Y2

1) 5, 1
2) −2.41, 0.41
3) −2.41, 5
4) 5, only

Intersect
x=−2.41
x=5

*window's good

4. Which value, to the *nearest tenth*, is *not* a solution of $p(x) = q(x)$ if $p(x) = x^3 + 3x^2 - 3x - 1$ and $q(x) = 3x + 8$? Intersect

Y1, Y2

1) −3.9
2) −1.1
3) 2.1
4) 4.7

x=−3.9
x=−1.1
x=2.1

*adjust y max

5. If $f(x) = 3|x| - 1$ and $g(x) = 0.03x^3 - x + 1$, an approximate solution for the equation $f(x) = g(x)$ is

Y1 | Y2 Intersect

1) 1.96
2) 11.29
3) $(-0.99, 1.96)$
4) $(11.29, 32.87)$

x=−.99 x=11.29
x=−1.11

*adjust y max

6. If $p(x) = 2\ln(x) - 1$ and $m(x) = \ln(x+6)$, then what is the solution for $p(x) = m(x)$?

Y1 | Y2 Intersect

1) 1.65
2) 3.14
3) 5.62
4) no solution

x=5.62

*window's good

7. If ~~$f(x) = g(x)$~~ $f(x) = 3|x| - 1$ and $g(x) = 0.03x^3 - x + 1$, ~~an approximate solution~~ for the equation $f(x) = g(x)$ is

1) 1.96
2) ~~11.29~~
3) ~~$(-0.99, 1.96)$~~
4) $(11.29, 32.87)$

8. Given: $h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2$ Y1 Intersect

$k(x) = -|0.7x| + 5$ Y2

State the solutions to the equation $h(x) = k(x)$, rounded to the *nearest hundredth.*

$x = -5.17$
$x = -1.13$
$x = 1.75$

9. If $f(t) = 325e^{-.0735t} + 75$ and $g(t) = 375e^{-.0817t} + 75$, for what value of t does $f(t) = g(t)$ rounded to the *nearest tenth*? Y1 Y2 Intersect

Zoom Fit

$t = 17.5$

10. A technology company is comparing two plans for speeding up its technical support time. Plan A can be modeled by the function $A(x) = 15.7(0.98)^x$ and plan B can be modeled by the function $B(x) = 11(0.99)^x$ where x is the number of customer service representatives employed by the company and $A(x)$ and $B(x)$ represent the average wait time, in minutes, of each customer. To the *nearest integer*, solve the equation $A(x) = B(x)$. Y1 Y2

Intersect

*adjust X max and y max

$x = 35$

11. Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is $P(x) = \log(x - 4)$, where x is the number of visits per week in thousands and $P(x)$ is the website's popularity rating. Y1

An alternative rating model is represented by $R(x) = \frac{1}{2}x - 6$, where x is the number of visits per week in thousands. For what number of weekly visits will the two models provide the same rating? Y2

Intersect

*adjust y max

$x = 14$
14,000

12. The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time. State when $V(t) = Z(t)$, to the *nearest hundredth*, Y1 Y2

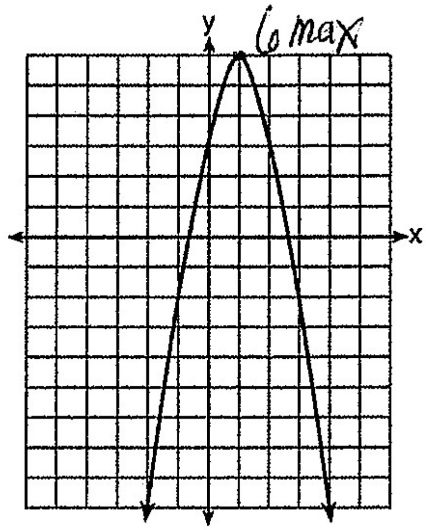
Intersect

*Zoom Fit

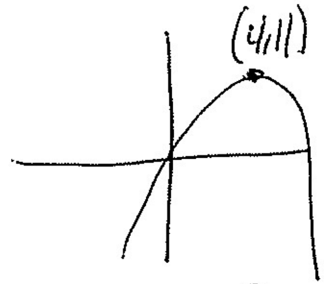
$t = 1.95$

Key Points
To compare key points, find the key point for each function. Use the graph, the table (2nd graph), and the calculate menu (2nd Trace).

1. Let f be the function represented by the graph below.



2nd Calc: maximum



Or

x	y
1	13/2
2	9
3	21/2
4	11 → max
5	21/2
6	9
7	13/2

Let g be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$. Determine which function has the larger maximum value. Justify your answer.

g(4) 11>6

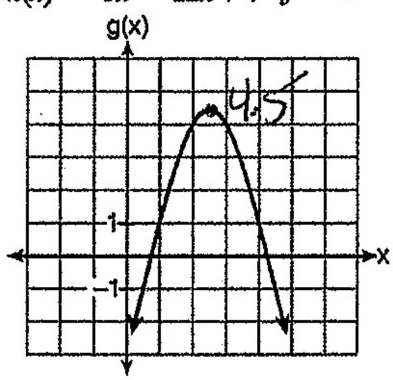
2. Which quadratic function has the largest maximum?

1) $h(x) = (3-x)(2+x)$ 2nd Calc: max: 6.25 3) $k(x) = -5x^2 - 12x + 4$ 2nd Calc: max: 11.2

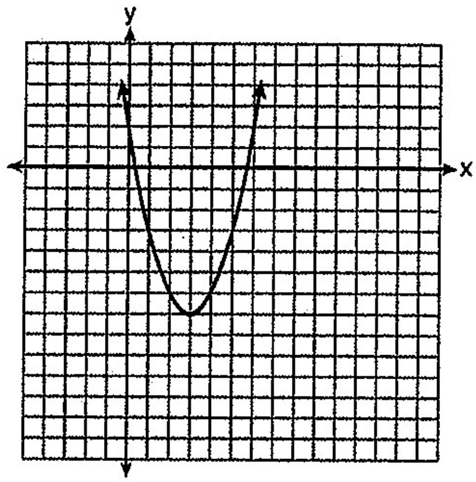
x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

2) ≈9.5

4)



3. The graph representing a function is shown below.



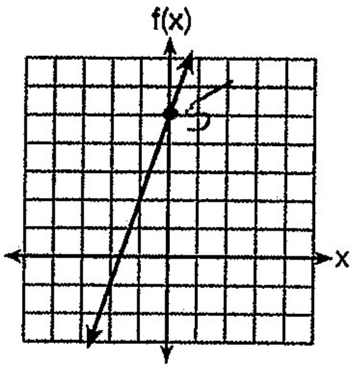
Which function has a minimum that is *less* than the one shown in the graph?

2nd Calc MM

1) $y = x^2 - 6x + 7$ -2
2) $y = |x+3| - 6$ -2
3) $y = x^2 - 2x - 10$ -11 $\quad -11 < -7$
4) $y = |x-8| + 2$ $\quad 2$

$x=0$

4. Which function has the greatest y-intercept?
1) $f(x) = 3x + 0$ $\quad 0$
2) $2x + 3y = 12$ $\quad 4$
3) the line that has a slope of 2 and passes through $(1,-4)$ -6
4) f(x)



2) $2x+3y=12$
$-2x \quad -2x$
$\frac{3y}{3} = \frac{-2x+12}{3}$
$y = -\frac{2}{3}x + 4$

3) $y - y_1 = m(x - x_1)$
$y + 4 = 2(0-1)$
$y + 4 = -2$
$-4 \quad -4$
$y = -6$

$x=0$

5. Which graph has the greatest y-intercept?

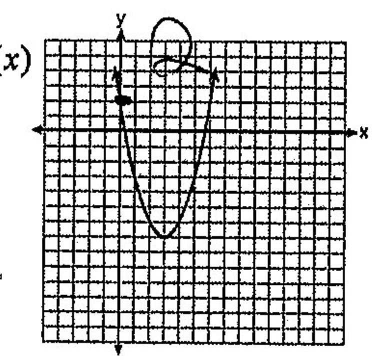
x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

5

$$g(x) = \left(\frac{1}{2}\right)^{x+1} + 3$$

$(0, 3.5)$

$h(x)$



f(x)

$5 > 3.5 > 2$

Finding Key Points of Polynomial Functions Using TI
-Type equation into Y=
-2nd Trace (Calc)

For the following polynomial functions, find the zeros, relative minima, and relative maxima rounded to the *nearest tenth*.

1. $f(x) = x^3 + 3x^2 - x - 2$

Zeros: -3.1, $-.8$, $-.9$

relative max $(-2.2, 4.1)$

relative min $(.2, -2.1)$

2. $f(x) = x^3 + 8x^2 + 3x - 8$

Zeros: -7.5, -1.4, $.8$

relative max $(-5.1, 52.1)$

relative min $(-.2, -8.3)$

3. An estimate of the number of milligrams of a medication in the bloodstream t hours after 400 mg has been taken can be modeled by the function below.

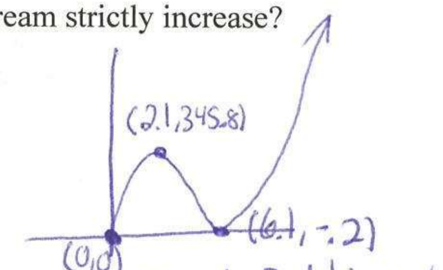
$$I(t) = 0.5t^4 + 3.45t^3 - 96.65t^2 + 347.7t, \quad Y=$$

adjust Y max

where $0 \le t \le 6$

Over what time interval does the amount of medication in the bloodstream strictly increase?

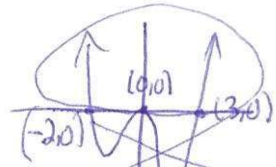
1) 0 to 2 hours ✓
2) 0 to 3 hours ✗
3) 2 to 6 hours ✗
4) 3 to 6 hours ✗



2nd Trace to find key points

4. Given $f(x) = x^4 - x^3 - 6x^2$, for what values of x will $f(x) > 0$?

1) $x < -2$, only
2) $x < -2$ or $x > 3$
3) $x < -2$ or $0 \le x \le 3$
4) $x > 3$, only

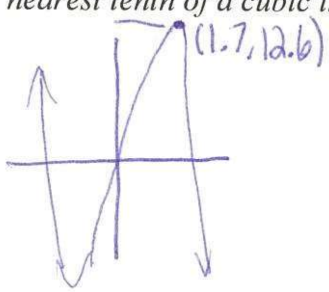


$(-\infty, -2)$
$(3, \infty)$

5. The function $v(x) = x(3 - x)(x + 4)$ models the volume, in cubic inches, of a rectangular solid for $0 \le x \le 3$.

x min: 0
x max: 3

To the *nearest tenth of a cubic inch*, what is the maximum volume of the rectangular solid?



adjust window

2nd Trace: maximum

12.6

6. Consider $f(x) = 4x^2 + 6x - 3$, and $p(x)$ defined by the graph below. The difference between the values of the maximum of p and minimum of f is

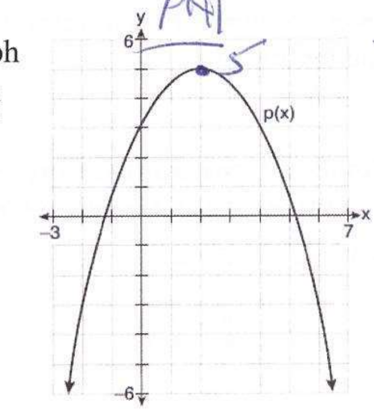
5 $\quad$ -5.25

1) 0.25 $\quad$ 3) 3.25

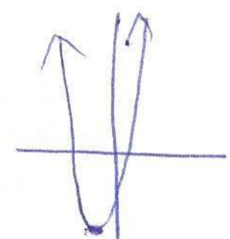
2) 1.25 $\quad$ 4) 10.25

$5 + {}^{+}5.25 = 10.25$

p(x)



f(x)



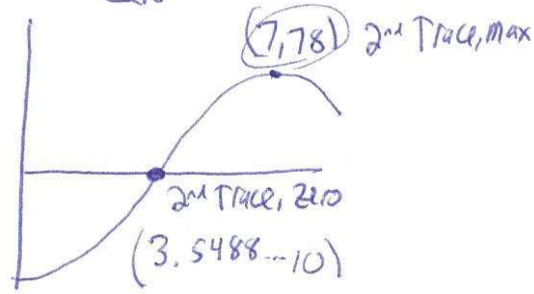
2nd Trace, minimum

$(-.75, -5.25)$

7. A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. Creating more products doesn't always increase profits because it requires additional costs, such as building a larger facility or hiring more workers. The manufacturer determines the profit, $p(x)$, in thousands of dollars, as a function of the number of sweatshirts sold, x, in thousands. This function, p, is given below. Over the given interval, state the coordinates of the maximum of p and round all values to the *nearest integer*. Explain what this point represents in terms of the number of sweatshirts sold and profit. Determine how many sweatshirts, to the *nearest whole sweatshirt*, the manufacturer would need to produce in order to first make a positive profit. Justify your answer.

Zero

$p(x) = -x^3 + 11x^2 - 7x - 69$



When 7,000 sweatshirts are sold, the maximum profit of \$78,000.

$3.5488\ldots(1000) = 3{,}549$ sweatshirts

Inverse of a function $f^{-1}(x)$:
Algebraically
Switch x and y, solve for y
Graphically
Y1: Type in original function
Y2: Type in each answer
Y3: x
Look for symmetry to $y = x$

1. What is the inverse of the function $y = 2x - 3$?

1) $y = \frac{x+3}{2}$
2) $y = \frac{x}{2} + 3$
3) $y = -2x + 3$
4) $y = \frac{1}{2x-3}$

$x = 2y - 3$
$+3 \quad +3$
$\frac{x+3}{2} = \frac{2y}{2}$

2. If a function is defined by the equation y = 3x + 2, which equation defines the inverse of this function?

1) x = $\frac{1}{3}y + \frac{1}{2}$
2) y = $\frac{1}{3}x + \frac{1}{2}$
3) y = $\frac{1}{3}x - \frac{2}{3}$
4) y = -3x – 2

$x = 3y + 2$
$-2 \quad -2$
$\frac{x-2}{3} = \frac{3y}{3}$
$\frac{x-2}{3} = y$
$\frac{1}{3}x - \frac{2}{3} = y$

3. If $f(x) = 5x - 7$, find $f^{-1}(x)$

$y = 5x - 7$
$x = 5y - 7$
$+7 \quad +7$
$\frac{x+7}{5} = \frac{5y}{5}$
$\frac{x+7}{5} = y$
$f^{-1}(x) = \frac{x+7}{5}$

4. What is $g^{-1}(x)$ if $g(x) = 3x + 6$

$y = 3x + 6$
$x = 3y + 6$
$-6 \quad -6$
$\frac{x-6}{3} = \frac{3y}{3}$
$\frac{x-6}{3} = y$
$g^{-1}(x) = \frac{x-6}{3}$

5. What is the inverse of $y = \frac{1}{2}x + 2$?

$2(x) = (\frac{1}{2}y) + (2)2$
$2x = y + 4$
$-4 \quad -4$
$y = 2x - 4$

6. What is $h^{-1}(x)$ if $h(x) = x^2 + 2$

$y = x^2 + 2$

$x = y^2 + 2$
$-2 \quad -2$

$\sqrt{x-2} = \sqrt{y^2}$

$\sqrt{x-2} = y$

$h^{-1}(x) = \sqrt{x-2}$

7. What is the inverse of the function $y = 4x + 5$?

1) $x = \frac{1}{4}y - \frac{5}{4}$

2) $y = \frac{1}{4}x - \frac{5}{4}$

3) $y = 4x - 5$

4) $y = \frac{1}{4x + 5}$

$x = 4y + 5$
$-5 \quad -5$

$\frac{x-5}{4} = \frac{4y}{4}$

$\frac{1}{4}x - \frac{5}{4} = y$

$Y1 = 4x + 5$
$Y2 = \frac{1}{4}x - \frac{5}{4}$
$Y3 = x$

8. What is the inverse of $f(x) = -6(x - 2)$?

1) $f^{-1}(x) = -2 - \frac{x}{6}$

2) $f^{-1}(x) = 2 - \frac{x}{6}$

3) $f^{-1}(x) = \frac{1}{-6(x-2)}$

4) $f^{-1}(x) = 6(x + 2)$

$y = -6(x-2)$

$\frac{x}{-6} = \frac{-6(y-2)}{-6}$

$-\frac{x}{6} = y - 2$
$+2 \quad +2$

$2 - \frac{x}{6} = y$

$Y1 = -6(x-2)$
$Y2 = 2 - \frac{x}{6}$
$Y3 = x$

9. Given $f(x) = \frac{1}{2}x + 8$, which equation represents the inverse, $g(x)$?

1) $g(x) = 2x - 8$

2) $g(x) = 2x - 16$

3) $g(x) = -\frac{1}{2}x + 8$

4) $g(x) = -\frac{1}{2}x - 16$

$y = \frac{1}{2}x + 8$

$2(x = (\frac{1}{2}y) + (8))2$

$2x = y + 16$
$-16 \quad -16$

$2x - 16 = y$

$Y1 = \frac{1}{2}x + 8$
$Y2 = 2x - 16$
$Y3 = x$

10. The inverse of the function $f(x) = \frac{x+1}{x-2}$ is

1) $f^{-1}(x) = \frac{x+1}{x+2}$

2) $f^{-1}(x) = \frac{2x+1}{x-1}$

3) $f^{-1}(x) = \frac{x+1}{x-2}$

4) $f^{-1}(x) = \frac{x-1}{x+1}$

$y = \frac{x+1}{x-2}$

$x = \frac{y+1}{y-2}$

$x(y-2) = y+1$

$xy - 2x = y + 1$
$-y + 2x \quad -y + 2x$

$xy - y = 2x + 1$

$\frac{y(x-1)}{x-1} = \frac{2x+1}{x-1}$

$y = \frac{2x+1}{x-1}$

$Y1 = \frac{x+1}{x-2}$
$Y2 = \frac{2x+1}{x-1}$
$Y3 = x$

11. What is the inverse of $f(x) = \frac{x}{x+2}$, where $x \neq -2$?

1) $f^{-1}(x) = \frac{2x}{x-1}$

2) $f^{-1}(x) = \frac{-2x}{x-1}$

3) $f^{-1}(x) = \frac{x}{x-2}$

4) $f^{-1}(x) = \frac{-x}{x-2}$

$y = \frac{x}{x+2}$

$x = \frac{y}{y+2}$

$x(y+2) = y$

$xy + 2x = y$
$-y - 2x \quad -y \quad -2x$

$xy - y = -2x$

$\frac{y(x-1)}{x-1} = \frac{-2x}{x-1}$

$y = \frac{-2x}{x-1}$

$Y1 = \frac{x}{x+2}$
$Y2 = \frac{-2x}{x-1}$
$Y3 = x$

Name Schlansky
Mr. Schlansky

Even
$f(x) = f(-x)$
symmetric to the y-axis
(y-axis cuts graph in half)

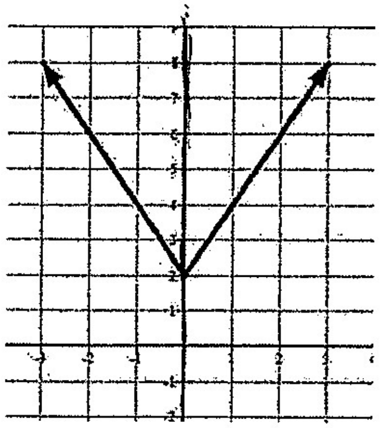
Odd
$f(-x) = -f(x)$
symmetric to the origin
(turn upside down and image is the same)

Date ______________
Algebra II

Even and Odd Functions

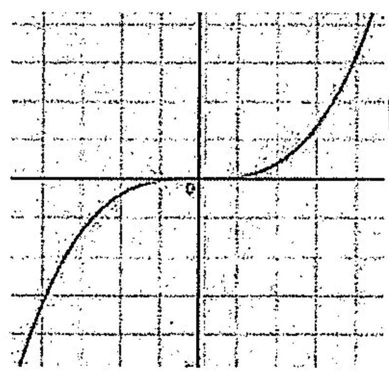
Determine graphically whether the following functions are even, odd, or neither

1.



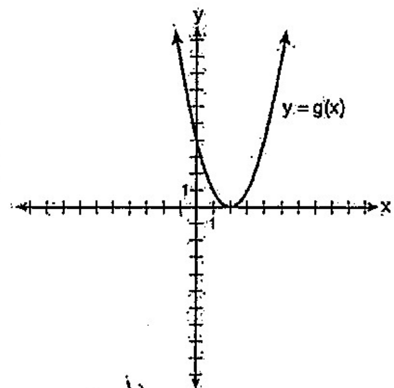
even because it's symmetric to the y-axis

2.



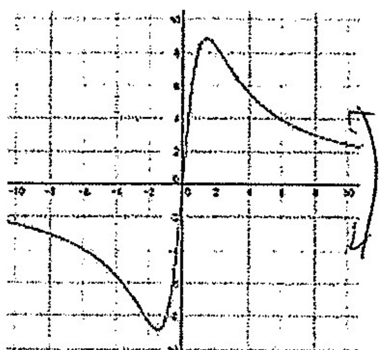
odd because it's symmetric to the origin

3.



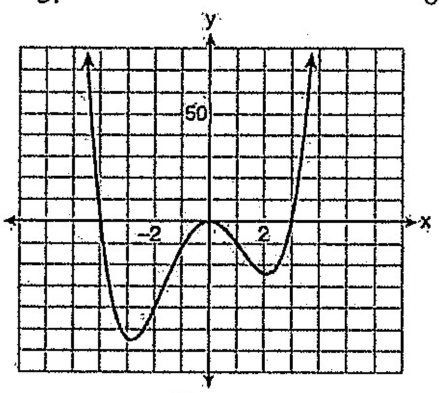
neither

4.



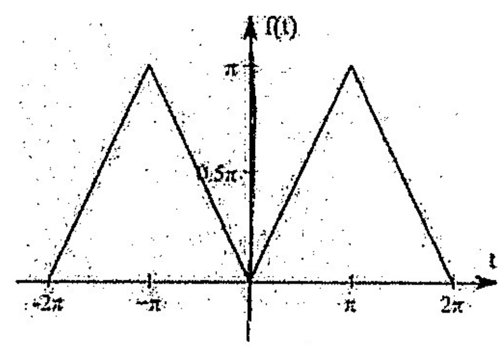
odd because it's symmetric to the origin

5.



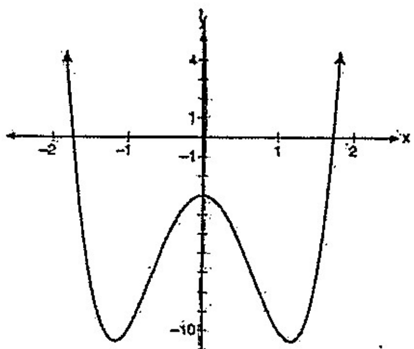
neither

6.



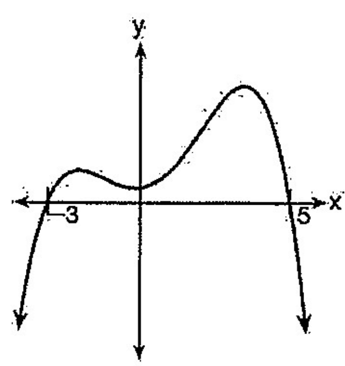
even because it's symmetric to the y-axis

7.



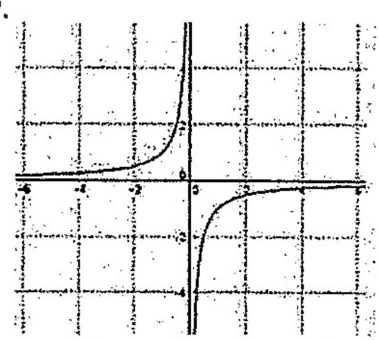
even because it's symmetric to the y-axis

8.



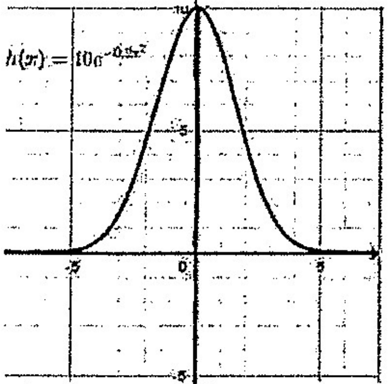
neither

9.



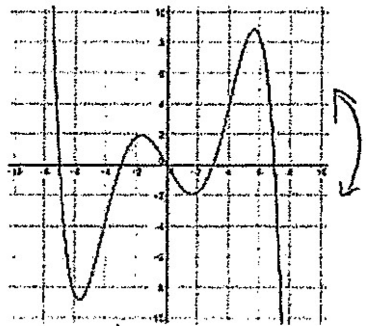
odd because it's symmetric to the origin

10.



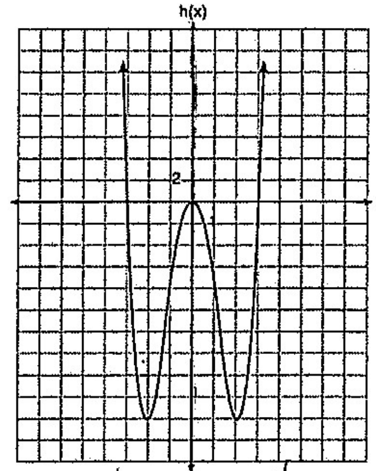
even because it's symmetric to the y-axis

11.



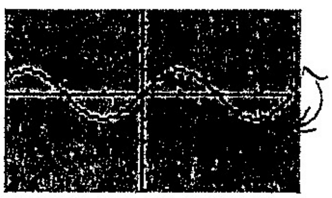
odd because it's symmetric to the origin

12.



even because it's symmetric to the y-axis

13.

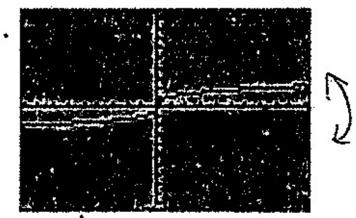


odd because it's symmetric to the origin

14.

even because it's symmetric to the y-axis

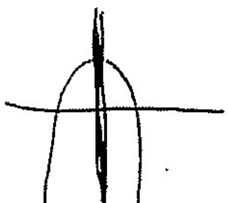
15.



odd because it's symmetric to the origin.

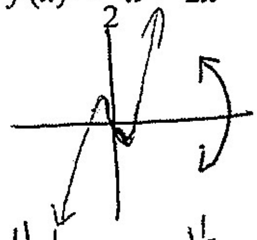
Type into y=

16. $f(x) = -x^4 + 4$



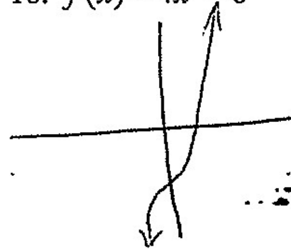
even because it's symmetric to the y-axis

17. $f(x) = \frac{1}{2}x^5 - 2x$



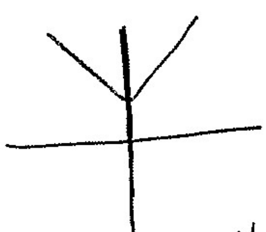
odd because it's symmetric to the origin

18. $f(x) = 4x^3 - 6$



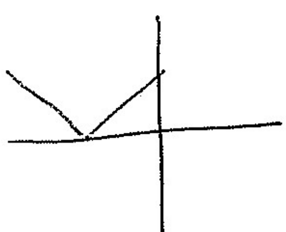
neither

19. $f(x) = |x| + 4$



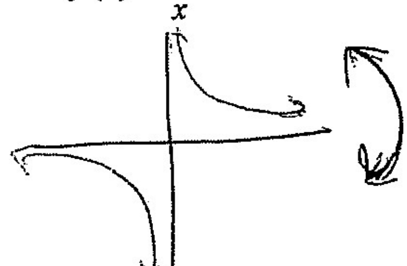
even because it's symmetric to the y-axis

20. $f(x) = |x + 4|$



neither

21. $f(x) = \frac{10}{x}$



odd because it's symmetric to the origin

Transforming Functions
Translations (+ or -)

If adding to f(x), the graph moves up or down
If adding to x, the graph moves left or right (the opposite direction in which you would think)

$y = f(x) + a$ moves UP a units

$y = f(x) - a$ moves DOWN a units

$y = f(x + a)$ moves LEFT a units

$y = f(x - a)$ moves RIGHT a units

Reflections (Negative)

$y = -f(x)$ is a reflection over the x axis (negate the y)

$y = f(-x)$ is a reflection over the y axis (negate the x)

1. If $g(x) = f(x-4)+2$, how is the graph of $f(x)$ translated to form the graph of $g(x)$?

right 4 up 2

2. If $h(x) = f(x+1)-3$, how is the graph of $f(x)$ translated to form the graph of $g(x)$?

left 1 down 3

3. How is the parent function transformed to create $f(x) = |x+3| - 2$?

left 3 down 2

4. How is the parent function transformed to create $f(x) = (x-4)^2 + 3$?

right 4 up 3

5. Relative to the graph of $y = 3\sin x$, what is the shift of the graph of $y = 3\sin\left(x + \frac{\pi}{3}\right)$?

1) $\frac{\pi}{3}$ right 2) $\frac{\pi}{3}$ left 3) $\frac{\pi}{3}$ up 4) $\frac{\pi}{3}$ down

left $\frac{\pi}{3}$

6. Given the parent function $p(x) = \cos x$, which phrase best describes the transformation used to obtain the graph of $g(x) = \cos(x + a) - b$, if a and b are positive constants?

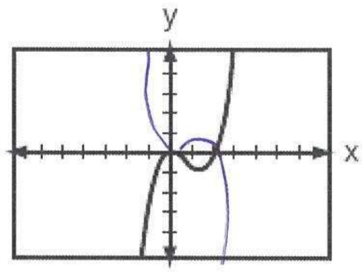
1) right a units, up b units
2) right a units, down b units
3) left a units, up b units
4) left a units, down b units

left a down b

7. If $f(x) = \log_3 x$ and $g(x)$ is the image of $f(x)$ after a translation five units to the left, which equation represents $g(x)$?

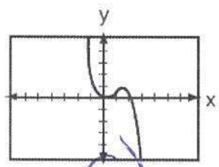
1) $g(x) = \log_3(x+5)$ left 5
2) $g(x) = \log_3 x + 5$
3) $g(x) = \log_3(x-5)$
4) $g(x) = \log_3 x - 5$

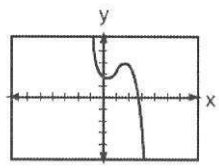
8. The accompanying graph represents the equation $y = f(x)$.

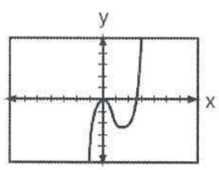


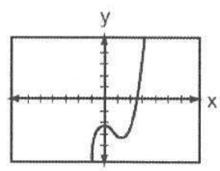
Which graph represents $g(x)$, if $g(x) = -f(x)$?

reflect over x-axis

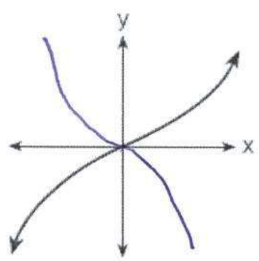

(1)


(3)


(2)

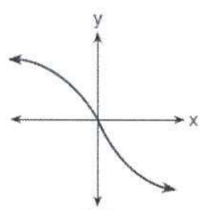

(4)

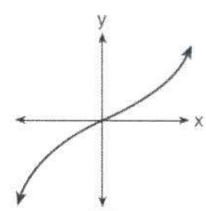
9. The graph below represents $f(x)$.

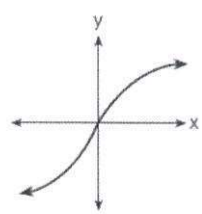


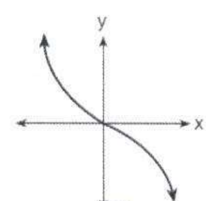
Which graph best represents $f(-x)$?

Reflect over y-axis

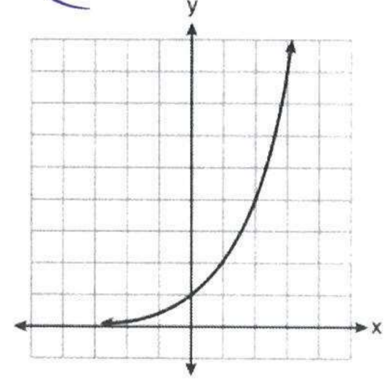

(1)


(3)

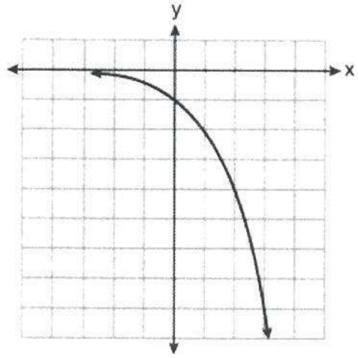

(2)


(4)

10. Consider the function $y = h(x)$, defined by the graph to the right. Which equation could be used to represent the graph shown below?



1) $y = h(x) - 2$
2) $y = h(x-2)$
3) $y = -h(x)$
4) $y = h(-x)$



reflect over x-axis

Average rate of change: $\dfrac{f(b)-f(a)}{b-a}$ or $\dfrac{y_2-y_1}{x_2-x_1}$

Always create a table!

1) If given table, circle values in the table.
2) If given a graph, pull y values from the graph.
3) If given an equation, type into y= and pull the values from the table.

Context: "On average, from a to b, the y topic is increasing/decreasing by AROC y units per x unit"

Intervals:

If given graph, the steepest slope is the greatest average rate of change. The flattest slope is the smallest average rate of change. If you cannot tell, find the average rate of change for each interval.

If given table, calculate the average rate of change for each interval.

1. The function $h(x)$ is given in the table below. Which of the following gives its average rate of change over the interval $2 \le x \le 6$?

(1) $-\frac{3}{2}$

(2) $\frac{6}{4}$

(3) $-\frac{7}{6}$

(4) -1

x	$h(x)$
0	10
2	9
4	6
6	3

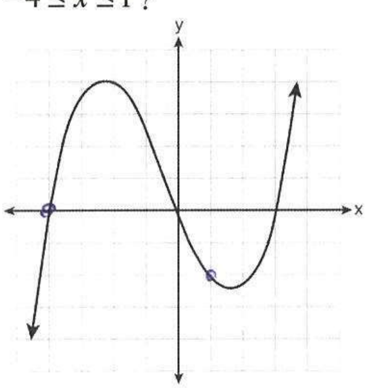
$\dfrac{3-9}{6-2} = \dfrac{-3}{2}$

2. What is the average rate of change from 0 to 2?

x	f(x)
0	1
1	2
2	5
3	7

$\dfrac{5-1}{2-0} = 2$

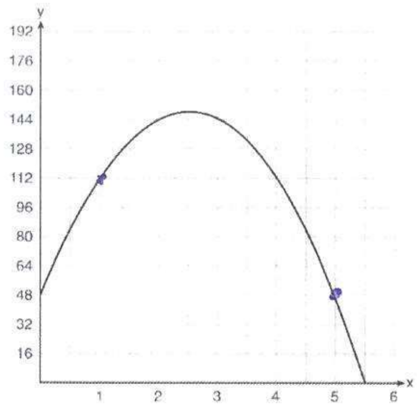
3. The graph of $p(x)$ is shown below. What is the average rate of change over the interval $-4 \le x \le 1$?



X	y
-4	0
1	-2

$\dfrac{-2-0}{1--4} = -\dfrac{2}{5}$

4. A ball is thrown into the air from the edge of a 48-foot-high cliff so that it eventually lands on the ground. The graph below shows the height, y, of the ball from the ground after x seconds. What is the average rate of change of the ball between 1 and 5 seconds?



x	y
1	112
5	48

$$\frac{48-112}{5-1} = -16$$

5. For the function $f(x) = 3^x$, find the average rate of change over the interval -5 to -1 rounded to the nearest thousandth.

y=

x	y
-5	.00412
-1	.3333

$$\frac{.3333-.00412}{-1--5} = .082$$

6. Find the average rate of change of the function $f(t) = 2500(0.97)^{4t}$ over the interval $10 \le t \le 15$ rounded to the nearest tenth.

y=

x	y
10	739.28
15	402.02

$$\frac{402.02-739.28}{15-10} = -67.5$$

7. The distance needed to stop a car after applying the brakes varies directly with the square of the car's speed. The table below shows stopping distances for various speeds. Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.

Speed (mph)	10	20	30	40	50	60	70
Distance (ft)	6.25	25	56.25	100	156.25	225	306.25

x	y
50	156.25
70	306.25

$$\frac{306.25-156.25}{70-50} = 7.5$$

On average, from 50 mph to 70 mph, the braking distance increases by 7.5 ft per mph

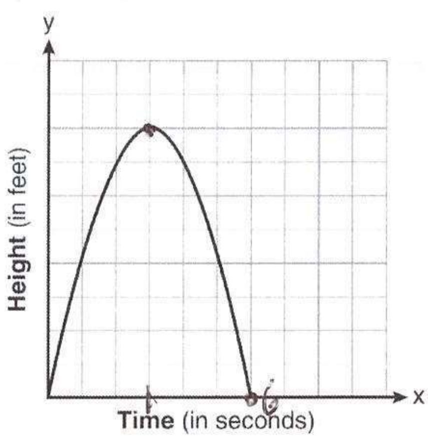
8. The average monthly high temperature in Buffalo, in degrees Fahrenheit, can be modeled by the function $B(t) = 25.29\sin(0.4895t - 1.9752) + 55.2877$, where t is the month number (January = 1). State, to the *nearest tenth*, the average monthly rate of temperature change between August and November. Explain its meaning in the given context.

x	y
8	78.866
11	48.598

$$\frac{48.598 - 78.866}{11 - 8}$$

-10.1 On average, from August to November, the average high monthly temperature in Buffalo decreases by 10.1° per month.

9. The graph below represents the parabolic path of a ball kicked by a young child. Find the average rate of change from 3 to 6 seconds. Explain its meaning in the context of the problem.

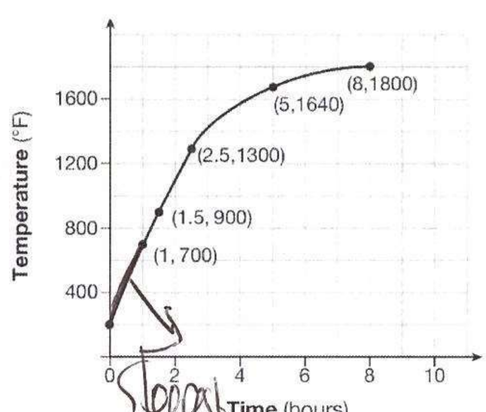


x	y
3	8
6	0

$$\frac{0-8}{6-3} = -\frac{8}{3}$$

On average, from 3 seconds to 6 seconds, the height of the ball decreased by $\frac{8}{3}$ ft per second.

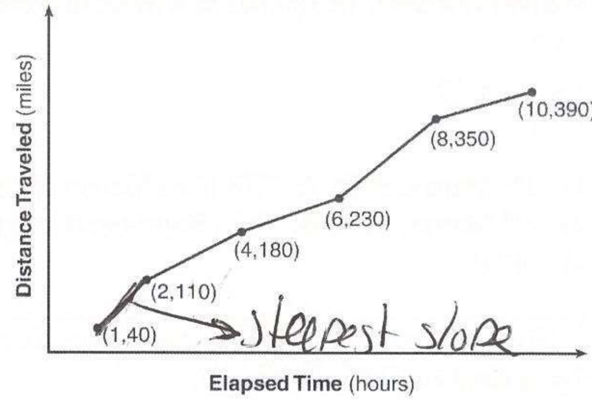
10. Firing a piece of pottery in a kiln takes place at different temperatures for different amounts of time. The graph below shows the temperatures in a kiln while firing a piece of pottery after the kiln is preheated to 200°F.
During which time interval did the temperature in the kiln show the greatest average rate of change?
1) 0 to 1 hour
2) 1 hour to 1.5 hours
3) 2.5 hours to 5 hours
4) 5 hours to 8 hours



Steepest slope

11. The Jamison family kept a log of the distance they traveled during a trip, as represented by the graph below.

During which interval was their average speed the greatest?
1) the first hour to the second hour
2) the second hour to the fourth hour
3) the sixth hour to the eighth hour
4) the eighth hour to the tenth hour



12. The table below shows the year and the number of households in a building that had high-speed broadband internet access.

Number of Households	11	16	23	33	42	47
Year	2002	2003	2004	2005	2006	2007

For which interval of time was the average rate of change the *smallest*?
1) 2002 - 2004
2) 2003 - 2005
3) 2004 - 2006
4) 2005 - 2007

1) | x | y |
|---|---|
| 2002 | 11 |
| 2004 | 23 |

$\frac{23-11}{2004-2002} = 6$

2) | x | y |
|---|---|
| 2003 | 16 |
| 2005 | 33 |

$\frac{33-16}{2005-2003} = 8.5$

3) | x | y |
|---|---|
| 2004 | 23 |
| 2006 | 42 |

$\frac{42-23}{2006-2004} = 9.5$

4) | x | y |
|---|---|
| 2005 | 33 |
| 2007 | 47 |

$\frac{47-33}{2007-2005} = 7$

13. Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of *B* dollars after *m* months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after *m* months. Over which interval of time is her average rate of change for the balance on her credit card account the greatest?

m	B
0	1000.00
10	1172.00
19	1352.00
36	1770.80
60	2591.90
69	2990.00
72	3135.80
73	3186.00

1) $\frac{2591.90-1172}{60-10} = 28.398$

2) $\frac{2990-1352}{69-19} = 32.76$

3) $\frac{3135.80-1770.80}{72-36} = 37.916$

4) $\frac{3186-2591.90}{73-60} = 45.7$

1) month 10 to month 60
2) month 19 to month 69
3) month 36 to month 72
4) month 60 to month 73

3 X 3 Linear Systems

Matrix Method: (Elimination will be in the equations packet)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$$

1) 2^{nd} Matrix, Edit, A, 3X3 (Coefficient of the left hand side)
2) 2^{nd} Matrix, Edit, B, 3X1 (Right hand side)
3) $A^{-1}B$

---

1. Which value is contained in the solution of the system shown below?

$2x + y - z = 1$

$x - 2y + z = 0$

$3x - y + 2z = 7$

1) 0 ~~3)~~ 2
2) -1 4) -3

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & -1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \\ 7 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

---

2. Which value is *not* contained in the solution of the system shown below?

1) −2
~~2)~~ 2
3) 3
4) −3

$a + 5b - c = -20$

$4a - 5b + 4c = 19$

$-a - 5b - 5c = 2$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 5 & -1 \\ 4 & -5 & 4 \\ -1 & -5 & -5 \end{pmatrix}^{-1} \begin{pmatrix} -20 \\ 19 \\ 2 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix}$

---

3. Which value is contained in the solution of the system shown below?

$3x + y + z = -4$

$x - 2y + z = -5$

$2x + 3y - 2z = -9$

3) -3 3) -5
~~4)~~ -4 4) -9

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & -2 & 1 \\ 2 & 3 & -2 \end{pmatrix}^{-1} \begin{pmatrix} -4 \\ -5 \\ -9 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \\ 5 \end{pmatrix}$

---

4. Which value is *not* contained in the solution of the system shown below?

$4x - 5y + 2z = 130$

$3x + 2y - 7z = -99$

$10x - 6y - 4z = 112$

1) -8 3) 10
2) -12 4) 15

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 & -5 & 2 \\ 3 & 2 & -7 \\ 10 & -6 & -4 \end{pmatrix}^{-1} \begin{pmatrix} 130 \\ -99 \\ 112 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ -12 \\ 15 \end{pmatrix}$

---

5. What is the solution of the system shown below?

$6x - 3y + 2z = 78$

$4x + 2y - 5z = -40$

$-3x - 4y - 3z = -41$

1) $x = 2,\ y = -4,\ z = 6$
~~2)~~ $x = 7,\ y = -4,\ z = 12$
3) $x = 78,\ y = -40,\ z = -41$
4) $x = 6,\ y = 2,\ z = -3$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 & -3 & 2 \\ 4 & 2 & -5 \\ -3 & -4 & -3 \end{pmatrix}^{-1} \begin{pmatrix} 78 \\ -40 \\ -41 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ -4 \\ 12 \end{pmatrix}$

---

6. For the system shown below, what is the value of z?

$y = -2x + 14$
$+2x \quad +2x$

$3x - 4z = 2$

$3x - y = 16$

$2x + y + 0z = 14$

$3x + 0y - 4z = 2$

$3x - y + 0z = 16$

1) 5 3) 6
2) 2 ~~4)~~ 4

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & 1 & 0 \\ 3 & 0 & -4 \\ 3 & -1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 14 \\ 2 \\ 16 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 4 \end{pmatrix}$

Exponents
FOLLOW THE FOLLOWING ORDER! STRUCTURE IS IMPERATIVE!!!

1) Radicals are fractional exponents (Fractional exponent = $\frac{power}{root}$)
2) Get rid of parenthesis (exponent outside parenthesis goes to everything inside)
Negative exponents are fractions (Move whatever is being raised to the negative power)
Clean it up (Multiply, divide/reduce, evaluate/put into radical)
*Add exponents when multiplying. Subtract exponents when dividing. Use a calculator for fractions.

Negative exponents are fractions! $x^{-2} = \frac{1}{x^2}$

If exponent is outside parenthesis, everything gets it $\left(\frac{xy}{z}\right)^3 = \frac{x^3y^3}{z^3}$

Rewrite the following as radicals

1. $x^{\frac{2}{3}}$ $\frac{p}{r}$

$\sqrt[3]{x^2}$

2. $x^{\frac{3}{4}}$ $\frac{p}{r}$

$\sqrt[4]{x^3}$

3. $x^{\frac{5}{6}}$ $\frac{p}{r}$

$\sqrt[6]{x^5}$

4. $x^{\frac{1}{3}}$ $\frac{p}{r}$

$\sqrt[3]{x}$

5. $x^{\frac{3}{2}}$ $\frac{p}{r}$

$\sqrt{x^3}$

6. $x^{\frac{1}{2}}$ $\frac{p}{r}$

$\sqrt{x}$

Rewrite the following using fractional exponents

7. $\sqrt[3]{x^4}$

$x^{\frac{4}{3}}$

8. $\sqrt[5]{x^3}$

$x^{\frac{3}{5}}$

9. $\sqrt[4]{x^7}$

$x^{\frac{7}{4}}$

10. $\sqrt[2]{x^3}$

$x^{\frac{3}{2}}$

11. $\sqrt[6]{x^5}$

$x^{\frac{5}{6}}$

12. $\sqrt[2]{x^1}$

$x^{\frac{1}{2}}$

13 ~~3~~. Explain what a rational exponent, such as $\frac{5}{2}$ means. Use this explanation to evaluate $9^{\frac{5}{2}}$. $\frac{\text{Power}}{\text{root}}$

Radicals are fractional exponents

$\frac{\text{power}}{\text{root}}$

$(\sqrt[2]{9})^5$

$3^5 = 243$

14 ~~6~~. Explain how $125^{\frac{4}{3}}$ can be evaluated using properties of rational exponents to result in an integer answer.

Radicals are fractional exponents

$\frac{\text{power}}{\text{root}}$

$(\sqrt[3]{125})^4$

$5^4 = 625$

15 ~~7~~. Explain how $(-8)^{\frac{4}{3}}$ can be evaluated using properties of rational exponents to result in an integer answer.

Radicals are fractional exponents

$\frac{\text{power}}{\text{root}}$

$(\sqrt[3]{-8})^4$

$(-2)^4 = 16$

16 ~~8~~. Write $\sqrt[3]{x^1} \bullet \sqrt[2]{x^1}$ as a single term with a rational exponent.

$x^{\frac{1}{3}} \cdot x^{\frac{1}{2}} = x^{\frac{5}{6}}$ $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$

17 ~~9~~. Simplify $\frac{\sqrt[3]{x^2} \bullet \sqrt[2]{x^5}}{\sqrt[6]{x^1}}$

$\frac{2}{3} + \frac{5}{2} = \frac{19}{6}$

$\frac{x^{\frac{2}{3}} \cdot x^{\frac{5}{2}}}{x^{\frac{1}{6}}}$ $\frac{x^{\frac{19}{6}}}{x^{\frac{1}{6}}} = x^3$ $\frac{19}{6} - \frac{1}{6} = 3$

1~~0~~8. Kenzie believes that for $x \geq 0$, the expression $\left(\sqrt[7]{x^2}\right)\left(\sqrt[5]{x^3}\right)$ is equivalent to $\sqrt[35]{x^6}$. Is she correct? Justify your response algebraically.

$$\frac{2}{7}+\frac{3}{5}=\frac{31}{35}$$

$$\left(x^{\frac{2}{7}}\right)\left(x^{\frac{3}{5}}\right) \neq x^{\frac{6}{35}}$$

$$x^{\frac{31}{35}} \neq x^{\frac{6}{35}}$$

No!

19 ~~11~~. Justify why $\dfrac{\sqrt[3]{x^2y^5}}{\sqrt[4]{x^3y^4}}$ is equivalent to $x^{\frac{-1}{12}}y^{\frac{2}{3}}$ using properties of rational exponents, where $x \neq 0$ and $y \neq 0$.

$$\frac{\left(x^2y^5\right)^{\frac{1}{3}}}{\left(x^3y^4\right)^{\frac{1}{4}}} = x^{-\frac{1}{12}}y^{\frac{2}{3}}$$

$$\frac{x^{\frac{2}{3}}y^{\frac{5}{3}}}{x^{\frac{3}{4}}y^{1}} = x^{\frac{-1}{12}}y^{\frac{2}{3}} \qquad \frac{2}{3}-\frac{3}{4}=\frac{-1}{12}$$

$$\frac{5}{3}-1=\frac{2}{3}$$

$$x^{-\frac{1}{12}}y^{\frac{2}{3}} = x^{-\frac{1}{12}}y^{\frac{2}{3}}$$

✓

20 ~~12~~. For n and $p > 0$, is the expression $\left(p^2n^{\frac{1}{2}}\right)^8 \sqrt[2]{p^5n^4}$ equivalent to $p^{18}n^6\sqrt{p}$? Justify your answer.

$$\left(p^2n^{\frac{1}{2}}\right)^8\left(p^5n^4\right)^{\frac{1}{2}} = p^{18}n^6p^{\frac{1}{2}}$$

$$p^{16}n^4p^{\frac{5}{2}}n^2 = p^{\frac{37}{2}}n^6$$

$$p^{\frac{37}{2}}n^6 = p^{\frac{37}{2}}n^6$$

21 ~~13~~. Use the properties of rational exponents to determine the value of y for the equation:

$\dfrac{\sqrt[3]{x^8}}{\left(x^4\right)^{\frac{1}{3}}} = x^y,\ x > 1$

$$\frac{x^{\frac{8}{3}}}{x^{\frac{4}{3}}} = x^y \qquad x^{\frac{4}{3}} = x^y$$

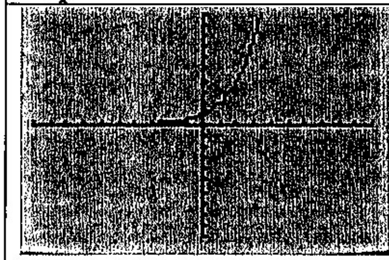
$$\frac{4}{3} = y$$

22 ~~14~~. Given that $\left(\dfrac{y^{\frac{17}{8}}}{y^{\frac{5}{4}}}\right)^{-4} = y^n$, where $y > 0$, determine the value of n.

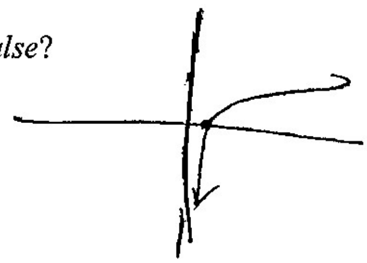
$$\frac{y^{-\frac{17}{2}}}{y^{-5}} = y^n \qquad \frac{y^5}{y^{\frac{17}{2}}} = y^n \qquad -\frac{7}{2} = n$$

$$y^{-\frac{7}{2}} = y^n$$

Graphing Exponential and Logarithmic Functions

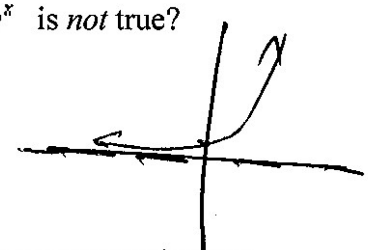
Exponential	**Logarithmic**
	
Horizontal Asymptote at $y = 0$	Vertical Asymptote at $x = 0$
Passes through $(0,1)$	Passes through $(1,0)$
Domain is all real numbers	Domain is all positive real numbers
Range is all positive real numbers	Range is all real numbers
Exponents and logarithms are inverses of each other!!!!!!!!!!!	

1. Which statement about the graph of $c(x) = \log_6 x$ is *false*?



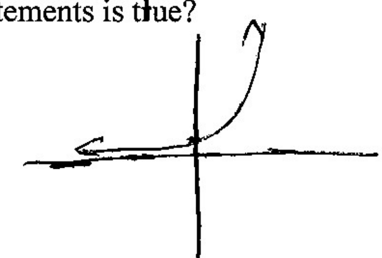
1) The asymptote has equation $y = 0$. $x=0$
2) The graph has no y-intercept.
3) The domain is the set of positive reals.
4) The range is the set of all real numbers.

2. Which statement about the graph of the equation $y = e^x$ is *not* true?



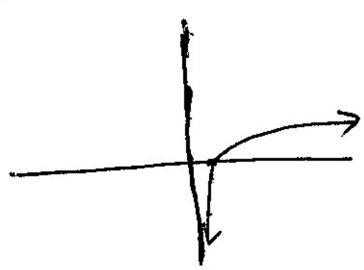
1) It is asymptotic to the x-axis.
2) The domain is the set of all real numbers.
3) It lies in Quadrants I and II.
4) It passes through the point $(e, 1)$. $(0,1)$

3. Given the equation $f(x) = \pi^x$, which of the following statements is true?



1) The graph passes through $(\pi,1)$
2) The domain is $[0,\infty)$
3) The graph passes through $(0,1)$
4) The range is all real numbers

4. Which statement is ~~false~~ true regarding the equation $f(x) = \log_a x$?



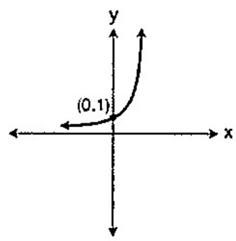
1) The range is $[0,\infty)$ $(-\infty,\infty)$
2) The graph passes through $(0,1)$ $(1,0)$
3) The domain is all real numbers $(0,\infty)$
4) The equation of the asymptote is $x = 0$ ✓

logs and exponential are inverses of each other

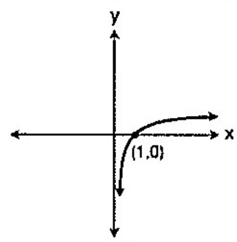
5. What is the inverse of the function $y = \log_3 x$? $y=3^x$

1) $y = x^3$ 2) $y = \log_x 3$ 3) $y = 3^x$ 4) $x = 3^y$

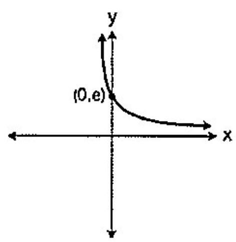
6. Which graph shows the inverse of $y = e^x$? $y = \ln x$



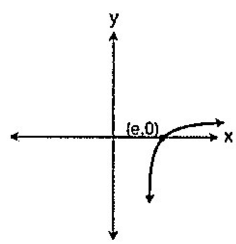
(1)



(3)



(2)



(4)

7. If $f(x) = a^x$ where $a > 1$, then the inverse of the function is

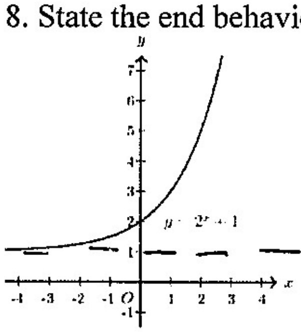
1) $f^{-1}(x) = \log_x a$

2) $f^{-1}(x) = a \log x$

3) $f^{-1}(x) = \log_a x$

4) $f^{-1}(x) = x \log a$

8. State the end behavior of the function below.



$x \to -\infty, f(x) \to 1$

$x \to \infty, f(x) \to \infty$

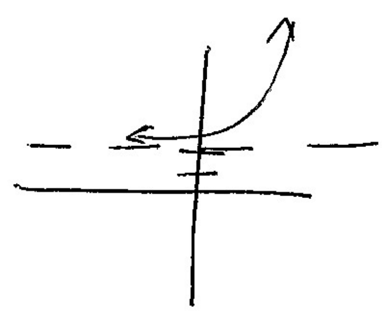
9. Given $f(x) = 3^{x-1} + 2$, as $x \to -\infty$

1) $f(x) \to -1$

2) $f(x) \to 0$

3) $f(x) \to 2$

4) $f(x) \to -\infty$



Graphing Functions

1) Type equation into $Y =$
2) 2nd Graph (Table)
   *Plot points in given domain or that fit on the given graph

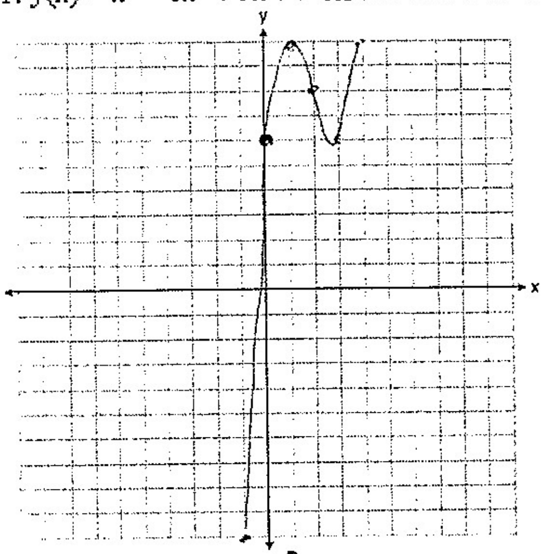
- Domain: no arrows. No domain: arrows.

Exponential and Logarithmic Graphs should include asymptotes.

Graph the following equations (Include domain and asymptotes if necessary)

1. $f(x) = x^3 - 6x^2 + 9x + 6$ on the domain $-1 \le x \le 4$.

X	y
-1	-10
0	6
1	10
2	8
3	6
4	10

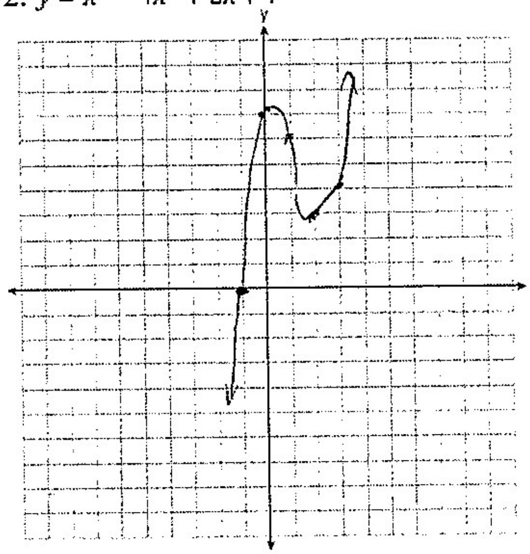


$x \to -\infty,\ f(x) \to -\infty$

$x \to \infty,\ f(x) \to \infty$

2. $y = x^3 - 4x^2 + 2x + 7$

X	y
-1	0
0	7
1	6
2	3
3	4



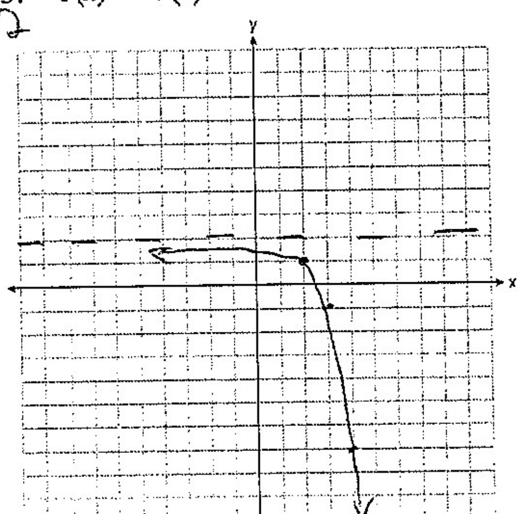
$x \to -\infty,\ f(x) \to -\infty$

$x \to \infty,\ f(x) \to \infty$

3. $c(x) = -9(3)^{x-4} + 2$

$y=2$

X	y
	2
	2
	2
	2
	2
2	1
3	-1
4	-7



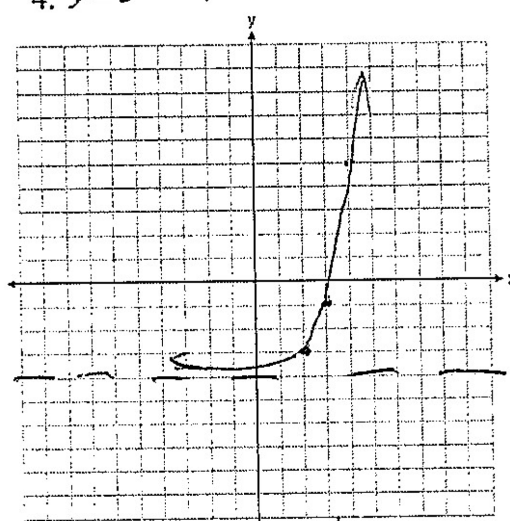
$x \to -\infty,\ f(x) \to 2$

$x \to \infty,\ f(x) \to -\infty$

4. $y = 3^{x-2} - 4$

$y=-4$

X	y
	-4
	-4
	-4
	-4
2	-3
3	-1
4	5



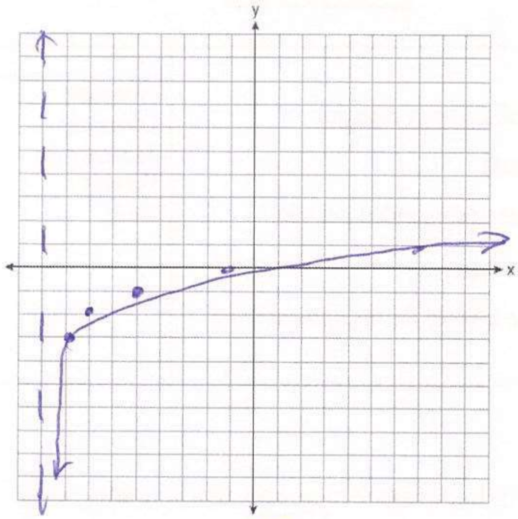
$x \to -\infty,\ f(x) \to -4$

$x \to \infty,\ f(x) \to \infty$

$x = -9$

x	y
-9	Error
-8	-3
-7	-2
-5	-1
-1	0
7	1

5. $y = \log_2(x+9) - 3$



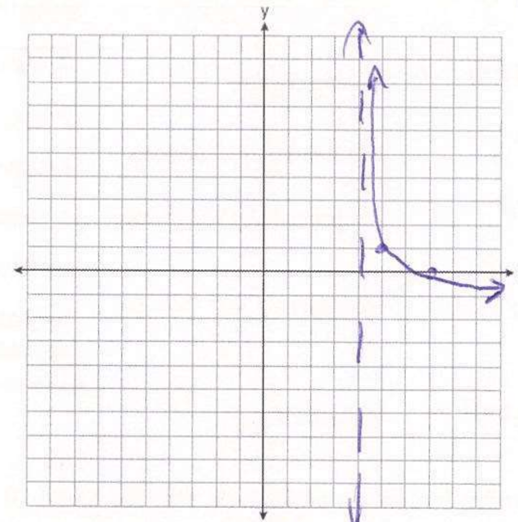
$x \to -9,\ f(x) \to -\infty$

$x \to \infty,\ f(x) \to \infty$

6. $y = -\log_3(x-4) + 1$

$x = 4$

x	y
4	Error
5	1
7	0



$x \to 4,\ f(x) \to \infty$

$x \to \infty,\ f(x) \to -\infty$

7. On the set of axes below, graph $y = f(x)$ and $y = g(x)$ for the given functions.

$$f(x) = x^3 - 3x^2$$

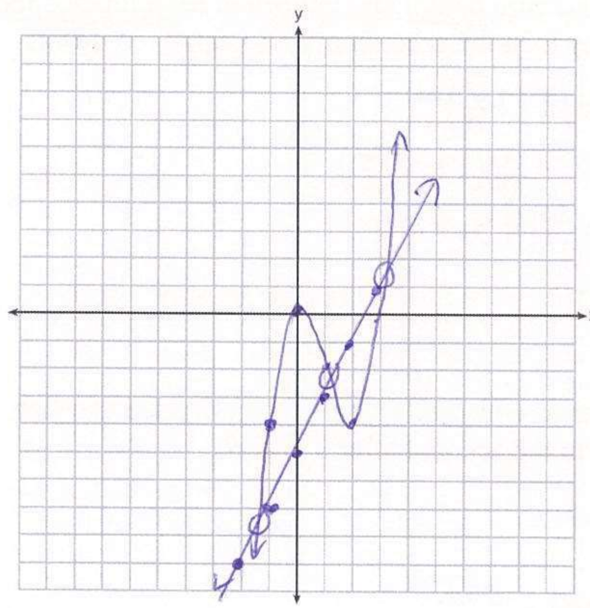
$$g(x) = 2x - 5$$

$f(x)$

x	y
-1	-4
0	0
1	-2
2	-4
3	0

$g(x)$

x	y
-2	-9
-1	-7
0	-5
1	-3
2	-1
3	1



State the number of solutions to the equation $f(x) = g(x)$.

3

Modeling Exponential Functions:

$A = P(1 \pm r)^t$	Nothing Below!
$A = P\left(1+\frac{r}{n}\right)^{nt}$	Compounding (Not Continuous)
$A = Pe^{rt}$	Compounding Continuously
$A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$	Half Life
$A = P(1 \pm r)^{\frac{t}{h}}$	Irregular Time

		n
A = after amount		
P = principal (initial/starting) amount	Annually	1
r = rate (as a decimal)	Quarterly	4
n = number of times compounded per year	Monthly	12
t = time (that is passing)	Weekly	52
h = half life or time it takes for the percent to be applied	Daily	365

1. Cassandra bought an antique dresser for $500. If the value of her dresser increases 6% annually, what will be the value of Cassandra's dresser at the end of 3 years to the *nearest dollar*?

P

$A = A$
$P = 500$
$r = .06$
$t = 3$

$A = P(1 \pm r)^t$
$A = 500(1+.06)^3$
$A = 596$

2. A bank account is opened with $3000 and interest is compounded monthly at an interest rate of 4.2%. How much money is in the account after 8 years?

P n=12 n formula

$A = A$
$P = 3000$
$r = .042$
$n = 12$
$t = 8$

$A = P(1+\frac{r}{n})^{nt}$
$A = 3000(1+\frac{.042}{12})^{12(8)}$
$A = 4195.56$

3. If a bank account is opened with $4000 and is compounded at a rate of 5.2% continuously, how much money will be in the account after 3 years?

P Pert

$A = A$
$P = 4000$
$r = .052$
$t = 3$

$A = Pe^{rt}$
$A = 4000e^{.052(3)}$
$A = 4675.30$

4. The half-life of mendelevium-258 is 51.5 days. To the *nearest hundredth of a gram*, how much mendelevium-258 will remain after 12 days?

of a 4000 gram sample

$A = A$

$P = 4000$

$t = 12$

$h = 51.5$

$A = P(\frac{1}{2})^{\frac{t}{h}}$

$A = 4000(\frac{1}{2})^{\frac{12}{51.5}}$

$A = 3403.43$

5. Phil is trying to get himself back into shape and wants to ease his way back into distance running. He will start by running 2 miles each day but every four days, he will increase his distance by 60%. How many miles will Phil be running after 10 days rounded to the *nearest mile*?

illegular time

$A = A$

$P = 2$

$r = .60$

$t = 10$

$h = 4$

$A = P(1 \pm r)^{\frac{t}{h}}$

$A = 2(1 + .60)^{\frac{10}{4}}$

$A = 6$

6. Sal has a savings account. He opened the account 6 years ago by putting in $3000. If the interest is compounded daily at a rate of 5.6%, how much money is in the account now?

n-formula

$A = A$

$P = 3000$

$r = .056$

$n = 365$

$t = 6$

$A = P(1 + \frac{r}{n})^{nt}$

$A = 3000(1 + \frac{.056}{365})^{365(6)}$

$A = 4197.91$

7. The amount of ants in a colony doubles every 8 days. If there are initially 275 ants, how many ants, to the nearest ant, will be in the colony after 30 days?

illegular h

$A = A$

$P = 275$

$t = 30$

$h = 8$

$A = P(2)^{\frac{t}{h}}$

$A = 275(2)^{\frac{30}{8}}$

$A = 3700$ ants

8. How much money is in a bank account opened 7.5 years ago with $3125.67 that is compounded continuously with an interest rate of 5.26%?

t

Pert

$A = A$

$P = 3125.67$

$r = .0526$

$t = 7.5$

$A = Pe^{rt}$

$A = 3125.67e^{.0526(7.5)}$

$A = 4636.38$

9. A certain car depreciates at a rate of 15% each year. If the car was initially worth $8125, what is the value of the car, rounded to the nearest cent, 11 years later?

p

t

$A = A$

$P = 8125$

$r = .15$

$t = 11$

$A = P(1 \pm r)^t$

$A = 8125(1 - .15)^{11}$

$A = 1359.66$

10. The half life of an element is 27 hours. If there were initially 4.2 kg of the substance, how much will remain after 2 days? Round your answer to the nearest hundredth of a kg.

h p

t=48

$A = A$

$P = 4.2$

$t = 48$

$h = 27$

$A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$

$A = 4.2\left(\frac{1}{2}\right)^{\frac{48}{27}}$

$A = 1.22$ Kg

11. The population of Schlansky, Arizona increases by 18% every 3.2 years. If the population is currently 2750, what will be the population, to the nearest person, 12 years from now?

h illegular time

P t

$A = A$

$P = 2750$

$r = .18$

$t = 12$

$h = 3.2$

$A = P(1 \pm r)^{\frac{t}{h}}$

$A = 2750(1 + .18)^{\frac{12}{3.2}}$

$A = 5,116$

Equivalent Exponential Forms (Absorbing the Exponent)

If you have a value in the exponent, absorb it into the parenthesis.

To interpret an exponential function, the initial value is in front of the parenthesis and $(1 \pm rate)$ is what is inside the parenthesis. If it is less than 1, it is decreasing. If it is more than 1, it is increasing.

Express each of the following functions with an exponent of *t*. Round values to the nearest thousandth.

1. $A = 12{,}000(1.025)^{12t}$

$A = 12{,}000(1.025^{12})^t$

$A = 12{,}000(1.345)^t$

2. $A = 17{,}000(.889)^{9.4t}$

$A = 17{,}000(.889^{9.4})^t$

$A = 17{,}000(.331)^t$

3. $A = 11{,}185(.764)^{\frac{t}{12}}$

$A = 11{,}185(.764^{\frac{1}{12}})^t$

$A = 11{,}185(.978)^t$

4. $A = 125{,}000(.785)^{\frac{t}{4}}$

$A = 125{,}000(.785^{\frac{1}{4}})^t$

$A = 125{,}000(.941)^t$

5. Iridium-192 is an isotope of iridium and has a half-life of 73.83 days. If a laboratory experiment begins with 100 grams of Iridium-192, the number of grams, A, of Iridium-192 present after t days would be $A = 100\left(\frac{1}{2}\right)^{\frac{t}{73.83}}$. Which equation approximates the amount of Iridium-192 present after t days?

1) $A = 100\left(\frac{73.83}{2}\right)^t$

2) $A = 100\left(\frac{1}{147.66}\right)^t$

3) $A = 100(0.990656)^t$

4) $A = 100(0.116381)^t$

$A = 100(\frac{1}{2}^{\frac{1}{73.83}})^t$

$A = 100(.990656)^t$

6. The amount of a substance, $A(t)$, that remains after t days can be given by the equation $A(t) = A_0(0.5)^{\frac{t}{0.0803}}$, where A_0 represents the initial amount of the substance. An equivalent form of this equation is

1) $A(t) = A_0(0.000178)^t$

2) $A(t) = A_0(0.945861)^t$

3) $A(t) = A_0(0.04015)^t$

4) $A(t) = A_0(1.08361)^t$

$A(t) = A_0(0.5^{\frac{1}{.0803}})^t$

$A(t) = A_0(.000178)^t$

1.78E-4 means

1.78×10^{-4}

$A=220\left(\frac{1}{2}^{\frac{1}{12}}\right)^t \quad A=220(.94387..)^t$

7. A certain pain reliever is taken in 220 mg dosages and has a half-life of 12 hours. The function $A = 220\left(\frac{1}{2}\right)^{\frac{t}{12}}$ can be used to model this situation, where A is the amount of pain reliever in milligrams remaining in the body after t hours. According to this function, which statement is true? decreases by ≈6%.

1) Every hour, the amount of pain reliever remaining is cut in half.
2) In 12 hours, there is no pain reliever remaining in the body. never 0
3) In 24 hours, there is no pain reliever remaining in the body. never 0
4) In 12 hours, 110 mg of pain reliever is remaining. $A=220(.94387..)^{12} \approx 110$

8. An equation to represent the value of a car after t months of ownership is $v = 32,000(0.81)^{\frac{t}{12}}$. Which statement is *not* correct?

$V=32000(.81^{\frac{1}{12}})^t$

$V=32,000(.98259..)^t$

1) The car lost approximately 19% of its value each month. ≈2%
2) The car maintained approximately 98% of its value each month.
3) The value of the car when it was purchased was \$32,000.
4) The value of the car 1 year after it was purchased was \$25,920. $V=32,000(.98259..)^{1} \approx 25,920$

9. The value of an investment account, $v(t)$, can be modeled by the equation $v(t) = 500(1.15)^{3.2t}$ after t years. Which of the following statements must be true?

$V(t)=500(1.15^{3.2})^t$

$V(t)=500(1.56..)^t$

1) The account is increasing approximately 15% each year.
2) The account is increasing approximately 56% each year
3) There will be \$1216.80 in the account after two years $V(2)=500(1.56..)^2=1223...$
4) It will take 3.68 years for the account to double $500(1.56)^{3.68}$ = 2592 more than doubled

10. The amount of a substance, $A(t)$, in grams, remaining after t days is modeled by $A(t) = 50(0.5)^{\frac{t}{3}}$. Which statement is false?

$A(t)=50(.5^{\frac{1}{3}})^t$

$A(t)=50(.793..)^t$

1) In 20 days, there is no substance remaining. never none remaining
2) After two half-lives, there is 25% of the substance remaining.
3) The amount of the substance remaining can also be modeled by $A(t) = 50(2)^{\frac{-t}{3}}$. $50(2^{\frac{-1}{3}})^t \quad 50(.793..)^t$
4) After one week, there is less than 10g of the substance remaining.

$A(7)=50(.793..)^7=9.92..$

11. If $f(t) = 50(.5)^{\frac{t}{5715}}$ represents a mass, in grams, of carbon-14 remaining after t years, which statement(s) must be true?

I. The mass of the carbon-14 is decreasing by half each year. $f(t)=50(.5^{\frac{1}{5715}})^t$
II. The mass of the original sample is 50 g.

$f(t)=50(.99987..)^t$

1) I, only
2) II, only
3) I and II
4) neither I nor II

Converting Rates (For example, from annual to monthly)

1) Start with $A = P(1 \pm r)^t$ (If it is not given in that form)

2) Raise $(1 \pm r)^{\frac{1}{n}}$ to find the new rate (Subtract the one and multiply by 100 to find the percent rate)

3) -If the variable is the original unit (years for example), the exponent is *nt*.
-If the variable matches the unit (months for example), then the exponent is *m*.

1. Stephanie found that the number of white-winged cross bills in an area can be represented by the formula $C = 550(1.08)^t$, where *t* represents the number of years since 2010. Which equation correctly represents the number of white-winged cross bills in terms of the monthly rate of population growth?

1) $C = 550(1.00643)^t$
3) $C = 550(1.00643)^{\frac{t}{12}}$
2) $C = 550(1.00643)^{12t}$
4) $C = 550(1.00643)^{t+12}$

Monthly rate 12 times per year

$1.08^{\frac{1}{12}}$
1.00643

2. On average, college seniors graduating in 2012 could compute their growing student loan debt using the function $D(t) = 29,400(1.068)^t$, where *t* is time in years. Which expression is equivalent to $29,400(1.068)^t$ and could be used by students to identify an approximate daily interest rate on their loans?

1) $29,400\left(1.068^{\frac{1}{365}}\right)^t$
3) $\cancel{29,400\left(1 + \frac{0.068}{365}\right)^t}$
2) $\cancel{29,400\left(\frac{1.068}{365}\right)^{365t}}$
4) $29,400\left(1.068^{\frac{1}{365}}\right)^{365t}$

daily rate 365 times per year.

$1.068^{\frac{1}{365}}$

3. The value of a stock after t years can be modeled by the function $V = 2500(1.14)^t$ after *t* years. Which function would represent the weekly rate of increase after *w* weeks?

1) $V = 2500(1.14)^w$
3) $V = 2500(1.0025)^w$
2) $V = 2500(1.14)^{52w}$
4) $V = 2500(1.0025)^{52w}$

$1.14^{\frac{1}{52}}$
1.0025

Weekly rate 1 time per week

4. The value of a home after t years can be modeled by the function $A = 525000(1.36)^t$ after *t* years. Which function would represent the monthly rate of increase after *m* ~~weeks~~ months?

2) $A = 525000(1.36)^m$
3) $A = 525000(1.026)^m$
2) $A = 525000(1.36)^{12m}$
4) $A = 525000(1.026)^{12m}$

$1.36^{\frac{1}{12}}$
1.026

Monthly rate 1 time per month

5. A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, $B(t)$, can be represented by the function $B(t) = 750(1.16)^t$, where the t represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function

1) $B(t) = 750(1.012)^t$

3) $B(t) = 750(1.012)^{12t}$ monthly rate 12 times per year

2) $B(t) = 750(1.16)^{12t}$

4) $B(t) = 750(1.16)^{\frac{t}{12}}$

$1.16^{\frac{1}{12}}$

1.012

6. Mia has a student loan that is in deferment, meaning that she does not need to make payments right now. The balance of her loan account during her deferment can be represented by the function $f(x) = 35,000(1.0325)^x$, where x is the number of years since the deferment began. If the bank decides to calculate her balance showing a monthly growth rate, an approximately equivalent function would be

1) $f(x) = 35,000(1.0027)^{12x}$ monthly rate 12 times per year

3) $f(x) = 35,000(1.0325)^{12x}$

2) $f(x) = 35,000(1.0027)^{\frac{x}{12}}$

4) $f(x) = 35,000(1.0325)^{\frac{x}{12}}$

$1.0325^{\frac{1}{12}}$

1.0027

7. A student studying public policy created a model for the population of Detroit, where the population decreased 25% over a decade. He used the model $P = 714(0.75)^d$, where P is the population, in thousands, d decades after 2010. Another student, Suzanne, wants to use a model that would predict the population after y years. Suzanne's model is best represented by

1) $P = 714(0.6500)^y$

3) $P = 714(0.9716)^y$ yearly rate 1 time per year

2) $P = 714(0.8500)^y$

4) $P = 714(0.9750)^y$

$.75^{\frac{1}{10}}$

$.9716$

$1+.0715=1.0715$

8. Each year, the amount of students in Eastbury High School increases by 7.15%. Which of the following expressions could be used to find the *weekly* rate of increase of Eastbury High School after w weeks?

1) $\cancel{(1.0715)^w}$

2) $\cancel{(1.0715)^{52w}}$

$1.0715^{\frac{1}{52}}$

3) $(1.0715^{\frac{1}{52}})^{52w}$

4) $(1.0715^{\frac{1}{52}})^{w}$ weekly rate 1 time per week

9. Each year, the amount of students in Eastbury High School increases by 7.15%. Which of the following expressions could be used to find the *weekly* rate of increase of Eastbury High School after t years?

1) $(1.0715^{\frac{1}{52}})^t$

2) $(1.0715^{\frac{1}{52}})^{52t}$ weekly rate 52 times per year

3) $\cancel{(1.0715)^{52t}}$

4) $\cancel{(1.0715)^t}$

$1.0715^{\frac{1}{52}}$

$1+.0525=1.0525$

10. Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let *m* represent months.]

1) $(1.0525)^m$ 3) $(1.00427)^m$ monthly rate 1 time per month

2) $(1.0525)^{\frac{12}{m}}$ 4) $(1.00427)^{\frac{m}{12}}$

$1.0525^{\frac{1}{12}}$

1.00427

$1+.092=1.092$

11. Rasmus invested $65,000 in the stock market and makes an average of 9.2% each year on his investments. Which equation could be used to find his monthly percent increase after *t* years?

1) $v=65000(1.092)^t$ 3) $v=65000(1.0074)^t$

2) $v=65000(1.0074)^{12t}$ 4) $v=65000(1.092)^{12t}$

monthly rate 12 times per year

$1.092^{\frac{1}{12}}$

1.0074

$1+.032=1.032$

12. Over the past several years, the value of a stock has increased by 3.2% each year. The value of the stock is now $87.24. Which of the following equations does not represent the value of the stock after *t* years or *m* months?

1) $a(t)=87.24(1.032)^t$ ✓ 3) ~~$a(m)=87.24(1.0026)^{12m}$~~

2) $a(t)=87.24(1.0026)^{12t}$ ✓ 4) $a(m)=87.24(1.0026)^m$ ✓

you don't get the monthly rate 12 times per month

$1.032^{\frac{1}{12}}$

1.0026

13. According to the USGS, an agency within the Department of Interior of the United States, the frog population in the U.S. is decreasing at the rate of 3.79% per year. A student created a model, $P=12,150(0.962)^t$, to estimate the population in a pond after *t* years. The student then created a model that would predict the population after *d* decades. This model is best represented by

longer time period

1) $P=12,150(0.461)^d$ 3) $P=12,150(0.996)^d$

2) $P=12,150(0.679)^d$ 4) $P=12,150(0.998)^d$

decade rate 1 time per decade

$.962^{10}$

$.6788$

Sequences:
Arithmetic: add a constant difference, Geometric: multiply by a common ratio
Explicit Formulas (From Reference Sheet)

Arithmetic: $a_n = a_1 + (n-1)d$ Geometric: $a_n = a_1(r)^{n-1}$

If initial or a_0 is given, $(n-1)$ becomes *n*. Same formulas as Algebra I modeling.

Arithmetic: $a_n = a_0 + nd$ Geometric: $a_n = a_0(r)^n$

Recursive Formula

$a_1 =$

$a_n = a_{n-1}$

Write an explicit AND recursive equation for the following sequences and find the tenth term.
Round to the nearest tenth if necessary

1. 19, 16, 13, 10 … arithmetic
-3 -3 -3
explicit
$a_n = a_1 + (n-1)d$
$a_n = 19 + (n-1)(-3)$
$a_1 = 19 - 3n + 3$
$a_n = -3n + 22$
recursive
$a_1 = 19$
$a_n = a_{n-1} - 3$
$a_{10} = -3(10) + 22$
$a_{10} = -8$

2. 2,8,32,128,… geometric
·4 ·4 ·4
explicit
$a_n = a_1(r)^{n-1}$
$a_n = 2(4)^{n-1}$
recursive
$a_1 = 2$
$a_n = 4a_{n-1}$
$a_{10} = 2(4)^{10-1}$
$a_{10} = 524288$

3. 3, -12, 48, -192, ... geometric
·-4 ·-4 ·-4
explicit
$a_n = a_1(r)^{n-1}$
$a_n = 3(-4)^{n-1}$
recursive
$a_1 = 3$
$a_n = -4a_{n-1}$
$a_{10} = 3(-4)^{10-1}$
$a_{10} = -786432$

4. 63, 57, 51, 45, ... arithmetic
-6 -6 -6
explicit re
$a_n = a_1 + (n-1)d$
$a_n = 63 + (n-1)(-6)$
$a_n = 63 - 6n + 6$
$a_n = -6n + 69$

5. 329.6, 376.8, 424, 471.2,…
+47.2 +47.2
$376.8 - 329.6 = 47.2$
$424 - 376.8 = 47.2$
arithmetic
explicit
$a_n = a_1 + (n-1)d$
$a_n = 329.6 + (n-1)47.2$
$a_n = 329.6 + 47.2n - 47.2$
$a_n = 47.2n + 282.4$
recursive
$a_1 = 329.6$
$a_n = a_{n-1} + 47.2$
$a_{10} = 47.2(10) + 282.4$
$a_{10} = 754.4$

6. 120, 192, 307.2, 491.52
·1.6 ·1.6
explicit
$a_n = a_1(r)^{n-1}$
$a_n = 120(1.6)^{n-1}$
recursive
$a_1 = 120$
$a_n = 1.6a_{n-1}$
$a_{10} = 120(1.6)^{10-1}$
$a_{10} = 8246.3$
$\frac{192}{120} = 1.6$
$\frac{307.2}{192} = 1.6$
geometric

7. 5400, 4050, 3037.5, 2278.125
·.75 ·.75
$\frac{4050}{5400} = .75$
$\frac{3037.5}{4050} = .75$
geometric
explicit
$a_n = a_1(r)^{n-1}$
$a_n = 5400(.75)^{n-1}$
recursive
$a_1 = 5400$
$a_n = .75a_{n-1}$
$a_{10} = 5400(.75)^{10-1}$
$a_{10} = 405.5$

8. 5205.20, 4208.15, 3211.1, 2214.05
-997.05 -997.05
explicit
$a_n = a_1 + (n-1)d$
$a_n = 5205.20 + (n-1)(-997.05)$
$a_n = 5205.20 - 997.05n + 997.05$
$a_n = -997.05n + 6202.25$
recursive
$a_1 = 5205.20$
$a_n = a_{n-1} - 997.05$
$4208.15 - 5205.20$
-997.05
$3211.1 - 4208.15$
-997.05
arithmetic
$a_{10} = -997.05(10) + 6202.25$
$a_{10} = -3768.25$

9. The values below represent the cost of an ice cream sundae with one through four toppings.

$4.75 $5.50 $6.25 $7.00

Write an explicit and recursive function that can be used to determine the cost of an ice cream cone with n toppings.

$5.50 - 4.75 = .75$

$6.25 - 5.50 = .75$

arithmetic

explicit

$a_n = a_1 + (n-1)d$

$a_n = 4.75 + (n-1).75$

recursive

$a_1 = 4.75$

$a_n = a_{n-1} + .75$

arithmetic

10. A theater with 15 rows has 10 seats in the first row, 12 seats in the second row, 14 seats in the third row, and so on. Write an explicit and recursive formula that can be used to determine the number of seats in the nth row of the theater.

+2 +2

explicit

$a_n = a_1 + (n-1)d$

$a_n = 10 + (n-1)2$

recursive

$a_1 = 10$

$a_n = a_{n-1} + 2$

11. Dana began an exercise program using a FitBit to measure her distance walked on her treadmill, in miles, per week. The following table shows her progress over three weeks.

$\frac{11.7}{9} = 1.3$

Week	1	2	3
Distance Walked on Treadmill (miles)	9	11.7	15.21

$\frac{15.21}{11.7} = 1.3$

geometric

If she continues to progress in this manner, which of the listed statements could model the number of miles Dana walks on her treadmill, a_n, in terms of n, the number of weeks?

1) $a_n = 9(1.3)^n$

2) $a_n = 9 + 2.7(n-1)$

3) $a_1 = 9$, $a_n = 1.3a_{n-1}$

4) $a_1 = 9$, $a_n = 2.7 + a_{n-1}$

12. The population of Jamesburg for the years 2010-2013, respectively, was reported as follows:
250,000 250,937 251,878 252,822

How can this sequence be recursively modeled?

1) $j_n = 250,000(1.00375)^{n-1}$

2) $j_n = 250,000 + 937^{n-1}$

not recursive

3) $j_1 = 250,000$, $j_n = 1.00375j_{n-1}$

4) $j_1 = 250,000$, $j_n = j_{n-1} + 937$

$\frac{250937}{250000} = 1.00375$

$\frac{251878}{250937} = 1.00375$

geometric

Name Schlansky
Mr. Schlansky

Date ____________
Algebra II

Evaluating Recursive Sequences

1. Find the first 4 terms of the sequence $a_n = a_{n-1} + 4$ where $a_1 = -1$.

$a_2 = a_1 + 4$　　$a_3 = a_2 + 4$　　$a_4 = a_3 + 4$　　$-1, 3, 7, 11$

$a_2 = -1 + 4$　　$a_3 = 3 + 4$　　$a_4 = 7 + 4$

$a_2 = 3$　　$a_3 = 7$　　$a_4 = 11$

2. Find the first 4 terms of the sequence $a_n = 4a_{n-1}$ where $a_1 = 12$.

$12, 48, 192, 768$

$a_2 = 4a_1$　　$a_3 = 4a_2$　　$a_4 = 4a_3$

$a_2 = 4(12)$　　$a_3 = 4(48)$　　$a_4 = 4(192)$

$a_2 = 48$　　$a_3 = 192$　　$a_4 = 768$

$-3, 13, -35, 109$

3. Find the first four terms of the recursive sequence $a_1 = -3$, $a_n = 4 - 3a_{n-1}$

$a_2 = 4 - 3a_1$　　$a_3 = 4 - 3a_2$　　$a_4 = 4 - 3a_3$

$a_2 = 4 - 3(-3)$　　$a_3 = 4 - 3(13)$　　$a_4 = 4 - 3(-35)$

$a_2 = 13$　　$a_3 = -35$　　$a_4 = 109$

4. If $a_n = 3a_{n-1} - 4$ and $a_1 = 9$, find a_5

$a_2 = 3a_1 - 4$　　$a_3 = 3a_2 - 4$　　$a_4 = 3a_3 - 4$　　$a_5 = 3a_4 - 4$

$a_2 = 3(9) - 4$　　$a_3 = 3(23) - 4$　　$a_4 = 3(65) - 4$　　$a_5 = 3(191) - 4$

$a_2 = 23$　　$a_3 = 65$　　$a_4 = 191$　　$a_5 = 569$

5. Find the 8[th] term for the sequence where $a_n = 5a_{n-1} + 2n$ where $a_5 = 3$

$a_6 = 5a_5 + 2(\underline{6})$　　$a_7 = 5a_6 + 2(\underline{7})$　　$a_8 = 5a_7 + 2(\underline{8})$

$a_6 = 5(3) + 12$　　$a_7 = 5(27) + 14$　　$a_8 = 5(149) + 16$

$a_6 = 27$　　$a_7 = 149$　　$a_8 = 761$

6. Find the first four terms of the recursive sequence defined below.

$a_1 = -3$

$a_n = a_{(n-1)} - n$

$a_2 = a_1 - 2 \quad a_3 = a_2 - 3 \quad a_4 = a_3 - 4$

$a_2 = -3-2 \quad a_3 = -5-3 \quad a_4 = -8-4$

$a_2 = -5 \quad a_3 = -8 \quad a_4 = -12$

$-3, -5, -8, -12$

7. A sequence is defined recursively by $f(1) = 16$ and $f(n) = f(n-1) + 2n$. Find $f(4)$.

(1) 32 (2) 30 (3) 28 ~~(4)~~ 34

$f(2) = f(1) + 2(2) \quad f(3) = f(2) + 2(3) \quad f(4) = f(3) + 2(4)$

$f(2) = 16+4 \quad f(3) = 20+6 \quad f(4) = 26+8$

$f(2) = 20 \quad f(3) = 26 \quad f(4) = 34$

8. Find the third term in the recursive sequence $a_{k+1} = 2a_k - 1$, where $a_1 = 3$.

$a_2 = 2a_1 - 1 \quad a_3 = 2a_2 - 1$

$a_2 = 2(3) - 1 \quad a_3 = 2(5) - 1$

$a_2 = 5 \quad a_3 = 9$

$\cdot 2, +1$

9. Which recursively defined function represents the sequence 3, 7, 15, 31, ...?

1) $f(1) = 3,\ f(n+1) = 2^{f(n)} + 3$

2) $f(1) = 3,\ f(n+1) = 2^{f(n)} - 1$

~~3)~~ $f(1) = 3,\ f(n+1) = 2f(n) + 1$

4) $f(1) = 3,\ f(n+1) = 3f(n) - 2$

10. What is the fourth term of the sequence defined by $a_1 = 3xy^5$

$$a_n = \left(\frac{2x}{y}\right)a_{n-1}?$$

1) $12x^3y^3$

2) $24x^2y^4$

~~3)~~ $24x^4y^2$

4) $48x^5y$

$a_2 = \left(\frac{2x}{y}\right)a_1 \quad a_3 = \left(\frac{2x}{y}\right)(a_2) \quad a_4 = \left(\frac{2x}{y}\right)a_3$

$a_2 = \frac{2x}{y}(3xy^5) \quad a_3 = \left(\frac{2x}{y}\right)(6x^2y^4) \quad a_4 = \left(\frac{2x}{y}\right)(12x^3y^3)$

$a_2 = 6x^2y^4 \quad a_3 = 12x^3y^3 \quad a_4 = 24x^4y^2$

Modeling Sequences

***r* is the common ratio (what you're multiplying by). If you're increasing or decreasing by a percent,** *common ratio* $= 1 \pm$ *rate*.

For example: Increases by 12% each year, common ratio is $1 + .12 = 1.12$
Decreases by 20% each year, common ratio is $1 - .20 = .80$

(Refer back to previous page for formulas)

1. The formula below can be used to model which scenario?

$$a_1 = 3000$$

$$a_n = 0.80a_{n-1}$$ 1−.2 decrease by 20%

1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
3) A bank account starts with a deposit of \$3000, and each year it grows by 80%.
4) The initial value of a specialty toy is \$3000, and its value each of the following years is 20% less.

add 20, no percent

2. Which situation *cannot* be modeled by the formula $f(n) = f(n-1) + 20$ with $f(1) = 10$?

1) Nancy put \$10 in her piggy bank on the first day and then added \$20 daily to her piggy bank.
2) Jay has a box of ten crayons and his teacher gives him twenty new crayons each month for good behavior.
3) Buzz has ten apples and that number increases by 20% per week.
4) Teresa has a block of metal that is 10°F and she heats it up at a rate of 20°F per minute.

1−.25, decrease by 25%.

3. The sequence defined by $r_1 = 15$ and $r_n = 0.75r_{n-1}$ best models which scenario?

1) Gerry's \$15 allowance is increased by \$0.75 each week.
2) A store that has not sold a \$15 item reduces the price by \$0.25 each week until someone purchases it.
3) A 15-gram sample of a chemical compound decays at a rate of 75% per hour.
4) A picture with an area of 15 square inches is reduced by 25% over and over again to make a proportionally smaller picture.

subtract 6, no percent

4. Which situation ~~cannot~~ can be modeled by the formula $a_n = a_{n-1} - 6$ with $a_0 = 1000$?

1) A bank account with an initial balance of \$1000 increases by 6% each year.
2) Taylor is assigned 1000 SAT problems and completes 6 each day.
3) The starting population of fish in a pond is 1000 and the population decreases by 6% each day.
4) Jessica has \$1000 saved and saves an additional \$6 each week.

5. The first day the MathSchlansky posts a video to his YouTube channel, it receives 4 views. Each day after that, the number of views increases by 7%. Which sequence can be used to determine the number of views his video receives after n days? $1+.07=1.07$

(1) $a_1 = 4$
$a_n = a_{n-1} + 7$

(2) $a_1 = 4$
$a_n = a_{n-1} + 1.07$

(3) $a_1 = 4$
$a_n = .07a_{n-1}$

(4) $a_1 = 4$
$a_n = 1.07a_{n-1}$

6. The height of Jenny's sunflower when she planted it was 6 inches. The sunflower grows by 0.25 inches per day. Which formula can be used to determine the height, in inches, of Jenny's sunflower on day n?

$+.25$

(1) $h_0 = 6$
$h_n = 0.25a_{n-1}$

(2) $h_0 = 6$
$h_n = 6 + 0.25h_{n-1}$

(3) $h_0 = 6$
$h_n = h_{n-1} + 0.25$

(4) $h_0 = 6$
$h_n = 6h_{n-1} + 0.25$

$.3$

7. A population of bacteria triples every day. If on the first day there are 300 bacteria in a Petri dish, which recursive sequence can be used to determine the population on day n?

1) $b_1 = 300$
$b_n = 3b_{n-1}$

2) $b_1 = 300$
$b_n = b_{n-1} + 3$

3) $b_1 = 300$
$b_n = 300(3b_{n-1})$

4) $b_1 = 300$
$b_n = \frac{1}{3}b_{n-1}$

$1+.016=1.016$

8. Daniela invested \$2000 in a stock that increases by 1.6% each week. Which of the following recursive sequences represents the value of her stock after n weeks?

1) $a_0 = 2000$
$a_n = a_{n-1} + 1.6$

2) $a_0 = 2000$
$a_n = a_{n-1} + 1.016$

3) $a_0 = 2000$
$a_n = 1.6a_{n-1}$

4) $a_0 = 2000$
$a_n = 1.016a_{n-1}$

$1-.2=.8$ $+80$

9. A tree farm initially has 150 trees. Each year, 20% of the trees are cut down and 80 seedlings are planted. Which recursive formula models the number of trees, a_n, after n years?

1) $a_1 = 150$
$a_n = a_{n-1}(0.2) + 80$

2) $a_1 = 150$
$a_n = a_{n-1}(0.8) + 80$

3) $a_n = 150(0.2)^n + 80$

4) $a_n = 150(0.8)^n + 80$

Series

Series is the sum of a sequence

To write a series explicitly: $S_n = \dfrac{a_1 - a_1(r)^n}{1-r}$ where r is the common ratio ($1 \pm rate$)

To write a series using summations: $\displaystyle\sum_{n=1}^{n} a_1(r)^{n-1}$ or $\displaystyle\sum_{n=0}^{n} a_0(r)^n$

Always use explicit unless it says otherwise

1. The sum of the first 20 terms of the series $-2 + 6 - 18 + 54 - \ldots$ is

S_n $\cdot -3 \cdot -3$

1) -610
2) -59
3) 1,743,392,200
4) 2,324,522,934

$a_1 = -2$

$r = -3$

$S_n = \dfrac{a_1 - a_1(r)^n}{1-r}$

$S_{20} = \dfrac{-2 - (-2)(-3)^{20}}{1 - -3}$

2. Brian deposited 1 cent into an empty non-interest bearing bank account on the first day of the month. He then additionally deposited 3 cents on the second day, 9 cents on the third day, and 27 cents on the fourth day. What would be the total amount of money in the account at the end of the 20th day if the pattern continued?

S_n

1) \$11,622,614.67
2) \$17,433,922.00
3) \$116,226,146.80
4) \$1,743,392,200.00

$\cdot 3 \quad \cdot 3 \quad \cdot 3$

$.01, .03, .09, .27$

$a_1 = .01$

$r = 3$

$n = 20$

$S_n = \dfrac{a_1 - a_1(r)^n}{1-r}$

$S_{20} = \dfrac{.01 - .01(3)^{20}}{1-3} = 17,433,922.00$

3. Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year. Find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

a_1 1.04 n

$S_n = \dfrac{a_1 - a_1(r)^n}{1-r}$ $S_{15} = \dfrac{33,000 - 33,000(1.04)^{15}}{1 - 1.04}$

$S_{15} = 660,778.39$

4. Dee is planning on decreasing the amount of time she eats fast food per month. After the first month, she ate fast food 42 times. Each month, she eats at fast food restaurants 10% less than the previous month. How many total times does she eat fast food in the first four months?

a_1 $1 - .10 = .90$ n

S_n

$S_n = \dfrac{a_1 - a_1(r)^n}{1-r}$

$S_4 = \dfrac{42 - 42(.90)^4}{1 - .90}$

$S_4 = 144$

5. A fisherman harvests 350 kilograms of crab on Monday. From Monday to Friday, the fisherman harvests 8% less kilograms of crab per day. To the *nearest tenth of a kilogram*, what is the total amount of crab harvested between Monday and Friday?

a_1

$1-.08$

$.92$

S_n $n=5$

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_5=\frac{350-350(.92)^5}{1-.92}$$

$$S_5=1491.5$$

6. Kina earns a $27,000 salary for the first year of work at her job. She earns annual increases of 2.5%. What is the total amount, to the *nearest cent*, that Kina will earn for the first eight years at this job?

a_1

$1+.025$

1.025

S_n n

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_8=\frac{27{,}000-27{,}000(1.025)^8}{1-1.025}$$

$$S_8=235875.13$$

7. This year, public parks in New York State will receive funds of $2.4 million. Every year afterward, New York State park funding will be improved by 3%. Determine the total amount of money, to the *nearest million dollars*, New York State parks will receive in funding for the first four years?

a_1 $1+.03=1.03$

S_n n

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_4=\frac{2.4-2.4(1.03)^4}{1-1.03}$$

$S_4 \approx \$10$ mil

8. The initial push of a child on a swing causes the swing to travel a total of 6 feet. Each successive swing travels 80% of the distance of the previous swing. Determine the total distance, to the *nearest hundredth of a foot*, a child travels in the first five swings.

a_1 $r=.80$ S_n n

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_5=\frac{6-6(.8)^5}{1-.8}$$

$$S_5=20.17$$

9. Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

$1+.10=1.10$

a_1

n

1) $\sum_{n=1}^{6} 8(1.10)^{n-1}$

2) $\sum_{n=1}^{6} 8(1.10)^{n}$

3) $\cancel{\dfrac{8+8(1.10)^6}{0.90}}$

4) $\cancel{\dfrac{8-8(0.10)^n}{1.10}}$

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_6=\frac{8-8(1.10)^6}{1-1.10}$$

$$S_6=\frac{8-8(1.10)^6}{-.10}$$

$$\sum_{n=1}^{6} a_1(r)^{n-1}$$

$$\sum_{n=1}^{6} 8(1.10)^{n-1}$$

10. In his first year running track, Usain earned 8 medals. He increases his amount of medals by 25% each year. Which of the following expressions *cannot* be used to determine how many medals Usain will have after four years of high school?

$1+.25$

1.25

a_1

n

1) $\dfrac{8-8(1.25)^4}{-.25}$

2) $8(1.25)^0+8(1.25)^1+8(1.25)^2+8(1.25)^3$

3) $\sum_{n=1}^{4} 8(1.25)^{n-1}$

4) $\dfrac{8-8(.25)^4}{1-.25}$

$$S_4=\frac{8-8(1.25)^4}{1-1.25}$$

$$S_4=\frac{8-8(1.25)^4}{-.25}$$

$$\sum_{n=1}^{4} a_1(r)^{n-1}$$

$$\sum_{n=1}^{4} 8(1.25)^{n-1}$$

11. A company fired several employees in order to save money. The amount of money the company saved per year over five years following the loss of employees is shown in the table below.

Year	Amount Saved (in dollars)
1	59,000
2	64,900
3	71,390
4	78,529
5	86,381.9

$\dfrac{64900}{59000}=1.1$

$\dfrac{71390}{64900}=1.1$

$r=1.1$

$a_1=59,000$

Which expression determines the total amount of money saved by the company over 5 years?

1) $\dfrac{59,000-59,000(1.1)^5}{1-1.1}$

2) $\dfrac{59,000-59,000(0.1)^5}{1-0.1}$

3) $\sum_{n=1}^{5} 59,000(1.1)^{n}$

4) $\sum_{n=1}^{5} 59,000(0.1)^{n-1}$

$$S_5=\frac{59,000-59,000(1.1)^5}{1-1.1}$$

$$\sum_{n=1}^{5} 59,000(1.1)^{n-1}$$

Name Schlansky
Mr. Schlansky

Date ____________
Algebra II

Mortgage and Annuities

1. Jim is looking to buy a vacation home for \$172,600 near his favorite southern beach. The formula to compute a mortgage payment, M, is $M = P \bullet \frac{r(1+r)^N}{(1+r)^N - 1}$ where P is the principal amount of the loan, r is the monthly interest rate, and N is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage. With a \$20,000 down payment, determine Jim's mortgage payment, rounded to the *nearest dollar*.

M = mortgage payment = M
P = principal amount of loan = 152,600
r = monthly interest rate = .00305
N = # of monthly payments = 15(12) = 180

principal = total cost − down payment
$P = 172{,}600 - 20{,}000$
$P = 152{,}600$

$$M = 152{,}600 \cdot \frac{.00305(1+.00305)^{180}}{(1+.00305)^{180} - 1}$$

$$M \stackrel{\$}{=} 1102.94$$

to find down payment, find P first!

Algebraically determine and state the down payment, rounded to the *nearest dollar*, that Jim needs to make in order for his mortgage payment to be \$1100.

M = 1100
P = P
r = .00305
n = 180

$$1100 = P\left(\frac{.00305(1+.00305)^{180}}{(1+.00305)^{180} - 1}\right)$$

type whole thing into calculator

$$\frac{1100}{.007\ldots} = \frac{P(.007\ldots)}{.007\ldots}$$

$$152{,}193\ldots = P$$

$D = T - P$

$D = 172{,}600 - 152{,}193\ldots$

$D \stackrel{\$}{=} 20{,}407$

2. Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of \$21,000 and a \$1000 down payment, to the *nearest cent.*

$$P_n = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

P_n = present amount borrowed $= 21{,}000 - 1{,}000 = 20000$
n = number of monthly pay periods $= 5(12) = 60$
PMT = monthly payment $= x$
i = interest rate per month $= .00625$

$$20000 = x\left(\frac{1-(1.00625)^{-60}}{.00625}\right)$$

$$\frac{20{,}000}{49.9\ldots} = \frac{x(\cancel{49.9\ldots})}{\cancel{49.9\ldots}}$$

$$400.76 = x$$

P=T-D

The affordable monthly payment is \$300 for the same time period. Determine an appropriate down payment, to the *nearest dollar*.

$P_n = x$

$n = 5(12) = 60$

$PMT = 300$

$i = .00625$

$$x = 300\left(\frac{1-(1.00625)^{-60}}{.00625}\right)$$

$$x = \cancel{300(}\ 14971\ldots$$

$$P = T - D$$

$$14971\ldots = 21{,}000 - D$$
$$-21000 \quad -21{,}000$$

$$\frac{-6028\ldots}{-1} = \frac{-D}{-1}$$

$$\$6028 = D$$

3. Monthly mortgage payments can be found using the formula below:

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{n}}{\left(1+\frac{r}{12}\right)^{n}-1}$$

M = monthly payment → m
P = amount borrowed 220,000 − 100,000 = 120,000
r = annual interest rate .048
n = number of monthly payments 15(12) = 180

The Banks family would like to purchase a home for $220,000. They qualified for an annual interest rate of 4.8%. If they put make a down payment of $100,000 and plan to spend 15 years to repay the loan, what will be the monthly payment rounded to the *nearest* cent?

$$m = \frac{120{,}000\left(\frac{.048}{12}\right)\left(1+\frac{.048}{12}\right)^{180}}{\left(1+\frac{.048}{12}\right)^{180}-1}$$

$$m = 936.50$$

If they want their monthly payment to be $1500, what would their down payment have to be?

m = 1500
P = x
r = .048
n = 180

$$1500 = x\left(\frac{\left(\frac{.048}{12}\right)\left(1+\frac{.048}{12}\right)^{180}}{\left(1+\frac{.048}{12}\right)^{180}-1}\right)$$

$$\frac{1500}{.0078\ldots} = \frac{x(\cancel{.0078\ldots})}{\cancel{.0078\ldots}}$$

$$192205\ldots = x$$

D = T − P

D = 220,000 − 192,205

D = 27,794.43

4. Malia wants to renovate the kitchen in her house and estimates that it will cost $39,000 to do so. She plans to make a down payment of $5,000 and then finance the rest at 0.25% interest per month over a ten-year period.
Use the following formula to determine Malia's monthly payment to the *nearest cent.*

$$P_n = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

P_n = present amount borrowed $= 39{,}000 - 5{,}000 = 34{,}000$
n = number of monthly pay periods $10(12) = 120$
PMT = monthly payment $= x$
i = interest rate per month $= .0025$

$$34{,}000 = x\left(\frac{1-(1+.0025)^{-120}}{.0025}\right)$$

$$\frac{34{,}000}{103\ldots} = \frac{x(\cancel{103\ldots})}{\cancel{103\ldots}}$$

$$\$328.31 = x$$

Malia can reasonably only afford a monthly payment of $275 per month ~~at most Malia's parents decide to help her with the cost of her new kitchen.~~ What would her down payment have to be in order for her monthly payment to be $275?

find P

$P_n = P$

$n = 120$

$pmT = 275$

$i = .0025$

$$P = 275\left(\frac{1-(1+.0025)^{-120}}{.0025}\right)$$

$$P = 28479\ldots$$

$$D = T - P$$

$$D = 39{,}000 - 28479\ldots$$

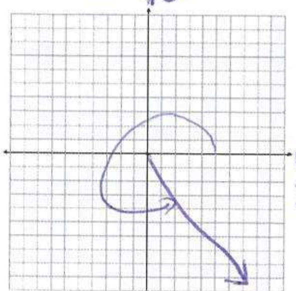
$$D = \$10{,}520.52$$

Sketching Radian Angles

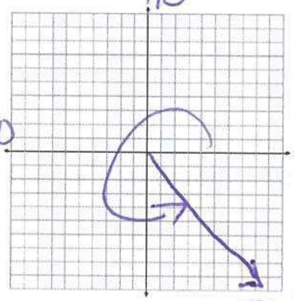
Radians to degrees: Multiply by $\frac{180}{\pi}$

Sketch the following angles on the grid

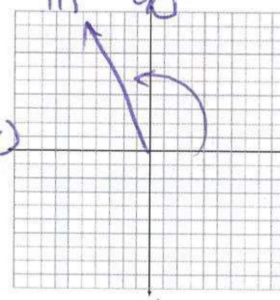
1. $\theta = \frac{5\pi}{3} \cdot \frac{180}{\pi} = 300°$



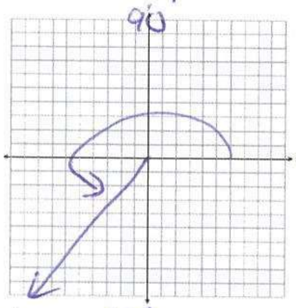
2. $\theta = \frac{7\pi}{4} \cdot \frac{180}{\pi} = 315°$



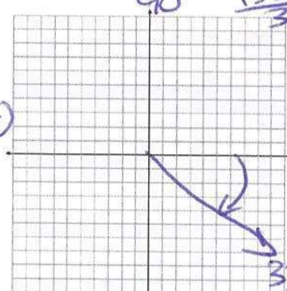
3. $\theta = 2 \cdot \frac{180}{\pi} \approx 115°$



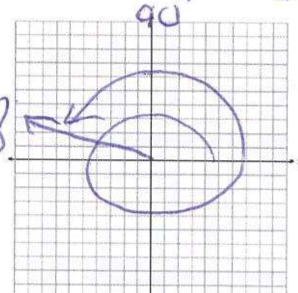
4. $\theta = 4.1 \cdot \frac{180}{\pi} \approx 235°$



5. $\theta = -\frac{\pi}{6} \cdot \frac{180}{\pi} = -30°$ $\frac{+360}{330}$

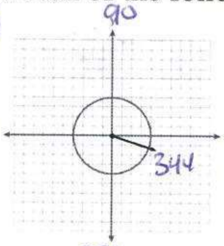


6. $\theta = 9.2 \cdot \frac{180}{\pi} \approx 527$ $\frac{-360}{167}$

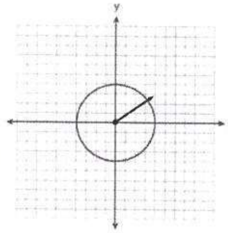


7. Which of the following sketches would represent 6 radians? $6 \cdot \frac{180}{\pi} \approx 344°$

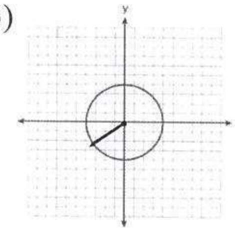
1)



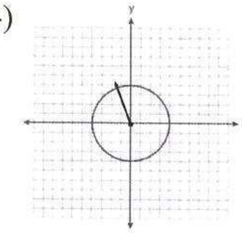
2)



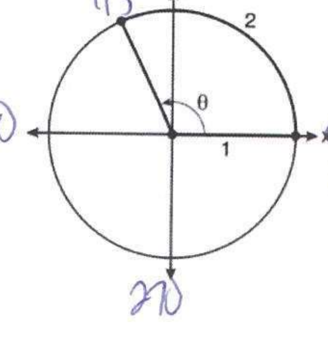
3)



4)



8. An angle, θ, is rotated counterclockwise on the unit circle, with its terminal side in the second quadrant, as shown in the diagram below.



Which value represents the radian measure of angle θ?

1) 1 $\cdot \frac{180}{\pi} \approx 57$

2) 2 $\cdot \frac{180}{\pi} \approx 115$

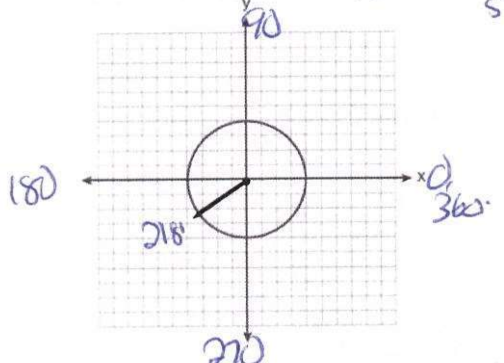
3) ~~65.4~~

4) ~~114.6~~ degrees

9. Which angle is sketched on the grid below? below?

~~1) 2.4 radians~~ $2.4 \cdot \frac{180}{\pi} \approx 137°$ 3) 3.8 radians $\approx 218°$ $3.8 \cdot \frac{180}{\pi}$

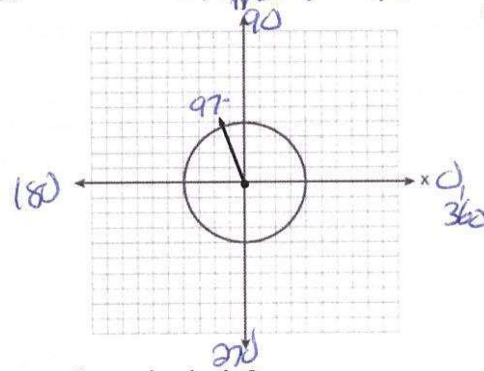
2) 4.5 radians $4.5 \cdot \frac{180}{\pi} \approx 258°$ ~~4) 5.2 radians~~ $\approx 300°$ $5.2 \cdot \frac{180}{\pi}$



10. Which angle is sketched on the grid

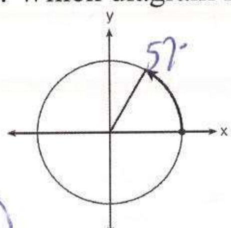
~~1) 1 radian~~ $1 \cdot \frac{180}{\pi} \approx 57°$ 3) 3 radians $3 \cdot \frac{180}{\pi} \approx 172°$

2) 1.7 radians $1.7 \cdot \frac{180}{\pi} \approx 97$ ~~4) 4.1 radians~~ ≈ 235 $4.1 \cdot \frac{180}{\pi}$

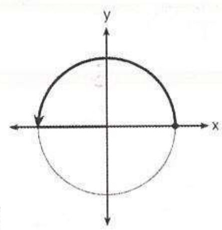


10. Which diagram shows an angle rotation of 1 radian on the unit circle?

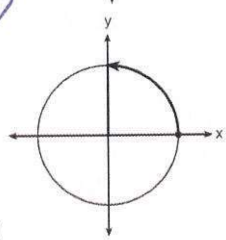
$1 \cdot \frac{180}{\pi} \approx 57°$



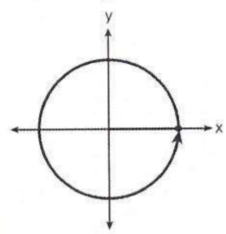
1)



3)



2)

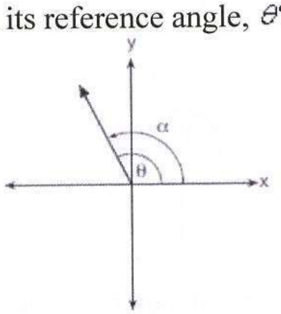


4)

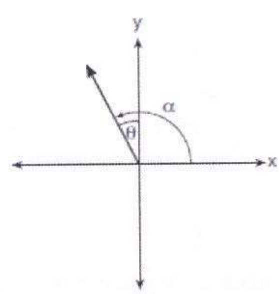
11. Which diagram represents an angle, α, measuring $\frac{13\pi}{20}$ radians drawn in standard position, and its reference angle, θ?

$\frac{13\pi}{20} \cdot \frac{180}{\pi} = 117°$

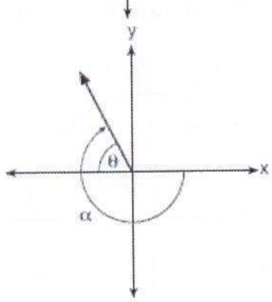
1)



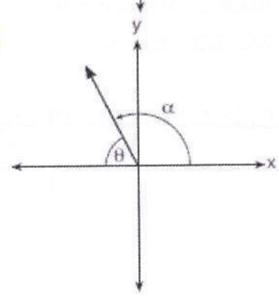
3)



2)



4)



Pythagorean Theorem

Look out for hidden right triangles where you may need to use $a^2 + b^2 = c^2$

a and *b* are the legs

c is the hypotenuse

Know your Pythagorean Triples!

3,4,5

5,12,13

7,24,25

8,15,17

9,40,41

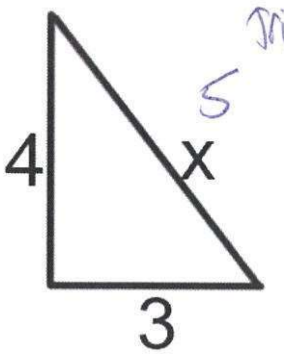
leaving your answer in radical form

Find the missing side of each right triangle ~~rounding to the nearest tenth~~

1.

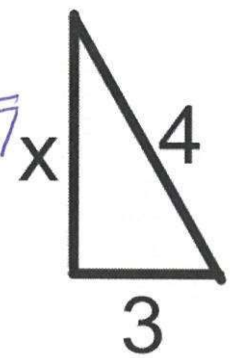
Triple

5



2.

$\sqrt{7}$



$a^2+b^2=c^2$

$3^2+x^2=4^2$

$9+x^2=16$

$-9 \quad -9$

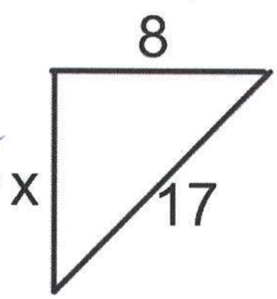
$\sqrt{x^2}=\sqrt{7}$

$x=\sqrt{7}$

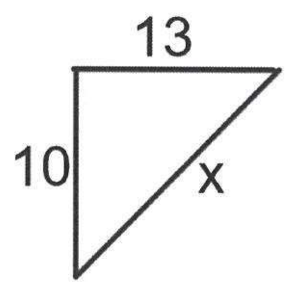
3.

Triple

15



4.



$a^2+b^2=c^2$

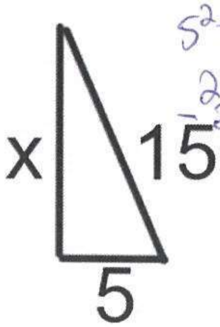
$10^2+13^2=x^2$

$100+169=x^2$

$\sqrt{269}=\sqrt{x^2}$

$\sqrt{269}=x$

5.



$a^2+b^2=c^2$

$5^2+x^2=15^2$

$25+x^2=225$

$-25 \quad -25$

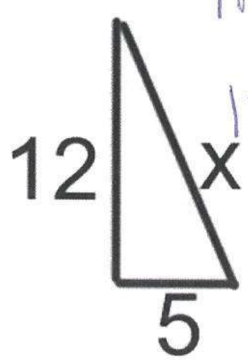
$\sqrt{x^2}=\sqrt{200}$

$x=\sqrt{200}$

6.

Triple

13



Rationalizing the Denominator

To rationalize the denominator, multiply top and bottom by the radical

When multiplying a radical by itself, the radical cancels out

Rationalize the following denominators

1. $\frac{2}{\sqrt{5}} \frac{\sqrt{5}}{\sqrt{5}}$ $\frac{2\sqrt{5}}{5}$

2. $\frac{-7}{\sqrt{11}} \frac{\sqrt{11}}{\sqrt{11}}$ $\frac{-7\sqrt{11}}{11}$

3. $\frac{3}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}}$ $\frac{3\sqrt{2}}{2}$

4. $\frac{6}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}}$ $\frac{\overset{2}{\cancel{6}}\sqrt{3}}{\underset{1}{\cancel{3}}}$ $2\sqrt{3}$

5. $\frac{4}{\sqrt{6}} \frac{\sqrt{6}}{\sqrt{6}}$ $\frac{\overset{2}{\cancel{4}}\sqrt{6}}{\underset{3}{\cancel{6}}}$

$\frac{2\sqrt{6}}{3}$

6. $\frac{-5}{\sqrt{10}} \frac{\sqrt{10}}{\sqrt{10}}$ $\frac{-\overset{1}{\cancel{5}}\sqrt{10}}{\underset{2}{\cancel{10}}}$ $\frac{-\sqrt{10}}{2}$

Trig Ratios with Triangles

If an angle passes through a point or sin/cos/tan = $\frac{something}{something}$, make a right triangle and use SOHCAHTOA

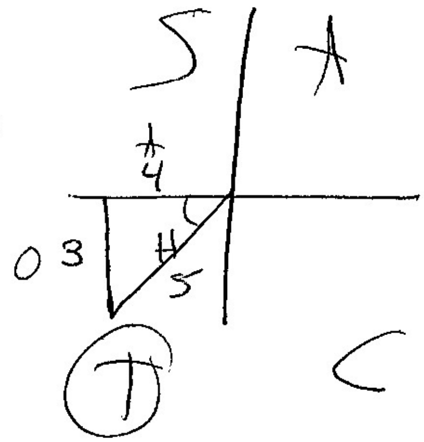
Any point on the unit circle is $(\cos\theta, \sin\theta)$

Know your Pythagorean triples: {3,4,5}, {5, 12, 13}, {8, 15, 17}, {7, 24, 25}

Reciprocal trig function pairs:

$\csc\theta$	$\sec\theta$	$\tan\theta$
$\sin\theta$	$\cos\theta$	$\cot\theta$

1. If $\sin\theta = -\frac{3 \; O}{5 \; H}$ and θ is in Quadrant III, find:



a) $\cos\theta$ $-\frac{4}{5}$

b) $\sin\theta$ $-\frac{3}{5}$

c) $\tan\theta$ $\frac{3}{4}$

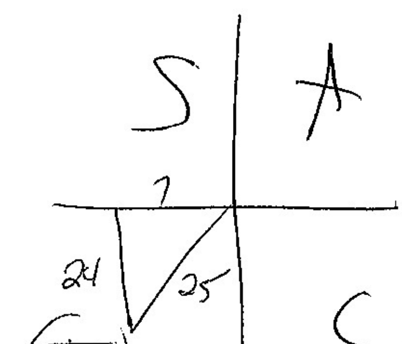
d) $\sec\theta$ $-\frac{5}{4}$

e) $\csc\theta$ $-\frac{5}{3}$

f) $\cot\theta$ $\frac{4}{3}$

$3^2+b^2=5^2$
$9+b^2=25$
$-9 \quad -9$
$\sqrt{b^2}=\sqrt{16}$
$b=4$

2. If $\tan\theta = \frac{24 \; O}{7 \; A}$ and θ is in Quadrant III, find:



a) $\cos\theta$ $-\frac{7}{25}$

b) $\sin\theta$ $-\frac{24}{25}$

c) $\tan\theta$ $\frac{24}{7}$

d) $\sec\theta$ $-\frac{25}{7}$

e) $\csc\theta$ $-\frac{25}{24}$

f) $\cot\theta$ $\frac{7}{24}$

$7^2+24^2=c^2$
$49+576=c^2$
$\sqrt{625}=\sqrt{c^2}$
$25=c$

3. Angle θ is in standard position and $(4,-7)$ is a point on the terminal side of θ. Find:

a) $\cos\theta$

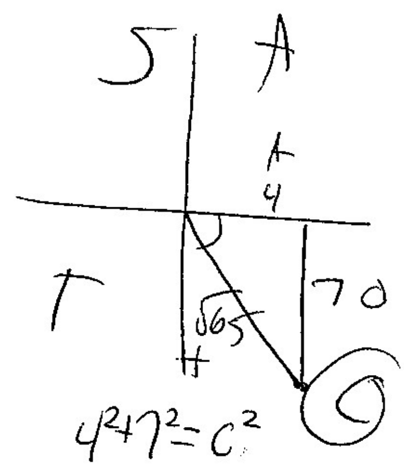
$\frac{4}{\sqrt{65}}\frac{\sqrt{65}}{\sqrt{65}}$ $\frac{4\sqrt{65}}{65}$

b) $\sin\theta$

$\frac{7}{\sqrt{65}}\frac{\sqrt{65}}{\sqrt{65}}$ $-\frac{7\sqrt{65}}{65}$

c) $\tan\theta$

$-\frac{7}{4}$



d) $\sec\theta$

$\frac{\sqrt{65}}{4}$

e) $\csc\theta$

$-\frac{\sqrt{65}}{7}$

f) $\cot\theta$

$-\frac{4}{7}$

$4^2+7^2=c^2$

$16+49=c^2$

$\sqrt{65}=\sqrt{c^2}$

$\sqrt{65}=c$

4. Angle θ is in standard position and $(-5,-12)$ is a point on the terminal side of θ. Find:

a) $\cos\theta$

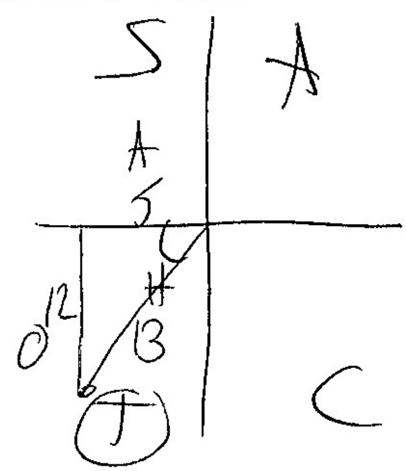
$-\frac{5}{13}$

b) $\sin\theta$

$-\frac{12}{13}$

c) $\tan\theta$

$\frac{12}{5}$



d) $\sec\theta$

$-\frac{13}{5}$

e) $\csc\theta$

$-\frac{13}{12}$

f) $\cot\theta$

$\frac{5}{12}$

$5^2+12^2=c^2$

$25+144=c^2$

$\sqrt{169}=\sqrt{c^2}$

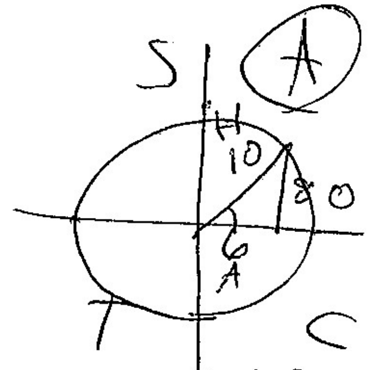
$13=c$

5. A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ, intercepts the circle in Quadrant I at point C. The y-coordinate of point C is 8. Find:

a) $\cos\theta$ $\frac{6}{10}$

b) $\sin\theta$ $\frac{8}{10}$

c) $\tan\theta$ $\frac{8}{6}$



d) $\sec\theta$ $\frac{10}{6}$

e) $\csc\theta$ $\frac{10}{8}$

f) $\cot\theta$ $\frac{6}{8}$

$a^2+8^2=10^2$

$a^2+64=100$

$-64\ -64$

$\sqrt{a^2}=\sqrt{36}$

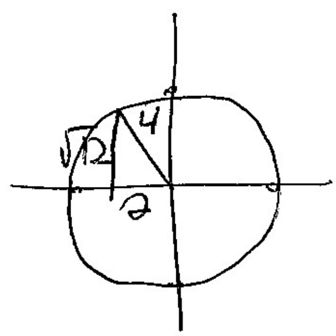
$a=6$

6. A circle centered at the origin has a radius of 4 units. The terminal side of an angle, θ, intercepts the circle in Quadrant II at point P. The x-coordinate of point P is 2. Find:

a) $\cos\theta$ $\frac{2}{4}$

b) $\sin\theta$ $\frac{\sqrt{12}}{4}$

c) $\tan\theta$ $\frac{\sqrt{12}}{2}$



d) $\sec\theta$ $\frac{4}{2}=2$

e) $\csc\theta$ $\frac{4\sqrt{12}}{\sqrt{12}\sqrt{12}}=\frac{4\sqrt{12}}{12}$

f) $\cot\theta$ $\frac{2\sqrt{12}}{\sqrt{12}\sqrt{12}}\ \frac{2\sqrt{12}}{12}$

$2^2+b^2=4^2$

$4+b^2=16$

$-4\ \ -4$

$\sqrt{b^2}=\sqrt{12}$

$b=\sqrt{12}$

Name Schlansky
Mr. Schlansky

Date ____________
Algebra II

Advanced Trig Ratios Regents Practice

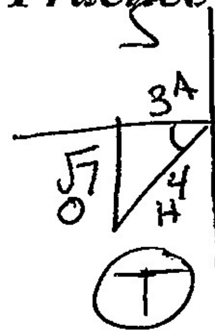
$a^2+b^2=c^2$
$3^2+b^2=4^2$
$9+b^2=16$
$-9 \quad -9$
$\sqrt{b^2}=\sqrt{7}$
$b=\sqrt{7}$

7. If $\cos\theta = -\frac{3}{4}$ ($\frac{A}{H}$) and θ is in Quadrant III, then $\sin\theta$ is equivalent to

1) $-\frac{\sqrt{7}}{4}$
2) $\frac{\sqrt{7}}{4}$
3) $-\frac{5}{4}$
4) $\frac{5}{4}$

$\sin\theta = \frac{O}{H}$

$\sin\theta = -\frac{\sqrt{7}}{4}$

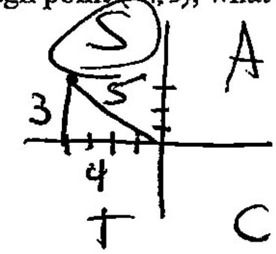


8. If the terminal side of angle θ, in standard position, passes through point (−4,3), what is the numerical value of $\sin\theta$?

1) $\frac{3}{5}$
2) $\frac{4}{5}$
3) $-\frac{3}{5}$
4) $-\frac{4}{5}$

$\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{3}{5}$



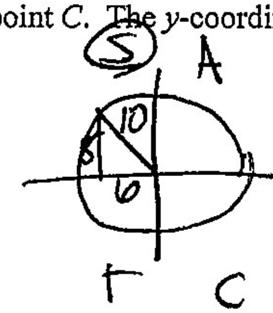
9 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ, intercepts the circle in Quadrant II at point C. The y-coordinate of point C is 8. What is the value of $\cos\theta$?

1) $-\frac{3}{5}$
2) $-\frac{3}{4}$
3) $\frac{3}{5}$
4) $\frac{4}{5}$

$\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{6}{10}$

$\cos\theta = -\frac{3}{5}$



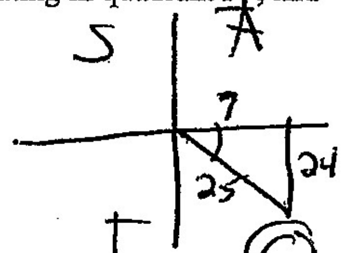
$a^2+b^2=c^2$
$a^2+8^2=10^2$
$a^2+64=100$
$-64 \quad -64$
$\sqrt{a^2}=\sqrt{36}$
$a=6$

10. Given $\cos\theta = \frac{7}{25}$ ($\frac{A}{H}$), where θ is an angle in standard position terminating in quadrant IV, and $\sin^2\theta + \cos^2\theta = 1$, what is the value of $\tan\theta$?

1) $-\frac{24}{25}$
2) $-\frac{24}{7}$
3) $\frac{24}{25}$
4) $\frac{24}{7}$

$\tan\theta = \frac{O}{A}$

$\tan\theta = -\frac{24}{7}$



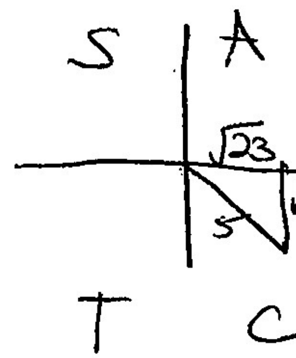
11. Given that $\sin^2\theta + \cos^2\theta = 1$ and $\sin\theta = -\frac{\sqrt{2}}{5}$ ($\frac{O}{H}$) → could be III or IV, what is a possible value of $\cos\theta$?

1) $\frac{5+\sqrt{2}}{5}$
2) $\frac{\sqrt{23}}{5}$
3) $\frac{3\sqrt{3}}{5}$
4) $\frac{\sqrt{35}}{5}$

$\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{\sqrt{23}}{5}$

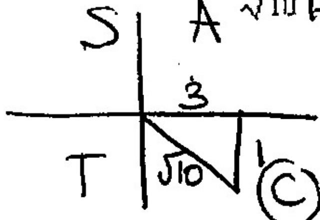
$a^2+b^2=c^2$
~~a^2+5^2~~
$a^2+\sqrt{2}^2=5^2$
$a^2+2=25$
$-2 \quad -2$
$\sqrt{a^2}=\sqrt{23}$



$\cos^{+}$ $\tan^{-}$

12 ~~6~~. Given $\cos A = \frac{3}{\sqrt{10}}$ $\frac{A}{H}$ and $\cot A = -3$, determine the value of $\sin A$ in radical form.

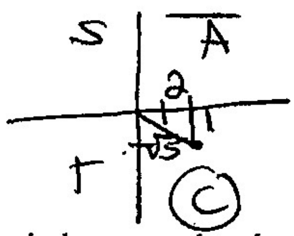
$\tan A = -\frac{1}{3}$ $\frac{O}{A}$



$\sin A = \frac{O}{H}$

$\sin A = \frac{1}{\sqrt{10}}\frac{\sqrt{10}}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$

13 ~~7~~. An angle, θ, is in standard position and its terminal side passes through the point $(2,-1)$. Find the *exact* value of $\sin\theta$.



$a^2+b^2=c^2$

$2^2+1^2=c^2$

$4+1=c^2$

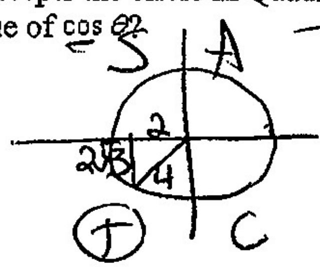
$\sqrt{5}=\sqrt{c^2}$

$\sqrt{5}=c$

$\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{1}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

14 ~~8~~. A circle centered at the origin has a radius of 4 units. The terminal side of an angle, θ, intercepts the circle in Quadrant III at point P. The x-coordinate of point P is 2. What is the value of $\cos\theta$?



$a^2+b^2=c^2$

$2^2+b^2=4^2$

$4+b^2=16$

$-4 \quad -4$

$\sqrt{b^2}=\sqrt{12}$

$b=\sqrt{4}\sqrt{3}$

$b=2\sqrt{3}$

$\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{2}{4}$

$\cos\theta = -\frac{1}{2}$

15 ~~9~~. The terminal side of θ, an angle in standard position, intersects the unit circle at $P\left(-\frac{1}{3}\frac{A}{H}, \frac{\sqrt{8}}{3}\right)$ $\frac{O}{H}$

$\cos\theta, \sin\theta$

What is the value of $\sec\theta$?

1) -3

2) $-\frac{3\sqrt{8}}{8}$

3) $-\frac{1}{3}$

4) $-\frac{\sqrt{8}}{3}$

If $\cos\theta = -\frac{1}{3}$

$\sec\theta = -3$

16 ~~10~~. Point $M\left(t, \frac{4}{7}\right)$ is located in the second quadrant on the ~~unit circle~~. Determine the exact value of t.

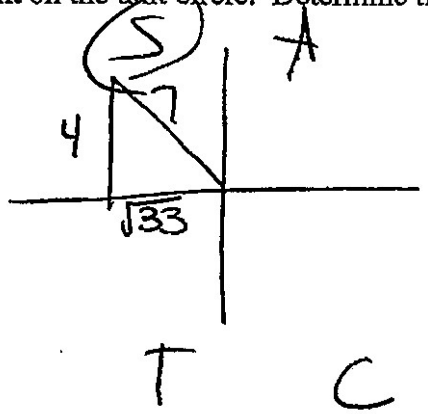
$\cos\theta, \sin\theta$

$\sin\theta = \frac{4}{7}$ $\frac{O}{H}$

It is asking for $\cos\theta$.

$\cos\theta = \frac{A}{H}$

$\cos\theta = -\frac{\sqrt{33}}{7}$



$a^2+b^2=c^2$

$a^2+4^2=7^2$

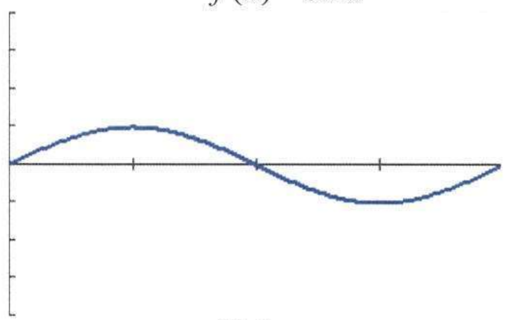
$a^2+16=49$

$-16 \quad -16$

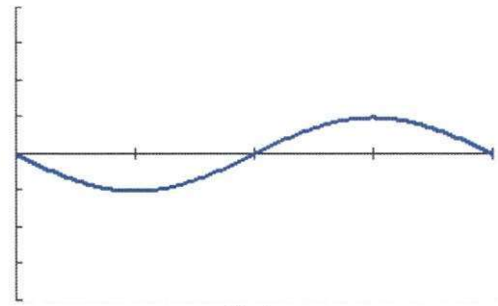
$\sqrt{a^2}=\sqrt{33}$

Trig Graphs:
Know what your waves look like!

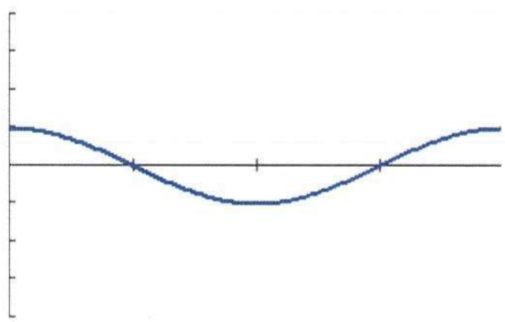
$f(x) = \sin x$



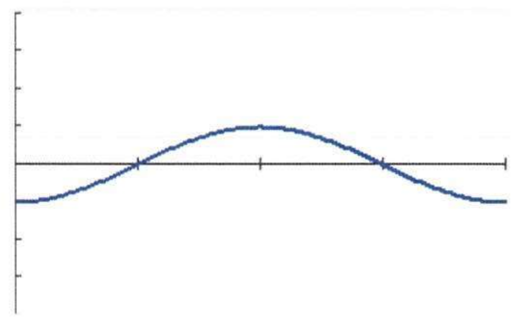
$f(x) = -\sin x$



$f(x) = \cos x$



$f(x) = -\cos x$



AMPSINFREQXSHIFT
Amplitude: Distance from the midline to minimum or maximum
Frequency: How many waves from 0 to 2π
Period: (Wavelength): How long it takes to make one full cycle
Shift/Midline: y value of the midline. The average value of the function.

To graph, list:
Amplitude
$\pm\sin/\cos$
Frequency
Shift/Midline

$$Period = \frac{2\pi}{frequency}$$

y-axis: Plot midline. Count amplitude above and below from the midline.
x-axis: Make 4 dashes on x-axis. Label the 4^{th} dash with the period.
Plot the 5 points for the appropriate wave.

To write the equation, find:

$Frequency = \dfrac{2\pi}{period}$ and $midline = \dfrac{\min + \max}{2}$

Substitute components into $y = amp \sin freqxshift$
If multiple choice, cross out answers with incorrect components!

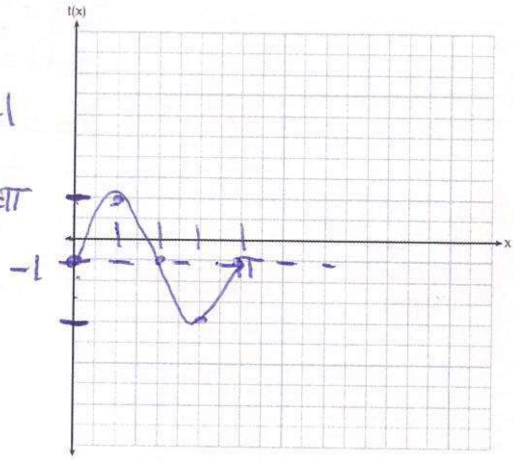
Graph one full wave of the following trigonometric functions

amp sin freq x shift

1. $y = 3\sin 2x - 1$

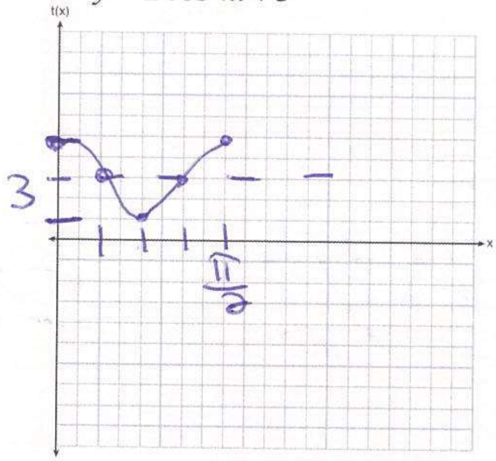
amp=3
+sin
freq=2
shift=-1

$p = \frac{2\pi}{2} = \pi$



amp sin freq x shift

2. $y = 2\cos 4x + 3$



amp=2
+cos
freq=4
shift=3

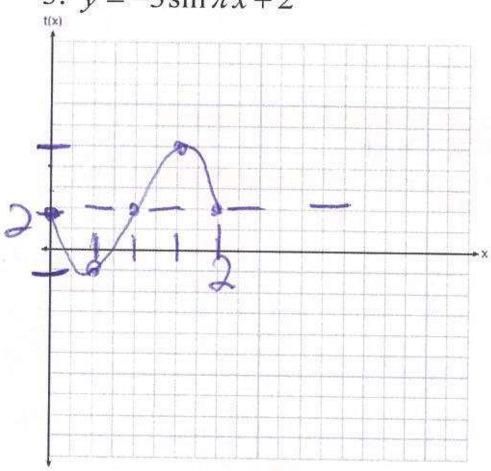
$p = \frac{2\pi}{4} = \frac{\pi}{2}$

amp sin freq x shift

3. $y = -3\sin \pi x + 2$

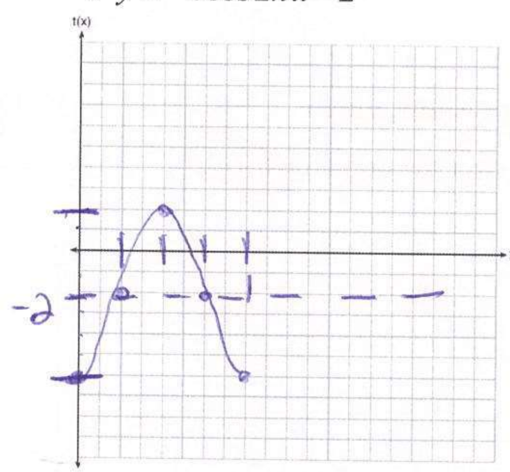
amp=3
-sin
freq=π
shift=2

$p = \frac{2\pi}{\pi} = 2$



amp sin freq x shift

4. $y = -4\cos 2\pi x - 2$



amp=4
-cos
freq=2π
shift=-2

$p = \frac{2\pi}{2\pi} = 1$

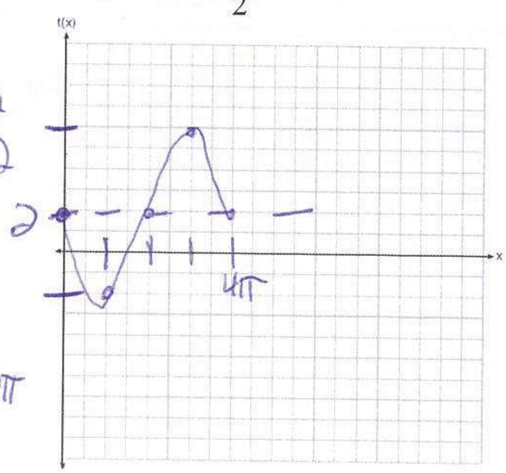
amp sin freq x shift

5. $y = -4\sin \frac{1}{2}x + 2$

amp=4
-sin
freq=½
shift=2

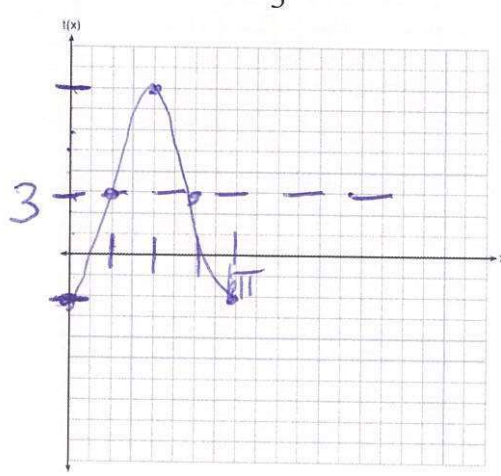
$p = \frac{2\pi}{\frac{1}{2}}$

$2\pi \cdot \frac{2}{1} = 4\pi$



amp sin freq x shift

6. $y = -5\cos \frac{1}{3}x + 3$



amp=5
-cos
freq= ⅓
shift=3

$p = \frac{2\pi}{\frac{1}{3}}$

$2\pi \cdot \frac{3}{1} = 6\pi$

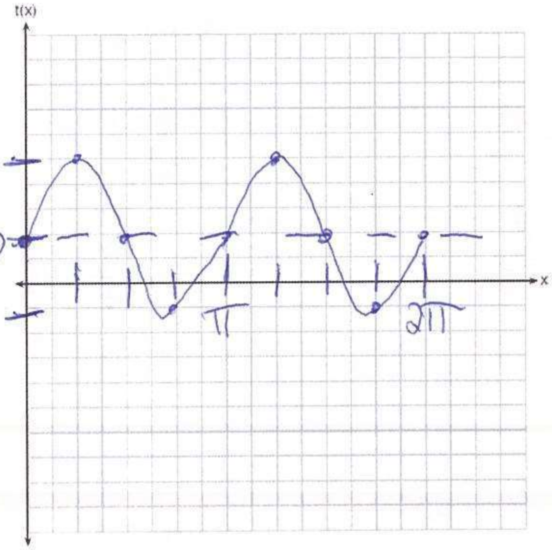
Graph the following two functions over the domain $[0, 2\pi]$ on the set of axes below.

amp sin freq x shift

7. $t(x) = 3\sin(2x) + 2$

amp=3
+sin
freq=2
shift=2

$p = \frac{2\pi}{2} = \pi$

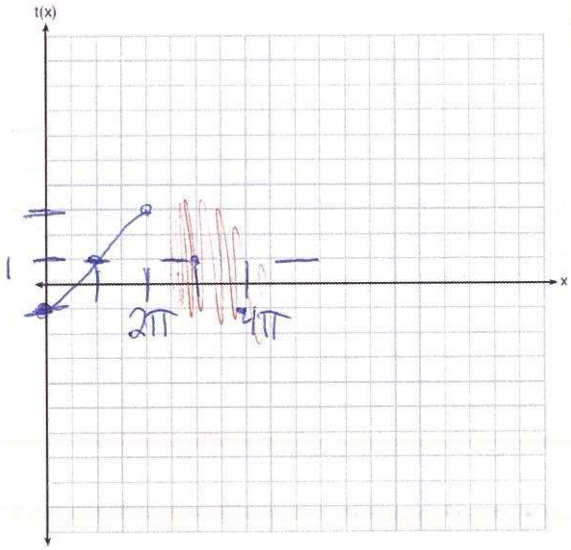


amp sin freq x shift

8. $y = -2\cos\frac{1}{2}x + 1$

amp = 2
− cos
freq = $\frac{1}{2}$
shift = 1
$p = \frac{2\pi}{\frac{1}{2}}$

$2\pi \cdot \frac{2}{1} = 4\pi$



9. On the set of axes below, graph

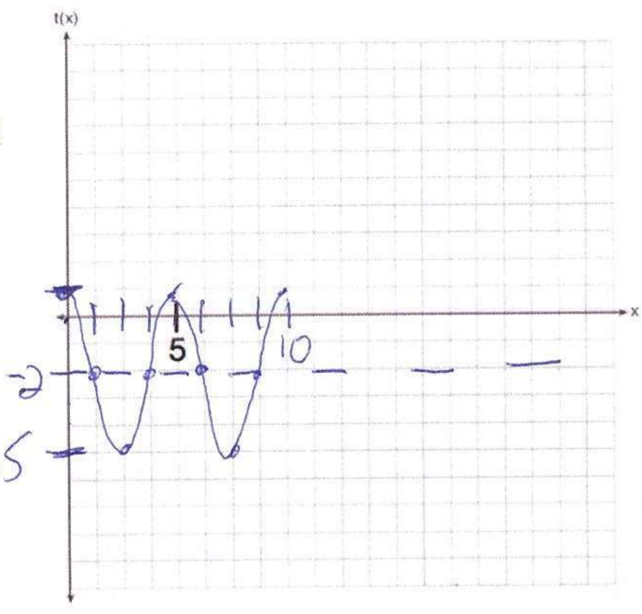
amp sin freq x shift

$y = 3\cos\frac{2\pi}{5}x - 2$ over the domain $[0,10]$

amp=3
+cos
freq=$\frac{2\pi}{5}$
shift=−2

$p = \frac{2\pi}{\frac{2\pi}{5}}$

$2\pi \cdot \frac{5}{2\pi} = 5$



10. On the set of axes below, graph

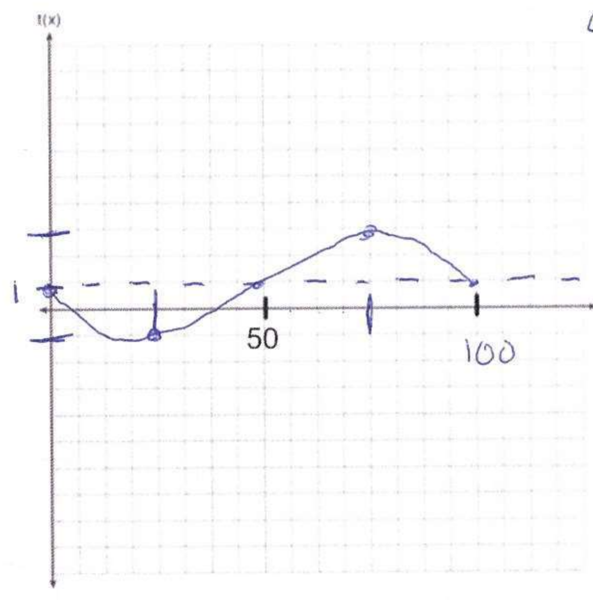
amp sin freq x shift

$y = -2\sin\frac{\pi}{50}x + 1$ over the domain $[0,100]$

amp=2
− sin
freq=$\frac{\pi}{50}$
shift=1

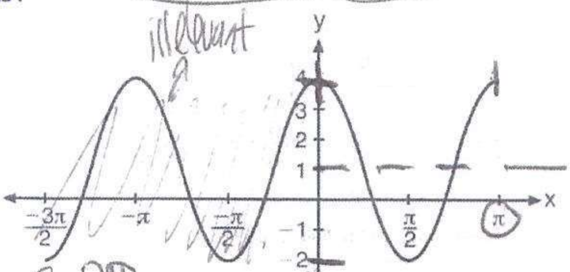
$p = \frac{2\pi}{\frac{\pi}{50}}$

$2\pi \cdot \frac{50}{\pi} = 100$



11.

$y = 3\cos 2x + 1$



amp = 3
+cos
freq = 2
shift = 1

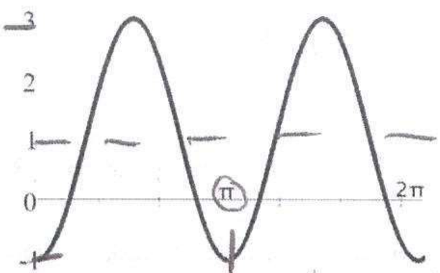
$f = \frac{2\pi}{p}$

$f = \frac{2\pi}{\pi} = 2$

midline $= \frac{\min + \max}{2}$

$= \frac{-2+4}{2} = 1$

12.



amp = 2
−cos
freq = 2
shift = 1

$y = -2\cos 2x + 1$

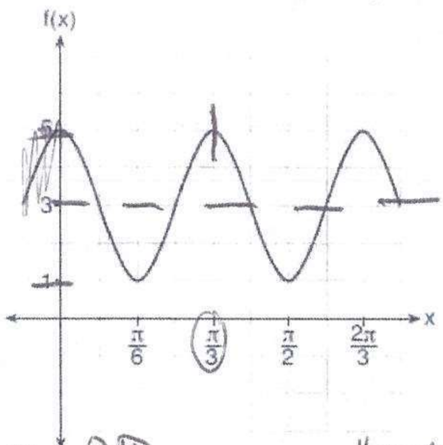
$f = \frac{2\pi}{p}$

$f = \frac{2\pi}{\pi} = 2$

midline $= \frac{\min + \max}{2}$

$= \frac{-1+3}{2} = 1$

13.



amp = 2
+cos
freq = 6
shift = 3

$y = 2\cos 6x + 3$

$f = \frac{2\pi}{p}$

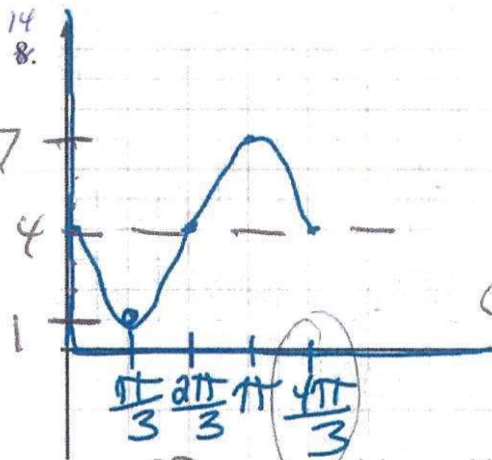
$f = \frac{2\pi}{\frac{\pi}{3}}$ $\quad \frac{2\pi}{1} \cdot \frac{3}{\pi}$

$f = 6$

midline $= \frac{\min + \max}{2}$

$= \frac{1+5}{2} = 3$

14



amp = 3
−sin
freq = $\frac{3}{2}$
shift = 4

$y = -3\sin \frac{3}{2}x + 4$

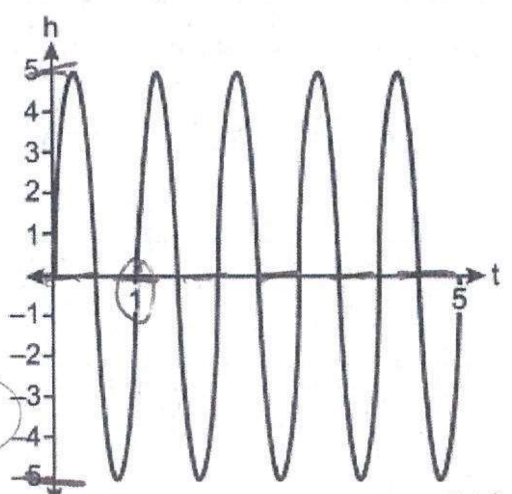
$f = \frac{2\pi}{p}$

$f = \frac{2\pi}{\frac{4\pi}{3}}$ $\quad \frac{2\pi}{1} \cdot \frac{3}{4\pi} = \frac{3}{2}$

midline $= \frac{\min + \max}{2}$

$= \frac{1+7}{2} = 4$

15.



amp = 5
+sin
freq = 2π
shift = 0

$y = 5\sin 2\pi x$

$f = \frac{2\pi}{p}$

$f = \frac{2\pi}{1} = 2\pi$

midline $= \frac{\min + \max}{2}$

$= \frac{-5+5}{2} = 0$

16.

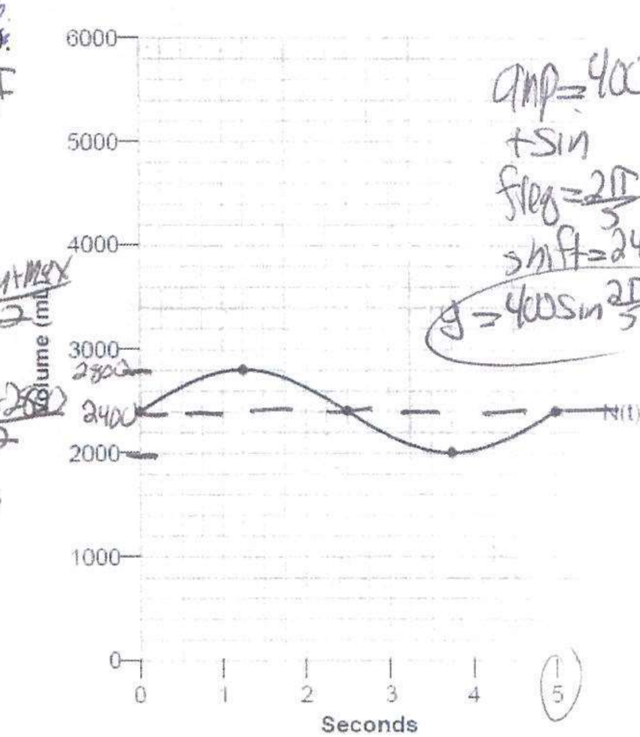
$f = \frac{2\pi}{p}$

$f = \frac{2\pi}{5}$

midline $= \frac{\min + \max}{2}$

midline $= \frac{2000 + 2800}{2}$

$= 2400$



amp = 400
+sin
freq = $\frac{2\pi}{5}$
shift = 2400

$y = 400\sin \frac{2\pi}{5}x + 2400$

17 ~~14~~. The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.

If the depth, d, is measured in feet and time, t, is measured in hours since midnight, what is an equation for the depth of the water at the marker?

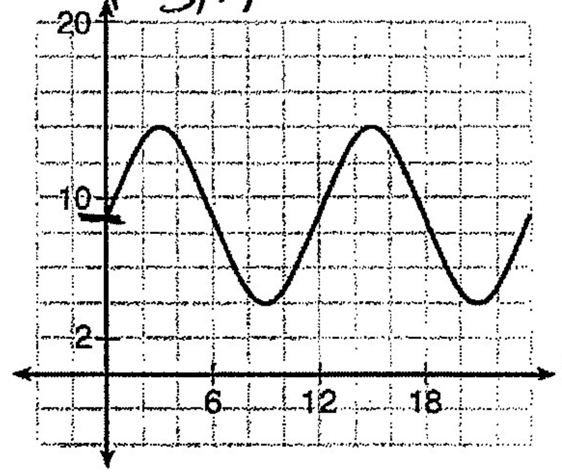
+ sin

1) $\cancel{d = 5\cos\left(\frac{\pi}{6}t\right) + 9}$ not + sin

2) $\cancel{d = 9\cos\left(\frac{\pi}{6}t\right) + 5}$

3) $\cancel{d = 9\sin\left(\frac{\pi}{6}t\right) + 5}$ midline not 5

4) $d = 5\sin\left(\frac{\pi}{6}t\right) + 9$



18 ~~12~~. Which equation is graphed in the diagram below?

not −cos

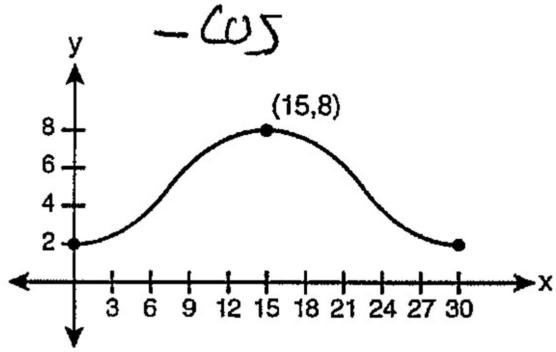
−cos

1) $\cancel{y = 3\cos\left(\frac{\pi}{30}x\right) + 8}$

2) $\cancel{y = 3\cos\left(\frac{\pi}{15}x\right) + 5}$

3) $\cancel{y = -3\cos\left(\frac{\pi}{30}x\right) + 8}$ midline not 8

4) $y = -3\cos\left(\frac{\pi}{15}x\right) + 5$



19 ~~13~~. Which equation is represented by the graph below?

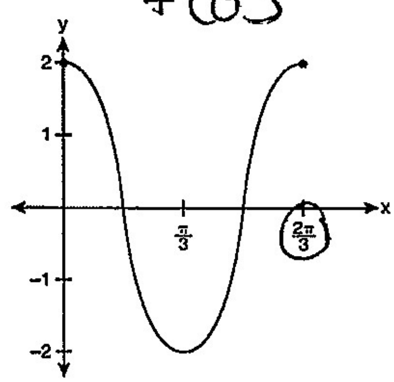
+ cos

1) $y = 2\cos 3x$

2) $\cancel{y = 2\sin 3x}$

3) $\cancel{y = 2\cos\frac{2\pi}{3}x}$ $p = \frac{2\pi}{3}$, not freq

4) $\cancel{y = 2\sin\frac{2\pi}{3}x}$ not + cos



Name Schlansky
Mr. Schlansky

Date ____________
Algebra II

Sinusoidal Applications

amp sin freq x shift

1. Which statement is *incorrect* for the graph of the function $y = -3\cos\left[\frac{\pi}{3}(x-4)\right] + 7$?

1) The period is 6.
2) The amplitude is 3.
3) The range is [4,10].
4) The midline is $y = -4$.

$y = 7$

amp = 3
−cos
freq = $\frac{\pi}{3}$
shift = 7

10, 7, 4, 6

$p = \frac{2\pi}{f}$

$p = \frac{2\pi}{\frac{\pi}{3}}$

$p = \frac{2\pi}{1} \cdot \frac{3}{\pi}$

$p = 6$

2. Which function's graph has a period of 8 and reaches a maximum height of 1 if at least one full period is graphed?

1) $y = -4\cos\left(\frac{\pi}{4}x\right) - 3$
2) $y = -4\cos\left(\frac{\pi}{4}x\right) + 5$
3) $y = -4\cos(8x) - 3$
4) $y = -4\cos(8x) + 5$

1, −3, −7

9, 5, 1

1, 8

$f = \frac{2\pi}{p}$

$f = \frac{2\pi}{8}$

$f = \frac{\pi}{4}$

3. The equation below can be used to model the height of a tide in feet, $H(t)$, on a beach at t hours.

amp sin freq x shift

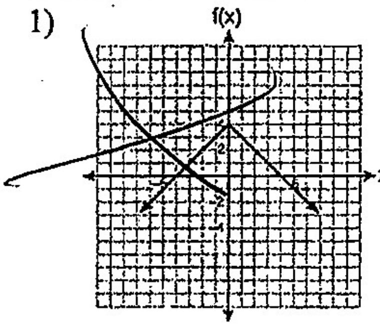
$$H(t) = 4.8\sin\left(\frac{\pi}{6}(t+3)\right) + 5.1$$

Using this function, the amplitude of the tide is

1) $\frac{\pi}{6}$
2) 4.8
3) 3
4) 5.1

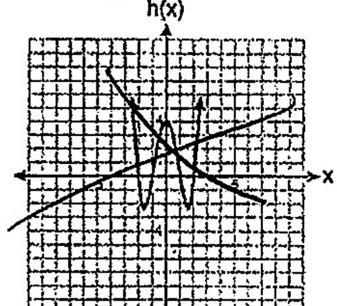
4. Which function has a maximum y-value of 4 and a midline of $y = 1$?

must be trig graph

1)


2) $g(x) = -3\cos(x) + 1$

4, 1, −2

3)


4) $j(x) = 4\sin(x) + 1$

5, 1, −3

5. The depth of the water, $d(t)$, in feet, on a given day at Thunder Bay, t hours after midnight is modeled by $d(t) = 5\sin\left(\frac{\pi}{6}(t-5)\right)+7$. Which statement about the Thunder Bay tide is *false*?

amp sin freq x shift

1) A low tide occurred at 2 a.m.
2) The maximum depth of the water was 12 feet.
3) The water depth at 9 a.m. was approximately 11 feet.
4) The difference in water depth between high tide and low tide is 14 feet. 10 feet

amp=5
+sin
freq $=\frac{\pi}{6}$

shift=7

$p=\frac{2\pi}{\frac{\pi}{6}}$

$\frac{2\pi}{1}\cdot\frac{6}{\pi}=12$

12

7

2

6. A person's lung capacity can be modeled by the function $C(t) = 250\sin\left(\frac{2\pi}{5}t\right)+2450$, where $C(t)$ represents the volume in mL present in the lungs after t seconds. State the maximum value of this function over one full cycle, and explain what this value represents.

amp sin freq x shift

2700

2450

2200

2700. the maximum volume present in the lungs is 2700 mL.

7. Based on climate data that have been collected in Bar Harbor, Maine, the average monthly temperature, in degrees F, can be modeled by the equation $B(x) = 23.914\sin(0.508x - 2.116) + 55.300$. The same governmental agency collected average monthly temperature data for Phoenix, Arizona, and found the temperatures could be modeled by the equation $P(x) = 20.238\sin(0.525x - 2.148) + 86.729$. Which statement can *not* be concluded based on the average monthly temperature models x months after starting data collection?

B(x)

9.214

55.3

31.386

1) The average monthly temperature amp variation is more in Bar Harbor than in Phoenix. 23.914 > 20.238
2) The midline average monthly temperature for Bar Harbor is lower than the midline temperature for Phoenix. 55.3 < 86.729
3) The maximum average monthly temperature for Bar Harbor is 79° F, to the nearest degree. 106.967 P(x)
4) The minimum average monthly temperature for Phoenix is 20° F, to the nearest degree. 86.729 66.491 66.491

8. The average monthly temperature of a city can be modeled by a cosine graph. Melissa has been living in Phoenix, Arizona, where the average annual temperature is 75°F. She would like to move, and live in a location where the average annual temperature is 62°F. When examining the graphs of the average monthly temperatures for various locations, Melissa should focus on the

average = midline

1) amplitude
2) horizontal shift
3) period
4) midline

9. Tides are a periodic rise and fall of ocean water. On a typical day at a seaport, to predict the ~~time of~~ the next high tide, the most important value to have would be the

1) time between consecutive low tides
2) time when the tide height is 20 feet
3) average depth of water over a 24-hour period
4) difference between the water heights at low and high tide

period

amp sin ~~flag~~ shift

10. Consider the function $h(x) = 2\sin(3x) + 1$ and the function q represented in the table below. Determine which function has the *smaller* minimum value for the domain $[-2, 2]$. Justify your answer.

3

1

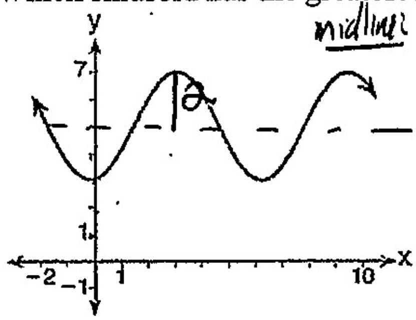
-1

q(x) has the smaller minimum.

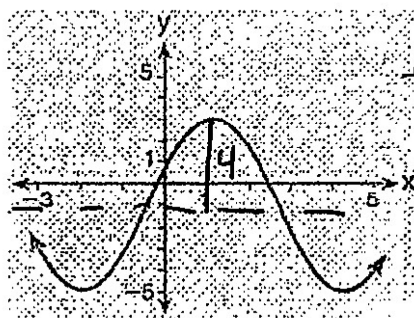
$-8 < -1$

x	$q(x)$
-2	-8
-1	0
0	0
1	-2
2	0

11. Which sinusoid has the greatest amplitude?

1)


midline: $\frac{3+7}{2} = 5$

3)


midline $\frac{3+-5}{2} = -1$

2) $y = 3\sin(\theta - 3) + 5$

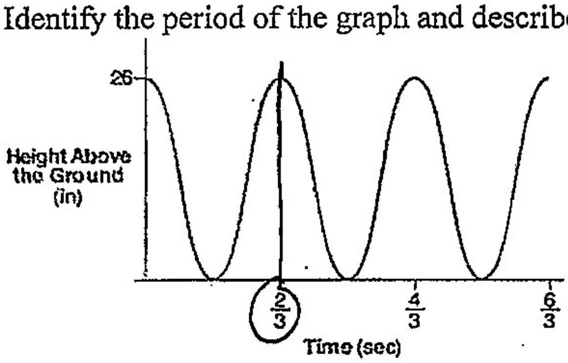
3

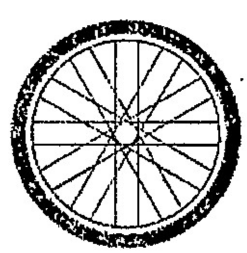
4) $y = -5\sin(\theta - 1) - 3$

5

12. The graph below represents the height above the ground, h, in inches, of a point on a triathlete's bike wheel during a training ride in terms of time, t, in seconds.

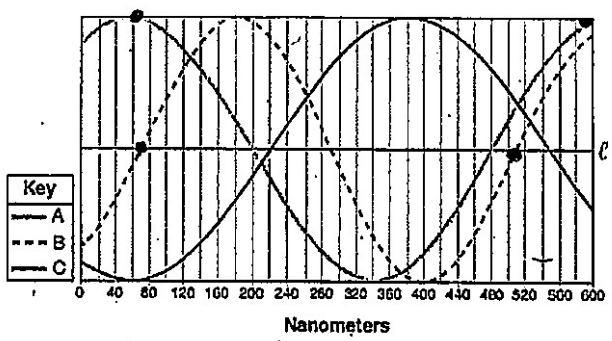
Identify the period of the graph and describe what the period represents in this context.





$\frac{2}{3}$. It takes the wheel $\frac{2}{3}$ of a second to make one complete rotation.

13. Visible light can be represented by sinusoidal waves. Three visible light waves are shown in the graph below. The midline of each wave is labeled ℓ. Based on the graph, which light wave has the longest period? Justify your answer.



C. It is the only one that can't fit one full cycle on the graph.

14. The Sea Dragon, a pendulum ride at an amusement park, moves from its central position at rest according to the trigonometric function $P(t) = -10\sin(\frac{\pi}{3}t)$, where t represents time, in seconds. How many seconds does it take the pendulum to complete one full cycle?

1) 5
2) 6
3) 3
4) 10

period

$P = \frac{2\pi}{f}$

$P = \frac{2\pi}{\frac{\pi}{3}}$ $\quad \frac{2\cancel{\pi}}{1} \cdot \frac{3}{\cancel{\pi}} = 6$

15. A wave displayed by an oscilloscope is represented by the equation $y = 3\sin x$. What is the period of this function?

1) 2π
2) 2
3) 3
4) 3π

$P = \frac{2\pi}{f}$

$P = \frac{2\pi}{1} = 2\pi$

16. The height above ground for a person riding a Ferris wheel after t seconds is modeled by $h(t) = 150\sin\left(\frac{\pi}{45}t + 67.5\right) + 160$ feet. How many seconds does it take to go from the bottom of the wheel to the top of the wheel? half of a full cycle

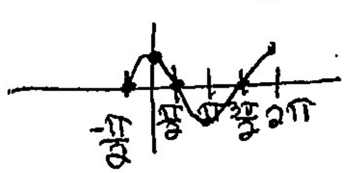
1) 10
2) 45
3) 90
4) 150

$P = \frac{2\pi}{f}$ $\quad P = \frac{2\pi}{\frac{\pi}{45}}$ $\quad \boxed{\frac{1}{2}(90) \\ 45}$

$P = \frac{2\cancel{\pi}}{1} \cdot \frac{45}{\cancel{\pi}} = 90$

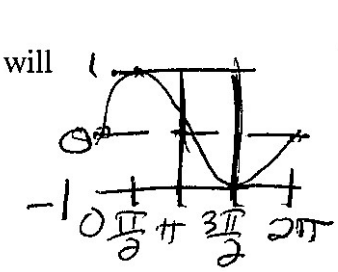
17. As θ increases from $-\frac{\pi}{2}$ to 0 radians, the value of $\cos\theta$ will $\quad P = \frac{2\pi}{1} = 2\pi$

1) decrease from 1 to 0
2) decrease from 0 to -1
3) increase from -1 to 0
4) increase from 0 to 1



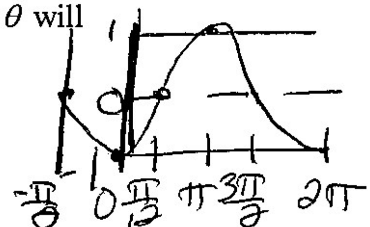
$\frac{2\pi}{4} = \frac{\pi}{2}$

17. As θ increases from π to $\frac{3\pi}{2}$ radians, the graph of $y = \sin\theta$ will



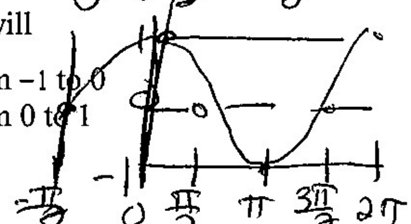
1) Decrease from 1 to 0
2) Decrease from 0 to -1
3) Increase from -1 to 0
4) Increase from 0 to 1

18. As θ increases from $-\frac{\pi}{2}$ to 0 radians, the graph of $y = -\cos\theta$ will



1) Decrease from 1 to 0
2) Decrease from 0 to -1
3) Increase from -1 to 0
4) Increase from 0 to 1

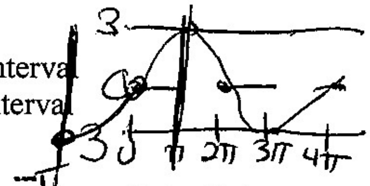
19. As θ increases from $-\frac{\pi}{2}$ to 0 radians, the value of $\cos\theta$ will



1) decrease from 1 to 0
2) decrease from 0 to -1
3) increase from -1 to 0
4) increase from 0 to 1

$\frac{4\pi}{4\pi}$

20. Given $p(\theta) = 3\sin\left(\frac{1}{2}\theta\right)$ on the interval $-\pi < \theta < \pi$, the function p



1) decreases, then increases
2) increases, then decreases
3) decreases throughout the interval
4) increases throughout the interval

$p = \frac{2\pi}{\frac{1}{2}}$

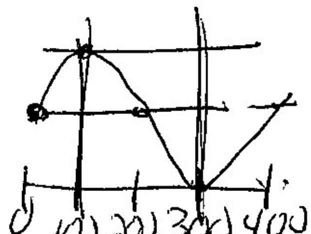
$p = 2\pi \cdot \frac{2}{1} = 4\pi$

21. A sine function increasing through the origin can be used to model light waves. Violet light has a wavelength of 400 nanometers. Over which interval is the height of the wave *decreasing*, only?

p

$\frac{400}{4} = 100$

1) (0, 200)
2) (100, 300)
3) (200, 400)
4) (300, 400)



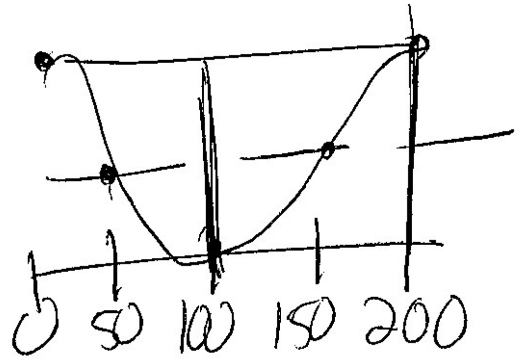
22. A cosine function decreasing through the origin has a frequency of $\frac{\pi}{100}$. What is the first positive interval where the wave is increasing?

$p = \frac{2\pi}{\frac{\pi}{100}}$

$\frac{200}{4} = 50$

$p = 2\cancel{\pi} \cdot \frac{100}{\cancel{\pi}} = 200$

(100, 200)



Probability with $\cap$ (and) and $\cup$ (or)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

If given the two events are independent, use $P(A \cap B) = P(A) \bullet P(B)$

To express as a percent, divide and multiply by 100.

1. The probability of event A is .27. The probability of event B is .36. The probability of both events happening is .11. What is the probability that event A or event B happens?

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = .27 + .36 - .11$

$P(A \cup B) = .52$

2. The probability of event A happening is 14% and the probability of event B happening is 18%, The probability that event A or event B happens is 20%. What is the probability that event A and event B happens?

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(A \cap B) = .14 + .18 - .20$

$P(A \cap B) = .12$

3. On a given school day, the probability that Nick oversleeps is 48% and the probability he has a pop quiz is 25%. Assuming these two events are independent, what is the probability that Nick oversleeps and has a pop quiz on the same day?

$P(A \cap B) = P(A) \cdot P(B)$

$P(A \cap B) = .48 \cdot .25$

$P(A \cap B) = .12$

4. The probability that a student in Jacqua High School is in band is $\frac{127}{466}$ and the probability that a student is on the track team is $\frac{82}{466}$. If the probability that they are on the track team and in band is $\frac{74}{466}$, what is the probability that they are on the track ~~and~~ or in band?

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{127}{466} + \frac{82}{466} - \frac{74}{466}$

$P(A \cup B) = \frac{135}{466}$

5. Over the past 30 nights, Baxter barked 8 nights and cried 15 nights. He barked or cried 11 nights. How many nights did he bark and cry?

$p(A \cap B) = p(A) + p(B) - p(A \cup B)$

$p(A \cap B) = \frac{8}{30} + \frac{15}{30} - \frac{11}{30}$

$p(A \cap B) = \frac{12}{30}$

6. Suppose events A and B are independent and $P(A \text{ and } B)$ is 0.2. Which statement could be true?

1) $P(A) = 0.4$, $P(B) = 0.3$, $P(A \text{ or } B) = 0.5$
2) $P(A) = 0.8$, $P(B) = 0.25$
3) $P(A|B) = 0.2$, $P(B) = 0.2$
4) $P(A) = 0.15$, $P(B) = 0.05$

$p(A \cap B) = p(A) \cdot p(B)$

$.2 = .8 \cdot .25$

$.2 = .2$ ✓

7. The probability that Chloe the cardinal shows up in the Schlansky's backyard is $\frac{12}{19}$. The probability that Chloe shows up in the Silverman's backyard is $\frac{10}{17}$. If the probability that Chloe shows up in the Schlansky's backyard or the Silverman's backyard is $\frac{12}{16}$, what is the probability that Chloe shows up in both backyards?

and

$p(A \cap B) = p(A) + p(B) - p(A \cup B)$

$p(A \cap B) = \frac{12}{19} + \frac{10}{17} - \frac{12}{16}$

$p(A \cap B) = \frac{607}{1292}$

8. A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

and

$p(A \cap B) = p(A) + p(B) - p(A \cup B)$

$p(A \cap B) = \frac{649}{1376} + \frac{433}{1376} - \frac{974}{1376}$

$p(A \cap B) = \frac{27}{344}$

Probability with Two Way Tables

Conditional Probabilities: Circle the row/column that contains the condition. Condition always comes after the phrase given that. You will not always see the phrase given that.
"And" is not conditional.

One-hundred employees of a company were asked their opinion on paying high salaries to the CEO. Their responses are summarized in the following contingency table. Express the following probabilities as fractions and rounded to the nearest percent.

	In Favor	Against	
Male	15	45	60
Female	4	36	40
	19	81	100

1. P(male and in favor)

$\frac{15}{100}$

2. P(female and against)

$\frac{36}{100}$

3. P(male)

$\frac{60}{100}$

4. P(in favor)

$\frac{19}{100}$

5. P(male given that in favor)

$\frac{15}{19}$

6. P(against given that male)

$\frac{45}{60}$

7. P(in favor given that male)

$\frac{15}{60}$

8. P(female given that against)

$\frac{36}{81}$

9. Probability a male is in favor

$\frac{15}{60}$

10. Probability a female is against

$\frac{36}{40}$

11. A statistics class surveyed some students during one lunch period to obtain opinions about television programming preferences. The results of the survey are summarized in the table below.

Programming Preferences

	Comedy	Drama	
Male	70	35	105
Female	48	42	90
	118	77	195

What is the probability that a student is male and prefers comedy?

$\frac{70}{195}$

What is the probability that a male student would prefer comedy?

$\frac{70}{105}$

What is the probability that a student is male? 1 thing

$\frac{105}{195}$

What is the probability that a student is female given that they like drama?

$\frac{42}{77}$

12. A public opinion poll was taken to explore the relationship between age and support for a candidate in an election. The results of the poll are summarized in the table below.

Age	For	Against	No Opinion	
21–40	30	12	8	50
41–60	20	40	15	75
Over 60	25	35	15	75
	75	87	38	200

What is the probability that someone has no opinion? 1 thing

$\frac{38}{200}$

What is the probability that someone is over 60 and against?

$\frac{35}{200}$

What is the probability that someone is for the candidate given that they are between 21-40?

$\frac{30}{50}$

13 ~~14~~. A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

Favorite Type of Program			
	Sports	Reality Show	Comedy Series
Senior	83	110	67
Freshmen	119	103	54

260
276

202 213 121 536

A student response is selected at random from the results. State the *exact* probability the student response is from a freshman, given the student prefers to watch reality shows on television.

$\frac{103}{213}$

14 ~~15~~. At Berkeley Central High School, a survey was conducted to see if students preferred cheeseburgers, pizza, or hot dogs for lunch. The results of this survey are shown in the table below.

	Cheeseburgers	Pizza	Hot Dogs
Females	32	44	24
Males	36	30	34

100
100

68 74 58 200

Based on this survey, what percent of the students preferred pizza?

1) 30
2) 37
3) 44
4) 74

$\frac{74}{200} = 37\%$

15 ~~16~~. A middle school conducted a survey of students to determine if they spent more of their time playing games or watching videos on their tablets. The results are shown in the table below.

	Playing Games	Watching Videos	Total
Boys	138	46	184
Girls	54	142	196
Total	192	188	380

Of the students who spent more time playing games on their tablets, approximately what percent were boys?

1) 41
2) 56
3) 72
4) 75

$\frac{138}{192} = 71.875\%$

16 ~~17~~. A survey was given to 12th-grade students of West High School to determine the location for the senior class trip. The results are shown in the table below.

	Niagara Falls	Darien Lake	New York City	
Boys	56	74	103	233
Girls	71	92	88	251
	127	166	191	484

To the *nearest percent*, what percent of the boys chose Niagara Falls?

1) 12
2) 24
3) 44
4) 56

$\frac{56}{233} = 24\%$

17 ~~18~~. Jenna took a survey of her senior class to see whether they preferred pizza or burgers. The results are summarized in the table below.

	Pizza	Burgers	
Male	23	42	65
Female	31	26	57
	54	68	122

Of the people who preferred burgers, approximately what percentage were female?

1) 21.3
2) 38.2
3) 45.6
4) 61.9

$\frac{26}{68} \simeq 38.2$

18 ~~19~~. Students were asked to name their favorite sport from a list of basketball, soccer, or tennis. The results are shown in the table below.

	Basketball	Soccer	Tennis	
Girls	42	58	20	120
Boys	84	41	5	130
	126	99	25	250

What percentage of the students chose soccer as their favorite sport?

1) 39.6%
2) 41.4%
3) 50.4%
4) 58.6%

$\frac{99}{250} = 39.6\%$

Independence

If events are independent:

$P(A \cap B) = P(A) \bullet P(B)$ → No condition, denominator is always total total. This formula is generally easier to use.

$P(A) = P(A / B)$

1. The results of a poll of 200 students are shown in the table below:

	Preferred Music Style			
	Techno	**Rap**	**Country**	
Female	54	25	27	106
Male	36	40	18	94
	90	65	45	200

A = male
B = techno
(doesn't matter which you pick)

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{36}{200} \neq \frac{94}{200} \cdot \frac{90}{200}$

Not Independent

2. At a local mall, 125 people were asked how they choose to pay for their merchandise. The data is shown in the table below:

	Credit Card	Cash	
Male	40	10	50
Female	60	15	75
	100	25	125

A = male
B = cash
(doesn't matter which you pick)

Does the data suggest that the gender and type of payment are independent of each other? Explain your answer.

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{10}{125} = \frac{50}{125} \cdot \frac{25}{125}$ ✓

Independent because $P(A \cap B) = P(A) \cdot P(B)$

3. One-hundred employees of a company were asked their opinion on paying high salaries to the CEO. Their responses are summarized in the following contingency table.

	In Favor	Against	
Male	15	45	60
Female	4	36	40
	19	81	100

A = female
B = against
(doesn't matter which you pick)

Based on the data, are gender an opinion on salaries independent of each other? Justify your answer.

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{36}{100} \neq \frac{40}{100} \cdot \frac{81}{100}$

Not Independent

4. The results of a survey of the student body at Central High School about television viewing preferences are shown below.

	Comedy Series	Drama Series	Reality Series	Total
Males	95	65	70	230
Females	80	70	110	260
Total	175	135	180	490

Are the events "student is a male" and "student prefers reality series" independent of each other? Justify your answer.

A B

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{70}{490} \neq \frac{230}{490} \cdot \frac{180}{490}$

Not Independent

5. Given events A and B, such that $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cup B) = 0.8$, determine whether A and B are independent or dependent.

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(A \cap B) = .6 + .5 - .8$

$P(A \cap B) = .3$

$P(A \cap B) = P(A) \cdot P(B)$

$.3 = .6 \cdot .5$

Independent

$P(A)=P(A|B) \quad P(B)=P(B|A)$

6. Given events A and B, such that $P(A)=0.8$, $P(B)=0.6$, and $P(A/B)=0.6$. Determine whether A and B are independent. Explain your answer.

$P(A|B) \neq P(A)$ No

$.6 \neq .8$

7. A fast-food restaurant analyzes data to better serve its customers. After its analysis, it discovers that the events *D*, that a customer uses the drive-thru, and *F*, that a customer orders French fries, are independent. The following data are given in a report:

$P(F) = 0.8$

$P(F \cap D) = 0.456$

Given this information, $P(F|D)$ is

1) 0.344
2) 0.3648
3) 0.57
4) 0.8

$P(F|D)=P(F)$

$.8 = .8$

$P(P)=.5 \quad P(R)=.4$

8. Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

1) independent
2) dependent
3) mutually exclusive
4) complements

$P(R|P)=.4$

~~P(R|B~~ $P(R|P)=P(R)$

$.4=.4$

9. Out of the 28 days in February, Jackie made coffee 24 days. Out of the 14 days it rained, Jackie made coffee 12 times. Are the events "Jackie makes coffee" and "it rains" independent of each other? Explain your answer.

$P(C)=\frac{24}{28} \quad P(C|R)=\frac{12}{14}$

$P(C)=P(C|R)$

$\frac{24}{28}=\frac{12}{14}$ Yes!

$\frac{6}{7}=\frac{6}{7}$

10. The probability that Gary and Jane have a child with blue eyes is 0.25, and the probability that they have a child with blond hair is 0.5. The probability that they have a child with both blue eyes and blond hair is 0.125. Given this information, the events blue eyes and blond hair are

I: dependent
II: independent
III: mutually exclusive $P(A \cap B)=0$

1) I, only
2) II, only
3) I and III
4) II and III

$P(\text{Blue})=.25$

$P(\text{Blonde})=.5$

$P(\text{Blue and Blonde})=.125$

$P(A \cap B)=P(A) \cdot P(B)$

$.125=.25 \cdot .5$

$.125=.125$ Independent

Normal Distributions

Normally distributed: Use *normalcdf* (2^{nd} VARS (Distr), 2:normalcdf)

1. The weights of bags of Graseck's Chocolate Candies are normally distributed with a mean of 4.3 ounces and a standard deviation of 0.05 ounces. What is the probability that a bag of these chocolate candies weighs less than 4.27 ounces?

1) 0.2257
2) 0.2743
3) 0.7257
4) 0.7757

normal cdf
lower = 0
upper = 4.27
μ = 4.3
σ = .05

.2743

2. The weight of a bag of pears at the local market averages 8 pounds with a standard deviation of 0.5 pound. The weights of all the bags of pears at the market closely follow a normal distribution. Determine what percentage of bags, to the *nearest integer*, weighed *less* than 8.25 pounds.

Mean

normal cdf
lower = 0
upper = 8.25
μ = 8
σ = 0.5
.6914(100) *because percent
69%

3. The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18. Determine the number of students who scored between 200 and 245.

normal cdf
lower = 200
upper = 245
μ = 225
σ = 8

.9929..(1200) *because asking for #

1191

4. The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the *nearest whole percent*, is

1) 6
2) 48
3) 68
4) 95

normal cdf
lower = 64
upper = 69.5
μ = 64
σ = 2.75

.477..(100) *because percent

48%

5. The lifespan of a 60-watt lightbulb produced by a company is normally distributed with a mean of 1450 hours and a standard deviation of 8.5 hours. If a 60-watt lightbulb produced by this company is selected at random, what is the probability that its lifespan will be between 1440 and 1465 hours?

1) 0.3803
2) 0.4612
3) 0.8415
4) 0.9612

normal cdf
lower = 1440
upper = 1465
μ = 1450
σ = 8.5
.8415

6. The weights of students on the boys cross country team is normally distributed with a mean of 135.3 pounds and a standard deviation of 2.8 pounds. Jackson believes that the probability of a student being between 132 and 134 is greater than the probability of a student being between 135 and 136.5 pounds. Is Jackson correct? Justify your answer.

normal cdf
lower = 132
upper = 134
μ = 135.3
σ = 2.8
.2019...

normal cdf
lower = 135
upper = 136.5
μ = 135.3
σ = 2.8
.2085...

No, .2019... < .2085...

7. The number of hours students spent studying for their Regents exam is normally distributed with a mean of 14 hours and a standard deviation of 3.2 hours. If a student is randomly selected, what is the probability that they spent less than 5 hours studying? What is the probability that a student spent more than 22 hours studying? Round your answer to the nearest tenth of a percent.

normal cdf
lower = 22
upper = 999999
μ = 14
σ = 3.2

.006...(100) *because percent
0.6%

normal cdf
lower = 0
upper = 5
μ = 14
σ = 3.2

.002...(100) *because percent
0.2%

8. The scores on a math test are normally distributed with a mean of 76.2 and a standard deviation of 4.7. If 248 students took the exam, approximately how many students got between a 70 and an 80?

normal cdf
lower = 70
upper = 80
μ = 76.2
σ = 4.7

.697..(248) *because asking for #
173

9. The number of hours of sleep employees at a company get per night is normally distributed with a mean of 7.1 hours and a standard deviation of 1.4 hours. If an employee is randomly selected, what is the probability they sleep between 5 and 8 hours each night? Round your answer to the *nearest percent*. If there are 2500 employees at the company, approximately how many of them, to the nearest person, got less than 5 hours of sleep?

normal cdf
lower = 5
upper = 8
μ = 7.1
σ = 1.4
.673...(100) *because percent
67%

normal cdf
lower = 0
upper = 5
μ = 7.1
σ = 1.4
.0668..(2500) *because asking for #
167

Statistical Studies

A *survey* is a type of observational study that gathers data by asking people a number of questions.

A good (unbiased) sample should be **randomly** selected where every member of the population has a chance of being chosen.

An *observational study* records the values of variables for members of a sample. NO TREATMENT IS ADMINISTERED.

A *controlled experiment* **randomly selects** a sample and **randomly assigns** members of the sample to a treatment and control group. The treatment group receives a treatment while the control group receives a placebo (if possible).

1. Which scenario is best described as an observational study?

1) For a class project, students in Health class ask every tenth student entering the school if they eat breakfast in the morning.
survey
2) A social researcher wants to learn whether or not there is a link between attendance and grades. She gathers data from 15 school districts.
3) A researcher wants to learn whether or not there is a link between children's daily amount of physical activity and their overall energy level. During lunch at the local high school, she distributed a short questionnaire to students in the cafeteria.
survey
4) Sixty seniors taking a course in Advanced Algebra Concepts are randomly divided into two classes. One class uses a graphing calculator all the time, and the other class never uses graphing calculators. A guidance counselor wants to determine whether there is a link between graphing calculator use and students' final exam grades.
Controlled experiment

2. A doctor wants to test the effectiveness of a new drug on her patients. She separates her sample of patients into two groups and administers the drug to only one of these groups. She then compares the results. Which type of study *best* describes this situation?

1) census
2) survey
3) observation
4) controlled experiment

3. Which task is *not* a component of an observational study?

1) The researcher decides who will make up the sample.
2) The researcher analyzes the data received from the sample.
3) The researcher gathers data from the sample, using surveys or taking measurements.
4) The researcher divides the sample into two groups, with one group acting as a control group.

Controlled experiment

4. Which statement about statistical analysis is *false*?

1) Experiments can suggest patterns and relationships in data.
2) Experiments can determine cause and effect relationships.
3) Observational studies can determine cause and effect relationships.
4) Observational studies can suggest patterns and relationships in data.

Only controlled experiments can establish causal relationships

5. A market research firm needs to collect data on viewer preferences for local news programming in Buffalo. Which method of data collection is most appropriate?

1) census
2) survey
3) observation
4) controlled experiment

6. A school cafeteria has five different lunch periods. The cafeteria staff wants to find out which items on the menu are most popular, so they give every student in the first lunch period a list of questions to answer in order to collect data to represent the school. Which type of study does this represent?

1) observation
2) controlled experiment
3) population survey → census
4) sample survey

7. Howard collected fish eggs from a pond behind his house so he could determine whether sunlight had an effect on how many of the eggs hatched. After he collected the eggs, he divided them into two tanks. He put both tanks outside near the pond, and he covered one of the tanks with a box to block out all sunlight. State whether Howard's investigation was an example of a controlled experiment, an observation, or a survey. Justify your response.

Controlled experiment.
He applied a treatment (Sunlight).

8. Darryl conducted a study comparing the statistics of baseball players in the steroid era compared to the non steroid era. Would this investigation be an example of a controlled experiment, an observation, or a survey? Justify your response.

Observational Study. He did not divide into two groups and apply a treatment.

Surveys (Choosing a sample)
A good sample is random. For example, every fifth student walking in the building.
A bad sample is bias.

1. Which statement(s) about statistical studies is true?

I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying.

II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school. × 9th graders don't drive

III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites. × students is not a sample for adults

IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don't like math. × not all students take calculus

1) I, only 2) II, only 3) I and III 4) III and IV

2. Which survey is *least* likely to contain bias?
1) surveying a sample of people leaving a movie theater to determine which flavor of ice cream is the most popular
2) surveying the members of a football team to determine the most watched TV sport football players like football
3) surveying a sample of people leaving a library to determine the average number of books a person reads in a year People at the library read books
4) surveying a sample of people leaving a gym to determine the average number of hours a person exercises per week People at the gym exercise more than most

3. A survey is to be conducted in a small upstate village to determine whether or not local residents should fund construction of a skateboard park by raising taxes. Which segment of the population would provide the most unbiased responses?
1) a club of local skateboard enthusiasts they will all say yes
2) senior citizens living on fixed incomes they will all say no
3) a group opposed to any increase in taxes they will all say no
4) every tenth person 18 years of age or older walking down Main St.

4. A survey is being conducted about American's favorite musicians. Which of the following survey methods would most likely produce a random sample?
1) Asking every 20^{th} person at a Green Day concert → they all like rock music
2) Asking every 10^{th} person at a vintage record store → they all like old music
3) Asking every 10^{th} person at the Westbury Public Library → this is only one community in America
4) Sending out surveys to random households across the country.

5. Which statement about data collection is most accurate?

1) A survey about parenting styles given to every tenth student entering the library will provide unbiased results. students "shouldn't" be parents
2) An observational study allows a researcher to determine the cause of an outcome. only controlled experiment
3) Margin of error increases as sample size increases.
4) A survey collected from a random sample of students in a school can be used to represent the opinions of the school population.

6. Which method of collecting data would most likely result in an unbiased random sample?

1) selecting every third teenager leaving a movie theater to answer a survey about entertainment they like going to the movies
2) placing a survey in a local newspaper to determine how people voted in the 2004 presidential election people won't send it back
3) selecting students by the last digit of their school ID number to participate in a survey about cafeteria food
4) surveying honor students taking Trigonometry to determine the average amount of time students in a school spend doing homework each night Honors students do more homework

7. A survey is to be completed to determine how much time adults spend working out. Which sample would be the best in order to represent the given population?

1) Every 5th student entering the local high school not adults
2) Every 10th person entering a local gym they work out more than most
3) Every 8th person walking down main street
4) Every 3rd woman entering a nail salon only women sampled. ~~women who~~ also only women who get their nails done.

8. The yearbook staff has designed a survey to learn about student opinions on how the yearbook could be improved for this year. If they want to distribute this survey to 100 students and obtain the most reliable data, they should survey

1) Every third student sent to the office
2) Every third student to enter the library
3) Every third student to enter the gym for the basketball game

not everyone goes to these things

4) Every third student arriving at school in the morning

Name Schlansky
Mr. Schlansky

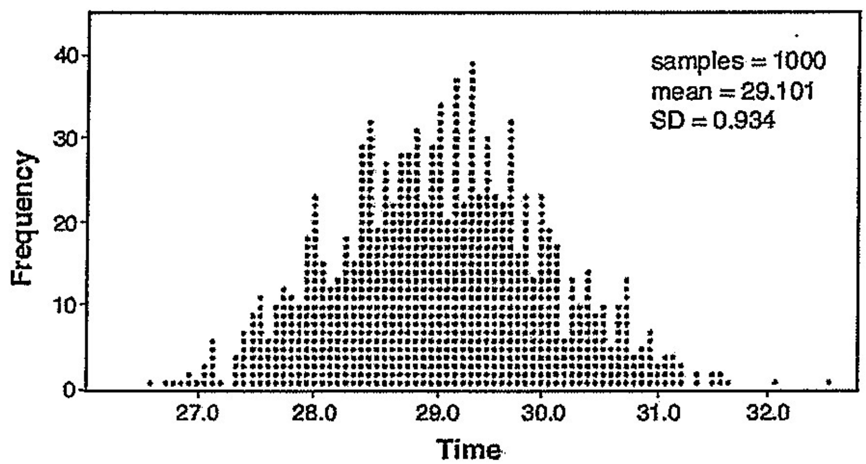
Date
Algebra II

Sample Distributions Part III

1. A radio station claims to its advertisers that the mean number of minutes commuters listen to the station is 30. The station conducted a survey of 500 of their listeners who commute. The sample statistics are shown below.

$\bar{x}$	29.11
s_x	20.718

A simulation was run 1000 times based upon the results of the survey. The results of the simulation appear below.



Based on the simulation results, is the claim that commuters listen to the station on average 30 minutes plausible? Explain your response including an interval containing the middle 95% of the data, rounded to the *nearest hundredth.*

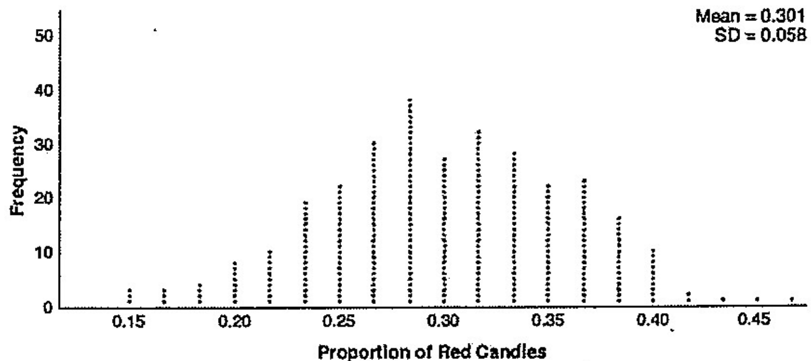
$CI = \bar{x} \pm 2S_x$

$= 29.101 + 2(.934) = 30.97$ $[27.23, 30.97]$

$= 29.101 - 2(.934) = 27.23$

Yes, 30 is inside the confidence interval.

2 ~~3~~. Mary bought a pack of candy. The manufacturer claims that 30% of the candies manufactured are red. In her pack, 14 of the 60 candies are red. She ran a simulation of 300 samples, assuming the manufacturer is correct. The results are shown below.



confidence interval

Based on the simulation, determine the middle 95% of plausible values that the proportion of red candies in a pack is within. Based on the simulation, is it unusual that Mary's pack had 14 red candies out of a total of 60? Explain.

CI = mean ± 2(standard deviation)

CI = .301 ± 2(.058)

CI = .301 + 2(.058) = .417

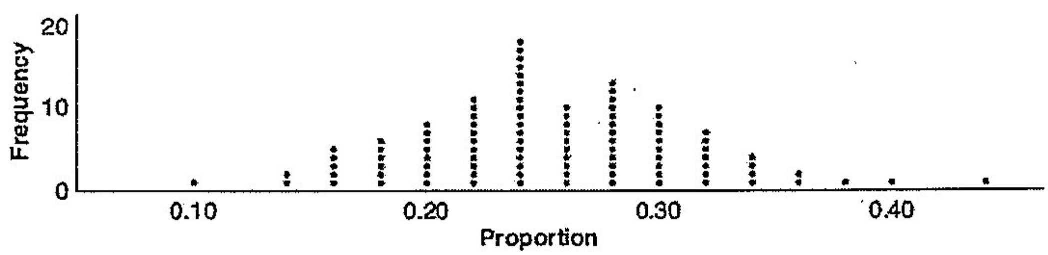
.301 − 2(.058) = .185

[.185, .417]

$\frac{14}{60} = .2\overline{3}$

No, it is usual because $.2\overline{3}$ is in the confidence interval.

3 ~~4~~. A group of students was trying to determine the proportion of candies in a bag that are blue. The company claims that 24% of candies in bags are blue. A simulation was run 100 times with a sample size of 50, based on the premise that 24% of the candies are blue. The approximately normal results of the simulation are shown in the dot plot below.



The simulation results in a mean of 0.254 and a standard deviation of 0.060. Based on this simulation, what is a plausible interval containing the middle 95% of the data? A student found that 18 out of 50 of the candies were blue. Use statistical evidence to explain why this is an expected value.

CI = mean ± 2(standard deviation)

CI = .254 ± 2(.060)

.254 + 2(.060) = .374

.254 − 2(.060) = .134

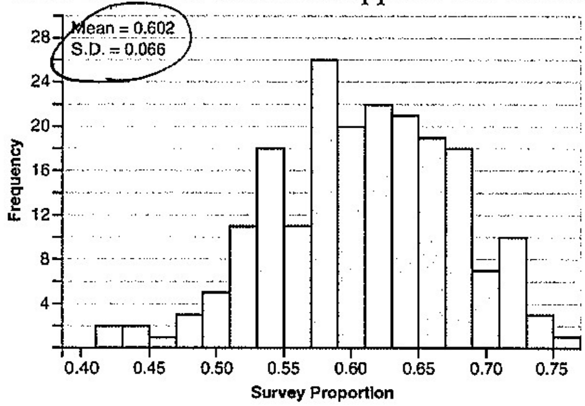
[.134, .374]

$\frac{18}{50} = .36$

.36 is inside the confidence interval.

4. Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band. A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.

Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.* Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50%-50% split. Explain what statistical evidence supports this concern.



CI = Mean ± 2(SD)

CI = .602 + 2(.066) = .73

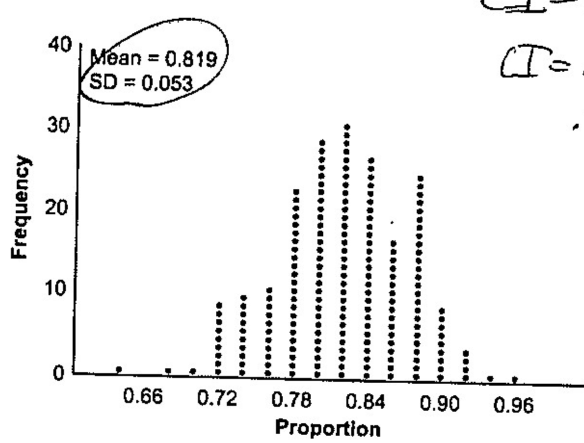
.602 − 2(.066) = .47

[.47, .73]

.5 is inside the confidence interval

5. State officials claim 82% of a community want to repeal the 30 mph speed limit on an expressway. A community organization devises a simulation based on the claim that 82% of the community supports the repeal. Each dot on the graph below represents the proportion of community members who support the repeal. The graph shows 200 simulated surveys, each of sample size 60.

CI = mean ± 2(SD)



CI = .819 + 2(.053) = .925

.819 − 2(.053) = .713

[.713, .925]

Based on the simulation, determine an interval containing the middle 95% of plausible proportions. Round your answer to the *nearest thousandth.* The community organization conducted its own sample survey of 60 people and found 70% supported the repeal. Based on the results of the simulation, explain why the organization should question the State officials' claim.

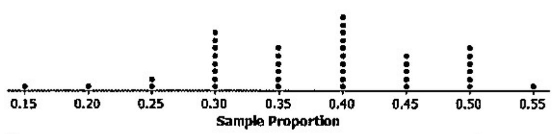
.7 is not inside the confidence interval.

Fair

A coin (or other object) is fair if the theoretical probability (.5 for a coin) is in the confidence interval of the *actual coin/object.*

A coin (or other object) is fair if the actual outcome is in the confidence interval of a *fair coin/object.*

1. A student wanted to decide whether or not a particular coin was fair. She flipped the coin 20 times, calculated the proportion of heads, and repeated this process a total of 40 times. Below is the sampling distribution of sample proportions of heads. The mean and standard deviation of the sampling distribution are 0.379 and 0.091, respectively. Do you think this was a fair coin? Why or why not?



CI = mean ± 2(SD)

CI = .379 + 2(.091) = .561

.379 − 2(.091) = .197

[.197, .561]

Yes, .5 (theoretical probability) is inside the confidence interval

2. A spinner with 10 sectors labeled 1-10 is spun and it lands on the number one 2 times out of 50. David believes the spinner is unfair to he repeats the process many times and creates a sample distribution. He finds that the mean is .098 and the standard deviation is .04. Is the spinner fair? Explain your answer.

[.018, .178]

$\frac{1}{10} = .1$

CI = mean ± 2(SD)

= .098 + 2(.04) = .178

.098 − 2(.04) = .018

Yes, .1 (theoretical probability) is inside the confidence interval.

3. Juanita rolls a 6 sided die and recorded that it landed on 6 five times out of 50. She questioned whether the die was fair so she ran a computer simulation of 1000 samples of 50 rolls of her die. The mean of the simulation was .094 with a standard deviation of .028. Is her die fair? Explain your answer.

[.038, .15]

$\frac{1}{6} = .1\overline{6}$

CI = mean ± 2(SD)

.094 + 2(.028) = .15

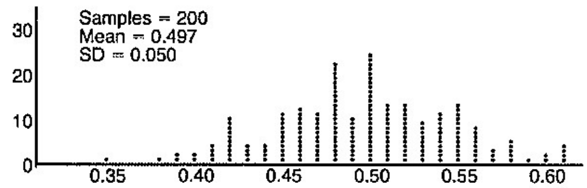
.094 − 2(.028) = .038

No, $.1\overline{6}$ (theoretical probability) is not inside the confidence interval

$\frac{73}{100} = .73$

4. Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

Given the results of her coin flips and of her computer simulation, which statement is most accurate?



1) 73 of the computer's next 100 coin flips will be heads.
2) 50 of her next 100 coin flips will be heads.
3) Her coin is not fair. Her result was not in the CI of a fair coin.
4) Her coin is fair.

~~The threshold p~~

$CI = .497 + 2(.050) = .597$

$.497 - 2(.050) = .397$

$[.397, .597]$

5. Juanita rolls a 6 sided die and recorded that it landed on 6 five times out of 50. She questioned whether the die was fair so she ran a computer simulation of 1000 samples of 50 rolls of a fair die. The mean of the simulation was .159 with a standard deviation of .102. Is her die fair? Explain your answer.

$\frac{6}{50} = .12$

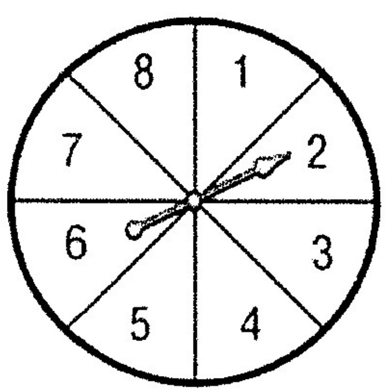
$CI = .159 + 2(.102) = .363$

$.159 - 2(.102) = -.045$

$[-.045, .363]$

Yes, her result was inside the CI of a fair die.

6. A spinner below is spun and it landed on the number "2" 3 times out of 50. A computer simulation of 500 samples of 50 spins of a fair spinner was spun. The mean of the simulation was .128 and the standard deviation was .07. Is the spinner fair? Explain your answer.



$CI = .128 + 2(.07) = .268$

$\frac{3}{50} = .06$

$.128 - 2(.07) = -.012$

$[-.012, .268]$

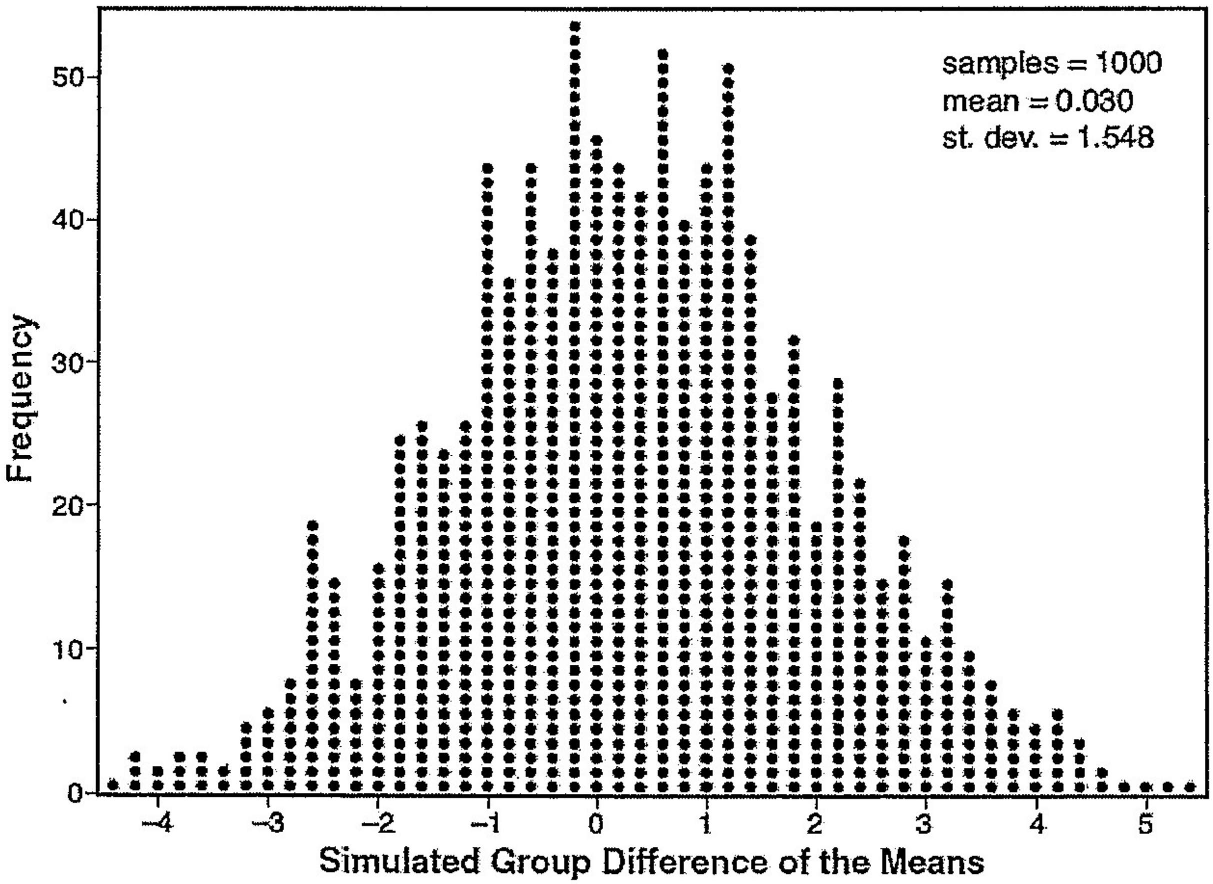
Yes, the result was inside the CI of a fair spinner.

4. Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

	Scented Paper	Unscented Paper
$\bar{x}$	23	18
s_x	2.898	2.408

$23-18=5$

Calculate the difference in means in the experimental test grades (scented -unscented). A simulation was conducted in which the subjects' scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.



Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the *nearest hundredth*. Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.

$CI=\bar{x}\pm 2s_x$

$CI=.03+2(1.548)=3.13$

$.03-2(1.548)=-3.07$

$[-3.07, 3.13]$

2.

The effects of caffeine on the body have been extensively studied. In one experiment, researchers trained a sample of male college students to tap their fingers at a rapid rate. The sample was then divided at random into two groups of 10 students each. Each student drank the equivalent of about two cups of coffee, which included about 200 mg of caffeine for the students in one group but was decaffeinated coffee for the second group. After a 2-hour period, each student was tested to measure finger tapping rate (taps per minute). The students did not know whether or not their drinks included caffeine and the person measuring the tap rates was also unaware of the groups. The finger-tapping rates measured in this experiment are summarized in the table below.

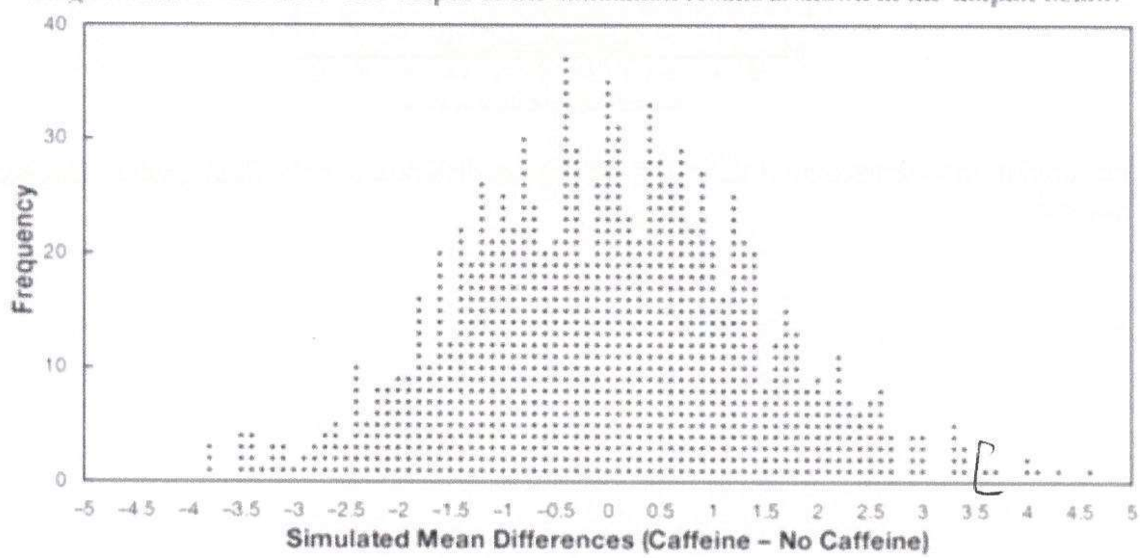
											Mean
Caffeine	246	248	250	252	248	250	246	248	245	250	248.3
No Caffeine	242	245	244	248	247	248	242	244	246	242	244.8

Calculate the mean difference (Caffeine – No Caffeine) and interpret your answer in the context of the problem.

$248.3 - 244.8 = 3.5$

On average, the students in the caffeine group tapped 3.5 more times per minute than the non-caffeine group.

The researchers then took the twenty finger-tapping rates and rerandomized them 1,000 times using simulation software. The output of the simulation results is shown in the dotplot below.



$\frac{7}{1000} = .7\%$

Does the simulation data support the conclusion that caffeine causes an increase in average finger-tapping rate? Justify your answer.

Yes, the mean difference occurred less than 5% of the time as it outside the confidence interval.

~~**To determine if a treatment is effective**~~

~~1) **Find the mean difference between the treatment and control group**~~

~~2) Rerandomize the sample many times and record the mean differences on a dot plot~~

~~**If the mean difference falls within the confidence interval (more than 5%), the treatment is not effective.**~~

~~**If the mean difference falls outside the confidence interval (less than 5%), the treatment is effective.**~~

3. Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year. A summary of the two groups' final grades is shown below:

	Group 1	Group 2
$\bar{x}$	80.16	83.8
S_x	6.9	5.2

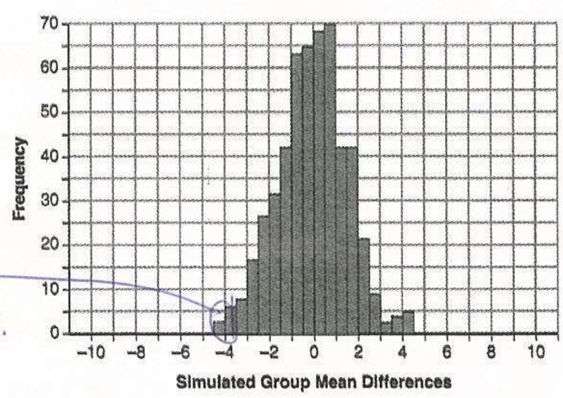
$80.16 - 83.8 = -3.64$

On average, students in group 1 scored 3.64 points lower than group 2.

Calculate the mean difference in the final grades (group 1 – group 2) and explain its meaning in the context of the problem. A simulation was conducted in which the students' final grades were rerandomized 500 times. The results are shown below.

$6+3 = \frac{9}{500} = .018$

1.8%



Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.

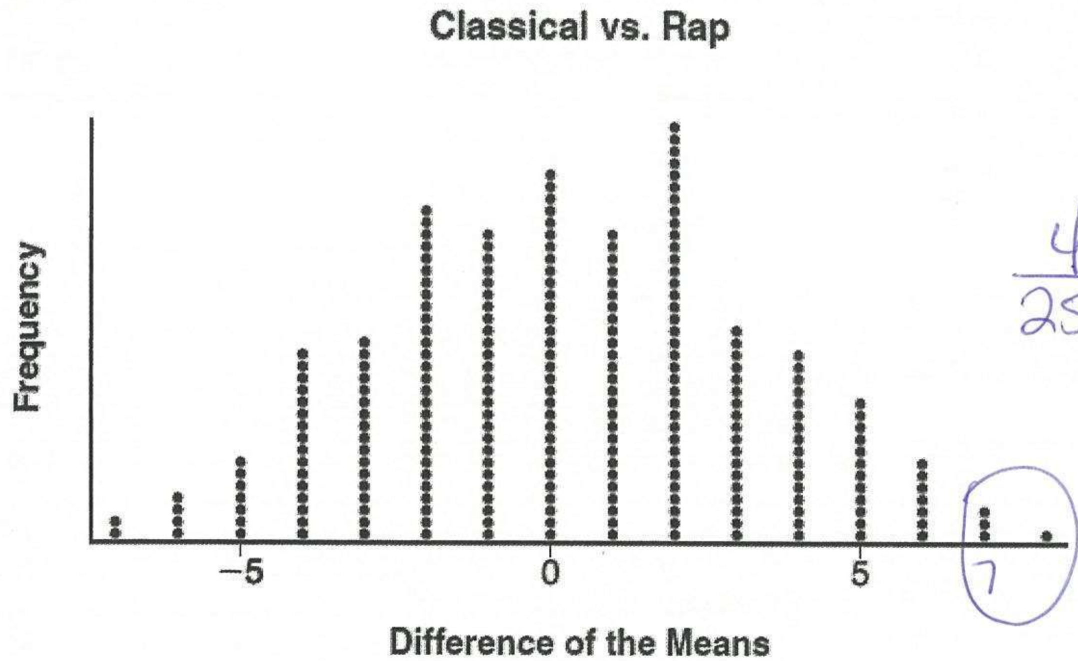
Yes, -3.64 or more extreme occured in less than 5% of the simulations so it is outside of the confidence interval.

4. To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

Classical: 74, 83, 77, 77, 84, 82, 90, 89
Rap: 77, 80, 78, 74, 69, 72, 78, 69

He found the average of the two groups and subtracted them. On average, the students listening to classical music scored 7 points higher than those listening to rap.

John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer. To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.



$\frac{4}{250} = .016$

1.6%

Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.

Yes, 7 occurred in less than 5% of the simulations. 7 is not in the confidence interval.

Exponential Regression Equations

1) **Stat, Edit**
2) **Input x column into L1 and y column into L2**
3) **Stat, Calc, 0: ExpReg**
4) **READ AND ROUND CAREFULLY**

1. Consider the data in the table below.
State an exponential regression equation to model these data, rounding all values to the *nearest thousandth.*

x	1	2	3	4	5	6
y	3.9	6	11	18.1	28	40.3

ExpReg
$y=a(b)^x$
$y=2.459(1.616)^x$

2. A runner is using a nine-week training app to prepare for a "fun run." The table below represents the amount of the program completed, *A*, and the distance covered in a session, *D*, in miles.

A	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{8}{9}$	1
D	2	2	2.25	3	3.25

ExpReg
$y=a(b)^x$
$y=1.223(2.652)^x$

Based on these data, write an exponential regression equation, rounded to the *nearest thousandth*, to model the distance the runner is able to complete in a session as she continues through the nine-week program.

3. A cup of coffee is left out on a countertop to cool. The table below represents the temperature, $F(t)$, in degrees Fahrenheit, of the coffee after it is left out for t minutes.

t	0	5	10	15	20	25
F(t)	180	144	120	104	93.3	86.2

Based on these data, write an exponential regression equation, $F(t)$, to model the temperature of the coffee. Round all values to the *nearest thousandth.*

ExpReg
$y=a(b)^x$
$F(t)=169.136(.971)^t$

Exponential Regression Equations * Make sure you write an equation with a y and an x

1) **Stat, Edit**
2) **Input x column into L1 and y column into L2**
3) **Stat, Calc, 0: ExpReg**
4) **READ AND ROUND CAREFULLY**

ExpReg

4. The accompanying table shows the number of bacteria present in a certain culture over a 5-hour period, where x is the time, in hours, and y is the number of bacteria.

x	y
0	1,000
1	1,049
2	1,100
3	1,157
4	1,212
5	1,271

ExpReg

$y=a(b)^x$

$y=999.9725(1.0493)^x$

$y=999.9725(1.0493)^{6.5}$

$y=1367$

Write an exponential regression equation for this set of data, rounding all values to *four decimal places*. Using this equation, determine the number of whole bacteria present after 6.5 hours.

5. The accompanying table shows the amount of water vapor, y, that will saturate 1 cubic meter of air at different temperatures, x.

Amount of Water Vapor That Will Saturate 1 Cubic Meter of Air at Different Temperatures

Air Temperature (x) (°C)	Water Vapor (y) (g)
−20	1
−10	2
0	5
10	9
20	17
30	29
40	50

ExpReg

$y=a(b)^x$

$y=4.194(1.068)^x$

$y=4.194(1.068)^{50}$

$y=112.5$

Write an exponential regression equation for this set of data, rounding all values to the *nearest thousandth*. Using this equation, predict the amount of water vapor that will saturate 1 cubic meter of air at a temperature of 50°C, and round your answer to the *nearest tenth of a gram*.

6. Jean invested \$380 in stocks. Over the next 5 years, the value of her investment grew, as shown in the accompanying table.

Years Since Investment (x)	Value of Stock, in Dollars (y)
0	380
1	395
2	411
3	427
4	445
5	462

Write the exponential regression equation for this set of data, rounding all values to *two decimal places*. Using this equation, find the value of her stock, to the *nearest dollar*, 10 years after her initial purchase.

ExpReg

$y=a(b)^x$

$y=379.92(1.04)^x$

$y=379.92(1.04)^{10}=562$

Complex Formulas

List what each variable represents and carefully substitute the appropriate values in

1. After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below. If the value of k is .066, write an equation to determine the temperature of the turkey after *t* hours. Determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m.

T_0 T_a $t=7$

$T = T_a + (T_0 - T_a)e^{-kt}$

$325 =$ T_a = the temperature surrounding the object

$68 =$ T_0 = the initial temperature of the object

t = the time in hours $=7$

T = the temperature of the object after t hours

$.066 =$ k = decay constant

$T=325+(68-325)e^{-.066t}$

$T=325+(68-325)e^{-.066(7)}$

$T=163$

2. The loudness of sound is measured in units called decibels (dB). These units are measured by first assigning an intensity I_0 to a very soft sound that is called the threshold sound. The sound to be measured is assigned an intensity, *I*, and the decibel rating, *d*, of this sound is found using $d = 10\log\frac{I}{I_0}$. The threshold sound audible to the average person is 1.0×10^{-12} W/m² (watts per square meter). Consider the following sound level classifications. How would a sound with intensity 6.3×10^{-3} W/m² be classified?

I_0

1) moderate I
2) loud
3) very loud
4) deafening

Moderate	45-69 dB
Loud	70-89 dB
Very loud	90-109 dB
Deafening	>110 dB

I_0 = threshold Sound

I = Intensity

d = decibel rating

$d = 10\log\frac{6.3\times10^{-3}}{1.0\times10^{-12}} = 97..$

3. A baseball is hit straight up from a height of 6 feet with an initial velocity of 90 feet per second. The equation that models the height of the ball, *s*, as a function of time, *t*, is $s = -16t^2 + v_0t + s_0$ where v_0 is the initial velocity and s_0 is the initial height. How high is the ball after 4 seconds?

s_0 v_0 t

s = height of ball

4 = t = time

90 = V_0 = initial velocity

6 = S_0 = initial height

$s=-16(4)^2+90(4)+6$

$s=110$ feet

4. The Fahrenheit temperature, $F(t)$, of a heated object at time t, in minutes, can be modeled by the function below. F_s is the surrounding temperature, F_0 is the initial temperature of the object, and k is a constant.

F_0 $$F(t) = F_s + (F_0 - F_s)e^{-kt}$$

Hot chocolate at a temperature of 200°F is poured into a container. The room temperature is kept at a constant 68°F and k = 0.05. What will be the temperature of the hot chocolate after 10 minutes has passed to the *nearest* degree.

$F(t)$ = temp of object

68 F_s = surrounding temp

200 F_0 = initial temp

.05 k = constant

10 t = time

$F(t) = 68 + (200-68)e^{-.05(10)}$

$F(t) = 148°$

5. A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, t, in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation $t = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum in meters and g is a constant of 9.81 m/s². The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the *nearest tenth of a second*, the time it takes this pendulum to complete one swing.

g

L

t = t = time

67 = L = Length of Pendulum

9.81 = g = 9.81

$t = 2\pi\sqrt{\frac{67}{9.81}}$

$t = 16.4$

d

6. Kayla puts \$3,000 into an account at the beginning of each year in order to save for a down payment on a house she plans to purchase in 8 years. She decides to invest in a savings account which gets 3.5% interest, compounded at the end of each year. Assume she make the same deposit on January 1st each of the 8 years and makes no other deposits or withdrawals throughout the year. Use the formula, $A = \frac{d}{r}((1+r)^t - 1)$

t

where A is the amount of money in the account after t years, d is the number of dollars invested at the beginning of each year, and r is the annual interest rate of the account, expressed as a decimal. How much money will Kayla have available for her down payment?

A = A = amount of money after

3000 = d = dollars at beginning of each year

.035 = r = annual interest rate

8 = t = years

$A = \frac{3000}{.035}((1+.035)^8 - 1)$

$A = 27,155.06$

Factoring:

Greatest Common Factor: GCF(   )

Difference of Two Squares: $(\sqrt{_1}+\sqrt{_2})(\sqrt{_1}-\sqrt{_2})$

Trinomials: (x   )(x   )

1) First sign comes down
2) The two signs must multiply for the last sign
3) Find two numbers that multiply to the last number and add/subtract to the middle number

Bridge Method: (Trinomial with a leading coefficient bigger than 1)

1) Build a bridge between the first and last numbers (Multiply)
2) Factor Trinomial Normally
3) Pay the toll (Divide by the leading coefficient)

*If possible, reduce the fraction

If they divide nicely, divide them

If not, put the denominator in front of the variable inside the parenthesis

Grouping: (4 Terms or More)

1) Look for a pattern in the exponents to determine the groups. **You cannot have two terms with the same exponent in the same group.**
2) Factor out the GCF in each group

*You should be left with the same factor. If signs are reversed, factor out a negative

3) Combine coefficients and keep like term.

***Factor further if necessary**

Sum/Difference of Two Cubes

SOAP for signs (Same, Opposite, Always Positive)

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

Substitution Trinomials:

1) Replace binomial with y
2) Factor normally
3) Substitute back

*Factor further if possible

Factor each expression

1. $\frac{4x}{4} + \frac{8}{4}$

$4(x+2)$

2. $\frac{12x}{6} + \frac{18}{6}$

$6(2x+3)$

3. $\frac{x^2}{x} - \frac{7x}{x}$

$x(x-7)$

4. $\frac{2x^2}{2x} - \frac{4xy}{2x}$

$2x(x-2y)$

5. $\frac{5x^2y}{5x} - \frac{20x}{5x}$

$5x(xy-4)$

6. $\sqrt{x^2} - \sqrt{64}$ DOTS

$(x+8)(x-8)$

7. $\sqrt{y^2} - \sqrt{36}$ DOTS

$(y+6)(y-6)$

8. $\sqrt{4t^2} - \sqrt{25}$ DOTS

$(2t+5)(2t-5)$

9. $\sqrt{9x^2} - \sqrt{16y^4}$ DOTS

$(3x+4y^2)(3x-4y^2)$

10. $\sqrt{36} - \sqrt{25x^2}$ DOTS

$(6+5x)(6-5x)$

11. $\sqrt{100y^4} - \sqrt{49t^6}$ DOTS

$(10y^2+7t^3)(10y^2-7t^3)$

12. $1 - 9x^8y^4$ DOTS

$(1+3x^4y^2)(1-3x^4y^2)$

13. $x^2 + 4x - 12$ 1,12 2,6 3,4

$(x+6)(x-2)$

14. $y^2 + 3y + 2$ 1,2

$(y+2)(y+1)$

$1,15$
$3,5$

15. $m^2 - 8m + 15$

$(m-5)(m-3)$

$1,20$
$2,10$
$4,5$

16. $x^2 - 8x - 20$

$(x-10)(x+2)$

$1,14$
$2,7$

17. $y^2 + 5y - 14$

$(y+7)(y-2)$

$1,12$
$2,6$
$3,4$

18. $x^2 + 1x - 12$

$(x+4)(x-3)$

$1,10$
$2,5$

19. $x^2 - 3x - 10$

$(x-5)(x+2)$

$1,12$
$2,6$
$3,4$

20. $x^2 - 7x + 12$

$(x-4)(x-3)$

$1,36$ $4,9$
$2,18$ $6,6$
$3,12$

21. $x^2 - 9x - 36$

$(x-12)(x+3)$

$1,110$ $10,11$
$2,55$
$5,22$

22. $y^2 - 21y + 110$

$(y-11)(y-10)$

$1,12$
$2,6$
$3,4$

23. $x^4 + 4x^2 - 12$

$(x^2+6)(x^2-2)$

$1,9$
$3,3$

24. $x^6 - 6x^3 + 9$

$(x^3-3)(x^3-3)$

$1,9$
$3,3$

25. $x^4 - 8x^2 - 9$

$(x^2-9)(x^2+1)$

$(x+3)(x-3)$

$1,2$

26. $x^4 + x^2 - 2$

$(x^2+2)(x^2-1)$ DOTS

$(x^2+2)(x+1)(x-1)$

27. $\frac{2x^2}{2} - \frac{50}{2}$

$2(x^2-25)$ DOTS

$2(x+5)(x-5)$

28. $\frac{2x^2}{2} - \frac{8x}{2} - \frac{10}{2}$

$2(x^2-4x-5)$

$2(x-5)(x+1)$

29. $\frac{3x^2}{3} + \frac{9x}{3} - \frac{12}{3}$

$3(x^2+3x-4)$

$3(x+4)(x-1)$

30. $\frac{6x^2}{6} - \frac{54}{6}$

$6(x^2-9)$ DOTS

$6(x+3)(x-3)$

31. $2x^2+14x+24$

$\frac{2x^2}{2}+\frac{14x}{2}+\frac{24}{2}$

$2(x^2+7x+12)$

$2(x+4)(x+3)$

32. $5x^2-500$

$\frac{5x^2}{5}-\frac{500}{5}$

$5(x^2-100)$

$5(x+10)(x-10)$

33. $ax^2-2ax-8a$

$\frac{ax^2}{a}-\frac{2ax}{a}-\frac{8a}{a}$

$a(x^2-2x-8)$

$a(x-4)(x+2)$

34. yx^2-64y

$\frac{yx^2}{y}-\frac{64y}{y}$

$y(x^2-64)$

$y(x+8)(x-8)$

35. $12x^2-75$

$\frac{12x^2}{3}-\frac{75}{3}$

$3(4x^2-25)$

$3(2x+5)(2x-5)$

36. x^4-81

DOTS $(x^2-9)(x^2+9)$

DOTS $(x+3)(x-3)(x^2+9)$

37. $2y^2-5y-7$ PT

$y^2-5y-14$

$(y-\frac{7}{2})(y+\frac{2}{2})$

$(2y-7)(y+1)$

38. $2x^2+15x-8$ PT

$x^2+15x-16$

$(x+\frac{16}{2})(x-\frac{1}{2})$

$(x+8)(2x-1)$

39. $2x^2+7x-4$ PT

x^2+7x-8

$(x+\frac{8}{2})(x-\frac{1}{2})$

$(x+4)(2x-1)$

40. $6x^2-11x-10$ PT

$x^2-11x-60$

$(x-\frac{15}{6})(x+\frac{4}{6})$ *reduce

$(x-\frac{5}{2})(x+\frac{2}{3})$

$(2x-5)(3x+2)$

41. $2x^2-9x-18$ PT

$x^2-9x-36$

$(x-\frac{12}{2})(x+\frac{3}{2})$

42. $3x^2+2x-8$ PT

$x^2+2x-24$

$(x+\frac{6}{3})(x-\frac{4}{3})$

$(x+2)(3x-4)$

43. $8x^2+7x-1$ PT

x^2+7x-8

$(x+\frac{8}{8})(x-\frac{1}{8})$

$(x+1)(8x-1)$

44. $6x^2+x-12$ PT

x^2+x-72

$(x+\frac{9}{6})(x-\frac{8}{6})$ *reduce

$(x+\frac{3}{2})(x-\frac{4}{3})$

$(2x+3)(3x-4)$

45. $(x^3+6x^2)(-3x-18)$

$\quad x^2 \quad x^2 \quad -3 \quad -3$

$x^2(x+6)-3(x+6)$

$(x^2-3)(x+6)$

46. $(x^3+10x^2)(-9x-90)$

$\quad x^2 \quad x^2 \quad -9 \quad -9$

$x^2(x+10)-9(x+10)$

$(x^2-9)(x+10)$

$(x+3)(x-3)(x+10)$

47. $(x^3+3x^2)(-9x-27)$

$\quad x^2 \quad x^2 \quad -9 \quad -9$

$x^2(x+3)-9(x+3)$

$(x^2-9)(x+3)$

$(x+3)(x-3)(x+3)$

48. $(8x^3+12x^2)(-2x-3)$

$\quad 4x^2 \quad 4x^2 \quad -1 \quad -1$

$4x^2(2x+3)-1(2x+3)$

$(4x^2-1)(2x+3)$

$(2x+1)(2x-1)(2x+3)$

49. $(x^3-3x^2+2x)(4x^2-12x+8)$

$\quad x \quad x \quad x$

$x(x^2-3x+2)+4(x^2-3x+2)$

$(x+4)(x^2-3x+2)$

$(x+4)(x-2)(x-1)$

50. $(3x^3+x^2)(-12x^2-4x)(-63x-21)$

$\quad x^2 \quad x^2 \quad -4x \quad -4x \quad -21 \quad -21$

$x^2(3x+1)-4x(3x+1)-21(3x+1)$

$(x^2-4x-21)(3x+1)$

$(x-7)(x+3)(3x+1)$

51. $(x^2+5x)^2-2(x^2+5x)-24$ $\quad y=x^2+5x$

$y^2-2y-24$

$(y-6)(y+4)$

$(x^2+5x-6)(x^2+5x+4)$

$(x+6)(x-1)(x+4)(x+1)$

52. $(x^2-2x)^2-11(x^2-2x)+24$ $\quad y=x^2-2x$

$y^2-11y+24$

$(y-8)(y-3)$

$(x^2-2x-8)(x^2-2x-3)$

$(x-4)(x+2)(x-3)(x+1)$

$a=y \quad b=5$

53. y^3-125

$(a^3-b^3)=(a-b)(a^2+ab+b^2)$

$=(y-5)(y^2+5y+25)$

$a=z \quad b=4$

54. z^3+64

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$z^3+64=(z+4)(z^2-4z+16)$

$2x \quad y^2$

55. $8x^3+y^6$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$8x^3+y^6=(2x+y^2)(4x^2-2xy^2+y^4)$

$a=y^3 \quad b=6x$

56. y^9-216x^3

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$y^9-216x^3=(y^3-6x)(y^6+6xy^3+36x^2)$

Reducing Rational Expressions

1) Factor

2) Cancel Common Factors

*If a factor is written backwards with a minus sign, they cancel to -1.

Express each of the following in simplest form

GCF / DOTS 1. $\frac{2x+6}{x^2-9}$ $\quad \frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \frac{2}{x-3}$

GCF / Tri 2. $\frac{10-5x}{x^2+2x-8}$ $\quad \frac{5\cancel{(2-x)}(-1)}{(x+4)\cancel{(x-2)}}$ $\quad \frac{-5}{x+4}$

GCF / GCF 3. $\frac{6x+18}{6x+12}$ $\quad \frac{\cancel{6}(x+3)}{\cancel{6}(x+2)} = \frac{x+3}{x+2}$

Tricky Tri / DOTS 4. $\frac{2x^2+x-6}{9-4x^2}$ $\quad \frac{(x+2)\cancel{(2x-3)}(-1)}{(3+2x)\cancel{(3-2x)}}$

$2x^2+x-6$ PT

x^2+x-12

$(x+\frac{4}{2})(x-\frac{3}{2})$

$(x+2)(2x-3)$

$\frac{-1(x+2)}{3+2x}$

Tri / Grouping 5. $\frac{x^2+3x+2}{x^3+2x^2+8x+16}$ $\quad \frac{\cancel{(x+2)}(x+1)}{(x^2+8)\cancel{(x+2)}}$

$\left(\frac{x^3}{x^2}+\frac{2x^2}{x^2}\right)\left(\frac{+8x}{8}+\frac{16}{8}\right)$

$x^2(x+2)+8(x+2)$

$(x^2+8)(x+2)$

$\frac{x+1}{x^2+8}$

Tricky Tri / DOTS 6. $\frac{3x^2+7x-6}{4-9x^2}$ $\quad \frac{(x+3)\cancel{(3x-2)}(-1)}{(2+3x)\cancel{(2-3x)}}$

$3x^2+7x-6$ PT

$x^2+7x-18$

$(x+\frac{9}{3})(x-\frac{2}{3})$

$(x+3)(3x-2)$

$\frac{-(x+3)}{2+3x}$

GCF/Tri / GCF/DOTS 7. $\frac{2x^4+4x^3-6x^2}{4x^3-36x}$ $\quad \frac{1\cancel{2}x^{\cancel{2}}\cancel{(x+3)}(x-1)}{2\cancel{4}\cancel{x}\cancel{(x+3)}(x-3)}$

$\frac{2x^4}{2x^2}+\frac{4x^3}{2x^2}-\frac{6x^2}{2x^2}$

$2x^2(x^2+2x-3)$

$2x^2(x+3)(x-1)$

$\frac{4x^3}{4x}-\frac{36x}{4x}$

$4x(x^2-9)$

$4x(x+3)(x-3)$

$\frac{x(x-1)}{2(x-3)}$

Grouping / GCF 8. $\frac{2x^3+x^2-18x-9}{3x-x^2}$ $\quad \frac{(x+3)\cancel{(x-3)}(2x+1)(-1)}{x\cancel{(3-x)}}$

$\left(\frac{2x^3}{x^2}+\frac{x^2}{x^2}\right)\left(\frac{-18x}{-9}-\frac{9}{-9}\right)$

$x^2(2x+1)-9(2x+1)$

$(x^2-9)(2x+1)$

$(x+3)(x-3)(2x+1)$

$\frac{-1(x+3)(2x+1)}{x}$

Solving Quadratic Equations By Factoring

1) Bring everything to one side
2) Factor
3) Set each factor equal to zero

Divide away an integer GCF if possible.
A variable GCF would have to stay in front and would produce an answer of 0.

1. $y^2 - 5y - 6 = 0$

$(y-6)(y+1)=0$

$y-\cancel{6}=0 \quad y+\cancel{1}=0$

$+\cancel{6}\ +6 \quad -\cancel{1}\ -1$

$y=6 \quad y=-1$

2. $x^2 + 4x = 0$

$x(x+4)=0$

$x=0 \quad x+\cancel{4}=0$

$-\cancel{4}\ -4$

$x=-4$

3. $a^2 - 8a = 20$

$-20\ -20$

$a^2-8a-20=0$

$(a-10)(a+2)=0$

$a-\cancel{10}=0 \quad a+\cancel{2}=0$

$+\cancel{10}\ +10 \quad -\cancel{2}\ -2$

$a=10 \quad a=-2$

4. $3x^2 = 48$

$-48\ -48$

$\frac{3x^2-48=0}{3 \quad 3}$

$x^2-16=0$

$(x+4)(x-4)=0$

$x+\cancel{4}=0 \quad x-\cancel{4}=0$

$-\cancel{4}\ -4 \quad +\cancel{4}\ +4$

$x=-4 \quad x=4$

5. $x^2 - 6x = -8$

$+8\ +8$

$x^2-6x+8=0$

$(x-4)(x-2)=0$

$x-\cancel{4}=0 \quad x-\cancel{2}=0$

$+\cancel{4}\ +4 \quad +\cancel{2}\ +2$

$x=4 \quad x=2$

6. $\frac{3x^2 + 3x - 6 = 0}{3 \qquad\qquad 3}$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x+\cancel{2}=0 \quad x-\cancel{1}=0$

$-\cancel{2}\ -2 \quad +\cancel{1}\ +1$

$x=-2 \quad x=1$

7. $n^2 = 3n + 18$

$-3n-18\ -3n-18$

$n^2-3n-18=0$

$(n-6)(n+3)=0$

$n-\cancel{6}=0 \quad n+\cancel{3}=0$

$+\cancel{6}\ +6 \quad -\cancel{3}\ -3$

$n=6 \quad n=-3$

8. $2x^2 + 3x = 5$

$-5\ -5$ pt

$2x^2+3x-5=0$

$x^2+3x-10=0$

$(x+5)(x-2)=0$

$\quad 2 \qquad\ 2$

$(2x+5)(x-1)=0$

$2x+\cancel{5}=0 \quad x-\cancel{1}=0$

$-\cancel{5}\ -5 \quad +\cancel{1}\ +1$

$\frac{2x}{2}=\frac{-5}{2} \quad x=1$

$x=-\frac{5}{2}$

9. $x^2 - 6x = 2x + 20$

$-2x-20 \quad -2x-20$

$x^2-8x-20=0$

$(x-10)(x+2)=0$

$x-10=0 \quad x+2=0$

$+10 \ +10 \quad -2 \ -2$

$x=10 \quad x=-2$

10. $x^2 + 2(x-4) = 3x - 8$

$x^2+2x-8=3x-8$

$-3x+8 \quad -3x+8$

$x^2-1x=0$

$x(x-1)=0$

$x=0 \quad x-1=0$

$+1 \ +1$

$x=1$

11. $4(x^2+2x) = 8x + 64$

$4x^2+8x=8x+64$

$-8x-64 \quad -8x-64$

$\frac{4x^2-64=0}{4 \quad 4}$

$x^2-16=0$

$(x+4)(x-4)=0$

$x+4=0 \quad x-4=0$

$-4 \ -4 \quad +4 \ +4$

$x=-4 \quad x=4$

12. $4x^2 + 4x = 3$

$-3 \quad -3$ pt

$4x^2+4x-3=0$

$x^2+4x-12$

$(x+6)(x-2)$

$\frac{}{4} \quad \frac{}{4}$

$(x+\frac{3}{2})(x-\frac{1}{2})$

$(2x+3)(2x-1)=0$

$2x+3=0 \quad 2x-1=0$

$-3 \ -3 \quad +1 \ +1$

$\frac{2x}{2}=\frac{-3}{2} \quad \frac{2x}{2}=\frac{1}{2}$

$x=-\frac{3}{2} \quad x=\frac{1}{2}$

13. $x^2 + 5x = -4(x+5)$

$x^2+5x=-4x-20$

$+4x+20 \quad +4x+20$

$x^2+9x+20=0$

$(x+5)(x+4)=0$

$x+5=0 \quad x+4=0$

$-5 \ -5 \quad -4 \ -4$

$x=-5 \quad x=-4$

14. $4x^2 = 25$

$-25 \ -25$

$4x^2-25=0$

$(2x+5)(2x-5)=0$

$2x+5=0 \quad 2x-5=0$

$-5 \ -5 \quad +5 \ +5$

$\frac{2x}{2}=\frac{-5}{2} \quad \frac{2x}{2}=\frac{5}{2}$

$x=-\frac{5}{2} \quad x=\frac{5}{2}$

15. $3x^2 + 5x = 2x + 60$

$-2x-60 \quad -2x-60$

$\frac{3x^2+3x-60}{3}=\frac{0}{3}$

$x^2+x-20=0$

$(x+5)(x-4)=0$

$x+5=0 \quad x-4=0$

$-5 \ -5 \quad +4 \ +4$

$x=-5 \quad x=4$

16. $8m^2 + 20m = 12$

$-12 \ -12$

$\frac{8m^2+20m-12}{4}=\frac{0}{4}$

pt

$2m^2+5m-3=0$

m^2+5m-6

$\frac{(m+6)(m-1)}{2 \quad 2}$

$(m+3)(2m-1)=0$

$m+3=0 \quad 2m-1=0$

$-3 \ -3 \quad +1 \ +1$

$m=-3 \quad \frac{2m}{2}=\frac{1}{2}$

$m=\frac{1}{2}$

Reducing Radicals

1) Separate into two radicals (perfect squares and non-perfect squares)
2) Take the square root of the perfect square

*If there is a negative inside the radical, bring it outside and make it an *i*

Reduce the following radicals

1. $\sqrt{12}$

$\sqrt{4}\sqrt{3}$

$2\sqrt{3}$

2. $\sqrt{-50}$

$i\sqrt{50}$

$i\sqrt{25}\sqrt{2}$

$5i\sqrt{2}$

3. $\sqrt{-45}$

$i\sqrt{45}$

$i\sqrt{9}\sqrt{5}$

$3i\sqrt{5}$

4. $\sqrt{75}$

$\sqrt{25}\sqrt{3}$

$5\sqrt{3}$

5. $\sqrt{-20}$

$i\sqrt{20}$

$i\sqrt{4}\sqrt{5}$

$2i\sqrt{5}$

6. $\sqrt{-54}$

$i\sqrt{54}$

$i\sqrt{9}\sqrt{6}$

$3i\sqrt{6}$

7. $\sqrt{162}$

$\sqrt{81}\sqrt{2}$

$9\sqrt{2}$

8. $\sqrt{-32}$

$i\sqrt{32}$

$i\sqrt{16}\sqrt{2}$

$4i\sqrt{2}$

Perfect Squares

1
4
9
16
25
36
49
64
81
100

Solving Quadratic Equations Using the Quadratic Formula
1) Bring everything to one side. Keep the leading coefficient positive.
If you cannot factor, USE QUADRATIC FORMULA!

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) $ax^2 + bx + c = 0$
2) List a, b, and c values
3) Substitute values into quadratic formula
4) Type what's inside the radical into the calculator
5) REDUCE THE RADICAL off to the side (If possible)
6) Break the fraction apart into separate fractions

Solve the following equations and express your answer in *simplest a+bi form.*

1. $x^2 + 4x = -8$

$+8 \quad +8$

$a=1$, $b=4$, $c=8$

$x^2+4x+8=0$

$x = \frac{-4 \pm \sqrt{(4)^2-4(1)(8)}}{2(1)}$

$x = \frac{-4 \pm \sqrt{-32}}{2}$

$\sqrt{-32}$, $i\sqrt{32}$, $i\sqrt{16}\sqrt{2}$, $4i\sqrt{2}$

$x = \frac{-4 \pm 4i\sqrt{2}}{2}$

$x = -2 \pm 2i\sqrt{2}$

2. $4x^2 + 2x = -1$

$+1 \quad +1$

$a=4$, $b=2$, $c=1$

$4x^2+2x+1=0$

$x = \frac{-2 \pm \sqrt{(2)^2-4(4)(1)}}{2(4)}$

$x = \frac{-2 \pm \sqrt{-12}}{8}$

$\sqrt{-12}$, $i\sqrt{12}$, $i\sqrt{4}\sqrt{3}$, $2i\sqrt{3}$

$x = \frac{-2 \pm 2i\sqrt{3}}{8}$

$x = -\frac{1}{4} \pm \frac{1}{4}i\sqrt{3}$

3. $2x^2 - 6x = -5$

$+5 \quad +5$

$a=2$, $b=-6$, $c=5$

$2x^2-6x+5=0$

$x = \frac{6 \pm \sqrt{(-6)^2-4(2)(5)}}{2(2)}$

$x = \frac{6 \pm \sqrt{-4}}{4}$

$\sqrt{-4}$, $i\sqrt{4}$, $2i$

$x = \frac{6 \pm 2i}{4}$

$x = \frac{3}{2} \pm \frac{1}{2}i$

4. $3x^2 = 4x - 2$

$-4x+2 \quad -4x+2$

$3x^2-4x+2=0$

$a=3$, $b=-4$, $c=2$

$x = \frac{4 \pm \sqrt{(-4)^2-4(3)(2)}}{2(3)}$

$x = \frac{4 \pm \sqrt{-8}}{6}$

$\sqrt{-8}$, $i\sqrt{8}$, $i\sqrt{4}\sqrt{2}$, $2i\sqrt{2}$

$x = \frac{4 \pm 2i\sqrt{2}}{6}$

$x = \frac{2}{3} \pm \frac{1}{3}i\sqrt{2}$

5. $x^2 + 2x = -8$

$+8 \quad +8$

$a=1$, $b=2$, $c=8$

$x^2+2x+8=0$

$x = \frac{-2 \pm \sqrt{(2)^2-4(1)(8)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{-28}}{2}$

$\sqrt{-28}$, $i\sqrt{28}$, $i\sqrt{4}\sqrt{7}$, $2i\sqrt{7}$

$x = \frac{-2 \pm 2i\sqrt{7}}{2}$

$x = -1 \pm i\sqrt{7}$

6. $3x^2 + 6 = 5x$

$-5x \quad -5x$

$3x^2-5x+6=0$

$a=3$, $b=-5$, $c=6$

$x = \frac{5 \pm \sqrt{(-5)^2-4(3)(6)}}{2(3)}$

$x = \frac{5 \pm \sqrt{-47}}{6}$

$\sqrt{-47}$, $i\sqrt{47}$

$x = \frac{5 \pm i\sqrt{47}}{6}$

$x = \frac{5}{6} \pm \frac{i\sqrt{47}}{6}$

7. A solution of the equation $2x^2 + 3x + 2 = 0$ is

1) $-\frac{3}{4} + \frac{1}{4}i\sqrt{7}$ 3) $-\frac{3}{4} + \frac{1}{4}\sqrt{7}$

2) ~~$-\frac{3}{4} + \frac{1}{4}i$~~ 4) ~~$\frac{1}{2}$~~

$a=2$, $b=3$, $c=2$

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(2)}}{2(2)}$

$\sqrt{-7}$, $i\sqrt{7}$

$x = \frac{-3 \pm \sqrt{-7}}{4}$

$x = \frac{-3 \pm i\sqrt{7}}{4}$

$x = \frac{-3}{4} \pm \frac{i\sqrt{7}}{4}$

8. The solutions to the equation $-\frac{1}{2}x^2 = -6x + 20$ are

$-2 \quad -2 \quad -2$

1) $-6 \pm 2i$
2) $-6 \pm 2\sqrt{19}$
3) $6 \pm 2i$
4) $6 \pm 2\sqrt{19}$

$x^2 = 12x - 40$

$-12x + 40 \quad -12x + 40$

$x^2 - 12x + 40 = 0$

$a = 1$, $b = -12$, $c = 40$

$x = \frac{12 \pm \sqrt{(-12)^2 - 4(1)(40)}}{2(1)}$

$\sqrt{-16}$, $i\sqrt{16}$, $4i$

$x = \frac{12 \pm \sqrt{-16}}{2}$

$x = \frac{12 \pm 4i}{2}$

$x = 6 \pm 2i$

9. Which equation has roots of $3 + i$ and $3 - i$?

1) $x^2 - 6x + 10 = 0$ 3) $x^2 - 10x + 6 = 0$

~~2)~~ $x^2 + 6x - 10 = 0$ ~~4)~~ $x^2 + 10x - 6 = 0$ c must be +

1) $a=1$, $b=-6$, $c=10$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(10)}}{2}$

$x = \frac{6 \pm \sqrt{-4}}{2}$

$\sqrt{-4}$, $i\sqrt{4}$, $2i$

$x = \frac{6 \pm 2i}{2}$

$x = 3 \pm i$

10. If a solution of $2(2x - 1) = 5x^2$ is expressed in simplest $a + bi$ form, the value of b is

1) $\frac{\sqrt{6}}{5}i$ 3) $\frac{1}{5}i$

2) $\frac{\sqrt{6}}{5}$ 4) $\frac{1}{5}$

$a = 5$, $b = -4$, $c = 2$

$4x - 2 = 5x^2$

$-4x + 2 \quad -4x + 2$

$0 = 5x^2 - 4x + 2$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(5)(2)}}{2(5)}$

$\sqrt{-24}$, $i\sqrt{24}$, $i\sqrt{4}\sqrt{6}$, $2i\sqrt{6}$

$x = \frac{4 \pm \sqrt{-24}}{10}$

$x = \frac{4 \pm 2i\sqrt{6}}{10}$

$x = \frac{2}{5} \pm \frac{1}{5}i\sqrt{6}$

$\frac{1}{5}i\sqrt{6} = \frac{\sqrt{6}}{5}i$

11. Algebraically determine the roots, in simplest $a + bi$ form, to the equation below.

$x^2 - 2x + 7 = 4x - 10$

$-4x + 10 \quad -4x + 10$

$x^2 - 6x + 17 = 0$

$a = 1$, $b = -6$, $c = 17$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(17)}}{2(1)}$

$\sqrt{-32}$, $i\sqrt{32}$, $i\sqrt{16}\sqrt{2}$, $4i\sqrt{2}$

$x = \frac{6 \pm \sqrt{-32}}{2}$

$x = \frac{6 \pm 4i\sqrt{2}}{2}$

$x = 3 \pm 2i\sqrt{2}$

Polynomial Equations

1) Bring everything to one side. Keep the leading coefficient positive.
2) Factor
3) Set each factor equal to zero

If you end up with $(x^2 + a)$, use isolate/square root method.

To find the roots/zeros algebraically, replace $f(x)$ with 0.

1. Solve $x^3 + 5x^2 = 4x + 20$ algebraically.

$-4x-20 \quad -4x-20$

$x^3+5x^2-4x-20=0$

$x^2 \quad x^2 \quad -4 \quad -4$

$x^2(x+5)-4(x+5)=0$

$(x^2-4)(x+5)=0$

$(x+2)(x-2)(x+5)=0$

$x+2=0$ $\quad x-2=0$ $\quad x+5=0$

$-2 \; -2$ $\quad +2 \; +2$ $\quad -5 \; -5$

$x=-2$ $\quad x=2$ $\quad x=-5$

2. Algebraically determine the zeros of the function below.

$r(x) = 3x^3 + 12x^2 - 3x - 12$

$0 = (3x^3+12x^2-3x-12)$

$3x^2 \quad 3x^2 \quad -3 \quad -3$

$0=3x^2(x+4)-3(x+4)$

$0=(3x^2-3)(x+4)$

$0=3(x^2-1)(x+4)$

$0=3(x+1)(x-1)(x-4)$

$3\neq0$ $\quad x+1=0$ $\quad x-1=0$ $\quad x-4=0$

$-1 \; -1$ $\quad +1 \; +1$ $\quad +4 \; +4$

$x=-1$ $\quad x=1$ $\quad x=4$

3. Solve for all values of x:

$x^4 - 6x^2 = -8$

$+8 \quad +8$

$x^4-6x^2+8=0$

$(x^2-4)(x^2-2)=0$

$(x+2)(x-2)(x^2-2)=0$

$x+2=0$ $\quad x-2=0$ $\quad x^2-2=0$

$-2 \; -2$ $\quad +2 \; +2$ $\quad +2 \; +2$

$x=-2$ $\quad x=2$ $\quad \sqrt{x^2}=\sqrt{2}$

$x=\pm\sqrt{2}$

4. Find algebraically the zeros for $p(x) = x^3 + x^2 - 4x - 4$.

$0=(x^3+x^2-4x-4)$

$x^2 \quad x^2 \quad -4 \quad -4$

$0=x^2(x+1)-4(x+1)$

$(x^2-4)(x+1)=0$

$(x+2)(x-2)(x+1)=0$

$x+2=0$ $\quad x-2=0$ $\quad x+1=0$

$-2 \; -2$ $\quad +2 \; +2$ $\quad -1 \; -1$

$x=-2$ $\quad x=2$ $\quad x=-1$

5. Solve the equation $2x^3 - x^2 - 8x + 4 = 0$ algebraically for all values of x.

$x^2 \quad x^2 \quad -4 \quad -4$

$x^2(2x-1)-4(2x-1)=0$

$(x^2-4)(2x-1)=0$

$(x+2)(x-2)(2x-1)=0$

$x+2=0$ $\quad x-2=0$ $\quad 2x-1=0$

$-2 \; -2$ $\quad +2 \; +2$ $\quad +1 \; +1$

$x=-2$ $\quad x=2$ $\quad 2x=1$

$\frac{2x}{2}=\frac{1}{2}$

$x=\frac{1}{2}$

6. Solve for all values of x:

$x^4-5x^2-36=0$

$(x^2-9)(x^2+4)=0$

$x^2-9=0 \quad x^2+4=0$

$+9 \; +9 \quad -4 \; -4$

$\sqrt{x^2}=\sqrt{9} \quad \sqrt{x^2}=\sqrt{-4}$

$x=\pm 3 \quad x=\pm 2i$

7. ~~Find alg~~ebraically the zeros of $p(x)=x^3-3x^2+4x-12$

$0=(x^3-3x^2)(+4x-12)$

$x^2 \; x^2 \; 4 \; 4$

$x^2(x-3)+4(x-3)=0$

$(x^2+4)(x-3)=0$

$x^2+4=0 \quad x-3=0$

$-4 \; -4 \quad +3 \; +3$

$\sqrt{x^2}=\sqrt{-4}$ $x=3$

$x=\pm 2i$

8. What are the zeros of $P(m)=(m^2-4)(m^2+1)$?

$0=(m^2-4)(m^2+1)$

$m^2-4=0 \quad m^2+1=0$

$+4 \; +4 \quad -1 \; -1$

$\sqrt{m^2}=\sqrt{4} \quad \sqrt{m^2}=\sqrt{-1}$

$m=\pm 2$ $m=\pm i$

9. Algebraically find the zeros for $f(x)=x^4-4x^3-9x^2+36x$

$0=x^4-4x^3-9x^2+36x$

$0=x[(x^3-4x^2)(-9x+36)]$

$x^2 \; x^2 \; -9 \; -9$

$0=x[x^2(x-4)-9(x-4)]$

$0=x[(x^2-9)(x-4)]$

$0=x(x+3)(x-3)(x-4)$

$x=0 \quad x+3=0 \quad x-3=0 \quad x-4=0$

$-3 \; -3 \quad +3 \; +3 \quad +4 \; +4$

$x=-3 \quad x=3 \quad x=4$

10. Solve algebraically for all values of x: $x^4+4x^3+4x^2=-16x$

$+16x \quad +16x$

$x^4+4x^3+4x^2+16x=0$

$x[(x^3+4x^2)(4x+16)]$

$x^2 \; x^2 \; 4 \; 4$

$x[x^2(x+4)+4(x+4)]$

$x(x^2+4)(x+4)=0$

$x=0 \quad x^2+4=0 \quad x+4=0$

$-4 \; -4 \quad -4 \; -4$

$\sqrt{x^2}=\sqrt{-4} \quad x=-4$

$x=\pm 2i$

Radical Equations

1) Isolate
2) Square both sides
3) Check

1. $\sqrt{2x+1} + 4 = 8$

$-4 \quad -4$

$(\sqrt{2x+1})^2 = (4)^2$

$2x+1=16$

$-1 \quad -1$

$2x=15$

$\frac{2x}{2}=\frac{15}{2}$

$x=7.5$ ✓

2. $\sqrt{x-5} + x = 7$

$-x \quad -x$

$(\sqrt{x-5})^2=(7-x)^2$

$x-5=(7-x)^2$

$x-5=x^2-14x+49$

$-x+5 \quad -x+5$

$0=x^2-15x+54$

$0=(x-9)(x-6)$

$\cancel{x=9}$ ✗ $\quad x=6$ ✓

	7	$-x$
7	49	$-7x$
$-x$	$-7x$	x^2

$x^2-14x+49$

3. $(\sqrt{56-x})^2=(x)^2$

$56-x=x^2$

$-56+x \quad -56+x$

$0=x^2+x-56$

$0=(x+8)(x-7)$

$\cancel{x=-8}$ ✗ $\quad x=7$ ✓

4. $\sqrt{2x-7} + x = 5$

$-x \quad -x$

$(\sqrt{2x-7})^2=(5-x)^2$

$2x-7=(5-x)^2$

$2x-7=x^2-10x+25$

$-2x+7 \quad -2x+7$

$0=x^2-12x+32$

$0=(x-8)(x-4)$

$\cancel{x=8}$ ✗ $\quad x=4$ ✓

	5	$-x$
5	25	$-5x$
$-x$	$-5x$	x^2

$x^2-10x+25$

5. $(\sqrt{5x+29})^2=(x+3)^2$

$5x+29=(x+3)^2$

$5x+29=x^2+6x+9$

$-5x-29 \quad -5x-29$

$0=x^2+x-20$

$0=(x+5)(x-4)$

$\cancel{x=-5}$ ✗ $\quad x=4$ ✓

	x	$+3$
x	x^2	$+3x$
$+3$	$+3x$	$+9$

x^2+6x+9

6. $(\sqrt{2x-4})^2=(x-2)^2$

$2x-4=(x-2)^2$

$2x-4=x^2-4x+4$

$-2x+4 \quad -2x+4$

$0=x^2-6x+8$

$0=(x-4)(x-2)$

$x=4$ ✓ $\quad x=2$ ✓

	x	-2
x	x^2	$-2x$
-2	$-2x$	$+4$

x^2-4x+4

7. $\sqrt{x^2+x-1} + 11x = 7x+3$

$-11x \quad -11x$

$(\sqrt{x^2+x-1})^2=(-4x+3)^2$

$x^2+x-1=16x^2-24x+9$

$-x^2-x+1 \quad -x^2-x+1$

$\frac{0}{5}=\frac{15x^2}{5}-\frac{25x}{5}+\frac{10}{5}$

$0=3x^2-5x+2$ PF

x^2-5x+6

$(x-3)(x-2)$ → $\frac{(x-3)}{3}\frac{(x-2)}{3}$

$(x-1)(3x-2)=0$

$\cancel{x=1}$ ✗ $\quad 3x-2=0$

$+2 \quad +2$

$\frac{3x}{3}=\frac{2}{3}$

$x=\frac{2}{3}$ ✓

	$-4x$	$+3$
$-4x$	$16x^2$	$-12x$
$+3$	$-12x$	$+9$

$x^2-24x+9$

8. $3\sqrt{x} - 2x = -5$

$+2x \quad +2x$

$\frac{3\sqrt{x}}{3}=\frac{(2x-5)}{3}$

$(\sqrt{x})^2=\left(\frac{2x-5}{3}\right)^2$

$9(x)=\frac{(4x^2-20x+25)}{9}9$

$9x=4x^2-20x+25$

$-9x \quad -9x$

$0=4x^2-29x+25$ PF

$0=x^2-29x+100$

$0=(x-25)(x-4)$

$\frac{\ }{4} \quad \frac{\ }{4}$

$0=(4x-25)(x-1)$

$4x-25=0 \quad \cancel{x=1}$ ✗

$+25 \quad +25$

$\frac{4x}{4}=\frac{25}{4}$

$x=\frac{25}{4}$

	$2x$	-5
$2x$	$4x^2$	$-10x$
-5	$-10x$	$+25$

$4x^2-20x+25$

5. $\sqrt{5x+29}=x+3$

6. $\sqrt{2x-4}=x-2$

7. $\sqrt{x^2+x-1}+11x=7x+3$

8. $3\sqrt{x}-2x=-5$

	$2x$	-5
$2x$	$4x^2$	$-10x$
-5	$-10x$	$+25$

$4x^2-20x+25$

9. $\sqrt{49-10x}+\cancel{5}=2x$

$-5 \quad -5$

$(\sqrt{49-10x})^2=(2x-5)^2$

$49-10x=(2x-5)(2x-5)$

$49-10x=4x^2-20x+25$

$-49+10x \qquad +10x-49$

$\frac{0}{2}=\frac{4x^2-10x-24}{2}$ PT

$0=2x^2-5x-12$

$x^2-5x-24$

$(x-\frac{8}{2})(x+\frac{3}{2})$

$(x-4)(2x+3)=0$

$x-4=0$	$2x+3=0$
$+4 \quad +4$	$-3 \quad -3$
$x=4$ ✓	$\frac{2x}{2}=\frac{-3}{2}$
	$\cancel{x=-\frac{3}{2}}$

	11	$-x$
11	121	$-11x$
$-x$	$-11x$	$+x^2$

$x^2-22x+121$

10. $(\sqrt{4x+1})^2=(11-x)^2$

$4x+1=(11-x)(11-x)$

$4x+1=x^2-22x+121$

$-4x-1 \qquad -4x-1$

$0=x^2-26x+120$

$0=(x-20)(x-6)$

$x-20=0$	$x-6=0$
$+20 \quad +20$	$+6 \quad +6$
$\cancel{x=20}$	$x=6$ ✓

Fractional Equations: MULTIPLY BY THE LCD

To find a common denominator:

1) Factor (if necessary)

2) Put all of your factors together

1. $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$3 - x = -1$

$-3 \quad -3$

$-x = -4$

$x = 4$

2. $\frac{5x}{2} = \frac{1}{x} + \frac{x}{4}$

$2x(5x) = 4 + x^2$

$10x^2 = 4 + x^2$

$-x^2 - 4 \quad -4 - x^2$

$9x^2 - 4 = 0$

$(3x+2)(3x-2) = 0$

$3x+2=0 \quad | \quad 3x-2=0$

$3x = -2 \quad 3x = 2$

$x = -\frac{2}{3} \quad x = \frac{2}{3}$

3. $\frac{3}{x} + \frac{x}{x+2} = -\frac{2}{x+2}$

$3(x+2) + x^2 = -2x$

$3x + 6 + x^2 = -2x$

$+2x \quad +2x$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x+3=0 \quad | \quad x+2=0$

$-3 \; -3 \quad -2 \; -2$

$~~x=-3~~ \quad ~~x=-2~~$

4. $\frac{x}{x-1} = \frac{2}{x} + \frac{1}{x-1}$

$x^2 = 2(x-1) + x$

$x^2 = 2x - 2 + x$

$x^2 = 3x - 2$

$-3x+2 \quad -3x+2$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x-2=0 \quad | \quad x-1=0$

$+2 \; +2 \quad +1 \; +1$

$x = 2 \quad ~~x=1~~$

✓

5. $\frac{3x+25}{x+7} - 5 = \frac{3}{x}$

$x(3x+25) - 5x(x+7) = 3(x+7)$

$3x^2 + 25x - 5x^2 - 35x = 3x + 21$

$-2x^2 - 10x = 3x + 21$

$+2x^2 + 10x \quad +2x^2 + 10x$

PT

$0 = 2x^2 + 13x + 21$

$x^2 + 13x + 42$

$(x+7)(x+6)$

$0 = (2x+7)(x+3)$

$2x+7=0 \quad x+3=0$

$-7 \; -7 \quad -3 \; -3$

$\frac{2x}{2} = \frac{-7}{2} \quad x = -3$

$x = -\frac{7}{2}$

✓ ✓

6. $\frac{2}{n^2} + \frac{3}{n} = \frac{4}{n^2}$

$2 + 3n = 4$

$-2 \quad -2$

$\frac{3n}{3} = \frac{2}{3}$

$n = \frac{2}{3}$

✓

F1: 8
F2: x
F3: x+1
LCD: 8x(x+1)

7. $\left(\frac{7}{2x}\right)-\left(\frac{2}{x+1}\right)=\left(\frac{1}{4}\right)$

$28(x+1)-16x=2x(x+1)$

$28x+28-16x=2x^2+2x$

$12x+28=2x^2+2x$

$-12x-28 \qquad -12x-28$

$0=2x^2-10x-28$

$0=x^2-5x-14$

$0=(x-7)(x+2)$

$x-7=0 \qquad x+2=0$

$x=7 \qquad x=-2$

8. $\left(\frac{1}{x-2}\right)+\left(\frac{4}{x+5}\right)=\left(\frac{7}{x^2+3x-10}\right)$

F1: x-2
F2: x+5
LCD: (x-2)(x+5)

$1(x+5)+4(x-2)=7$

$1x+5+4x-8=7$

$5x-3=7$

$+3 \quad +3$

$\frac{5x}{5}=\frac{10}{5}$

$\cancel{x=2}$

No Solution

9. $\left(\frac{x}{x+2}\right)+\left(\frac{1}{x^2-4}\right)=\left(\frac{4}{x-2}\right)(x+2)(x-2)$

$x(x-2)+1=4(x+2)$

$x^2-2x+1=4x+8$

$-4x-8 \quad -4x-8$

$x^2-6x-7=0$

$(x-7)(x+1)=0$

$x-7=0 \qquad x+1=0$

$x=7 \qquad x=-1$

10. $\left(\frac{30}{x^2-9}\right)+1=\left(\frac{5}{x-3}\right)(x+3)(x-3)$

	x	+3
x	x^2	+3x
-3	-3x	-9

x^2-9

$30+(x+3)(x-3)=5(x+3)$

$30+x^2-9=5x+15$

$x^2+21=5x+15$

$-5x-15 \quad -5x-15$

$x^2-5x+6=0$

$(x-3)(x-2)=0$

$x-3=0 \qquad x-2=0$

$\cancel{x=3} \qquad x=2$

F1: 2
F2: b+3
F3: b-3
LCD: 2(b+3)(b-3)

11. $\left(\frac{1}{b-3}\right)-\left(\frac{3}{2b+6}\right)=\left(\frac{b}{b^2-9}\right)$

$2(b+3)-3(b-3)=2b$

$2b+6-3b+9=2b$

$-b+15=2b$

$+b \qquad +b$

$\frac{15}{3}=\frac{3b}{3}$

$5=b$

12. $\left(\frac{2}{x+3}\right)-\left(\frac{3}{4-x}\right)=\left(\frac{2x-2}{x^2-x-12}\right)$

$2(x-4)+3(x+3)=2x-2$

$2x-8+3x+9=2x-2$

$5x+1=2x-2$

$-2x \qquad -2x$

$3x+1=-2$

$-1 \qquad -1$

$\frac{3x}{3}=\frac{-3}{3}$

$x=-1$

Exponential Equations

Isolate the base

a) Constants: Take the appropriate root of both sides or raise each side to the reciprocal power

b) Variables: Take the log of both sides

Solve for x and round your answers to the nearest tenth:

1. $10^{x} = 182$

$\log 10^{x} = \log 182$

$\frac{x \log 10}{\log 10} = \frac{\log 182}{\log 10}$

$x = 2.3$

2. $15^{2n} = 245$

$\log 15^{2n} = \log 245$

$\frac{2n \log 15}{2 \log 15} = \frac{\log 245}{2 \log 15}$

$n = 1.02$

3. $3(5)^{2x} = 60$

$\frac{3(5)^{2x}}{3} = \frac{60}{3}$

$\log 5^{2x} = \log 20$

$\frac{2x \log 5}{2 \log 5} = \frac{\log 20}{2 \log 5}$

$x = .9$

4. $4^{x} - 5 = 12$

$+5 \quad +5$

$\log 4^{x} = \log 17$

$\frac{x \log 4}{\log 4} = \frac{\log 17}{\log 4}$

$x = 2.0$

5. $2(3)^{2x} + 8 = 18$

$-8 \quad -8$

$\frac{2(3)^{2x}}{2} = \frac{10}{2}$

$\log 3^{2x} = \log 5$

$\frac{2x \log 3}{2 \log 3} = \frac{\log 5}{2 \log 3}$

$x = .7$

6. $8 + 2(4)^{-5x} = 14$

$-8 \quad -8$

$\frac{2(4)^{-5x}}{2} = \frac{6}{2}$

$\log 4^{-5x} = \log 3$

$\frac{-5x \log 4}{-5 \log 4} = \frac{\log 3}{-5 \log 4}$

$x = -.2$

7. $1 - 2(3)^{2x} = -5$

$-1 \quad -1$

$\frac{-2(3)^{2x}}{-2} = \frac{-6}{-2}$

$\log 3^{2x} = \log 3$

$\frac{2x \log 3}{2 \log 3} = \frac{\log 3}{2 \log 3}$

$x = \frac{1}{2}$

8. $256 + 3(2)^{6x} = 2700$

$-256 \quad -256$

$\frac{3(2)^{6x}}{3} = \frac{2444}{3}$

$\log 2^{6x} = \log 814.\overline{6}$

$\frac{6x \log 2}{6 \log 2} = \frac{\log 814.\overline{6}}{6 \log 2}$

$x = 1.6$

log log

9. $e^{4x} = 12$

$\frac{4x\log e}{4\log e} = \frac{\log 12}{4\log e}$

$x = .6$

10. $1.2(3)^{-2x} + \cancel{15} = 195$

$-15 \quad -15$

$\frac{1.2(3)^{-2x}}{1.2} = \frac{180}{1.2}$

$\log 3^{-2x} \overset{\log}{=} 150$

$\frac{-2x\log 3}{-2\log 3} = \frac{\log 150}{-2\log 3}$

$x = -2.28$

11. $18 - 4(6)^{\frac{1}{3}x} = 16$

$-18 \quad -18$

$\frac{-4(6)^{\frac{1}{3}x}}{-4} = \frac{-2}{-4}$

$\log 6^{\frac{1}{3}x} \overset{\log}{=} .5$

$3\left(\frac{1}{3}x\log 6\right) = (\log .5)3$

$\frac{x\log 6}{\log 6} = \frac{3\log .5}{\log 6}$

$x = -1.16$

12. $12 + 3(1.2)^{\frac{x}{2}} = 100$

$-12 \quad -12$

$\frac{3(1.2)^{\frac{x}{2}}}{3} = \frac{88}{3}$

$\log 1.2^{\frac{x}{2}} \overset{\log}{=} \frac{88}{3}$

$2\left(\frac{x}{2}\log 1.2\right) = \left(\log\frac{88}{3}\right)2$

$\frac{x\log 1.2}{\log 1.2} = \frac{2\log\frac{88}{3}}{\log 1.2}$

$x = 37.1$

13. $\left(x^5\right)^{\frac{1}{5}} = (7)^{\frac{1}{5}}$

$x = 1.5$

14. $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = (4)^{\frac{3}{2}}$

$x = 8$

15. $x^{\frac{4}{3}} - \cancel{1} = 5$

$+1 \quad +1$

$\left(x^{\frac{4}{3}}\right)^{3/4} = (6)^{3/4}$

$x = 3.8$

16. $4x^{\frac{2}{3}} - \cancel{5} = 20$

$+5 \quad +5$

$\frac{4x^{\frac{2}{3}}}{4} = \frac{25}{4}$

$\left(x^{\frac{2}{3}}\right)^{3/2} = \left(\frac{25}{4}\right)^{3/2}$

$x = \frac{125}{8}$

Quadratic Systems of Equations Algebraically

1) Isolate at least one variable in one of the equations
2) Substitute one equation into the other (set them equal if you solved both equations for the same variables).
3) Solve equation (Mr. x^2 /Polynomial Equations)
4) Substitute answers into one of the original equations to find the second variable.

1. $y = x^2 - 5$
   $y = 3x - 1$

$x^2 - 5 = 3x - 1$
$-3x + 1 \quad -3x + 1$
$x^2 - 3x - 4 = 0$
$(x-4)(x+1) = 0$
$x - 4 = 0 \quad | \quad x + 1 = 0$
$+4 \ +4 \quad | \quad -1 \ -1$
$x = 4 \quad x = -1$

$x = 4$
$y = 3x - 1$
$y = 3(4) - 1$
$y = 11$
$(4, 11)$

$x = -1$
$y = 3x - 1$
$y = 3(-1) - 1$
$y = -4$
$(-1, -4)$

2. $x^2 + y^2 = 2$
   $y + 2 = x$

$(y+2)^2 + y^2 = 2$
$y^2 + 4y + 4 + y^2 = 2$
$2y^2 + 4y + 4 = 2$
$-2 \ -2$
$\frac{2y^2 + 4y + 2}{2} = \frac{0}{2}$
$y^2 + 2y + 1 = 0$
$(y+1)(y+1) = 0$
$y = -1$

$y + 2$: y^2, $+2y$, $+2y$, $+4$
$y^2 + 4y + 4$

$y = -1$
$x = y + 2$
$x = -1 + 2$
$x = 1$
$(1, -1)$

3. $x^2 + y^2 = 25$
   $y + 5 = 2x$
   $-5 \quad -5$
$y = 2x - 5$

$2x \ -5$: $4x^2$, $-10x$, $-10x$, $+25$
$4x^2 - 20x + 25$

$x^2 + (2x-5)^2 = 25$
$x^2 + 4x^2 - 20x + 25 = 25$
$-25 \ -25$
$5x^2 - 20x = 0$
$5x(x-4) = 0$
$\frac{5x}{5} = \frac{0}{5} \quad | \quad x - 4 = 0$
$+4 \ +4$
$x = 0 \quad x = 4$
$y = 2x - 5 \quad y = 2x - 5$
$y = 2(0) - 5 \quad y = 2(4) - 5$
$y = -5 \quad y = 3$

$(0, -5)$
$(4, 3)$

4. $y = 2x^2 - 7x + 4$
   $y = 11 - 2x$

$11 - 2x = 2x^2 - 7x + 4$
$-11 + 2x \quad +2x - 11$
$0 = 2x^2 - 5x - 7$ pt
$0 = x^2 - 5x - 14$
$(x - \frac{7}{2})(x + \frac{2}{2})$
$0 = (2x-7)(x+1)$
$2x - 7 = 0 \quad | \quad x + 1 = 0$
$+7 \ +7 \quad -1 \ -1$
$\frac{2x}{2} = \frac{7}{2} \quad x = -4$
$x = 3.5$

$x = 3.5 \quad x = -4$
$y = 11 - 2x \quad y = 11 - 2x$
$y = 11 - 2(3.5) \quad y = 11 - 2(-4)$
$y = 4 \quad y = 19$
$(3.5, 4) \quad (-4, 19)$

5. $(x+2)^2 + (y-4)^2 = 40$

$y = x+2$

	x	$+2$
x	x^2	$+2x$
$+2$	$+2x$	$+4$

x^2+4x+4

	x	-2
x	x^2	$-2x$
-2	$-2x$	$+4$

x^2-4x+4

$(x+2)^2+(x+2-4)^2=40$

$(x+2)^2+(x-2)^2=40$

$(x+2)(x+2)+(x-2)(x-2)=40$

$x^2+\cancel{4x}+4+x^2\cancel{-4x}+4=40$

$2x^2+\cancel{8}=40$

$\quad -8 \quad -8$

$\frac{2x^2}{2}=\frac{32}{2}$

$\sqrt{x^2}=\sqrt{16}$

$x=\pm 4$

$x=4 \qquad x=-4$

$y=x+2 \qquad y=x+2$

$y=4+2 \qquad y=-4+2$

$y=6 \qquad y=-2$

$(4,6) \qquad (-4,-2)$

6. $(x-2)^2 + (y-3)^2 = 16$

$x+y-1=0$

$-x \quad +1 \quad -x+1$

$y=-x+1$

	x	-2
x	x^2	$-2x$
-2	$-2x$	$+4$

x^2-4x+4

	$-x$	-2
$-x$	x^2	$+2x$
-2	$+2x$	$+4$

x^2+4x+4

$(x-2)^2+(-x+1-3)^2=16$

$(x-2)^2+(-x-2)^2=16$

$x^2-4x+4+x^2+4x+4=16$

$2x^2+\cancel{8}=16$

$\quad -8 \quad -8$

$\frac{2x^2}{2}=\frac{8}{2}$

$\sqrt{x^2}=\sqrt{4}$

$x=\pm 2$

$x=2 \qquad x=-2$

$y=-x+1 \qquad y=-x+1$

$y=-2+1 \qquad y=2+1$

$y=-1 \qquad y=3$

$(2,-1) \qquad (-2,3)$

7. $x+y=5$

$-x \quad -x \quad y=5-x$

$(x+3)^2 + (y-3)^2 = 53$

	x	$+3$
x	x^2	$+3x$
$+3$	$+3x$	$+9$

x^2+6x+9

	2	$-x$
2	4	$-2x$
$-x$	$-2x$	$+x^2$

x^2-4x+4

$(x+3)^2+(5-x-3)^2=53$

$(x+3)^2+(2-x)^2=53$

$x^2+6x+9+x^2-4x+4=53$

$2x^2+2x+13=53$

$\quad -53 \quad -53$

$\frac{2x^2+2x-40}{2}=\frac{0}{2}$

$x^2+x-20=0$

$(x+5)(x-4)=0$

$x+\cancel{5}=0 \quad | \quad x-\cancel{4}=0$

$-5 \; -5 \quad | \quad +4 \; +4$

$x=-5 \qquad x=4$

$x=-5 \qquad x=4$

$y=5-x \qquad y=5-x$

$y=5-5 \qquad y=5-4$

$y=10 \qquad y=1$

$(-5,10) \qquad (4,1)$

8. $(x-3)^2 + (y+2)^2 = 16$

$2x+2y=10$

$-2x \quad -2x$

$\frac{2y}{2}=\frac{-2x+10}{2}$

$y=-x+5$

	x	-3
x	x^2	$-3x$
-3	$-3x$	$+9$

x^2-6x+9

	$-x$	$+7$
$-x$	x^2	$-7x$
$+7$	$-7x$	$+49$

$x^2-14x+49$

$(x-3)^2+(-x+5+2)^2=16$

$(x-3)^2+(-x+7)^2=16$

$x^2-6x+9+x^2-14x+49=16$

$2x^2-20x+58=16$

$\quad -16 \quad -16$

$\frac{2x^2-20x+42}{2}=\frac{0}{2}$

$x^2-10x+21=0$

$(x-7)(x-3)=0$

$x-\cancel{7}=0 \quad | \quad x-\cancel{3}=0$

$+7 \; +7 \quad | \quad +3 \; +3$

$x=7 \qquad x=3$

$x=7 \qquad x=3$

$y=-x+5 \qquad y=-x+5$

$y=-7+5 \qquad y=-3+5$

$y=-2 \qquad y=2$

$(7,-2) \qquad (3,2)$

Linear Systems In Three Variables
Elimination Method:

1) Choose two pairs of equations and get the same variable to cancel
2) Use Addition Method to solve the system with your two new equations
3) Substitute those two answers into one of the original equations to find your third variables

*Make sure all variables are in order on the left hand side and all constants are on the right hand side.

1. Solve the following system of equations algebraically for all values of x, y, and z:

A $x + 3y + 5z = 45$
B $6x - 3y + 2z = -10$
C $-2x + 3y + 8z = 72$

A and B
$x + 3y + 5z = 45$
$+\ 6x - 3y + 2z = -10$
D $7x + 7z = 35$

B and C
$6x - 3y + 2z = -10$
$+\ -2x + 3y + 8z = 72$
E $4x + 10z = 62$

D and E
$-4(7x + 7z = 35)$
$7(4x + 10z = 62)$
$-28x - 28z = -140$
$28x + 70z = 434$
$\frac{42z}{42} = \frac{294}{42}$
$z = 7$

$7x + 7z = 35$
$7x + 7(7) = 35$
$7x + 49 = 35$
$-49 \quad -49$
$\frac{7x}{7} = \frac{-14}{7}$
$x = -2$

$x + 3y + 5z = 45$
$-2 + 3y + 5(7) = 45$
$-2 + 3y + 35 = 45$

$3y + 33 = 45$
$-33 \quad -33$
$\frac{3y}{3} = \frac{12}{3}$
$y = 4$

2. Solve the following system of equations algebraically for all values of x, y, and z:

A $x + 2y - 3z = -2$
B $2x - 2y + z = 7$
C $x + y + 2z = -4$

A and B
$x + 2y - 3z = -2$
$+\ 2x - 2y + z = 7$
D $3x - 2z = 5$

B and C
$1(2x - 2y + z = 7)$
$2(x + y + 2z = -4)$
$2x - 2y + z = 7$
$+\ 2x + 2y + 4z = -8$
E $4x + 5z = -1$

D and E
$5(3x - 2z = 5)$
$2(4x + 5z = -1)$
$15x - 10z = 25$
$+\ 8x + 10z = -2$
$\frac{23x}{23} = \frac{23}{23}$
$x = 1$

$4x + 5z = -1$
$4(1) + 5z = -1$
$4 + 5z = -1$
$-4 \quad -4$
$\frac{5z}{5} = \frac{-5}{5}$
$z = -1$

$x + 2y - 3z = -2$
$1 + 2y - 3(-1) = -2$
$1 + 2y + 3 = -2$
$2y + 4 = -2$
$-4 \quad -4$
$\frac{2y}{2} = \frac{-6}{2}$
$y = -3$

3. Solve the following system of equations algebraically for all values of x, y, and z:

A $2x + 3y - 4z = -1$

B $x - 2y + 5z = 3$

C $-4x + y + z = 16$

A and B

$1(2x+3y-4z=-1)$

$-2(x-2y+5z=3)$

$2x+3y-4z=-1$

$+\ -2x+4y-10z=-6$

D $7y-14z=-7$

A and C

$2(2x+3y-4z=-1)$

$1(-4x+y+z=16)$

$4x+6y-8z=-2$

$+\ -4x+y+z=16$

E $7y-7z=14$

D and E

$-1(7y-14z=-7)$

$7y-7z=14$

$-7y+14z=7$

$+\ 7y-7z=14$

$7z=21$

$\frac{7z}{7}=\frac{21}{7}$

$z=3$

$7y-7z=14$

$7y-7(3)=14$

$7y-21=14$

$+21\quad +21$

$7y=35$

$\frac{7y}{7}=\frac{35}{7}$

$y=5$

$x-2y+5z=3$

$x-2(5)+5(3)=3$

$x-10+15=3$

$x+5=3$

$-5\quad -5$

$x=-2$

4. Solve the following system of equations algebraically for all values of a, b, and c.

A $a + 4b + 6c = 23$

B $a + 2b + c = 2$

$6b + 2c = a + 14$

$-a\qquad -a$

C $-a+6b+2c=14$

A and C

$a+4b+6c=23$

$+\ -a+6b+2c=14$

D $10b+8c=37$

B and C

$a+2b+c=2$

$+\ -a+6b+2c=14$

E $8b+3c=16$

D and E

$-3(10b+8c=37)$

$8(8b+3c=16)$

$-30b-24c=-111$

$64b+24c=128$

$34b=17$

$\frac{34b}{34}=\frac{17}{34}$

$b=.5$

$10b+8c=37$

$10(.5)+8c=37$

$5+8c=37$

$-5\qquad -5$

$8c=32$

$\frac{8c}{8}=\frac{32}{8}$

$c=4$

$a+4b+6c=23$

$a+4(.5)+6(4)=23$

$a+2+24=23$

$a+26=23$

$-26\quad -26$

$a=-3$

Modeling Exponential Functions With Algebra

$A=P(1\pm r)^t$	Basic Exponential (Nothing Below)
$A=P\left(1\pm\frac{r}{n}\right)^{nt}$	Compounding (not continuously)
$A=Pe^{rt}$	Compounding continuously or just continuously
$A=P\left(\frac{1}{2}\right)^{\frac{t}{h}}$, $A=P(2)^{\frac{t}{h}}$, $A=P(1\pm r)^{\frac{t}{h}}$	Half Life, Double Time, Irregular Time

1. A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

t P A

constant exponential equation

$A=135000$ $A=P(1+r)^t$

$P=100000$ $\frac{135000}{100000}=\frac{100000(1+r)^5}{100000}$

$r=r$ $(1.35)^{\frac{1}{5}}=((1+r)^5)^{\frac{1}{5}}$

$t=5$

$1.06..=1+r$

$-1 \quad -1$

$100(.06..)=r$

$6\% = r$

2. A car that was bought for \$24,320 is worth \$9,200 after 7 years. To the *nearest percent*, what is the annual rate of depreciation?

P A t

$A=9200$ $A=P(1-r)^t$

$P=24320$ $\frac{9200}{24320}=\frac{24320(1-r)^7}{24320}$

$r=r$

$t=7$

$(.378..)^{\frac{1}{7}}=((1-r)^7)^{\frac{1}{7}}$

$.8703=1-r$

$-1 \quad -1$

$\frac{-.12..}{-1}=\frac{-r}{-1}$

$(.1296..)100=r$

$13\%=r$

3. Determine, to the *nearest year*, how long it will take \$750 invested at an annual rate of 3% to triple.

P r $P\cdot3$

$A=3(750)$ $A=P(1\pm r)^t$

$P=750$ $\frac{3(750)}{750}=\frac{750(1+.03)^t}{750}$

$r=.03$

$t=t$

$\log 3=\log(1.03)^t$

$\frac{\log 3}{\log 1.03}=\frac{t\log 1.03}{\log 1.03}$

$37=t$

4. A local university has a current enrollment of 12,000 students. The enrollment is increasing continuously at a rate of 2.5% each year. Which logarithm is equal to the number of years it will take for the population to increase to 15,000 students?

1) $\frac{\ln 1.25}{0.25}$ 3) $\frac{\ln 1.25}{2.5}$

2) $\frac{\ln 3000}{0.025}$ 4) $\frac{\ln 1.25}{0.025}$

$A=Pe^{rt}$

$A=15000$

$P=12000$

$r=.025$

$t=$

$\frac{15000}{12000}=\frac{12000e^{.025t}}{12000}$

$\ln 1.25=\ln e^{.025t}$

$\frac{\ln 1.25}{.025}=\frac{.025t\ln e}{.025}$

5. Susie invests $500 in an account that is compounded continuously at an annual interest rate of 5%. Approximately how many years will it take for Susie's money to double?

P 2P

$A = 2(500)$ $A = Pe^{rt}$ $\ln 2 = .05t \ln e$
$P = 500$ $\frac{2(500)}{500} = \frac{500e^{.05t}}{500}$ $\frac{\ln 2}{.05} = \frac{.05t}{.05}$
$r = .05$ $\ln 2 = \ln e^{.05t}$ $14 = t$
$t = t$

6. The number of bacteria present in a Petri dish can be modeled by the function $N = 50e^{3t}$, where N is the number of bacteria present in the Petri dish after t hours. Using this model, determine, to the *nearest hundredth*, the number of hours it will take for N to reach 30,700.

$\frac{30700}{50} = \frac{50e^{3t}}{50}$ $\frac{\ln 614}{3} = \frac{3t \ln e}{3}$
$\ln 614 = \ln e^{3t}$ $2.14 = t$

7. One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the *nearest day*, the amount of time needed before the amount of I–131 in the patient's body is approximately 7 milligrams.

h P A

$A = 7$ $A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$ $\log\frac{7}{20} = \log\left(\frac{1}{2}\right)^{\frac{t}{8.02}}$ $12 = t$
$P = 20$ $\frac{7}{20} = \frac{20\left(\frac{1}{2}\right)^{\frac{t}{8.02}}}{20}$ $8.02\log\left(\frac{7}{20}\right) = \frac{t}{8.02}\log\left(\frac{1}{2}\right) 8.02$
$t = t$ $\frac{8.02\log\left(\frac{7}{20}\right)}{\log\frac{1}{2}} = \frac{t\log\frac{1}{2}}{\log\frac{1}{2}}$
$h = 8.02$

8. The half-life of a radioactive substance is 15 years. Write an equation that can be used to determine the amount, $s(t)$, of 200 grams of this substance that remains after t years. Determine algebraically, to the *nearest year*, how long it will take for $\frac{1}{10}$ of this substance to remain.

h P

$A = S(t)$ $A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$ $A = \frac{1}{10}(200)$ $50 = t$
$P = 200$ $S(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{15}}$ $\log\frac{1}{10} = \log\left(\frac{1}{2}\right)^{\frac{t}{15}}$
$t = t$ $\frac{\frac{1}{10}(200)}{200} = \frac{200\left(\frac{1}{2}\right)^{\frac{t}{15}}}{200}$ $15\left(\log\frac{1}{10}\right) = \left(\frac{t}{15}\log\frac{1}{2}\right)15$
$h = 15$ $\frac{15\log\frac{1}{10}}{\log\frac{1}{2}} = \frac{t\log\frac{1}{2}}{\log\frac{1}{2}}$

9. Determine, to the *nearest tenth of a year*, how long it would take an investment to double at a $3\frac{3}{4}$% interest rate, compounded continuously.

3.75% $A=Pe^{rt}$

$A=2P$
$P=P$
$r=.0375$
$t=t$

$\frac{2P}{P}=\frac{Pe^{.0375t}}{P}$

$\ln 2=\ln e^{.0375t}$

$\frac{\ln 2}{.0375}=\frac{.0375t \ln e}{.0375}$

$18.5=t$

10. Carla wants to start a college fund for her daughter Lila. She puts \$63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, $C(t)$, that represents the amount of money in the account t years after the account is opened, given that no more money is deposited into or withdrawn from the account. Calculate algebraically the number of years it will take for the account to reach \$100,000, to the *nearest hundredth of a year*.

$A=C(t)$
$P=63{,}000$
$r=.0255$
$n=12$
$t=t$

$A=P(1+\frac{r}{n})^{nt}$

$C(t)=63{,}000(1+\frac{.0255}{12})^{12t}$

$C(t)=63{,}000(1.002125)^{12t}$

$\frac{100000}{63000}=\frac{63000(1.002125)^{12t}}{63000}$

$\log\frac{100}{63}=\log 1.002125^{12t}$

$\frac{\log\frac{100}{63}}{12\log 1.002125}=\frac{12t\log 1.002125}{12\log 1.002125}$

$18.14=t$

11. When observed by researchers under a microscope, a smartphone screen contained approximately 11,000 bacteria per square inch. Bacteria, under normal conditions, double in population every 20 minutes.

a) Assuming an initial value of 11,000 bacteria, write a function, $p(t)$, that can be used to model the population of bacteria, p, on a smartphone screen, where t represents the time in minutes after it is first observed under a microscope.

b) Using $p(t)$ from part a, determine algebraically, to the *nearest hundredth of a minute*, the amount of time it would take for a smartphone screen that was not touched or cleaned to have a population of 1,000,000 bacteria per square inch.

$A=p(t)$
$P=11{,}000$
$t=t$
$h=20$

$A=P(2)^{\frac{t}{h}}$

$p(t)=11{,}000(2)^{\frac{t}{20}}$

$\frac{1{,}000{,}000}{11{,}000}=\frac{11{,}000(2)^{\frac{t}{20}}}{11{,}000}$

$\log\frac{1000}{11}=\log 2^{\frac{t}{60}}$

$60\left(\log\frac{1000}{11}\right)=\left(\frac{t}{60}\log 2\right)60$

$\frac{60\log\frac{1000}{11}}{\log 2}=\frac{t\log 2}{\log 2}$

$390.38=t$

12. Seth's parents gave him \$5000 to invest for his 16th birthday. He is considering two investment options. Option *A* will pay him 4.5% interest compounded annually. Option *B* will pay him 4.6% compounded quarterly. Write a function of option *A* and option *B* that calculates the value of each account after *n* years. Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option *B* will earn than option *A* to the *nearest cent*. Algebraically determine, to the *nearest tenth of a year*, how long it would take for option *B* to double Seth's initial investment.

P $n=1$ $n=4$

$A = A(n)$, $P = 5000$, $r = .045$, $n = 1$, $t = n$

Option A

$A = P(1+\frac{r}{n})^{nt}$

$A = 5000(1+\frac{.045}{1})^{1n}$

$A(n) = 5000(1.045)^n$

$A(6) = 5000(1.045)^6$

$A(6) = 6511.30$

$$\begin{array}{r} 6587.87 \\ -6511.30 \\ \hline \$67.57 \end{array}$$

$2(5000) = 5000(1.0115)^{4n}$

$\frac{2(5000)}{5000} = \frac{5000(1.0115)^{4n}}{5000}$

$\log 2 = \log 1.0115^{4n}$

$\log 2 = 4n \log 1.0115$

$\frac{\log 2}{4\log 1.0115} = \frac{4n\log 1.0115}{4\log 1.0115}$

$15.2 = n$

Option B $A = B(n)$

$A = P(1+\frac{r}{n})^{nt}$, $P = 5000$, $r = .046$, $n = 4$, $t = n$

$B(n) = 5000(1+\frac{.046}{4})^{4n}$

$B(n) = 5000(1.0115)^{4n}$

$B(6) = 5000(1.0115)^{4(6)}$

$B(6) = 6578.87$

13. A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m. Write an equation in the form $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ that models this situation, where h is the constant representing the number of hours in the half-life, A_0 is the initial mass, and A is the mass t hours after 3 p.m. Using this equation, solve for h, to the *nearest ten thousandth*. Determine when the mass of the radioactive substance will be 40 g. Round your answer to the *nearest tenth of an hour*.

P A $t = 5$

$A = 100$, $A_0 = 140$, $t = 5$, $h = h$

$100 = 140(\frac{1}{2})^{\frac{5}{h}}$

$\frac{100}{140} = \frac{140(\frac{1}{2})^{\frac{5}{h}}}{140}$

$\log\frac{5}{7} = \log\frac{1}{2}^{\frac{5}{h}}$

$h(\log\frac{5}{7}) = (\frac{5}{h}\log\frac{1}{2})h$

$h\log\frac{5}{7} = 5\log\frac{1}{2}$

$\frac{h\log\frac{5}{7}}{\log\frac{5}{7}} = \frac{5\log\frac{1}{2}}{\log\frac{5}{7}}$

$h = 10.3002$

$40 = 140(\frac{1}{2})^{\frac{t}{10.3002}}$

$\frac{40}{140} = \frac{140(\frac{1}{2})^{\frac{t}{10.3002}}}{140}$

$A = 40$, $A_0 = 140$, $t = t$, $h = 10.3002$

$\log\frac{2}{7} = \log\frac{1}{2}^{\frac{t}{10.3002}}$

$10.3002(\log\frac{2}{7}) = (\frac{t}{10.3002}\log\frac{1}{2})10.3002$

$\frac{10.3002\log(\frac{2}{7})}{\log\frac{1}{2}} = \frac{t\log\frac{1}{2}}{\log\frac{1}{2}}$

$18.6 = t$

Graphing Functions

1) Type equation(s) into Y= in calculator
2) The domain (what you're graphing between) will either be given in the problem or on the graph. If not, you must find an appropriate window in your calculator and use that as your domain.
3) Determine your scale. $scale \geq \dfrac{\text{max}}{\#\ of\ boxes}$
4) Plot Points

*For follow up questions, you may need to solve equations graphical

1. Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is $P(x) = \log(x-4)$, where x is the number of visits per week in thousands and $P(x)$ is the website's popularity rating. According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the *nearest tenth*? Graph $y = P(x)$ on the axes below.

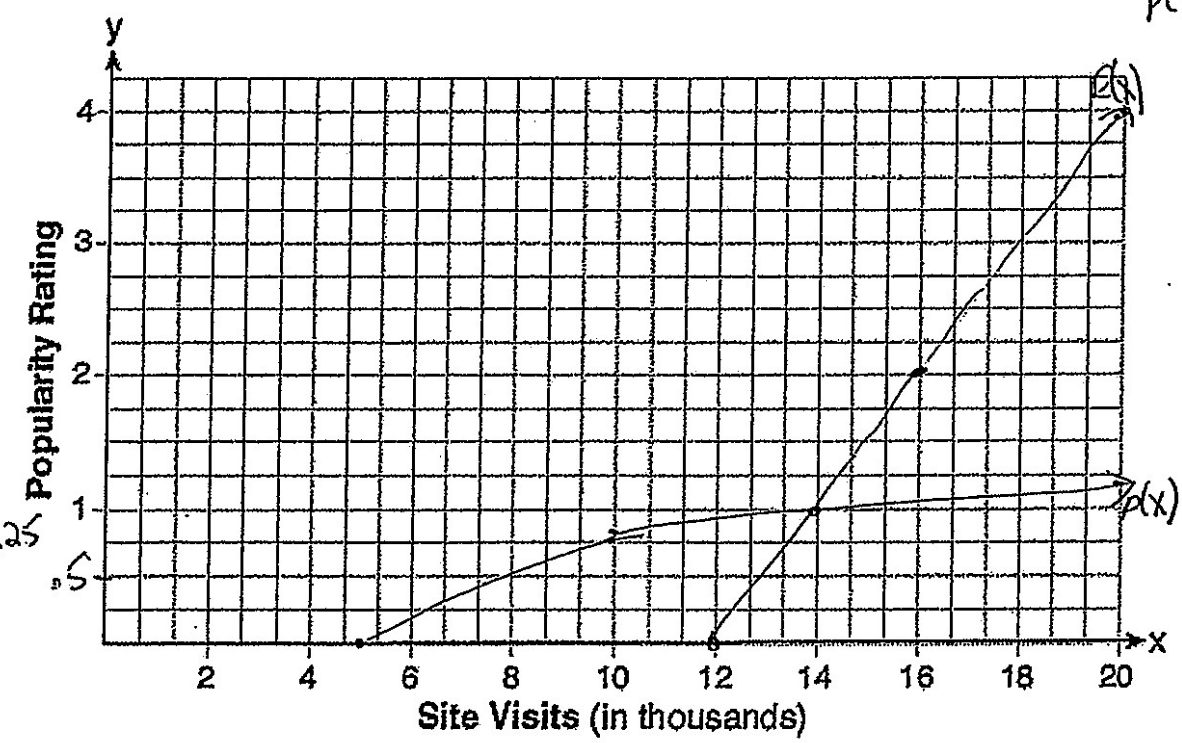
$P(16) = \log(16-4)$

$P(16) = 1.1$

$P(x)$

x	y
5	0
10	.77815
14	1
20	1.2

scale $= \frac{1}{4} = .25$



An alternative rating model is represented by $R(x) = \frac{1}{2}x - 6$, where x is the number of visits per week in thousands. Graph $R(x)$ on the same set of axes. For what number of weekly visits will the two models provide the same rating?

$P(x) = R(x)$

2nd Trace, Intersect

$(14, 1)$

14,000 weekly visits

$R(x)$

x	y
0	-6
4	-4
8	-2
12	0
16	2
20	4

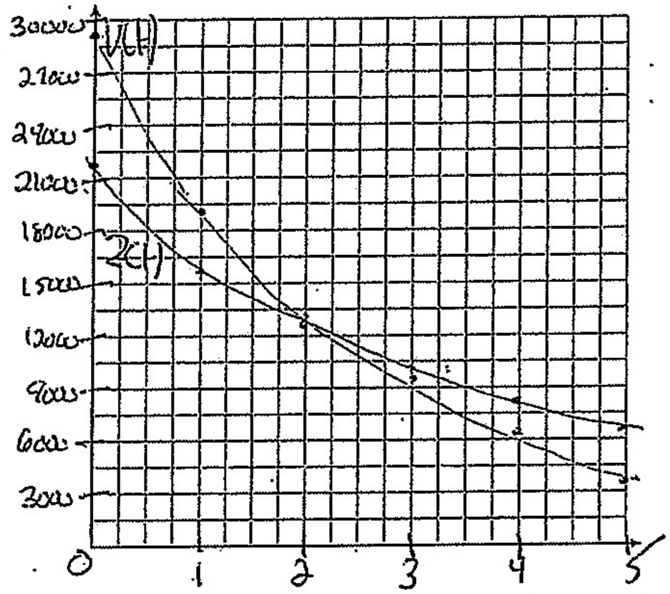
2 ~~##~~. The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time. Graph $V(t)$ and $Z(t)$ over the interval $0 \le t \le 5$, on the set of axes below.

V(t)

X	y
0	28483
1	19492
2	13326
3	9114.8
4	6237.6
5	4264.4

Z(t)

X	y
0	22151
1	17234
2	13408
3	10431
4	8115.6
5	6313.4



Scale

$x \geq \frac{5}{20}$ $\quad$ $y \geq \frac{28483}{20}$

$x \geq .25$ $\quad$ $y \geq 1424.15$

$y = 1500$

State when $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem. Zach takes out an insurance policy that requires him to pay a \$3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the *nearest year*, how long will it take Zach to cancel this policy? Justify your answer.

3000

$t = 1.95$

After 1.95 years, the value of the loans will be the same (\$13569.29)

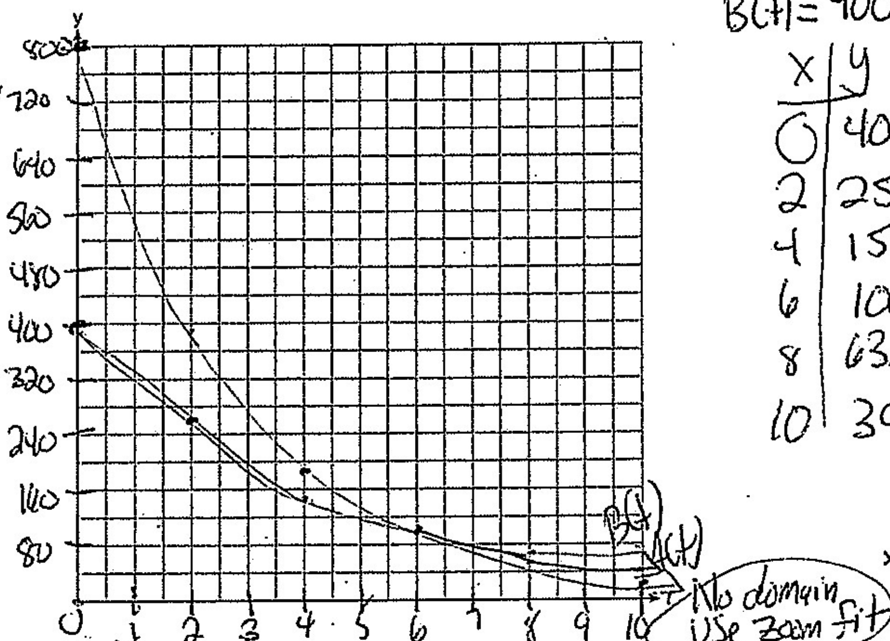
$Z(t) = 22151.327(0.778)^t$

$3000 = 22151.327(0.778)^t$

3 ~~12~~. Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 N_0 milligrams of a drug with a decay rate of 0.347 r. Patient B, $B(t)$, is given 400 N_0 milligrams of another drug with a decay rate of 0.231 r. Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.

$A(t) = 800e^{-.347t}$

x	y
0	800
2	399.66
4	199.66
6	99.744
8	49.83
10	24.894



$B(t) = 400e^{-.231t}$

x	y
0	400
2	252.01
4	158.77
6	100.03
8	63.021
10	39.105

x and y min = 0

To the *nearest hour*, t, when does the amount of the given drug remaining ~~in patient~~ B begin to exceed the amount of the given drug remaining in patient A? The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

ame
Trace
ntersect
(5.98, 100.60)
hours

Scale

$x \geq \frac{10}{20}$ $\quad y \geq \frac{800}{20}$

$x \geq .5$ $\quad y \geq 40$

$x = .5$ $\quad y = 40$

$A(t) = .15(800)$

$.15(800) = 800e^{-.347t}$

$Y_1 \qquad Y_2$ Find intersection

or

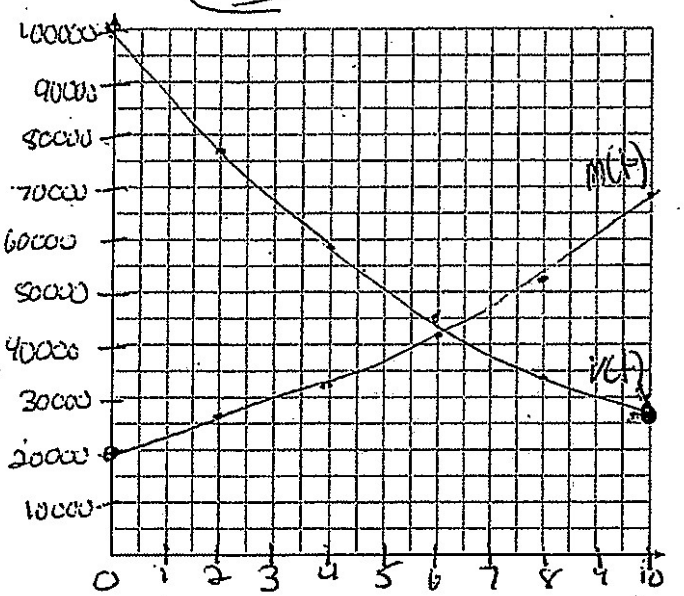
$\frac{.15(800)}{800} = \frac{800e^{-.347t}}{800}$

$\ln .15 = \ln e^{-.347t}$

$\frac{\ln .15}{-.347\ln e} = \frac{-.347t\ln e}{-.347\ln e}$

$5.5 = t$

4 ~~18~~. The value of Tom's bank account is currently 100000 and is decreasing according to the equation $V(t)=100000(.876)^t$. The amount of money he has paid for his mortgage can be represented by the equation $M(t)=20000(1.1304)^t$. Graph and label $V(t)$ and $M(t)$ over the interval [0,10].



x	y
0	100000
2	76738
4	58887
6	45188
8	34676
10	26610

x	y
0	20000
2	25556
4	32656
6	41728
8	53320
10	68132

Scale

$x \geq \frac{10}{20}$ $\qquad$ $y \geq \frac{100000}{20}$

$x \geq .5$ $\qquad$ $y \geq 5000$

$x = .5$ $\qquad$ $y = 5000$

After how many years will the value of Tom's bank account be equal to the amount of money he paid for his mortgage? Round your answer to the *nearest tenth of a year*. Tom will open a new bank account when the value of his account has decreased by 72%. After how many years, to the *nearest hundredth of a year*, will that happen?

$1-.72$

$.28$

2nd Trace, Intersect

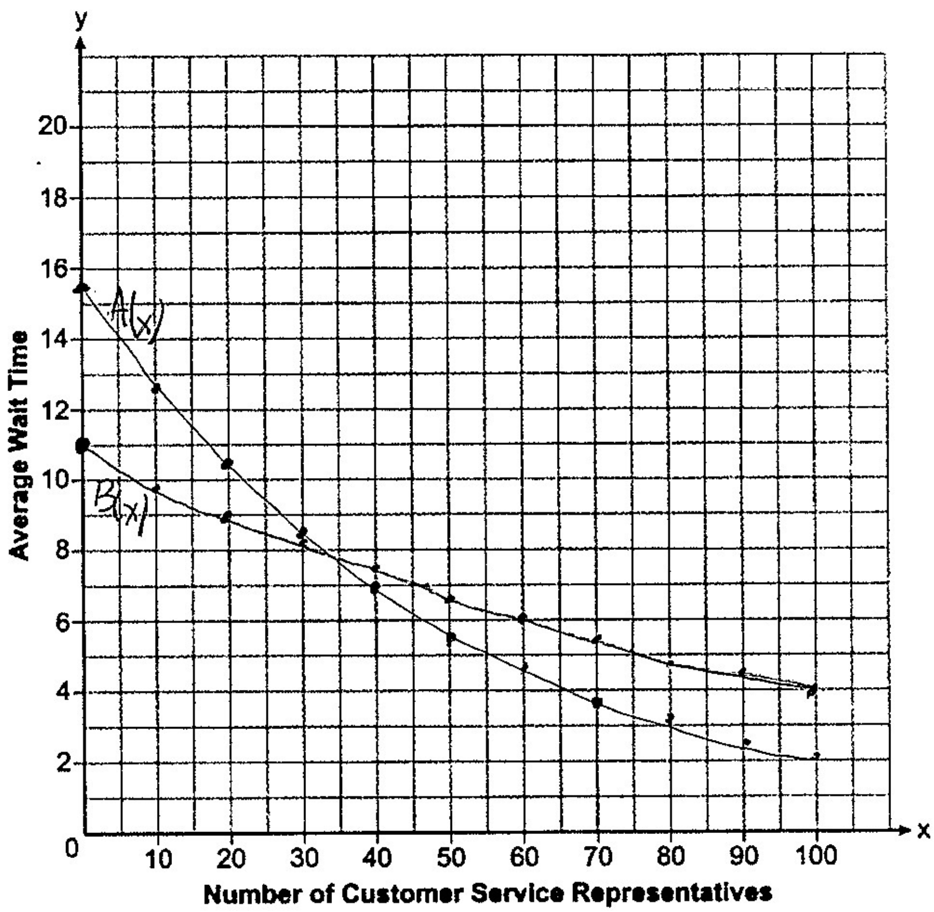
(6.3, ~~43~~ 43356.8)

6.3 years

5. A technology company is comparing two plans for speeding up its technical support time. Plan A can be modeled by the function $A(x) = 15.7(0.98)^x$ and plan B can be modeled by the function $B(x) = 11(0.99)^x$ where x is the number of customer service representatives employed by the company and $A(x)$ and $B(x)$ represent the average wait time, in minutes, of each customer. Graph $A(x)$ and $B(x)$ in the interval $0 \le x \le 100$ on the set of axes below.

A(x)

x	y
0	15.7
10	12.8
20	10.5
30	8.6
40	7.0
50	5.7
60	4.7
70	3.8
80	3.1
90	2.5
100	2.1



B(x)

x	y
0	11
10	9.9
20	9.0
30	8.1
40	7.4
50	6.7
60	6.0
70	5.4
80	4.9
90	4.5
100	4.0

To the *nearest integer*, solve the equation $A(x) = B(x)$. Determine, to the *nearest minute*, $B(100) - A(100)$. Explain what this value represents in the given context.

41 42 intersect

$x = 35$

$B(100) - A(100)$

$4.0 - 2.1$

1.9

If they hire 100 customer service representatives, the average wait time time would be 1.9 minutes longer with Plan B.

P

6. Tony is evaluating his retirement savings. He currently has $318,000 in his account, which earns an interest rate of 7% compounded annually. He wants to determine how much he will have in the account in the future, even if he makes no additional contributions to the account. Write a function, $A(t)$, to represent the amount of money that will be in his account in t years. Graph $A(t)$ where $0 \le t \le 20$ on the set of axes below.

$A = P(1+r)^t$

$A = A(t)$

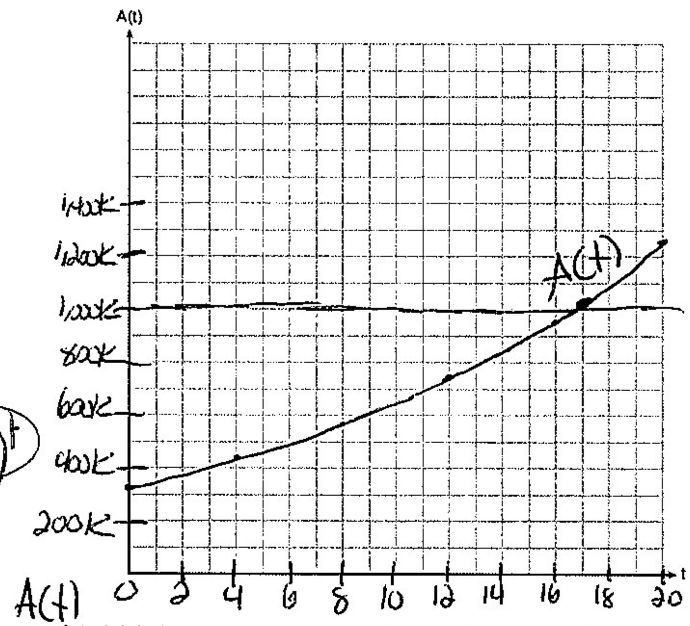
$P = 318{,}000$

$r = .07$

$t = t$

$A = 318{,}000(1+.07)^t$

$A(t) = 318{,}000(1.07)^t$



A(t)

x	y
0	318,000
4	416833
8	546383
12	716197
16	938788
20	1230000

Scale $\ge \frac{\text{max}}{\#\text{ of boxes}}$

$\ge \frac{1230000}{20}$

≥ 61500

Tony's goal is to save $1,000,000. Determine algebraically, to the *nearest year*, how many years it will take for him to achieve his goal. Explain how your graph of $A(t)$ confirms your answer. Scale = 100,000

$\frac{1{,}000{,}000}{318{,}000} = \frac{318{,}000(1.07)^t}{318{,}000}$

$\log \frac{500}{159} = \log 1.07^t$

$\frac{\log \frac{500}{159}}{\log 1.07} = \frac{t \log 1.07}{\log 1.07}$

$17 = t$

17 is the first year the graph crosses 1,000,000.

7. The resting blood pressure of an adult patient can be modeled by the function P below, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds.

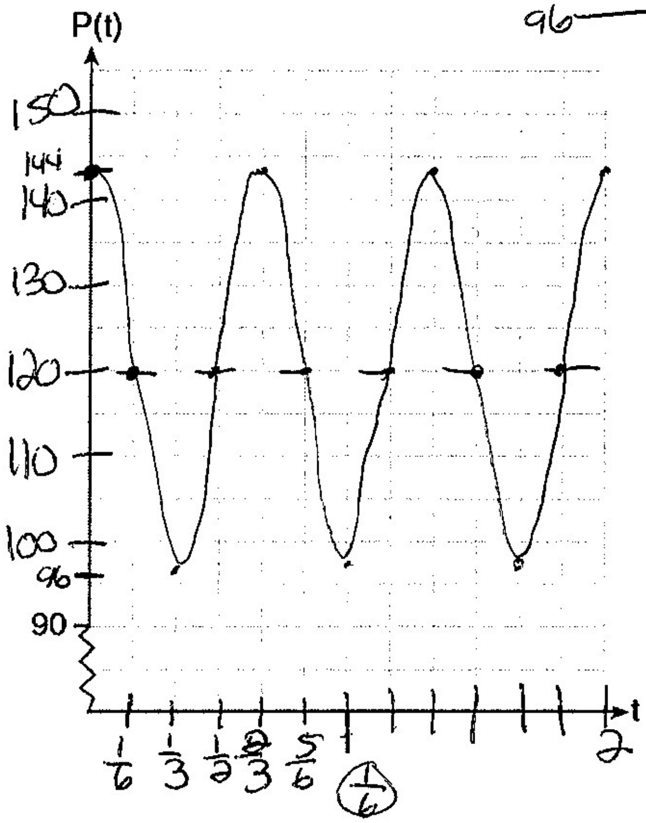
$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \le t \le 2$.

amp sin freq x shift +
$24\cos(3\pi t) + 120$
amp = 24
+ cos
freq = 3π
shift = 120

$p = \frac{2\pi}{f}$
$p = \frac{2\pi}{3\pi}$
$p = \frac{2}{3}$

144 ———
120 — — —
96 ———



Determine the period of P. Explain what this value represents in the given context. Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.

$P = \frac{2}{3}$

It takes $\frac{2}{3}$ of a second for blood pressure to drop down and come back up.

High Blood Pressure
$144 > 140$ and $96 > 90$

8. Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function $f(t) = -13\cos(0.8\pi t) + 13$, where t represents the time (in seconds) since the nail first became caught in the tire. Determine the period of $f(t)$. Interpret what the period represents in this context. On the grid below, graph *at least one* cycle of $f(t)$ that includes the y-intercept of the function.

amp sin freq x shift

$-13\cos 0.8\pi t + 13$

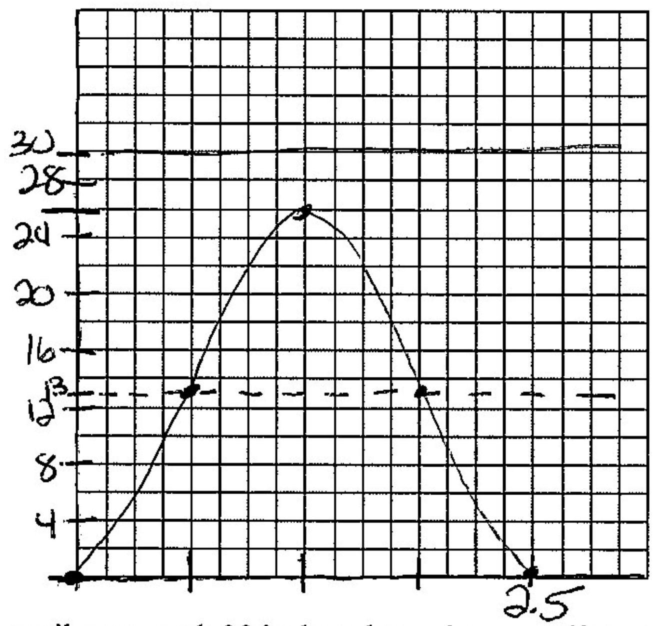
amp = 13

$-\cos$

freq = 0.8π

shift = 13

$p = \frac{2\pi}{0.8\pi} = 2.5$



26

13

0

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

Period = 2.5

It takes 2.5 seconds for the tire to complete one full revolution

No, the maximum height is 26 inches

Exponential Regression Equations with Equation Solving

1. Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

Hours (x)	**Average Number of Spores** (y)
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380

ExpReg

$y=a(b)^x$

$y=4.168(3.981)^x$

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*. The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest quarter hour*, that the meat can be kept at room temperature safely.

$\frac{100}{4.168}=\frac{\cancel{4.168}(3.981)^x}{\cancel{4.168}}$

$\log 23.99 = \log 3.981^x$

$\frac{\log 23.99\ldots}{\log 3.981} = \frac{x\cancel{\log 3.981}}{\cancel{\log 3.981}}$

$2.30 = x$

$2.25 = x$

2. The table below gives air pressures in kPa at selected altitudes above sea level measured in kilometers.

x	**Altitude** (km)	0	1	2	3	4	5
y	**Air Pressure** (kPa)	101	90	79	70	62	54

Write an exponential regression equation that models these data rounding all values to the *nearest thousandth*. Use this equation to algebraically determine the altitude, to the *nearest hundredth* of a kilometer, when the air pressure is 29 kPa.

ExpReg

$y=a(b)^x$

$y=101.523(.883)^x$

$\underset{y}{29}$

$\frac{29}{101.523}=\frac{\cancel{101.523}(.883)^x}{\cancel{101.523}}$

$\log .2856\ldots = \log .883^x$

$\frac{\log .2856\ldots}{\log .883} = \frac{x\cancel{\log .883}}{\cancel{\log .883}}$

$10.07 = x$

Complex Formulas with Equation Solving
List what each variable represents and CAREFULLY substitute into the given formula.
Solve the equation using the appropriate Algebra skills

1. The Fahrenheit temperature, $F(t)$, of a heated object at time t, in minutes, can be modeled by the function below. F_s is the surrounding temperature, F_0 is the initial temperature of the object, and k is a constant.

$$F(t) = F_s + (F_0 - F_s)e^{-kt}$$

Coffee at a temperature of 195°F (F_0) is poured into a container. The room temperature is kept at a constant 68°F (F_s) and $k = 0.05$. Coffee is safe to drink when its temperature is, at most, 120°F ($F(t)$). To the *nearest minute*, how long will it take until the coffee is safe to drink?

120 $F(t)$ = temp of object
68 F_s = temp surrounding
195 F_0 = temp initial
.05 $K = .05$
t t = time

$120 = 68 + (195-68)e^{-.05t}$
$-68 \quad -68$
$\frac{52}{127} = \frac{127e^{-.05t}}{127}$
$\ln\frac{52}{127} = \ln e^{-.05t}$

$\frac{\ln\frac{52}{127}}{-.05} = \frac{-.05t \cancel{\ln e}}{-.05}$

$18 = t$

2. The speed of a tidal wave, s, in hundreds of miles per hour, can be modeled by the equation $s = \sqrt{t} - 2t + 6$, where t represents the time from its origin in hours. Algebraically determine the time when $s = 0$. How much faster was the tidal wave traveling after 1 hour than 3 hours, to the *nearest mile per hour*? Justify

radical equation

0 = s = speed
t = time

$0 = \sqrt{t} - 2t + 6$
$+2t-6 \quad +2t-6$
$(2t-6)^2 = (\sqrt{t})^2$
$(2t-6)(2t-6) = t$
$4t^2 - 24t + 36 = t$
$-t \quad -t$
$4t^2 - 25t + 36 = 0$

	$2t$	-6
$2t$	$4t^2$	$-12t$
-6	$-12t$	$+36$

$4t^2 - 24t + 36$

$t^2 - 25t + 144$
$(t-16)(t-9)$
$\frac{\;}{4} \quad \frac{\;}{4}$
$(t-4)(4t-9) = 0$

$t-4=0$
$+4 \; +4$
$t = 4$

$4t-9=0$
$+9 \; +9$
$\frac{4t}{4} = \frac{9}{4}$
$\cancel{t = 2.25}$

$s(1) = \sqrt{1} - 2(1) + 6 = 5$
$s(3) = \sqrt{3} - 2(3) + 6 = \sqrt{3}$
$5 - \sqrt{3} = \boxed{3}$

*both of these questions would be full credit with y1 y2 intersect

3. A formula for work problems involving two people is shown below.

$$\frac{1}{t_1}+\frac{1}{t_2}=\frac{1}{t_b}$$

Fred t_1 = the time taken by the first person to complete the job = 8
Barney t_2 = the time taken by the second person to complete the job = 6
t_b = the time it takes for them working together to complete the job t_b

Fred and Barney are carpenters who build the same model desk. It takes Fred eight hours to build the desk while it only takes Barney six hours. Write an equation that can be used to find the time it would take both carpenters working together to build a desk. Determine, to the *nearest tenth of an hour*, how long it would take Fred and Barney working together to build a desk.

F1: 48
F2: t_b
LCD: $48t_b$

$$6 \cdot 48t_b\left(\frac{1}{8}\right)+8 \cdot 48t_b\left(\frac{1}{6}\right)=\left(\frac{1}{t_b}\right)48t_b$$

$$6t_b+8t_b=48$$

$$\frac{14t_b}{14}=\frac{48}{14}$$

$$t_b=3.4$$

4. Objects cool at different rates based on the formula below.

$T=(T_0-T_R)e^{-rt}+T_R$ T = temp of shirt

T_0: initial temperature = 400

T_R: room temperature = 75

r: rate of cooling of the object = .0735

5 t: time in minutes that the object cools to a temperature, T

Mark makes T-shirts using a hot press to transfer designs to the shirts. He removes a shirt from a press that heats the shirt to 400°F. T_0 The rate of cooling for the shirt is 0.0735 and the room temperature is 75°F. T_R Using this information, write an equation for the temperature of the shirt, T, after t minutes. Use the equation to find the temperature of the shirt, to the *nearest degree*, after t five minutes. At the same time, Mark's friend Jeanine removes a hoodie from a press that heats the hoodie to 450°F. T_0 After eight minutes, the hoodie measured 270°F. T The room temperature is still 75°F. Determine the rate of cooling of the hoodie, to the *nearest ten thousandth*. The T-shirt and hoodie were removed at the same time. Determine when the temperature will be the same, to the *nearest minute*.

$$T=(400-75)e^{-.0735t}+75$$
$$T=(400-75)e^{-.0735(5)}+75$$
$$T=300°$$

Jeanine
$T=270$
$T_0=450$
$T_R=75$
$r=r$
$t=8$

$$270=(450-75)e^{-8r}+75$$
$$-75 \qquad -75$$
$$\frac{195}{375}=\frac{375e^{-8r}}{375}$$
$$\ln .52=\ln e^{-8r}$$
$$\frac{\ln .52}{-8}=\frac{-8r\ln e}{-8}$$
$$.0817=r$$

y1: $(400-75)e^{-.0735t}+75=$
$(450-75)e^{-.0817t}+75$ y2

$t=17.5$ intersect

5. A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, t, in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation $t = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum in meters and g is a constant of 9.81 m/s^2. The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the *nearest tenth of a second*, the time it takes this pendulum to complete one swing. Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the *nearest tenth of a meter*, the length of this pendulum.

g

L

t = t = time

67 = L = Length of Pendulum

9.81 = g = 9.81

$t = 2\pi\sqrt{\frac{67}{9.81}}$

$t = 16.4$ sec

$t = 9.6$

$L = L$

$g = 9.81$

$\frac{9.6}{2\pi} = \frac{2\pi}{2\pi}\sqrt{\frac{L}{9.81}}$

$(1.527...)^2 = \left(\sqrt{\frac{L}{9.81}}\right)^2$

$9.81\,(2.33...) = \left(\frac{L}{9.81}\right)9.81$

$22.9 = L$

6. Sarah is fighting a sinus infection. Her doctor prescribed a nasal spray and an antibiotic to fight the infection. The active ingredients, in milligrams, remaining in the bloodstream from the nasal spray, $n(t)$, and the antibiotic, $a(t)$, are modeled in the functions below, where t is the time in hours since the medications were taken.

$$n(t) = \frac{t+1}{t+5} + \frac{18}{t^2 + 8t + 15}$$

$$a(t) = \frac{9}{t+3}$$

$t=0$

Determine which drug is made with a greater initial amount of active ingredient. Justify your answer. Sarah's doctor told her to take both drugs at the same time. Determine algebraically the number of hours after taking the medications when both medications will have the same amount of active ingredient remaining in her bloodstream.

equal

n(t) = mg of nasal spray

a(t) = mg of antibiotic

t = time

$n(0) = \frac{0+1}{0+5} + \frac{18}{(0)^2+8(0)+15} = \frac{7}{5}$

$a(0) = \frac{9}{0+3} = 3$ antibiotic

F1: $t+5$

F2: $t+3$

LCD: $(t+5)(t+3)$

t^2+4t+3

$\frac{t+1}{t+5} + \frac{18}{t^2+8t+15} = \frac{9}{t+3}$ $(t+5)(t+3)$

$(t+3)(t+5)$

$(t+1)(t+3) + 18 = 9(t+5)$

$t^2+4t+3+18 = 9t+45$

$t^2+4t+21 = 9t+45$

$-9t-45 \quad -9t-45$

$t^2-5t-24=0$

$(t-8)(t+3)=0$

$t-8=0$ $t+3=0$

$+8\ +8$ $-3\ -3$

$t=8$ $\cancel{t=-3}$

7. The Beaufort Wind Scale was devised by British Rear Admiral Sir Francis Beaufort, in 1805 based upon observations of the effects of the wind. Beaufort numbers, B, are determined by the equation $B = 1.69\sqrt{s+4.45} - 3.49$, where s is the speed of the wind in mph, and B is rounded to the nearest integer from 0 to 12.

B = Beaufort numbers
s = speed of wind

Beaufort Wind Scale	
Beaufort Number	**Force of Wind**
0	Calm
1	Light air
2	Light breeze
3	Gentle breeze
4	Moderate breeze
5	Fresh breeze
6	Steady breeze
7	Moderate gale
8	Fresh gale
9	Strong gale
10	Whole gale
11	Storm
12	Hurricane

Using the table above, classify the force of wind at a speed of 30 mph. Justify your answer. In 1946, the scale was extended to accommodate strong hurricanes. A strong hurricane received a B value of exactly 15. Algebraically determine the value of s, to the *nearest mph*. Any B values that round to 10 receive a Beaufort number of 10. Using technology, find an approximate range of wind speeds, to the *nearest mph*, associated with a Beaufort number of 10.

$B = B$
$S = 30$
$B = 1.69\sqrt{30+4.45} - 3.49$
$B = 6.42..$
Steady Breeze

$B = 15$
$S = S$
$15 = 1.69\sqrt{S+4.45} - 3.49$
$+3.49 \qquad +3.49$
$18.49 = 1.69\sqrt{S+4.45}$
$\overline{1.69} \qquad \overline{1.69}$
$(10.9\ldots)^2 = (\sqrt{S+4.45})^2$
$119.. = S+4.45$
$-4.45 \qquad -4.45$
$115 = S$

$B = 9.5 \quad B = 10.49$

X	y
54	9.43
55	9.54
56	9.65
↓	
64	10.49
65	10.59

55-64

Writing Equations FIRST!

Read carefully! If it asks for an equation, write the equation with two variables. Use the given variables if necessary.

AFTER you write your equation, re-write the equation substituting in the appropriate variables.

1. Tony is evaluating his retirement savings. He currently has $318,000 in his account, which earns an interest rate of 7% compounded annually. He wants to determine how much he will have in the account in the future, even if he makes no additional contributions to the account. Write a function, $A(t)$, to represent the amount of money that will be in his account in t years. Determine algebraically, to the *nearest year*, how many years it will take for him to achieve his goal.

P

end of question

Tony's goal is to save $1,000,000. A

$A = A(t)$
$P = 318{,}000$
$r = .07$
$n = 1$
$t = t$

$A = P(1+\frac{r}{n})^{nt}$

$\cancel{A(t) = 318{,}000(1+\frac{.07}{1})^{1t}}$

$A(t) = 318{,}000(1.07)^t$

$\frac{1{,}000{,}000}{318{,}000} = \frac{318{,}000(1.07)^t}{318{,}000}$

$\log 3.. \overset{\log}{=} 1.07^t$

$\frac{\log 3..}{\log 1.07} = \frac{t \log 1.07}{\log 1.07}$

$17 = t$

2. The half-life of a radioactive substance is 15 years. Write an equation that can be used to determine the amount, $s(t)$, of 200 grams of this substance that remains after t years. Determine algebraically, to the *nearest year*, how long it will take for $\frac{1}{10}$ of this substance to remain.

end of question

P

$A = s(t)$
$P = 200$
$t = t$
$h = 15$

$A = P(\frac{1}{2})^{\frac{t}{h}}$

$s(t) = 200(\frac{1}{2})^{\frac{t}{15}}$

$A = \frac{1}{10}(200)$

$\frac{\frac{1}{10}(200)}{200} = \frac{200(\frac{1}{2})^{\frac{t}{15}}}{200}$

$\log \frac{1}{10} \overset{\log}{=} \frac{1}{2}^{\frac{t}{15}}$

$15\left(\log \frac{1}{10} = \frac{t}{15} \log \frac{1}{2}\right)15$

$\frac{15 \log \frac{1}{10}}{\log \frac{1}{2}} = \frac{t \log \frac{1}{2}}{\log \frac{1}{2}}$

$50 = t$

3. On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year. Write two functions, $P(x)$ and $F(x)$, that represent the number of palm trees and flamingos on this island, respectively, x years from now. State the solution to the equation $P(x) = F(x)$, rounded to the *nearest year*. Interpret the meaning of this value within the given context.

end of question

41.42 Zoom Fit Intersect

Palm tree

$A = P(1 \pm r)^t$

$A = P(x)$
$P = 500$
$r = .03$
$t = t$

$P(x) = 500(1-.03)^t$

flamingo

$A = Pe^{rt}$

$A = F(x)$
$P = 200$
$r = .02$
$t = t$

$F(x) = 200e^{.02t}$

$x = 18$

In 18 years, the population of Palm Trees and Flamingos will be the same.

4. When observed by researchers under a microscope, a smartphone screen contained approximately 11,000 bacteria per square inch. Bacteria, under normal conditions, double in population every 20 minutes. Assuming an initial value of 11,000 bacteria, write a function, $p(t)$, that can be used to model the population of bacteria, p, on a smartphone screen, where t represents the time in minutes after it is first observed under a microscope. | end of question Determine algebraically, to the *nearest hundredth of a minute*, the amount of time it would take for a smartphone screen that was not touched or cleaned to have a population of 1,000,000 bacteria per square inch.

$\underset{h}{20}$ irregular

$A = p(t)$
$P = 11{,}000$
$t = t$
$h = 20$

$A = P(2)^{\frac{t}{h}}$

$p(t) = 11{,}000(2)^{\frac{t}{20}}$

$\frac{1{,}000{,}000}{11{,}000} = \frac{11{,}000(2)^{\frac{t}{20}}}{11{,}000}$

$\log \frac{1000}{11} = \log 2^{\frac{t}{20}}$

$20\left(\log \frac{1000}{11}\right) = \left(\frac{t}{20}\log 2\right)20$

$\frac{20\log\left(\frac{1000}{11}\right)}{\log 2} = \frac{t\log 2}{\log 2}$

$390.38 = t$

5. The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested \$8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously. Write a function, $A(t)$, for Abby's account and a function, $B(t)$, for Brett's account that calculates the value of each account after t years. | end of question Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*. | end of question Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

n = 4, Pert

Abby

$A = P(1 + \frac{r}{n})^{nt}$

$A = A(t)$
$P = 8000$
$r = .042$
$n = 4$
$t = t$

$A = 8000(1 + \frac{.042}{4})^{4t}$

$A(t) = 8000(1.0105)^{4t}$

Brett

$B(t) = 3(8000)$

$A = Pe^{rt}$

$A = B(t)$
$P = 8000$
$r = .039$
$t = t$

$B(t) = 8000e^{.039t}$

$A(18) = 8000(1.0105)^{4(18)} = 16970.90$

$B(18) = 8000e^{.039(18)} = 16142.27$

$\$828.63$

Abby will have \$828.63 more

$\frac{3(8000)}{8000} = \frac{8000e^{.039t}}{8000}$

$\ln 3 = \ln e^{.039t}$

$\frac{\ln 3}{.039} = \frac{.039t}{.039}$

$28.2 = t$

6. Given the geometric series $300 + 360 + 432 + 518.4 + \ldots$, write a geometric series formula, S_n, for the sum of the first n terms. | end of question Use the formula to find the sum of the first 10 terms, to the *nearest tenth*.

n

$\frac{360}{300} = 1.2$

$\frac{432}{360} = 1.2$

$r = 1.2$
$a_1 = 300$

$S_n = \frac{a_1 - a_1(r)^n}{1 - r}$

$S_n = \frac{300 - 300(1.2)^n}{1 - 1.2}$

$S_{10} = \frac{300 - 300(1.2)^{10}}{1 - 1.2}$

$S_{10} = 7787.6$

Common Core High School Math Reference Sheet
(Algebra I, Geometry, Algebra II)

CONVERSIONS

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilograms
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

FORMULAS

Triangle	$A = \frac{1}{2}bh$	Pythagorean Theorem	$a^2 + b^2 = c^2$
Parallelogram	$A = bh$	Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Circle	$A = \pi r^2$	Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Circle	$C = \pi d$ or $C = 2\pi r$	Geometric Sequence	$a_n = a_1 r^{n-1}$
General Prisms	$V = Bh$	Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Cylinder	$V = \pi r^2 h$	Radians	$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$
Sphere	$V = \frac{4}{3}\pi r^3$	Degrees	$1 \text{ degree} = \frac{\pi}{180} \text{ radians}$
Cone	$V = \frac{1}{3}\pi r^2 h$	Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$
Pyramid	$V = \frac{1}{3}Bh$		