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Common Core Algebra II Common Regents Questions Packet!

Mr. Schlansky

Comparing Expressions

Use Multiple Choice Strategy with Variables for each option

1. Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?

I $(m+p)^2 = m^2 + 2mp + p^2$

II $(x+y)^3 = x^3 + 3xy + y^3$

III $(a^2+b^2)^2 = (a^2-b^2)^2 + (2ab)^2$

1) I, only
2) I and II
3) II and III
4) I and III

2. Which expression(s) are equivalent to $\frac{x^2-4x}{2x}$, where $x \neq 0$?

I. $\frac{x}{2} - 2$ II. $\frac{x-4}{2}$ III. $\frac{x-1}{2} - \frac{3}{2}$

1) II, only
2) I and II
3) II and III
4) I, II, and III

3. Which of the following identities hold true for all real numbers?

I. $(x^2+2y)^2 = x^4 + 4x^2y + 4y^2$

II. $(x^2-4z^3)(x^2+4z^3) = x^4 - 16z^6$

III. $(x+y)(x^2-xy-y^2) = x^3 - y^3$

1) I, only
2) I and II only
3) II and III only
4) I and III only

4. Which factorizations are correct?

I. $a^3 + 27b^3 = (a+3b)(a^2-3ab+9b^2)$

II. $c^3 - 6c^2 + 8c + 5c^2 - 30c + 40 = (c-2)(c-4)(c+5)$

III. $1 - x^4 = (1+x)^2(1-x)^2$

1) I, only
2) I and II only
3) II and III only
4) I, II, and III

5. Which factorization is *incorrect*?

1) $4k^2 - 49 = (2k+7)(2k-7)$
2) $a^3 - 8b^3 = (a-2b)(a^2+2ab+4b^2)$
3) $m^3 + 3m^2 - 4m + 12 = (m-2)^2(m+3)$
4) $t^3 + 5t^2 + 6t + t^2 + 5t + 6 = (t+1)(t+2)(t+3)$

6. Which expression has been rewritten correctly to form a true statement?

1) $(x+2)^2 + 2(x+2) - 8 = (x+6)x$
2) $x^4 + 4x^2 + 9x^2y^2 - 36y^2 = (x+3y)^2(x-2)^2$
3) $x^3 + 3x^2 - 4xy^2 - 12y^2 = (x-2y)(x+3)^2$
4) $(x^2-4)^2 - 5(x^2-4) - 6 = (x^2-7)(x^2-6)$

Dividing Polynomials: (Synthetic Division)

Negative the value of what you are dividing by and put it outside

Bring the first number down

Multiply, Add, Multiply, Add, etc.

Decrease the first terms exponent by 1, the last number is the remainder. The remainder goes over the divisor.

(Put 0 as a placeholder if necessary)

Divide each of the following polynomials

1. $\dfrac{2x^3+5x^2-31x-84}{x+3}$

$$\begin{array}{r|rrrr} -3 & 2 & 5 & -31 & -84 \\ & & -6 & 3 & 84 \\ \hline & 2 & -1 & -28 & 0 \end{array}$$

$2x^2-x-28$

2. $\dfrac{x^4-2x^2-7x+12}{x+6}$

$$\begin{array}{r|rrrrr} -6 & 1 & 0 & -2 & -7 & 12 \\ & & -6 & 36 & 204 & -1182 \\ \hline & 1 & -6 & 34 & 197 & -1170 \end{array}$$

$x^3-6x^2+34x+197-\frac{1170}{x+6}$

3. $\dfrac{x^3+5x^2-1}{x+2}$

$$\begin{array}{r|rrrr} -2 & 1 & 5 & 0 & -1 \\ & & -2 & -6 & 12 \\ \hline & 1 & 3 & -6 & 11 \end{array}$$

$x^2+3x-6+\frac{11}{x+2}$

4. $\dfrac{4x^3+12x^2-5}{x+5}$

$$\begin{array}{r|rrrr} -5 & 4 & 12 & 0 & -5 \\ & & -20 & 40 & -200 \\ \hline & 4 & -8 & 40 & -205 \end{array}$$

$4x^2-8x+40-\frac{205}{x+5}$

5. $\dfrac{6x^3-5x+3}{x-3}$

$$\begin{array}{r|rrrr} 3 & 6 & 0 & -5 & 3 \\ & & 18 & 54 & 147 \\ \hline & 6 & 18 & 49 & 150 \end{array}$$

$6x^2+18x+49+\frac{150}{x-3}$

6. $\dfrac{5x^3-60}{x-2}$

$$\begin{array}{r|rrrr} 2 & 5 & 0 & 0 & -60 \\ & & 10 & 20 & 40 \\ \hline & 5 & 10 & 20 & -20 \end{array}$$

$5x^2+10x+20-\frac{20}{x-2}$

7. $\dfrac{x^2+x-4}{x-3}$

$$\begin{array}{r|rrr} 3 & 1 & 1 & -4 \\ & & 3 & 12 \\ \hline & 1 & 4 & 8 \end{array}$$

$x+4+\frac{8}{x-3}$

8. $\dfrac{-3x^2+10x-6}{x+1}$

$$\begin{array}{r|rrr} -1 & -3 & 10 & -6 \\ & & 3 & -13 \\ \hline & -3 & 13 & -19 \end{array}$$

$-3x+13-\frac{19}{x+1}$

To determine if a binomial is a factor:
Find the remainder! (Use remainder theorem)
If the remainder is 0, it is a factor
If the remainder is not 0, it is not a factor

1. Is $x-6$ a factor of $p(x)=x^3-6x^2+4x-1$? Explain your answer.

$P(6)=(6)^3-6(6)^2-4(6)-1$
$P(6)=23$
No, the remaindor is not 0

2. Is $x+2$ a factor of $p(x)=x^3-3x^2-8x+4$? Explain your answer.

$P(-2)=\cancel{x}(-2)^3-3(-2)^2-8(-2)+4$
$P(-2)=0$ Yes, the remainder is 0

$2x+1=0$
$-1\ -1$
$2x=-1$
$\frac{2x}{2}=\frac{-1}{2}$
$x=-\frac{1}{2}$

3. Is $2x+1$ a factor of $p(x)=2x^2+5x+2$? Explain your answer.

$P(-\frac{1}{2})=2(-\frac{1}{2})^2+5(-\frac{1}{2})+2$
$P(-\frac{1}{2})=0$ Yes, the remainder is 0.

4. Determine if $x-5$ is a factor of $2x^3-4x^2-7x-10$. Explain your answer.

$P(5)=2(5)^3-4(5)^2-7(5)-10$
$P(5)=105$ No, the remainder is not 0

5. Determine if $x+4$ is a factor of $p(x)=x^4-6x^3-4x^2+54x-45$. Explain your answer.

$P(-4)=(-4)^4-6(-4)^3-4(-4)^2+54(-4)-45$
$P(-4)=315$ No, the remainder is not 0

6. Determine if $x+3$ is a factor of $p(x)=x^4+7x^3+9x^2-21x-36$. Explain your answer.

$P(-3)=(-3)^4+7(-3)^3+9(-3)^2-21(-3)-36$
$P(-3)=0$ Yes, the remainder is 0

7. Use an appropriate procedure to show that $x-4$ is a factor of the function $f(x)=2x^3-5x^2-11x-4$. Explain your answer.

$f(4)=2(4)^3-5(4)^2-11(4)-4$
$f(4)=0$

x-4 is a factor because when divided into the polynomial, the remainder is 0.

8. Which binomial is a factor of $x^4 - 4x^2 - 4x + 8$?

~~1)~~ $x-2$ $p(2)=0$
2) $x+2$ $p(-2)=16$
3) $x-4$ $p(4)=184$
4) $x+4$ $p(-4)=216$

9. Which binomial is *not* a factor of the expression $x^3 - 11x^2 + 16x + 84$?

1) $x+2$ $p(-2)=0$
~~2)~~ $x+4$ $p(-4)=-220$
3) $x-6$ $p(6)=0$
4) $x-7$ $p(7)=0$

10. Which binomial is *not* a factor of the expression $x^3 - 6x^2 - 49x - 66$?

1) $x-11$ $p(11)=0$
2) $x+2$ $p(-2)=0$
~~3)~~ $x+6$ $p(-6)=-204$
4) $x+3$ $p(-3)=0$

11. Which binomial is a factor of the expression $x^3 - 7x - 6$?

1) $x+3$ $p(-3)=-12$
2) $x-1$ $p(1)=-12$
3) $x-2$ $p(2)=-12$
~~4)~~ $x+2$ $p(-2)=0$

12. Which binomial is *not* a factor of the expression $x^3 - 4x^2 - 25x + 28$?

~~1)~~ $x+6$ $p(-6)=-182$
2) $x-7$ $p(7)=0$
3) $x-1$ $p(1)=0$
4) $x+4$ $p(-4)=0$

13. Which binomial is a factor of the expression $x^4 + 4x^2 - 32$?

1) $x+8$ $p(-8)=4320$
2) $x-8$ $p(8)=4320$
3) $x-1$ $p(1)=-27$
~~4)~~ $x+2$ $p(-2)=0$

14. Which binomial is not a factor of $p(x) = 2x^3 + 7x^2 - 5x - 4$?

1) $x+4$ $p(-4)=0$
~~2)~~ $x+1$ $p(-1)=6$
3) $x-1$ $p(1)=0$
4) $2x+1$ $p(-\frac{1}{2})=0$
$2x+1=0$

15. Given $r(x) = x^3 - 4x^2 + 4x - 6$, find the value of $r(2)$. What does your answer tell you about $x-2$ as a factor of $r(x)$? Explain.

$r(2)=(2)^3-4(2)^2+4(2)-6$
$r(2)=-6$

$x-2$ is not a factor because the remainder is not 0.

Sketching Polynomial Graphs with Factors and Zeros

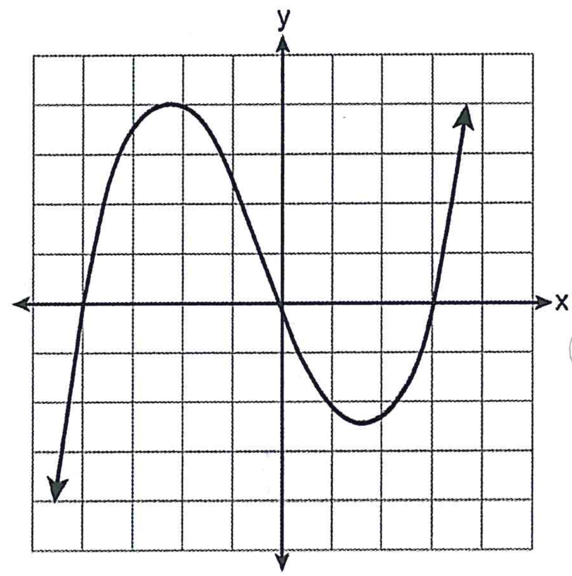
If a is a zero, $p(a) = 0$, $x - a$ is a factor, and the polynomial is divisible by $x - a$. Once you have one of the four pieces of information, you have all four.

The zeros are the x intercepts

1. When g(x) is divided by $x + 4$, the remainder is 0. Given $g(x) = x^4 + 3x^3 - 6x^2 - 6x + 8$, which conclusion about g(x) is true? → x+4 is a factor
-4 is a zero

1) $g(4) = 0$
2) $g(-4) = 0$
3) $x - 4$ is a factor of $g(x)$.
4) No conclusion can be made regarding $g(x)$.

2. The graph of $p(x)$ is shown below.



Which of the following statements must be true?

I. -4, 0, *and* 3 *are zeros of the polynomial*

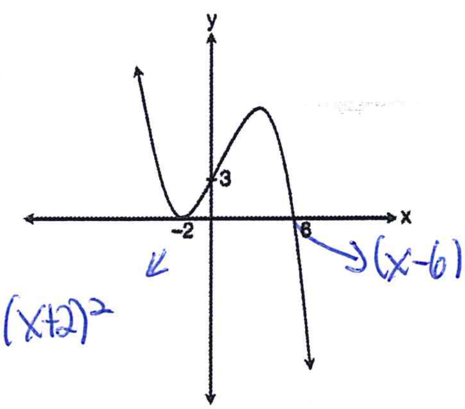
II. $(x+4)$, x, *and* $(x-3)$ *are factors of the polynomial*

III. $p(-4) = 0$, $p(0) = 0$, $p(3) = 0$

IV. The polynomial is divisible by $(x+4)$, x, *and* $(x-3)$

1) I and II
2) II only
3) I only
4) I, II, III, and IV

3. **The graph below shows the polynomial $y = p(x)$.**

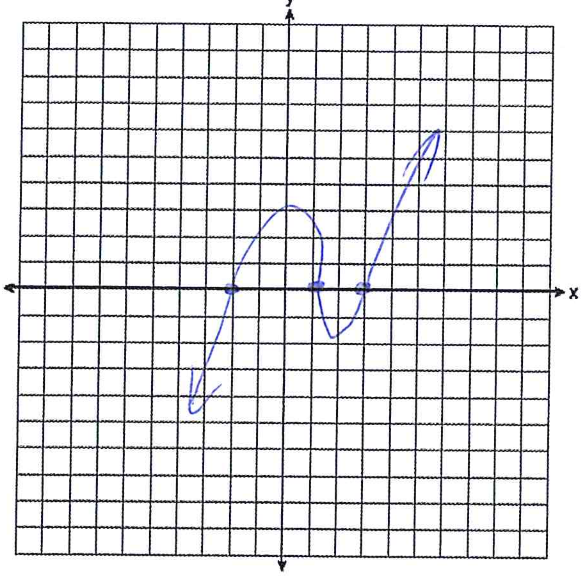


The factors of $p(x)$ are

(1) $(x+2)$, $(x-3)$, and $(x+6)$
(2) $(x-2)$, $(x+3)$, and $(x+6)$
(3) $(x-2)$, $(x-2)$, and $(x+6)$
(4) $(x+2)$, $(x+2)$, and $(x-6)$

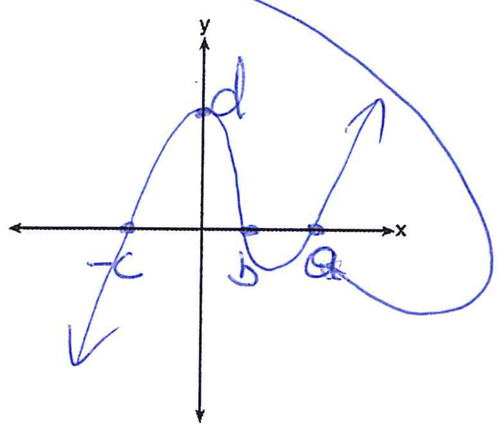
4. On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and -2.

hit the
x-axis

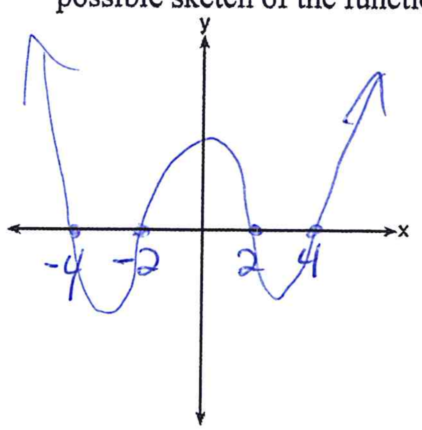


5. On the axes below, sketch a possible function $p(x) = (x - a)(x - b)(x + c)$, where a, b, and c are positive, $a > b$, and $p(x)$ has a positive y-intercept of d. Label all intercepts.

$x = a$ $x = b$ $x = -c$



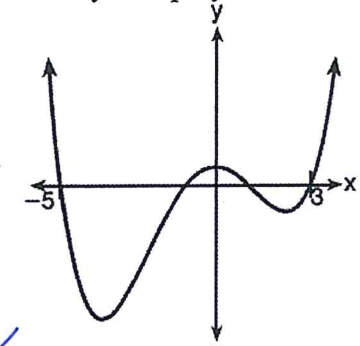
6. The zeros of a quartic polynomial function are 2, −2, 4, and −4. Use the zeros to construct a possible sketch of the function, on the set of axes below.



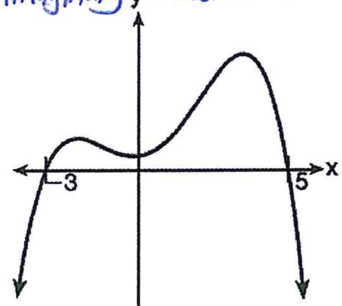
7. A 4th degree polynomial has zeros -5, 3, i, and $-i$. Which graph could represent the function defined by this polynomial?

imaginary $\Rightarrow$ don't touch x-axis

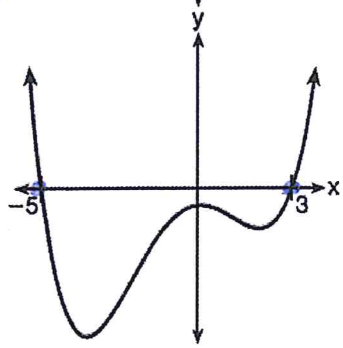
1)



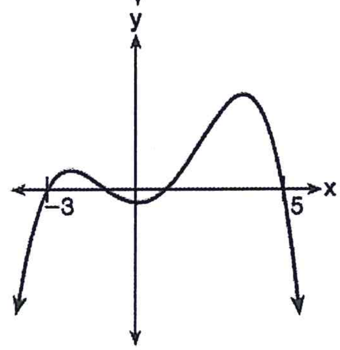
3)



2)



4)



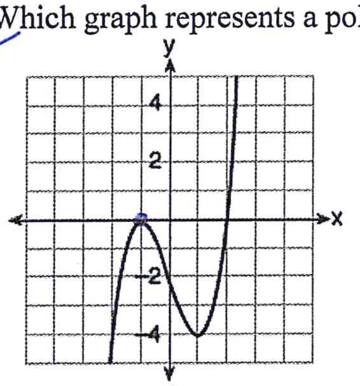
8. Which graph represents a polynomial function that contains $x^2 + 2x + 1$ as a factor?

$\rightarrow (x+1)^2$

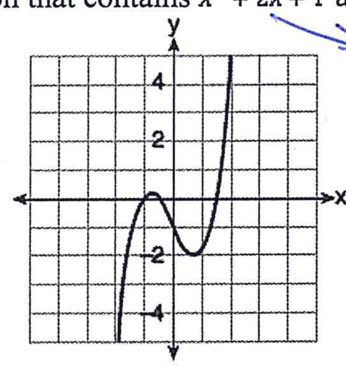
$\Downarrow$

double root at -1

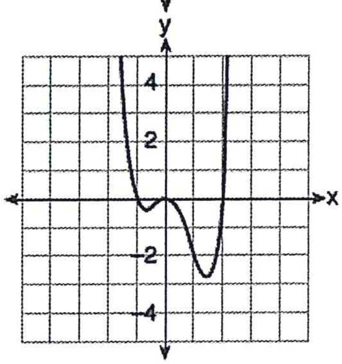
1)



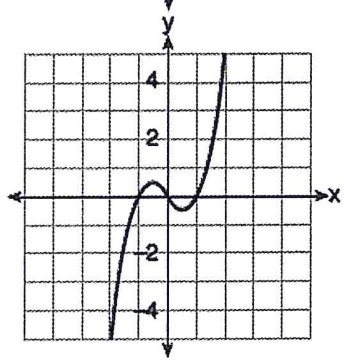
3)



2)



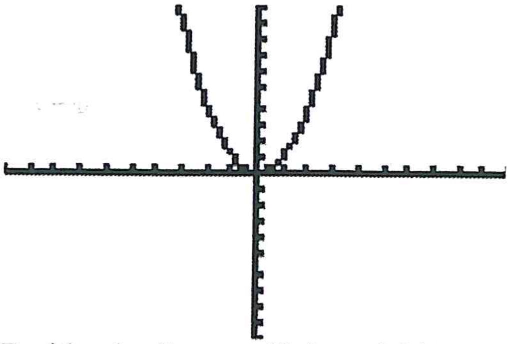
4)



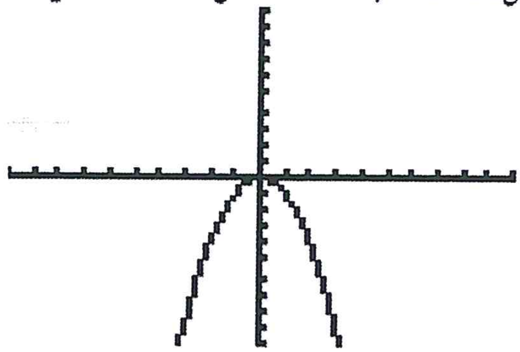
Sketching Polynomial Graphs Given Equation

End Behavior

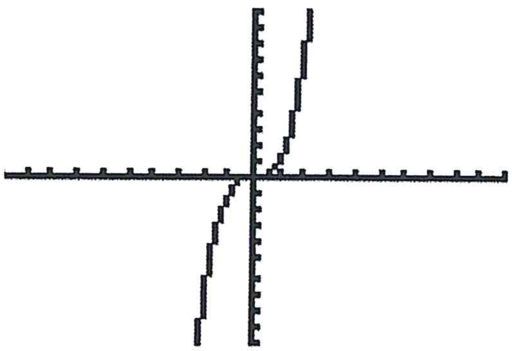
Positive leading coefficient, Even Degree



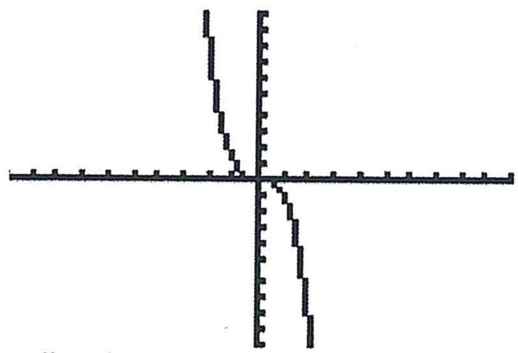
Negative leading coefficient, Even Degree



Positive leading coefficient, Odd Degree



Negative leading coefficient, Odd Degree



Even degree begins and ends pointing in same direction
Odd degree begins and ends point in opposite directions
Positive leading coefficient slopes up at the end
Negative leading coefficient slopes down at the end

The degree is the largest exponent
The y-intercept is the constant term

To find zeros:
Set equations equal to zero and solve polynomial equation.
If there is an odd amount of multiple roots, the graph passes through the x-axis
If there is an even amount of multiple roots, the graph bounces off the x-axis.

There is a maximum of $n-1$ relative minima/maxima for a polynomial of degree n

1. $f(x) = x^3 + 2x^2 - 9x - 18$ Positive odd

Degree: 3

y-intercept: -18

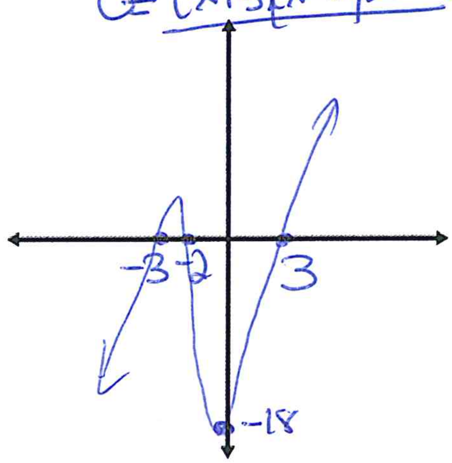
x-intercepts (zeros): $x = -3, 3, -2$

$0 = \frac{x^3 + 2x^2}{x^2} \frac{-9x - 18}{-9}$

$x^2(x+2) - 9(x+2)$

$(x^2 - 9)(x+2)$

$0 = (x+3)(x-3)(x+2)$



End Behavior:

left $x \to -\infty$, $f(x) \to -\infty$ down

right $x \to \infty$, $f(x) \to \infty$ up

Intervals where function is increasing:

$(-\infty, \approx -2.5)$ $(\approx .5, \infty)$

Intervals where function is decreasing:

$(\approx -2.5, \approx .5)$

Intervals where function is positive:

$(-3, -2)$ $(3, \infty)$

Intervals where function is negative:

$(-2, 3)$

Local extrema:

1 local max

1 local min

2. $f(x) = x^4 - 10x^2 + 9$ Positive even

Degree: 4

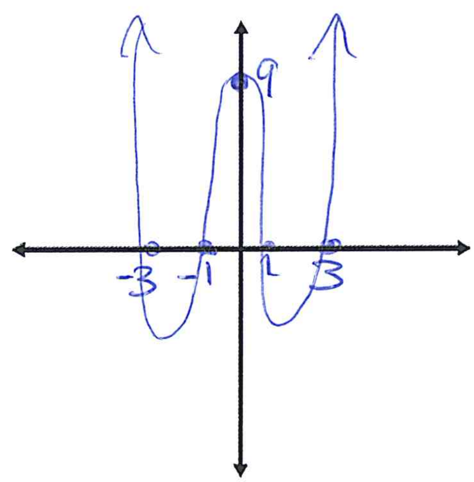
y-intercept: 9

x-intercepts (zeros): $0 = x^4 - 10x^2 + 9$

$(x^2 - 9)(x^2 - 1)$

$0 = (x+3)(x-3)(x+1)(x-1)$

$x = -3, 3, -1, 1$



End Behavior:

left $x \to -\infty$, $f(x) \to \infty$ up

right $x \to \infty$, $f(x) \to \infty$ up

Intervals where function is increasing:

$(\approx -2, \approx 0)$ $(\approx 2, \infty)$

Intervals where function is decreasing:

$(-\infty, \approx -2)$ $(\approx 0, \approx 2)$

Intervals where function is positive:

$(-\infty, -3)$ $(-1, 1)$ $(3, \infty)$

Intervals where function is negative:

$(-3, -1)$ $(1, 3)$

Local extrema:

1 local max

2 local mins

negative odd

3. $p(x) = -x^3 - 3x^2 + 4x + 12$

Degree: 3

y-intercept: 12

x-intercepts (zeros): $\frac{0}{-1} = \frac{-x^3 - 3x^2 + 4x + 12}{-1}$

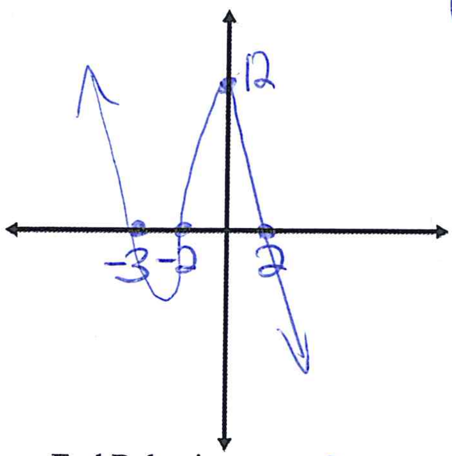
$0 = (x^3 + 3x^2)(-4x - 12)$

$x^2(x+3) - 4(x+3)$

$(x^2 - 4)(x+3)$

$(x+2)(x-2)(x+3)$

$x = -2, 2, -3$



End Behavior:

left $x \to -\infty$, $f(x) \to \infty$ up

right $x \to \infty$, $f(x) \to -\infty$ down

Intervals where function is increasing:

$(\approx -2.5, \approx 0)$

Intervals where function is decreasing:

$(-\infty, \approx -2.5)$ $(\approx 0, \infty)$

Intervals where function is positive:

$(-\infty, -3)$ $(-2, 2)$

Intervals where function is negative:

$(-3, -2)$ $(2, \infty)$

Local extrema:

1 local max

1 local min

negative even

4. $f(x) = -x^4 + 3x^3 + 10x^2$

Degree: 4

y-intercept: 0

x-intercepts (zeros): $\frac{0}{-1} = \frac{-x^4 + 3x^3 + 10x^2}{-1}$

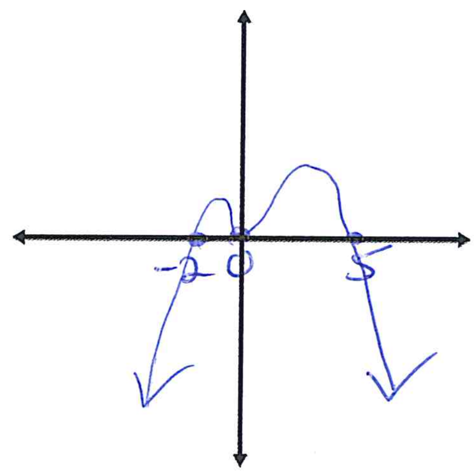
$0 = \frac{x^4}{x^2} - \frac{3x^3}{x^2} - \frac{10x^2}{x^2}$

$x^2(x^2 - 3x - 10)$

$x^2(x-5)(x+2)$

$x = 0, 5, -2$

double root



End Behavior:

left $x \to -\infty$, $f(x) \to$ down $-\infty$

right $x \to \infty$, $f(x) \to -\infty$ down

Intervals where function is increasing:

$(-\infty, \approx -1)$ $(0, \approx 2.5)$

Intervals where function is decreasing:

$(\approx -1, 0)$ $(\approx 2.5, \infty)$

Intervals where function is positive:

$(-2, 0)$ $(0, 5)$

Intervals where function is negative:

$(-\infty, -2)$ $(5, \infty)$

Local extrema:

2 local max

1 local min

Complex Numbers:

Treat i like a normal variable except know that $i^2 = -1$ and $i^3 = -i$

a + b*i* form simply means there will be an *i* in the answer

Use Multiple Choice Strategy if Multiple Choice!!!!!!!!!

I would use MC Strategy for this whole page

1. Given i is the imaginary unit, $(2 - yi)^2$ in simplest form is

1) $y^2 - 4yi + 4$
2) $-y^2 - 4yi + 4$
3) $-y^2 + 4$
4) $y^2 + 4$

$(2-yi)(2-yi)$

	2	$-yi$
2	4	$-2yi$
$-yi$	$-2yi$	$+y^2i^2$

$4-4yi+y^2i^2$

$4-4yi+y^2(-1)$

$4-4yi-y^2$

2. The expression $(3 - 7i)^2$ is equivalent to

1) $-40 + 0i$
2) $-40 - 42i$
3) $58 + 0i$
4) $58 - 42i$

$(3-7i)(3-7i)$

	3	$-7i$
3	9	$-21i$
$-7i$	$-21i$	$+49i^2$

$9-42i+49i^2$

$9-42i+49(-1)$

$9-42i-49$

$-40-42i$

3. The expression $(x + i)^2 - (x - i)^2$ is equivalent to

1) 0
2) -2
3) $-2 + 4xi$
4) $4xi$

$(x+i)(x+i) - (x-i)(x-i)$

	x	$+i$
x	x^2	xi
$+i$	xi	i^2

	x	$-i$
x	x^2	$-xi$
$-i$	$-xi$	i^2

$(x^2+2xi+i^2)-(x^2-2xi+i^2)$

Keep change change

$x^2+2xi+i^2$

$+ -x^2+2xi-i^2$

$4xi$

4. The expression $6xi^3(-4xi + 5)$ is equivalent to

1) $2x - 5i$
2) $-24x^2 - 30xi$
3) $-24x^2 + 30x - i$
4) $26x - 24x^2i - 5i$

$6x(-i)(-4xi+5)$

$-6xi(-4xi+5)$

$24x^2i^2-30xi$

$24x^2(-1)-30xi$

$-24x^2-30xi$

5. Which expression is equivalent to $(3k - 2i)^2$, where i is the imaginary unit?

1) $9k^2 - 4$
2) $9k^2 + 4$
3) $9k^2 - 12ki - 4$
4) $9k^2 - 12ki + 4$

	$3k$	$-2i$
$3k$	$9k^2$	$-6ki$
$-2i$	$-6ki$	$+4i^2$

$9k^2-12ki+4i^2$

$9k^2-12ki+4(-1)$

$9k^2-12ki-4$

6. Which expression is equivalent to $(2x - i)^2 - (2x - i)(2x + 3i)$ where i is the imaginary unit and x is a real number?

1) $-4 - 8xi$
2) $-4 - 4xi$
3) 2
4) $8x - 4i$

	$2x$	$-i$
$2x$	$4x^2$	$-2xi$
$-i$	$-2xi$	$+i^2$

$4x^2-4xi+i^2$

	$2x$	$-i$
$2x$	$4x^2$	$-2xi$
$+3i$	$6xi$	$-3i^2$

$4x^2+4xi-3i^2$

Keep change change

$(4x^2-4xi+i^2)-(4x^2+4xi-3i^2)$

$4x^2-4xi+i^2$

$+ -4x^2-4xi+3i^2$

$-8xi+4i^2$

$-8xi+4(-1)$

$-8xi-4$

7. If $A = -3 + 5i$, $B = 4 - 2i$, and $C = 1 + 6i$, where i is the imaginary unit, then $A - BC$ equals

1) $5 - 17i$
2) $5 + 27i$
3) $-19 - 17i$
4) $-19 + 27i$

$(-3+5i)-(4-2i)(1+6i)$

$(-3+5i)-(16+22i)$

$-3+5i$

$+ -16-22i$

$-19-17i$

	4	$-2i$
1	4	$-2i$
$6i$	$24i$	$-12i^2$

$4+22i-12i^2$

$4+22i-12(-1)$

$4+22i+12$

$16+22i$

	x	$+3i$
x	x^2	$+3xi$
$+3i$	$+3xi$	$+9i^2$

	$2x$	$-3i$
$2x$	$4x^2$	$-6xi$
$-3i$	$-6xi$	$+9i^2$

$(x^2+6xi+9i^2)-(4x^2-12xi+9i^2)$

8. Where i is the imaginary unit, the expression $(x+3i)^2-(2x-3i)^2$ is equivalent to

1) $-3x^2$
2) $-3x^2-18$
3) $-3x^2+18xi$
4) $-3x^2-6xi-18$

$x^2+6xi+\cancel{9i^2}$

$+\ -4x^2+12xi-\cancel{9i^2}$

$-3x^2+18xi$

9. If x is a real number, express $2xi(i-4i^2)$ in simplest $a+bi$ form.

$2xi(i-4(-1))$

$2xi(i+4)$

$2xi^2+8xi$

$2x(-1)+8xi$

$-2x+8xi$

10. Express $(1-i)^3$ in $a+bi$ form.

$(1-i)(1-i)(1-i)$

	1	$-i$
1	1	$-i$
$-i$	$-i$	$+i^2$

$1-2i+i^2$

$\cancel{1}-2i+\cancel{(-1)}$

$-2i$

$-2i(1-i)$

$-2i+2i^2$

$-2i+2(-1)$

$-2i-2$

$-2-2i$

11. Simplify $xi(i-7i)^2$, where i is the imaginary unit.

$xi(-6i)^2$

$xi(36i^2)$

$xi(36(-1))$

$xi(-36)$

$-36xi$

12. Write $(5+2yi)(4-3i)-(5-2yi)(4-3i)$ in $a+bi$ form, where y is a real number.

	5	$+2yi$
4	20	$+8yi$
$-3i$	$-15i$	$-6yi^2$

$20+8yi-15i-6yi^2$

	5	$-2yi$
4	20	$-8yi$
$-3i$	$-15i$	$+6yi^2$

$20-8yi-15i+6yi^2$

$(20+8yi-15i-6yi^2)-(20-8yi-15i+6yi^2)$

$\cancel{20}+8yi-\cancel{15i}-6yi^2$

$+\ -\cancel{20}+8yi+\cancel{15i}-6yi^2$

$16yi-12yi^2$

$16yi-12y(-1)$

$16yi+12y$

$12y+16yi$

13. Elizabeth tried to find the product of $(2+4i)$ and $(3-i)$, and her work is shown below.

$$(2+4i)(3-i)$$
$$=6-2i+12i-4i^2$$
$$=6+10i-4i^2$$
$$=6+10i-4(1)$$
$$=6+10i-4$$
$$=2+10i$$

Identify the error in the process shown and determine the correct product of $(2+4i)$ and $(3-i)$.

$i^2=-1$ not 1

$6+10i-4(-1)$

$6+10i+4$

$10+10i$

Definition of a Parabola: A parabola is the set of all points equidistant between a point (focus) and a line (directrix).
The vertex is directly in between the focus and the directrix. USE GRAPH PAPER AND COUNT!

$$\frac{(x-v)^2}{4p} = y - t$$

$(v,t) = vertex$

$p = distance\ from\ vertex\ to\ focus$

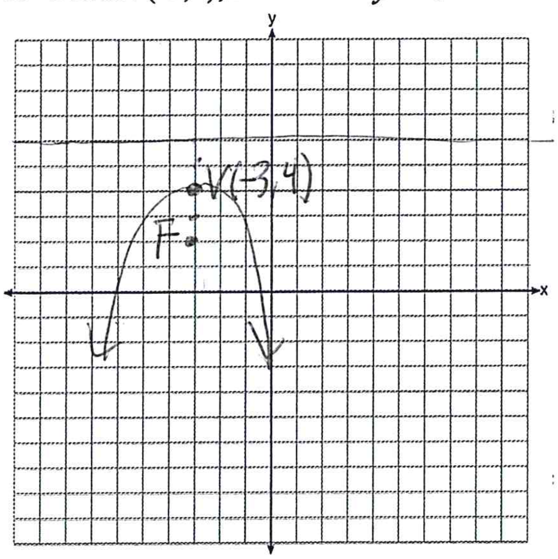
*p is positive when parabola opens up and negative when parabola opens down

You might have to cross multiply or isolate y

If given equation, pull the vertex out!

For each of the following problems, state the coordinate of the focus and vertex, the equation of the directrix and the parabola in three different forms.

1. Focus: (-3,2), Directrix: $y = 6$



$$\frac{(x-v)^2}{4p} = y - t$$

$v=-3$

$t=4$

$p=-2$

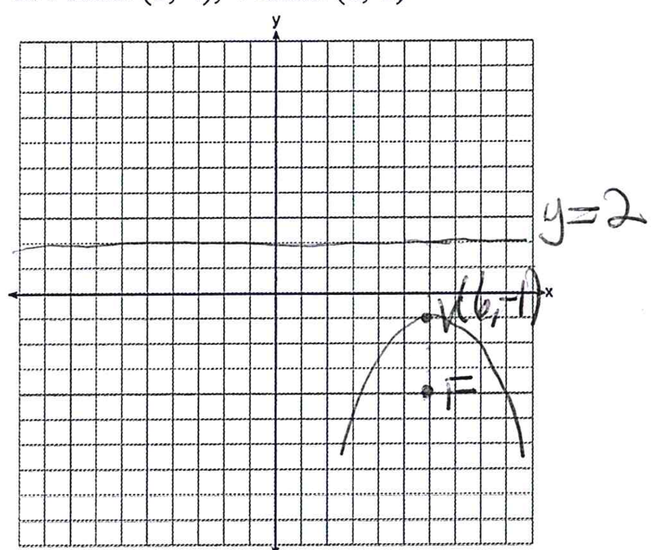
$$\frac{(x+3)^2}{4(-2)} = y-4$$

$$\frac{(x+3)^2}{-8} = y-4$$

$$-8(y-4)=(x+3)^2$$

$$y=\frac{(x+3)^2}{-8}+4$$

2. Focus: (6,-4), Vertex: (6,-1)



$$\frac{(x-v)^2}{4p} = y - t$$

$v=6$

$t=-1$

$p=-3$

$$\frac{(x-6)^2}{4(-3)} = y+1$$

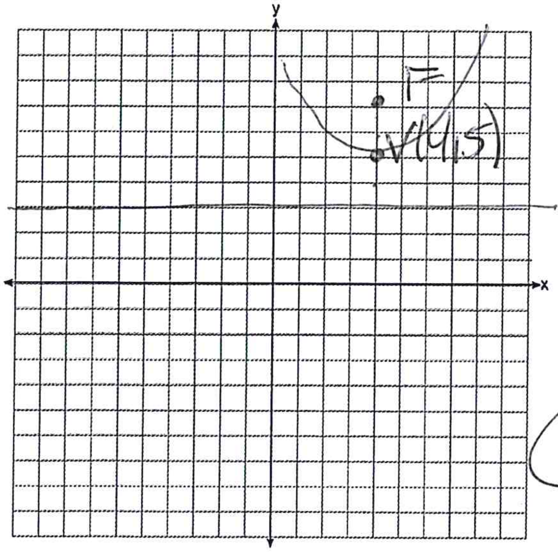
$$\frac{(x-6)^2}{-12} = y+1$$

$$-12(y+1)=(x-6)^2$$

$$y=\frac{(x-6)^2}{-12}-1$$

3. Write an equation for the set of points equidistant from $y = 3$ and $(4, 7)$ in three different forms.

directrix focus



$\frac{(x-v)^2}{4p} = y-t$ $\quad V=4$ $\quad t=5$ $\quad p=2$

$\frac{(x-4)^2}{4(2)} = y-5$

$\frac{(x-4)^2}{8} = y-5$

$8(y-5)=(x-4)^2$

$y = \frac{(x-4)^2}{8} + 5$

4. Which equation represents the set of points equidistant from line ℓ and point R shown on the graph below?

1) $y = -\frac{1}{8}(x+2)^2 + 1$

2) $y = -\frac{1}{8}(x+2)^2 - 1$

3) $y = -\frac{1}{8}(x-2)^2 + 1$

4) $y = -\frac{1}{8}(x-2)^2 - 1$

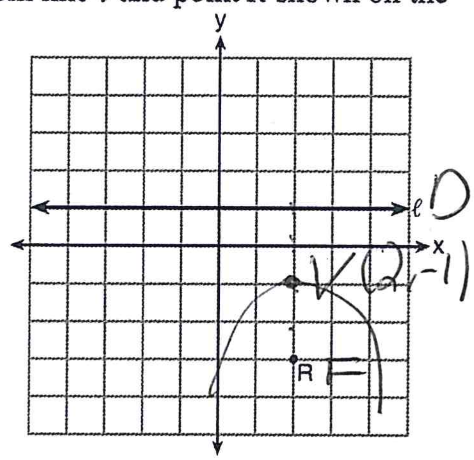
$V=2$ $\quad t=-1$ $\quad p=-2$

$\frac{(x-v)^2}{4p} = y-t$

$\frac{(x-2)^2}{4(-2)} = y+1$

$\frac{(x-2)^2}{-8} = y+1$
$\quad -1 \quad -1$

$\frac{(x-2)^2}{-8} - 1 = y$



5. Which equation represents the equation of the parabola with focus $(-3, 3)$ and directrix $y = 7$?

1) $y = \frac{1}{8}(x+3)^2 - 5$

2) $y = \frac{1}{8}(x-3)^2 + 5$

3) $y = -\frac{1}{8}(x+3)^2 + 5$

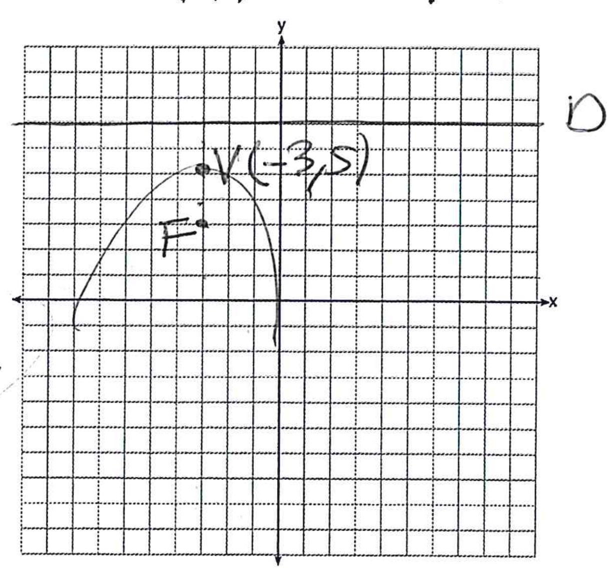
4) $y = -\frac{1}{8}(x-3)^2 + 5$

$\frac{(x-v)^2}{4p} = y-t$ $\quad V=-3$ $\quad t=5$ $\quad p=-2$

$\frac{(x+3)^2}{4(-2)} = y-5$

$\frac{(x+3)^2}{-8} = y-5$
$\quad +5 \quad +5$

$y = \frac{(x+3)^2}{-8} + 5$



6. Which equation represents a parabola with a focus of $(-2,5)$ and a directrix of $y = 9$?

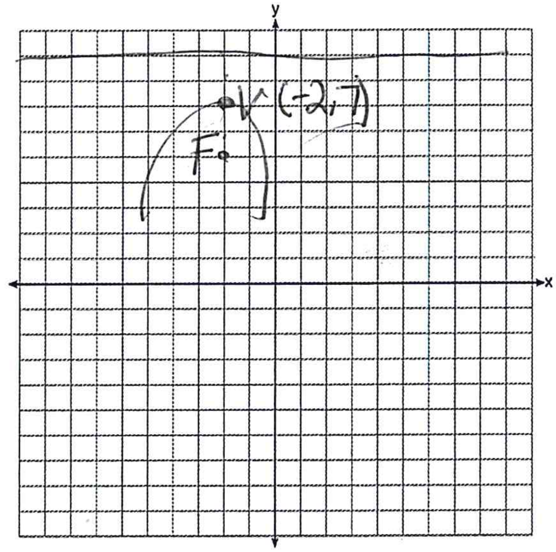
1) $(y-7)^2 = 8(x+2)$
2) $(y-7)^2 = -8(x+2)$
3) $(x+2)^2 = 8(y-7)$
4) $(x+2)^2 = -8(y-7)$

$\frac{(x-v)^2}{4p} = y-t$ $v=-2$, $t=7$, $p=-2$

$\frac{(x+2)^2}{4(-2)} = y-7$

$\frac{(x+2)^2}{-8} = \frac{y-7}{1}$

$-8(y-7) = (x+2)^2$



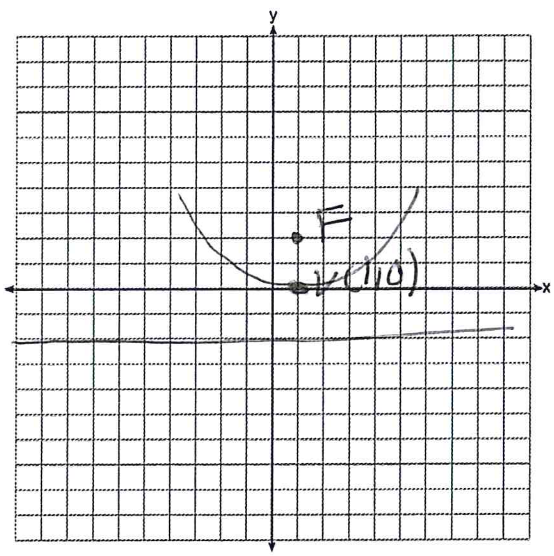
7. A parabola has its focus at $(1,2)$ and its directrix is $y = -2$. The equation of this parabola could be

1) $y = 8(x+1)^2$
2) $y = \frac{1}{8}(x+1)^2$
3) $y = 8(x-1)^2$
4) $y = \frac{1}{8}(x-1)^2$

$\frac{(x-v)^2}{4p} = y-t$ $v=1$, $t=0$, $p=2$

$\frac{(x-1)^2}{4(2)} = y-0$

$\frac{(x-1)^2}{8} = y$



8. Which equation represents a parabola with a focus of $(0,4)$ and a directrix of $y = 2$?

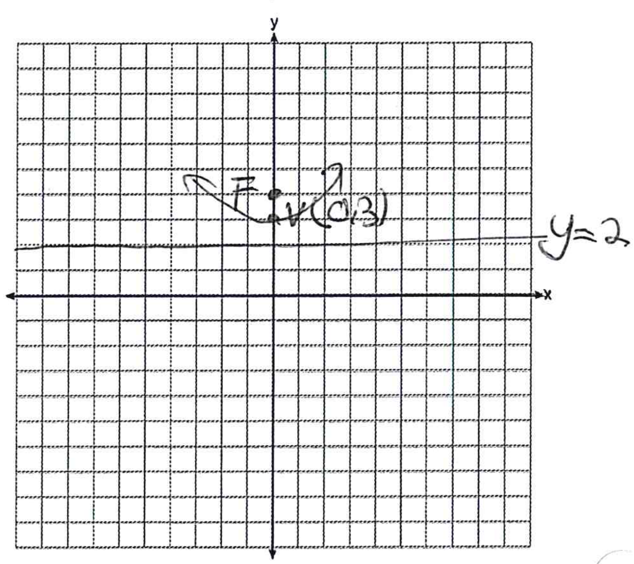
1) $y = x^2 + 3$
2) $y = -x^2 + 1$
3) $y = \frac{x^2}{2} + 3$
4) $y = \frac{x^2}{4} + 3$

$\frac{(x-v)^2}{4p} = y-t$ $v=0$, $t=3$, $p=1$

$\frac{(x-0)^2}{4(1)} = y-3$

$\frac{x^2}{4} = y - \cancel{3}$
$+3 \qquad +3$

$\frac{x^2}{4} + 3 = y$



- negate what's with x
- don't negate y (if y is isolated)
- negate y if y is not isolated.

9. What is the vertex of the following parabolas:

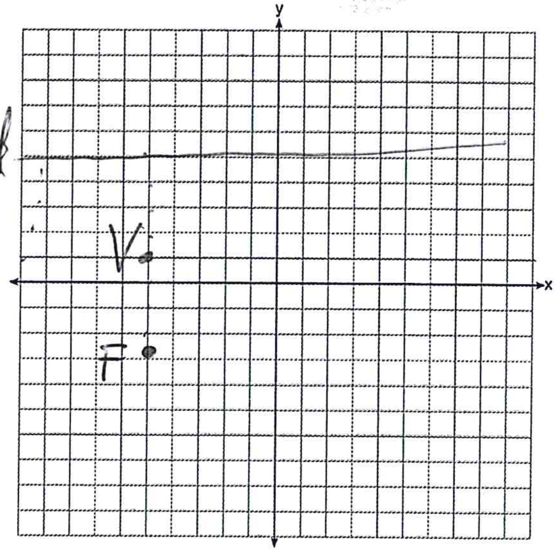
a) $y=\frac{1}{2}(x-3)^2+4$ b) $(y-2)=\frac{1}{4}(x+1)^2$ c) $y=-\frac{1}{8}(x+9)^2-1$ d) $(x+3)^2=\frac{(y-2)}{4}$

(3,4) (-1,2) (-9,-1) (-3,2)

10. The equation of a parabola is $y=a(x+5)^2+1$. If the focus is $(-5,-3)$, what is the equation of the directrix?

V(-5,1)

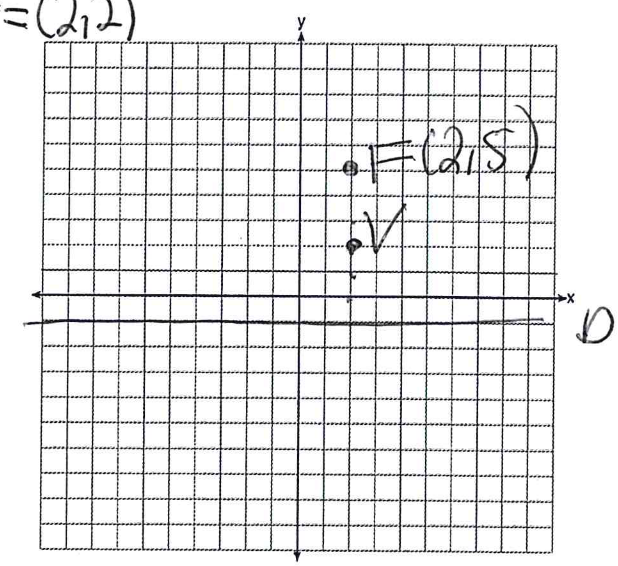
y=5



11. The parabola described by the equation $y=\frac{1}{12}(x-2)^2+2$ has the directrix at $y=-1$. The focus of the parabola is

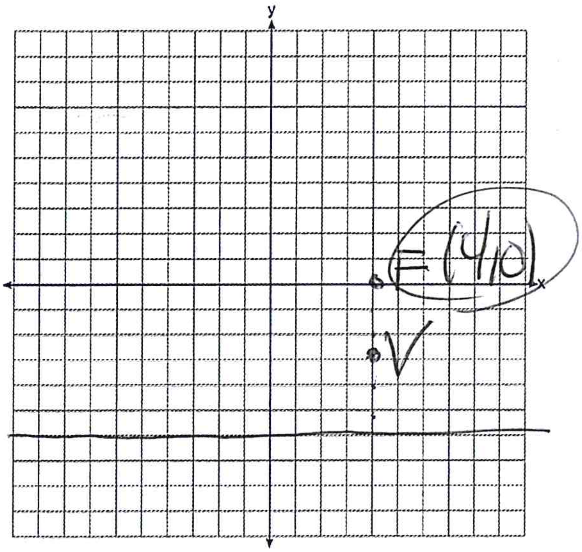
1) $(2,-1)$
2) $(2,2)$
3) $(2,3)$
4) $(2,5)$

V=(2,2)



12. The directrix of the parabola $12(y+3)=(x-4)^2$ has the equation $y=-6$. Find the coordinates of the focus of the parabola.

$(4,-3)$ V



13. What is the equation of the directrix for the parabola $-8(y-3)=(x+4)^2$?

~~1)~~ $y=5$ 3) $y=-2$

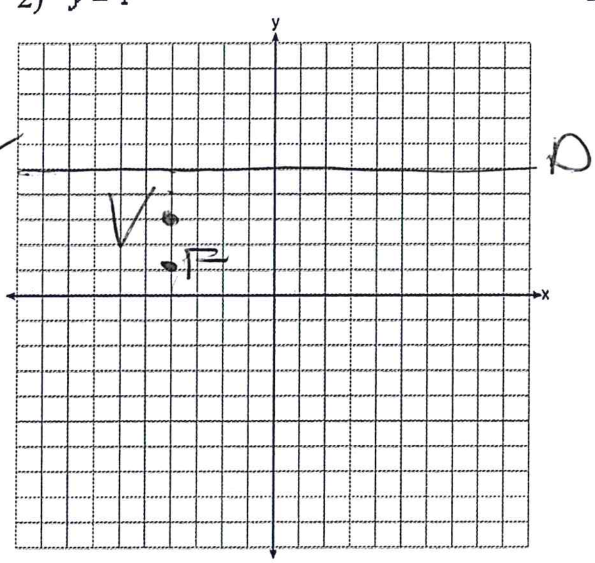
2) $y=1$ 4) $y=-6$

V(-4,3)

$$\frac{4p}{4}=\frac{-8}{4}$$

$p=-2$

y=5



Solving Systems of Equations Graphically Using TI-84+ ($f(x) = g(x)$)

1) Type equations into Y_1 and Y_2
2) Zoom 6 (Standard) is your standard window. Adjust window OR try Zoom 0(Fit) if you don't see what you want to see.
3) 2^{nd} Trace (Calc), 5 (Intersect)
4) Place cursor over point of intersection, hit enter, enter, enter. Repeat the process for any other points of intersection.

***The solutions to the system of equations are the x values of the intersections.**

1. When $g(x) = \frac{2}{x+2}$ and $h(x) = \log(x+1)+3$ are graphed on the same set of axes, which coordinates best approximate their point of intersection? $Y_1 = \frac{2}{x+2}$ $Y_2 = \log(x+1)+3$

1) $(-0.9, 1.8)$ 3) $(1.4, 3.3)$
2) $(-0.9, 1.9)$ 4) $(1.4, 3.4)$

*window's good

2nd Trace (Calc)
5: Intersect

2. To the *nearest tenth*, the value of x that satisfies $2^x = -2x + 11$ is

1) 2.5 3) 5.8
2) 2.6 4) 5.9

Y_1 Y_2

2nd Trace (Calc)
5: intersect

*window's good

3. Which value, to the *nearest tenth*, is *not* a solution of $p(x) = q(x)$ if $p(x) = x^3 + 3x^2 - 3x - 1$ and $q(x) = 3x + 8$?

$Y_1 = x^3+3x^2-3x-1$ $Y_2 = 3x+8$

1) -3.9 3) 2.1
2) -1.1 4) 4.7

2nd Trace (Calc)
5: intersect

*adjust y max

4. If $f(x) = 3|x| - 1$ and $g(x) = 0.03x^3 - x + 1$, an approximate solution for the equation $f(x) = g(x)$ is

1) 1.96 3) $(-0.99, 1.96)$
2) 11.29 4) $(11.29, 32.87)$

$Y_1 = 3|x|-1$ $Y_2 = .03x^3 - x + 1$
2nd Trace (Calc)
5: intersect

*adjust x max and y max

5. If $p(x) = 2\ln(x) - 1$ and $m(x) = \ln(x+6)$, then what is the solution for $p(x) = m(x)$?

1) 1.65 3) 5.62
2) 3.14 4) no solution

$Y_1 = 2\ln(x) - 1$
$Y_2 = \ln(x+6)$

2nd Trace (Calc)
5: Intersect

*window's good

6. For which values of x, rounded to the *nearest hundredth*, will $|x^2 - 9| - 3 = \log_3 x$?

1) 2.29 and 3.63 3) 2.84 and 3.17
2) 2.37 and 3.54 4) 2.92 and 3.06

Y_1 Y_2

2nd Trace (Calc)
5: Intersect

*window's good

7. Given: $h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2$ Y_1

$k(x) = -|0.7x| + 5$ Y_2

2nd Trace (Calc)
5: Intersect

State the solutions to the equation $h(x) = k(x)$, rounded to the *nearest hundredth*.

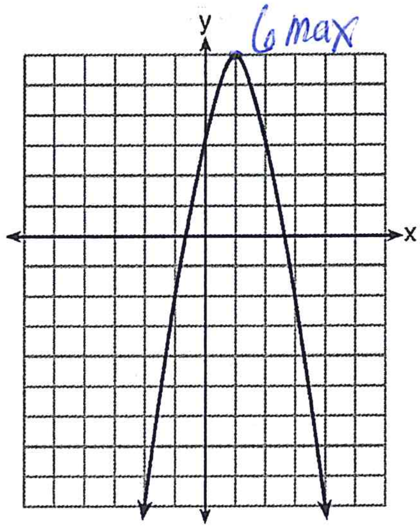
$x = -5.17$ $x = -1.13$ $x = 1.75$

*window's good

Key Points

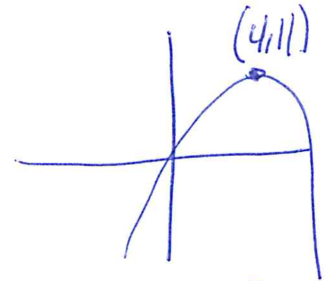
To compare key points, find the key point for each function. Use the graph, the table (2nd graph), and the calculate menu (2nd Trace).

1. Let f be the function represented by the graph below.



2nd Calc: maximum

(4, 11)



or

X	y
1	13/2
2	9
3	21/2
4	11 → max
5	21/2
6	9
7	13/2

Let g be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$. Determine which function has the larger maximum value. Justify your answer.

g(x) 11 > 6

2. Which quadratic function has the largest maximum?

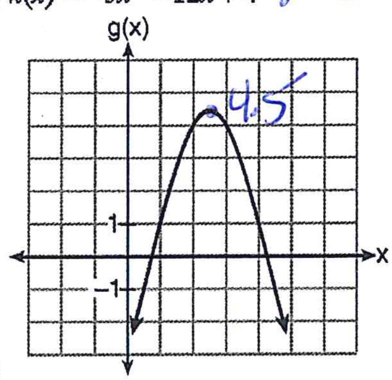
1) $h(x) = (3-x)(2+x)$ 2nd Calc: max: 6.25 3) $k(x) = -5x^2 - 12x + 4$ 2nd Calc: max: 11.2

2)

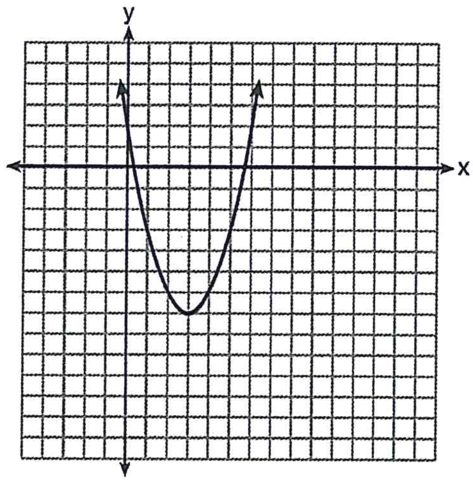
x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

≈ 9.5

4)



3. The graph representing a function is shown below.



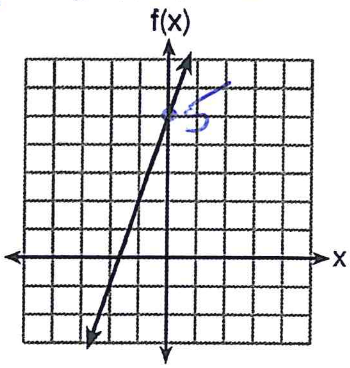
Which function has a minimum that is *less* than the one shown in the graph?

2nd calc min.

1) $y = x^2 - 6x + 7$ -2
2) $y = |x + 3| - 6$ -2
3) $y = x^2 - 2x - 10$ -11 $\quad -11 < -7$
4) $y = |x - 8| + 2$ 2

$x=0$

4. Which function has the greatest y-intercept?
1) $f(x) = 3x$ $+0$ $\quad 0$
2) $2x + 3y = 12$ $\quad 4$
3) the line that has a slope of 2 and passes through $(1, -4)$ -6
4)



2) $2x+3y=12$
$-2x \quad -2x$
$\frac{3y}{3}=\frac{-2x+12}{3}$
$y=-\frac{2}{3}x+4$

3) $y-y_1=m(x-x_1)$
$y+4=2(0-1)$
$y+4=-2$
$-4 \quad -4$
$y=-6$

$x=0$

5. Which graph has the greatest y-intercept?

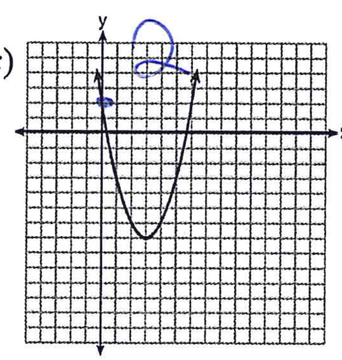
x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

5

$$g(x) = \left(\frac{1}{2}\right)^{x+1} + 3$$

$(0, 3.5)$

$h(x)$



f(x)

$5 > 3.5 > 2$

Inverse of a function $f^{-1}(x)$:

Switch x and y, solve for y

1. What is the inverse of the function $y = 2x - 3$?

(1) $y = \frac{x+3}{2}$ (3) $y = -2x + 3$

(2) $y = \frac{x}{2} + 3$ (4) $y = \frac{1}{2x-3}$

$x = 2y - 3$
$+3 \quad +3$
$\frac{x+3}{2} = \frac{2y}{2}$
$\frac{x+3}{2} = y$

2. If a function is defined by the equation y = 3x + 2, which equation defines the inverse of this function?

(1) $x = \frac{1}{3}y + \frac{1}{2}$ (3) $y = \frac{1}{3}x - \frac{2}{3}$

(2) $y = \frac{1}{3}x + \frac{1}{2}$ (4) y = -3x – 2

$x = 3y + 2$
$-2 \quad -2$
$\frac{1x-2}{3} = \frac{3y}{3}$

$y = \frac{1}{3}x - \frac{2}{3}$

3. If $f(x) = x^2$, find $f^{-1}(x)$

$y = x^2$

$\sqrt{x} = \sqrt{y^2}$

$\sqrt{x} = y$

$f^{-1}(x) = \sqrt{x}$

4. If $f(x) = 5x - 7$, find $f^{-1}(x)$

$y = 5x - 7$

$x = 5y - 7$
$+7 \quad +7$
$\frac{x+7}{5} = \frac{5y}{5}$

$y = \frac{1}{5}x + \frac{7}{5}$
$f^{-1}(x) = \frac{1}{5}x + \frac{7}{5}$

5. What is $g^{-1}(x)$ if $g(x) = 3x + 6$

$y = 3x + 6$

$x = 3y + 6$
$-6 \quad -6$
$\frac{x-6}{3} = \frac{3y}{3}$

$y = \frac{1}{3}x - 2$

6. What is $h^{-1}(x)$ if $h(x) = x^2 + 2$

$y = x^2 + 2$

$x = y^2 + 2$
$-2 \quad -2$
$\sqrt{x-2} = \sqrt{y^2}$
$\sqrt{x-2} = y$

$h^{-1}(x) = \sqrt{x-2}$

7. What is the inverse of $y = \frac{1}{2}x + 2$?

$2(x) = \left(\frac{1}{2}y + 2\right)2$

$2x = y + 4$

$-4 \quad -4$

$2x - 4 = y$

8. For the function $f(x) = (x-3)^3 + 1$, find $f^{-1}(x)$.

$y = (x-3)^3 + 1$

$x = (y-3)^3 + 1$

$-1 \quad -1$

$\sqrt[3]{x-1} = \sqrt[3]{(y-3)^3}$

$\sqrt[3]{x-1} = y - 3$

$+3 \quad +3$

$y = \sqrt[3]{x-1} + 3$

$f^{-1}(x) = \sqrt[3]{x-1} + 3$

9. What is the inverse of $f(x) = -6(x-2)$?

1) $f^{-1}(x) = -2 - \frac{x}{6}$

2) $f^{-1}(x) = 2 - \frac{1x}{6}$

3) $f^{-1}(x) = \frac{1}{-6(x-2)}$

4) $f^{-1}(x) = 6(x+2)$

$y = -6(x-2)$

$x = -6(y-2)$

$x = -6y + 12$

$-12 \quad -12$

$\frac{x-12}{-6} = \frac{-6y}{-6}$

$y = -\frac{1}{6}x + 2$

10. Given $f(x) = \frac{1}{2}x + 8$, which equation represents the inverse, $g(x)$?

1) $g(x) = 2x - 8$

2) $g(x) = 2x - 16$

3) $g(x) = -\frac{1}{2}x + 8$

4) $g(x) = -\frac{1}{2}x - 16$

$y = \frac{1}{2}x + 8$

$2(x) = \left(\frac{1}{2}y + 8\right)2$

$2x = y + 16$

$-16 \quad -16$

$2x - 16 = y$

11. Given $f^{-1}(x) = -\frac{3}{4}x + 2$, which equation represents $f(x)$?

1) $f(x) = \frac{4}{3}x - \frac{8}{3}$

2) $f(x) = -\frac{4}{3}x + \frac{8}{3}$

3) $f(x) = \frac{3}{4}x - 2$

4) $f(x) = -\frac{3}{4}x + 2$

$y = -\frac{3}{4}x + 2$

$-4(x) = \left(-\frac{3}{4}y + 2\right)-4$

$-4x = 3y - 8$

$+8 \quad +8$

$\frac{-4x + 8}{3} = \frac{3y}{3}$

$y = -\frac{4}{3}x + \frac{8}{3}$

Even and Odd Functions

Even Functions:

$f(x) = f(-x)$

The graph is symmetric to the y axis

If it is a polynomial function, all of the exponents are even.

Constant terms have an exponent of 0. ($5 = 5x^0$)

Odd Functions:

$f(-x) = -f(x)$

The graph is symmetric to the origin

If it is a polynomial function, all of the exponents are odd.

1. $f(x) = -6x^3 - 8x$

odd
Symmetric
to origin

2. $f(x) = 10x^2 + 8x^4 - 4x$

neither

3. $f(x) = -7x^8 + 7$

even
Symmetric
to y axis

4. $f(x) = |x| + 4$

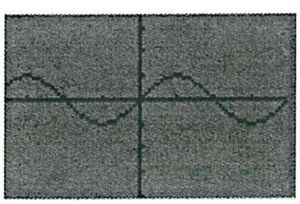
even
symmetric to
y axis

5. $f(x) = |x + 4|$

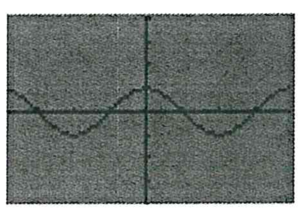
neither

6. $f(x) = \dfrac{10}{x}$

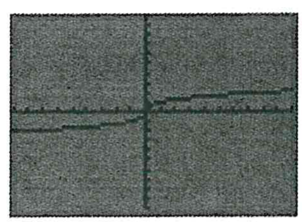
odd
symmetric
to origin

7. 

odd
Symmetric
to origin

8. 

even
Symmetric
to y axis

9. 

odd
Symmetric
to origin

Translating Functions

If adding to f(x), the graph moves up or down

If adding to x, the graph moves left or right (the opposite direction in which you would think)

$y = f(x) + a$ moves UP a units

$y = f(x) - a$ moves DOWN a units

$y = f(x + a)$ moves LEFT a units

$y = f(x - a)$ moves RIGHT a units

1. Relative to the graph of $y = 3\sin x$, what is the shift of the graph of $y = 3\sin\left(x + \frac{\pi}{3}\right)$? left $\frac{\pi}{3}$

1) $\frac{\pi}{3}$ right

2) $\frac{\pi}{3}$ left

3) $\frac{\pi}{3}$ up

4) $\frac{\pi}{3}$ down

2. Given the parent function $p(x) = \cos x$, which phrase best describes the transformation used to obtain the graph of $g(x) = \cos(x + a) - b$, if a and b are positive constants?

1) right a units, up b units

2) right a units, down b units

3) left a units, up b units

4) left a units, down b units

left a down b

3. The function $f(x) = \sqrt{x}$. Which function represents a shift of the graph left 3 units?

(1) $f(x-3) = \sqrt{x-3}$ right 3

(2) $f(x+3) = \sqrt{x+3}$ left 3

(3) $f(x)+3 = \sqrt{x}+3$ up 3

(4) $f(x)-3 = \sqrt{x}-3$ down 3

4. If $f(x) = \log_3 x$ and g(x) is the image of f(x) after a translation five units to the left, which equation represents g(x)?

1) $g(x) = \log_3(x+5)$ 5 left

2) $g(x) = \log_3 x + 5$ 5 up

3) $g(x) = \log_3(x-5)$ 5 right

4) $g(x) = \log_3 x - 5$ 5 down

5. Joey's math class is studying the basic quadratic function, $f(x) = x^2$. Each student is supposed to make two new functions by adding or subtracting a constant to the function. Joey chooses the functions $g(x) = x^2 - 5$ and $h(x) = x^2 + 2$. What transformations would map $f(x)$ to $g(x)$ and $f(x)$ to $h(x)$? down 5 up 2

(1) shift left 5, shift right 2

(2) shift right 5, shift left 2

(3) shift up 5, shift down 2

(4) shift down 5, shift up 2

Average rate of change: $\frac{f(b)-f(a)}{b-a}$, you may have to find $f(a)$ and $f(b)$ by typing into the calculator and using your table or using a graph.

"On average, the function is increasing/decreasing x units per unit of time."

1. Given the functions g(x), f(x), and h(x) shown below:

x	y
0	0
3	3

$\frac{f(b)-f(a)}{b-a}$

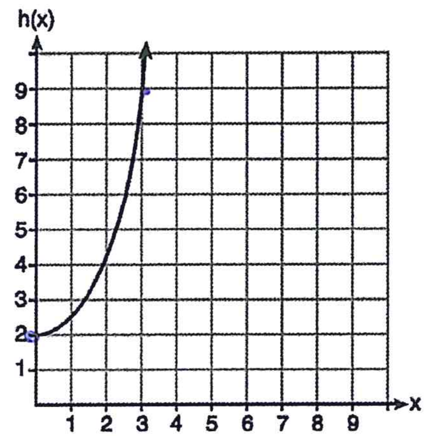
$\frac{3-0}{3-0}=1$

$g(x) = x^2 - 2x$

$\frac{7-1}{3-0}$ = 2

x	y
0	1
3	7

x	f(x)
0	1
1	2
2	5
3	7



x	y
0	2
3	9

$\frac{9-2}{3-0}=\frac{7}{3}$

The correct list of functions ordered from greatest to least by average rate of change over the interval $0 \le x \le 3$ is

1) f(x), g(x), h(x)
2) h(x), g(x), f(x)
3) g(x), f(x), h(x)
4) h(x), f(x), g(x)

2. Find the average rate of change of the function shown below that represents the amount of money in a savings account in Lender's Bank between weeks 2 and 4.

Week	Balance
1	\$128
2	\$144
3	\$157
4	\$175
5	\$184

$\frac{f(b)-f(a)}{b-a}$ $\quad \frac{175-144}{4-2}=\frac{31}{2}=15.50$

Explain what this rate of change means in the context of the problem.

On average, between week 2 and 4, the balance is increasing \$15.50 per week.

3. Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$$\frac{f(b)-f(a)}{b-a} \qquad \frac{80-1.25}{4-^-2} = 13.125$$

$g(x)$

$g(x) = 4x^3 - 5x^2 + 3$

x	y
−2	−49
4	179

$$\frac{179-^-49}{4-^-2} = 38$$

4. The distance needed to stop a car after applying the brakes varies directly with the square of the car's speed. The table below shows stopping distances for various speeds.

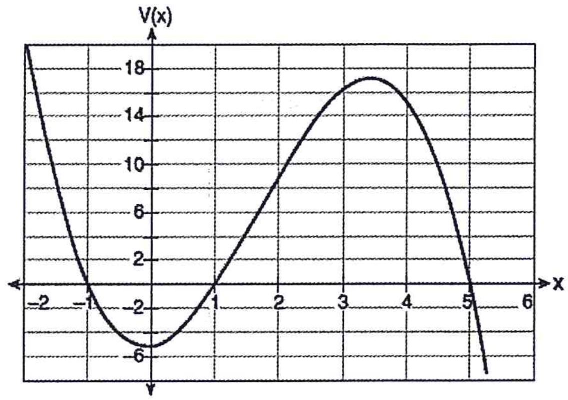
Speed (mph)	10	20	30	40	50	60	70
Distance (ft)	6.25	25	56.25	100	156.25	225	306.25

Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.

$$\frac{f(b)-f(a)}{b-a} \qquad \frac{306.25-156.25}{70-50} = 7.5 \text{ ft/mph}$$

On average, between 50 mph and 70 mph, the braking distance increases by 7.5 ft/mph.

5. A cardboard box manufacturing company is building boxes with length represented by $x+1$, width by $5-x$, and height by $x-1$. The volume of the box is modeled by the function below.



Over which interval is the volume of the box changing at the fastest average rate?

1) [1, 2]
2) [1,3.5]
3) [1,5]
4) [0, 3.5]

1)
x	y
1	0
2	9

$\frac{9-0}{2-1}=9$

2)
x	y
1	0
3.5	17

$\frac{17-0}{3.5-1}=6.8$

3)
x	y
1	0
5	0

$\frac{0-0}{5-1}=0$

4)
x	y
0	-5
3.5	17

$\frac{17--5}{3.5-0}=6.28..$

6. Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of B dollars after m months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after m months.

m	B
0	1000.00
10	1172.00
19	1352.00
36	1770.80
60	2591.90
69	2990.00
72	3135.80
73	3186.00

1) $\frac{2591.90-1172.00}{60-10}$ 28.398

2) $\frac{2990.00-1352.60}{69-19}$ 32.76

3) $\frac{3135.80-1770.80}{72-36}$ 37.916

4) $\frac{3186.00-2591.90}{73-60}$ 45.7

Over which interval of time is her average rate of change for the balance on her credit card account the greatest?

1) month 10 to month 60
2) month 19 to month 69
3) month 36 to month 72
4) month 60 to month 73

3 X 3 Linear Systems

Matrix Method: (Elimination will be in the equations packet)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$$

1) 2^{nd} Matrix, Edit, A, 3X3 (Coefficient of the left hand side)
2) 2^{nd} Matrix, Edit, B, 3X1 (Right hand side)
3) $A^{-1}B$

---

1. Which value is contained in the solution of the system shown below?

$2x + y - z = 1$

$x - 2y + z = 0$

$3x - y + 2z = 7$

1) 0   3) 2
2) -1   4) -3

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & -1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

2. Which value is *not* contained in the solution of the system shown below?

$a + 5b - c = -20$

$4a - 5b + 4c = 19$

$-a - 5b - 5c = 2$

1) -2
2) 2
3) 3
4) -3

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 5 & -1 \\ 4 & -5 & 4 \\ -1 & -5 & -5 \end{pmatrix}^{-1} \begin{pmatrix} -20 \\ 19 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix}$$

3. Which value is contained in the solution of the system shown below?

$3x + y + z = -4$

$x - 2y + z = -5$

$2x + 3y - 2z = -9$

3) -3   3) -5
4) -4   4) -9

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & -2 & 1 \\ 2 & 3 & -2 \end{pmatrix}^{-1} \begin{pmatrix} -4 \\ -5 \\ -9 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \\ 5 \end{pmatrix}$$

4. Which value is *not* contained in the solution of the system shown below?

$4x - 5y + 2z = 130$

$3x + 2y - 7z = -99$

$10x - 6y - 4z = 112$

1) -8   3) 10
2) -12   4) 15

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 & -5 & 2 \\ 3 & 2 & -7 \\ 10 & -6 & -4 \end{pmatrix}^{-1} \begin{pmatrix} 130 \\ -99 \\ 112 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ -12 \\ 15 \end{pmatrix}$$

5. What is the solution of the system shown below?

$6x - 3y + 2z = 78$

$4x + 2y - 5z = -40$

$-3x - 4y - 3z = -41$

1) $x = 2,\ y = -4,\ z = 6$
2) $x = 7,\ y = -4,\ z = 12$
3) $x = 78,\ y = -40,\ z = -41$
4) $x = 6,\ y = 2,\ z = -3$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 & -3 & 2 \\ 4 & 2 & -5 \\ -3 & -4 & -3 \end{pmatrix}^{-1} \begin{pmatrix} 78 \\ -40 \\ -41 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ -4 \\ 12 \end{pmatrix}$$

6. For the system shown below, what is the value of z?

$y = -2x + 14$
$+2x\ +2x$

$3x - 4z = 2$

$3x - y = 16$

$2x + y + 0z = 14$
$3x + 0y - 4z = 2$
$3x - y + 0z = 16$

1) 5   3) 6
2) 2   4) 4

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & 1 & 0 \\ 3 & 0 & -4 \\ 3 & -1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 14 \\ 2 \\ 16 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 4 \end{pmatrix}$$

Exponents

Negative exponents are fractions! $x^{-2} = \frac{1}{x^2}$

If exponent is outside parenthesis, everything gets it $\left(\frac{xy}{z}\right)^3 = \frac{x^3y^3}{z^3}$

Radicals are fractional exponents (Fractional exponent = $\frac{power}{root}$)

Get rid of parenthesis
Negative exponents are fractions (Move whatever is being raised to the negative power)
Clean it up (Multiply, divide, or put back into radical)

Use Multiple Choice Strategy if its multiple choice but that is not the focus of this section.

1. Use the properties of rational exponents to determine the value of y for the equation:

$$\frac{\sqrt[3]{x^8}}{\left(x^4\right)^{\frac{1}{3}}} = x^y,\ x > 1$$

$$\frac{x^{\frac{8}{3}}}{x^{\frac{4}{3}}} \quad x^{\frac{8}{3}-\frac{4}{3}} = x^{\frac{4}{3}}$$

2. Given the equal terms $\sqrt[3]{x^5}$ and y^7, determine and state y, in terms of x.

$$\left(x^{\frac{5}{3}}\right)^{\frac{1}{7}} = \left(y^7\right)^{\frac{1}{7}}$$

$$x^{\frac{5}{21}} = y$$

3. Write $\sqrt[3]{x} \bullet \sqrt{x}$ as a single term with a rational exponent.

$$x^{\frac{1}{3}} \cdot x^{\frac{1}{2}}$$

$$x^{\frac{1}{3}+\frac{1}{2}}$$

$$x^{\frac{5}{6}}$$

4. For $x \neq 0$, which expressions are equivalent to one divided by the sixth root of x? $\frac{1}{\sqrt[6]{x}}$

I. $\frac{\sqrt[6]{x}}{\sqrt[3]{x}}$ II. $\frac{x^{\frac{1}{6}}}{x^{\frac{1}{3}}}$ III. $x^{\frac{-1}{6}}$

$x^{-\frac{1}{6}} = \frac{1}{x^{\frac{1}{6}}} = \frac{1}{\sqrt[6]{x}}$

1) I and II, only
2) I and III, only
3) II and III, only
~~4)~~ I, II, and III

$\text{I} \ \frac{x^{\frac{1}{6}}}{x^{\frac{1}{3}}} = x^{-\frac{1}{6}}$ $\text{II} \ \frac{x^{\frac{1}{6}}}{x^{\frac{1}{3}}} = x^{-\frac{1}{6}}$ $\text{III} \ x^{-\frac{1}{6}}$

5. If $n = \sqrt[2]{a^5}$ and $m = a$, where $a > 0$, an expression for $\frac{n}{m}$ could be

1) $a^{\frac{5}{2}}$
2) a^4
3) $\sqrt[3]{a^2}$
~~4)~~ $\sqrt{a^3}$

$n = a^{\frac{5}{2}}$ $m = a$

$\frac{n}{m} = \frac{a^{\frac{5}{2}}}{a^1}$

$a^{\frac{5}{2}-1} = a^{\frac{3}{2}} = \sqrt[2]{a^3}$

2. Simplify $\frac{\sqrt[3]{x^2} \bullet \sqrt{x^5}}{\sqrt[6]{x}}$

$\frac{x^{\frac{2}{3}} \cdot x^{\frac{5}{2}}}{x^{\frac{1}{6}}}$ $\frac{x^{\frac{19}{6}}}{x^{\frac{1}{6}}} = x^{\frac{18}{6}} = x^3$

$\frac{2}{3} + \frac{5}{2} = \frac{19}{6}$

7. Explain how $(-8)^{\frac{4}{3}}$ can be evaluated using properties of rational exponents to result in an integer answer.

Radicals are fractional exponents $\left(\frac{\text{power}}{\text{root}}\right)$

Cubed root of -8 is -2 and $(-2)^4$ is 16

$\sqrt[3]{-8}^{\,4}$

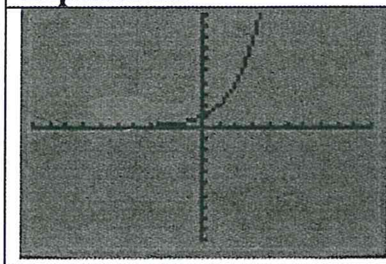
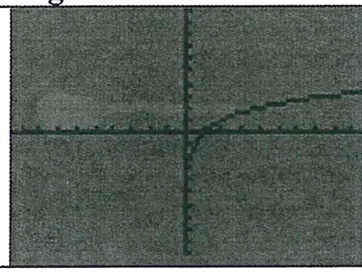
8. Explain why $81^{\frac{3}{4}}$ equals 27.

Radicals are fractional exponents $\left(\frac{\text{power}}{\text{root}}\right)$

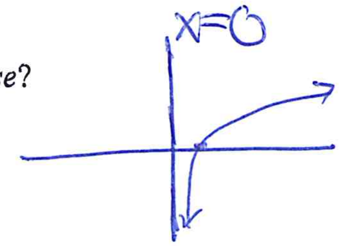
$\sqrt[4]{81}^{\,3}$ the fourth root of 81 is 3

and 3^3 is 27.

Graphing Exponential and Logarithmic Functions

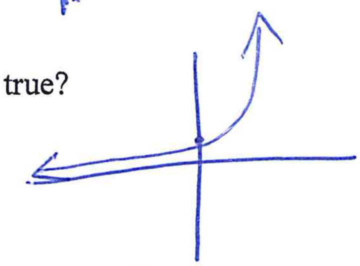
Exponential	**Logarithmic**
	
Horizontal Asymptote at $y = 0$	Vertical Asymptote at $x = 0$
Passes through $(0,1)$	Passes through $(1,0)$
Domain is all real numbers	Domain is all positive real numbers
Range is all positive real numbers	Range is all real numbers
Exponents and logarithms are inverses of each other!!!!!!!!!!!	

1. Which statement about the graph of $c(x) = \log_6 x$ is *false*?



1) The asymptote has equation $y = 0$. X=0
2) The graph has no y-intercept.
3) The domain is the set of positive reals.
4) The range is the set of all real numbers.

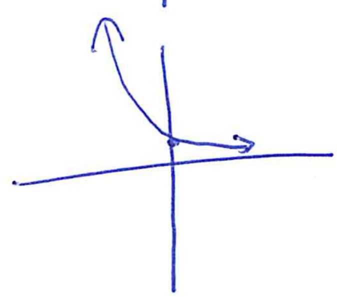
2. Which statement about the graph of the equation $y = e^x$ is *not* true?



1) It is asymptotic to the x-axis.
2) The domain is the set of all real numbers.
3) It lies in Quadrants I and II.
4) It passes through the point $(e, 1)$.

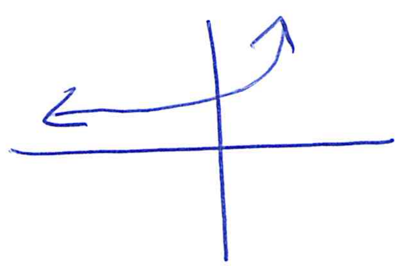
(0,1)

3. Which statement is true about the graph of $f(x) = \left(\frac{1}{8}\right)^x$?



1) The graph is always increasing.
2) The graph is always decreasing.
3) The graph passes through $(1, 0)$. (0,1)
4) The graph has an asymptote, $x = 0$. y=0

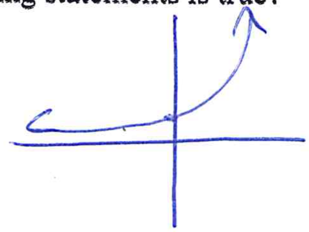
4. If the function $g(x) = ab^x$ represents exponential growth, which statement about $g(x)$ is *false*?



1) $a > 0$ and $b > 1$
2) The y-intercept is $(0, a)$.
3) The asymptote is $y = 0$.
4) The x-intercept is $(b, 0)$. it doesn't have an x intercept

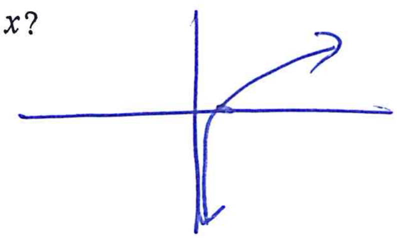
5. Given the equation $f(x) = \pi^x$, which of the following statements is true?

1) The graph passes through $(\pi, 1)$
2) The domain is $[0, \infty)$
3) The graph passes through $(0,1)$
4) The range is all real numbers



6. Which statement is false regarding the equation $f(x) = \log_a x$?

1) The range is $[0, \infty)$
2) The graph passes through $(0,1)$ (1,0)
3) The domain is all real numbers
4) The equation of the asymptote is $x = 0$



7. Which of the following equations does not have an asymptote of $y = 0$?

1) $f(x) = -a^x$ 2) $f(x) = a^{x-4}$ 3) $f(x) = a^x - 3$ 4) $f(x) = a^{2x}$

the only way to move a horizontal asymptote is with a vertical shift.

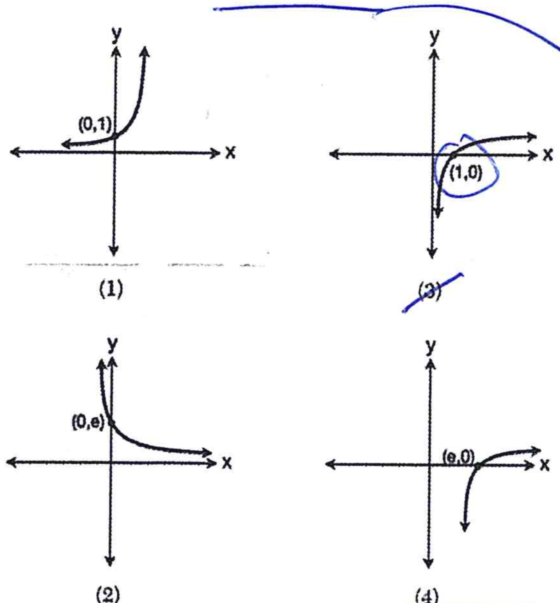
8. What is the inverse of the function $y = \log_3 x$?

1) $y = x^3$ 2) $y = \log_x 3$ 3) $y = 3^x$ 4) $x = 3^y$

the inverse of logs is exponential. same base

9. Which graph shows the inverse of $y = e^x$?

the inverse of exponential is logs



10. If $f(x) = a^x$ where $a > 1$, then the inverse of the function is

1) $f^{-1}(x) = \log_x a$
2) $f^{-1}(x) = a \log x$
3) $f^{-1}(x) = \log_a x$
4) $f^{-1}(x) = x \log a$

the inverse of exponentials is logs. Same base!

Graphing Functions

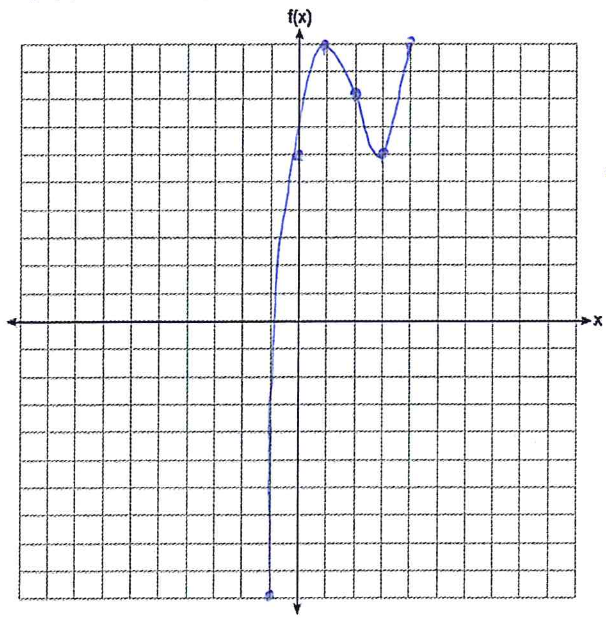
1) Type equation into $Y =$
2) 2^{nd} Graph (Table)

    *Plot points in given domain or that fit on the given graph

- Domain: no arrows. No domain: arrows.
- $scale \geq \dfrac{\max imum}{\#\ of\ boxes}$

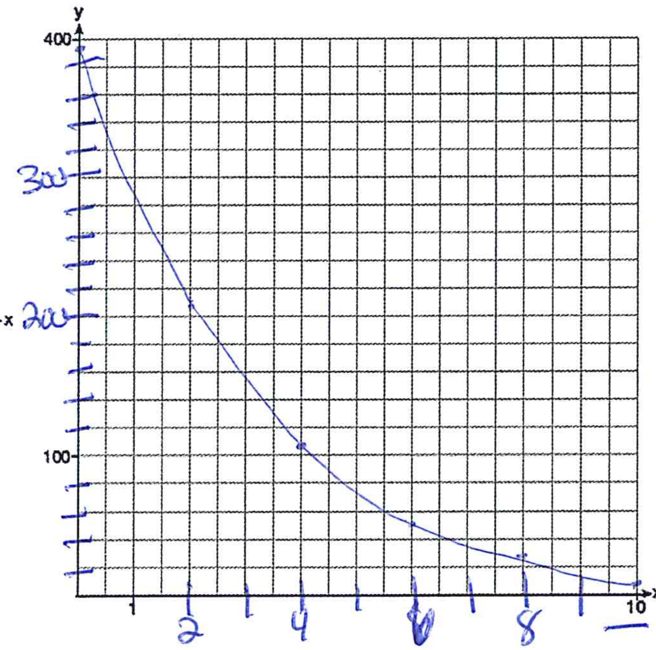
1. On the grid below, graph the function $f(x) = x^3 - 6x^2 + 9x + 6$ on the domain $-1 \leq x \leq 4$.

x	y
-1	-10
0	6
1	10
2	8
3	6
4	10



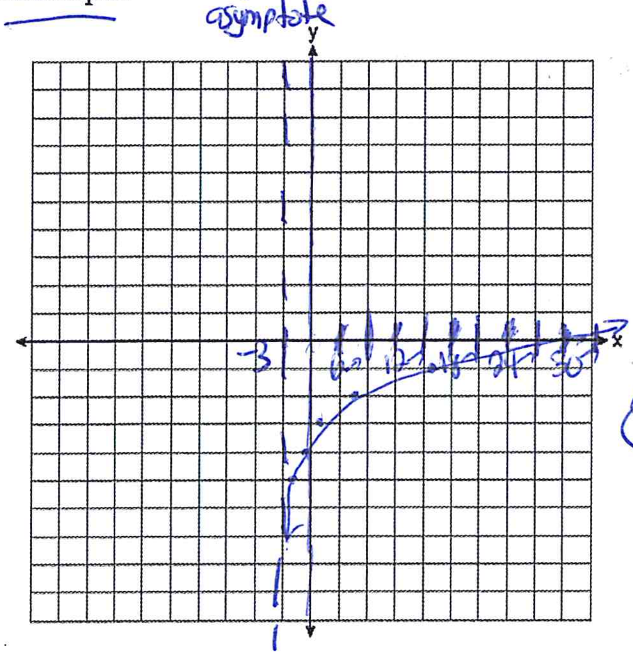
2. Graph $y = 400(.85)^{2x} - 6$ on the set of axes below.

x	y
0	394
2	202.8
4	103
6	51
8	23.7
10	9.5



3. Graph $y = \log_2(x + 3) - 5$ on the set of axes below. Use an appropriate scale to include *both* intercepts.

x=-3
asymptote



x	y
-2	-5
-1	-4
1	-3
5	-2
13	-1
29	0

$scale \geq \dfrac{max}{\#\ of\ boxes}$

$scale \geq \dfrac{29}{10}$

$scale \geq 2.9$

$scale = 3$

COMPOUNDING Interest: $A = P\left(1 \pm \frac{r}{n}\right)^{nt}$, where A is the current amount, P is the initial amount, r is the rate as a decimal (divide by 100), n is the number of times compounded (yearly =1, semiannually = 2, quarterly = 4, monthly = 12, weekly = 52, daily = 365) and t is time.
COMPOUNDING CONTINUOUSLY: $A = Pe^{rt}$

1. The table below shows three different investment options in which Lauren can invest $7,000.

Option	Annual Interest Rate	Frequency of Compounding
A	6.5%	Annually
B	6.38%	Continuously
C	6.46%	Weekly

Which option will allow Lauren to earn the most money over the course of a four-year period? Justify your answer.

A

$A=P(1+\frac{r}{n})^{nt}$

A=A
P=7000
r=.065
n=1
t=4

$A=7000(1+\frac{.065}{1})^{1(4)}$
A=9005.26

B

$A=Pe^{rt}$
A=A
P=7000
r=.0638
t=4

$A=7000e^{.0638(4)}$
A=9035.04

C

$A=P(1+\frac{r}{n})^{nt}$
$A=7000(1+\frac{.0646}{52})^{52(4)}$
A=A
P=7000
r=.0646
n=52
t=4

A=9062.54
option C

2. The table below shows three different investment options in which Lauren can invest $3,200.

Option	Annual Interest Rate	Frequency of Compounding
A	4.9%	Quarterly
B	4.81%	Continuously
C	4.85%	Monthly

Which option will allow Lauren to earn the most money over the course of a four-year period? Justify your answer.

A

$A=P(1+\frac{r}{n})^{nt}$

A=A
P=3200
r=.049
n=4
t=4

$A=3200(1+\frac{.049}{4})^{4(4)}$
A=3888.25
Option A

B

$A=Pe^{rt}$
A=A
P=3200
r=.0481
t=4

$A=3200e^{.0481(4)}$
A=3878.90

C

$A=P(1+\frac{r}{n})^{nt}$
A=A
P=3200
r=.0485
n=12
t=4

$A=3200(1+\frac{.0485}{12})^{12(4)}$
A=3883.59

3. Moe opened a bank account with \$3100 4 years ago at an interest rate of 6.1% that is compounded continuously. How much money is in Moe's bank account now?

$A = X$ $\quad A = Pe^{rt}$

$P = 3100$ $\quad A = 3100e^{.061(4)}$

$r = .061$ $\quad A = 3956.67$

$t = 4$

4. Max opens a bank account with \$2100. If interest is compounded quarterly at an interest rate of 7%, how much interest will Max have earned after 3 years?

$A = X$ $\quad A = P(1+\frac{r}{n})^{nt}$ $\quad$ Interest $= A - P$

$P = 2100$ $\quad x = 2100(1+\frac{.07}{4})^{4(3)}$ $\quad$ Interest $= 2586.02 - 2100 = 486.02$

$r = .07$

$n = 4$ $\quad x = 2586.02$

$t = 3$

5. The amount of a compound decreases by 8% per day at a continuous rate. How much of a 100 gram sample will remain after 1 week to the nearest tenth of a gram?

week → 7 days

$A = X$ $\quad A = Pe^{rt}$

$P = 100$ $\quad x = 100e^{-.08(7)}$

$r = -.08$ $\quad x = 57.1$

$t = 7$

6. Seth's parents gave him \$5000 to invest for his 16th birthday. He is considering two investment options. Option *A* will pay him 4.5% interest compounded annually. Option *B* will pay him 4.6% compounded quarterly. Write a function of option *A* and option *B* that calculates the value of each account after *n* years. Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option *B* will earn than option *A* to the *nearest cent*.

Option A

$A = P(1+r)^t$

$A = A(n)$ $\quad A(n) = 5000(1.045)^n$

$P = 5000$ $\quad A(6) = 6511.30$

$r = .045$

$t = n$

Option B

$A = P(1+\frac{r}{n})^{nt}$

$A = B(n)$ $\quad B(n) = 5000(1+\frac{.046}{4})^{4n}$

$P = 5000$ $\quad B(n) = 5000(1.0115)^{4n}$

$r = .046$ $\quad B(6) = 6578.87$

$n = 4$

$t = n$

$6578.87 - 6511.30$

67.57

Converting Rates (For example, from annual to monthly)

1) Start with $A = P(1 \pm r)^t$ (If it is not given in that form)

2) Raise $(1 \pm r)^{\frac{1}{n}}$ to find the new rate (Subtract the one and multiply by 100 to find the percent rate)

3) -If the variable is the original unit (years for example), the exponent is *nt*.
-If the variable matches the unit (months for example), then the exponent is *m*.

1. Each year, the amount of students in Eastbury High School increases by 7.15%. Which of the following expressions could be used to find the *weekly* rate of increase of Eastbury High School after *w* weeks?

$\left(\left(1.0715\right)^{\frac{1}{52}}\right)^{w}$

1) $(1.0715)^w$ 2) $(1.0715^{\frac{1}{52}})^{52w}$ ~~3)~~ $(1.0715^{\frac{1}{52}})^w$ 4) $(1.0715)^{52w}$

2. Each year, the amount of students in Eastbury High School increases by 7.15%. Which of the following expressions could be used to find the *weekly* rate of increase of Eastbury High School after *t* years?

$\left(\left(1.0715\right)^{\frac{1}{52}}\right)^{52t}$

1) $(1.0715^{\frac{1}{52}})^t$ ~~2)~~ $(1.0715^{\frac{1}{52}})^{52t}$ 3) $(1.0715)^{52t}$ 4) $(1.0715)^t$

3. A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, $B(t)$, can be represented by the function $B(t) = 750(1.16)^t$, where the *t* represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function

$1.16^{\frac{1}{12}} = 1.012$

1) $B(t) = 750(1.012)^t$ ~~3)~~ $B(t) = 750(1.012)^{12t}$

2) $B(t) = 750(1.16)^{12t}$ 4) $B(t) = 750(1.16)^{\frac{t}{12}}$

4. Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let *m* represent months.]

$1.0525^{\frac{1}{12}} = 1.00427$

1) $(1.0525)^m$ ~~3)~~ $(1.00427)^m$

2) $(1.0525)^{\frac{12}{m}}$ 4) $(1.00427)^{\frac{m}{12}}$

5. Rasmus invested $65,000 in the stock market and makes an average of 9.2% each year on his investments. Which equation could be used to find his monthly percent increase after *t* years?

1) $v = 65000(1.092)^t$ 3) $v = 65000(1.0074)^t$
~~2)~~ $v = 65000(1.0074)^{12t}$ 4) $v = 65000(1.092)^{12t}$

$(1.092)^{\frac{1}{12}} = 1.0074$

6. The population, $p(t)$, of a small county in Western New York has grown according to the formula $p(t)=87218(1.421)^t$ after t years. What is the *weekly* percent of increase rounded to the nearest hundredth of a percent?

$1.421^{\frac{1}{52}} = 1.006779819$

-1

$.006779819(100) = .68\%$

7. A student studying public policy created a model for the population of Detroit, where the population decreased 25% over a decade. He used the model $P=714(0.75)^d$, where P is the population, in thousands, d decades after 2010. Another student, Suzanne, wants to use a model that would predict the population after y years. Suzanne's model is best represented by

*10 years in a decade

1) $P=714(0.6500)^y$

2) $P=714(0.8500)^y$

3) $P=714(0.9716)^y$

4) $P=714(0.9750)^y$

$0.75^{\frac{1}{10}} = (.9716)^y$

8. Stephanie found that the number of white-winged cross bills in an area can be represented by the formula $C=550(1.08)^t$, where t represents the number of years since 2010. Which equation correctly represents the number of white-winged cross bills in terms of the monthly rate of population growth?

1) $C=550(1.00643)^t$

2) $C=550(1.00643)^{12t}$

3) $C=550(1.00643)^{\frac{t}{12}}$

4) $C=550(1.00643)^{t+12}$

$1.08^{\frac{1}{12}}$

$(1.00643)^{12t}$

9. On average, college seniors graduating in 2012 could compute their growing student loan debt using the function $D(t)=29,400(1.068)^t$, where t is time in years. Which expression is equivalent to $29,400(1.068)^t$ and could be used by students to identify an approximate daily interest rate on their loans?

1) $29,400\left(1.068^{\frac{1}{365}}\right)^t$

2) $29,400\left(\frac{1.068}{365}\right)^{365t}$

3) $29,400\left(1+\frac{0.068}{365}\right)^t$

4) $29,400\left(1.068^{\frac{1}{365}}\right)^{365t}$

$(1.068)^{\frac{1}{365}\,365t}$

10. A population of insects grows according to the formula $p(t)=2100(1.37)^t$ where $p(t)$ is the population of insects after t weeks. What is the daily interest rate rounded to the nearest tenth of a percent?

$1.37^{\frac{1}{7}} = 1.045999579$

-1

$.045999579(100)$

4.6%

Irregular Time (Half Life, Double Time, Or a given percent every x unit of time)

$A = P(1 \pm r)^{\frac{t}{h}}$ where h is the amount of time the rate is applied. For example, if the rate increases by 15% every 5 years, r = .15 and h = 5.

$A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$ for half life, $A = P(2)^{\frac{t}{h}}$ for double time

1. The half-life of mendelevium-258 is 51.5 days. Write an equation for the amount of mendelevium-258 remaining from an initial amount of 4000 grams after d days. To the *nearest hundredth of a gram*, how much mendelevium-258 will remain after 12 days?

$A = A$
$P = 4000$
$t = d$
$h = 51.5$

$A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$
$A = 4000\left(\frac{1}{2}\right)^{\frac{d}{51.5}}$

$A = 4000\left(\frac{1}{2}\right)^{\frac{12}{51.5}}$
$A = 3403.43$

2. The amount of ants in a colony doubles every 8 days. If there are initially 275 ants, write an equation for $a(t)$, the amount of ants in the colony after t days. How many ants, to the nearest ant, will be in the colony after 30 days?

$A = a(t)$
$P = 275$
$t = t$
$h = 8$

$A = P(2)^{\frac{t}{h}}$
$a(t) = 275(2)^{\frac{t}{8}}$

$a(30) = 275(2)^{\frac{30}{8}}$
$a(30) = 3700$ ants

3. Jabba went to the movies on Friday night and bought a large popcorn. Every 20 minutes, Jabba eats 40% of the remaining amount of popcorn in his bucket. If there were 967 pieces of popcorn initially in Jabba's bucket, write an equation for $a(t)$, the amount of popcorn left in Jabba's bucket after t minutes. How many pieces of popcorn, to the *nearest piece of popcorn*, will be left an hour and a half into the movie?

$A = a(t)$
$P = 967$
$r = .4$
$t = t$
$h = 20$

$A = P(1 \pm r)^{\frac{t}{h}}$
$a(t) = 967(1 - .4)^{\frac{t}{20}}$
$a(t) = 967(.6)^{\frac{t}{20}}$

90 minutes

$a(90) = 967(.6)^{\frac{90}{20}}$
$a(90) = 97$

4. A payday loan company makes loans between \$100 and \$1000 available to customers. Every 14 days, customers are charged 30% interest with compounding. In 2013, Remi took out a \$300 payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?

1) $300(.30)^{\frac{14}{365}}$
2) $300(1.30)^{\frac{14}{365}}$
3) $300(.30)^{\frac{365}{14}}$
4) $300(1.30)^{\frac{365}{14}}$

$A = A$
$P = 300$
$r = .3$
$t = 365$
$h = 14$

$A = P(1 \pm r)^{\frac{t}{h}}$
$A = 300(1.3)^{\frac{365}{14}}$

Sequences:
Arithmetic: add a constant difference, Geometric: multiply by a common ratio
Explicit Formulas (From Reference Sheet)

Arithmetic: $a_n = a_1 + (n-1)d$ Geometric: $a_n = a_1(r)^{n-1}$

If initial or a_0 is given, $(n-1)$ becomes n. Same formulas as Algebra I modeling.

Arithmetic: $a_n = a_0 + nd$ Geometric: $a_n = a_0(r)^n$

Recursive Formulas

Arithmetic: $a_1 =$, $a_n = a_{n-1} + d$ Geometric: $a_1 =$, $a_n = ra_{n-1}$

Use explicit to find a specific term.

1. Write an explicit AND recursive equation for the following sequence and find the tenth term.
19, 16, 13, 10 …
$-3\ -3\ -3$

$a_n = a_1 + (n-1)d$
$a_n = 19 + (n-1)(-3)$
$a_{10} = 19 + (10-1)(-3)$
$a_{10} = -8$

$a_1 = 19$
$a_n = a_{n-1} - 3$

2. Write an explicit AND recursive equation for the following sequence and find the ninth term.
2,8,32,128,…
$\cdot 4\ \cdot 4\ \cdot 4$

$a_n = a_1(r)^{n-1}$
$a_n = 2(4)^{n-1}$
$a_9 = 2(4)^{9-1}$
$a_9 = 131072$

$a_1 = 2$
$a_n = 4a_{n-1}$

3. Write an explicit AND recursive equation for the following sequence and find the eighth term.
2, 6, 18, 54, ...
$\cdot 3\ \cdot 3\ \cdot 3$

$a_n = a_1(r)^{n-1}$
$a_n = 2(3)^{n-1}$
$a_8 = 2(3)^{8-1}$
$a_8 = 4374$

$a_1 = 2$
$a_n = 3a_{n-1}$

4. Write an explicit AND recursive equation for the following sequence and find the 20th term.
63, 57, 51, 45, ...
$-6\ -6\ -6$

$a_n = a_1 + (n-1)d$
$a_n = 63 + (n-1)(-6)$
$a_{20} = 63 + (20-1)(-6)$
$a_{20} = -51$

$a_1 = 63$
$a_n = a_{n-1} - 6$

5. Write an explicit AND recursive equation for the following sequence and find the 7th term.
3, -12, 48, -192, ...
$\cdot -4\ \cdot -4\ \cdot -4$

$a_n = a_1(r)^{n-1}$
$a_n = 3(-4)^{n-1}$
$a_7 = 3(-4)^{7-1}$
$a_7 = 12288$

$a_1 = 3$
$a_n = -4a_{n-1}$

Evaluating Recursive Sequences

a_{n-1} means the previous term!

1) Start with the term after the one they give you
2) Substitute the previous term in for a_{n-1}

1. Find the first 4 terms of the sequence $a_n = 2a_{n-1} + 4$ where a $a_1 = 3$.

$a_2 = 2a_1 + 4$ $\quad a_3 = 2a_2 + 4$ $\quad a_4 = 2a_3 + 4$

$a_2 = 2(3) + 4$ $\quad a_3 = 2(10) + 4$ $\quad a_4 = 2(24) + 4$

$a_2 = 10$ $\quad a_3 = 24$ $\quad a_4 = 52$

3, 10, 24, 52

2. Find the first 4 terms of the recursive sequence $\begin{array}{l} a_1 = -3 \\ a_n = 4 - 3a_{n-1} \end{array}$

$a_2 = 4 - 3a_1$ $\quad a_3 = 4 - 3a_2$ $\quad a_4 = 4 - 3a_3$

$a_2 = 4 - 3(-3)$ $\quad a_3 = 4 - 3(13)$ $\quad a_4 = 4 - 3(-35)$

$a_2 = 13$ $\quad a_3 = -35$ $\quad a_4 = 109$

-3, 13, -35, 109

3. If $a_n = 3a_{n-1} - 4$ and $a_1 = 9$, find a_5

$a_2 = 3a_1 - 4$ $\quad a_3 = 3a_2 - 4$ $\quad a_4 = 3a_3 - 4$ $\quad a_5 = 3a_4 - 4$

$a_2 = 3(9) - 4$ $\quad a_3 = 3(23) - 4$ $\quad a_4 = 3(65) - 4$ $\quad a_5 = 3(191) - 4$

$a_2 = 23$ $\quad a_3 = 65$ $\quad a_4 = 191$ $\quad a_5 = 569$

4. A sequence is defined recursively by $f(1) = 16$ and $f(n) = f(n-1) + 2n$. Find $f(4)$.

(1) 32 (2) 30 (3) 28 (4) 34

n is the term you're finding

$f(2) = f(1) + 2(2)$ $\quad f(3) = f(2) + 2(3)$ $\quad f(4) = f(3) + 2(4)$

$f(2) = 16 + 4$ $\quad f(3) = 20 + 6$ $\quad f(4) = 26 + 8$

$f(2) = 20$ $\quad f(3) = 26$ $\quad f(4) = 34$

5. Find the first four terms of the recursive sequence defined below.

$a_1 = -3$

$a_n = a_{(n-1)} - n$

$a_2 = a_1 - 2$ $\quad a_3 = a_2 - 3$ $\quad a_4 = a_3 - 4$

$a_2 = -3 - 2$ $\quad a_3 = -5 - 3$ $\quad a_4 = -8 - 4$

$a_2 = -5$ $\quad a_3 = -8$ $\quad a_4 = -12$

-3, -5, -8, -12

Modeling Sequences

r **is the common ratio (what you're multiplying by). If you're increasing or decreasing by a percent,** *common ratio* $= 1 \pm rate$.

For example: Increases by 12% each year, common ratio is $1 + .12 = 1.12$

Decreases by 20% each year, common ratio is $1 - .20 = .80$

(Refer back to previous page for formulas)

1. The formula below can be used to model which scenario?

$$a_1 = 3000$$
$$a_n = 0.80a_{n-1}$$

1-.2 decreasing 20%

1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
3) A bank account starts with a deposit of \$3000, and each year it grows by 80%.
4) The initial value of a specialty toy is \$3000, and its value each of the following years is 20% less.

1-.25 decreasing 25%

2. The sequence defined by $r_1 = 15$ and $r_{n+1} = 0.75r_n$ best models which scenario?

1) Gerry's \$15 allowance is increased by \$0.75 each week.
2) A store that has not sold a \$15 item reduces the price by \$0.25 each week until someone purchases it.
3) A 15-gram sample of a chemical compound decays at a rate of 75% per hour.
4) A picture with an area of 15 square inches is reduced by 25% over and over again to make a proportionally smaller picture.

adding 20, no percent

3. Which situation *cannot* be modeled by the formula $f(n) = f(n-1) + 20$ with $f(1) = 10$?

1) Nancy put \$10 in her piggy bank on the first day and then added \$20 daily to her piggy bank.
2) Jay has a box of ten crayons and his teacher gives him twenty new crayons each month for good behavior.
3) Buzz has ten apples and that number increases by 20% per week.
4) Teresa has a block of metal that is 10°F and she heats it up at a rate of 20°F per minute.

4. The height of Jenny's sunflower when she planted it was 6 inches (h_0). The sunflower grows by 0.25 inches per day. Which formula can be used to determine the height, in inches, of Jenny's sunflower on day *n*?

+.25

(1) $h_0 = 6$, $h_n = 0.25a_{n-1}$

(3) $h_0 = 6$, $h_n = h_{n-1} + 0.25$

(2) $h_0 = 6$, $h_n = 6 + 0.25h_{n-1}$

(4) $h_0 = 6$, $h_n = 6h_{n-1} + 0.25$

$\cdot 3$ $\quad b_1$

5. A population of bacteria triples every day. If on the first day there are 300 bacteria in a Petri dish, which recursive sequence can be used to determine the population on day *n*?

(1) $b_1 = 300$, $b_n = 3b_{n-1}$

(2) $b_1 = 300$, $b_n = b_{n-1} + 3$

(3) $b_1 = 300$, $b_n = 300(3b_{n-1})$

(4) $b_1 = 300$, $b_n = \frac{1}{3}b_{n-1}$

1.015 $\quad P_0$

6. In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State *t* years after 2010?

1) $P_t = 19,378,000(1.5)^t$

2) $P_0 = 19,378,000$
$P_t = 19,378,000 + 1.015P_{t-1}$

3) $P_t = 19,378,000(1.015)^{t-1}$

4) $P_0 = 19,378,000$
$P_t = 1.015P_{t-1}$

Recursive
$P_0 = 19,378,000$
$P_t = 1.015P_{t-1}$

Explicit
$P_t = P_0(r)^t$
$P_t = 19,378,000(1.015)^t$
*If P_0, no t instead of $t-1$.

7. The values below represent the cost of an ice cream sundae with one through four toppings.

$4.75 $5.50 $6.25 $7.00

Write a recursive function that can be used to determine the cost of an ice cream cone with *n* toppings.

$5.50 - 4.75 = .75$ $\quad 6.25 - 5.50 = .75$
Common difference
arithmetic

$a_1 = 4.75$
$a_n = a_{n-1} + .75$

$\frac{5.50}{4.75} = 1.15..$ $\quad \frac{6.25}{5.50} = 1.13..$
Not a common ratio, not geometric

8. The population of Jamesburg for the years 2010-2013, respectively, was reported as follows:
250,000 250,937 251,878 252,822
How can this sequence be recursively modeled?

1) $j_n = 250,000(1.00375)^{n-1}$

2) $j_n = 250,000 + 937^{(n-1)}$

3) $j_1 = 250,000$
$j_n = 1.00375j_{n-1}$

4) $j_1 = 250,000$
$j_n = j_{n-1} + 937$

$250937 - 250000 = 937$ $\quad 251878 - 250937 = 941$
Not a common difference
not arithmetic

$a_1 = 250,000$
$a_n = 1.00375a_{n-1}$

$\frac{250937}{250000} = 1.00375$ $\quad \frac{251878}{250937} = 1.00375$
common ratio
geometric

Series is the sum of a sequence

To write a series explicitly: $S_n = \dfrac{a_1 - a_1(r)^n}{1-r}$ where r is the common ratio $(1 \pm rate)$

To write a series using summations: $\sum_{n=1}^{n} a_1(r)^{n-1}$ or $\sum_{n=0}^{n} a_0(r)^n$

Always use explicit unless it says otherwise

1. Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year. Write a geometric series formula, S_n, for Alexa's total earnings over n years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

a_1 $r=1.04$

$a_1 = 33{,}000$
$r = 1.04$

$S_n = \dfrac{a_1 - a_1(r)^n}{1-r}$

$S_n = \dfrac{33{,}000 - 33{,}000(1.04)^n}{1-1.04}$

$S_{15} = \dfrac{33{,}000 - 33{,}000(1.04)^{15}}{1-1.04}$

$S_{15} = 660778.39$

2. Dee is planning on decreasing the amount of time she eats fast food per month. After the first month, she ate fast food 42 times. Each month, she eats at fast food restaurants 10% less than the previous month. How many times does she eat fast food in the first four months?

a_1 $r = 1 - .1 = .9$

$a_1 = 42$
$r = .9$

$S_n = \dfrac{a_1 - a_1(r)^n}{1-r}$

$S_n = \dfrac{42 - 42(.9)^n}{1-.9}$

$S_4 = \dfrac{42 - 42(.9)^4}{1-.9}$

$S_4 = 144$ times

3. A fisherman harvests 350 kilograms of crab on Monday. From Monday to Friday, the fisherman harvests 8% less kilograms of crab per day. To the *nearest tenth of a kilogram*, what is the total amount of crab harvested between Monday and Friday?

a_1 5 days

$a_1 = 350$
$r = .92$

$r = 1 - .08 = .92$

$S_n = \dfrac{a_1 - a_1(r)^n}{1-r}$

$S_n = \dfrac{350 - 350(.92)^n}{1-.92}$

$S_5 = \dfrac{350 - 350(.92)^5}{1-.92}$

$S_5 = 1491.5$ Kg

4. Kina earns a $27,000 salary for the first year of work at her job. She earns annual increases of 2.5%. What is the total amount, to the *nearest cent*, that Kina will earn for the first eight years at this job?

a_1 $r = 1 + .025$

$a_1 = 27{,}000$
$r = 1.025$

$S_n = \dfrac{a_1 - a_1(r)^n}{1-r}$

$S_n = \dfrac{27{,}000 - 27{,}000(1.025)^n}{1-1.025}$

$S_8 = \dfrac{27{,}000 - 27{,}000(1.025)^8}{1-1.025}$

$S_8 = 235875.13$

5. Brian deposited 1 cent into an empty non-interest bearing bank account on the first day of the month. He then additionally deposited 3 cents on the second day, 9 cents on the third day, and 27 cents on the fourth day. What would be the total amount of money in the account at the end of the 20th day if the pattern continued?

1) \$11,622,614.67
2) \$17,433,922.00
3) \$116,226,146.80
4) \$1,743,392,200.00

1, 3, 9, 27, ...
$r=3$

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_{20}=\frac{.01-.01(3)^{20}}{1-3}=17433922$$

6. This year, public parks in New York State will receive funds of \$2.4 million. Every year afterward, New York State park funding will be improved by 3%. Write an equation to represent how much funding New York State parks will receive in n years. Use your expression to determine the total amount of money, to the *nearest million dollars,* New York State parks will receive in funding for the first four years?

a_1 → 1.03

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_n=\frac{2.4-2.4(1.03)^n}{1-1.03}$$

$$S_4=\frac{2.4-2.4(1.03)^4}{1-1.03}$$

$S_4=10$ mil

$r=1.1$

7. Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

a_1

1) $\sum_{n=1}^{6} 8(1.10)^{n-1}$
2) $\sum_{n=1}^{6} 8(1.10)^{n}$
3) $\dfrac{8-8(1.10)^6}{0.90}$
4) $\dfrac{8-8(0.10)^n}{1.10}$

Explicit

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_6=\frac{8-8(1.1)^6}{1-1.1}$$

$$S_6=\frac{8-8(1.1)^6}{-.1}$$

Summations

$$S_n=\sum_{n=1}^{6} a_1(r)^{n-1}$$

$$S_n=\sum_{n=1}^{6} 8(1.1)^{n-1}$$

$r=1.25$

8. In his first year running track, Usain earned 8 medals. He increases his amount of medals by 25% each year. Which of the following expressions *cannot* be used to determine how many medals Usain will have after four years of high school?

a_1

1) $\dfrac{8-8(1.25)^4}{-.25}$
2) $8(1.25)^0+8(1.25)^1+8(1.25)^2+8(1.25)^3$
3) $\sum_{n=1}^{4} 8(1.25)^{n-1}$
4) $\dfrac{8-8(.25)^4}{1-.25}$

Explicit

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_4=\frac{8-8(1.25)^4}{1-1.25}$$

$$S_4=\frac{8-8(1.25)^4}{-.25}$$

Summations

$$S_n=\sum_{n=1}^{4} a_1(r)^{n-1}$$

$$S_n=\sum_{n=1}^{4} 8(1.25)^{n-1}$$

Mortgage/Annuities
For these formulas, P represents the Principal/Amount Borrowed/Loan Amount
D = T – P (Down payment = Total Cost – Principal)
P = T – D (Principal = Total Cost – Down Payment)
To find down payment:
1) Find Principal
2) Substitute into D = T – P to find the down payment

If given down payment:
P = T – D and substitute P into mortgage formula.
*Principal may not be given as P. It may be a different variable

1. Jim is looking to buy a vacation home for $172,600 near his favorite southern beach. The formula to compute a mortgage payment, *M*, is $M = P \bullet \frac{r(1+r)^N}{(1+r)^N - 1}$ where *P* is the principal amount of the loan, *r* is the monthly interest rate, and *N* is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage. With no down payment, determine Jim's mortgage payment, rounded to the *nearest dollar*.

M = Mortgage Payment = X
P = Principal amount of loan = 172,600
r = monthly interest rate = .00305
N = # of monthly payments = 15(12) = 180

P = T − D
P = 172,600 − 0
P = 172,600

$x = 172600 \cdot \frac{.00305(1.00305)^{180}}{1.00305^{180} - 1}$

$x = \$1247$

Algebraically determine and state the down payment, rounded to the *nearest dollar*, that Jim needs to make in order for his mortgage payment to be $1100.

P = T − D
Need P

$1100 = x\left(\frac{.00305(1.00305)^{180}}{1.00305^{180} - 1}\right)$

M = 1100
P = x
r = .00305
N = 180

$\frac{1100}{.007\ldots} = \frac{x(.007\ldots)}{.007\ldots}$

152,193... = x

152,193... = 172,600 − D
−172,600 − 172,600
−20407 = −D
$20407 = D

2. Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the *nearest cent.*

$$P_n = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

P_n = present amount borrowed $= 21{,}000 - 1{,}000 = 20000$
n = number of monthly pay periods $= 5(12) = 60$
PMT = monthly payment $= x$
i = interest rate per month $= .00625$

$$20000 = x\left(\frac{1-(1.00625)^{-60}}{.00625}\right)$$

$$\frac{20{,}000}{49.9\ldots} = \frac{x(\cancel{49.9\ldots})}{\cancel{49.9\ldots}}$$

$$400.76 = x$$

P=T-D The affordable monthly payment is $300 for the same time period. Determine an appropriate down payment, to the *nearest dollar.*

$P_n = x$

$n = 5(12) = 60$

$PMT = 300$

$i = .00625$

$$x = 300\left(\frac{1-(1.00625)^{-60}}{.00625}\right)$$

$$x = \cancel{300(} \; 14971\ldots$$

$$P = T - D$$

$$14971\ldots = 21{,}000 - D$$
$$-21000 \quad -21{,}000$$

$$\frac{-6028\ldots}{-1} = \frac{-D}{-1}$$

$$\$6028 = D$$

$D = .2(380{,}000) = 76{,}000$

3. Mr. and Mrs. Jenkins just closed on a new home whose purchase price was $380,000. At the closing, they supplied a down payment of 20% of the purchase price. If on the day of the closing the ~~annual~~ monthly interest rate was .3125%, determine the Jenkins' monthly mortgage payment, to the *nearest cent*, if they were approved for a 30-year loan.

Use the formula $M = P \bullet \frac{r(1+r)^n}{(1+r)^n - 1}$ where M is the mortgage payment, P is the principal amount of the loan, r is the monthly interest rate, and n is the number of monthly payments.

$M = X$

$P = T - D \quad P = 380{,}000 - 76{,}000 = 304{,}000$

$r = .003125$

$n = 30(12) = 360$

$X = 304{,}000\left(\frac{.003125(1.003125)^{360}}{(1.003125)^{360} - 1}\right)$

$X = \$1407.87$

Algebraically determine and state the down payment, to the *nearest dollar*, Mr. and Mrs. Jenkins would need to initially supply in order to bring their monthly mortgage payment down to $1200.

$M = 1200$

$P = X$

$r = .003125$

$n = 30(12) = 360$

$1200 = X\left(\frac{.003125(1.003125)^{360}}{(1.003125)^{360} - 1}\right)$

$\frac{1200}{.00463\ldots} = \frac{X(\cancel{.00463\ldots})}{\cancel{.00463\ldots}}$

$259114\ldots = X$

$P = T - D$

$259114\ldots = 380{,}000 - D$

$-380{,}000 \quad -380{,}000$

$\frac{-120885\ldots}{-1} = \frac{-D}{-1}$

$D = 120885$

4. Astrid just purchased a new car for $30,000. She traded in her old car and used the money she received from it to make a $4,000 down payment on the car. To the *nearest cent*, what will be Astrid's monthly payment on her new car if her loan has an interest rate of 0.05% per month and the life of the loan is ten years? Use the formula $A = R\left(\frac{1-(1+i)^{-n}}{i}\right)$ where A = present amount borrowed, R = monthly payment, n = number of monthly pay periods, and I = monthly interest rate.

T

D

A = present amount borrowed = T − D 30,000 − 4,000 = 26,000

R = monthly payment = X

i = .0005

n = 10(12) = 120

$$26000 = x\left(\frac{1-(1.0005)^{-120}}{.0005}\right)$$

$$\frac{26{,}000}{116\ldots} = \frac{x(116\ldots)}{116\ldots}$$

223.29 = X

Astrid knows that she cannot afford a monthly payment of more than $200 for the same time period. By how much, to the *nearest dollar*, should she increase her down payment to satisfy this condition?

R

A = X

R = 200

i = .0005

n = 120

$$x = 200\left(\frac{1-(1.005)^{-120}}{.0005}\right)$$

x = 23288...

principal = total − down payment

23288... = 30000 − D

− 30000 − 30000

$$\frac{-6711..}{-1} = \frac{-D}{-1}$$

6711 = D

6711

− 4000

$2711

She will need to increase her down payment by $2711

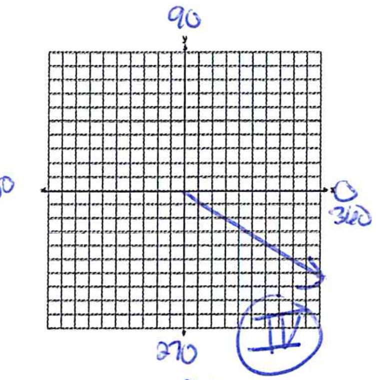
Sketching Radian Angles

Degrees to radians: Multiply by $\frac{\pi}{180}$

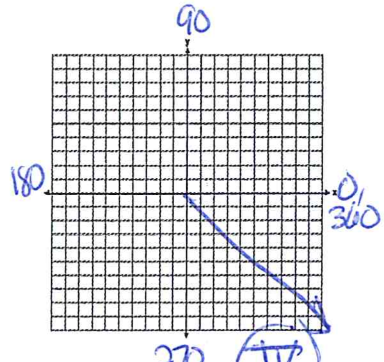
Radians to degrees: Multiply by $\frac{180}{\pi}$ OR replace π with 180

Sketch the following angles and state the quadrant

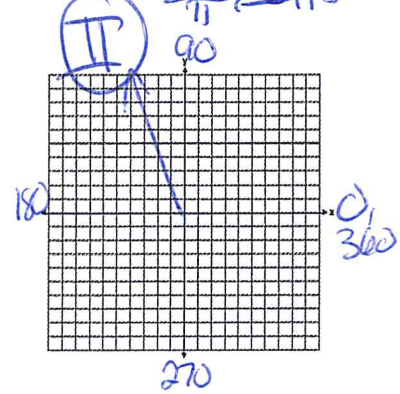
1. $\theta = \frac{5\pi}{3} \cdot \frac{180}{\pi} = 300°$



2. $\theta = \frac{7\pi}{4} \cdot \frac{180}{\pi} = 315$

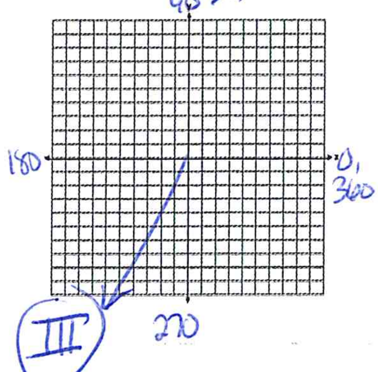


3. $\theta = 2 \cdot \frac{180}{\pi} \approx 115$



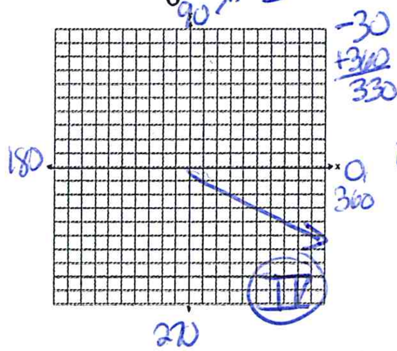
4. $\theta = \frac{10\pi}{3} \cdot \frac{180}{\pi} = 600°$

600°
−360
240°



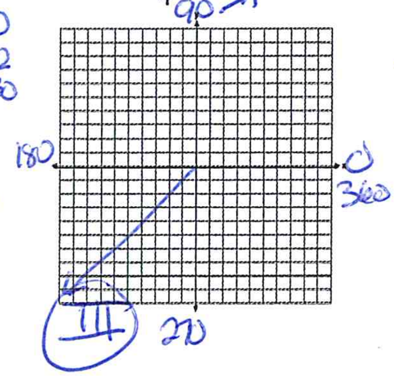
5. $\theta = -\frac{\pi}{6} \cdot \frac{180}{\pi} = -30$

−30
+360
330



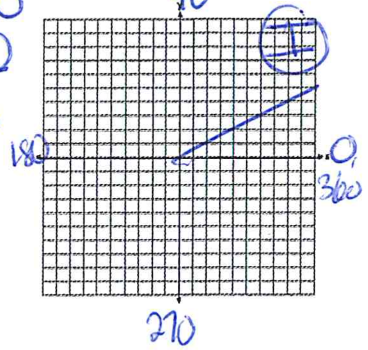
6. $\theta = \frac{13\pi}{4} \cdot \frac{180}{\pi} = 585$

585
−360
225

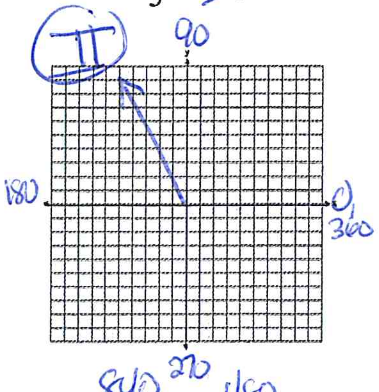


7. $\theta = \frac{-11\pi}{6} \cdot \frac{180}{\pi} = -330$

−330
+360
30



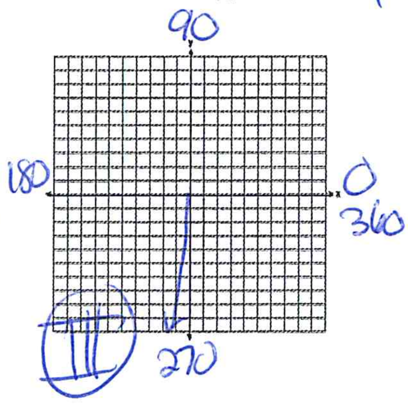
8. $\theta = \frac{14\pi}{3} \cdot \frac{180}{\pi} = 840$



840
−360
480

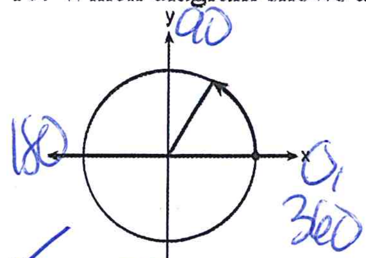
480
−360
120

9. $\theta = 4.7 \cdot \frac{180}{\pi} \approx 269$

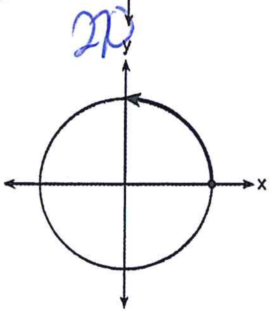


10. Which diagram shows an angle rotation of 1 radian on the unit circle?

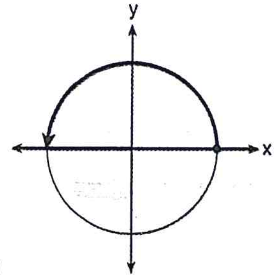
$$1 \cdot \frac{180}{\pi} \approx 57$$



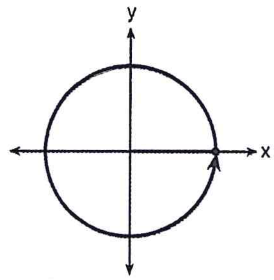
1)



2)



3)

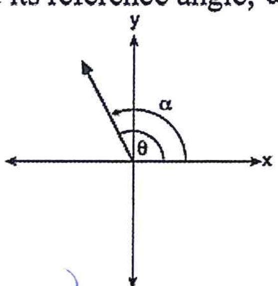


4)

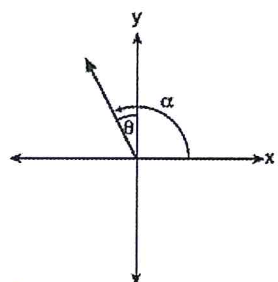
11. Which diagram represents an angle, α, measuring $\frac{13\pi}{20}$ radians drawn in standard position, and its reference angle, θ?

$$\frac{13\cancel{\pi}}{20} \cdot \frac{180}{\cancel{\pi}} = 117$$

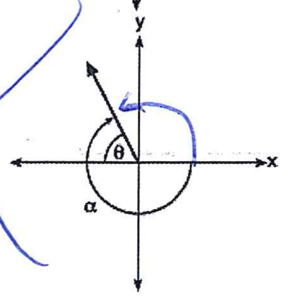
1)



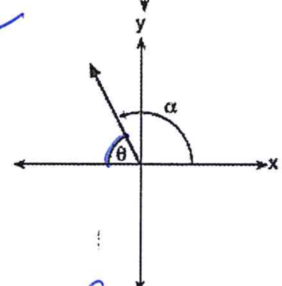
3)



2)



4)



reference angle is the acute angle formed between the angle and the x axis.

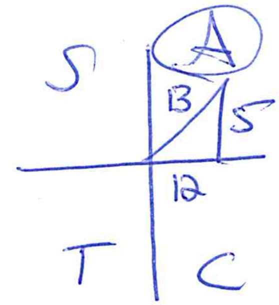
Trig Ratios

If an angle passes through a point or sin/cos/tan = $\frac{something}{something}$, make a right triangle and use SOHCAHTOA

Any point on the unit circle is $(\cos\theta, \sin\theta)$

Know your Pythagorean triples: {3,4,5}, {5, 12, 13}, {8, 15, 17}, {7, 24, 25}

1. If $\cos\theta = \frac{12\ A}{13\ H}$ and θ is in Quadrant I, find:



a) $\cos\theta$ $\frac{12}{13}$

b) $\sin\theta$ $\frac{5}{13}$

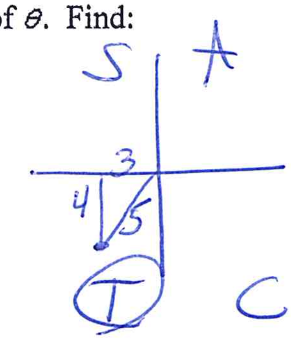
c) $\tan\theta$ $\frac{5}{12}$

d) $\sec\theta$ $\frac{13}{12}$

e) $\csc\theta$ $\frac{13}{5}$

f) $\cot\theta$ $\frac{12}{5}$

2. Angle θ is in standard position and (-3,-4) is a point on the terminal side of θ. Find:



a) $\cos\theta$ $-\frac{3}{5}$

b) $\sin\theta$ $-\frac{4}{5}$

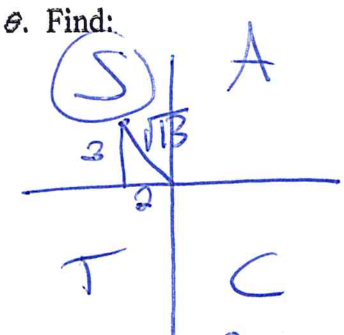
c) $\tan\theta$ $\frac{4}{3}$

d) $\sec\theta$ $-\frac{5}{3}$

e) $\csc\theta$ $-\frac{5}{4}$

f) $\cot\theta$ $\frac{3}{4}$

3. Angle θ is in standard position and (-2,3) is a point on the terminal side of θ. Find:



a) $\cos\theta$ $-\frac{2}{\sqrt{13}}$

b) $\sin\theta$ $\frac{3}{\sqrt{13}}$

c) $\tan\theta$ $-\frac{3}{2}$

d) $\sec\theta$ $-\frac{\sqrt{13}}{2}$

e) $\csc\theta$ $\frac{\sqrt{13}}{3}$

f) $\cot\theta$ $-\frac{2}{3}$

$2^2+3^2=x^2$

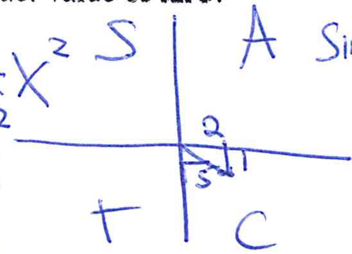
$4+9=x^2$

$\sqrt{13}=\sqrt{x^2}$

$\sqrt{13}=x$

4. An angle, θ, is in standard position and its terminal side passes through the point $(2,-1)$. Find the *exact* value of $\sin\theta$.

$2^2+1^2=x^2$
$4+1=x^2$
$\sqrt{5}=\sqrt{x^2}$
$\sqrt{5}=x$



$\sin\theta=\frac{-1}{\sqrt{5}}$

5. The terminal side of θ, an angle in standard position, intersects the unit circle at $P\left(-\frac{1}{3},-\frac{\sqrt{8}}{3}\right)$.

cos θ   sin θ

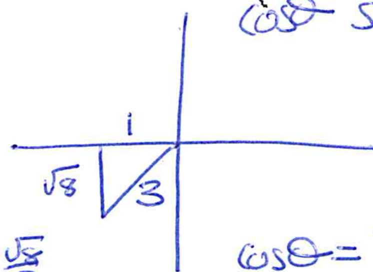
What is the value of $\sec\theta$?

~~1)~~ -3

2) $-\frac{3\sqrt{8}}{8}$

3) $-\frac{1}{3}$

4) $-\frac{\sqrt{8}}{3}$



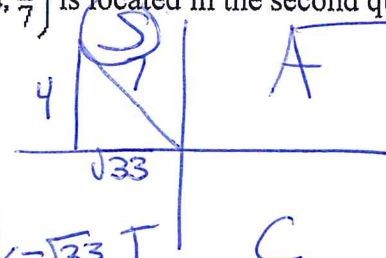
$\sin\theta=-\frac{\sqrt{8}}{3}$

$\cos\theta=\frac{-1}{3}$

$\sec\theta=-3$

cos θ, sin θ

6. Point $M\left(t,\frac{4}{7}\right)$ is located in the second quadrant on the unit circle. Determine the exact value of t.



$x^2+4^2=7^2$
$x^2+16=49$
$-16\quad -16$
$\sqrt{x^2}=\sqrt{33}$
$x=\sqrt{33}$

$\cos\theta=\frac{-\sqrt{33}}{7}$

$t=\frac{-\sqrt{33}}{7}$

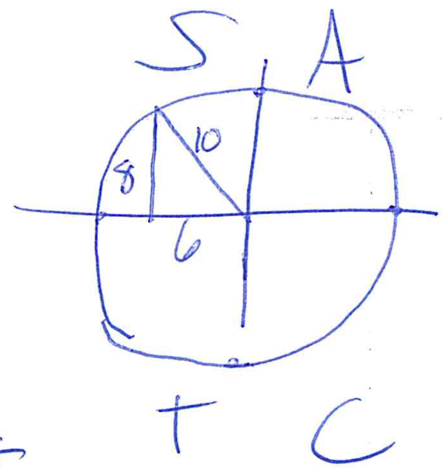
7. A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ, intercepts the circle in Quadrant II at point C. The y-coordinate of point C is 8. What is the value of $\cos\theta$?

~~1)~~ $-\frac{3}{5}$

2) $-\frac{3}{4}$

3) $\frac{3}{5}$

4) $\frac{4}{5}$



$x^2+8^2=10^2$
$x^2+64=100$
$-64\quad -64$
$\sqrt{x^2}=\sqrt{36}$
$x=6$

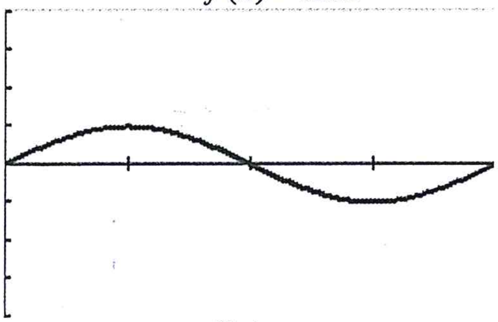
$\cos\theta=\frac{A}{H}$

$\cos\theta=\frac{-6}{10}$

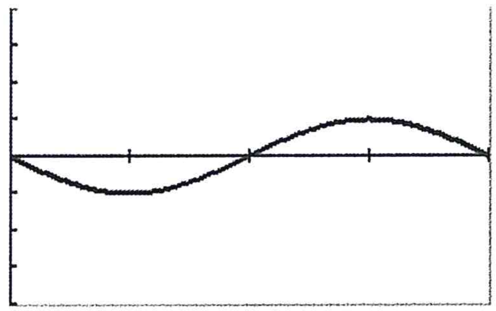
$\cos\theta=\frac{-3}{5}$

Trig Graphs:
Know what your waves look like!

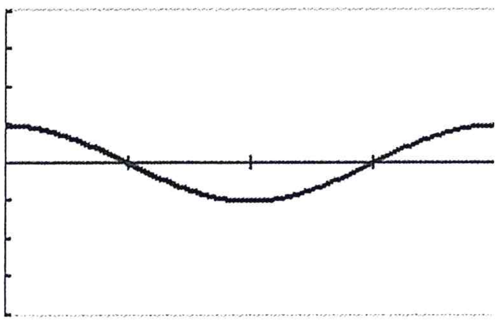
$f(x) = \sin x$



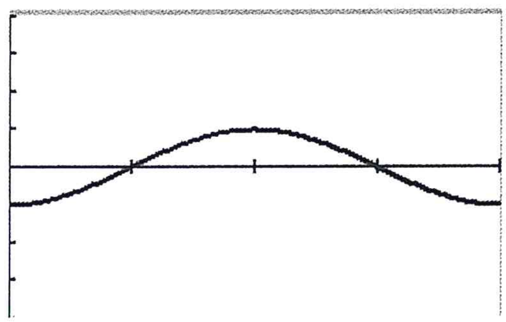
$f(x) = -\sin x$



$f(x) = \cos x$



$f(x) = -\cos x$



AMPSINFREQXSHIFT

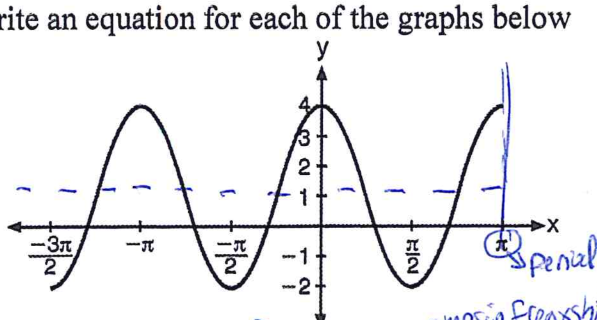
Amplitude: Distance from the midline to minimum or maximum
Frequency: How many waves from 0 to 2π
Period: (Wavelength): How long it takes to make one full cycle
Shift: y value of the midline. The average value of the function.

$$Period = \frac{2\pi}{frequency}, Frequency = \frac{2\pi}{period}$$

To graph: Draw a little picture! Find midline ($\frac{\min + \max}{2}$) and period! Make 4 dashes on x-axis and put period at the 4th dash. Divide that value by 4 to find the scale.

Write an equation for each of the graphs below

1.

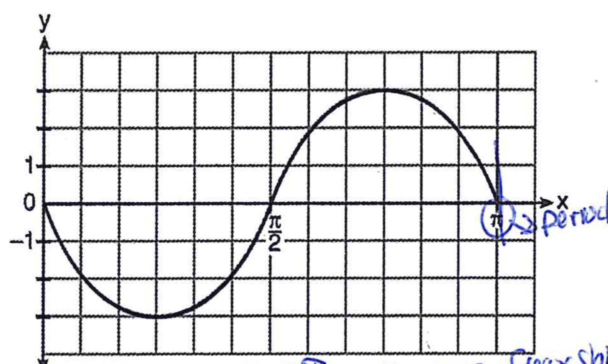


midline = $\frac{min+max}{2}$
midline = $\frac{-2+4}{2}$
midline = 1

$f = \frac{2\pi}{p}$
$f = \frac{2\pi}{\pi}$
$f = 2$

y = ampsin freqxshift
amp = 3
+cos
freq = 2
shift = 1
$y = 3\cos 2x + 1$

2.



midline = $\frac{min+max}{2}$
midline = $\frac{-3+3}{2}$
midline = 0

$f = \frac{2\pi}{p}$
$f = \frac{2\pi}{\pi}$
$f = 2$

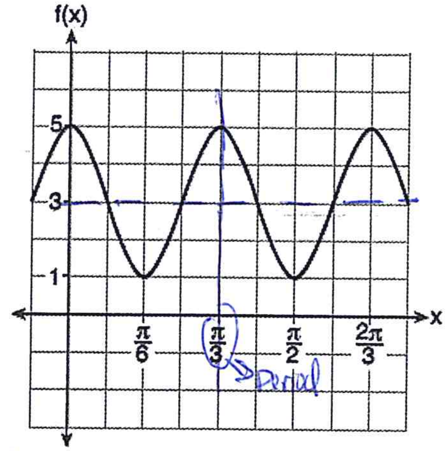
y = ampsinfreqxshift
amp = 3
−sin
freq = 2
shift = 0
$y = -3\sin 2x$

3. The function $f(x) = a\cos bx + c$ is plotted on the graph shown below.

mid=$\frac{min+max}{2}$ $f=\frac{2\pi}{p}$

mid=$\frac{1+5}{2}$ $f=\frac{2\pi}{\frac{\pi}{3}}$ / $\frac{2\pi}{1}\cdot\frac{3}{\pi}=6$

mid=3



What are the values of a, b, and c?

1) $a = 2, b = 6, c = 3$
2) $a = 2, b = 3, c = 1$
3) $a = 4, b = 6, c = 5$
4) $a = 4, b = \frac{\pi}{3}, c = 3$

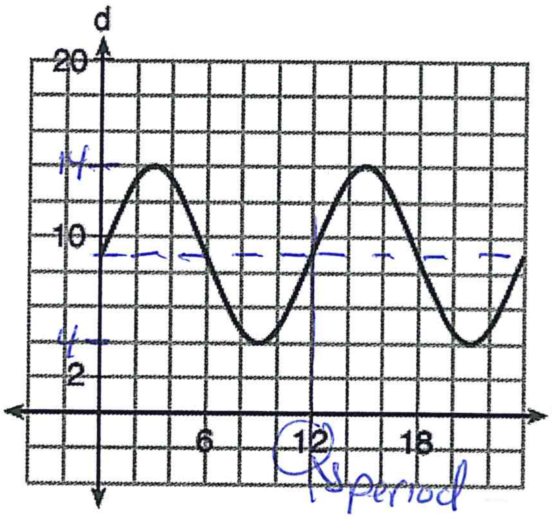
y=amp sin freq x shift

a amp=2 +cos

b freq=6

c shift=3

4. The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.



If the depth, d, is measured in feet and time, t, is measured in hours since midnight, what is an equation for the depth of the water at the marker?

1) $d = 5\cos\left(\frac{\pi}{6}t\right) + 9$
2) $d = 9\cos\left(\frac{\pi}{6}t\right) + 5$
3) $d = 9\sin\left(\frac{\pi}{6}t\right) + 5$
4) $d = 5\sin\left(\frac{\pi}{6}t\right) + 9$

mid $=\frac{min+max}{2}$

mid $=\frac{4+14}{2}=9$

$f=\frac{2\pi}{p}$

$f=\frac{2\pi}{12}$

$f=\frac{\pi}{6}$

y=amp sin freq x shift

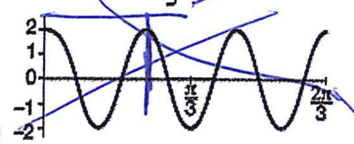
amp=5 +sin

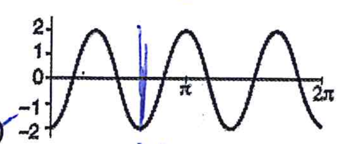
freq=$\frac{\pi}{6}$

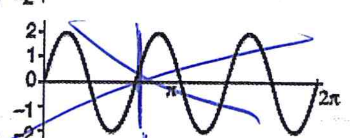
shift=9

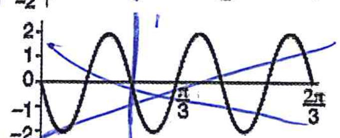
$y=5\sin\frac{\pi}{6}x+9$

5. Which graph represents a cosine function with no horizontal shift, an amplitude of 2, and a period of $\frac{2\pi}{3}$? → one full wave ends at $\frac{2\pi}{3}$

1) 

3) 

2) 

4) 

frequency=3 (3 full waves from 0 to 2π)

$p=\frac{2\pi}{f}$

$p=\frac{2\pi}{3}$

sine function

3 full waves end at $\frac{2\pi}{3}$

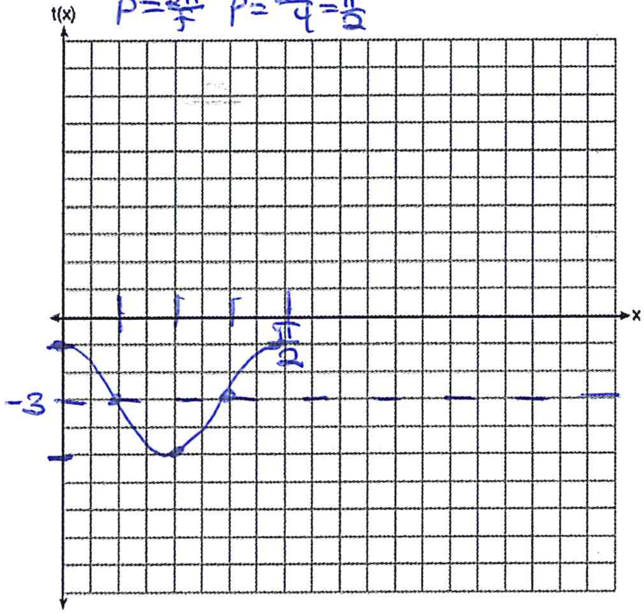
$y = amp \sin freq\, x\ shift$

Graph one cycle of the following functions on the axes provided

6. $y = 2\cos 4x - 3$

amp=2 +cos
freq=4
shift=-3

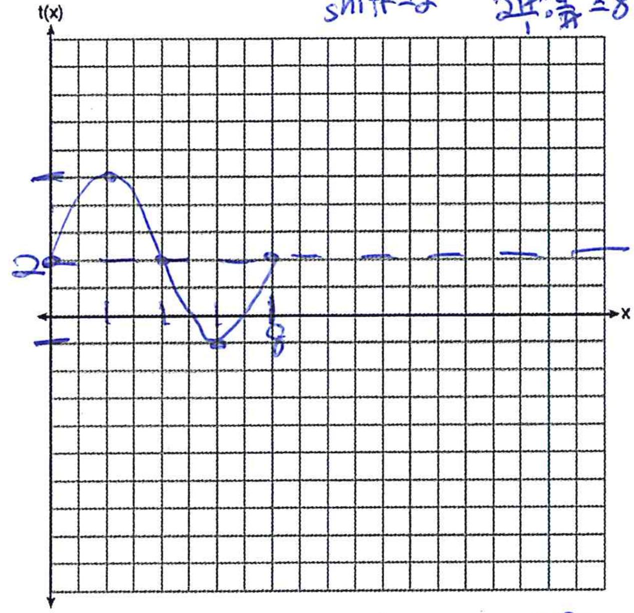
$P=\frac{2\pi}{f}$ $P=\frac{2\pi}{4}=\frac{\pi}{2}$



7. $y = 3\sin\frac{\pi}{4}x + 2$

amp=3 +sin
freq=$\frac{\pi}{4}$
shift=2

$P=\frac{2\pi}{f}$
$P=\frac{2\pi}{\frac{\pi}{4}}$
$\frac{2\pi}{1}\cdot\frac{4}{\pi}=8$

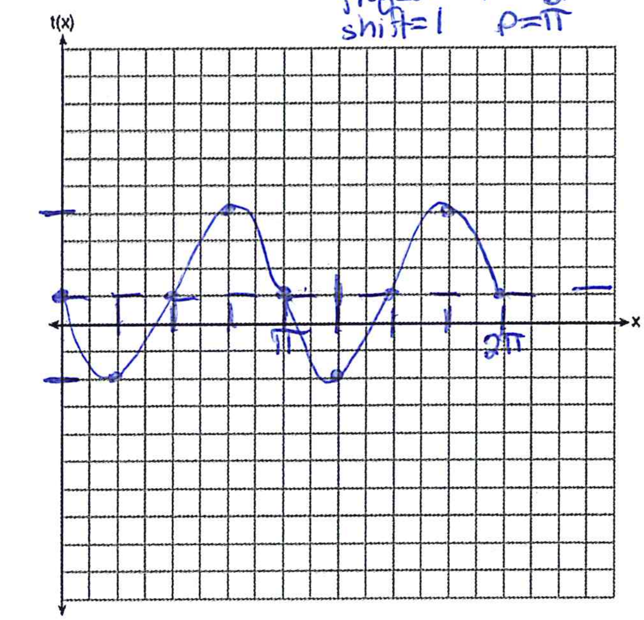


Graph the following functions over the interval $[0, 2\pi]$

8. $y = -3\sin 2x + 1$

amp=3 -sin
freq=2
shift=1

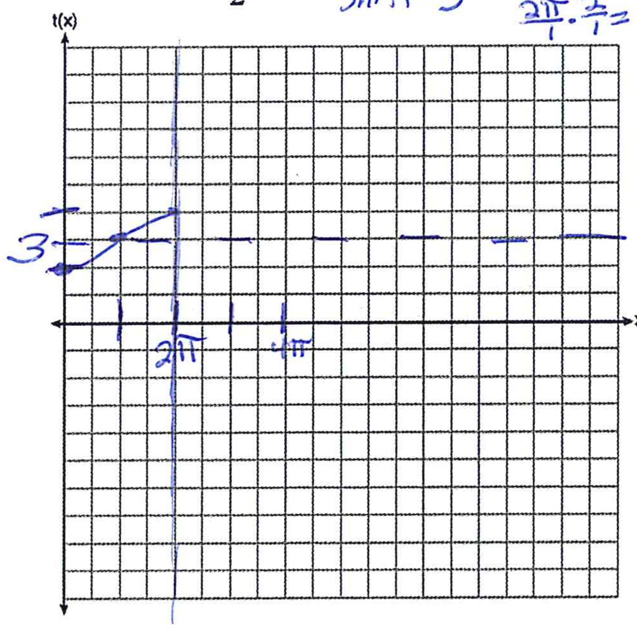
$P=\frac{2\pi}{f}$
$P=\frac{2\pi}{2}$
$P=\pi$



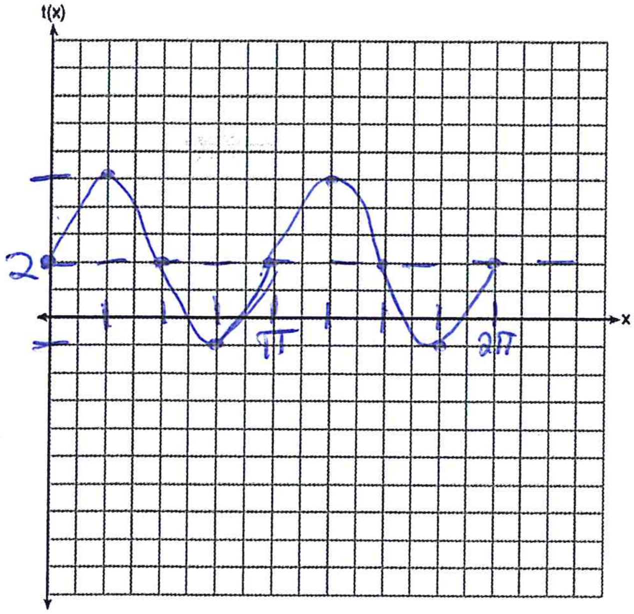
9. $y = -\cos\frac{1}{2}x + 3$

amp=1 -cos
freq=$\frac{1}{2}$
shift=3

$P=\frac{2\pi}{f}$
$=\frac{2\pi}{\frac{1}{2}}$
$\frac{2\pi}{1}\cdot\frac{2}{1}=4\pi$

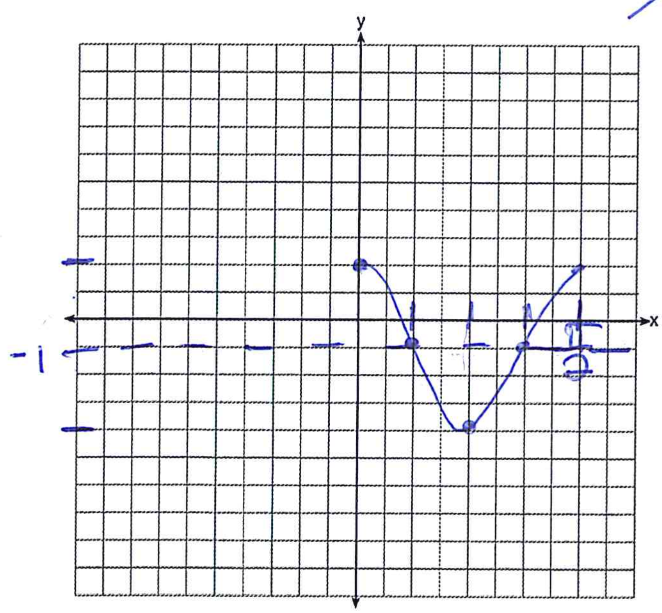


10. Graph $t(x) = 3\sin(2x) + 2$ over the domain $[0, 2\pi]$ on the set of axes below.



amp = 3
+sin
freq = 2
shift = 2
$P = \frac{2\pi}{f}$
$P = \frac{2\pi}{2}$
$P = \pi$

11. On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point $(0, 2)$. → starts at max



Probability
Conditional Probabilities: Circle the row/column that contains the condition. Condition always comes after the phrase given that. You will not always see the phrase given that.
"And" is not conditional.

1. The set of data in the table below shows the results of a survey on the number of messages that people of different ages text on their cell phones each month.

Age Group	Text Messages per Month			
	0–10	11–50	Over 50	
15–18	4	37	68	
19–22	6	25	87	
23–60	25	47	157	229

If a person from this survey is selected at random, what is the probability that the person texts over 50 messages per month given that the person is between the ages of 23 and 60?

1) $\frac{157}{229}$
2) $\frac{157}{312}$
3) $\frac{157}{384}$
4) $\frac{157}{456}$

$\frac{157}{229}$

2. A statistics class surveyed some students during one lunch period to obtain opinions about television programming preferences. The results of the survey are summarized in the table below.

Programming Preferences

	Comedy	Drama	
Male	70	35	105
Female	48	42	90
	118	77	195

Condition

What percentage of the school's male students would prefer comedy?
Based on the sample, predict how many of the school's 351 males would prefer comedy. Justify your answer.

$\frac{70}{105} = 66.\overline{6}\%$

~~66% of 351~~

~~.67(351) =~~

$66.\overline{6}\%$ of 351

$.\overline{6}(351) = 234$

3. A public opinion poll was taken to explore the relationship between age and support for a candidate in an election. The results of the poll are summarized in the table below.

Age	For	Against	No Opinion	
21–40	30	12	8	50
41–60	20	40	15	
Over 60	25	35	15	

What percent of the 21-40 age group was for the candidate?

1) 15
2) 25
3) 40
4) 60

condition

$\frac{30}{50} = .6(100) = 60\%$

4. A radio station did a survey to determine what kind of music to play by taking a sample of middle school, high school, and college students. They were asked which of three different types of music they prefer on the radio: hip-hop, alternative, or classic rock. The results are summarized in the table below.

What percentage of college students prefer classic rock?

	Hip-Hop	Alternative	Classic Rock	
Middle School	28	18	4	
High School	22	22	6	
College	16	20	14	50
			24	

$\frac{14}{50} = .28$ condition 28%

What percentage of the students that prefer classic rock are college students?

$\frac{14}{24} = .58\overline{3}$ 58.3% condition

5. A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

	Favorite Type of Program		
	Sports	**Reality Show**	**Comedy Series**
Senior	83	110	67
Freshmen	119	103	54
		213	

A student response is selected at random from the results. State the *exact* probability the student response is from a freshman, given the student prefers to watch reality shows on television.

$\frac{103}{213}$

Independence

If events are independent:

$P(A \cap B) = P(A) \bullet P(B)$ → No condition, denominator is always total total. This formula is generally easier to use.

$P(A) = P(A / B)$

1. The results of a poll of 200 students are shown in the table below:

	Preferred Music Style			
	Techno	**Rap**	**Country**	
Female	54	25	27	106
Male	36	40	18	94
	90	65	45	200

A = male
B = techno
(doesn't matter which you pick)

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{36}{200} \neq \frac{94}{200} \cdot \frac{90}{200}$

Not Independent

2. At a local mall, 125 people were asked how they choose to pay for their merchandise. The data is shown in the table below:

	Credit Card	Cash	
Male	40	10	50
Female	60	15	75
	100	25	125

A = male
B = cash
(doesn't matter which you pick)

Does the data suggest that the gender and type of payment are independent of each other? Explain your answer.

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{10}{125} = \frac{50}{125} \cdot \frac{25}{125}$ ✓

Independent because $P(A \cap B) = P(A) \cdot P(B)$

3. One-hundred employees of a company were asked their opinion on paying high salaries to the CEO. Their responses are summarized in the following contingency table.

	In Favor	Against	
Male	15	45	60
Female	4	36	40
	19	81	100

A = female
B = against
(doesn't matter which you pick)

Based on the data, are gender an opinion on salaries independent of each other? Justify your answer.

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{36}{100} \neq \frac{40}{100} \cdot \frac{81}{100}$

Not Independent

4. The results of a survey of the student body at Central High School about television viewing preferences are shown below.

	Comedy Series	Drama Series	Reality Series	Total
Males	95	65	70	230
Females	80	70	110	260
Total	175	135	180	490

Are the events "student is a male" (A) and "student prefers reality series" (B) independent of each other? Justify your answer.

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{70}{490} \neq \frac{230}{490} \cdot \frac{180}{490}$

Not Independent

5. Given events A and B, such that $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cup B) = 0.8$, determine whether A and B are independent or dependent.

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(A \cap B) = .6 + .5 - .8$

$P(A \cap B) = .3$

$P(A \cap B) = P(A) \cdot P(B)$

$.3 = .6 \cdot .5$

Independent

Normal Distributions

Normally distributed: Use *normalcdf* (2^{nd} VARS (Distr), 2:normalcdf)

1. The weights of bags of Graseck's Chocolate Candies are normally distributed with a mean of 4.3 ounces and a standard deviation of 0.05 ounces. What is the probability that a bag of these chocolate candies weighs less than 4.27 ounces?

1) 0.2257
2) 0.2743
3) 0.7257
4) 0.7757

normal cdf
lower = 0
upper = 4.27
μ = 4.3
σ = .05
.2743

2. The weight of a bag of pears at the local market averages (mean) 8 pounds with a standard deviation of 0.5 pound. The weights of all the bags of pears at the market closely follow a normal distribution. Determine what percentage of bags, to the *nearest integer*, weighed *less* than 8.25 pounds.

normal cdf
lower = 0
upper = 8.25
μ = 8
σ = 0.5
.691462(100) *because percent
69%

3. The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18. Determine the number of students who scored between 200 and 245.

normal cdf
lower = 200
upper = 245
μ = 225
σ = 8
.9929...(1200) *because asking for #
1191

4. The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the *nearest whole percent*, is

1) 6
2) 48
3) 68
4) 95

normal cdf
lower = 64
upper = 69.5
μ = 64
σ = 2.75
.477...(100) *because percent
48%

5. The lifespan of a 60-watt lightbulb produced by a company is normally distributed with a mean of 1450 hours and a standard deviation of 8.5 hours. If a 60-watt lightbulb produced by this company is selected at random, what is the probability that its lifespan will be between 1440 and 1465 hours?

1) 0.3803
2) 0.4612
3) 0.8415
4) 0.9612

normal cdf
lower = 1440
upper = 1465
μ = 1450
σ = 8.5
.8415

6. The weights of students on the boys cross country team is normally distributed with a mean of 135.3 pounds and a standard deviation of 2.8 pounds. Jackson believes that the probability of a student being between 132 and 134 is greater than the probability of a student being between 135 and 136.5 pounds. Is Jackson correct? Justify your answer.

normal cdf
lower = 132
upper = 134
μ = 135.3
σ = 2.8
.2019..

normal cdf
lower = 135
upper = 136.5
μ = 135.3
σ = 2.8
.2085..

No, .2019.. < .2085..

7. The number of hours students spent studying for their Regents exam is normally distributed with a mean of 14 hours and a standard deviation of 3.2 hours. If a student is randomly selected, what is the probability that they spent less than 5 hours studying? What is the probability that a student spent more than 22 hours studying? Round your answer to the nearest tenth of a percent.

normal cdf
lower = 22
upper = 999999
μ = 14
σ = 3.2

.006..(100) *because percent
0.6%

normal cdf
lower = 0
upper = 5
μ = 14
σ = 3.2

.002...(100) *because percent
0.2%

8. The scores on a math test are normally distributed with a mean of 76.2 and a standard deviation of 4.7. If 248 students took the exam, approximately how many students got between a 70 and an 80?

normal cdf
lower = 70
upper = 80
μ = 76.2
σ = 4.7
.697..(248) *because asking for #
173

9. The number of hours of sleep employees at a company get per night is normally distributed with a mean of 7.1 hours and a standard deviation of 1.4 hours. If an employee is randomly selected, what is the probability they sleep between 5 and 8 hours each night? Round your answer to the *nearest percent*. If there are 2500 employees at the company, approximately how many of them, to the nearest person, got less than 5 hours of sleep?

normal cdf
lower = 5
upper = 8
μ = 7.1
σ = 1.4
.673..(100) *because percent
67%

normal cdf
lower = 0
upper = 5
μ = 7.1
σ = 1.4
.0668..(2500) *because asking for #
167

Statistical Studies

A *survey* is a type of observational study that gathers data by asking people a number of questions.

A good (unbiased) sample should be **randomly** selected where every member of the population has a chance of being chosen.

An *observational study* records the values of variables for members of a sample. NO TREATMENT IS ADMINISTERED.

A *controlled experiment* **randomly selects** a sample and **randomly assigns** members of the sample to a treatment and control group. The treatment group receives a treatment while the control group receives a placebo (if possible).

1. Which scenario is best described as an observational study?

1) For a class project, students in Health class ask every tenth student entering the school if they eat breakfast in the morning.
survey

2) A social researcher wants to learn whether or not there is a link between attendance and grades. She gathers data from 15 school districts.

3) A researcher wants to learn whether or not there is a link between children's daily amount of physical activity and their overall energy level. During lunch at the local high school, she distributed a short questionnaire to students in the cafeteria.
survey

4) Sixty seniors taking a course in Advanced Algebra Concepts are randomly divided into two classes. One class uses a graphing calculator all the time, and the other class never uses graphing calculators. A guidance counselor wants to determine whether there is a link between graphing calculator use and students' final exam grades.
controlled experiment

2. A doctor wants to test the effectiveness of a new drug on her patients. She separates her sample of patients into two groups and administers the drug to only one of these groups. She then compares the results. Which type of study *best* describes this situation?

1) census
2) survey
3) observation
4) controlled experiment

3. Which task is *not* a component of an observational study?

1) The researcher decides who will make up the sample.
2) The researcher analyzes the data received from the sample.
3) The researcher gathers data from the sample, using surveys or taking measurements.
4) The researcher divides the sample into two groups, with one group acting as a control group.

controlled experiment

4. Which statement about statistical analysis is *false*?

1) Experiments can suggest patterns and relationships in data.
2) Experiments can determine cause and effect relationships.
3) Observational studies can determine cause and effect relationships.
4) Observational studies can suggest patterns and relationships in data.

Only controlled experiments can establish causal relationships

5. A market research firm needs to collect data on viewer preferences for local news programming in Buffalo. Which method of data collection is most appropriate?

1) census
2) survey
3) observation
4) controlled experiment

6. A school cafeteria has five different lunch periods. The cafeteria staff wants to find out which items on the menu are most popular, so they give every student in the first lunch period a list of questions to answer in order to collect data to represent the school. Which type of study does this represent?

1) observation
2) controlled experiment
3) population survey → census
4) sample survey

7. Howard collected fish eggs from a pond behind his house so he could determine whether sunlight had an effect on how many of the eggs hatched. After he collected the eggs, he divided them into two tanks. He put both tanks outside near the pond, and he covered one of the tanks with a box to block out all sunlight. State whether Howard's investigation was an example of a controlled experiment, an observation, or a survey. Justify your response.

Controlled experiment.
He applied a treatment (Sunlight).

8. Darryl conducted a study comparing the statistics of baseball players in the steroid era compared to the non steroid era. Would this investigation be an example of a controlled experiment, an observation, or a survey? Justify your response.

Observational Study. He did not divide into two groups and apply a treatment.

Sample Distributions

To determine if something is usual or unusual, expected or unexpected:

Find the confidence interval!!

$Confidence\ Interval = mean \pm 2(Standard\ Deviation)$

$Margin\ of\ Error = 2(Standard\ Deviation)$

A coin (or other object) is fair if the population proportion (.5 for a coin) is in the confidence interval

As sample size increases:

The mean remains relatively the same (it may differ due to random chance).

The spread/variability decreases.

The standard deviation decreases.

1. In 2013, approximately 1.6 million students took the Critical Reading portion of the SAT exam. The mean score, the modal score, and the standard deviation were calculated to be 496, 430, and 115, respectively. Which interval reflects 95% of the Critical Reading scores? mean

middle SD

1) 430 ± 115
2) 430 ± 230
3) 496 ± 115
4) 496 ± 230

confidence interval

CI = mean ± 2(standard deviation)

$CI = 496 \pm 2(115)$

$= 496 \pm 230$

→ 360 seconds

2. Elizabeth waited for 6 minutes at the drive thru at her favorite fast-food restaurant the last time she visited. She was upset about having to wait that long and notified the manager. The manager assured her that her experience was very unusual and that it would not happen again. A study of customers commissioned by this restaurant found an approximately normal distribution of results. The mean wait time was 226 seconds and the standard deviation was 38 seconds. Given these data, and using a 95% level of confidence, was Elizabeth's wait time unusual? Justify your answer.

Find confidence interval

$CI = \bar{x} \pm 2s_x$

$CI = 226 + 2(38) = 302$

$226 - 2(38) = 150$

[150, 302]

Yes, 360 was outside the confidence interval

3. Jessica got 20 math problems for homework and complained to her teacher that this was an unusual amount of homework. Her teacher told her to look at the number of questions in all of her past homework assignments from the school year and find the range of the expected number of math problems. She found that the mean was 11.2 and the standard deviation was 3. Was Jessica correct that 20 math problems was unusual? Justify your answer.

Find confidence interval

$CI = \bar{x} \pm 2s_x$

$CI = 11.2 + 2(3) = 17.2$

$11.2 - 2(3) = 5.2$

[5.2, 17.2]

Yes, 20 is outside the confidence interval.

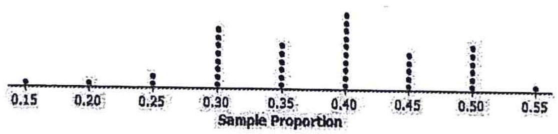
4. A student wanted to decide whether or not a particular coin was fair. She flipped the coin 20 times, calculated the proportion of heads, and repeated this process a total of 40 times. Below is the sampling distribution of sample proportions of heads. The mean and standard deviation of the sampling distribution are 0.379 and 0.091, respectively. Do you think this was a fair coin? Why or why not?

$CI = \bar{x} \pm 2S_x$

$CI = .379 + 2(.091) = .561$

$.379 - 2(.091) = .197$

$[.197, .561]$



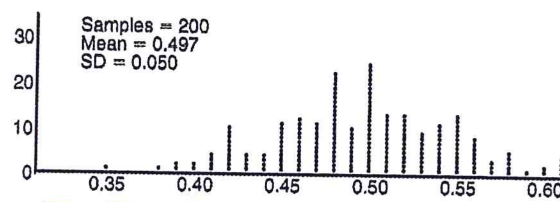
Yes, .5 is inside the confidence interval.

5. Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

$\frac{73}{100} = .73$

Given the results of her coin flips and of her computer simulation, which statement is most accurate?

1) 73 of the computer's next 100 coin flips will be heads.
2) 50 of her next 100 coin flips will be heads.
3) Her coin is not fair. .73 is not in the confidence interval
4) Her coin is fair.

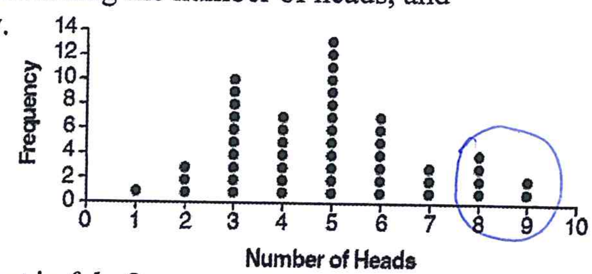


$CI = \bar{x} \pm 2S_x$ $CI = .497 + 2(.050) = .597$

$.497 - 2(.050) = .397$

$[.397, .597]$

6. The results of simulating tossing a coin 10 times, recording the number of heads, and repeating this 50 times are shown in the graph below.



Based on the results of the simulation, which statement is *false*?

1) Five heads occurred most often, which is consistent with the theoretical probability of obtaining a heads.
2) Eight heads is unusual, as it falls outside the middle 95% of the data.
3) Obtaining three heads or fewer occurred 28% of the time.
4) Seven heads is not unusual, as it falls within the middle 95% of the data.

$\frac{6}{50} = 12\%$ $12 > 5$ Inside CI

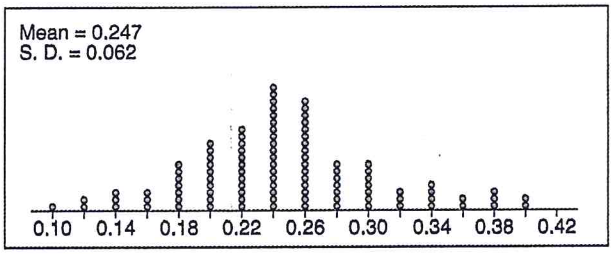
7. Suppose two sets of test scores have the same mean, but different standard deviations, σ_1 and σ_2, with $\sigma_2 > \sigma_1$. Which statement best describes the variability of these data sets?

1) Data set one has the greater variability.
2) Data set two has the greater variability.
3) The variability will be the same for each data set.
4) No conclusion can be made regarding the variability of either set.

The lower the sample size, the higher the standard deviation/variability.

Standard deviation and variability have a positive relationship.

8. Stephen's Beverage Company is considering whether to produce a new brand of cola. The company will launch the product if at least 25% of cola drinkers will buy the product. Fifty cola drinkers are randomly selected to take a blind taste-test of products *A*, *B*, and the new product. Nine out of fifty participants preferred Stephen's new cola to products *A* and *B*. The company then devised a simulation based on the requirement that 25% of cola drinkers will buy the product. Each dot in the graph shown below represents the proportion of people who preferred Stephen's new product, each of sample size 50, simulated 100 times.



Proportion Preferring Stephen's Product

Assume the set of data is approximately normal and the company wants to be 95% confident of its results. Does the sample proportion obtained from the blind taste-test, nine out of fifty, fall within the margin of error developed from the simulation? Justify your answer. The company decides to continue developing the product even though only nine out of fifty participants preferred its brand of cola in the taste-test. Describe how the simulation data could be used to support this decision.

$CI = \bar{x} \pm 2S_x$

$\frac{9}{50} = .18$

$CI = .247 + 2(.062) = .371$

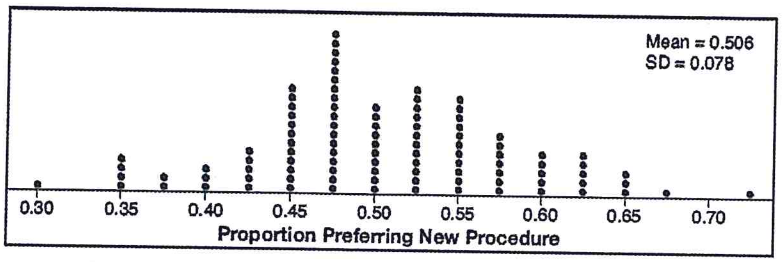
$.247 - 2(.062) = .123$

$[.123, .371]$

yes, .18 is in the confidence interval.

.25 is inside the confidence interval so it is an expected value.

9. Charlie's Automotive Dealership is considering implementing a new check-in procedure for customers who are bringing their vehicles for routine maintenance. The dealership will launch the procedure if 50% or more of the customers give the new procedure a favorable rating when compared to the current procedure. The dealership devises a simulation based on the minimal requirement that 50% of the customers prefer the new procedure. Each dot on the graph below represents the proportion of the customers who preferred the new check-in procedure, each of sample size 40, simulated 100 times.



Assume the set of data is approximately normal and the dealership wants to be 95% confident of its results. Determine an interval containing the plausible sample values for which the dealership will launch the new procedure. Round your answer to the *nearest hundredth.* Forty customers are selected randomly to undergo the new check-in procedure and the proportion of customers who prefer the new procedure is 32.5%. The dealership decides *not* to implement the new check-in procedure based on the results of the study. Use statistical evidence to explain this decision.

$CI = \overline{X} \pm 2S_x$

$= .506 + 2(.078) = .66$ $[.35, .66]$

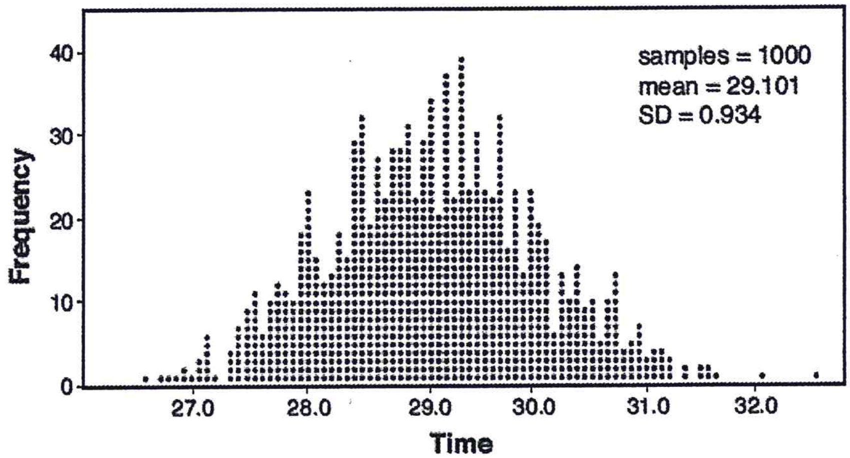
$.506 - 2(.078) = .35$

.325 is not in the confidence interval.

10. A radio station claims to its advertisers that the mean number of minutes commuters listen to the station is 30. The station conducted a survey of 500 of their listeners who commute. The sample statistics are shown below.

$\bar{x}$	29.11
s_x	20.718

A simulation was run 1000 times based upon the results of the survey. The results of the simulation appear below.



Based on the simulation results, is the claim that commuters listen to the station on average 30 minutes plausible? Explain your response including an interval containing the middle 95% of the data, rounded to the *nearest hundredth*.

$CI = \bar{X} \pm 2S_x$

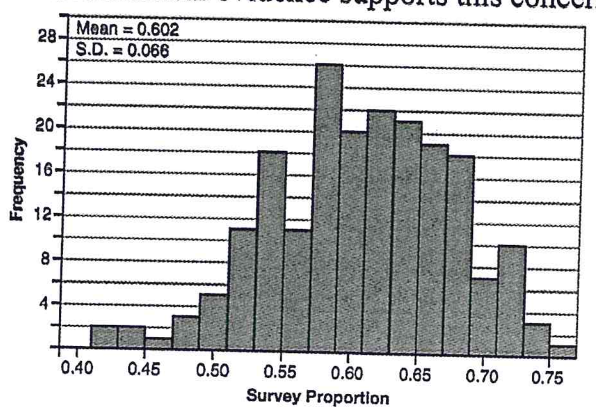
$= 29.101 + 2(.934) = 30.97$ [27.23, 30.97]

$29.101 - 2(.934) = 27.23$

Yes, 30 is inside the confidence interval.

11. Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band. A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.

Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*. Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50%-50% split. Explain what statistical evidence supports this concern.



$CI = \bar{x} \pm 2S_x$

$= .602 + 2(.066) = .73$

$= .602 - 2(.066) = .47$

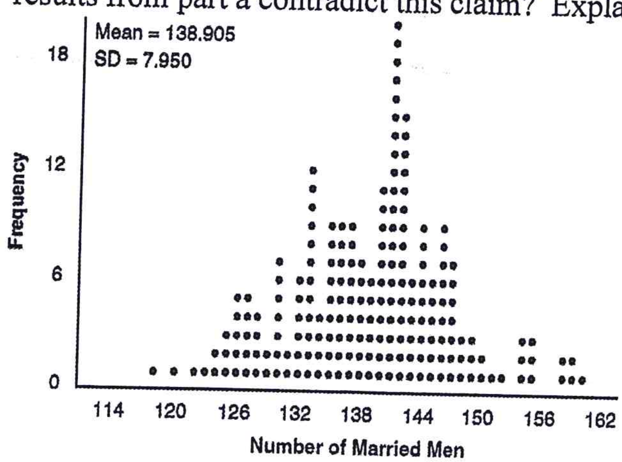
$[.47, .73]$

.5 is inside the confidence interval so 50% is an expected value.

12. In a random sample of 250 men in the United States, age 21 or older, 139 are married. The graph below simulated samples of 250 men, 200 times, assuming that 139 of the men are married.

a) Based on the simulation, create an interval in which the middle 95% of the number of married men may fall. Round your answer to the *nearest integer*.

b) A study claims "50 percent of men 21 and older in the United States are married." Do your results from part a contradict this claim? Explain.



$CI = \bar{x} \pm 2S_x$

$= 138.905 + 2(7.950) = 155$

$= 138.905 - 2(7.950) = 123$

$[123, 155]$

50% of $250 = 125$

No, 125 is inside the confidence interval so 50% is an expected value.

To determine if a treatment is effective

1) **Find the mean difference between the treatment and control group**
2) Rerandomize the sample many times and record the mean differences on a dot plot

If the mean difference falls within the confidence interval (more than 5%), the treatment is not effective.

If the mean difference falls outside the confidence interval (less than 5%), the treatment is effective.

1. Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year. A summary of the two groups' final grades is shown below:

$80.16 - 83.8 = -3.64$

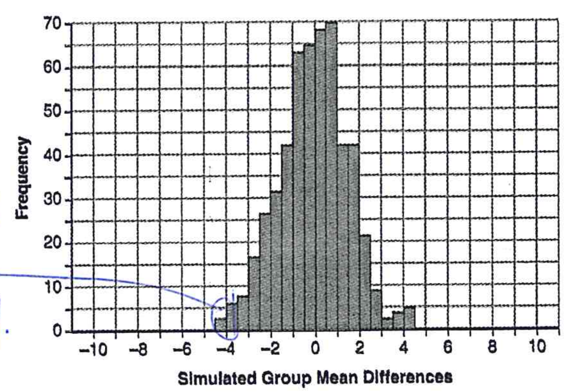
	Group 1	Group 2
$\bar{x}$	80.16	83.8
S_x	6.9	5.2

On average, students in group 1 scored 3.64 points lower than group 2.

Calculate the mean difference in the final grades (group 1 – group 2) and explain its meaning in the context of the problem. A simulation was conducted in which the students' final grades were rerandomized 500 times. The results are shown below.

$6+3 = \frac{9}{500} = .018$

1.8%



Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.

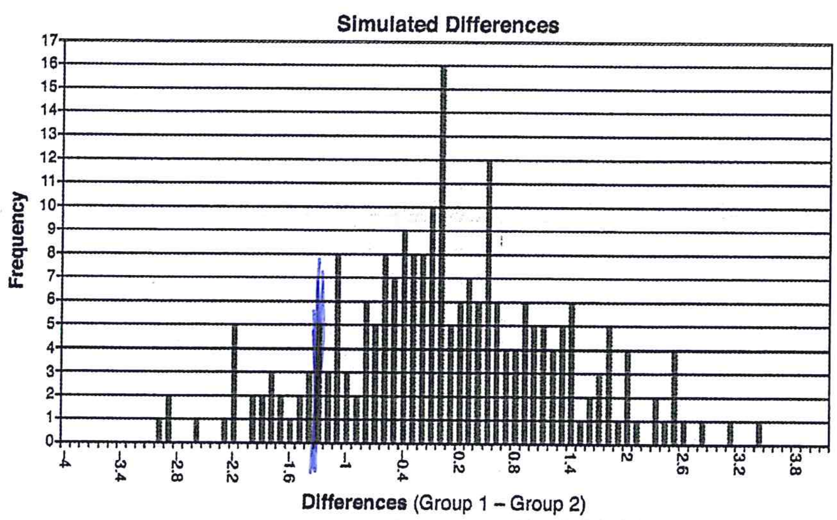
Yes, −3.64 or more extreme occured in less than 5% of the simulations so it is outside of the confidence interval.

2. Ayva designed an experiment to determine the effect of a new energy drink on a group of 20 volunteer students. Ten students were randomly selected to form group 1 while the remaining 10 made up group 2. Each student in group 1 drank one energy drink, and each student in group 2 drank one cola drink. Ten minutes later, their times were recorded for reading the same paragraph of a novel. The results of the experiment are shown below.

Group 1 (seconds)	Group 2 (seconds)
17.4	23.3
18.1	18.8
18.2	22.1
19.6	12.7
18.6	16.9
16.2	24.4
16.1	21.2
15.3	21.2
17.8	16.3
19.7	14.5
Mean = 17.7	Mean = 19.1

group 2 had some very fast readers

Ayva thinks drinking energy drinks makes students read faster. Using information from the experimental design or the results, explain why Ayva's hypothesis may be *incorrect*. Using the given results, Ayva randomly mixes the 20 reading times, splits them into two groups of 10, and simulates the difference of the means 232 times.



Ayva has decided that the difference in mean reading times is not an unusual occurrence. Support her decision using the results of the simulation. Explain your reasoning.

mean difference = 17.7 − 19.1
= −1.4

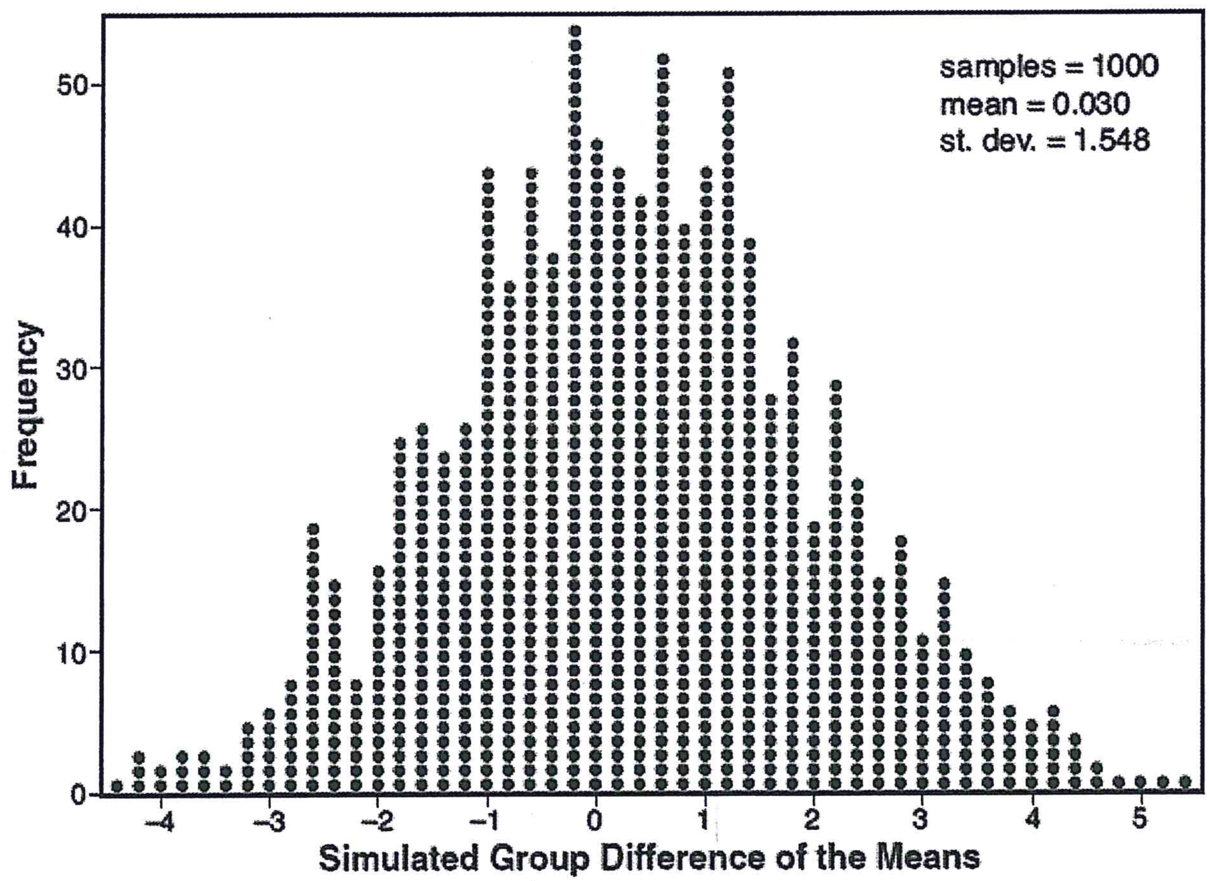
−1.4 or more occured more than 5% of the time. −1.4 is inside the confidence interval.

3. Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

Mean difference
$23-18=5$

	Scented Paper	Unscented Paper
$\bar{x}$	23	18
s_x	2.898	2.408

Calculate the difference in means in the experimental test grades (scented -unscented). A simulation was conducted in which the subjects' scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.



Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the *nearest hundredth*. Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.

$CI = \bar{x} \pm 2S_x$

$.03 + 2(1.548) = 3.13$

$.03 - 2(1.548) = -3.07$

$[-3.07, 3.13]$

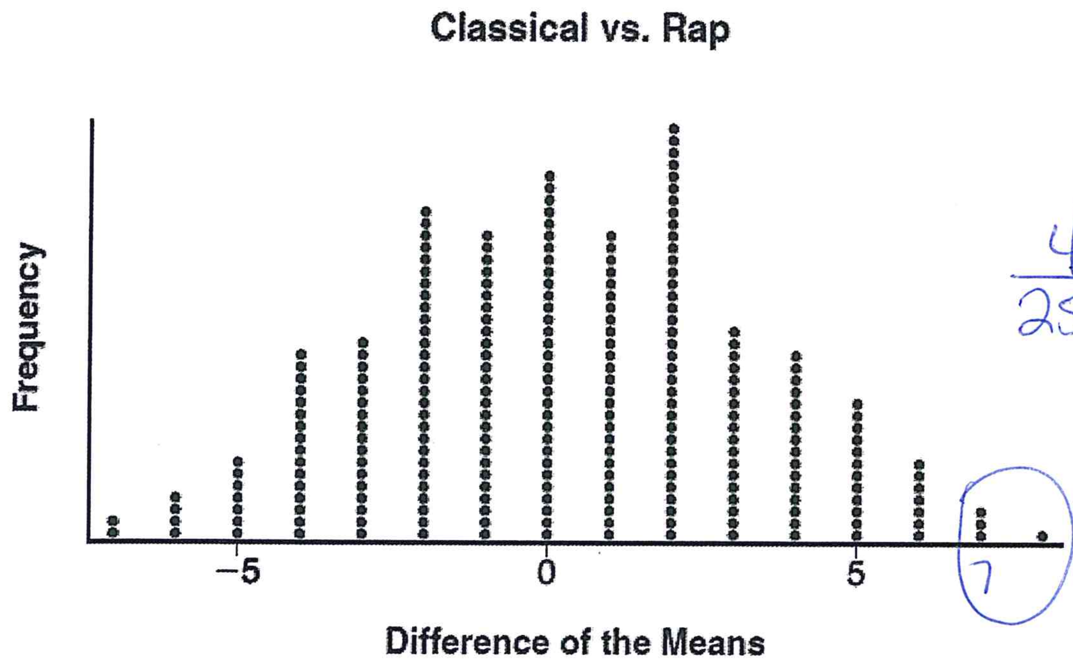
yes, 5 is not inside the confidence interval

4. To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

Classical: 74, 83, 77, 77, 84, 82, 90, 89
Rap: 77, 80, 78, 74, 69, 72, 78, 69

He found the average of the two groups and subtracted them. On average, the students listening to classical music scored 7 points higher than those listening to rap.

John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer. To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.



$\frac{4}{250} = .016$

1.6%

Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.

Yes, 7 occurred in less than 5% of the simulations. 7 is not in the confidence interval.

Exponential Regression Equations * make sure you write an equation with a y and an x

1) **Stat, Edit**
2) **Input x column into L1 and y column into L2**
3) **Stat, Calc, 0: ExpReg**
4) **READ AND ROUND CAREFULLY**

ExpReg

1. The accompanying table shows the number of bacteria present in a certain culture over a 5-hour period, where x is the time, in hours, and y is the number of bacteria.

x	y
0	1,000
1	1,049
2	1,100
3	1,157
4	1,212
5	1,271

ExpReg

$y = a(b)^x$

$y = 999.9725(1.0493)^x$

$y = 999.9725(1.0493)^{6.5}$

$y = 1367$

Write an exponential regression equation for this set of data, rounding all values to *four decimal places*. Using this equation, determine the number of whole bacteria present after 6.5 hours.

2. The accompanying table shows the amount of water vapor, y, that will saturate 1 cubic meter of air at different temperatures, x.

Amount of Water Vapor That Will Saturate 1 Cubic Meter of Air at Different Temperatures

Air Temperature (x) (°C)	Water Vapor (y) (g)
−20	1
−10	2
0	5
10	9
20	17
30	29
40	50

ExpReg

$y = a(b)^x$

$y = 4.194(1.068)^x$

$y = 4.194(1.068)^{50}$

$y = 112.5$

Write an exponential regression equation for this set of data, rounding all values to the *nearest thousandth*. Using this equation, predict the amount of water vapor that will saturate 1 cubic meter of air at a temperature of 50°C, and round your answer to the *nearest tenth of a gram*.

3. Jean invested \$380 in stocks. Over the next 5 years, the value of her investment grew, as shown in the accompanying table.

Years Since Investment (x)	Value of Stock, in Dollars (y)
0	380
1	395
2	411
3	427
4	445
5	462

Write the exponential regression equation for this set of data, rounding all values to *two decimal places*. Using this equation, find the value of her stock, to the *nearest dollar*, 10 years after her initial purchase.

ExpReg

$y = a(b)^x$

$y = 379.92(1.04)^x$

$y = 379.92(1.04)^{10} = 562$

Complex Formulas

List what each variable represents and carefully substitute the appropriate values in

1. After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + (T_0 - T_a)e^{-kt}$$

T_a = the temperature surrounding the object 325

T_0 = the initial temperature of the object 68

t = the time in hours 7

T = the temperature of the object after t hours T

k = decay constant .066

~~The turkey reaches the temperature of approximately 100° F after 2 hours.~~ If the value of k is .066, write an equation to determine the temperature of the turkey after t hours. Determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m. 8AM-3PM = 7 hours

$T = 325 + (68-325)e^{-.066t}$

$T = 325 + (68-325)e^{-.066(7)}$

$T = 163°$

2. The loudness of sound is measured in units called decibels (dB). These units are measured by first assigning an intensity I_0 to a very soft sound that is called the threshold sound. The sound to be measured is assigned an intensity, I, and the decibel rating, d, of this sound is found using $d = 10\log\frac{I}{I_0}$. The threshold sound audible to the average person is 1.0×10^{-12} W/m² (watts per square meter). Consider the following sound level classifications:

I_0

Moderate	45–69 dB
Loud	70–89 dB
Very loud	90–109 dB
Deafening	>110 dB

I_0 · threshold
I intensity
d decibal rating

How would a sound with intensity 6.3×10^{-3} W/m² be classified?
1) moderate
2) loud
3) very loud
4) deafening

I

$d = 10\log\frac{I}{I_0}$

$d = 10\log\frac{6.3\times10^{-3}}{1.0\times10^{-12}}$

$d = 97.9934..$

s_0 $\quad$ V_0

3. A baseball is hit straight up from a height of 6 feet with an initial velocity of 90 feet per second. The equation that models the height of the ball, s, as a function of time, t, is $s = -16t^2 + v_0 t + s_0$ where v_0 is the initial velocity and s_0 is the initial height. How high is the ball after 4 seconds?

t

$S = -16(4)^2 + 90(4) + 6$

$S =$ ~~110 feet~~ 110 feet

t

4. Kayla needs \$30,000 for a down payment on a house she plans to purchase in 8 years. She decides to invest in a savings account which gets 3.5% interest, compounded at the end of each year. Assume she make the same deposit on January 1st each of the 8 years and makes no other deposits or withdrawals throughout the year. Use the formula, $A = \frac{d}{r}((1+r)^t - 1)$ where A is the amount of money in the account after t years, d is the number of dollars invested at the beginning of each year, and r is the annual interest rate of the account, expressed as a decimal. How much money should Kayla put in the account at the beginning of each year to reach her goal?

$A = 30{,}000$

$t = 8$

$r = .035$

$d = d$

$30{,}000 = \frac{d}{.035}((1.035)^8 - 1)$

$\frac{30{,}000}{9.05\ldots} = \frac{9.05\ldots d}{9.05\ldots}$

$3314.30 = d$

5. A formula for work problems involving two people is shown below.

$$\frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{t_b}$$

t_1 = the time taken by the first person to complete the job

t_2 = the time taken by the second person to complete the job

t_b = the time it takes for them working together to complete the job

Fred and Barney are carpenters who build the same model desk. It takes Fred eight hours to build the desk while it only takes Barney six hours. Write an equation that can be used to find the time it would take both carpenters working together to build a desk. Determine, to the *nearest tenth of an hour*, how long it would take Fred and Barney working together to build a desk.

t_1 $\quad$ t_2

$6\ 48t_b$ $\quad$ $8\ 48t_b$

$\left(\frac{1}{8}\right) + \left(\frac{1}{6}\right) = \left(\frac{1}{t_b}\right) 48t_b$ multiply by LCD

$6t_b + 8t_b = 48$

$\frac{14t_b}{14} = \frac{48}{14}$

$t_b = 3.4$ hours

Profit

Profit = amount made – amount spent
Profit = revenue – cost
To find profit, revenue function – cost function
*Keep, change, change when subtracting polynomials

1. The profit function, $p(x)$, for a company is the cost function, $c(x)$, subtracted from the revenue function, $r(x)$. The profit function for the Acme Corporation is $p(x) = -0.5x^2 + 250x - 300$ and the revenue function is $r(x) = -0.3x^2 + 150x$. The cost function for the Acme Corporation is

1) $c(x) = 0.2x^2 - 100x + 300$
2) $c(x) = 0.2x^2 + 100x + 300$
3) $c(x) = -0.2x^2 + 100x - 300$
4) $c(x) = -0.8x^2 + 400x - 300$

$-0.3x^2 + 150x$
$+\ 0.5x^2 - 250x + 300$
$.2x^2 - 100x + 300$

$p(x) = r(x) - c(x)$
$+c(x) \qquad +c(x)$
$c(x) + p(x) = r(x)$
$-p(x) \qquad -p(x)$

$c(x) = r(x) - p(x)$
$c(x) = (-0.3x^2 + 150x) - (-0.5x^2 + 250x - 300)$
*you can use MC strategy at this point.

2. A manufacturing company has developed a cost model, $C(x) = 0.15x^3 + 0.01x^2 + 2x + 120$, where x is the number of items sold, in thousands. The sales price can be modeled by $S(x) = 30 - 0.01x$. Therefore, revenue is modeled by $R(x) = x \bullet S(x)$. The company's profit, $P(x) = R(x) - C(x)$, could be modeled by

1) $0.15x^3 + 0.02x^2 - 28x + 120$
2) $-0.15x^3 - 0.02x^2 + 28x - 120$
3) $-0.15x^3 + 0.01x^2 - 2.01x - 120$
4) $-0.15x^3 + 32x + 120$

$R(x) = x(30 - 0.01x)$

$P(x) = R(x) - C(x)$
$P(x) = x(30 - 0.01x) - (0.15x^3 + .01x^2 + 2x + 120)$
$P(x) = (30x - .01x^2) - (0.15x^3 + .01x^2 + 2x + 120)$

you can do MC strategy at this point

$30x - .01x^2$
$+\ -120 \quad -2x \quad -.01x^2 \quad -0.15x^3$
$-120 + 28x - .02x^2 - 0.15x^3$

3. A major car company analyzes its revenue, $R(x)$, and costs $C(x)$, in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years, x, using the given functions.

$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$
$$C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$$

The company's profits can be represented as the difference between its revenue and costs. Write the profit function, $P(x)$, as a polynomial in standard form.

$P(x) = R(x) - C(x)$
$P(x) = (550x^3 - 12,000x^2 + 83,000x + 7000) - (880x^3 - 21,000x^2 + 150,000x - 160,000)$

$550x^3 - 12,000x^2 + 83,000x + 7000$
$+\ -880x^3 + 21,000x^2 - 150,000x + 160,000$

$P(x) = -330x^3 + 9000x^2 - 67,000x + 167,000$