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Common Core Algebra II Regents Review Packet!

Mr. Schlansky

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Multiple Choice Strategy with Variables
If variables in the problems and answers:
10 STO → X, 15 STO → Y
Type in original problem, write down the value.
Type in each choice, write down the value.
If they match up, they are equivalent.
Check all four choices as more than one may be equivalent!

1. The expression $\frac{6x^3+17x^2+10x+2}{2x+3}$ equals $\frac{7802}{23}$

[✓ 1)] $3x^2+4x-1+\frac{5}{2x+3}$ $\frac{7802}{23}$ 3) $6x^2-x+13-\frac{37}{2x+3}$
2) $6x^2+8x-2+\frac{5}{2x+3}$ 4) $3x^2+13x+\frac{49}{2}+\frac{151}{2x+3}$

2. The expression $\frac{4x^3+5x+10}{2x+3}$ is equivalent to $\frac{4060}{23}$

1) $2x^2+3x-7+\frac{31}{2x+3}$ 3) $2x^2+2.5x+5+\frac{15}{2x+3}$
[✓ 2)] $2x^2-3x+7-\frac{11}{2x+3}$ $\frac{4060}{23}$ 4) $2x^2-2.5x-5-\frac{20}{2x+3}$

3. What is the completely factored form of $k^4-4k^2+8k^3-32k+12k^2-48$? 18432
1) $(k-2)(k-2)(k+3)(k+4)$ 3) $(k+2)(k-2)(k+3)(k+4)$
2) $(k-2)(k-2)(k+6)(k+2)$ [✓ 4)] $(k+2)(k-2)(k+6)(k+2)$ 18432

4. When factored completely, the expression $3x^3-5x^2-48x+80$ is equivalent to 2100
1) $(x^2-16)(3x-5)$ 2100 [✓ 3)] $(x+4)(x-4)(3x-5)$ 2100
2) $(x^2+16)(3x-5)(3x+5)$ 4) $(x+4)(x-4)(3x-5)(3x-5)$

not factored completely

5. Given i is the imaginary unit, $(2-yi)^2$ in simplest form is $-96-40i$
1) $y^2-4yi+4$ 3) $-y^2+4$
[✓ 2)] $-y^2-4yi+4$ 4) y^2+4
$-96-40i$

6. The expression $(x+i)^2-(x-i)^2$ is equivalent to $40i$
1) 0 3) -2
2) $-2+4xi$ [✓ 4)] $4xi$ $40i$

7. The expression $6xi^3(-4xi+5)$ is equivalent to $-2400-300i$
1) $2x-5i$ 3) $-24x^2+30x-i$
[✓ 2)] $-24x^2-30xi$ 4) $26x-24x^2i-5i$

$-2400-300i$

8. The expression $\frac{a^2b^{-3}}{a^{-4}b^2}$ $\frac{320}{243}$ is equivalent to

[✓ 1)] $\frac{a^6}{b^5}$ $\frac{320}{243}$ 3) $\frac{a^2}{b}$

2) $\frac{b^5}{a^6}$ 4) $a^{-2}b^{-1}$

9. Which expression is equivalent to $\frac{x^{-1}y^2}{x^2y^{-4}}$? $\frac{91125}{8}$

1) $\frac{x}{y^2}$ 2) $\frac{x^3}{y^6}$ 3) $\frac{y^2}{x}$ [✓ 4)] $\frac{y^6}{x^3}$ $\frac{91125}{8}$

10. What is the product of $\sqrt[3]{4a^2b^4}$ and $\sqrt[3]{16a^3b^2}$? 41774.. math: 4

[✓ 1)] $4ab^2(\sqrt[3]{a^2})$ 41774.. 3) $8ab^2\sqrt[3]{a^2}$

2) $4a^2b^3\sqrt[3]{a}$ 4) $8a^2b^3\sqrt[3]{a}$

11. The expression $\sqrt[4]{16x^2y^7}$ 723.. → 4 math: 5 is equivalent to

[✓ 1)] $2x^{\frac{1}{2}}y^{\frac{7}{4}}$ 723... 3) $4x^{\frac{1}{2}}y^{\frac{7}{4}}$

2) $2x^8y^{28}$ 4) $4x^8y^{28}$

12. For positive values of x, which expression is equivalent to $\sqrt{16x^2}\bullet x^{\frac{2}{3}}+\sqrt[3]{8x^5}$ 278..

[✓ 1)] $6(\sqrt[3]{x^5})$ 278.. 3) $4\sqrt[3]{x^2}+2\sqrt[3]{x^5}$

2) $6(\sqrt[5]{x^3})$ 4) $4\sqrt{x^3}+2\sqrt[5]{x^3}$

13. Written in simplest form, $\frac{c^2-d^2}{d^2+cd-2c^2}$ $-\frac{5}{7}$ where $c\neq d$, is equivalent to

1) $\frac{c+d}{d+2c}$ [✓ 3)] $\frac{-c-d}{d+2c}$ $-\frac{5}{7}$

2) $\frac{c-d}{d+2c}$ 4) $\frac{-c+d}{d+2c}$

14. The expression $\frac{-3x^2-5x+2}{x^3+2x^2}$ $-\frac{29}{100}$ can be rewritten as

1) $\frac{-3x-3}{x^2+2x}$ 3) $-3x^{-1}+1$

2) $\frac{-3x-1}{x^2}$ [✓ 4)] $-3x^{-1}+x^{-2}$ $-\frac{29}{100}$

Comparing Expressions
Use Multiple Choice Strategy with Variables for each option

1. Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?

I $(m+p)^2 = m^2 + 2mp + p^2$ 625=625 ✓

II $(x+y)^3 = x^3 + 3xy + y^3$ 15625 ≠ 4825 ✗

III $(a^2+b^2)^2 = (a^2-b^2)^2 + (2ab)^2$ 105625=105625 ✓

1) I, only 3) II and III
2) I and II [✓ 4) I and III]

3

2. Which expression(s) are equivalent to $\frac{x^2-4x}{2x}$, where $x \neq 0$? 3

3 3

I. $\frac{x}{2} - 2$ II. $\frac{x-4}{2}$ III. $\frac{x-1}{2} - \frac{3}{2}$

1) II, only
2) I and II
3) II and III
[✓ 4) I, II, and III]

3. Which of the following identities hold true for all real numbers?

$I.\ (x^2+2y)^2 = x^4 + 4x^2y + 4y^2$ 16900 = 16900 ✓

$II.\ (x^2-4z^3)(x^2+4z^3) = x^4 - 16z^6$ -182240000 = -182240000 ✓

$III.\ (x+y)(x^2-xy-y^2) = x^3 - y^3$ -6875 ≠ -2375 ✗

1) I, only 3) II and III only
[✓ 2) I and II only] 4) I and III only

4. Which factorizations are correct?

$I.\ a^3 + 27b^3 = (a+3b)(a^2-3ab+9b^2)$ 92125=92125 ✓

$II.\ c^3 - 6c^2 + 8c + 5c^2 - 30c + 40 = (c-2)(c-4)(c+5)$ 720=720 ✓

$III.\ 1 - x^4 = (1+x)^2(1-x)^2$ -9999 ≠ 9801 ✗

1) I, only 3) II and III only
[✓ 2) I and II only] 4) I, II, and III

5. Which factorization is *incorrect*?

1) $4k^2 - 49 = (2k+7)(2k-7)$ 351=351 ✓
2) $a^3 - 8b^3 = (a-2b)(a^2+2ab+4b^2)$ -26000 = -26000 ✓
[✓ 3)] $m^3 + 3m^2 - 4m + 12 = (m-2)^2(m+3)$ 1272 ≠ 6656 ✗
4) $t^3 + 5t^2 + 6t + t^2 + 5t + 6 = (t+1)(t+2)(t+3)$ 1716=1716 ✓

6. Which expression has been rewritten correctly to form a true statement?

[✓ 1)] $(x+2)^2 + 2(x+2) - 8 = (x+6)x$ 160=160 ✓ 3) $x^3 + 3x^2 - 4xy^2 - 12y^2 = (x-2y)(x+3)^2$ -10400 ≠ -3380

2) $x^4 + 4x^2 + 9x^2y^2 - 36y^2 = (x+3y)^2(x-2)^2$ 4) $(x^2-4)^2 - 5(x^2-4) - 6 = (x^2-7)(x^2-6)$ 8130 ≠ 8742

204800 ≠ 193600

Multiple Choice Strategy with Equations

-Store each potential answer (_____ STO → X)

-Type in equation

-1 is correct, 0 is incorrect

*Be sure to check all potential answers as most equations have multiple answers

1. The solution set of the equation $\sqrt{x+3} = 3 - x$ is

[✓ 1) $\{1\}$] 1 STO→X 1 $2=2$ ✓

2) $\{0\}$

3) $\{1, 6\}$ 6 STO→X 0 $3=-3$ ✗

4) $\{2, 3\}$

2. What is the solution set for the equation $\sqrt{5x + 29} = x + 3$?

[✓ 1) $\{4\}$] 4 STO→X 1 $7=7$ ✓

2) $\{-5\}$

3) $\{4, 5\}$ 5 STO→X 0 $\sqrt{54} \neq 8$ ✗

4) $\{-5, 4\}$ −5 STO→X 0 $2 \neq -2$ ✗

3. The solution set of $\sqrt{3x + 16} = x + 2$ is

1) $\{-3, 4\}$

2) $\{-4, 3\}$ −4 STO→ 0 $2 \neq -2$ ✗

[✓ 3) $\{3\}$] 3 STO→X 1 $5=5$ ✓

4) $\{-4\}$

4. The solution set of the equation $\sqrt{2x - 4} = x - 2$ is

1) $\{-2, -4\}$

[✓ 2) $\{2, 4\}$] 2 STO→X 1 $0=0$ ✓
4 STO→X 1 $2=2$ ✓

3) $\{4\}$

4) $\{\ \}$

5. What is the solution set of the equation $\frac{30}{x^2 - 9} + 1 = \frac{5}{x - 3}$?

1) $\{2,3\}$ 3 STO→X ERR

[✓ 2) $\{2\}$] 2 STO→X 1 $-5=-5$ ✓

3) $\{3\}$

4) $\{\ \}$

6. What is the solution set of the equation $\frac{3x+25}{x+7} - 5 = \frac{3}{x}$?

1) $\left\{\frac{3}{2}, 7\right\}$

2) $\left\{\frac{7}{2}, -3\right\}$

3) $\left\{-\frac{3}{2}, 7\right\}$

[✓ 4)] $\left\{-\frac{7}{2}, -3\right\}$ $-\frac{7}{2}$ STO→X $\frac{-6}{7}=\frac{-6}{7}$ ✓

-3 STO→X -1=-1 ✓

7. The solution set for the equation $\sqrt{56-x} = x$ is

1) $\{-8, 7\}$ -8 STO→X 8=-8 ✗

2) $\{-7, 8\}$

[✓ 3)] $\{7\}$ 7 STO→X 7=7 ✓

4) $\{\ \}$

8. Which is the solution to: $2(3)^{4x} + 1 = 11$?

[✓ 1)] $\frac{\log 5}{4\log 3}$ →X 11=11 ✓

3) $\frac{\log 3}{4\log 5}$

2) $\frac{4\log 5}{\log 3}$

4) $\frac{4\log 3}{\log 5}$

9. Which is the solution to: $256 + 4(2)^{6x} = 2700$?

1) $\frac{\ln 4}{6\ln 2}$

[✓ 3)] $\frac{\ln 611}{6\ln 2}$ →X 2700=2700 ✓

2) $\frac{6\ln 423}{\ln 4}$

4) $\frac{6\ln 2444}{\ln 4}$

10. Which is the solution to: $1 - 2(5)^{2x} = -5$?

1) $\frac{\ln 6}{2\ln 3}$

3) $\frac{2\ln 4}{\ln 3}$

2) $\frac{2\ln 5}{\ln 1}$

[✓ 4)] $\frac{\ln 3}{2\ln 5}$ →X -5=-5 ✓

Open Response Equations

1) Type in left hand side into Y1
2) Type in right hand side into Y2
3) Adjust window (if necessary)
4) 2nd Trace (Calc), 5: Intersect
5) The solution is the x value of the intersection

*You may want to divide both sides at the beginning to make the values smaller

1. Solve for all values of x: $\sqrt{x-5}+x=7$

Y1 Y2

Intersect

$x=6$

window's good

2. What is the solution set for the equation $\sqrt{56-x}=x$?

Y1 Y2

Intersect

$x=7$

window's good

3. What is the solution set for the equation $\sqrt{5x+29}=x+3$?

Y1 Y2

Intersect

$x=4$

window's good

4. Solve algebraically for x: $\sqrt{x^2+x-1}+11x=7x+3$

Y1 Y2

Intersect

$x=.\overline{6}$

window's good

5. What is the solution set of the equation $\frac{30}{x^2-9}+1=\frac{5}{x-3}$?

Y1 Y2

Intersect

$x=2$

window's good

6. What is the solution set of the equation $\frac{3x+25}{x+7}-5=\frac{3}{x}$?

Y1 Y2

Intersect

$x=-3$

window's good

7. What is the solution, if any, of the equation $\frac{2}{x+3}-\frac{3}{4-x}=\frac{2x-2}{x^2-x-12}$?

Y1 Y2

Intersect

$x=-1$

window's good

Y1 Y2

8. Solve for x: $\frac{1}{x}-\frac{1}{3}=-\frac{1}{3x}$ Intersect

window's good but hard to see. I would ZBOX

$x=4$

9. Solve the equation $2x^3 - x^2 - 8x = 4$ for all values of x.

Window's good Y1 | Y2 Intersect

$X = -2$
$X = .5$
$X = 2$

10. Solve for x: $x^3 + x^2 = 4x + 4$

Window's good Y1 | Y2 Intersect

$x = -2$
$x = -1$
$x = 2$

11. Solve for x: $x^3 - 2x^2 = x - 2$

Window's good Y1 | Y2 Intersect

$x = -1$
$x = 1$
$x = 2$

12. Solve for x and round your answer to the nearest thousandth: $\frac{1}{2}(1.8)^x = 7.5$

Window's good Y1 | Y2 Intersect $x = 4.607$

13. Solve for x and round your answer to the nearest thousandth: $2\left(\frac{1}{3}\right)^x = 4$

Window's good Y1 | Y2 Intersect $x = -.631$

14. Solve for x and round your answer to the nearest thousandth: $1 - 2(3)^{2x} = -5$

Window's good Y1 | Y2 Intersect $x = .5$

15. Solve $x^3 + 5x^2 = 4x + 20$.

adjust y max Y1 | Y2 Intersect

$x = -5$
$x = -2$
$x = 2$

16. Solve for all values of x: $x^4 + 4x^3 + 4x^2 = -16x$

adjust y max Y1 | Y2 Intersect

$X = 0$
$X = -4$

17. Solve for x and round your answer to the nearest hundredth: $4^x - 5 = 12$

adjust y max Y1 | Y2 Intersect $x = 2.04$

18. Solve for x and round your answer to the nearest hundredth: $8 + 2(4)^{x-5} = 14$

adjust y max Y1 | Y2 Intersect $x = 5.79$

Profit

Profit = revenue – cost, $p(x) = r(x) - c(x)$

Net worth = value of accounts - debt

*Keep, change, change when subtracting polynomials

*you can use mc strategy once it's set up

1. Mr. Schlansky's tutoring revenue can be represented by $r(x) = 25x^2 - 90x + 14$ and his costs can be represented by $c(x) = 12x^2 + 21x + 10$. If his profit can be determined using $p(x) = r(x) - c(x)$, write a polynomial function what would represent $p(x)$.

$p(x) = (25x^2 - 90x + 14) - (12x^2 + 21x + 10)$

$$\begin{array}{r} 25x^2 - 90x + 14 \\ -12x^2 - 21x - 10 \\ \hline \end{array}$$

$p(x) = 13x^2 - 111x + 4$

2. Stone Manufacturing has developed a cost model, $C(x) = 0.18x^3 + 0.02x^2 + 4x + 180$, where x is the number of sprockets sold, in thousands. The sales price can be modeled by $S(x) = 95.4 - 6x$ and the company's revenue by $R(x) = x \bullet S(x)$. The company's profits, $R(x) - C(x)$, could be modeled by

$x(95.4 - 6x)$

1) $0.18x^3 + 6.02x^2 + 91.4x + 180$
2) $0.18x^3 - 5.98x^2 - 91.4x + 180$
[✓ 3)] $-0.18x^3 - 6.02x^2 + 91.4x - 180$
4) $0.18x^3 + 5.98x^2 + 99.4x + 180$

$x(95.4 - 6x) - (.18x^3 + .02x^2 + 4x + 180)$

$95.4x - 6x^2 - .18x^3 - .02x^2 - 4x - 180$

$-.18x^3 - 6.02x^2 + 91.4x - 180$

3. Chet has \$1200 invested in a bank account modeled by the function $P(n) = 1200(1.002)^n$, where $P(n)$ is the value of his account, in dollars, after n months. Chet's debt is modeled by the function $Q(n) = 100n$, where $Q(n)$ is the value of debt, in dollars, after n months. After n months, which function represents Chet's net worth, $R(n)$?

1) $R(n) = 1200(1.002)^n + 100n$
2) $R(n) = 1200(1.002)^{12n} + 100n$
[✓ 3)] $R(n) = 1200(1.002)^n - 100n$
4) $R(n) = 1200(1.002)^{12n} - 100n$

$R(n) = P(n) - Q(n)$

$R(n) = 1200(1.002)^n - 100n$

4. A manufacturing company has developed a cost model, $C(x) = 0.15x^3 + 0.01x^2 + 2x + 120$, where x is the number of items sold, in thousands. The sales price can be modeled by $S(x) = 30 - 0.01x$. Therefore, revenue is modeled by $R(x) = x \bullet S(x)$. The company's profit, $P(x) = R(x) - C(x)$, could be modeled by

$x(30-.01x)$

1) $0.15x^3 + 0.02x^2 - 28x + 120$
[✓ 2)] $-0.15x^3 - 0.02x^2 + 28x - 120$
3) $-0.15x^3 + 0.01x^2 - 2.01x - 120$
4) $-0.15x^3 + 32x + 120$

$x(30-.01x) - (.15x^3+.01x^2+2x+120)$

$30x-.01x^2-.15x^3-.01x^2-2x-120$

$-.15x^3-.02x^2+28x-120$

5. A major car company analyzes its revenue, $R(x)$, and costs $C(x)$, in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years, x, using the given functions.

$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$
$$C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$$

The company's profits can be represented as the difference between its revenue and costs. Write the profit function, $P(x)$, as a polynomial in standard form.

$(550x^3-12,000x^2+83,000x+7000)-(880x^3-21,000x^2+150,000x-160,000)$

$550x^3-12,000x^2+83,000x+7000$

$-880x^3+21,000x^2-150,000x+160,000$

$-330x^3+9,000x^2-67,000x+167,000$

6. The profit function, $p(x)$, for a company is the cost function, $c(x)$, subtracted from the revenue function, $r(x)$. The profit function for the Acme Corporation is $p(x) = -0.5x^2 + 250x - 300$ and the revenue function is $r(x) = -0.3x^2 + 150x$. The cost function for the Acme Corporation is

[✓ 1)] $c(x) = 0.2x^2 - 100x + 300$
2) $c(x) = 0.2x^2 + 100x + 300$
3) $c(x) = -0.2x^2 + 100x - 300$
4) $c(x) = -0.8x^2 + 400x - 300$

$p(x) = r(x) - c(x)$

$c(x) = r(x) - p(x)$

$=(-0.3x^2+150x) - (-.5x^2+250x-300)$

$-.3x^2+150x$

$.5x^2-250x+300$

$+.2x^2-100x+300$

Dividing Polynomials: (Synthetic Division)
Negative the value of what you are dividing by and put it outside
Bring the first number down
Multiply, Add, Multiply, Add, etc.
Decrease the first terms exponent by 1, the last number is the remainder. The remainder goes over the divisor.
(Put 0 as a placeholder if necessary)

Divide each of the following polynomials

1. $\dfrac{2x^3+5x^2-31x-84}{x+3}$

$$\begin{array}{r|rrrr} -3 & 2 & 5 & -31 & -84 \\ & & -6 & 3 & 84 \\ \hline & 2 & -1 & -28 & 0 \end{array}$$

$2x^2-x-28$

2. $\dfrac{x^4-2x^2-7x+12}{x+6}$

$$\begin{array}{r|rrrrr} -6 & 1 & 0 & -2 & -7 & 12 \\ & & -6 & 36 & 204 & -1182 \\ \hline & 1 & -6 & 34 & 197 & -1170 \end{array}$$

$x^3-6x^2+34x+197-\frac{1170}{x+6}$

3. $\dfrac{x^3+5x^2-1}{x+2}$

$$\begin{array}{r|rrrr} -2 & 1 & 5 & 0 & -1 \\ & & -2 & -6 & 12 \\ \hline & 1 & 3 & -6 & 11 \end{array}$$

$x^2+3x-6+\frac{11}{x+2}$

4. $\dfrac{4x^3+12x^2-5}{x+5}$

$$\begin{array}{r|rrrr} -5 & 4 & 12 & 0 & -5 \\ & & -20 & 40 & -200 \\ \hline & 4 & -8 & 40 & -205 \end{array}$$

$4x^2-8x+40-\frac{205}{x+5}$

5. $\dfrac{6x^3-5x+3}{x-3}$

$$\begin{array}{r|rrrr} 3 & 6 & 0 & -5 & 3 \\ & & 18 & 54 & 147 \\ \hline & 6 & 18 & 49 & 150 \end{array}$$

$6x^2+18x+49+\frac{150}{x-3}$

6. $\dfrac{5x^3-60}{x-2}$

$$\begin{array}{r|rrrr} 2 & 5 & 0 & 0 & -60 \\ & & 10 & 20 & 40 \\ \hline & 5 & 10 & 20 & -20 \end{array}$$

$5x^2+10x+20-\frac{20}{x-2}$

7. $\dfrac{x^2+x-4}{x-3}$

$$\begin{array}{r|rrr} 3 & 1 & 1 & -4 \\ & & 3 & 12 \\ \hline & 1 & 4 & 8 \end{array}$$

$x+4+\frac{8}{x-3}$

8. $\dfrac{-3x^2+10x-6}{x+1}$

$$\begin{array}{r|rrr} -1 & -3 & 10 & -6 \\ & & 3 & -13 \\ \hline & -3 & 13 & -19 \end{array}$$

$-3x+13-\frac{19}{x+1}$

To determine if a binomial is a factor:
Find the remainder! (Use remainder theorem)
If the remainder is 0, it is a factor
If the remainder is not 0, it is not a factor

1. Is $x-6$ a factor of $p(x)=x^3-6x^2+4x-1$? Explain your answer.

$p(6)=(6)^3-6(6)^2+4(6)-1$ No, the remainder is not 0
$p(6)=23$

2. Is $x+2$ a factor of $p(x)=x^3-3x^2-8x+4$? Explain your answer.

$p(-2)=(-2)^3-3(-2)^2-8(-2)+4$ Yes, the remainder is 0
$p(-2)=0$

3. Is $2x+1$ a factor of $p(x)=2x^2+5x+2$? Explain your answer.

$2x+\cancel{1}=0$
$\cancel{-1}\ -1$
$\frac{\cancel{2}x}{\cancel{2}}=\frac{-1}{2}$
$x=\frac{-1}{2}$

$p(-\frac{1}{2})=2(-\frac{1}{2})^2+5(-\frac{1}{2})+2$ Yes, the remainder is 0
$p(-\frac{1}{2})=0$

4. Is $3x-2$ a factor of $p(x)=3x^3-2x^2-27x+18$? Explain your answer.

$3x\cancel{-2}=0$
$\cancel{+2}\ +2$
$\frac{\cancel{3}x}{\cancel{3}}=\frac{2}{3}$
$x=\frac{2}{3}$

$p(\frac{2}{3})=3(\frac{2}{3})^3-2(\frac{2}{3})^2-27(\frac{2}{3})+18$
$p(\frac{2}{3})=0$ Yes, the remainder is 0

5. Determine if $x-5$ is a factor of $2x^3-4x^2-7x-10$. Explain your answer.

$p(5)=2(5)^3-4(5)^2-7(5)-10$ No, the remainder is not 0
$p(5)=105$

6. Which binomial is a factor of x^4-4x^2-4x+8?

[✓ 1)] $x-2$ $p(2)=0$
2) $x+2$ $p(-2)=16$
3) $x-4$ $p(4)=184$
4) $x+4$ $p(-4)=216$

7. Which binomial is *not* a factor of the expression $x^3-11x^2+16x+84$?

1) $x+2$ $p(-2)=0$
[✓ 2)] $x+4$ $p(-4)=-220$
3) $x-6$ $p(6)=0$
4) $x-7$ $p(7)=0$

8. Which binomial is *not* a factor of the expression $x^3-6x^2-49x-66$?

1) $x-11$ $P(11)=0$
2) $x+2$ $p(-2)=0$
3) $x+6$ $P(-6)=-204$
4) $x+3$ $P(-3)=0$

9. Which binomial is a factor of the expression x^3-7x-6?

1) $x+3$ $P(-3)=-12$
2) $x-1$ $P(1)=-12$
3) $x-2$ $P(2)=-12$
[✓ 4)] $x+2$ $P(-2)=0$

10. Which binomial is *not* a factor of the expression $x^3-4x^2-25x+28$?

[✓ 1)] $x+6$ $P(-6)=-182$
2) $x-7$ $P(7)=0$
3) $x-1$ $P(1)=0$
4) $x+4$ $P(-4)=0$

11. Which binomial is not a factor of $p(x)=2x^3+7x^2-5x-4$?

1) $x+4$ $P(-4)=0$
[✓ 2)] $x+1$ $p(-1)=6$
3) $x-1$ $P(1)=0$
4) $2x+1=0$ $P(-\frac{1}{2})=0$

$-1\ \ -1$

$\frac{2x}{2}=\frac{-1}{2}$

12. Which binomial is not a factor of $p(x)=2x^3-5x^2+6x-2$?

1) $x-1$ $P(1)=1$
2) $x-2$ $P(2)=6$
3) $2x-1=0$ $P(\frac{1}{2})=0$
4) $2x+1=0$ $P(-\frac{1}{2})=-\frac{13}{2}$ → not a factor

$2x-1=0$ $\quad 2x+1=0$

$+1\ \ +1$ $\quad -1\ \ -1$

$\frac{2x}{2}=\frac{1}{2}$ $\quad \frac{2x}{2}=\frac{-1}{2}$

$x=\frac{1}{2}$ $\quad x=-\frac{1}{2}$

13. Given $P(x)=x^3-3x^2-2x+4$, which statement is true?

1) $(x-1)$ is a factor because $P(-1)=2$.
2) $(x+1)$ is a factor because $P(-1)=2$.
3) $(x+1)$ is a factor because $P(1)=0$.
[✓ 4)] $(x-1)$ is a factor because $P(1)=0$.

$P(-1)=(-1)^3-3(-1)^2-2(-1)+4$

$P(-1)=2$

$P(1)=(1)^3-3(1)^2-2(1)+4$

$P(1)=0$

↓

$x-1$

14. If $f(x)=2x^4-x^3-16x+8$, then $f\left(\frac{1}{2}\right)=0$

1) equals 0 and $2x+1$ is a factor of $f(x)$
[✓ 2)] equals 0 and $2x-1$ is a factor of $f(x)$
3) ~~does not equal 0 and $2x+1$ is not a factor of $f(x)$~~
4) ~~does not equal 0 and $2x-1$ is a factor of $f(x)$~~

$2x+1=0$ $\quad -1\ \ -1$ $\quad \frac{2x}{2}=\frac{-1}{2}$

$2x-1=0$ $\quad +1\ \ +1$ $\quad \frac{2x}{2}=\frac{1}{2}$

15. Consider the function $f(x)=2x^3+x^2-18x-9$. Which statement is true?

1) $2x-1$ is a factor of $f(x)$. $x=\frac{1}{2}$ $P(\frac{1}{2})=-\frac{35}{2}$
[✓ 2)] $x-3$ is a factor of $f(x)$. $x=3$ $P(3)=0$
3) $f(3)\neq f\left(-\frac{1}{2}\right)$
4) $f\left(\frac{1}{2}\right)=0$

$2x-1=0$

$+1\ \ +1$

$\frac{2x}{2}=\frac{1}{2}$

Finding k in a Polynomial Equation

Finding k in a Polynomial Equation

If $x+a$ is a factor then a is a zero. Replace $p(x)$ with 0 and x with a.

1. Consider the polynomial $p(x)=x^3+kx^2+x+6$. Find a value of k so that $x+1$ is a factor of P.

$P(-1)=0$

$0=(-1)^3+k(-1)^2+(-1)+6$

$0=-1+k-1+6$

$0=k+4$

$-4 \quad -4$

$\boxed{-4=k}$

2. Consider the polynomial $p(x)=x^3+kx-30$. Find a value of k so that $x+3$ is a factor of P.

$P(-3)=0$

$0=(-3)^3+k(-3)-30$

$0=-27-3k-30$

$0=-3k-57$

$+57 \quad +57$

$\frac{57}{-3}=\frac{-3k}{-3}$

$\boxed{-19=k}$

3. If $x-1$ is a factor of x^3-kx^2+2x, what is the value of k?

$P(1)=0$

$0=(+1)^3-k(1)^2+2(1)$

$0=1-k+2$

$0=-k+3$

$+k \quad +k$

$\boxed{k=3}$

4. The polynomial function $g(x)=x^3+ax^2-5x+6$ has a factor of $(x-3)$. Determine the value of a.

$P(3)=0$

$0=(3)^3+a(3)^2-5(3)+6$

$0=27+9a-15+6$

$0=9a+18$

$-18 \quad -18$

$\frac{-18}{9}=\frac{9a}{9}$

$\boxed{-2=a}$

Real zeros hit the x-axis
Imaginary zeros don't

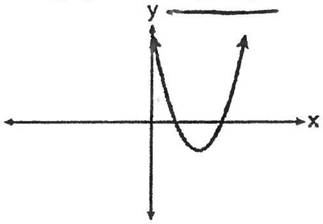
Name Schlansky
Mr. Schlansky

Date ____________
Algebra II

Imaginary Zeros

1. Which graph has imaginary roots?

1)



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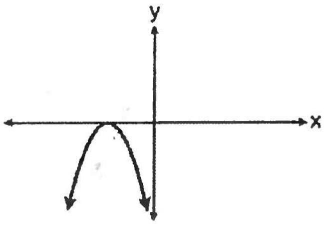
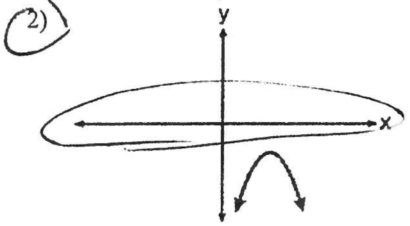


[✓ 2)]



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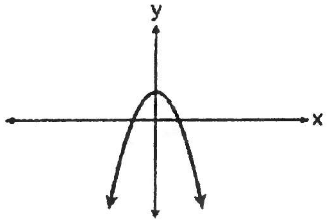
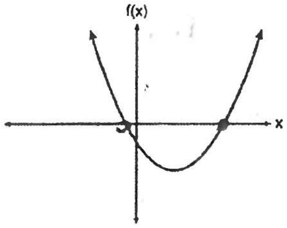


2. If $f(x)$ is represented by the graph below, Does $f(x)$ have imaginary roots? Explain your answer.



No, the roots are real because the graph hits the x-axis

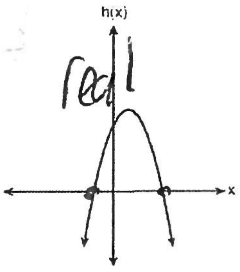
3. Which quadratic functions have imaginary roots?

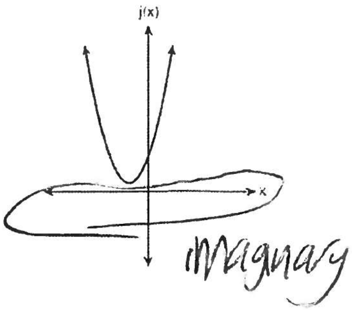
1) $h(x)$ only

[✓ 2) $j(x)$ only]

3) Both $j(x)$ and $h(x)$

4) Neither $j(x)$ or $h(x)$





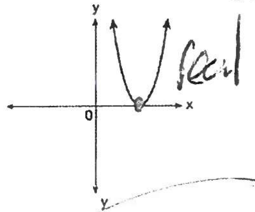
4. Does the equation $x^2 - 4x + 13 = 0$ have imaginary solutions? Justify your answer.

Type into y=

Yes, the graph doesn't touch the x-axis

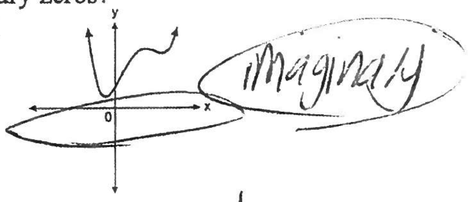
5. Which of the following graphs have imaginary zeros?

I



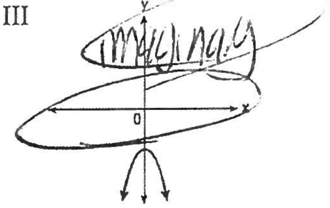
real

II



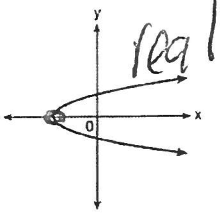
imaginary

III



imaginary

IV



real

1) I and IV
[✓ 2) II and III]
3) II only
4) III and IV

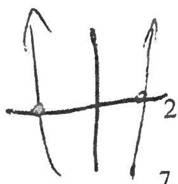
6. Which representation of a quadratic has imaginary roots?

1)

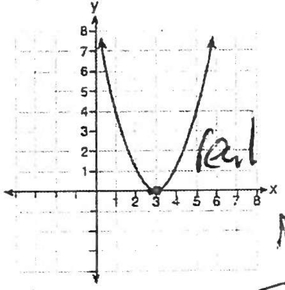
x	y
−2.5	2
−2.0	0
−1.5	−1
−1.0	−1
−0.5	0
0.0	2

real

real



3)



real

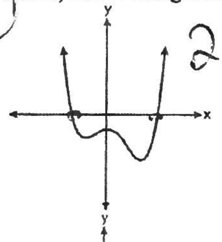
2) $2(x+3)^2 = 64$ $y=2(x+3)^2-64$
−64 −64

[✓ 4)] $2x^2 + 32 = 0$
imaginary

7. Which graph could represent a 4th degree polynomial function with a positive leading coefficient, 2 real zeros, and 2 imaginary zeros?

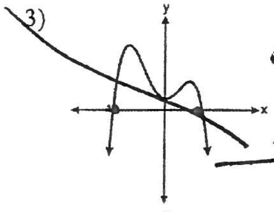
→ opens up

[✓ 1)]



2 real

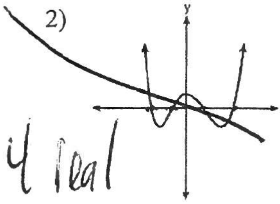
3)



2 real

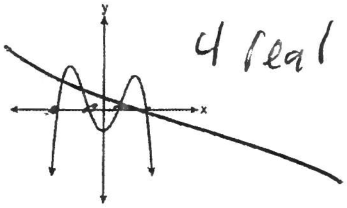
→ opens down

2)



4 real

4)



4 real

Writing Equations of Polynomial Functions

To write the equation of a polynomial function, list the factors. In order to list the factors, list the zeros (where the graph crosses the x-axis). If $x-a$ is a factor, then a is a zero (switch the sign to go back and forth between factors and zeros).

If a is a zero, $p(a)=0$, $x-a$ is a factor, and the polynomial is divisible by $x-a$. Once you have one of the four pieces of information, you have all four.

Single and Double Roots

Single roots pass through the x axis

Double roots bounce off the x axis

Real and Imaginary Roots

Real roots hit the x axis

Imaginary roots (roots with an *i*, do not hit the x axis.

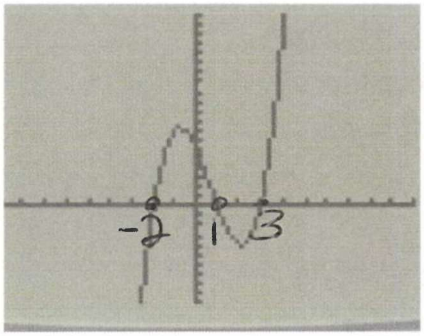
Factoring

You may need to completely or partially factor to put the equation into factored form

A perfect square trinomial factor leads to a double root

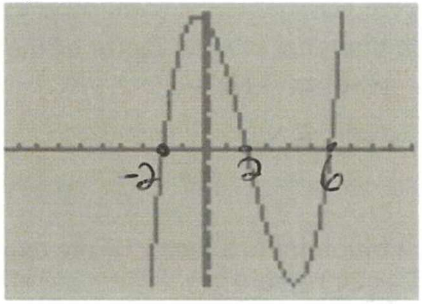
Write a possible equation for each of the following polynomials

1.



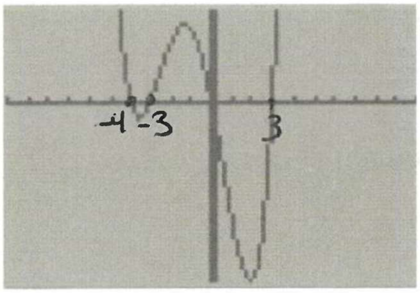
$p(x) = (x+2)(x-1)(x-3)$

2.



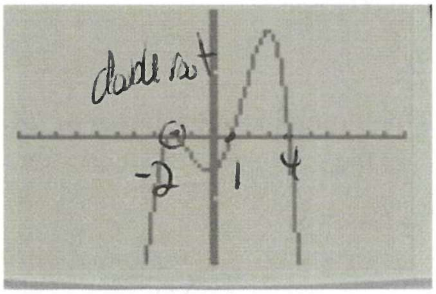
$p(x) = (x+2)(x-2)(x-6)$

3.



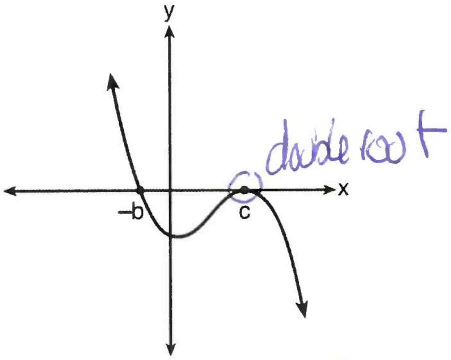
$p(x) = (x+4)(x+3)(x-3)$

4.



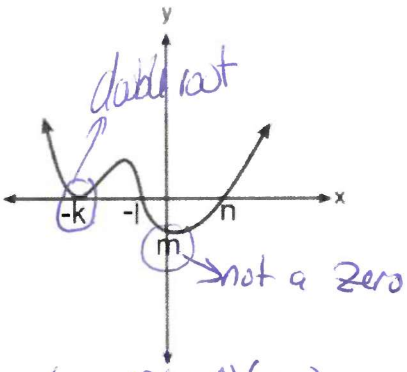
$p(x) = -(x+2)^2(x-1)(x-4)$

5.



$p(x) = -(x+b)(x-c)^2$

6.



$p(x) = (x+k)^2(x+l)(x-n)$

7. The graph below shows the polynomial $y = p(x)$.

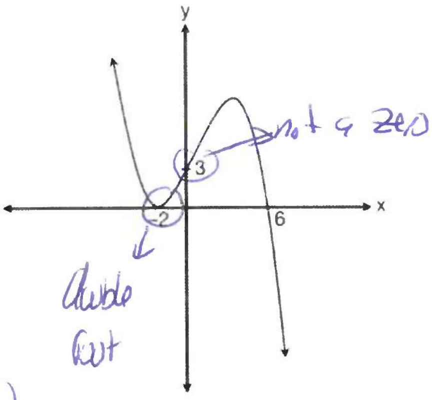
The factors of $p(x)$ are

(1) $(x+2)$, $(x-3)$, and $(x+6)$

(2) $(x-2)$, $(x+3)$, and $(x+6)$

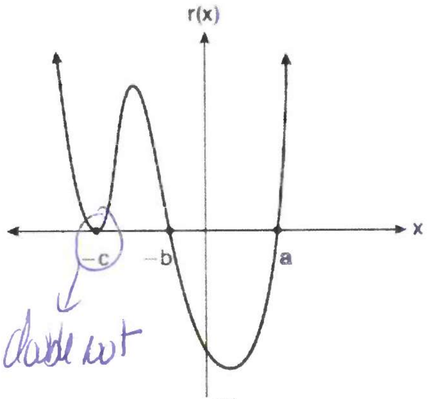
(3) $(x-2)$, $(x-2)$, and $(x+6)$

[✓ (4) $(x+2)$, $(x+2)$, and $(x-6)$]



$p(x) = (x+2)^2(x-6)$

8. A sketch of $r(x)$ is shown below.



An equation for $r(x)$ could be

1) $r(x) = (x-a)(x+b)(x+c)$

2) $r(x) = (x+a)(x-b)(x-c)^2$

3) $r(x) = (x+a)(x-b)(x-c)$

[✓ 4) $r(x) = (x-a)(x+b)(x+c)^2$]

9. The graph of a cubic polynomial function $p(x)$ is shown below.

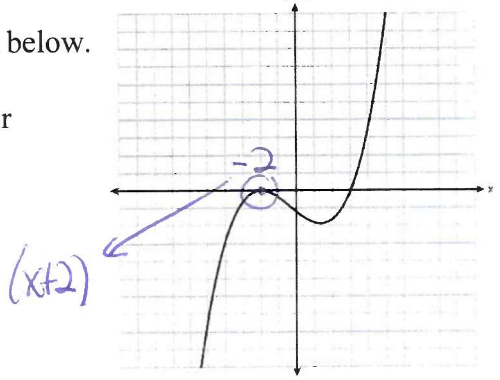
If $p(x)$ is written as a product of linear factors, which factor would appear twice?

1) $x-2$

[✓ 2) $x+2$]

3) $x-3$

4) $x+3$

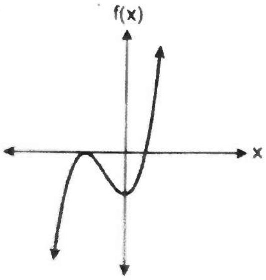


+ → opens up

10. Which graph best represents the graph of $f(x) = (x+a)^2(x-b)$, where a and b are positive real numbers?

negative double root, positive single root

[✓ 1)]



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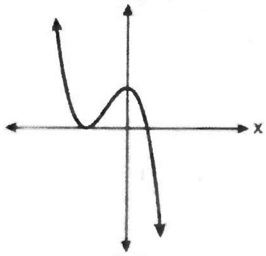
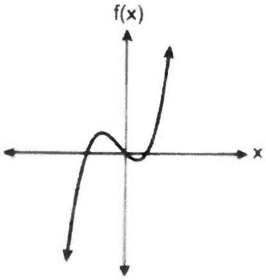


2)



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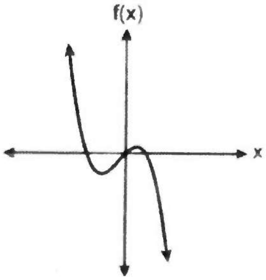
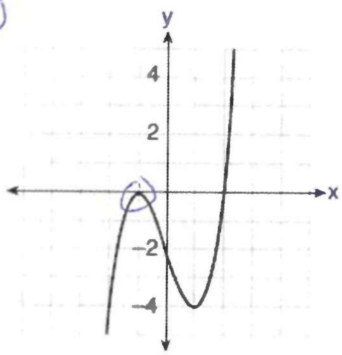


11. Which graph represents a polynomial function that contains $x^2 + 2x + 1$ as a factor?

$(x+1)(x+1)$

$(x+1)^2 \rightarrow -1$ is a double root

[✓ 1)]



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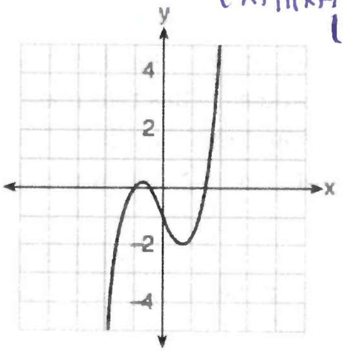

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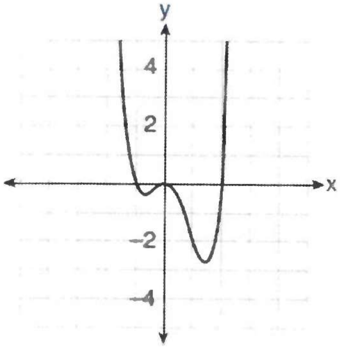

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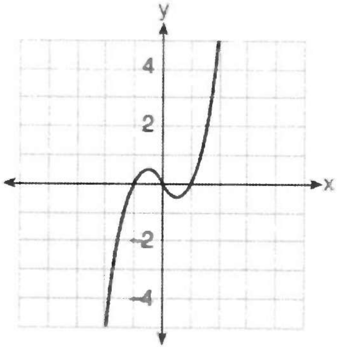

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12. If a, b, and c are all positive real numbers, which graph could represent the sketch of the graph of $p(x) = -a(x+b)\left(x^2 - 2cx + c^2\right)$?

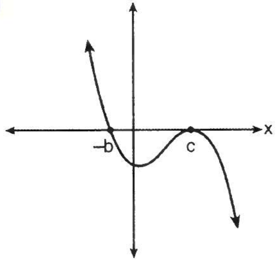
$(x-c)(x-c)$

$(x-c)^2$

↓

c is a double root

[✓ 1)]



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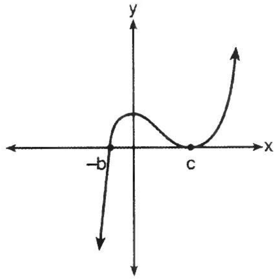

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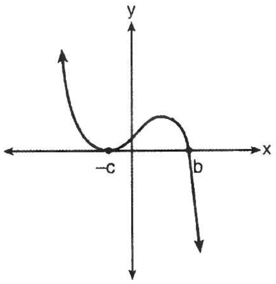

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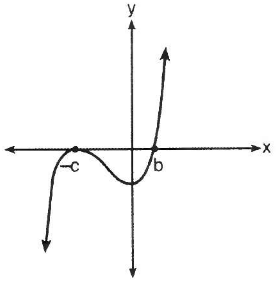


type into Y=

13. Consider the function $p(x) = 3x^3 + x^2 - 5x$ and the graph of $y = m(x)$ below.

Which statement is true?

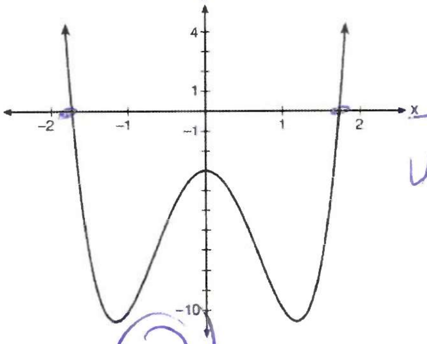
[✓ 1)] $p(x)$ has three real roots and $m(x)$ has two real roots.

2) $p(x)$ has one real root and $m(x)$ has two real roots.

3) $p(x)$ has two real roots and $m(x)$ has three real roots.

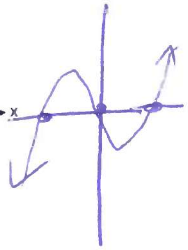
4) $p(x)$ has three real roots and $m(x)$ has four real roots.

m(x)



②

p(x)

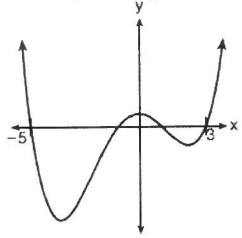


③

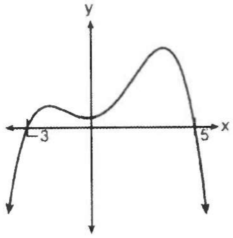
must hit only at −5 and 3

14. A 4th degree polynomial has zeros -5, 3, i, and $-i$. Which graph could represent the function defined by this polynomial?

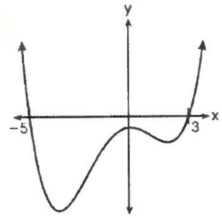
1)



3)



[✓ 2)]



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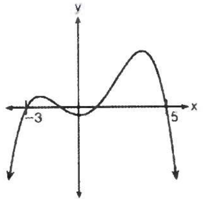


The zeros are where the graph hits the x-axis.
If x-a is a factor, then a is a zero.

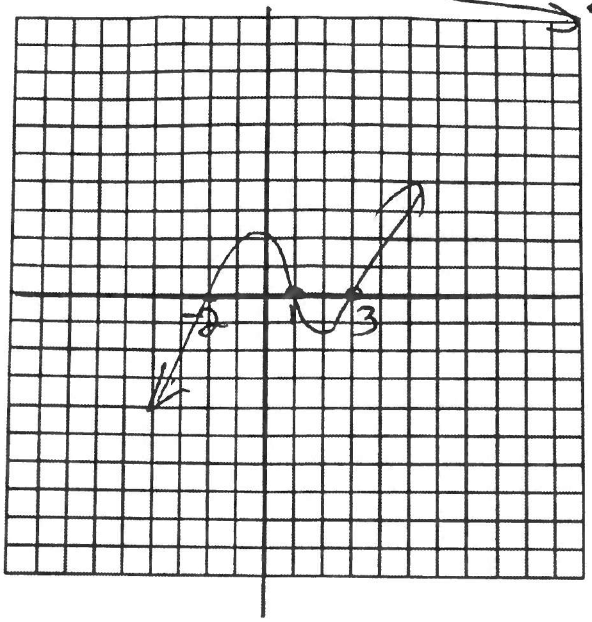
Name Schlansky
Mr. Schlansky

Date ____________
Algebra II

Sketching Polynomial Graphs Regents Practice

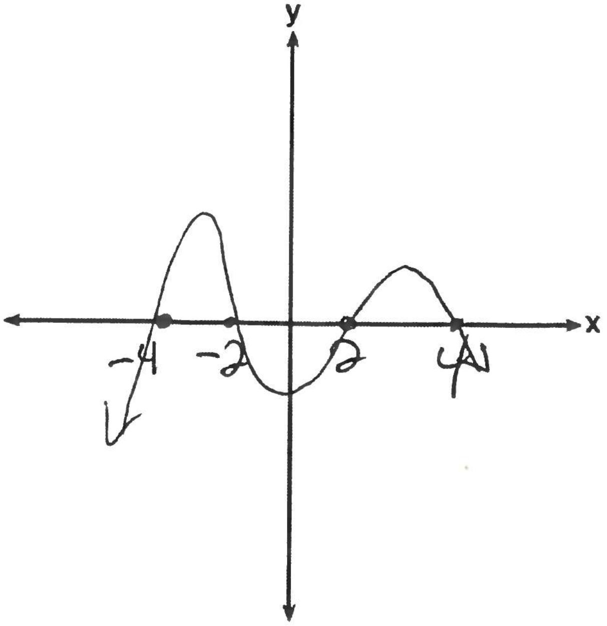
1. On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and -2.

Zeros hit the x-axis. Don't negate!



2. The zeros of a quartic polynomial function are 2, −2, 4, and −4. Use the zeros to construct a possible sketch of the function, on the set of axes below.

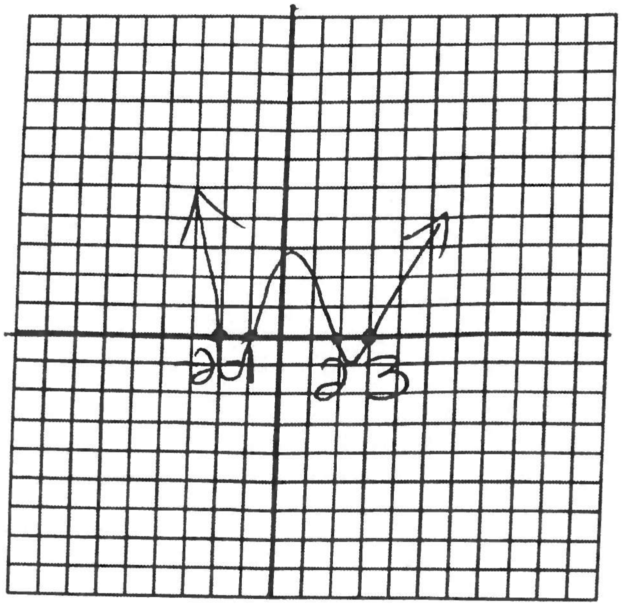
Zeros hit the x-axis
Don't negate



+2 and -2

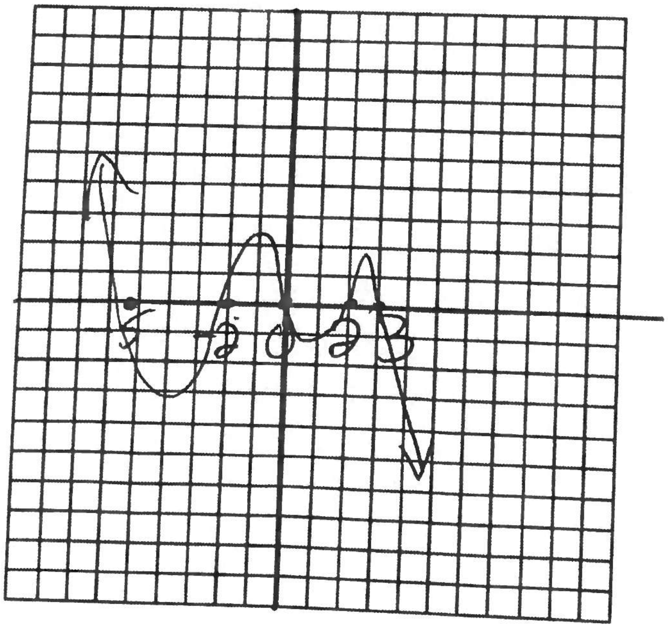
3. The zeros of a quartic polynomial function h are -1, ± 2, and 3. Sketch a graph of $y = h(x)$ on the grid below.

Zeos hit the X-axis. Don't negate



4. The zeros of a polynomial function are $-5, \pm 2, 0,$ *and* 3. Sketch a graph of the polynomial functions on the grid below.

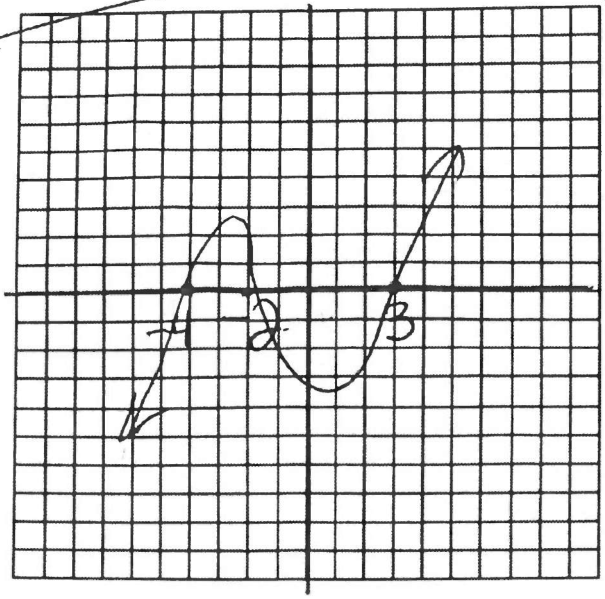
Zeos hit the X-axis. Don't negate



5. On the grid below, sketch a cubic polynomial whose factors are $x-3$, $x+4$, and $x+2$.

3 -4 -2 zeros

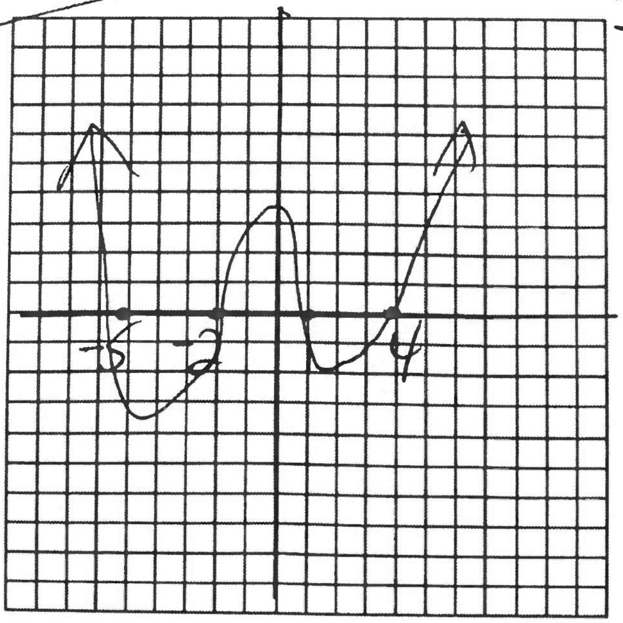
factors don't hit the x-axis, zeros do!



6. On the grid below, sketch a quartic polynomial whose factors are $x+5$, $x+2$, $x-1$, and $x-4$ → 4

-5 -2 1 zeros

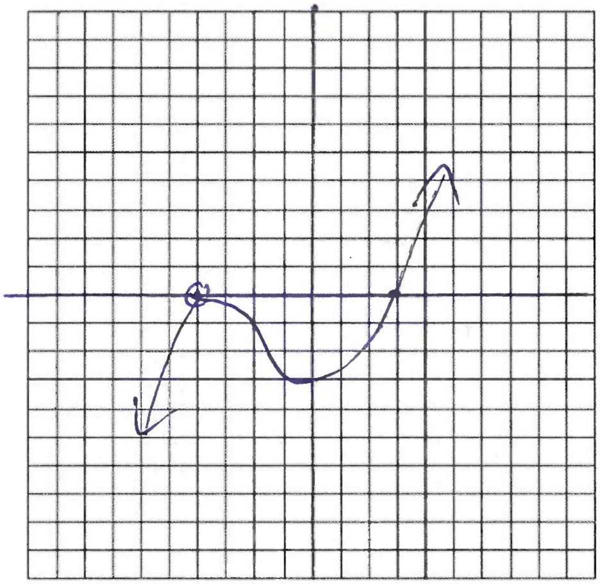
factors don't hit the x-axis, zeros do



7. On the grid below, sketch a cubic polynomial whose factors are $x-3$ and $x^2+8x+16$.

$(x+4)(x+4)$

$(x+4)^2$



Factors

$(x-3)(x+4)^2$

Zeros

3 and -4

↓

double root

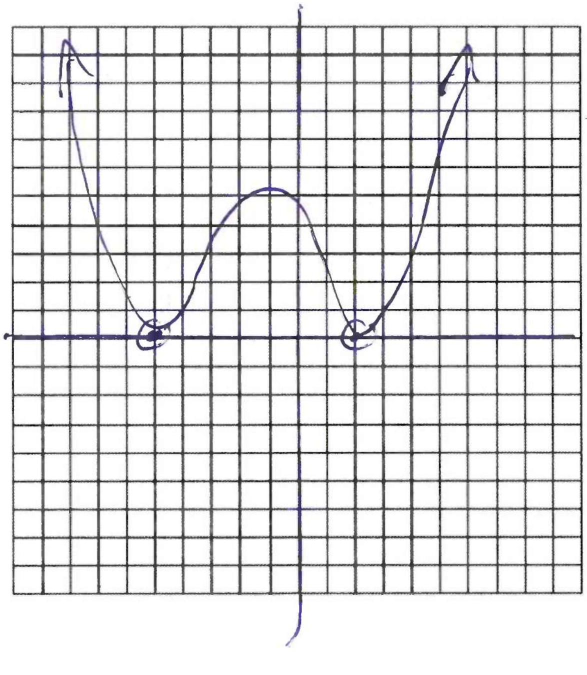
8. On the grid below, sketch a quartic polynomial whose factors are x^2-4x+4 and $x^2+10x+25$.

$(x-2)(x-2)$

$(x-2)^2$

$(x+5)(x+5)$

$(x+5)^2$



Factors

$(x+5)^2(x-2)^2$

Zeros

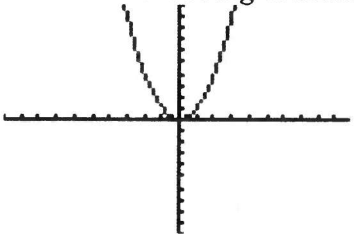
-5 and 2

↓ ↓

double root double root

Sketching Polynomial Graphs (Specific)

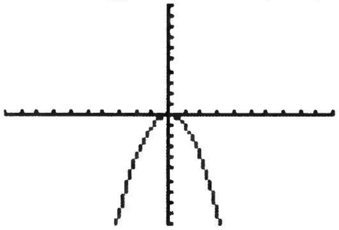
Positive leading coefficient, Even Degree



$x \to -\infty, f(x) \to \infty$

$x \to \infty, f(x) \to \infty$

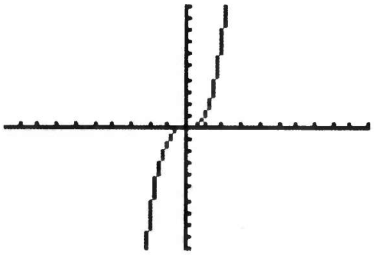
Negative leading coefficient, Even Degree



$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to -\infty$

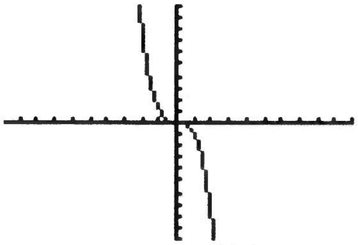
Positive leading coefficient, Odd Degree



$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to \infty$

Negative leading coefficient, Odd Degree



$x \to -\infty, f(x) \to \infty$

$x \to \infty, f(x) \to -\infty$

Sketch the shape and fill in the end behavior for each of the following polynomial equations

1. $f(x) \overset{+}{=} x^{\textcircled{3}} + 2x^2 - 9x - 18$ positive odd

$x \to -\infty,\ f(x) \to -\infty$

$x \to \infty,\ f(x) \to \infty$

2. $f(x) \overset{+}{=} x^{\textcircled{4}} - 10x^2 + 9$ positive even

$x \to -\infty,\ f(x) \to \infty$

$x \to \infty,\ f(x) \to \infty$

3. $p(x) = \textcircled{-}x^{\textcircled{3}} - 3x^2 + 4x + 12$ negative odd

$x \to -\infty,\ f(x) \to \infty$

$x \to \infty,\ f(x) \to -\infty$

4. $f(x) = \textcircled{-}x^{\textcircled{4}} + 3x^3 + 10x^2$ negative even

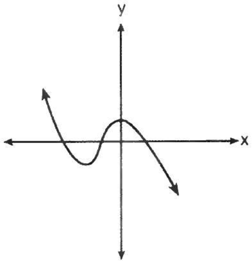
$x \to -\infty,\ f(x) \to -\infty$

$x \to \infty,\ f(x) \to -\infty$

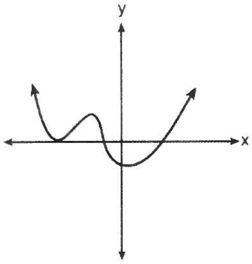
5. Which graph has the following characteristics?

- as $x \to \overset{\text{left}}{-\infty}$, $f(x) \to \overset{\text{down}}{-\infty}$
- as $x \to \overset{\text{right}}{\infty}$, $f(x) \to \overset{\text{up}}{\infty}$

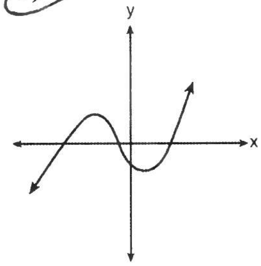
1)



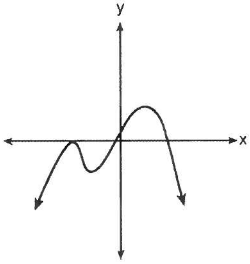
2)



[✓ 3)]



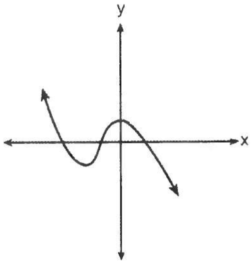
4)



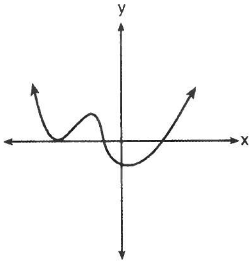
6. Which graph has the following characteristics?

- $x \to \overset{\text{left}}{-\infty}, f(x) \to \infty$ up
- $x \to \underset{\text{right}}{\infty}, f(x) \to \infty$ up

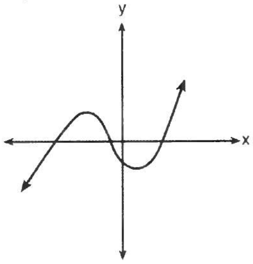
1)



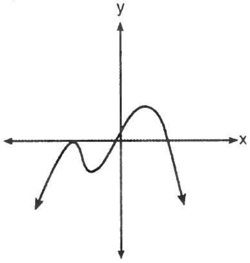
[✓ 2)]



3)



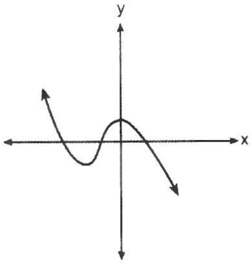
4)



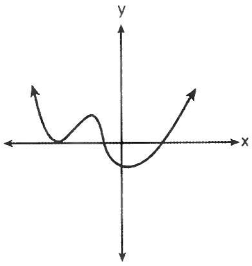
7. Which graph has the following characteristics?

- $x \overset{\text{left}}{\to} -\infty, f(x) \to \overset{\text{down}}{-\infty}$
- $x \overset{\text{right}}{\to} \infty, f(x) \to \overset{\text{down}}{-\infty}$

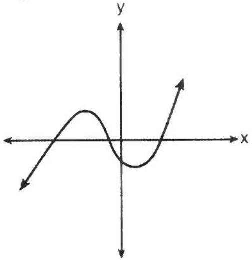
1)



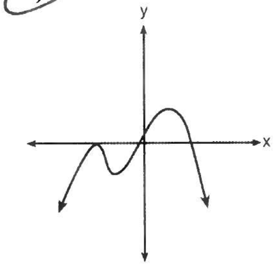
2)



3)



[✓ 4)]



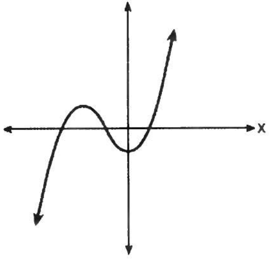
8. Consider the end behavior description below.

- as $\overset{\text{left}}{x \rightarrow -\infty}, \overset{\text{up}}{f(x) \rightarrow \infty}$
- as $\underset{\text{right}}{x \rightarrow \infty}, \underset{\text{down}}{f(x) \rightarrow -\infty}$

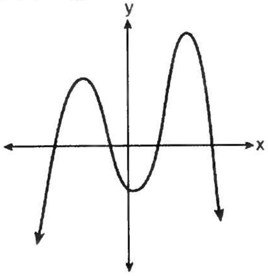
1) $f(x) = {}^{+}x^{\textcircled{4}} + 2x^2 + 1$

[✓ 3)] $f(x) = \textcircled{-}x^{\textcircled{3}} + 2x - 6$

2)



4)



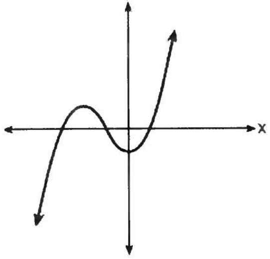
9. Consider the end behavior description below.

- as $\overset{\text{left}}{x \rightarrow -\infty}, \overset{\text{up}}{f(x) \rightarrow \infty}$
- as $\overset{\text{right}}{x \rightarrow \infty}, \overset{\text{up}}{f(x) \rightarrow \infty}$

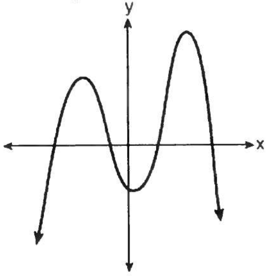
[✓ 1)] $f(x) = {}^{+}x^{\textcircled{4}} + 2x^2 + 1$

3) $f(x) = \textcircled{-}x^{\textcircled{3}} + 2x - 6$

2)



4)



10. Consider the end behavior description below.

- as $\overset{\text{left}}{x \rightarrow -\infty}, \overset{\text{up}}{f(x) \rightarrow \infty}$
- as $\overset{\text{right}}{x \rightarrow \infty}, \overset{\text{down}}{f(x) \rightarrow -\infty}$

Which function satisfies the given conditions?

1) $f(x) = \textcircled{-}x^{\textcircled{4}} + 3x^3 + 2x^2 - 1$

2) $f(x) = \textcircled{2}x^{\textcircled{3}} - 7x + 5$

[✓ 3)] $f(x) = \textcircled{-}7x^{\textcircled{5}} + 5x^4 + 8x^2 - 6$

[✓ 4)] $f(x) = \textcircled{-}8x^{\textcircled{7}} + 5x^5 - 11x^2 + 2x - 7$

OOPS

11. $(2-yi)^2$

$(2-yi)(2-yi)$

	2	$-yi$
2	4	$-2yi$
$-yi$	$-2yi$	$+y^2i^2$

$4-4yi+y^2i^2$

$4-4yi+y^2(-1)$

$\boxed{4-4yi-y^2}$

12. $(3-7i)^2$

$(3-7i)(3-7i)$

	3	$-7i$
3	9	$-21i$
$-7i$	$-21i$	$+49i^2$

$9-42i+49i^2$

$9-42i+49(-1)$

$9-42i-49$

$\boxed{-40-42i}$

1 13. $(3k-2i)^2$

$(3k-2i)(3k-2i)$

	$3k$	$-2i$
$3k$	$9k^2$	$-6ki$
$-2i$	$-6ki$	$+4i^2$

$9k^2-12ki+4i^2$

$9k^2-12ki+4(-1)$

$\boxed{9k^2-12ki-4}$

9

2 14. $(4x-3yi)^2$

$(4x-3yi)(4x-3yi)$

	$4x$	$-3yi$
$4x$	$16x^2$	$-12xyi$
$-3yi$	$-12xyi$	$+9y^2i^2$

$16x^2-24xyi+9y^2i^2$

$16x^2-24xyi+9y^2(-1)$

$\boxed{16x^2-24xyi-9y^2}$

3 15. $3xi(3-2i)$

$9xi-6xi^2$

$9xi-6x(-1)$

$\boxed{+6x+9xi}$

11

4 16. $5i+4i(2+3i)$

$5i+8i+12i^2$

$13i+12(-1)$

$\boxed{-12+13i}$

5 17. $2xi(i-4i^2)$

$2xi^2-8xi^3$

$2x(-1)-8x(-i)$

$\boxed{-2x+8xi}$

13

6 18. $6xi^3(-4xi+5)$

$6x(-i)(-4xi+5)$

$-6xi(-4xi+5)$

$24x^2i^2-30xi$

$24x^2(-1)-30xi$

$\boxed{-24x^2-30xi}$

7 19. $2i(\sqrt{-4}-4)$

$\sqrt{-4}$

$i\sqrt{4}$

$2i$

$2i(2i-4)$

$4i^2-8i$

$4(-1)-8i$

$\boxed{-4-8i}$

15

8 20. $-\frac{1}{2}i^3\left(\sqrt{-9}-4\right)-3i^2$

$\sqrt{-9}$

$i\sqrt{9}$

$3i$

$-\frac{1}{2}(-i)(3i-4)-3(-1)$

$\frac{1}{2}i(3i-4)+3$

$\frac{3}{2}i^2-2i+3$

$\frac{3}{2}(-1)-2i+3$

$-\frac{3}{2}-2i+3$

$\boxed{\frac{3}{2}-2i}$

Writing the Equation of a Parabola

Definition of a Parabola: A parabola is the set of all points equidistant between a point (focus) and a line (directrix).

The vertex is directly in between the focus and the directrix. USE GRAPH PAPER AND COUNT!

$$y = \frac{1}{4p}(x-v)^2 + t$$

$(v,t) = vertex$

$p = distance\ from\ vertex\ to\ focus$

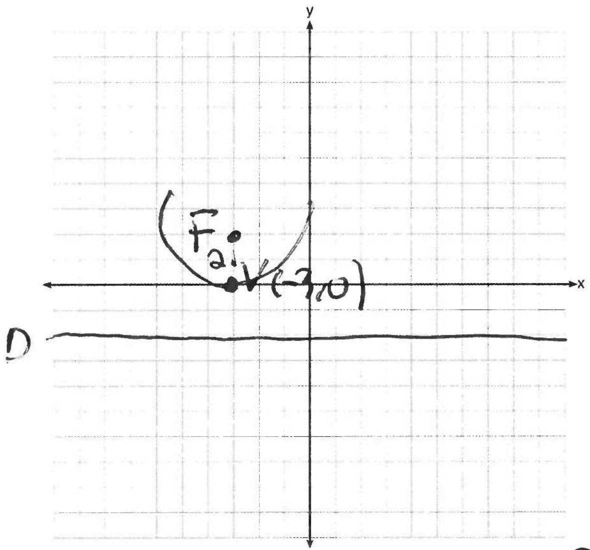
*p is positive when parabola opens up and negative when parabola opens down

You might have to manipulate the equation if it is multiple choice

If given equation, pull the vertex out!

For each of the following problems, state the coordinate of the focus and vertex, the equation of the directrix and the parabola in three different forms.

1. Focus: (-3,2), Directrix: $y = -2$



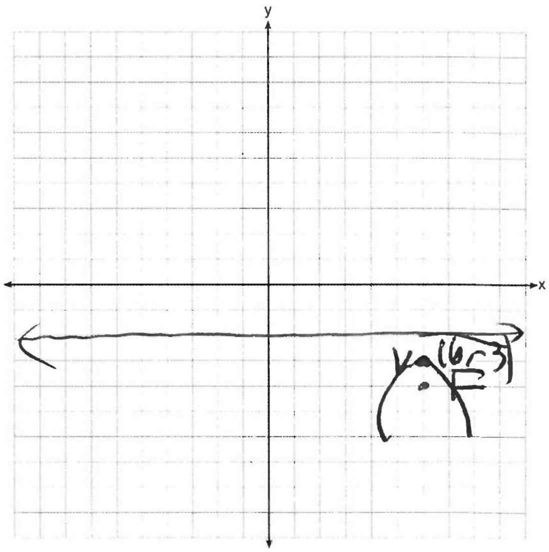
$y = \frac{1}{4p}(x-v)^2 + t$

p = 2
v = -3
t = 0

$y = \frac{1}{4(2)}(x+3)^2 + 0$

$y = \frac{1}{8}(x+3)^2$

2. Focus: (6,-4), Directrix: $y = 2$



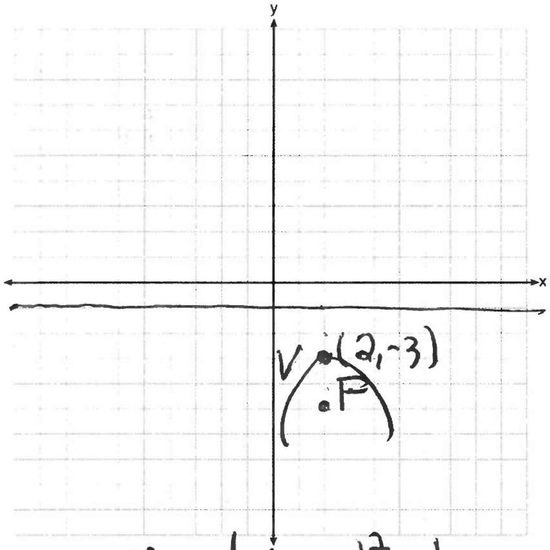
$y = \frac{1}{4p}(x-v)^2 + t$

p = -1
v = 6
t = -3

$y = \frac{1}{4(-1)}(x-6)^2 - 3$

$y = -\frac{1}{4}(x-6)^2 - 3$

3. Focus: (2,-5), Directrix: $y=-1$



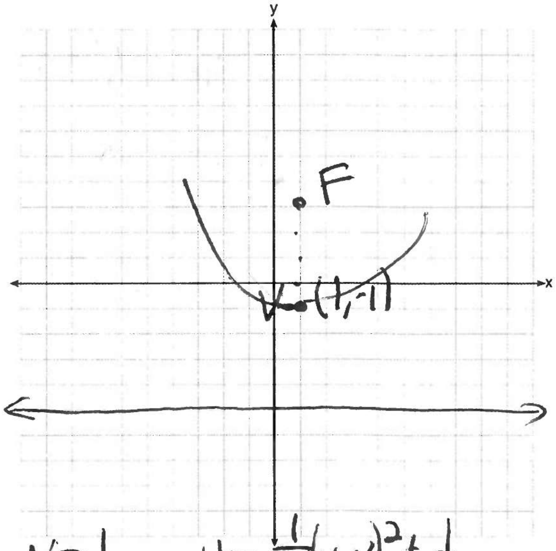
$V=2$
$t=-3$
$P=-2$

$y=\frac{1}{4p}(x-v)^2+t$

$y=\frac{1}{4(-2)}(x-2)^2-3$

$y=-\frac{1}{8}(x-2)^2-3$

4. Focus: (1,3), Directrix: $y=-5$



$V=1$
$t=-1$
$P=4$

$y=\frac{1}{4p}(x-v)^2+t$

$y=\frac{1}{4(4)}(x-1)^2-1$

$y=\frac{1}{16}(x-1)^2-1$

5. Which equation represents the set of points equidistant from line ℓ and point *R* shown on the graph below?

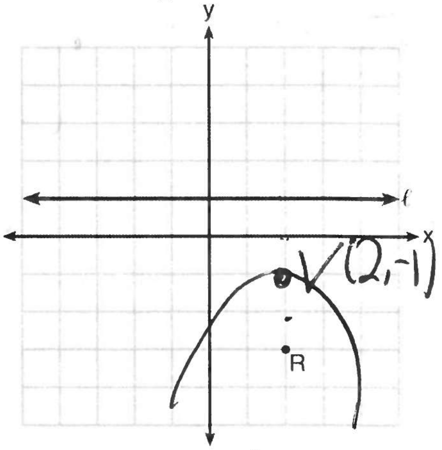
1) $y=-\frac{1}{8}(x+2)^2+1$
2) $y=-\frac{1}{8}(x+2)^2-1$
3) $y=-\frac{1}{8}(x-2)^2+1$
[✓ 4)] $y=-\frac{1}{8}(x-2)^2-1$

$y=\frac{1}{4p}(x-v)^2+t$

$V=2$
$t=-1$
$P=-2$

$y=\frac{1}{4(-2)}(x-2)^2-1$

$y=-\frac{1}{8}(x-2)^2-1$



6. Which equation represents the equation of the parabola with focus (−3, 3) and directrix $y=7$?

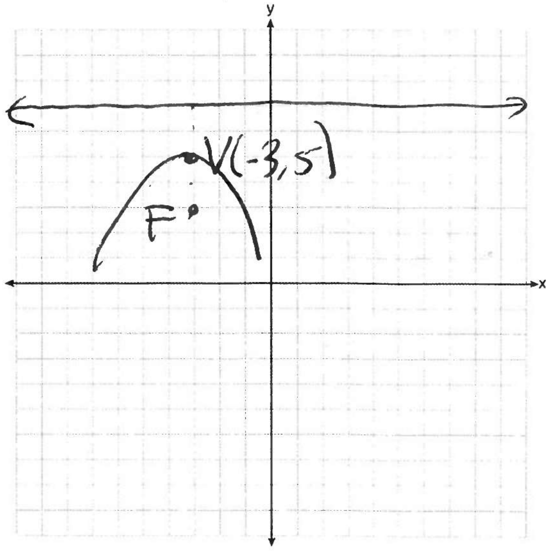
1) $y=\frac{1}{8}(x+3)^2-5$
[✓ 3)] $y=-\frac{1}{8}(x+3)^2+5$
2) $y=\frac{1}{8}(x-3)^2+5$
4) $y=-\frac{1}{8}(x-3)^2+5$

$V=-3$
$t=5$
$P=-2$

$y=\frac{1}{4p}(x-v)^2+t$

$y=\frac{1}{4(-2)}(x+3)^2+5$

$y=-\frac{1}{8}(x+3)^2+5$



7. Which equation represents a parabola with a focus of $(0, 4)$ and a directrix of $y = 2$?

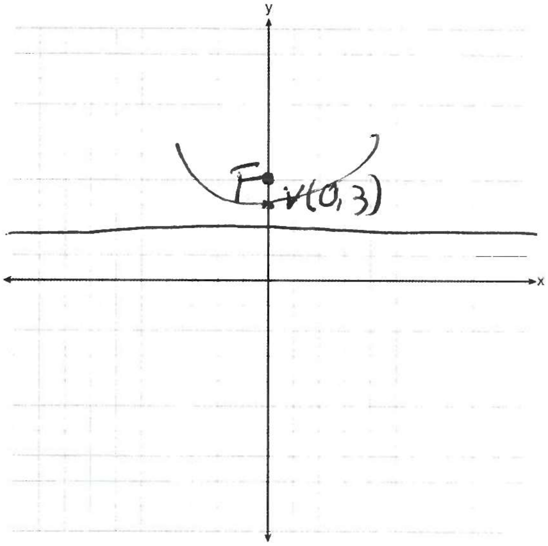
1) $y = x^2 + 3$
2) $y = -x^2 + 1$
3) $y = \frac{x^2}{2} + 3$
[✓ 4)] $y = \frac{x^2}{4} + 3$

$y=\frac{1}{4p}(x-v)^2+t$

$V= 0$
$t= 3$
$P= 1$

$y=\frac{1}{4(1)}(x-0)^2+3$

$y=\frac{1}{4}x^2+3$



8. Which equation represents a parabola with a focus of $(-2, 5)$ and a directrix of $y = 9$?

1) $(y-7)^2 = 8(x+2)$
2) $(y-7)^2 = -8(x+2)$
3) $(x+2)^2 = 8(y-7)$
[✓ 4)] $(x+2)^2 = -8(y-7)$

$y=\frac{1}{4p}(x-v)^2+t$

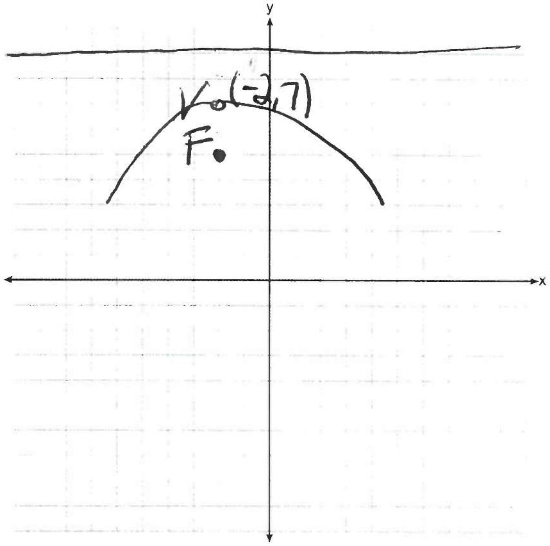
$V= -2$
$t=7$
$P= -2$

$y=\frac{1}{4(-2)}(x+2)^2+7$

$y=-\frac{1}{8}(x+2)^2+7$
$-7 \qquad -7$

$-8\,(y-7)=-8\left(\cancel{-\frac{1}{8}}(x+2)^2\right)$

$-8(y-7)=(x+2)^2$



9. A parabola has its focus at $(1, 2)$ and its directrix is $y = -2$. The equation of this parabola could be

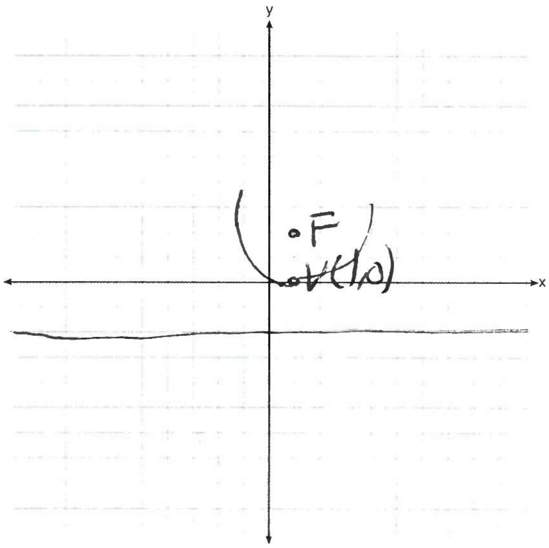
1) $y = 8(x+1)^2$
2) $y = \frac{1}{8}(x+1)^2$
3) $y = 8(x-1)^2$
[✓ 4)] $y = \frac{1}{8}(x-1)^2$

$y=\frac{1}{4p}(x-v)^2+t$

$P= 2$
$V= 1$
$t= 0$

$y=\frac{1}{4(2)}(x-1)^2+0$

$y=\frac{1}{8}(x-1)^2$



Given the Equation of a Parabola

If given equation, pull the vertex and p value out!

To find vertex, negate the x coordinate but not the y (unless it is in cross multiplied form).
To find p, divide the value by 4.
The vertex is directly in between the focus and the directrix. USE GRAPH PAPER AND COUNT!

$$y=\frac{1}{4p}(x-v)^2+t$$

$$(v,t)=vertex$$

$$p=dis\tan ce\ \ from\ \ vertex\ \ to\ \ focus$$

*p is positive when parabola opens up and negative when parabola opens down

Find the vertex and p value of the parabolas below

1. $y=\frac{1}{12}(x-5)^2-1$

(5,-1)
p= 3

2. $y=\frac{1}{8}(x+3)^2-4$

(-3,-4)
p=2

3. $y=-\frac{1}{16}(x+9)^2-8$

(-9,-8)
p=-4

4. $y=\frac{1}{4}(x+9)^2-3$

(-9,-3)
p=1

5. $y=-\frac{1}{12}(x-7)^2+1$

(7,1)
p=-3

6. $y=\frac{1}{20}x^2+5$

(0,5)
p=5

7. $12(y+2)=(x+3)^2$

(-3,-2)
p =3

8. $-4(y+1)=(x-2)^2$

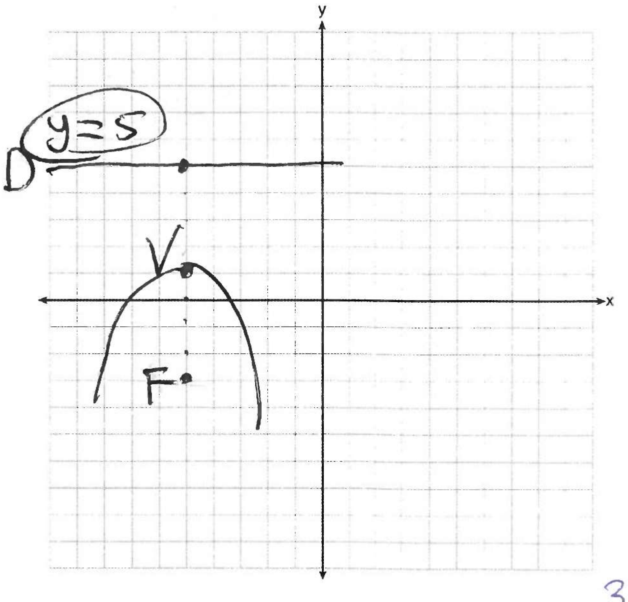
(2,-1)
p=-1

9. $24(y+1)=(x-7)^2$

(7,-1)
p=6

10. The equation of a parabola is $y=-\frac{1}{16}(x+5)^2+1$. If the focus is $(-5,-3)$, what is the equation of the directrix?

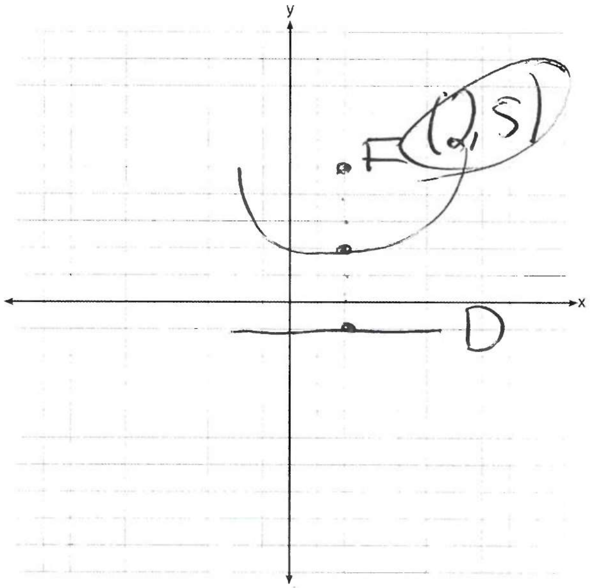
(-5,1)
p=-4



11. The parabola described by the equation $y = \frac{1}{12}(x-2)^2 + 2$ has the directrix at $y = -1$. What is the focus?

V: (2,2)

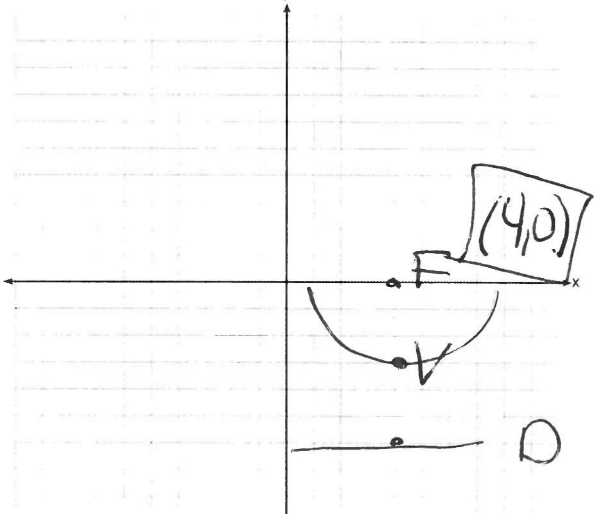
p=3



12. The directrix of the parabola $12(y+3) = (x-4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

V: (4,-3)

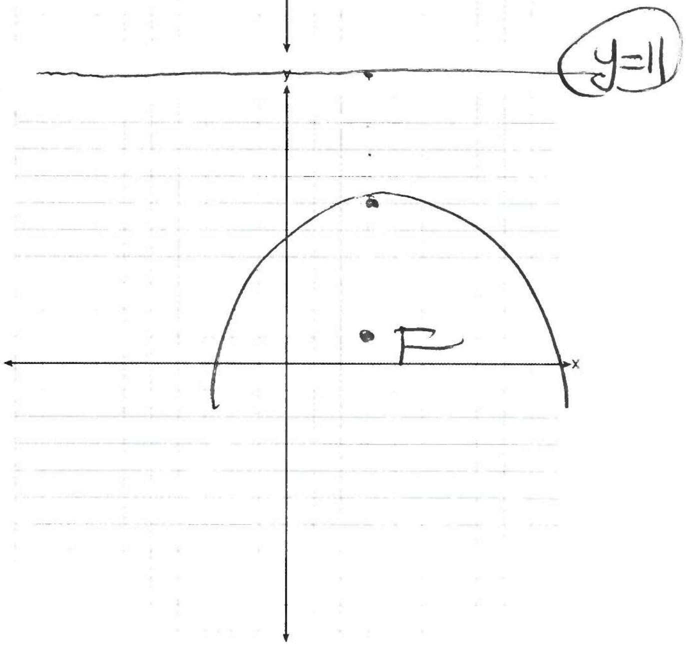
p=3



13. The parabola $y = -\frac{1}{20}(x-3)^2 + 6$ has its focus at (3, 1). Determine and state the equation of the directrix.

V:(3,6)

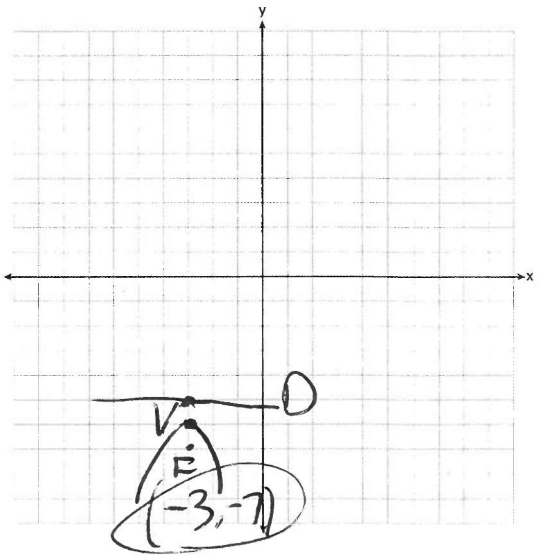
p=-5



14. The parabola $y=-\frac{1}{4}(x+3)^2-6$ has a directrix at $y=-5$. What is the focus?

V: (-3,-6)

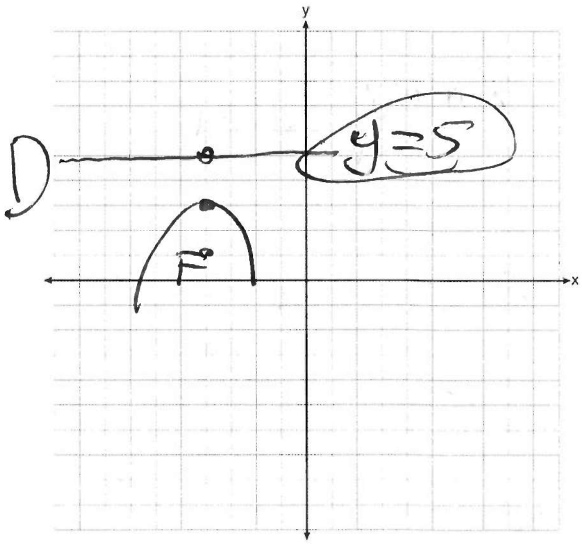
p=-1



15. What is the equation of the directrix for the parabola $-8(y-3)=(x+4)^2$?

V: (-4,3)

p=-2

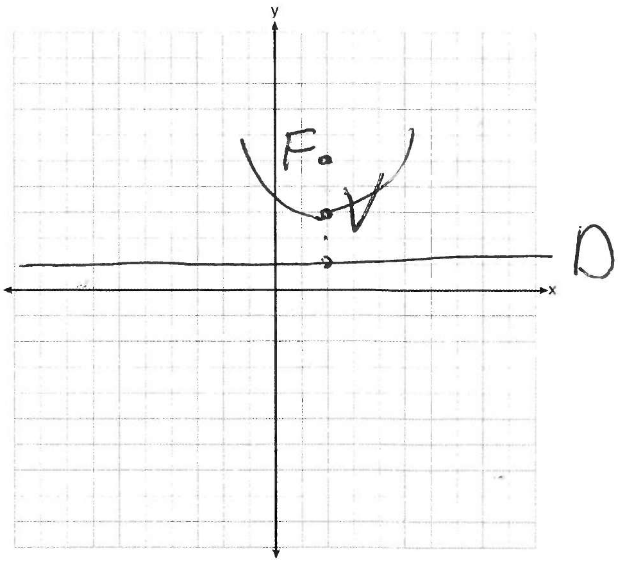


16. The parabola $8(y-3)=(x-2)^2$ has a focus of (2,5). What is the equation of the directrix?

V: (2,3)

p=2

y=1



Solving Systems of Equations Graphically Using TI-84+ ($f(x) = g(x)$)

1) Type equations into Y_1 and Y_2
2) Zoom 6 (Standard) is your standard window. Adjust window OR try Zoom 0(Fit) if you don't see what you want to see.
3) 2nd Trace (Calc), 5 (Intersect)
4) Place cursor over point of intersection, hit enter, enter, enter. Repeat the process for any other points of intersection.

***The solutions to the system of equations are the x values of the intersections.**

1\. To the *nearest tenth*, the value of x that satisfies $2^x = -2x + 11$ is Intersect

y_1 y_2

1) 2.5
[✓ 2) 2.6]
3) 5.8
4) 5.9

$x = 2.6$

*window's good

2\. For which values of x, rounded to the *nearest hundredth*, will $|x^2 - 9| - 3 = \log_3 x$? Intersect

y_1 y_2

[✓ 1) 2.29 and 3.63]
2) 2.37 and 3.54
3) 2.84 and 3.17
4) 2.92 and 3.06

$x = 2.29$
$x = 3.63$

*window's good

y_1 y_2

3\. For which approximate value(s) of x will $\log(x + 5) = |x - 1| - 3$?

1) 5, 1
2) −2.41, 0.41
[✓ 3) −2.41, 5]
4) 5, only

Intersect
$x = -2.41$
$x = 5$

*window's good

4\. Which value, to the *nearest tenth*, is *not* a solution of $p(x) = q(x)$ if $p(x) = x^3 + 3x^2 - 3x - 1$ and $q(x) = 3x + 8$? Intersect

y_1 y_2

1) −3.9
2) −1.1
3) 2.1
[✓ 4) 4.7]

$x = -3.9$
$x = -1.1$
$x = 2.1$

*adjust y max

5\. If $f(x) = 3|x| - 1$ and $g(x) = 0.03x^3 - x + 1$, an approximate solution for the equation $f(x) = g(x)$ is

y_1 y_2 Intersect

1) 1.96
[✓ 2) 11.29]
3) (−0.99, 1.96)
4) (11.29, 32.87)

$x = -.99$ $x = 11.29$
$x = -1.11$

*adjust y max

6\. If $p(x) = 2\ln(x) - 1$ and $m(x) = \ln(x + 6)$, then what is the solution for $p(x) = m(x)$?

y_1 y_2 Intersect

1) 1.65
2) 3.14
[✓ 3) 5.62]
4) no solution

$x = 5.62$

*window's good

7\. If ~~f(x) = g(x)~~ $f(x) = 3|x| - 1$ and $g(x) = 0.03x^3 - x + 1$, ~~an approximate solu~~tion for the equation $f(x) = g(x)$ is

1) 1.96
~~2) 11.29~~
3) ~~(−0.99, 1.96)~~
4) (11.29, 32.87)

8. Given: $h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2$ Y1 Intersect

$k(x) = -|0.7x| + 5$ Y2

State the solutions to the equation $h(x) = k(x)$, rounded to the *nearest hundredth.*

$x = -5.17$
$x = -1.13$
$x = 1.75$

9. If $f(t) = 325e^{-.0735t} + 75$ and $g(t) = 375e^{-.0817t} + 75$, for what value of *t* does $f(t) = g(t)$ rounded to the *nearest tenth*?

Y1 Y2 Intersect

Zoom Fit

$t = 17.5$

10. A technology company is comparing two plans for speeding up its technical support time. Plan *A* can be modeled by the function $A(x) = 15.7(0.98)^x$ and plan *B* can be modeled by the function $B(x) = 11(0.99)^x$ where *x* is the number of customer service representatives employed by the company and $A(x)$ and $B(x)$ represent the average wait time, in minutes, of each customer. To the *nearest integer*, solve the equation $A(x) = B(x)$.

Y1 Y2 Intersect

*adjust X max
and y max

$x = 35$

11. Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is $P(x) = \log(x - 4)$, where *x* is the number of visits per week in thousands and $P(x)$ is the website's popularity rating. An alternative rating model is represented by $R(x) = \frac{1}{2}x - 6$, where *x* is the number of visits per week in thousands. For what number of weekly visits will the two models provide the same rating?

Y1 Y2 Intersect

*adjust y max

$x = 14$
14,000

12. The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and *t* is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and *t* is the time in years, models the unpaid amount of Zach's loan over time. State when $V(t) = Z(t)$, to the *nearest hundredth*,

Y1 Y2 Intersect

*Zoom Fit

$t = 1.95$

Finding Key Points of Polynomial Functions Using TI

-Type equation into Y=
-2nd Trace (Calc)

1. Given the function $f(x) = x^3 + 3x^2 - x - 2$, find the zeros and relative extrema to the *nearest tenth.*

Zeros	Maximum	Minimum
-3.1	(-2.2,4.1)	(.2,-2.1)
-.7		
.9		

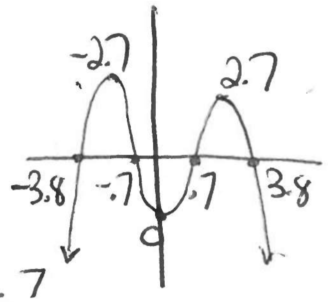
2. Given the function $f(x) = -x^3 - 2x^2 + 2x + 3$, find the zeros and relative extrema to the *nearest tenth.*

Zeros	Maximum	Minimum
-2.3	(.4,3.4)	(-1.7,-1.3)
-1		
1.3		

3. Over what intervals are $f(x) = -x^4 + 15x^2 - 7$:

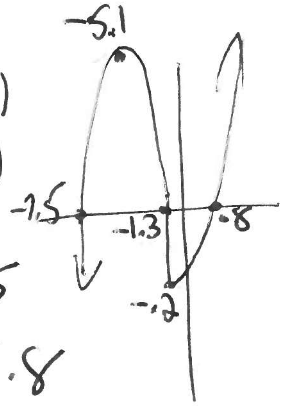
Increasing	Decreasing	Positive	Negative
(-∞,-2.7) (0,2.7)	(-2.7,0) (2.7,∞)	(-3.8,-.7) (.7,3.8)	(-∞,-3.8) (-.7,.7) (3.8,∞)
$x<-2.7$ $0<x<2.7$	$-2.7<x<0$ $x>2.7$	$-3.8<x<-.7$ $.7<x<3.8$	$x<-3.8$ $-.7<x\leq.7$ $x>3.8$

*adjust window



4. Over what intervals are $f(x) = x^3 + 8x^2 + 3x - 8$:

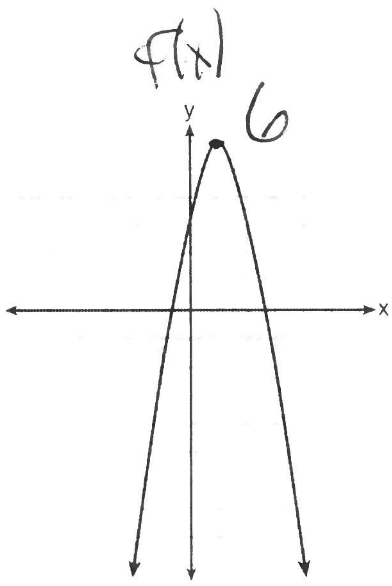
Increasing	Decreasing	Positive	Negative
(-∞,-5.1) (-.2,∞)	(-5.1,-.2)	(-7.5,-1.3) (.8,∞)	(-∞,-7.5) (-1.3,.8)
$x<-5.1$ $x>.2$	$-5.1<x<-.2$	$-7.5<x<-1.3$ $x>.8$	$x<-7.5$ $-1.3<x<.8$



5. Let *f* be the function represented by the graph below.

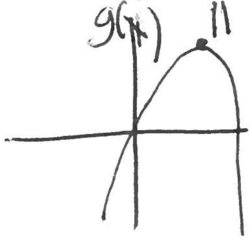
Let *g* be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$.

Determine which function has the larger maximum value. Justify your answer.



g(x)

11>6

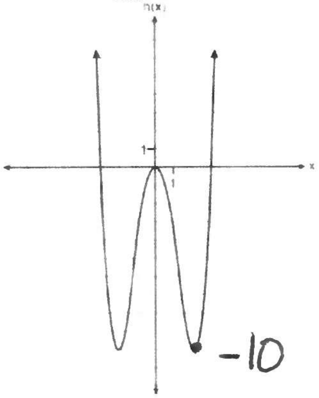


6. Which graph has a smaller relative minimum?

$g(x) = x^3 + 4x^2 - 2x - 10$

2nd Trace, minimum

-10.23



g(x) is smaller

7. Which quadratic function has the largest maximum?

1) $h(x) = (3 - x)(2 + x)$ 2nd Trace, max 6.25

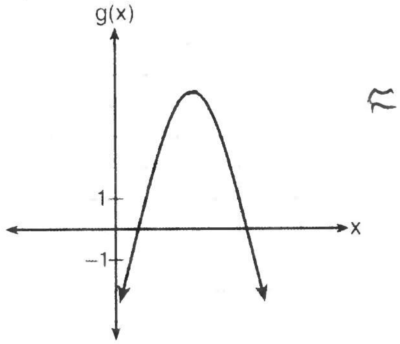
2)

x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

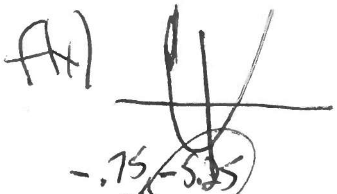
≈9.5

[✓ 3)] $k(x) = -5x^2 - 12x + 4$ 2nd Trace: max 11.2 *adjust window

4)



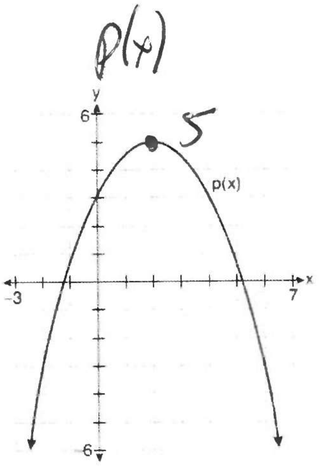
≈4.5



8. Consider $f(x) = 4x^2 + 6x - 3$, and $p(x)$ defined by the graph below. The difference between the values of the maximum of p and minimum of f is

1) 0.25 3) 3.25

2) 1.25 [✓ 4) 10.25]



max p – min f

5 – ⁻5.25

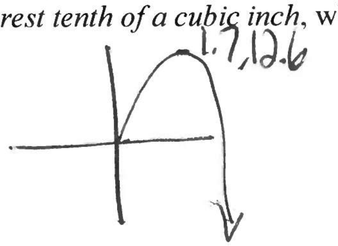
10.25

9. The function $v(x) = x(3 - x)(x + 4)$ models the volume, in cubic inches, of a rectangular solid for $0 \le x \le 3$. To the *nearest tenth of a cubic inch*, what is the maximum volume of the rectangular solid?

window:

x min: 0

x max: 3

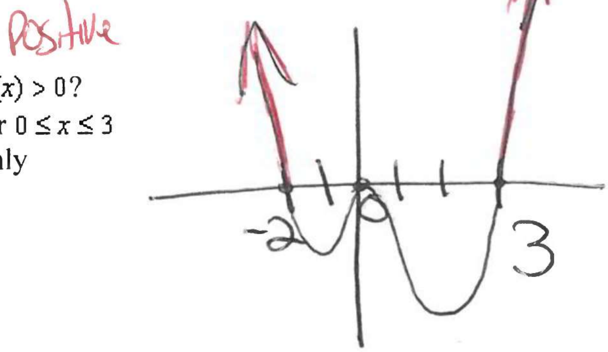


12.6

10. Given $f(x) = x^4 - x^3 - 6x^2$, for what values of x will $f(x) > 0$?

1) $x < -2$, only 3) $x < -2$ or $0 \le x \le 3$

[✓ 2) $x < -2$ or $x > 3$] 4) $x > 3$, only

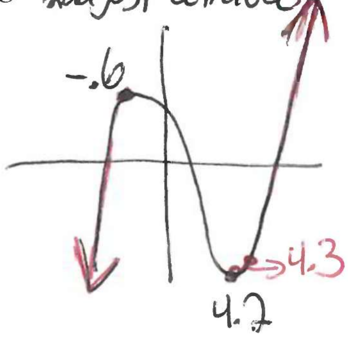


11. At which x value is the graph of $f(x) = 2x^3 - 11x^2 - 14x + 26$ *increasing*?

*adjust window

1) -.5 3) 1.7

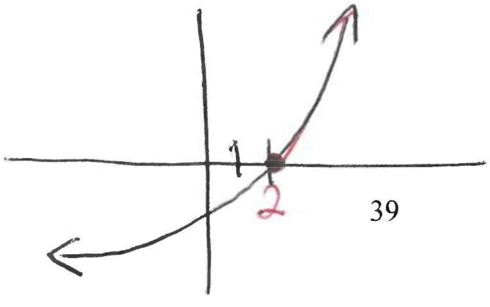
2) 3.9 [✓ 4) 4.3]



12. The graph of $y = 2^x - 4$ is positive on which interval?

1) $(-\infty, \infty)$ 3) $(0, \infty)$

[✓ 2) $(2, \infty)$] 4) $(-4, \infty)$



Graphing Polynomial Functions

1) Type equation into $Y =$
2) 2^{nd} Graph (Table)
   *Plot points in given domain or that fit on the given graph

- Domain: no arrows. No domain: arrows.

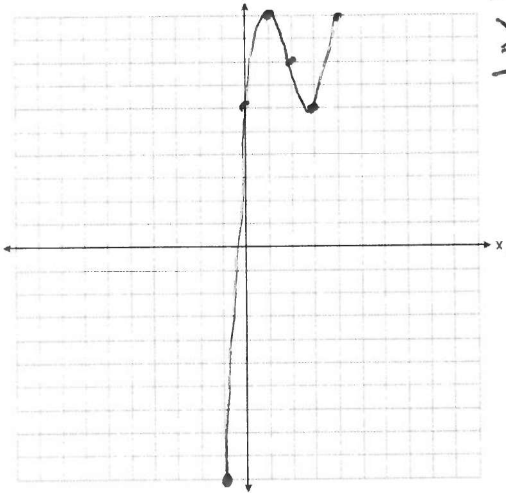
Exponential: $y = \textit{vertical shift}$ or y value that is repeated in the table

Logarithmic: $x = \textit{horizontal shift}$ or the x value that contains the last error

Graph the following equations (Include domain and asymptotes if necessary)

no arrows

1. $f(x) = x^3 - 6x^2 + 9x + 6$ on the domain $-1 \le x \le 4$.



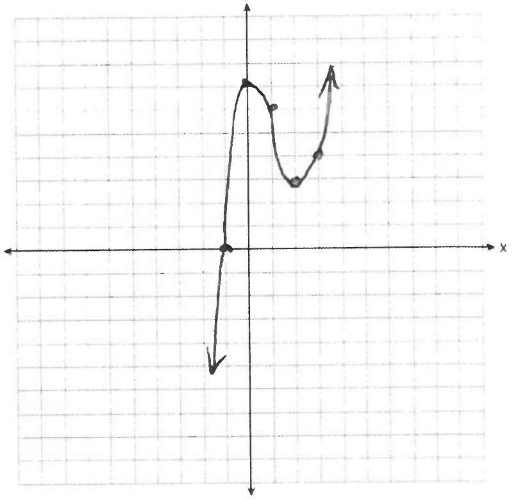
x	y
-1	-10
0	6
1	10
2	8
3	6
4	10

$x \to -\infty$, $f(x) \to -\infty$

$x \to \infty$, $f(x) \to \infty$

arrows!

2. $y = x^3 - 4x^2 + 2x + 7$



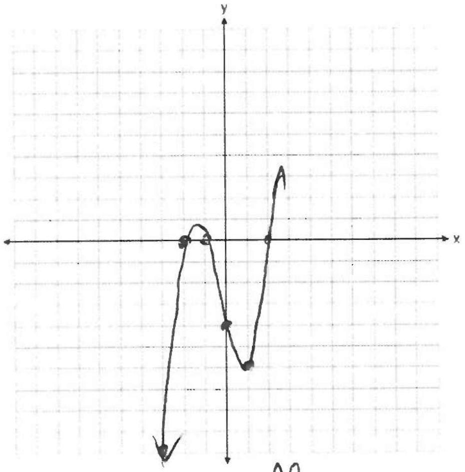
x	y
-1	0
0	7
1	6
2	3
3	4

$x \to -\infty$, $f(x) \to -\infty$

$x \to \infty$, $f(x) \to \infty$

arrows!

3. $p(x) = x^3 + x^2 - 4x - 4$



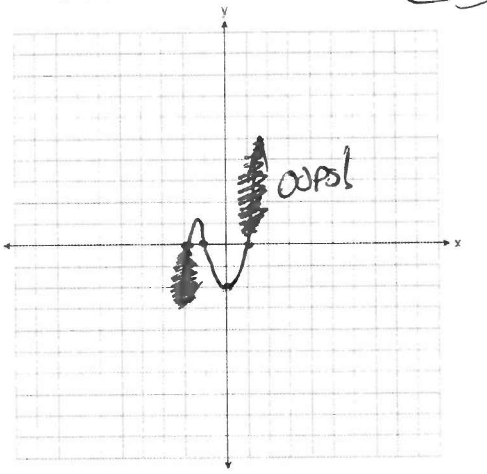
x	y
-3	-10
-2	0
-1	0
0	-4
1	-6
2	0

$x \to -\infty$, $f(x) \to -\infty$

$x \to \infty$, $f(x) \to \infty$

no arrows

4. $p(x) = x^3 + 2x^2 - x - 2$ from $-2 \le x \le 1$



x	y
-2	0
-1	0
0	-2
1	0

$x \to -\infty$, $f(x) \to -\infty$

$x \to \infty$, $f(x) \to \infty$

Inverse of a function $f^{-1}(x)$:

Algebraically

Switch x and y, solve for y

Graphically

Y1: Type in original function

Y2: Type in each answer

Y3: x

Look for symmetry to $y = x$

1. What is the inverse of the function $y = 2x - 3$?

[✓ 1)] $y = \frac{x+3}{2}$ 3) $y = -2x + 3$

2) $y = \frac{x}{2} + 3$ 4) $y = \frac{1}{2x-3}$

$x = 2y - 3$

$+3 \quad +3$

$\frac{x+3}{2} = \frac{2y}{2}$

2. If a function is defined by the equation y = 3x + 2, which equation defines the inverse of this function?

1) x = $\frac{1}{3}y + \frac{1}{2}$ [✓ 3)] $y = \frac{1}{3}x - \frac{2}{3}$

2) y = $\frac{1}{3}x + \frac{1}{2}$ 4) y = -3x – 2

$x = 3y + 2$

$-2 \quad -2$

$\frac{x-2}{3} = \frac{3y}{3}$

$\frac{x-2}{3} = y$

$\frac{1}{3}x - \frac{2}{3} = y$

3. If $f(x) = 5x - 7$, find $f^{-1}(x)$

$y = 5x - 7$

$x = 5y - 7$

$+7 \quad +7$

$\frac{x+7}{5} = \frac{5y}{5}$

$\frac{x+7}{5} = y$

$f^{-1}(x) = \frac{x+7}{5}$

4. What is $g^{-1}(x)$ if $g(x) = 3x + 6$

$y = 3x + 6$

$x = 3y + 6$

$-6 \quad -6$

$\frac{x-6}{3} = \frac{3y}{3}$

$\frac{x-6}{3} = y$

$g^{-1}(x) = \frac{x-6}{3}$

5. What is the inverse of $y = \frac{1}{2}x + 2$?

$2(x) = (\frac{1}{\cancel{2}}y) + (2)2$

$2x = y + 4$

$-4 \quad -4$

$y = 2x - 4$

If multiple choice:
1) Type original into y=
2) Copy down 3 nice points from table
3) Switch x and y
4) Type all 4 choices in and see which has the "switched" table x and y

$(\sqrt{x-2} = \sqrt{y})^2$

6. What is $h^{-1}(x)$ if $h(x)=x^2+2$

$y=x^2+2$

$x=y^2+2$
$-2 \quad -2$

$\sqrt{x-2}=y$

$h^{-1}(x)=\sqrt{x-2}$

7. What is the inverse of the function $y = 4x + 5$?

1) $x = \frac{1}{4}y - \frac{5}{4}$

2) $y = \frac{1}{4}x - \frac{5}{4}$ [✓]

3) $y = 4x - 5$

4) $y = \frac{1}{4x+5}$

$x=4y+5$
$-5 \quad -5$

$\frac{x-5}{4}=\frac{4y}{4}$

$\frac{1}{4}x-\frac{5}{4}=y$

x	y
-3	-2
1	-1
5	0

or $y=4x+5$ switch x and y

x	y
-2	-3
-1	1
0	5

$\rightarrow$

x	y
-3	-2
1	-1
5	0

8. What is the inverse of $f(x) = -6(x - 2)$?

1) $f^{-1}(x) = -2 - \frac{x}{6}$

2) $f^{-1}(x) = 2 - \frac{x}{6}$ [✓]

3) $f^{-1}(x) = \frac{1}{-6(x-2)}$

4) $f^{-1}(x) = 6(x + 2)$

$\frac{x}{-6}=\frac{-6(y-2)}{-6}$

$-\frac{x}{6}=y-2$
$+2 \quad +2$

$2-\frac{x}{6}=y$

x	y
6	1
0	2
-6	3

or $y=-6(x-2)$ switch x and y

x	y
1	6
2	0
3	-6

$\rightarrow$

x	y
6	1
0	2
-6	3

9. Given $f(x) = \frac{1}{2}x + 8$, which equation represents the inverse, $g(x)$?

1) $g(x) = 2x - 8$

2) $g(x) = 2x - 16$

3) $g(x) = -\frac{1}{2}x + 8$

4) $g(x) = -\frac{1}{2}x - 16$

$2(x=\frac{1}{2}y+8)2$

$2x=y+16$
$-16 \quad -16$

$2x-16=y$

x	y
6	-4
7	-2
8	0

or $y=\frac{1}{2}x+8$ switch x and y

x	y
-4	6
-2	7
0	8

$\rightarrow$

x	y
6	-4
7	-2
8	0

10. The inverse of the function $f(x) = \frac{x+1}{x-2}$ is

1) $f^{-1}(x) = \frac{x+1}{x+2}$

2) $f^{-1}(x) = \frac{2x+1}{x-1}$ [✓]

3) $f^{-1}(x) = \frac{x+1}{x-2}$

4) $f^{-1}(x) = \frac{x-1}{x+1}$

nope, not doing it

x	y
0	-1
-2	1
4	3

or $y=\frac{x+1}{x-2}$ switch x and y

x	y
-1	0
1	-2
3	4

$\rightarrow$

x	y
0	-1
-2	1
4	3

11. What is the inverse of $f(x) = \frac{x}{x+2}$, where $x \neq -2$?

okay, fine, but just [illegible]

1) $f^{-1}(x) = \frac{2x}{x-1}$

2) $f^{-1}(x) = \frac{-2x}{x-1}$ [✓]

3) $f^{-1}(x) = \frac{x}{x-2}$

4) $f^{-1}(x) = \frac{-x}{x-2}$

$(y+2)(x=\frac{y}{y+2})(y+2)$

$xy+2x=y$
$-y-2x \quad -y-2x$

$xy-y=-2x$

$\frac{y(x-1)}{x-1}=\frac{-2x}{x-1}$

x	y
3	-3
-1	-1
0	0

or $y=\frac{x}{x+2}$ switch x and y

x	y
-3	3
-1	-1
0	0

$\rightarrow$

x	y
3	-3
-1	-1
0	0

Name Schlansky
Mr. Schlansky

even
f(x)=f(-x)
symmetric to the y-axis
(y-axis cuts graph in half)

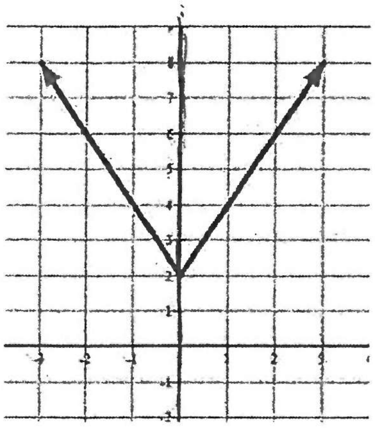
Odd
f(-x)=-f(x)
symmetric to the origin
(turn upside down and image is the same)

Date ____________________
Algebra II

Even and Odd Functions

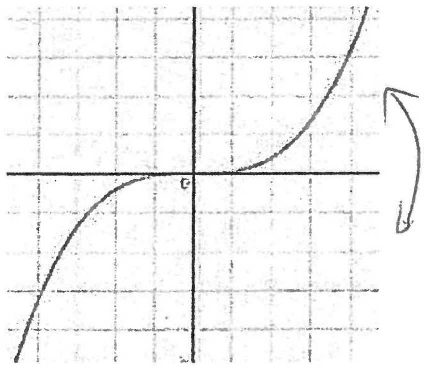
Determine graphically whether the following functions are even, odd, or neither

1.



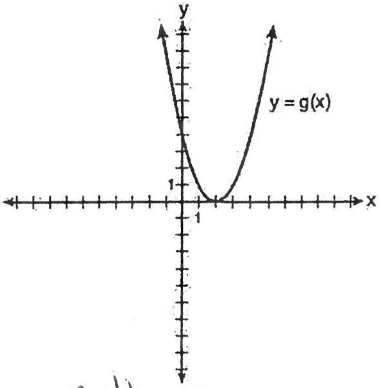
even because it's symmetric to the y-axis

2.



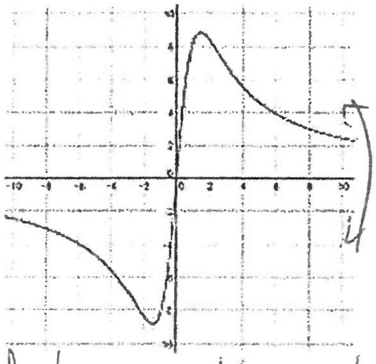
odd because it's symmetric to the origin

3.



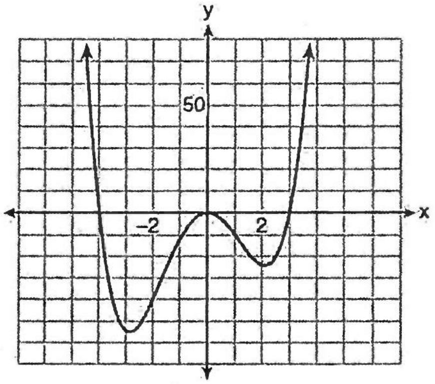
neither

4.



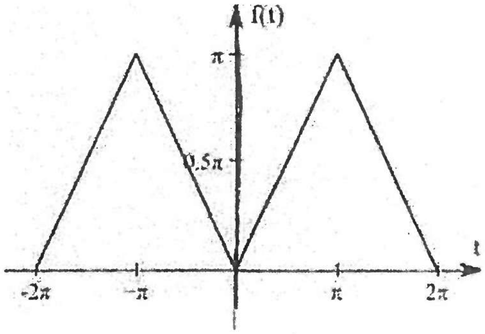
odd because it's symmetric to the origin

5.



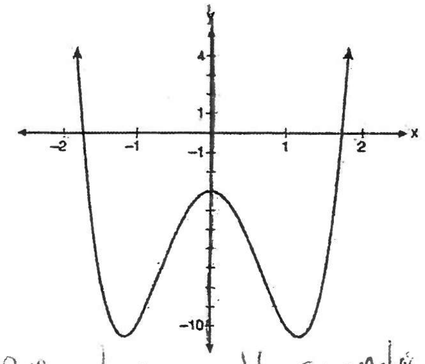
neither

6.



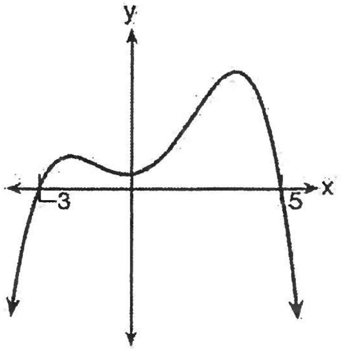
even because it's symmetric to the y-axis

7.



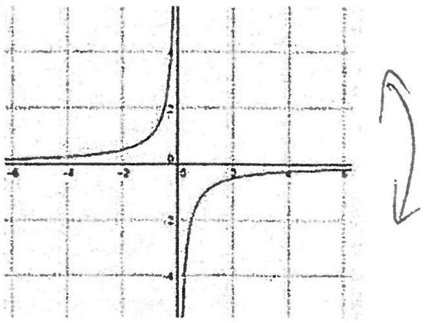
even because it's symmetric to the y-axis

8.



neither

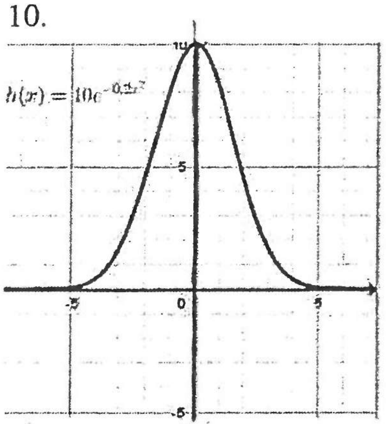
9.



odd because it's symmetric to the origin

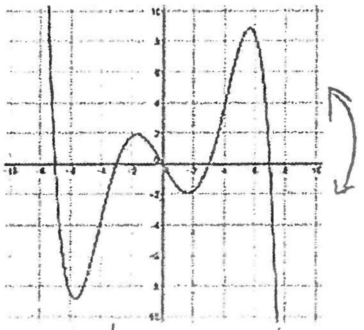
43
42

10.



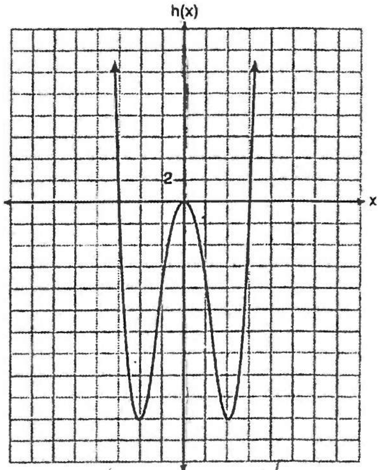
even because it's symmetric to the y-axis

11.



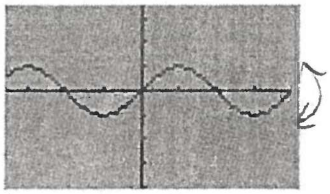
odd because it's symmetric to the origin

12.



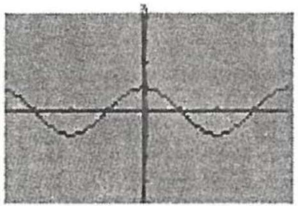
even because it's symmetric to the y-axis

13.



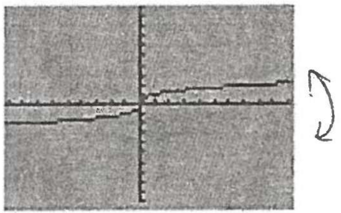
odd because it's symmetric to the origin

14.



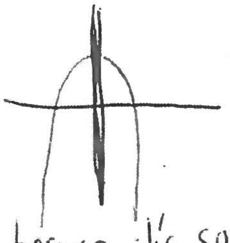
even because it's symmetric to the y-axis

15.



odd because it's symmetric to the origin.

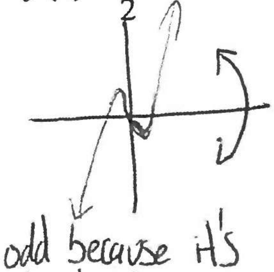
16. $f(x) = -x^4 + 4$



even because it's symmetric to the y-axis

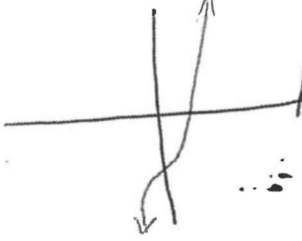
Type into y=

17. $f(x) = \frac{1}{2}x^5 - 2x$



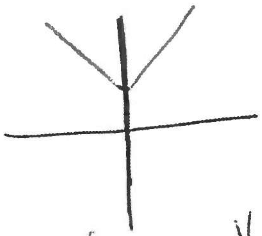
odd because it's symmetric to the origin

18. $f(x) = 4x^3 - 6$



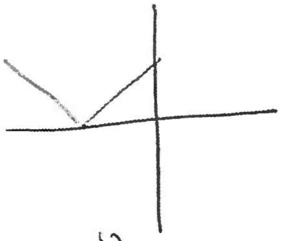
neither

19. $f(x) = |x| + 4$



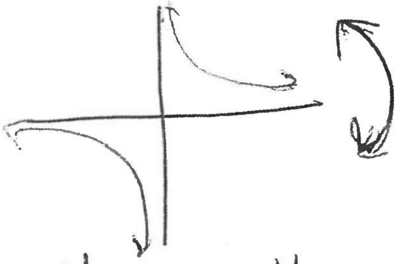
even because it's symmetric to the y-axis

20. $f(x) = |x + 4|$



neither

21. $f(x) = \frac{10}{x}$



odd because it's symmetric to the origin

Transforming Functions

Translations (+ or -)

If adding to f(x), the graph moves up or down

If adding to x, the graph moves left or right (the opposite direction in which you would think)

$y = f(x) + a$ moves UP a units

$y = f(x) - a$ moves DOWN a units

$y = f(x + a)$ moves LEFT a units

$y = f(x - a)$ moves RIGHT a units

Reflections (Negative)

$y = -f(x)$ is a reflection over the x axis (negate the y)

$y = f(-x)$ is a reflection over the y axis (negate the x)

1. If $g(x) = f(x-4)+2$, how is the graph of $f(x)$ translated to form the graph of $g(x)$?

right 4 up 2

2. If $h(x) = f(x+1) - 3$, how is the graph of $f(x)$ translated to form the graph of $g(x)$?

left 1 down 3

3. How is the parent function transformed to create $f(x) = |x+3| - 2$?

left 3 down 2

4. How is the parent function transformed to create $f(x) = (x-4)^2 + 3$?

right 4 up 3

5. Relative to the graph of $y = 3\sin x$, what is the shift of the graph of $y = 3\sin\left(x + \frac{\pi}{3}\right)$?

left $\frac{\pi}{3}$

1) $\frac{\pi}{3}$ right [✓ 2) $\frac{\pi}{3}$ left] 3) $\frac{\pi}{3}$ up 4) $\frac{\pi}{3}$ down

6. Given the parent function $p(x) = \cos x$, which phrase best describes the transformation used to obtain the graph of $g(x) = \cos(x + a) - b$, if *a* and *b* are positive constants?

1) right *a* units, up *b* units
2) right *a* units, down *b* units
3) left *a* units, up *b* units
[✓ 4) left *a* units, down *b* units]

left a down b

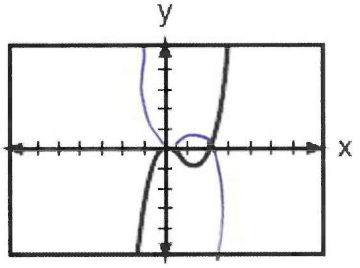
7. If $f(x) = \log_3 x$ and $g(x)$ is the image of $f(x)$ after a translation five units to the left, which equation represents $g(x)$?

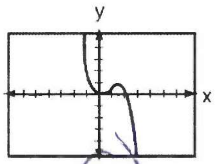
[✓ 1)] $g(x) = \log_3(x+5)$ → left 5
3) $g(x) = \log_3(x-5)$
2) $g(x) = \log_3 x + 5$
4) $g(x) = \log_3 x - 5$

8. The accompanying graph represents the equation $y = f(x)$.

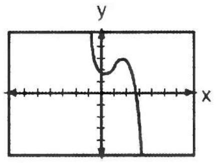
Which graph represents $g(x)$, if $g(x) = -f(x)$?

reflect over x-axis

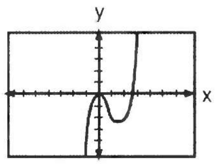




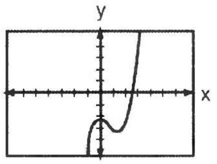
[✓ (1)]



(3)

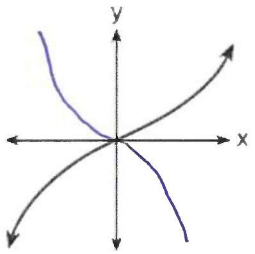


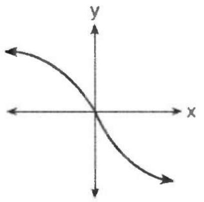
(2)



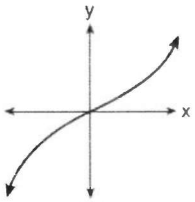
(4)

9. The graph below represents f(*x*).

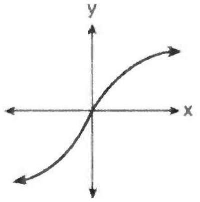




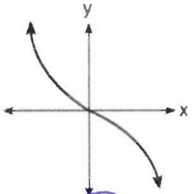
(1)



(3)



(2)

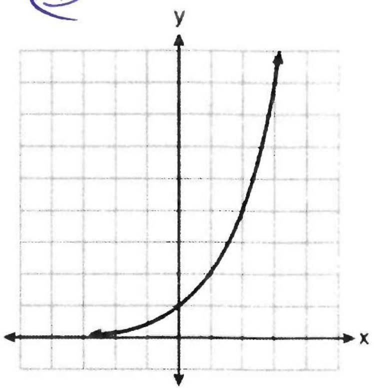


[✓ (4)]

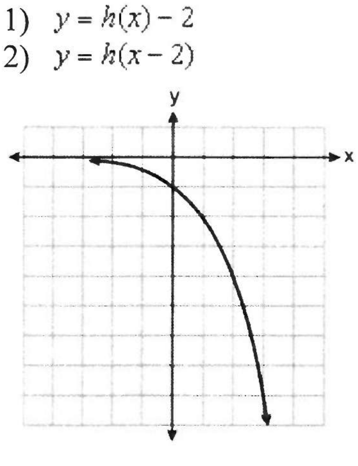
Which graph best represents f(-*x*)?

reflect over y-axis

10. Consider the function $y = h(x)$, defined by the graph to the right. Which equation could be used to represent the graph shown below?



1) $y = h(x) - 2$
[✓ 3)] $y = -h(x)$
2) $y = h(x-2)$
4) $y = h(-x)$



reflect over x-axis

Average rate of change: $\frac{f(b)-f(a)}{b-a}$ or $\frac{y_2-y_1}{x_2-x_1}$

Always create a table!

1) If given table, circle values in the table.
2) If given a graph, pull y values from the graph.
3) If given an equation, type into y= and pull the values from the table.

Context: "On average, from a to b, the y topic is increasing/decreasing by AROC y units per x unit"

Intervals:

If given graph, the steepest slope is the greatest average rate of change. The flattest slope is the smallest average rate of change. If you cannot tell, find the average rate of change for each interval.

If given table, calculate the average rate of change for each interval.

1. The function $h(x)$ is given in the table below. Which of the following gives its average rate of change over the interval $2 \le x \le 6$?

[✓ (1) $-\frac{3}{2}$]

(3) $-\frac{7}{6}$

(2) $\frac{6}{4}$

(4) -1

x	$h(x)$
0	10
2	9
4	6
6	3

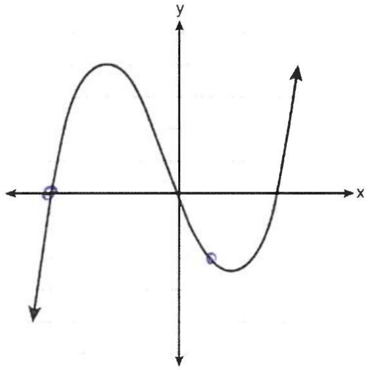
$\frac{3-9}{6-2} = \frac{-3}{2}$

2. What is the average rate of change from 0 to 2?

x	f(x)
0	1
1	2
2	5
3	7

$\frac{5-1}{2-0} = 2$

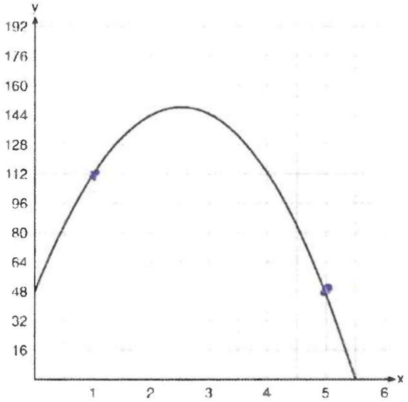
3. The graph of $p(x)$ is shown below. What is the average rate of change over the interval $-4 \le x \le 1$?



X	y
-4	0
1	-2

$\frac{-2-0}{1--4} = -\frac{2}{5}$

4. A ball is thrown into the air from the edge of a 48-foot-high cliff so that it eventually lands on the ground. The graph below shows the height, y, of the ball from the ground after x seconds. What is the average rate of change of the ball between 1 and 5 seconds?



x	y
1	112
5	48

$$\frac{48-112}{5-1} = -16$$

5. For the function $\overset{y=}{f(x)} = 3^x$, find the average rate of change over the interval -5 to -1 rounded to the nearest thousandth.

x	y
-5	.00412
-1	.3333

$$\frac{.3333 - .00412}{-1 - -5} = .082$$

6. Find the average rate of change of the function $f(t) = \overset{y=}{2500}(0.97)^{4t}$ over the interval $10 \le t \le 15$ rounded to the nearest tenth.

x	y
10	739.28
15	402.02

$$\frac{402.02 - 739.28}{15-10} = -67.5$$

7. The distance needed to stop a car after applying the brakes varies directly with the square of the car's speed. The table below shows stopping distances for various speeds. Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.

Speed (mph)	10	20	30	40	50	60	70
Distance (ft)	6.25	25	56.25	100	156.25	225	306.25

x	y
50	156.25
70	306.25

$$\frac{306.25 - 156.25}{70-50} = 7.5$$

On average, from 50 mph to 70 mph, the braking distance increases by 7.5 ft per mph

8. The average monthly high temperature in Buffalo, in degrees Fahrenheit, can be modeled by the function $B(t) = 25.29\sin(0.4895t - 1.9752) + 55.2877$, where t is the month number (January = 1). State, to the *nearest tenth*, the average monthly rate of temperature change between August and November. Explain its meaning in the given context.

8

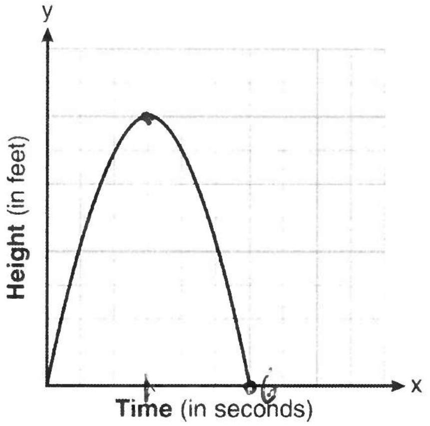
11

x	y
8	78.866
11	48.598

$$\frac{48.598 - 78.866}{11-8}$$

-10.1 On average, from August to November, the average high monthly temperature in Buffalo decreases by 10.1° per month.

9. The graph below represents the parabolic path of a ball kicked by a young child. Find the average rate of change from 3 to 6 seconds. Explain its meaning in the context of the problem.



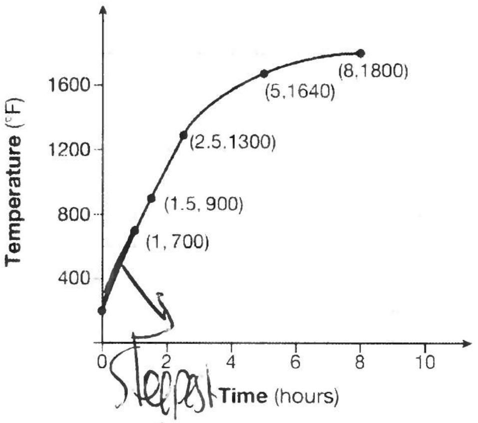
x	y
3	8
6	0

$$\frac{0-8}{6-3} = -\frac{8}{3}$$

On average, from 3 seconds to 6 seconds, the height of the ball decreased by $\frac{8}{3}$ ft per second.

10. Firing a piece of pottery in a kiln takes place at different temperatures for different amounts of time. The graph below shows the temperatures in a kiln while firing a piece of pottery after the kiln is preheated to 200°F.
During which time interval did the temperature in the kiln show the greatest average rate of change?

[✓ 1)] 0 to 1 hour
2) 1 hour to 1.5 hours
3) 2.5 hours to 5 hours
4) 5 hours to 8 hours

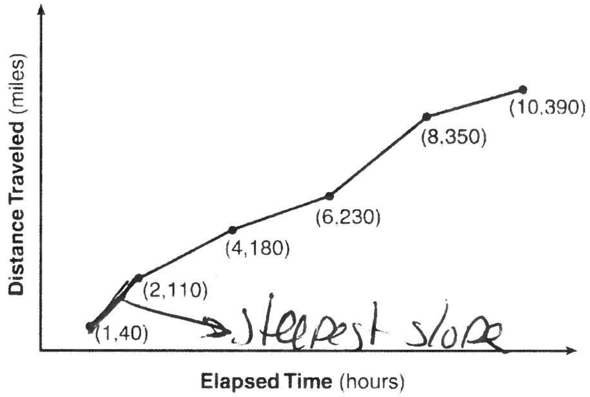


Steepest slope

11. The Jamison family kept a log of the distance they traveled during a trip, as represented by the graph below.

During which interval was their average speed the greatest?

[✓ 1)] the first hour to the second hour
2) the second hour to the fourth hour
3) the sixth hour to the eighth hour
4) the eighth hour to the tenth hour



12. The table below shows the year and the number of households in a building that had high-speed broadband internet access.

Number of Households	11	16	23	33	42	47
Year	2002	2003	2004	2005	2006	2007

For which interval of time was the average rate of change the *smallest*?

[✓ 1)] 2002 - 2004
2) 2003 - 2005
3) 2004 - 2006
4) 2005 - 2007

1)

X	y
2002	11
2004	23

$\boxed{\frac{23-11}{2004-2002}=6}$

2)

x	y
2003	16
2005	33

$\frac{33-16}{2005-2003}=8.5$

3)

x	y
2004	23
2006	42

$\frac{42-23}{2006-2004}=9.5$

4)

X	y
2005	33
2007	47

$\frac{47-33}{2007-2005}=7$

13. Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of *B* dollars after *m* months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after *m* months. Over which interval of time is her average rate of change for the balance on her credit card account the greatest?

m	B
0	1000.00
10	1172.00
19	1352.00
36	1770.80
60	2591.90
69	2990.00
72	3135.80
73	3186.00

1) $\frac{2591.90-1172}{60-10}=28.398$

3) $\frac{3135.80-1770.80}{72-36}=37.916$

2) $\frac{2990-1352}{69-19}=32.76$

4) $\frac{3186-2591.90}{73-60}\boxed{=45.7}$

1) month 10 to month 60
2) month 19 to month 69
3) month 36 to month 72
[✓ 4)] month 60 to month 73

3 X 3 Linear Systems

Matrix Method: (Elimination will be in the equations packet)

Apps
PlySmlt2
2

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = A^{-1}B$$

1) 2nd Matrix, Edit, A, 3X3 (Coefficient of the left hand side)
2) 2nd Matrix, Edit, B, 3X1 (Right hand side)
3) $A^{-1}B$

1. Which value is contained in the solution of the system shown below?

$2x + y - z = 1$

$x - 2y + z = 0$

$3x - y + 2z = 7$

1) 0
2) -1
3) 2 ✓
4) -3

$\begin{pmatrix}x\\y\\z\end{pmatrix} = A^{-1}B$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2&1&-1\\1&-2&1\\3&-1&2\end{pmatrix}^{-1}\begin{pmatrix}1\\0\\7\end{pmatrix}$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$

2. Which value is *not* contained in the solution of the system shown below?

$a + 5b - c = -20$

$4a - 5b + 4c = 19$

$-a - 5b - 5c = 2$

1) −2
2) 2 ✓
3) 3
4) −3

$\begin{pmatrix}x\\y\\z\end{pmatrix} = A^{-1}B$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1&5&-1\\4&-5&4\\-1&-5&-5\end{pmatrix}^{-1}\begin{pmatrix}-20\\19\\2\end{pmatrix}$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}-2\\-3\\3\end{pmatrix}$

3. Which value is contained in the solution of the system shown below?

$3x + y + z = -4$

$x - 2y + z = -5$

$2x + 3y - 2z = -9$

3) -3
3) -5
4) -4 ✓
4) -9

$\begin{pmatrix}x\\y\\z\end{pmatrix} = A^{-1}B$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}3&1&1\\1&-2&1\\2&3&-2\end{pmatrix}^{-1}\begin{pmatrix}-4\\-5\\-9\end{pmatrix}$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}-4\\3\\5\end{pmatrix}$

4. Which value is *not* contained in the solution of the system shown below?

$4x - 5y + 2z = 130$

$3x + 2y - 7z = -99$

$10x - 6y - 4z = 112$

1) -8
2) -12
3) 10
4) 15

$\begin{pmatrix}x\\y\\z\end{pmatrix} = A^{-1}B$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}4&-5&2\\3&2&-7\\10&-6&-4\end{pmatrix}^{-1}\begin{pmatrix}130\\-99\\112\end{pmatrix}$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}10\\-12\\15\end{pmatrix}$

5. What is the solution of the system shown below?

$6x - 3y + 2z = 78$

$4x + 2y - 5z = -40$

$-3x - 4y - 3z = -41$

1) $x = 2,\ y = -4,\ z = 6$
2) $x = 7,\ y = -4,\ z = 12$ ✓
3) $x = 78,\ y = -40,\ z = -41$
4) $x = 6,\ y = 2,\ z = -3$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = A^{-1}B$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}6&-3&2\\4&2&-5\\-3&-4&-3\end{pmatrix}^{-1}\begin{pmatrix}78\\-40\\-41\end{pmatrix}$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}7\\-4\\12\end{pmatrix}$

6. For the system shown below, what is the value of z?

$y = -2x + 14$
$+2x\ \ +2x$

$3x - 4z = 2$

$3x - y = 16$

$2x + y + 0z = 14$
$3x + 0y - 4z = 2$
$3x - y + 0z = 16$

1) 5
2) 2
3) 6
4) 4 ✓

$\begin{pmatrix}x\\y\\z\end{pmatrix} = A^{-1}B$

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2&1&0\\3&0&-4\\3&-1&0\end{pmatrix}^{-1}\begin{pmatrix}14\\2\\16\end{pmatrix}$

$\boxed{\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}6\\2\\4\end{pmatrix}}$

Exponents

FOLLOW THE FOLLOWING ORDER! STRUCTURE IS IMPERATIVE!!!

1) Radicals are fractional exponents (Fractional exponent = $\frac{power}{root}$)

2) Get rid of parenthesis (exponent outside parenthesis goes to everything inside)
Negative exponents are fractions (Move whatever is being raised to the negative power)
Clean it up (Multiply, divide/reduce, evaluate/put into radical)
*Add exponents when multiplying. Subtract exponents when dividing. Use a calculator for fractions.

Negative exponents are fractions! $x^{-2} = \frac{1}{x^2}$

If exponent is outside parenthesis, everything gets it $\left(\frac{xy}{z}\right)^3 = \frac{x^3y^3}{z^3}$

Rewrite the following as radicals

1. $x^{\frac{2}{3}}$ $\sqrt[3]{X^2}$

2. $x^{\frac{3}{4}}$ $\sqrt[4]{X^3}$

3. $x^{\frac{5}{6}}$ $\sqrt[6]{X^5}$

4. $x^{\frac{1}{3}}$ $\sqrt[3]{X}$

5. $x^{\frac{3}{2}}$ $\sqrt{X^3}$

6. $x^{\frac{1}{2}}$ $\sqrt{X}$

Rewrite the following using fractional exponents

7. $\sqrt[3]{x^4}$ $X^{\frac{4}{3}}$

8. $\sqrt[5]{x^3}$ $X^{3/5}$

9. $\sqrt[4]{x^7}$ $X^{\frac{7}{4}}$

10. $\sqrt[2]{x^3}$ $X^{\frac{3}{2}}$

11. $\sqrt[6]{x^5}$ $X^{\frac{5}{6}}$

12. $\sqrt[2]{x}^{1}$ $X^{\frac{1}{2}}$

Express with a rational exponent

13. $\sqrt[4]{x^3} \bullet \sqrt[2]{x^5}$

$x^{\frac{3}{4}} \cdot x^{\frac{5}{2}}$

$\frac{3}{4} + \frac{5}{2} = \frac{13}{4}$ $\boxed{x^{\frac{13}{4}}}$

14. $\sqrt[3]{b^5} \bullet \sqrt[4]{b^1}$

$\boxed{b^{\frac{23}{12}}}$

$b^{\frac{5}{3}} \cdot b^{\frac{1}{4}}$

$\frac{5}{3} + \frac{1}{4} = \frac{23}{12}$

15. $\frac{\sqrt[6]{x^5}}{\sqrt[3]{x^2}}$ $\frac{5}{6} - \frac{2}{3} = \frac{1}{6}$

$\frac{x^{\frac{5}{6}}}{x^{\frac{2}{3}}}$ $\boxed{x^{\frac{1}{6}}}$

16. $\frac{\sqrt[2]{m^7}}{\sqrt[5]{m^2}}$

$\frac{m^{\frac{7}{2}}}{m^{\frac{2}{5}}}$ $\frac{7}{2} - \frac{2}{5} = \frac{31}{10}$

$\boxed{m^{\frac{31}{10}}}$

17. $\frac{\sqrt[3]{x^2} \bullet \sqrt[2]{x^5}}{\sqrt[6]{x}}$ $\frac{2}{3} + \frac{5}{2} = \frac{19}{6}$

$\frac{19}{6} - \frac{1}{6} = 3$

$\frac{x^{\frac{2}{3}} \cdot x^{\frac{5}{2}}}{x^{\frac{1}{6}}} = \frac{x^{\frac{19}{6}}}{x^{\frac{1}{6}}} = \boxed{x^3}$

18. $\frac{x^1\sqrt[2]{x^3}}{\sqrt[3]{x^5}}$

$1 + \frac{3}{2} = \frac{5}{2}$

$\frac{5}{2} - \frac{5}{3} = \frac{5}{6}$

$\frac{x^1 \cdot x^{\frac{3}{2}}}{x^{\frac{5}{3}}}$ $\frac{x^{\frac{5}{2}}}{x^{\frac{5}{3}}} = x^{\frac{5}{6}}$

19. $a^1\sqrt[5]{a^4}$

$a^1 \cdot a^{\frac{4}{5}}$ $1 + \frac{4}{5} = \frac{9}{5}$

$\boxed{a^{\frac{9}{5}}}$

20. $2xy^2\sqrt[3]{x^2y}$

$1 + \frac{2}{3} = \frac{5}{3}$

$2 + \frac{1}{3} = \frac{7}{3}$

$2x^1y^2(x^{\frac{2}{3}}y^{\frac{1}{3}})$

$\boxed{2x^{\frac{5}{3}}y^{\frac{7}{3}}}$

21 10. Kenzie believes that for $x \geq 0$, the expression $\left(\sqrt[7]{x^2}\right)\left(\sqrt[5]{x^3}\right)$ is equivalent to $\sqrt[35]{x^6}$. Is she correct? Justify your response algebraically.

$$\left(x^{\frac{2}{7}}\right)\left(x^{\frac{3}{5}}\right) \neq x^{\frac{6}{35}}$$

$$\frac{2}{7}+\frac{3}{5}=\frac{31}{35}$$

$$x^{\frac{31}{35}} \neq x^{\frac{6}{35}}$$

No!

22 11. Justify why $\dfrac{\sqrt[3]{x^2y^5}}{\sqrt[4]{x^3y^4}}$ is equivalent to $x^{\frac{-1}{12}}y^{\frac{2}{3}}$ using properties of rational exponents, where $x \neq 0$ and $y \neq 0$.

$$\frac{\left(x^2y^5\right)^{\frac{1}{3}}}{\left(x^3y^4\right)^{\frac{1}{4}}} = x^{-\frac{1}{12}}y^{\frac{2}{3}}$$

$$\frac{x^{\frac{2}{3}}y^{\frac{5}{3}}}{x^{\frac{3}{4}}y^{1}} = x^{\frac{-1}{12}}y^{\frac{2}{3}} \qquad \frac{2}{3}-\frac{3}{4}=\frac{-1}{12} \qquad \frac{5}{3}-1=\frac{2}{3}$$

$$x^{-\frac{1}{12}}y^{\frac{2}{3}} = x^{-\frac{1}{12}}y^{\frac{2}{3}}$$

✓

23 ~~20~~ 12. For n and $p > 0$, is the expression $\left(p^2n^{\frac{1}{2}}\right)^8\sqrt[3]{p^5n^4}^{\,1}$ equivalent to $p^{18}n^6\sqrt{p}$? Justify your answer.

$$\left(p^2n^{\frac{1}{2}}\right)^8\left(p^5n^4\right)^{\frac{1}{3}} = p^{18}n^6p^{\frac{1}{2}}$$

$$p^{16}n^4p^{\frac{5}{2}}n^2 = p^{\frac{37}{2}}n^6$$

$$p^{\frac{37}{2}}n^6 = p^{\frac{37}{2}}n^6$$

24 ~~21~~ 13. Use the properties of rational exponents to determine the value of y for the equation:

$$\frac{\sqrt[3]{x^8}}{\left(x^4\right)^{\frac{1}{3}}} = x^y,\ x > 1$$

$$\frac{x^{\frac{8}{3}}}{x^{\frac{4}{3}}} = x^y \qquad x^{\frac{4}{3}} = x^y$$

$$\frac{4}{3} = y$$

25 ~~22~~ 14. Given that $\left(\dfrac{y^{\frac{17}{8}}}{y^{\frac{5}{4}}}\right)^{-4} = y^n$, where $y > 0$, determine the value of n.

$$\frac{y^{-\frac{17}{2}}}{y^{-5}} = y^n$$

$$\frac{y^5}{y^{\frac{17}{2}}} = y^n \qquad -\frac{7}{2} = n$$

$$y^{-\frac{7}{2}} = y^n$$

26. Explain what a rational exponent, such as $\frac{5}{2}$ means. Use this explanation to evaluate $9^{\frac{5}{2}}$.

radicals are fractional exponents

$\frac{\text{power}}{\text{root}}$

$9^{\frac{5}{2}} = (\sqrt[2]{9})^5$

$\boxed{3^5 = 243}$

27. Explain how $125^{\frac{4}{3}}$ can be evaluated using properties of rational exponents to result in an integer answer.

radicals are fractional exponents

$\frac{\text{power}}{\text{root}}$

$125^{\frac{4}{3}} = (\sqrt[3]{125})^4$

$\boxed{5^4 = 625}$

28. Explain how $(-8)^{\frac{4}{3}}$ can be evaluated using properties of rational exponents to result in an integer answer.

radicals are fractional exponents

$\frac{\text{power}}{\text{root}}$

$(-8)^{\frac{4}{3}} = (\sqrt[3]{-8})^4$

$\boxed{(-2)^4 = 16}$

Express in simplest form:

29. $\frac{2x^{-2}y^{-2}}{4y^{-5}}$

$\frac{2y^5}{4x^2y^2}$

$\boxed{\frac{1y^3}{2x^2}}$

Radicals are fractional exponents
get rid of parentheses
negative exponents are fractions

clean it up MD

30. $(5^{-2}a^3b^{-4})^{-1}$

$\frac{5^2a^{-3}b^4}{1}$ $\frac{5^2b^4}{a^3}$ $\boxed{\frac{25b^4}{a^3}}$

31. $\frac{(3x^{-2}y^2)^2}{9x^{-3}y^{-3}}$

$\frac{3^2x^{-4}y^4}{9x^{-3}y^{-3}}$ $\frac{3^2y^4x^3y^3}{9x^4}$

$\frac{\cancel{9}x^3y^7}{\cancel{9}x^4} = \boxed{\frac{y^7}{x}}$

32. $\frac{3x^{-4}y^5}{(2x^3y^{-7})^{-2}}$

$\frac{3x^{-4}y^5}{2^{-2}x^{-6}y^{14}}$ $\frac{3y^5 2^2 x^6}{x^4y^{14}}$

$\frac{3(4)x^6y^5}{x^4y^{14}}$ $\boxed{\frac{12x^2}{y^9}}$

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Algebra II

Graphing Exponential and Logarithmic Functions

For the following equations, graph the equation and the asymptote. State the domain, range, equation of the asymptote, and end behavior.

1. $y = 2^x - 3$

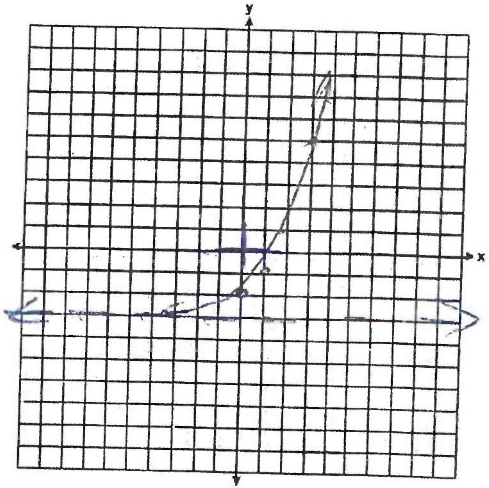
Domain: $(-\infty, \infty)$

Range: $(-3, \infty)$

Asymptote: $y = -3$

End Behavior:
$x \to -\infty,\ f(x) \to -3$
$x \to \infty,\ f(x) \to \infty$

X	y
0	-2
1	-1
2	1
3	5



2. $y = \frac{1}{2}^{x-3} + 1$

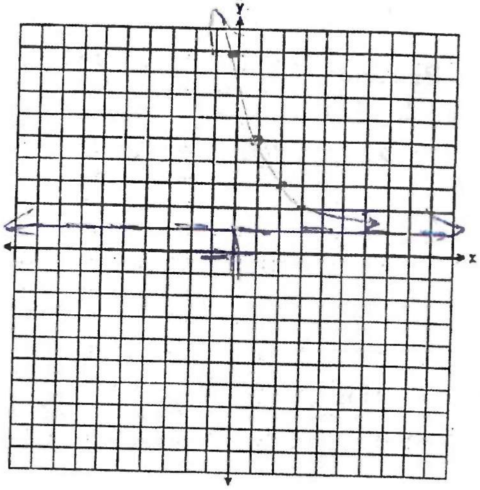
Domain: $(-\infty, \infty)$

Range: $(1, \infty)$

Asymptote: $y = 1$

End Behavior:
$x \to -\infty,\ f(x) \to \infty$
$x \to \infty,\ f(x) \to 1$

X	y
0	9
1	5
2	3
3	2



3. $y = -3^{x-2} + 4$

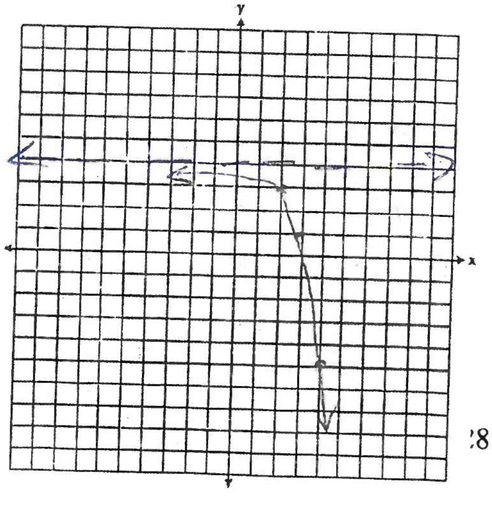
Domain: $(-\infty, \infty)$

Range: $(-\infty, 4)$

Asymptote: $y = 4$

End Behavior:
$x \to -\infty,\ f(x) \to 4$
$x \to \infty,\ f(x) \to -\infty$

X	y
2	3
3	1
4	-5



4 ~~10~~. $y = 4\log_{\frac{1}{2}}(x-3)+1$

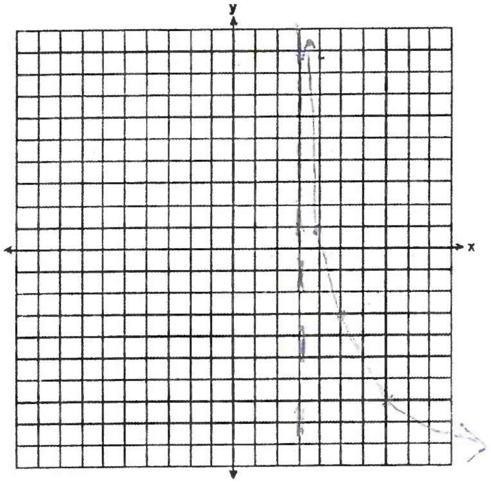
Domain: $(3,\infty)$

Range: $(-\infty,\infty)$

Asymptote: $x=3$

End Behavior:
$x \to 3,\ f(x) \to \infty$
$x \to \infty,\ f(x) \to -\infty$

x	y
3	ERROR
4	1
5	-3
7	-7



5 ~~11~~. $y = 3\log_4(x+1)-8$

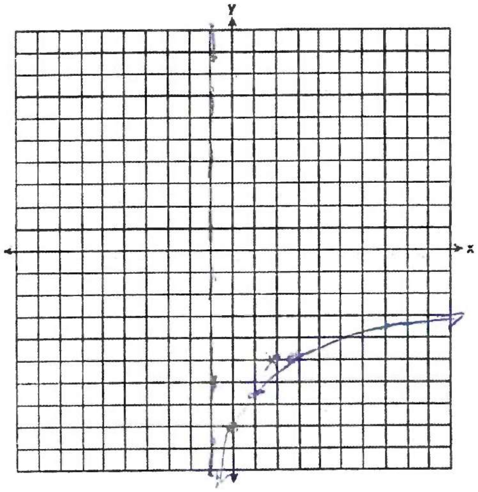
Domain: $(-1,\infty)$

Range: $(-\infty,\infty)$

Asymptote: $x=-1$

End Behavior:
$x \to -1,\ f(x) \to -\infty$
$x \to \infty,\ f(x) \to \infty$

x	y
-1	ERROR
0	-8
1	-6.5
3	-5
7	-3.5



6 ~~12~~. $y = -4\log_2(x+9)+4$

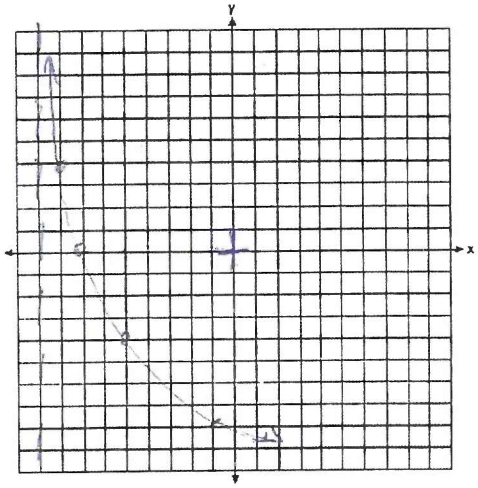
Domain: $(-9,\infty)$

Range: $(-\infty,\infty)$

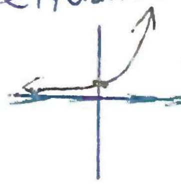
Asymptote: $x=-9$

End Behavior:
$x \to -9,\ f(x) \to \infty$
$x \to \infty,\ f(x) \to -\infty$

x	y
-9	ERROR
-8	4
-7	0
-5	-4
-1	-8

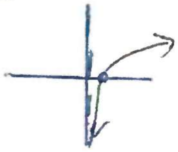


exponential



inverses of each other

logarithmic



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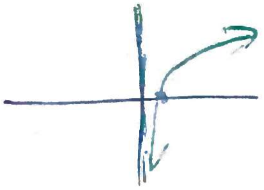
Algebra II

Exponential and Logarithmic Graphs Multiple Choice

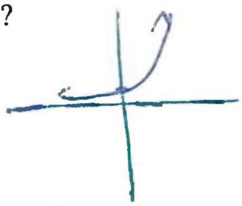
1\. Which statement about the graph of $c(x) = \log_6 x$ is *false*?

[✓ 1)] The asymptote has equation $y = 0$. $x=0$
2) The graph has no y-intercept.
3) The domain is the set of positive reals.
4) The range is the set of all real numbers.



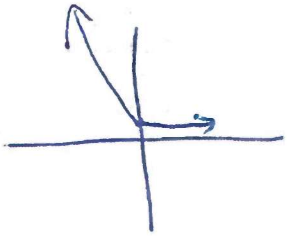
2\. Which statement about the graph of the equation $y = e^x$ is *not* true?

1) It is asymptotic to the x-axis.
2) The domain is the set of all real numbers.
3) It lies in Quadrants I and II.
[✓ 4)] It passes through the point $(e, 1)$. $(0,1)$



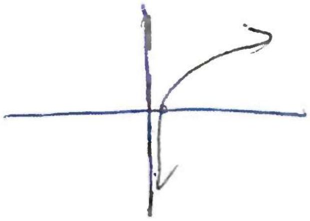
3\. Which statement is true about the graph of $f(x) = \left(\frac{1}{8}\right)^x$?

1) The graph is always increasing.
[✓ 2)] The graph is always decreasing.
3) The graph passes through $(1, 0)$.
4) The graph has an asymptote, $x = 0$.



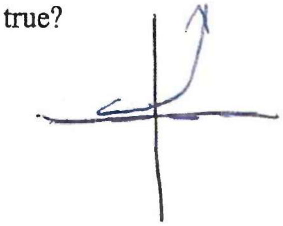
4\. Which statement is *true* regarding the equation $f(x) = \log_7 x$?

[✓ 1)] It is always increasing ✓
2) The graph passes through $(0,1)$ ×
3) The domain is all real numbers ✓
4) The equation of the asymptote is $y=0$ ×
$x=0$



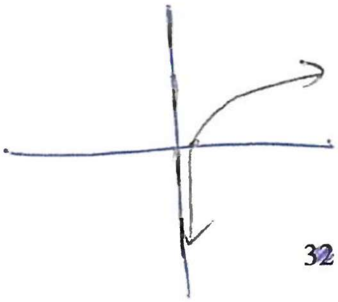
5\. Given the equation $f(x) = \pi^x$, which of the following statements is true?

1) The graph passes through $(\pi,1)$
2) The domain is $[0,\infty)$
[✓ 3)] The graph passes through $(0,1)$
4) The range is all real numbers



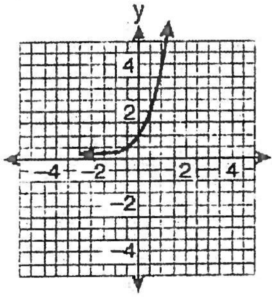
6\. Which statement is *false* regarding the equation $f(x) = \ln x$?

1) It passes through $(1, 0)$ ✓
[✓ 2)] It is always decreasing ×
3) The equation of the asymptote is $x=0$ ✓
4) Its range is $(-\infty, \infty)$ ✓

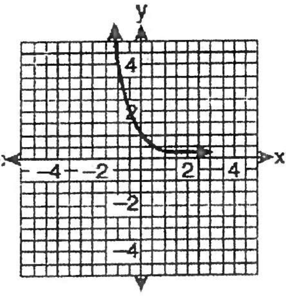


5 ~~7~~. If a function is defined by the equation $f(x) = 4^x$, which graph represents the inverse of this function?

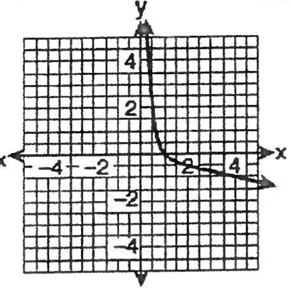
1)



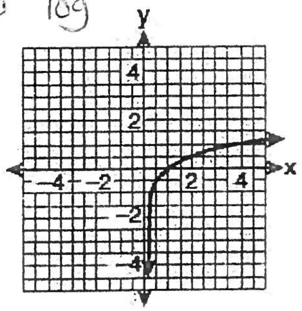
2)



3)

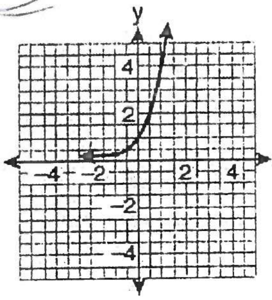


[✓ 4)] log

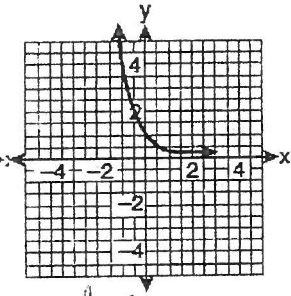


6 ~~8~~. If a function is defined by the equation $f(x) = \log_4 x$, which graph represents the inverse of this function? exponential

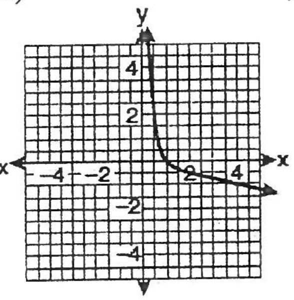
[✓ 1)]



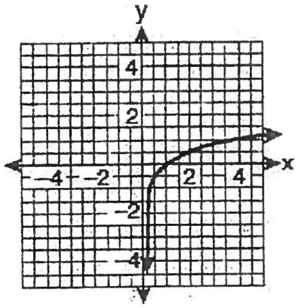
2)



3)

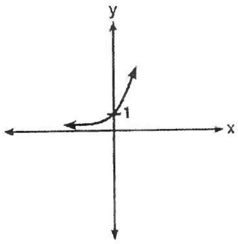


4)

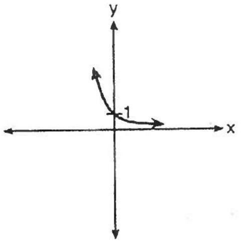


7 ~~9~~. Which sketch shows the inverse of $y = a^x$, where $a > 1$? log

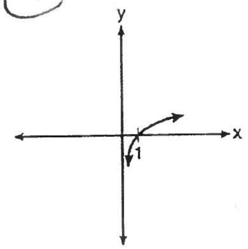
1)



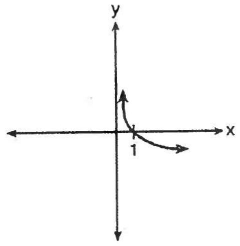
2)



[✓ 3)]



4)



8 ~~10~~. What is the inverse of the function $y = \log_3 x$? exponential

1) $y = x^3$ 2) $y = \log_x 3$ [✓ 3)] $y = 3^x$ 4) $x = 3^y$

9 ~~11~~. If $f(x) = a^x$ where $a > 1$, then the inverse of the function is log

1) $f^{-1}(x) = \log_x a$ [✓ 3)] $f^{-1}(x) = \log_a x$

2) $f^{-1}(x) = a \log x$ 4) $f^{-1}(x) = x \log a$

the base of
the log is
the base
of the exponent

Use the table or the shifts

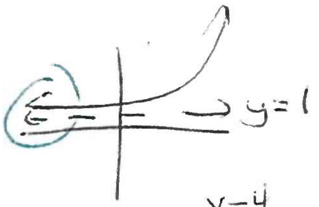
10 ~~12~~. The asymptote of the graph of $f(x) = 5\log(x + 4)$ is
[✓ 1) $y = 6$]
3) $x = 4$
2) $x = -4$
4) $y = 5$

11 ~~13~~. The asymptote of the graph of $j(x) = 2e^{x-4} - 1$ is
1) $x = 4$
[✓ 3) $y = -1$]
2) $x = -4$
4) $y = 2$

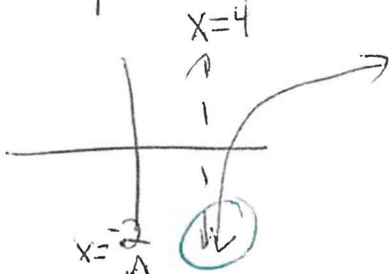
12 ~~14~~. The asymptote of the graph of $e(x) = \log_3(x - 5) + 1$ is
1) $y = 1$
3) $y = 5$
2) $x = 1$
[✓ 4) $x = 5$]

13 ~~15~~. The asymptote of the graph of $m(x) = -3(2)^{x+1} - 4$ is
1) $x = -1$
[✓ 3) $y = -4$]
2) $x = 3$
4) $y = -3$

14 ~~16~~. For the equation $f(x) = 2^{x-3} + 1$, as $x \to -\infty$
1) $f(x) \to -\infty$
3) $f(x) \to \infty$
[✓ 2) $f(x) \to 1$]
4) $f(x) \to 3$

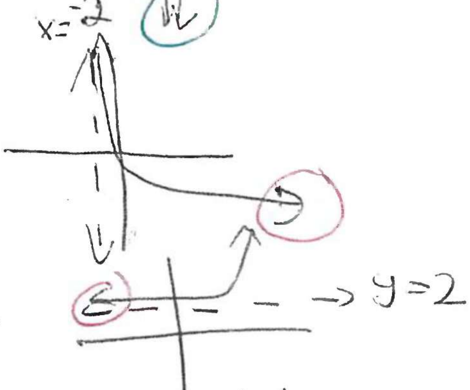


15 ~~17~~. For the equation $f(x) = \log_2(x - 4) + 3$, as $x \to 4$
[✓ 1) $f(x) \to -\infty$]
3) $f(x) \to \infty$
2) $f(x) \to 3$
4) $f(x) \to 4$

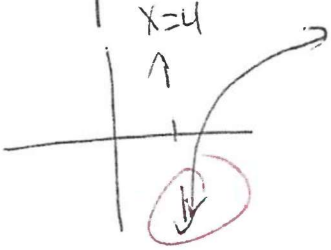


16 ~~18~~. For the equation $f(x) = -\log_3(x + 1) - 2$, as $x \to \infty$
[✓ 1) $f(x) \to -\infty$]
3) $f(x) \to \infty$
2) $f(x) \to -1$
4) $f(x) \to -2$

17 ~~19~~. Given $f(x) = 3^{x-1} + 2$, as $x \to -\infty$
1) $f(x) \to -1$
[✓ 3) $f(x) \to 2$]
2) $f(x) \to 0$
4) $f(x) \to -\infty$



18 ~~20~~. For the equation $f(x) = 3\ln(x - 4) + 1$, $\underline{f(x) \to -\infty}$ as
[✓ 1) $x \to 4$]
3) $x \to \infty$
2) $x \to 1$
4) $x \to -\infty$



1) Isolate the base < add/subtract first
divide last

2) log/ln of both sides

3) Bring exponent to the front

4) Divide to isolate x.

*Use Alpha X to divide

Name Schlansky Date ____________

Mr. Schlansky Algebra II

Variable Exponential Equations

Solve each of the following and round to the *nearest hundredth*.

1. $3^{2x} = 182$

$\log 3^{2x} = \log 182$

$\frac{2x\log 3}{2\log 3} = \frac{\log 182}{2\log 3}$

$\boxed{x = 2.37}$

2. $e^{2n} = 245$

$\ln e^{2n} = \ln 245$

$\frac{\cancel{2}n\ln e}{\cancel{2}\ln e} = \frac{\ln 245}{2\ln e}$

$\boxed{n = 2.75}$

3. $\frac{3(5)^{2x}}{3} = \frac{60}{3}$

$\log 5^{2x} \overset{\log}{=} 20$

$\frac{\cancel{2x}\log 5}{2\log 5} = \frac{\log 20}{2\log 5}$

$\boxed{x = .93}$

4. $\frac{20e^{4x}}{20} = \frac{120}{20}$

$\ln e^{4x} \overset{\ln}{=} 6$

$\frac{4x\ln e}{4} = \frac{\ln 6}{4}$

$\boxed{x = -.45}$

5. $\frac{250(1.04)^{4x}}{250} = \frac{500}{250}$

$\log 1.04^{4x} \overset{\log}{=} 2$

$\frac{4x\log 1.04}{4\log 1.04} = \frac{\log 2}{4\log 1.04}$

$\boxed{x = 4.42}$

6. $\frac{48e^{.12x}}{48} = \frac{60}{48}$

$\ln e^{.12x} \overset{\ln}{=} 1.25$

$\frac{.12x\ln e}{.12} = \frac{\ln 1.25}{.12}$

$x = 1.86$

7. $\frac{1.2(4)^{2x}}{1.2} = \frac{20}{1.2}$

$\log 4^{2x} \overset{\log}{=} 16.\overline{6}$

$\frac{2x\log 4}{2\log 4} = \frac{\log 16.\overline{6}}{2\log 4}$

$\boxed{x = 1.01}$

8. $\frac{400(.987)^{2.5x}}{400} = \frac{300}{400}$

$\log .987^{2.5x} \overset{\log}{=} .75$

$\frac{2.5x\log .987}{2.5\log .987} = \frac{\log .75}{2.5\log .987}$

$\boxed{x = 8.79}$

9. $2(3)^{2x}+8=18$

$-8 \quad -8$

$\frac{\cancel{2}(3)^{2x}}{\cancel{2}}=\frac{10}{2}$

$\log 3^{2x} \overset{\log}{=} 5$

$\frac{\cancel{2x}\log 3}{\cancel{2\log 3}}=\frac{\log 5}{2\log 3}$

$X=.73$

10. $4(2)^{3x}+3=15$

$-3 \quad -3$

$\frac{4(2)^{3x}}{4}=\frac{12}{4}$

$\log 2^{3x} \overset{\log}{=} 3$

$\frac{\cancel{3}x\log 2}{\cancel{3\log 2}}=\frac{\log 3}{3\log 2}$

$X=.53$

11. $8+2e^{-5x}=14$

$-8 \quad -8$

$\frac{2e^{-5x}}{2}=\frac{6}{2}$

$\ln e^{-5x} \overset{\ln}{=} 3$

$\frac{\cancel{-5}x\ln e}{\cancel{-5}}=\frac{\ln 3}{-5}$

$X=-.22$

12. $12+2(5)^{8x}=2000$

$-12 \quad -12$

$\frac{2(5)^{8x}}{2}=\frac{1988}{2}$

$\log 5^{8x} \overset{\log}{=} 994$

$\frac{\cancel{8}x\log 5}{\cancel{8\log 5}}=\frac{\log 994}{8\log 5}$

$X=.54$

13. $\frac{500e^{\frac{x}{2}}}{500}=\frac{200}{500}$

$\ln e^{(\frac{x}{2})} \overset{\ln}{=} .4$

$2(\frac{x}{\cancel{2}})\ln e=(\ln .4)2$

$X=2\ln .4$

$X=-1.83$

14. $\frac{2000(2)^{\frac{x}{4.2}}}{2000}=\frac{1500}{2000}$

$\log 2^{\frac{x}{4.2}} \overset{\log}{=} .75$

$4.2(\frac{x}{\cancel{4.2}}\log 2)=(\log .75)4.2$

$\frac{x\cancel{\log 2}}{\cancel{\log 2}}=\frac{4.2\log .75}{\log 2}$

$X=-1.74$

15. $1.2(3)^{\frac{x}{4.1}}+15=195$

$-15 \quad -15$

$\frac{\cancel{1.2}(3)^{\frac{x}{4.1}}}{\cancel{1.2}}=\frac{180}{1.2}$

$\log 3^{\frac{x}{4.1}} \overset{\log}{=} 150$

$4.1(\frac{x}{\cancel{4.1}}\log 3)=\overset{4.1}{\log 150}$

$\frac{x\cancel{\log 3}}{\cancel{\log 3}}=\frac{4.1\log 150}{\log 3}$

$X=18.70$

16. $18-4(6)^{\frac{x}{3}}=16$

$-18 \quad -18$

$\frac{-4(6)^{\frac{x}{3}}}{-4}=\frac{-2}{-4}$

$\log 6^{\frac{x}{3}} \overset{\log}{=} \frac{1}{2}$

$3(\frac{x}{\cancel{3}}\log 6)=(\log\frac{1}{2})3$

$\frac{x\cancel{\log 6}}{\cancel{\log 6}}=\frac{3\log\frac{1}{2}}{\log 6}$

$X=-1.16$

Exponential Regression Equations

1) **Stat, Edit**
2) **Input x column into L1 and y column into L2**
3) **Stat, Calc, 0: ExpReg**
4) **READ AND ROUND CAREFULLY**

1. Consider the data in the table below.
State an exponential regression equation to model these data, rounding all values to the *nearest thousandth.*

x	1	2	3	4	5	6
y	3.9	6	11	18.1	28	40.3

ExpReg
$y=a(b)^x$
$y=2.459(1.616)^x$

2. A runner is using a nine-week training app to prepare for a "fun run." The table below represents the amount of the program completed, *A*, and the distance covered in a session, *D*, in miles.

A	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{8}{9}$	1
D	2	2	2.25	3	3.25

ExpReg
$y=a(b)^x$
$y=1.223(2.652)^x$

$D=1.223(2.652)^A$

Based on these data, write an exponential regression equation, rounded to the *nearest thousandth*, to model the distance the runner is able to complete in a session as she continues through the nine-week program.

3. A cup of coffee is left out on a countertop to cool. The table below represents the temperature, *F*(*t*), in degrees Fahrenheit, of the coffee after it is left out for *t* minutes.

t	0	5	10	15	20	25
F(t)	180	144	120	104	93.3	86.2

Based on these data, write an exponential regression equation, *F*(*t*), to model the temperature of the coffee. Round all values to the *nearest thousandth.*

ExpReg
$y=a(b)^x$

$F(t)=169.136(.971)^t$

Exponential Regression Equations with Equation Solving

4. Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

Hours (x)	**Average Number of Spores (y)**
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380

ExpReg

$y=a(b)^x$

$y=4.168(3.981)^x$

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*. The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest quarter hour*, that the meat can be kept at room temperature safely.

$$\frac{100}{4.168}=\frac{4.1\cancel{68}(3.981)^x}{4.168}$$

$$\log 23.99 = \log 3.981^x$$

$$\frac{\log 23.99-}{\log 3.981} = \frac{x\log 3.9\cancel{81}}{\log 3.981}$$

$$2.30 = x$$

$$\boxed{2.25 = x}$$

5. The table below gives air pressures in kPa at selected altitudes above sea level measured in kilometers.

x	**Altitude** (km)	0	1	2	3	4	5
y	**Air Pressure** (kPa)	101	90	79	70	62	54

Write an exponential regression equation that models these data rounding all values to the *nearest thousandth*. Use this equation to algebraically determine the altitude, to the *nearest hundredth* of a kilometer, when the air pressure is $\underset{y}{29}$ kPa.

ExpReg

$y=a(b)^x$

$y=101.523(.883)^x$

$$\frac{29}{101.523}=\frac{101.5\cancel{23}(.883)^x}{101.523}$$

$$\log .2856.. = \log .883^x$$

$$\frac{\log .2856..}{\log .883} = \frac{x\log .8\cancel{83}}{\log .883}$$

$$10.07 = x$$

Name Schlansky
Mr. Schlansky

Date ____________
Algebra II

Newton's Law of Heating and Cooling

1. The Fahrenheit temperature of a heated object can be modeled by the function below.

$$F(t) = F_S + (F_0 - F_S)e^{-kt}$$

$F(t)$ = the temperature of the object after t minutes $=150$
t = time in minutes $=t$
F_S = the surrounding temperature $=68$
F_0 = the initial temperature of the object $=200$
k = a constant $=.05$

Hot chocolate at a temperature of 200°F (initial) is poured into a container. The room temperature is kept at a constant 68°F (surrounding) and $k = 0.05$.

After how much time, to the *nearest minute*, will the temperature of the hot chocolate be 150°F (temperature after time passes)?

$150=68+(200-68)e^{--.05t}$ t
$-68-68$
$\frac{82}{132}=\frac{132e^{-.05t}}{132}$

$\ln\frac{41}{66}=\ln e^{-.05t}$
$\frac{\ln\frac{41}{66}}{-.05}=\frac{-.05t\ln e}{-.05}$

$\boxed{10=t}$

After how much time, to the *nearest tenth of a minute*, will the temperature of the hot chocolate be 120°F?

$F(t)=120$
$t=t$
$F_S=68$
$F_0=200$
$K=.05$

$120=68+(200-68)e^{--.05t}$
$-68-68$
$\frac{52}{132}=\frac{132e^{-.05t}}{132}$

$\ln\frac{13}{33}=\ln e^{--.05t}$
$\frac{\ln\frac{13}{33}}{-.05}=\frac{\cancel{-.05}t\ln e}{\cancel{-.05}}$

$\boxed{18.6=t}$

2. The Fahrenheit temperature, $F(t)$, of a heated object at time t, in minutes, can be modeled by the function below. F_s is the surrounding temperature, F_0 is the initial temperature of the object, and k is a constant.

$$F(t) = F_S + (F_0 - F_S)e^{-kt}$$

Coffee at a temperature of 195°F (initial) is poured into a container. The room temperature is kept at a constant 68°F (surrounding temp) and $k = 0.05$. Coffee is safe to drink when its temperature is, at most, 120°F ($F(t)$). To the *nearest minute*, how long will it take until the coffee is safe to drink?

$F(t)$ = temperature after time passes $=120$
t = time in minutes $=t$
F_s = sullounding temp $=68$
F_0 = initial temp $=195$
k = constant $=.05$

$120=68+(195-68)e^{-.05t}$
$-68-68$
$\frac{52}{127}=\frac{127e^{-.05t}}{127}$
$\ln\frac{52}{127}=\ln e^{-.05t}$
$\frac{\ln\frac{52}{127}}{-.05}=\frac{\cancel{-.05}t\ln e}{\cancel{-.05}}$

$\boxed{18=t}$

surrounding initial

3. After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + \left(T_0 - T_a\right)e^{-kt}$$

$100 = 325 + (68 - 325)e^{-k(2)}$

$-325 \quad -325$

$\frac{-225}{-257} = \frac{-2\cancel{8}7e^{-2k}}{-257}$

$\ln\frac{225}{257} = \ln e^{-2k}$

$\frac{\ln\frac{225}{251}}{-2} = \frac{-2k\ln e}{-2}$

$.066 = k$

T_a = the temperature surrounding the object $= 325$

T_0 = the initial temperature of the object $= 68$

t = the time in hours $= 2$

T = the temperature of the object after t hours $= 100$

k = decay constant $= k$

The turkey reaches the temperature of approximately 100° F after 2 hours. Find the value of k, to the *nearest thousandth*. Determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m.

$T_a = 325$

$T_0 = 68$

$t = 8am - 3PM = 7$

$T = T$

$k = .066$

$T = 325 + (68 - 325)e^{-.066(7)}$

$\boxed{T = 163°}$

surrounding initial

4. Empanadas are taken out of an oven when they reached a temperature of 168°F and put on the kitchen table at room temperature (68°F). After 8 minutes, the temperature of the empanadas is 125°F. The temperature of a cooled object can be given by the formula below:

$$T = T_a + \left(T_0 - T_a\right)e^{-kt}$$

T = the temperature of the object after t minutes $= 125$

t = time in minutes $= 8$

T_a = the surrounding temperature $= 68$

T_0 = the initial temperature of the object $= 168$

k = decay constant $= k$

$125 = 68 + (168 - 68)e^{-k(8)}$

$-68 \quad -68$

$\frac{57}{100} = \frac{100e^{-8k}}{100}$

$\ln\frac{57}{100} = \ln e^{-8k}$

$\frac{\ln\left(\frac{57}{100}\right)}{-8} = \frac{-8k\ln e}{-8}$

Find the value of k, rounded to the *nearest thousandth*. Using your value of k, to the *nearest minute*, how long will it take for the empanadas to reach 100°F?

$\boxed{.070 = k}$

$T = 100$

$t = t$

$T_a = 68$

$T_0 = 168$

$k = .070$

$100 = 68 + (168 - 68)e^{-.070t}$

$-68 \quad -68$

$\frac{32}{100} = \frac{100e^{-.070t}}{100}$

$\ln\frac{8}{25} = \ln e^{-.070t}$

$\frac{\ln\frac{8}{25}}{-.070} = \frac{-\cancel{.070}t\ln e}{-\cancel{.070}}$

$\boxed{16 = t}$

$t=1.5$ surrounding air temp

5. Megan is performing an experiment in a lab where the air temperature is a constant 73°F and the liquid is 237°F. One and a half hours later, the temperature of the liquid is 112°F. Newton's law of cooling states $T(t) = T_a + (T_0 - T_a)e^{-kt}$ where:

initial

$T(t)$: temperature, °F, of the liquid at t hours $= 112$

$t = 1.5$

T_a: air temperature $= 73$

T_0: initial temperature of the liquid $= 237$

k: constant $= K$

$T(t)$

$$112 = 73 + (237-73)e^{-k(1.5)}$$
$$-73 \quad -73$$
$$\frac{39}{164} = \frac{164}{164}e^{-1.5k}$$
$$\ln\frac{39}{164} \ln = e^{-1.5k}$$
$$\frac{\ln\frac{39}{164}}{-1.5} = \frac{-1.5k \, \ln e}{-1.5}$$
$$.958 = K$$

Determine the value of k, to the *nearest thousandth*, for this liquid. Determine the temperature of the liquid using your value for k, to the *nearest degree*, after two and a half hours. Megan needs the temperature of the liquid to be 80°F to perform the next step in her experiment. Use your value for k to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

$t=2.5$ $T(t)$

$T(t) = T$

$t = 2.5$

$T_a = 73$

$T_0 = 237$

$k = .458$

$$T = 73 + (237-73)e^{-.958(2.5)}$$
$$T = 88°$$

$T(t) = ~~[illegible]~~ 80$

$t = t$

$T_a = 73$

$T_0 = 237$

$k = .958$

$$80 = 73 + (237-73)e^{-.958t}$$
$$-73 \quad -73$$
$$\frac{7}{164} = \frac{\cancel{164}}{164}e^{-.958t}$$
$$\ln\frac{7}{164} = \ln e^{-.958t}$$
$$\frac{\ln\frac{7}{164}}{-.958} = \frac{-.958t \, \ln e}{-.958}$$
$$\boxed{3.3 = t}$$

6. Objects cool at different rates based on the formula below.

$T = (T_0 - T_R)e^{-rt} + T_R$

T_0: initial temperature $= 400$

T_R: room temperature $= 75$

r: rate of cooling of the object $= .0735$

t: time in minutes that the object cools to a temperature, $T = 5$

$t = T$

Mark makes T-shirts using a hot press to transfer designs to the shirts. He removes a shirt from a press that heats the shirt to 400°F. The rate of cooling for the shirt is 0.0735 and the room temperature is 75°F. Find the temperature of the shirt, to the *nearest degree*, after five minutes. At the same time, Mark's friend Jeanine removes a hoodie from a press that heats the hoodie to 450°F. After eight minutes, the hoodie measured 270°F. The room temperature is still 75°F. Determine the rate of cooling of the hoodie, to the *nearest ten thousandth*. The T-shirt and hoodie were removed at the same time. Determine when the temperature will be the same, to the *nearest minute*.

T_R initial t T initial t r equal intersect

$$T = (400-75)e^{-.0735(5)} + 75$$
$$\boxed{T = 300°}$$

$T = 270$

$T_0 = 450$

$T_R = 75$

$r = r$

$t = 8$

$$270 = (450-75)e^{-r(8)} + 75$$
$$-75 \quad -75$$
$$\frac{195}{375} = \frac{375}{375}e^{-8r}$$
$$\ln\frac{13}{25} = \ln e^{-8r}$$
$$\frac{\ln\frac{13}{25}}{-8} = \frac{-8r \, \ln e}{-8}$$
$$\boxed{.0817 = r}$$

$$75 + (400-75)e^{-.0735t} = (450-75)e^{-.0817t} + T_R$$
$$Y1 = (400-75)e^{-.0735t} + 75$$
$$Y2 = (450-75)e^{-.0817t} + 75$$

Intersect 2nd Trace

$\boxed{t = 17 \text{ minutes}}$

Name Schlansky
Mr. Schlansky

Date ________________
Algebra II

Modeling Exponential Functions Practice

$A = P(1\pm r)^t$	Nothing Below!
$A = P\left(1+\frac{r}{n}\right)^{nt}$	Compounding (Not Continuous)
$A = Pe^{rt}$	Compounding Continuously
$A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$	Half Life
$A = P(1\pm r)^{\frac{t}{h}}$	Irregular Time

A = after amount

P = principal (initial/starting) amount

r = rate (as a decimal)

n = number of times compounded per year

t = time (that is passing)

h = half life or time it takes for the percent to be applied

	n
Annually	1
Quarterly	4
Monthly	12
Weekly	52
Daily	365

n=12
n for mnth

1. Jackie deposits $26,000 (P) into a savings account with interest compounded monthly at a rate of 4.6% (r) each year. Write an equation for $A(t)$, the value of her account after t years. Use your equation to determine how much money will be in her account after 4 years?

$A=A(t)$
$P=26{,}000$
$r=.046$
$n=12$
$t=t$

$A=P(1+\frac{r}{n})^{nt}$
$A(t)=26{,}000(1+\frac{.046}{12})^{12t}$
$A(t)=26{,}000(1.0038\overline{3})^{12t}$

$t=4$
$A(4)=26{,}000(1.0038\overline{3})^{12(4)}$
$A(4)=\$31{,}241.42$

h illegular time

2. The population of Schlansky, Arizona increases (+) by 18% (r) every 3.2 years. If the population is currently 2750 (p), write an equation for $p(t)$, the population after t years. Using your equation, what will be the population, to the *nearest person*, 12 years (t) from now?

$A=P(t)$
$P=2750$
$r=.18$
$t=t$
$h=3.2$

$A=P(1\pm r)^{\frac{t}{h}}$
$p(t)=2750(1+.18)^{\frac{t}{3.2}}$
$p(t)=2750(1.18)^{\frac{t}{3.2}}$

$p(12)=2750(1.18)^{\frac{12}{3.2}}$
$p(12)=5116$

Pert

3. A bank account is opened with $2700 (P) and interest is compounded continuously at a rate of 3.76% (r) per year. Write an equation for $b(t)$, the balance of the account after t years. Using your equation, what will be the balance of the account after 8.1 years (t)?

$A=b(t)$
$P=2700$
$r=~~3~~.0376$
$t=t$

$A=Pe^{rt}$
$b(t)=2700e^{.0376t}$

$b(8.1)=2700e^{.0376(8.1)}$
$b(8.1)=\$3661.28$

nothing!

4. A certain car depreciates at a rate of 14% each year. If the car was initially worth $22,500, write an equation for $v(t)$, the value of the account after t years. Using your equation, what is the value of the car, rounded to the *nearest cent*, 12 years later?

$A = v(t)$
$P = 22{,}500$
$r = .14$
$t = t$

$A = P(1 \pm r)^t$
$v(t) = 22{,}500(1 - .14)^t$
$\boxed{v(t) = 22{,}500(.86)^t}$

$v(12) = 22{,}500(.86)^{12}$
$\boxed{v(12) = \$3682.68}$

half life

5. The half life of an element is 73 minutes. If there were initially 7.4 kg of the substance, write an equation for $a(t)$, the amount of the substance remaining after t minutes. Using your equation, to the *nearest hundredth of a kg*, how much will remain after 110 minutes?

$A = a(t)$
$P = 7.4$
$t = t$
$h = 73$

$A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$
$\boxed{a(t) = 7.4\left(\frac{1}{2}\right)^{\frac{t}{73}}}$

$a(110) = 7.4\left(\frac{1}{2}\right)^{\frac{110}{73}}$
$\boxed{= 2.60}$

nothing!

6. Skylar bought an antique mirror for $800. If the value of her mirror increases 6% annually, write an equation for $v(t)$, the value of her mirror after t years. Using your equation, determine the value of Skylar's mirror at the end of 4 years to the *nearest dollar*?

$A = v(t)$
$P = 800$
$r = .06$
$t = t$

$A = P(1 \pm r)^t$
$v(t) = 800(1 + .06)^t$
$\boxed{v(t) = 800(1.06)^t}$

$v(4) = 800(1.06)^4$
$v(4) = 1009|981$
1009
$\boxed{\$1010}$

n formula $n = 4$

7. A bank account is opened with $1500 and interest is compounded quarterly at an interest rate of 3.1%. Write an equation for $b(t)$, the balance of the account after t years. Using your equation, how much money will be in the account after 7 years?

$A = b(t)$
$P = 1500$
$r = .031$
$n = 4$
$t = t$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$
$b(t) = 1500\left(1 + \frac{.031}{4}\right)^{4t}$
$\boxed{b(t) = 1500(1.00775)^{4t}}$

$b(7) = 1500(1.00775)^{4(7)}$
$\boxed{b(7) = 1861.96}$

Name Schlansky
Mr. Schlansky

Date ______________
Algebra II

Exponential Modeling Finding t

P n formula n=52

8. Megan opens a savings account with \$5,000 in it. If interest is compounded weekly at a rate of 4.3%, write an equation for $b(t)$, the balance of her account after t years. Using your equation, how long (to the nearest tenth) will it take for Megan's money to reach \$8,000? $b(t)$

$A=b(t)$
$P=5{,}000$
$r=.043$
$n=52$
$t=t$

$A=P(1+\frac{r}{n})^{nt}$

$b(t)=5{,}000(1+\frac{.043}{52})^{52t}$

$b(t)=5{,}000(1.000826923)^{52t}$

$\frac{8000}{5000}=\frac{5000(1.000826923)^{52t}}{5000}$

$\log 1.6=\log 1.000826923^{52t}$

$\frac{\log 1.6}{52\log 1.000826923}=\frac{52t\log 1.000826923}{52\log 1.000826923}$

$11=t$

half life

9. One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days (h). A patient is injected with 20 (P) milligrams of I–131. Create an equation for $a(t)$, the amount of Iodine-131 remaining after t days. Determine, to the *nearest day*, the amount of time needed before the amount of I–131 in the patient's body is approximately 7 milligrams ($a(t)$).

$A=a(t)$
$P=20$
$t=t$
$h=8.02$

$A=P(\frac{1}{2})^{\frac{t}{h}}$

$a(t)=20(\frac{1}{2})^{\frac{t}{8.02}}$

$\frac{7}{20}=\frac{20(\frac{1}{2})^{\frac{t}{8.02}}}{20}$

$\log .35=\log\frac{1}{2}^{\frac{t}{8.02}}$

$8.02(\log .35=\frac{t}{8.02}\log(\frac{1}{2}))$

$\frac{8.02\log .35}{\log(\frac{1}{2})}=\frac{t\log(\frac{1}{2})}{\log(\frac{1}{2})}$

$12=t$

Pert

10. Tyler opens a bank account with \$5,450 (P) with an annual interest rate of 5.3% (r) compounded continuously. Write an equation for $b(t)$, the balance of Tyler's account after t years. Using your equation, to the *nearest hundredth of a year*, how long will it take for Tyler's account to triple?

$A=b(t)$
$P=5450$
$r=.053$
$t=t$

$A=Pe^{rt}$

$b(t)=5450e^{.053t}$

$b(t)=3(5450)$
$b(t)=16350$

$\frac{16350}{5450}=\frac{5450e^{.053t}}{5450}$

$\ln 3=\ln e^{.053t}$

$\frac{\ln 3}{.053}=\frac{.053t\ln e}{.053}$

$20.73=t$

illegular time

n

P r

11 4. Jessica deposits $2000 into a bank account where 4% interest is given every 2.4 years. Write an equation for $v(t)$, the value of Jessica's account after t years. Using your equation, to the *nearest tenth of a year*, how long will it take for Jessica's investment to reach $5000?

A = v(t)
P = 2000
r = .04
t = t
n = 2.4

$A=P(1\pm r)^{\frac{t}{n}}$

$v(t)=2000(1+.04)^{\frac{t}{2.4}}$

$\boxed{v(t)=2000(1.04)^{\frac{t}{2.4}}}$

v(t)

$\frac{5000}{2000}=\frac{2000(1.04)^{\frac{t}{2.4}}}{2000}$

$\log 2.5=\log 1.04^{\frac{t}{2.4}}$

$2.4(\log 2.5)=\left(\frac{t}{\cancel{2.4}}\log 1.04\right)\cancel{2.4}$

$\frac{2.4\log 2.5}{\log 1.04}=\frac{t\log 1.04}{\log 1.04}$

$\boxed{t=56.1}$

P r

12 5. Manny opens a savings account with $6,400.00 with a 5.2% interest rate that is compounded quarterly. Write an equation for $b(t)$, the balance of the account after t years. Using your equation, to the nearest *tenth of a year,* how long will it take for Manny's balance to double?

n formula n=4

A = b(t)
P = 6400
r = .052
n = 4
t = t

$A=P(1+\frac{r}{n})^{nt}$

$b(t)=6400(1+\frac{.052}{4})^{4t}$

$\boxed{b(t)=6400(1.013)^{4t}}$

b(t) = 2(6400) = 12800

$\frac{12800}{6400}=\frac{6400(1.013)^{4t}}{6400}$

$\log 2=\log 1.013^{4t}$

$\frac{\log 2}{4\log 1.013}=\frac{4t\log 1.013}{4\log 1.013}$

$\boxed{13.4=t}$

illegular time

13 P

6. Christopher is preparing for the Nassau County Spelling Bee. Currently, Christopher knows 1200 words and will learn 20% more words every 4 days. Write an equation, $A(t)$, to represent how many words Christopher will be able to spell after t days. After how many days, to the *nearest day,* will Christopher be able to spell 5000 words?

r n

A = A(t)
P = 1200
r = .2
t = t
n = 4

$A=P(1\pm r)^{\frac{t}{n}}$

$A(t)=1200(1+.2)^{\frac{t}{4}}$

$\boxed{A(t)=1200(1.2)^{\frac{t}{4}}}$

A(t)

$\frac{5000}{1200}=\frac{1200(1.2)^{\frac{t}{4}}}{1200}$

$\log\frac{25}{6}=\log 1.2^{\frac{t}{4}}$

$4\left(\log\frac{25}{6}\right)=\left(\frac{t}{\cancel{4}}\log 1.2\right)\cancel{4}$

$\frac{4\log\frac{25}{6}}{\log 1.2}=\frac{t\log 1.2}{\log 1.2}$

$\boxed{31=t}$

P Pe^{rt} r

7. If a bank account was opened with \$3000 and interest is compounded continuously at 5.2%. Write an equation for $v(t)$, the value of the account after t years. To the *nearest hundredth of a year*, how long will it take for the value of the account to reach \$4000?

$v(t)$

$A = v(t)$

$P = 3000$

$r = .052$

$t = t$

$A = Pe^{rt}$

$v(t) = 3000e^{.052t}$

$\frac{4000}{3000} = \frac{3000e^{.052t}}{3000}$

$\ln\frac{4}{3} = \ln e^{.052t}$

$\frac{\ln\frac{4}{3}}{.052} = \frac{.052t \ln e}{.052}$

$5.53 = t$

nothing below

P t r

8. Danielle bought a basketball card for \$2125 its value is increasing by 4.1% each year. Create an equation for $v(t)$, the value of the basketball card after t years. Using your equation, how long to the nearest year, will it take for the value of the basketball card to reach \$10000?

$v(t)$

$A = v(t)$

$P = 2125$

$r = .041$

$t = t$

$A = P(1 \pm r)^t$

$v(t) = 2125(1 + .041)^t$

$v(t) = 2125(1.041)^t$

$\frac{10{,}000}{2125} = \frac{2125(1.041)^t}{2125}$

$\log 4.7 = \log 1.041^t$

$\frac{\log 4.7}{\log 1.041} = \frac{t \log 1.041}{\log 1.041}$

$39 = t$

P n=12 n formula

9. Miguel opened a bank account with \$1000 and interest is compounded monthly at a rate of 8.1%. Write an equation to represent $b(t)$, the balance of Miguel's account after t years. Using your equation, how much time, to the *nearest year*, will it take for Miguel's money to triple?

r

$b(t) = 3(1000) = 3000$

$A = b(t)$

$P = 1000$

$r = .081$

$n = 12$

$t = t$

$A = P(1 + \frac{r}{n})^{nt}$

$b(t) = 1000(1 + \frac{.081}{12})^{12t}$

$b(t) = 1000(1.00675)^{12t}$

$\frac{3000}{1000} = \frac{1000(1.00675)^{12t}}{1000}$

$\log 3 = \log 1.00675^{12t}$

$\frac{\log 3}{12\log 1.00675} = \frac{12t \log 1.00675}{12\log 1.00675}$

$14 = t$

$A=P(1\pm r)^t$

Name Schlansky
Mr. Schlansky

Date ____________
Algebra II

Finding Exponential Rate

1. A bank account opened up 3 years ago with an initial balance of \$12000 now has a balance of \$12824. Find the annual growth rate, to the *nearest tenth of a percent.*

$A=12824$
$P=12{,}000$
$r=r$
$t=3$

$\frac{12824}{12{,}000}=\frac{12{,}000(1+r)^3}{12{,}000}$

$\sqrt[3]{1.068..}=\sqrt[3]{(1+r)^3}$

$1.022=1+r$
$-1 \quad -1$

$.022=r$

$2.2\%=r$

2. Jack bought a new car in 2010 for \$16100. In 2018, the car is now worth \$6125. What is the annual rate of decrease to the *nearest percent*?

$A=6125$
$P=16100$
$r=r$
$t=8$

$\frac{6125}{16100}=\frac{16100(1-r)^8}{16100}$

$\sqrt[8]{.38}=\sqrt[8]{(1-r)^8}$

$.8862..=1-r$
$-1 \quad -1$

$\frac{-.11379..}{-1}=\frac{-r}{-1}$

$.11379=r$

$11\%=r$

3. A collectible toy was bought 15 years ago for \$5 and is now worth \$42. Find the annual growth rate to the *nearest tenth of a percent.*

$A=42$
$P=5$
$r=r$
$t=15$

$\frac{42}{5}=\frac{5(1+r)^{15}}{5}$

$\sqrt[15]{8.4}=\sqrt[15]{(1+r)}$

$1.152=1+r$
$-1 \quad -1$

$.152..=r$

15.2%

4. A colony of 120 timberwolves increased to 245 over a 6 year span. Assuming exponential growth, what was the annual growth rate to the *nearest percent*?

$A=245$
$P=120$
$r=r$
$t=6$

$\frac{245}{120}=\frac{120(1+r)^6}{120}$

$\sqrt[6]{2.0416}=\sqrt[6]{(1+r)^6}$

$1.126...=1+r$
$-1 \quad -1$

$.126..=r$

13%

Equivalent Exponential Forms (Absorbing the Exponent)

If you have a value in the exponent, absorb it into the parenthesis.

To interpret an exponential function, the initial value is in front of the parenthesis and $(1 \pm rate)$ is what is inside the parenthesis. If it is less than 1, it is decreasing. If it is more than 1, it is increasing.

Express each of the following functions with an exponent of *t*. Round values to the nearest thousandth.

1. $A = 12{,}000(1.025)^{12t}$

$A = 12{,}000(1.025^{12})^{t}$

$A = 12{,}000(1.345)^{t}$

2. $A = 17{,}000(.889)^{9.4t}$

$A = 17{,}000(.889^{9.4})^{t}$

$A = 17{,}000(.331)^{t}$

3. $A = 11{,}185(.764)^{\frac{t}{12}}$

$A = 11{,}185(.764^{\frac{1}{12}})^{t}$

$A = 11{,}185(.978)^{t}$

4. $A = 125{,}000(.785)^{\frac{t}{4}}$

$A = 125{,}000(.785^{\frac{1}{4}})^{t}$

$A = 125{,}000(.941)^{t}$

5. Iridium-192 is an isotope of iridium and has a half-life of 73.83 days. If a laboratory experiment begins with 100 grams of Iridium-192, the number of grams, *A*, of Iridium-192 present after *t* days would be $A = 100\left(\frac{1}{2}\right)^{\frac{1\ t}{73.83}}$. Which equation approximates the amount of Iridium-192 present after *t* days?

1) $A = 100\left(\frac{73.83}{2}\right)^{t}$

[✓ 3)] $A = 100(0.990656)^{t}$

2) $A = 100\left(\frac{1}{147.66}\right)^{t}$

4) $A = 100(0.116381)^{t}$

$A = 100\left(\frac{1}{2}^{\frac{1}{73.83}}\right)^{t}$

$A = 100(.990656)^{t}$

6. The amount of a substance, $A(t)$, that remains after *t* days can be given by the equation $A(t) = A_0(0.5)^{\frac{1\ t}{0.0803}}$, where A_0 represents the initial amount of the substance. An equivalent form of this equation is

[✓ 1)] $A(t) = A_0(0.000178)^{t}$

3) $A(t) = A_0(0.04015)^{t}$

2) $A(t) = A_0(0.945861)^{t}$

4) $A(t) = A_0(1.08361)^{t}$

$A(t) = A_0(0.5^{\frac{1}{.0803}})^{t}$

$A(t) = A_0(.000178)^{t}$

1.78E-4 means

1.78×10^{-4}

$A=220(\frac{1}{2}^{\frac{1}{12}})^t$ $A=220(.94387.)^t$

7. A certain pain reliever is taken in 220 mg dosages and has a half-life of 12 hours. The function $A = 220\left(\frac{1}{2}\right)^{\frac{t}{12}}$ can be used to model this situation, where *A* is the amount of pain reliever in milligrams remaining in the body after *t* hours. According to this function, which statement is true? decreases by ≈6%.

X 1) Every hour, the amount of pain reliever remaining is cut in half.

X 2) In 12 hours, there is no pain reliever remaining in the body. never 0

X 3) In 24 hours, there is no pain reliever remaining in the body. never 0

[✓ 4)] In 12 hours, 110 mg of pain reliever is remaining. $A=220(.94387..)^{12} \approx 110$

8. An equation to represent the value of a car after *t* months of ownership is $v = 32,000(0.81)^{\frac{t}{12}}$. Which statement is *not* correct?

$V=32,000(.81^{\frac{1}{12}})^t$

$V=32,000(.98259..)^t$

X [✓ 1)] The car lost approximately 19% of its value each month. ≈2%

✓ 2) The car maintained approximately 98% of its value each month.

✓ 3) The value of the car when it was purchased was $32,000.

✓ 4) The value of the car 1 year after it was purchased was $25,920. $V=32,000(.98259..)^t \approx 25,920$

9. The value of an investment account, $v(t)$, can be modeled by the equation $v(t) = 500(1.15)^{3.2t}$ after *t* years. Which of the following statements must be true?

$V(t)=500(1.15^{3.2})^t$

$V(t)=500(1.56..)^t$

1) The account is increasing approximately 15% each year. X

[✓ 2)] The account is increasing approximately 56% each year ✓

3) There will be $1216.80 in the account after two years $V(2)=500(1.56.)^2=1223...$ X

4) It will take 3.68 years for the account to double X $500(1.56)^{3.68}$ = 2592 more than doubled

10. The amount of a substance, $A(t)$, in grams, remaining after *t* days is modeled by $A(t) = 50(0.5)^{\frac{t}{3}}$. Which statement is false?

$A(t)=50(.5^{\frac{1}{3}})^t$

$A(t)=50(.793..)^t$

X [✓ 1)] In 20 days, there is no substance remaining. never none remaining

✓ 2) After two half-lives, there is 25% of the substance remaining.

✓ 3) The amount of the substance remaining can also be modeled by $A(t) = 50(2)^{\frac{-t}{3}}$. $50(2^{\frac{-1}{3}})^t$ $50(.793..)^t$

✓ 4) After one week, there is less than 10g of the substance remaining.

$A(7)=50(.793..)^7=9.92..$

11. If $f(t) = 50(.5)^{\frac{t}{5715}}$ represents a mass, in grams, of carbon-14 remaining after *t* years, which statement(s) must be true?

I. The mass of the carbon-14 is decreasing by half each year. X $f(t)=50(.5^{\frac{1}{5715}})^t$

II. The mass of the original sample is 50 g. ✓

$f(t)=50(.999878..)^t$

1) I, only

[✓ 2)] II, only

3) I and II

4) neither I nor II

1. Stephanie found that the number of white-winged cross bills in an area can be represented by the formula $C = 550(1.08)^t$, where t represents the number of years since 2010. Which equation correctly represents the number of white-winged cross bills in terms of the monthly rate of population growth?

$1.08^{\frac{1}{12}} = 1.00643$

1) $C = 550(1.00643)^t$
[✓ 2)] $C = 550(1.00643)^{12t}$
3) $C = 550(1.00643)^{\frac{t}{12}}$
4) $C = 550(1.00643)^{t+12}$

Monthly rate 12 times per year

2 ~~4~~. The value of a stock after t years can be modeled by the function $V = 2500(1.14)^t$ after t years. Which function would represent the weekly rate of increase after w weeks?

$1.14^{\frac{1}{52}} = 1.0025$

1) $V = 2500(1.14)^w$
2) $V = 2500(1.14)^{52w}$
[✓ 3)] $V = 2500(1.0025)^w$
4) $V = 2500(1.0025)^{52w}$

Weekly rate once per week

3 ~~5~~. The value of a home after t years can be modeled by the function $A = 525000(1.36)^t$ after t years. Which function would represent the monthly rate of increase after m months?

$1.36^{\frac{1}{12}} = 1.026$

1) $A = 525000(1.36)^m$
2) $A = 525000(1.36)^{12m}$
[✓ 3)] $A = 525000(1.026)^m$
4) $A = 525000(1.026)^{12m}$

Monthly rate once per month

4 ~~6~~. A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, $B(t)$, can be represented by the function $B(t) = 750(1.16)^t$, where the t represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function

$1.16^{\frac{1}{12}} = 1.012$

1) $B(t) = 750(1.012)^t$
2) $B(t) = 750(1.16)^{12t}$
[✓ 3)] $B(t) = 750(1.012)^{12t}$
4) $B(t) = 750(1.16)^{\frac{t}{12}}$

Monthly rate 12 times per year

7 ~~47~~. Mia has a student loan that is in deferment, meaning that she does not need to make payments right now. The balance of her loan account during her deferment can be represented by the function $f(x) = 35,000(1.0325)^x$, where x is the number of years since the deferment began. If the bank decides to calculate her balance showing a monthly growth rate, an approximately equivalent function would be

$1.0325^{\frac{1}{12}} = 1.0027$

[✓ 1)] $f(x) = 35,000(1.0027)^{12x}$
2) $f(x) = 35,000(1.0027)^{\frac{x}{12}}$
3) $f(x) = 35,000(1.0325)^{12x}$
4) $f(x) = 35,000(1.0325)^{\frac{x}{12}}$

monthly rate 12 times per year

8. The population of Schlansky, Utah is increasing according to the formula $p(t) = 10421(1.23)^t$ after t years. Which expression can represent the weekly growth rate, after w weeks?

1) $10421(1.23)^{52w}$
2) $10421(1.004)^{52w}$
3) $10421(1.23)^{w}$
[✓ 4) $10421(1.004)^{w}$]

$1.23^{\frac{1}{52}} = 1.004$

weekly rate one time per week

9. On average, college seniors graduating in 2012 could compute their growing student loan debt using the function $D(t) = 29,400(1.068)^t$, where t is time in years. Which expression is equivalent to $29,400(1.068)^t$ and could be used by students to identify an approximate daily interest rate on their loans?

1) $29,400\left(\boxed{1.068^{\frac{1}{365}}}\right)^{t}$
2) $29,400\left(\frac{1.068}{365}\right)^{365t}$
3) $29,400\left(1+\frac{0.068}{365}\right)^{t}$
[✓ 4) $29,400\left(\boxed{1.068^{\frac{1}{365}}}\right)^{365t}$]

$1.068^{\frac{1}{365}}$

daily rate 365 times per year

10. A student studying public policy created a model for the population of Detroit, where the population decreased 25% over a decade. He used the model $P = 714(0.75)^d$, where P is the population, in thousands, d decades after 2010. Another student, Suzanne, wants to use a model that would predict the population after y years. Suzanne's model is best represented by

1) $P = 714(0.6500)^y$
2) $P = 714(0.8500)^y$
[✓ 3) $P = 714(0.9716)^y$]
4) $P = 714(0.9750)^y$

$.75^{\frac{1}{10}} = .9716$

yearly rate one time per year

11. Each year, the amount of students in Eastbury High School increases by 7.15%. Which of the following expressions could be used to find the *weekly* rate of increase of Eastbury High School after w weeks?

1) $(1.0715)^{w}$
2) $(1.0715)^{52w}$
3) $(1.0013)^{52w}$
[✓ 4) $(1.0013)^{w}$]

$1 + .0715$
1.0715 +
$1.0715^{\frac{1}{52}} =$
1.0013

weekly rate once per week

12. Each year, the amount of students in Eastbury High School increases by 7.15%. Which of the following expressions could be used to find the *weekly* rate of increase of Eastbury High School after t years?

1) $(1.0013)^{t}$
[✓ 2) $(1.0013)^{52t}$]
3) $(1.0715)^{52t}$
4) $(1.0715)^{t}$

$1 + .0715$
1.0715 +
$1.0715^{\frac{1}{52}}$
1.0013

weekly rate 52 times per year

13. Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let m represent months.]

1) $(1.0525)^{m}$ [✓ 3)] $(1.00427)^{m}$ → monthly rate one time per month.

2) $(1.0525)^{\frac{12}{m}}$ 4) $(1.00427)^{\frac{m}{12}}$

$1+.0525$

1.0525^{t}

$1.0525^{\frac{1}{12}}$

1.00427

14. Rasmus invested $65,000 in the stock market and makes an average of 9.2% each year on his investments. Which equation could be used to find his monthly percent increase after t years?

1) $v = 65000(1.092)^{t}$ 3) $v = 65000(1.0074)^{t}$

[✓ 2)] $v = 65000(1.0074)^{12t}$ 4) $v = 65000(1.092)^{12t}$

monthly rate 12 times per year

$65{,}000(1+.092)^{t}$

$65{,}000(1.092)^{t}$

$1.092^{\frac{1}{12}}$

1.0074

15. Blake's currently has 240 Pokemon cards and is increasing by 12.4% each year. Which expression represents her *weekly* rate after w weeks?

1) $240(1.124)^{52w}$ 3) $240(1.002)^{52w}$

2) $240(1.124)^{w}$ [✓ 4)] $240(1.002)^{w}$

$240(1+.124)^{t}$

$240(1.124)^{t}$

$1.124^{\frac{1}{52}}$

1.002

weekly rate one time per week.

16. Cameron's YouTube video currently has 1200 views and the views are increasing by 23% each week. Which expression represents her *daily* rate after t weeks?

1) $1200(1.23)^{52t}$ 3) $1200(1.03)^{t}$

2) $1200(1.23)^{7t}$ [✓ 4)] $1200(1.03)^{7t}$

$1200(1+.23)^{t}$

$1200(1.23)^{t}$

$1.23^{\frac{1}{365}}$

1.03

daily rate 7 times per week

17. Over the past several years, the value of a stock has increased by 3.2% each year. The value of the stock is now $87.24. Which of the following equations does not represent the value of the stock after t years or m months?

yearly rate one time per year

1) $a(t) = 87.24(1.032)^{t}$ ✓ [✓ 3)] ✗ $a(m) = 87.24(1.0026)^{12m}$ monthly rate 12 times per month

2) $a(t) = 87.24(1.0026)^{12t}$ ✓ 4) $a(m) = 87.24(1.0026)^{m}$

monthly rate 12 times per year

✓ monthly rate one time per month

$87.24(1+.032)^{t}$

$87.24(1.032)^{t}$

$1.032^{\frac{1}{12}}$

1.0026

18. According to the USGS, an agency within the Department of Interior of the United States, the frog population in the U.S. is decreasing at the rate of 3.79% per year. A student created a model, $P = 12,150(0.962)^{t}$, to estimate the population in a pond after t years. The student then created a model that would predict the population after d decades. This model is best represented by

1) $P = 12,150(0.461)^{d}$ $.461^{\frac{1}{10}} = .925$ 3) $P = 12,150(0.996)^{d}$ $.996^{\frac{1}{10}}$ $.999$

[✓ 2)] $P = 12,150(0.679)^{d}$ $.679^{\frac{1}{10}} = .962$ 4) $P = 12,150(0.998)^{d}$ $.998^{\frac{1}{10}}$ $.999$

years to decades opposite direction

Sequences:

Arithmetic: add a constant difference, Geometric: multiply by a common ratio

Explicit Formulas (From Reference Sheet)

Arithmetic: $a_n = a_1 + (n-1)d$ Geometric: $a_n = a_1(r)^{n-1}$

If initial or a_0 is given, $(n-1)$ becomes n. Same formulas as Algebra I modeling.

Arithmetic: $a_n = a_0 + nd$ Geometric: $a_n = a_0(r)^n$

Recursive Formula

$a_1 =$

$a_n = a_{n-1}$

Write an explicit AND recursive equation for the following sequences and find the tenth term.
Round to the nearest tenth if necessary

1. 19, 16, 13, 10 … arithmetic

-3 -3 -3

explicit

$a_n = a_1 + (n-1)d$

$a_n = 19 + (n-1)(-3)$

$a_1 = 19 - 3n + 3$

$a_n = -3n + 22$

recursive

$a_1 = 19$

$a_n = a_{n-1} - 3$

$a_{10} = -3(10) + 22$

$a_{10} = -8$

2. 2,8,32,128,… geometric

·4 ·4 ·4

explicit

$a_n = a_1(r)^{n-1}$

$a_n = 2(4)^{n-1}$

recursive

$a_1 = 2$

$a_n = 4a_{n-1}$

$a_{10} = 2(4)^{10-1}$

$a_{10} = 524288$

3. 3, -12, 48, -192, … geometric

·-4 ·-4 ·-4

explicit

$a_n = a_1(r)^{n-1}$

$a_n = 3(-4)^{n-1}$

recursive

$a_1 = 3$

$a_n = -4a_{n-1}$

$a_{10} = 3(-4)^{10-1}$

$a_{10} = -786432$

4. 63, 57, 51, 45, ... arithmetic

-6 -6 -6

explicit

$a_n = a_1 + (n-1)d$

$a_n = 63 + (n-1)(-6)$

$a_n = 63 - 6n + 6$

$a_n = -6n + 69$

re

5. 329.6, 376.8, 424, 471.2,…

+47.2 +47.2

376.8 - 329.6 = 47.2

424 - 376.8 = 47.2

arithmetic

explicit

$a_n = a_1 + (n-1)d$

$a_n = 329.6 + (n-1)47.2$

$a_n = 329.6 + 47.2n - 47.2$

$a_n = 47.2n + 282.4$

recursive

$a_1 = 329.6$

$a_n = a_{n-1} + 47.2$

$a_{10} = 47.2(10) + 282.4$

$a_{10} = 754.4$

6. 120, 192, 307.2, 491.52

·1.6 ·1.6

$\frac{192}{120} = 1.6$

$\frac{307.2}{192} = 1.6$

geometric

explicit

$a_n = a_1(r)^{n-1}$

$a_n = 120(1.6)^{n-1}$

recursive

$a_1 = 120$

$a_n = 1.6a_{n-1}$

$a_{10} = 120(1.6)^{10-1}$

$a_{10} = 8246.3$

7. 5400, 4050, 3037.5, 2278.125

·.75 ·.75

$\frac{4050}{5400} = .75$

$\frac{3037.5}{4050} = .75$

geometric

explicit

$a_n = a_1(r)^{n-1}$

$a_n = 5400(.75)^{n-1}$

recursive

$a_1 = 5400$

$a_n = .75a_{n-1}$

$a_{10} = 5400(.75)^{10-1}$

$a_{10} = 405.5$

8. 5205.20, 4208.15, 3211.1, 2214.05

-997.05 -997.05

4208.15 - 5205.20

-997.05

3211.1 - 4208.15

-997.05

arithmetic

explicit

$a_n = a_1 + (n-1)d$

$a_n = 5205.20 + (n-1)(-997.05)$

$a_n = 5205.20 - 997.05n + 997.05$

$a_n = -997.05n + 6202.25$

recursive

$a_1 = 5205.20$

$a_n = a_{n-1} - 997.05$

$a_{10} = -997.05(10) + 6202.25$

$a_{10} = -3768.25$

percents
If less than 1: decreasing
If more than 1: increasing

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Algebra II

Recursive Sequences Regents Practice

9 1. The formula below can be used to model which scenario?

$$a_1 = 3000$$

$$a_n = 0.80a_{n-1}$$ → decreasing by 20%.

1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
3) A bank account starts with a deposit of \$3000, and each year it grows by 80%.
[✓ 4)] The initial value of a specialty toy is \$3000, and its value each of the following years is 20% less.

10 2. The formula below can be used to model which scenario?

$$a_0 = 92.2$$

$$a_n = 1.015a_{n-1}$$ → increasing by 1.5%

1) The initial population of a county is 92.2 thousand and it is increasing by 15% each year.
[✓ 2)] The initial population of a county is 92.2 thousand and it is increasing by 1.5% each year.
3) The population after one year is 92.2 thousand and it is increasing by 15% each year.
4) The population after one year is 92.2 thousand and it is increasing by 1.5% each year.

11 3. The sequence defined by $r_1 = 15$ and $r_n = 0.75r_{n-1}$ best models which scenario? → decreasing by 25%

1) Gerry's \$15 allowance is increased by \$0.75 each week.
2) A store that has not sold a \$15 item reduces the price by \$0.25 each week until someone purchases it.
3) A 15-gram sample of a chemical compound decays at a rate of 75% per hour.
[✓ 4)] A picture with an area of 15 square inches is reduced by 25% over and over again to make a proportionally smaller picture.

12 4. The sequence defined by $a_1 = 20$ and $a_n = 1.05a_{n-1}$ best models which scenario? → increasing by 5%

1) Jamal scored 20 baskets the first week and scores 5 more baskets each week.
[✓ 2)] Julie made \$20 her first month working and earns 5% more each month.
3) Samantha creates 20 paintings the first year and makes 50% more paintings each year.
4) Jennifer's flower is 20 inches tall on day 1 and increases by .05 inches each day.

$\rightarrow$ add 20

13 5. Which situation *cannot* be modeled by the formula $a_n = a_{n-1} + 20$ with $a_1 = 10$?

1) Nancy put $10 in her piggy bank on the first day and then added $20 daily to her piggy bank.
2) Jay has a box of ten crayons and his teacher gives him twenty new crayons each month for good behavior. ✓
[✓ 3)] Buzz has ten apples and that number increases by 20% per week. × $1.2a_{n-1}$
4) Teresa has a block of metal that is 10°F and she heats it up at a rate of 20°F per minute. ✓

14 6. Which situation *can* be modeled by the formula $a_n = 1.025a_{n-1}$ with $a_0 = 100$?

$\rightarrow$ increase by 2.5%

1) Devin has $100 saved and he will increase that amount by $2.50 each week.
2) Catherine has 100 Pokemon cards and gets 25% more each week.
[✓ 3)] Lucas has 100 points and each week increases by 2.5%.
4) Olivia's plant is 100 cm tall and it grows .025 cm each week.

$\rightarrow$ decrease by 6

15 7. Which situation *cannot* be modeled by the formula $a_n = a_{n-1} - 6$ with $a_0 = 1000$?

1) A bank account with an initial balance of $1000 increases by 6% each year.
[✓ 2)] Taylor is assigned 1000 SAT problems and completes 6 each day.
3) The starting population of fish in a pond is 1000 and the population decreases by 6% each day.
4) Jessica has $1000 saved and saves an additional $6 each week.

16 8. The height of Jenny's sunflower when she planted it was 6 inches. The sunflower grows by 0.25 inches per day. Which formula can be used to determine the height, in inches, of Jenny's sunflower on day *n*?

$a_{n-1} + .25$

(1) $h_0 = 6$, $h_n = 0.25a_{n-1}$

[✓ (3)] $h_0 = 6$, $h_n = h_{n-1} + 0.25$

(2) $h_0 = 6$, $h_n = 6 + 0.25h_{n-1}$

(4) $h_0 = 6$, $h_n = 6h_{n-1} + 0.25$

$3a_{n-1}$

17 9. A population of bacteria triples every day. If on the first day there are 300 bacteria in a Petri dish, which recursive sequence can be used to determine the population on day *n*?

[✓ 1)] $b_1 = 300$, $b_n = 3b_{n-1}$

3) $b_1 = 300$, $b_n = 300(3b_{n-1})$

2) $b_1 = 300$, $b_n = b_{n-1} + 3$

4) $b_1 = 300$, $b_n = \frac{1}{3}b_{n-1}$

$a_{n-1} - 100$

18 10. A lumber yard has 1500 2" by 4" pieces of wood that need to be transported to a construction site. A truck can take 100 pieces of wood per trip. Which sequence can be used to determine the number of pieces of wood left at the lumberyard after *n* trips?

[✓ (1)] $a_0 = 1500$, $a_n = a_{n-1} - 100$

(3) $a_0 = 1500$, $a_n = 1500 - 100a_{n-1}$

(2) $a_0 = 1500$, $a_n = 100 - a_{n-1}$

(4) $a_0 = 1500$, $a_n = 100 - 1500a_{n-1}$

1+.016 $1.016a_{n-1}$

19 11. Daniela invested $2000 in a stock that increases by 1.6% each week. Which of the following recursive sequences represents the value of her stock after n weeks?

1) $a_0 = 2000$, $a_n = a_{n-1} + 1.6$

3) $a_0 = 2000$, $a_n = 1.6a_{n-1}$

2) $a_0 = 2000$, $a_n = a_{n-1} + 1.016$

[✓ 4)] $a_0 = 2000$, $a_n = 1.016a_{n-1}$

a_{n-1} +1000

20 12. At her job, Pat earns $25,000 the first year and receives a raise of $1000 each year. The explicit formula for the *n*th term of this sequence is $a_n = 25,000 + (n-1)1000$. Which rule best represents the equivalent recursive formula?

1) $a_n = 24,000 + 1000n$

[✓ 3)] $a_1 = 25,000,\ a_n = a_{n-1} + 1000$

2) $a_n = 25,000 + 1000n$

4) $a_1 = 25,000,\ a_n = a_{n+1} + 1000$

1-.08

.92a_{n-1}

21 13. The average depreciation rate of a new boat is approximately 8% per year. If a new boat is purchased at a price of $75,000, which model is a recursive formula representing the value of the boat *n* years after it was purchased?

1) $a_n = 75,000(0.08)^n$

3) $a_n = 75,000(1.08)^n$

2) $a_0 = 75,000$, $a_n = (0.92)^n$

[✓ 4)] $a_0 = 75,000$, $a_n = 0.92(a_{n-1})$

1+.035

$1.035a_{n-1}$

22 14. An initial investment of $5000 in an account earns 3.5% annual interest. Which function correctly represents a recursive model of the investment after *n* years?

1) $A = 5000(0.035)^n$

3) $A = 5000(1.035)^n$

2) $a_0 = 5000$, $a_n = a_{n-1}(0.035)$

[✓ 4)] $a_0 = 5000$, $a_n = a_{n-1}(1.035)$

23 ~~15~~. MathSchlansky posts a video to his YouTube channel and it receives 4 views on the first day. Each day after that, the number of views increases by 7%. Which sequence can be used to determine the number of views his video receives after n days? $\rightarrow 1.07a_{n-1}$

1) $a_1 = 4$
$a_n = a_{n-1} + 7$

3) $a_1 = 4$
$a_n = .07a_{n-1}$

2) $a_1 = 4$
$a_n = a_{n-1} + 1.07$

[✓ 4)] $a_1 = 4$
$a_n = 1.07a_{n-1}$

$1 - .20$
$.8a_{n-1} + 80$

24 ~~16~~. A tree farm initially has 150 trees. Each year, 20% of the trees are cut down and 80 seedlings are planted. Which recursive formula models the number of trees, a_n, after n years?

1) $a_1 = 150$
$a_n = a_{n-1}(0.2) + 80$

3) $a_n = 150(0.2)^n + 80$

[✓ 2)] $a_1 = 150$
$a_n = a_{n-1}(0.8) + 80$

4) $a_n = 150(0.8)^n + 80$

25 ~~17~~. A recursive formula for the sequence $18, 9, 4.5, \ldots$ is

[✓ 1)] $g_1 = 18$
$g_n = \frac{1}{2} g_{n-1}$

2) $g_n = 18\left(\frac{1}{2}\right)^{n-1}$

3) $g_1 = 18$
$g_n = 2g_{n-1}$

4) $g_n = 18(2)^{n-1}$

$r = \frac{a_2}{a_1} = \frac{9}{18} = \frac{1}{2}$

$r = \frac{4.5}{9} = \frac{1}{2}$

$\frac{1}{2}a_{n-1}$

$r = \frac{30}{40} = \frac{3}{4}$

26 ~~18~~. A recursive formula for the sequence $40, 30, 22.5, \ldots$ is

1) $g_n = 40\left(\frac{3}{4}\right)^n$

3) $g_n = 40\left(\frac{3}{4}\right)^{n-1}$

2) $g_1 = 40$
$g_n = g_{n-1} - 10$

[✓ 4)] $g_1 = 40$
$g_n = \frac{3}{4} g_{n-1}$

$r = \frac{22.5}{30} = \frac{3}{4}$

$\frac{3}{4}a_{n-1}$

27 ~~19~~. A recursive formula for the sequence $64, 48, 36, \ldots$ is

1) $a_n = 64(0.75)^{n-1}$

3) $a_n = 64 + (n-1)(-16)$

2) $a_1 = 64$
$a_n = a_{n-1} - 16$

[✓ 4)] $a_1 = 64$
$a_n = 0.75a_{n-1}$

$r = \frac{48}{64} = \frac{3}{4}$

$r = \frac{36}{48} = \frac{3}{4}$

$\frac{3}{4}a_{n-1}$

28 ~~20~~. After Roger's surgery, his doctor administered pain medication in the following amounts in milligrams over four days.

Day (n)	1	2	3	4
Dosage (m)	2000	1680	1411.2	1185.4

How can this sequence best be modeled recursively?

1) $m_1 = 2000$

$m_n = m_{n-1} - 320$

2) $m_n = 2000(0.84)^{n-1}$

[✓ 3)] $m_1 = 2000$

$m_n = (0.84)m_{n-1}$

4) $m_n = 2000(0.84)^{n+1}$

$r = \frac{1680}{2000}$ $r = \frac{1411.2}{1680}$

$.84a_{n-1}$

$r = .84$ $r = .84$

29 ~~21~~. The population of Jamesburg for the years 2010-2013, respectively, was reported as follows:
250,000 250,937 251,878 252,822
How can this sequence be recursively modeled?

1) $j_n = 250,000(1.00375)^{n-1}$

3) $j_n = 250,000 + 937^{(n-1)}$

[✓ 2)] $j_1 = 250,000$

$j_n = 1.00375j_{n-1}$

4) $j_1 = 250,000$

$j_n = j_{n-1} + 937$

$r = \frac{250937}{250000} \approx 1.00375$

$r = \frac{251878}{250937} \approx 1.00375$

$1.00375a_{n-1}$

30 ~~22~~. Write a recursive formula for the sequence 6, 9, 13.5, 20.25, ...

$r = \frac{9}{6} = 1.5$

$r = \frac{13.5}{9} = 1.5$

$a_1 = 6$

$a_n = 1.5a_{n-1}$

31 ~~23~~. Write a recursive formula for the sequence 189, 63, 21, 7, ...

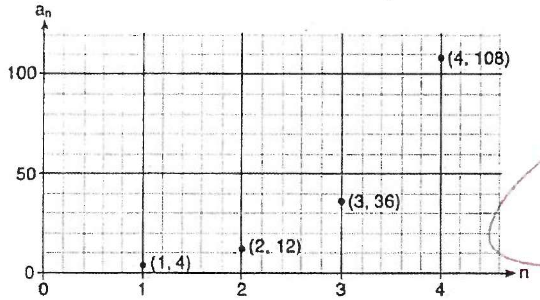
$r = \frac{63}{189} = \frac{1}{3}$

$r = \frac{21}{63} = \frac{1}{3}$

$a_1 = ~~6~~ 189$

$a_n = \frac{1}{3}a_{n-1}$

32 ~~24~~. Write a recursive formula, a_n, to describe the sequence graphed below.



4, 12, 36, 108

$r = \frac{12}{4} = 3$

$r = \frac{36}{12} = 3$

$a_1 = 4$

$a_n = 3a_{n-1}$

33 ~~25~~. The explicit formula $a_n = 6 + 6n$ represents the number of seats in each row in a movie theater, where n represents the row number. Rewrite this formula in recursive form.

$a_1 = 6 + 6(1) = 12$

$a_2 = 6 + 6(2) = 18$

$a_3 = 6 + 6(3) = 24$

$d = 18 - 12 = 6$

$d = 24 - 18 = 6$

$a_1 = 12$

$a_n = a_{n-1} + 6$

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Algebra II

Evaluating Recursive Sequences

1. Find the first 4 terms of the sequence $a_n = a_{n-1} + 4$ where $a_1 = -1$.

$a_2 = a_1 + 4$
$a_2 = -1 + 4$
$a_2 = 3$

$a_3 = a_2 + 4$
$a_3 = 3 + 4$
$a_3 = 7$

$a_4 = a_3 + 4$
$a_4 = 7 + 4$
$a_4 = 11$

$-1, 3, 7, 11$

2. Find the first 4 terms of the sequence $a_n = 4a_{n-1}$ where $a_1 = 12$.

$12, 48, 192, 768$

$a_2 = 4a_1$
$a_2 = 4(12)$
$a_2 = 48$

$a_3 = 4a_2$
$a_3 = 4(48)$
$a_3 = 192$

$a_4 = 4a_3$
$a_4 = 4(192)$
$a_4 = 768$

3. Find the first four terms of the recursive sequence $a_1 = -3$, $a_n = 4 - 3a_{n-1}$

$-3, 13, -35, 109$

$a_2 = 4 - 3a_1$
$a_2 = 4 - 3(-3)$
$a_2 = 13$

$a_3 = 4 - 3a_2$
$a_3 = 4 - 3(13)$
$a_3 = -35$

$a_4 = 4 - 3a_3$
$a_4 = 4 - 3(-35)$
$a_4 = 109$

4. If $a_n = 3a_{n-1} - 4$ and $a_1 = 9$, find a_5

$a_2 = 3a_1 - 4$
$a_2 = 3(9) - 4$
$a_2 = 23$

$a_3 = 3a_2 - 4$
$a_3 = 3(23) - 4$
$a_3 = 65$

$a_4 = 3a_3 - 4$
$a_4 = 3(65) - 4$
$a_4 = 191$

$a_5 = 3a_4 - 4$
$a_5 = 3(191) - 4$
$a_5 = 569$

5. Find the 8th term for the sequence where $a_n = 5a_{n-1} + 2n$ where $a_5 = 3$

$a_{\underline{6}} = 5a_5 + 2(\underline{6})$
$a_6 = 5(3) + 12$
$a_6 = 27$

$a_{\underline{7}} = 5a_6 + 2(\underline{7})$
$a_7 = 5(27) + 14$
$a_7 = 149$

$a_{\underline{8}} = 5a_7 + 2(\underline{8})$
$a_8 = 5(149) + 16$
$\boxed{a_8 = 761}$

6. Find the first four terms of the recursive sequence defined below.

$a_1 = -3$

$a_n = a_{(n-1)} - n$

$a_2 = a_1 - 2$

$a_2 = -3 - 2$

$a_2 = -5$

$a_3 = a_2 - 3$

$a_3 = -5 - 3$

$a_3 = -8$

$a_4 = a_3 - 4$

$a_4 = -8 - 4$

$a_4 = -12$

$-3, -5, -8, -12$

7. A sequence is defined recursively by $f(1) = 16$ and $f(n) = f(n-1) + 2n$. Find $f(4)$.

(1) 32 (2) 30 (3) 28 [✓ (4) 34]

$f(2) = f(1) + 2(2)$

$f(2) = 16 + 4$

$f(2) = 20$

$f(3) = f(2) + 2(3)$

$f(3) = 20 + 6$

$f(3) = 26$

$f(4) = f(3) + 2(4)$

$f(4) = 26 + 8$

$f(4) = 34$

8. Find the third term in the recursive sequence $a_{k+1} = 2a_k - 1$, where $a_1 = 3$.

$a_2 = 2a_1 - 1$

$a_2 = 2(3) - 1$

$a_2 = 5$

$a_3 = 2a_2 - 1$

$a_3 = 2(5) - 1$

$\boxed{a_3 = 9}$

$\cdot 2, +1$

9. Which recursively defined function represents the sequence 3, 7, 15, 31, . . .?

1) $f(1) = 3,\ f(n+1) = 2^{f(n)} + 3$
2) $f(1) = 3,\ f(n+1) = 2^{f(n)} - 1$
[✓ 3)] $f(1) = 3,\ f(n+1) = 2f(n) + 1$
4) $f(1) = 3,\ f(n+1) = 3f(n) - 2$

10. What is the fourth term of the sequence defined by $a_1 = 3xy^5$

$a_n = \left(\frac{2x}{y}\right)a_{n-1}$?

1) $12x^3y^3$
2) $24x^2y^4$
[✓ 3)] $24x^4y^2$
4) $48x^5y$

$a_2 = \left(\frac{2x}{y}\right)a_1$

$a_2 = \frac{2x}{y}(3xy^5)$

$a_2 = 6x^2y^4$

$a_3 = \left(\frac{2x}{y}\right)(a_2)$

$a_3 = \left(\frac{2x}{y}\right)(6x^2y^4)$

$a_3 = 12x^3y^3$

$a_4 = \left(\frac{2x}{y}\right)a_3$

$a_4 = \left(\frac{2x}{y}\right)(12x^3y^3)$

$a_4 = 24x^4y^2$

total: $S_n = \dfrac{a_1 - a_1(r)^n}{1-r}$

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Algebra II

Modeling Series

$r = 1.04$

a_1

1. Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year. Write a geometric series formula, S_n, for Alexa's total earnings over n years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent.*

n

$$S_n = \frac{a_1 - a_1(r)^n}{1-r}$$

$$S_{15} = \frac{33{,}000 - 33{,}000(1.04)^{15}}{1-1.04}$$

$$\boxed{S_n = \frac{33{,}000 - 33{,}000(1.04)^n}{1-1.04}}$$

$$\boxed{S_{15} = 660{,}778.39}$$

$r = 1.15$

a_1

2. Ross has a hobby of collecting comic books. He currently has 50 comic books and each year, he will increase his collection by 15%. Write a geometric series formula, S_n, for Ross' total amount of comic books after n years. Use this formula to find the total number of comic books Ross will have 12 years from now.

n

$$S_n = \frac{a_1 - a_1(r)^n}{1-r}$$

$$S_{12} = \frac{50 - 50(1.15)^{12}}{1-1.15}$$

$$\boxed{S_n = \frac{50 - 50(1.15)^n}{1-1.15}}$$

$$\boxed{S_{12} = 1450}$$

$a_1 = 42$

$r = .9$

3. Dee is planning on decreasing the amount of time she eats fast food per month. After the first month, she ate fast food 42 times. Each month, she eats at fast food restaurants 10% less than the previous month. Write a geometric series formula, S_n, for the total amount of fast food Dee eats after n months. Using your formula, how many total times does she eat fast food in the first four months? Round your answer to the nearest integer.

$n = 4$

$$S_n = \frac{a_1 - a_1(r)^n}{1-r}$$

$$S_4 = \frac{42 - 42(.9)^4}{1-.9}$$

$$\boxed{S_n = \frac{42 - 42(.9)^n}{1-.9}}$$

$$S_4 = 144$$

a_1

4. Kina earns a $27,000 salary for the first year of work at her job. She earns annual increases of 2.5%. What is the total amount, to the *nearest cent*, that Kina will earn for the first eight years at this job?

$r=1.025$

S_n n

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_8=\frac{27000-27000(1.025)^8}{1-1.025}$$

$$S_8=\$235,875.13$$

$r=1.03$ a_1

5. Rowan is training to run in a race. He runs 15 miles in the first week, and each week following, he runs 3% more than the week before. Using a geometric series formula, find the total number of miles Rowan runs over the first ten weeks of training, rounded to the *nearest thousandth*.

S_n n

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_{10}=\frac{15-15(1.03)^{10}}{1-1.03}$$

$$S_{10}=171.958$$

a_1

6. A 7-year lease for office space states that the annual rent is $85,000 for the first year and will increase by 6% each additional year of the lease. What will the total rent expense be for the entire 7-year lease?

$r=1.06$

n S_n

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_7=\frac{85,000-85,000(1.06)^7}{1-1.06}$$

$$S_7=713476.20$$

a_1

7. A fisherman harvests 350 kilograms of crab on Monday. From Monday to Friday, the fisherman harvests 8% less kilograms of crab per day. To the *nearest tenth of a kilogram*, what is the total amount of crab harvested between Monday and Friday?

$1-.08$

$r=.92$

S_n $n=5$

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_5=\frac{350-350(.92)^5}{1-.92}$$

$$S_5=1491.5$$

a_1 $r=.8$

8. A ball is dropped from a height of 32 feet. It bounces and rebounds 80% of the height from which it was falling. What is the total downward distance, in feet, the ball traveled up to the 12th bounce?

S_n n

$$S_n=\frac{a_1-a_1(r)^n}{1-r}$$

$$S_{12}=\frac{32-32(.8)^{12}}{1-.8}$$

$$S_{12}=149$$

9. Your parents want you to do some work around the house. You get them to agree to pay you $.01 on the first day, $.02 on the second day, $.04 on the third day, and so on.
At the end of the 30-day month, what is the total amount of money your parents have paid you, to the *nearest cent*? n

S_n

$r = \frac{.02}{.01} = 2$

$r = \frac{.04}{.02} = 2$

.01, .02, .04

$a_1 = .01$
$r = 2$
$n = 30$

$S_n = \frac{a_1 - a_1(r)^n}{1 - r}$

$S_{30} = \frac{.01 - .01(2)^{30}}{1 - 2}$

$S_{30} = 10,737,418.23$

a_1 $r=2$

10. On Sunday, the first day of the week, Natasha does 5 pushups. Each day, she doubles the amount of pushups she does. How many total pushups will Tasha complete at the end of the 7 day week?

n

$a_1 = 5$ S_n
$r = 2$
$n = 7$

$S_n = \frac{a_1 - a_1(r)^n}{1 - r}$

$S_7 = \frac{5 - 5(2)^7}{1 - 2}$

$S_7 = 635$

11. Samantha logged her weekly ~~running~~ distances in the table below. If the continues increasing her distance at this rate, what is the total amount of miles Samantha will have ran after 10 weeks to the nearest tenth of a mile?

S_n n

Week	Distance (In Miles)
1	12
2	14.4
3	17.28
4	20.736

$a_1 = 12$
$r = 1.2$
$n = 10$

$r = \frac{14.4}{12} = 1.2$

$r = \frac{17.28}{14.4} = 1.2$

$S_{10} = \frac{12 - 12(1.2)^{10}}{1 - 1.2}$

$S_{10} = 311.5$

12. Brian deposited 1 cent into an empty non-interest bearing bank account on the first day of the month. He then additionally deposited 3 cents on the second day, 9 cents on the third day, and 27 cents on the fourth day. What would be the total amount of money in the account at the end of the 20th day if the pattern continued?

n S_n

$a_1 = .01$
$a_2 = .03$
$a_3 = .09$
$a_4 = .27$

$r = \frac{.03}{.01} = 3$

$r = \frac{.09}{.03} = 3$

$a_1 = .01$
$r = 3$
$n = 20$

$S_{20} = \frac{.01 - .01(3)^{20}}{1 - 3}$

$S_{20} = 17,433,922$

Explicit

$$S_n = \frac{a_1 - a_1(r)^n}{1-r}$$

Summations

$$S_n = \sum_{n=1} a_1(r)^{n-1}$$

13. Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

[✓ 1)] $\sum_{x=1}^{6} 8(1.10)^{x-1}$

2) $\sum_{x=1}^{6} 8(1.10)^{x}$

Summations

3) $\frac{8-8(1.10)^6}{0.90}$

4) $\frac{8-8(0.10)^x}{1.10}$

explicit

explicit

$$S_6 = \frac{8-8(1.10)^6}{1-1.10}$$

$$S_6 = \frac{8-8(1.10)^6}{-.1}$$

Summation

$$\sum_{n=1}^{6} 8(1.1)^{n-1}$$

$a_1=8$
$r=1.10$
$n=6$

14. In his first year running track, Brendon earned 8 medals. He increases his amount of medals by 25% each year. Which of the following expressions can be used to determine how many total medals Brendon will have after four years of high school?

$r=1.25$ a_1

explicit

1) $\frac{8-8(0.25)^4}{-.25}$

3) $\frac{8-8(1.25)^4}{1-.25}$

Summation

2) $\sum_{n=1}^{4} 8(0.25)^{n-1}$

[✓ 4)] $\sum_{n=1}^{4} 8(1.25)^{n-1}$

explicit

$$S_4 = \frac{8-8(1.25)^4}{1-1.25} = \frac{8-8(1.25)^4}{-.25}$$

Summation

$$\sum_{n=1}^{4} 8(1.25)^{n-1}$$

$a_1=8$
$r=1.25$
$n=4$

15. A company fired several employees in order to save money. The amount of money the company saved per year over five years following the loss of employees is shown in the table below.

explicit

$$S_5 = \frac{59000-59000(1.1)^5}{1-1.1}$$

Summation

$$\sum_{n=1}^{5} 59000(1.1)^{n-1}$$

Year	Amount Saved (in dollars)
1	59,000
2	64,900
3	71,390
4	78,529
5	86,381.9

$a_1=59000$
$r=1.1$
$n=5$

$r=\frac{64900}{59000}=1.1$

$r=\frac{71390}{64900}=1.1$

Which expression determines the total amount of money saved by the company over 5 years?

n

[✓ 1)] $\frac{59,000-59,000(1.1)^5}{1-1.1}$

3) $\sum_{x=1}^{5} 59,000(1.1)^{x}$

2) $\frac{59,000-59,000(0.1)^5}{1-0.1}$

4) $\sum_{x=1}^{5} 59,000(0.1)^{x-1}$

explicit

Summation

P=amount of loan = total cost - down payment
n=# of monthly payments = 12(# of years)
r= interest rate (move decimal 2 places to left)
m=mortgage payment

Name Schlansky
Mr. Schlansky

Date ____________
Algebra II

Mortgage Problems

1. Jim is looking to buy a vacation home for total $172,600 near his favorite southern beach. The formula to compute a mortgage payment, M, is $M = P \bullet \dfrac{r(1+r)^N}{(1+r)^N - 1}$ where P is the principal amount of the loan, r is the monthly interest rate, and N is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage. With a $20,000 down payment, determine Jim's mortgage payment, rounded to the *nearest dollar*.

M= mortgage payment = m
P= Principal amount of loan = 172600 - 20000 = 152600
r= monthly interest rate = .00305
n = # of monthly payments = 12(15) = 180

$$m = 152600 \cdot \frac{.00305(1+.00305)^{180}}{(1+.00305)^{180} - 1}$$

$$\boxed{m = \$1103}$$

Algebraically determine and state the find P down payment, rounded to the *nearest dollar*, that Jim needs to make in order for his mortgage payment to be $900. m

m = 900
P = P
r = .00305
n = 180

$$900 = P \cdot \boxed{\frac{.00305(1+.00305)^{180}}{(1+.00305)^{180} - 1}} \rightarrow \text{type into calc}$$

$$\frac{900}{.007\ldots} = \frac{P \cdot (.007\ldots)}{.007\ldots}$$

$$124521\ldots = P$$

$$\begin{array}{r} 172{,}600 \\ -124{,}521 \\ \hline \boxed{\$48{,}078} \end{array}$$

2. Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the *nearest cent.*

$$P_n = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

P_n = present amount borrowed = 21,000 − 1,000 = 20000

n = number of monthly pay periods = 5(12) = 60

PMT = monthly payment = x

i = interest rate per month = .00625

$$20000 = x\left(\frac{1-(1.00625)^{-60}}{.00625}\right)$$

$$\frac{20{,}000}{49.9\ldots} = \frac{x(\cancel{49.9\ldots})}{\cancel{49.9\ldots}}$$

$$\boxed{400.76 = x}$$

P=T−D

The affordable monthly payment is $300 for the same time period. Determine an appropriate down payment, to the *nearest dollar*.

$P_n = x$

$n = 5(12) = 60$

$PMT = 300$

$i = .00625$

$$x = 300\left(\frac{1-(1.00625)^{-60}}{.00625}\right)$$

$x =$ ~~300(~~ 14971..

$$P = T - D$$

$$14971.. = 21{,}000 - D$$

$$-21000 \quad \cancel{-21{,}000}$$

$$\frac{-6028..}{-1} = \frac{-D}{-1}$$

$$\boxed{6028 = D}$$

3. Monthly mortgage payments can be found using the formula below:

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{n}}{\left(1+\frac{r}{12}\right)^{n}-1}$$

M = monthly payment → m
P = amount borrowed 220,000 − 100,000 = 120,000
r = annual interest rate .048
n = number of monthly payments 15(12) = 180

The Banks family would like to purchase a home for \$220,000. They qualified for an annual interest rate of 4.8%. If they put make a down payment of \$100,000 and plan to spend 15 years to repay the loan, what will be the monthly payment rounded to the *nearest* cent?

$$M = \frac{120{,}000\left(\frac{.048}{12}\right)\left(1+\frac{.048}{12}\right)^{180}}{\left(1+\frac{.048}{12}\right)^{180}-1}$$

$$m = 936.50$$

If they want their monthly payment to be \$1500, what would their down payment have to be?

$M = 1500$
$P = x$
$r = .048$
$n = 180$

$$1500 = x\left(\frac{\left(\frac{.048}{12}\right)\left(1+\frac{.048}{12}\right)^{180}}{\left(1+\frac{.048}{12}\right)^{180}-1}\right)$$

$$\frac{1500}{.0078..} = \frac{x(\cancel{.0078...})}{\cancel{.0078...}}$$

$$192205... = X$$

$D = T - P$

$D = 220{,}000 - 192{,}205$

$$\boxed{D = 27{,}794.43}$$

$n = 30(12) = 360$

4 ~~7~~. The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is \$152,500 and they will make a \$15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the *nearest dollar*.

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^n}{\left(1+\frac{r}{12}\right)^n - 1}$$

M = monthly payment $= m$

P = amount borrowed $152{,}500 - 15{,}250 = 137{,}250$

r = annual interest rate $.036$

n = total number of monthly payments $= 360$

$$m = \frac{137250\left(\frac{.036}{12}\right)\left(1+\frac{.036}{12}\right)^{360}}{\left(1+\frac{.036}{12}\right)^{360} - 1}$$

$$\boxed{m = 624}$$

5 8. Monthly mortgage payments can be found using the formula below, where M is the monthly payment, P is the amount borrowed, r is the annual interest rate, and n is the total number of monthly payments. If Adam takes out a 15-year mortgage, borrowing \$240,000 at an annual interest rate of 4.5%, What will his monthly payment be?

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^n}{\left(1+\frac{r}{12}\right)^n - 1}$$

M = monthly payment = m

P = amount borrowed = 240,000

$r = .045$

$n = 15(12) = 180$

$$m = \frac{240{,}000\left(\frac{.045}{12}\right)\left(1+\frac{.045}{12}\right)^{180}}{\left(1+\frac{.045}{12}\right)^{180} - 1}$$

$$\boxed{m = 1835.98}$$

6 9. Robert is buying a car that costs \$22,000. After a down payment of \$4000, he borrows the remainder from a bank, a six year loan at 6.24% annual interest rate. The following formula can be used to calculate his monthly loan payment. What will Robert's monthly payment be?

divide by 12 for monthly rate

$$R = \frac{(P)(i)}{1-(1+i)^{-t}}$$

R = monthly payment R

P = loan amount $22{,}000 - 4{,}000 = 18{,}000$

i = monthly interest rate $\frac{.0624}{12} = .0052$

t = time, in months $6(12) = 72$

$$R = \frac{18{,}000(.0052)}{1-(1+.0052)^{-72}}$$

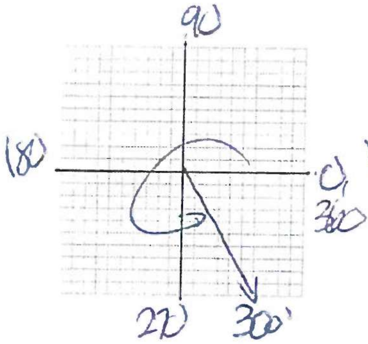
$$R = 300.36$$

Name Schlansky
Mr. Schlansky

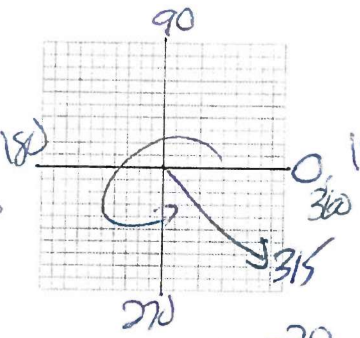
Date ____________
Algebra II

Sketching Radian Angles on the Grid

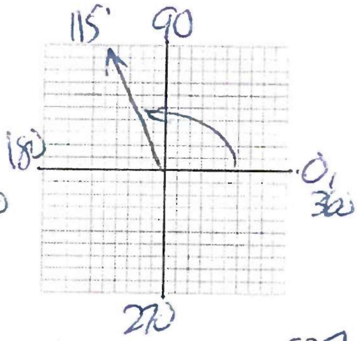
1. $\theta=\frac{5\cancel{\pi}}{3}\cdot\frac{180}{\cancel{\pi}}=300°$



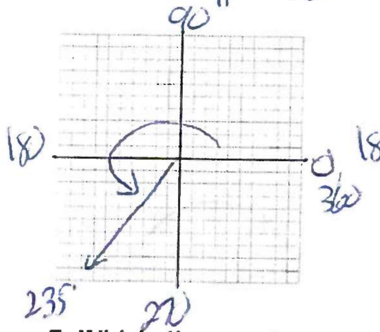
2. $\theta=\frac{7\cancel{\pi}}{4}\cdot\frac{180}{\cancel{\pi}}=315$



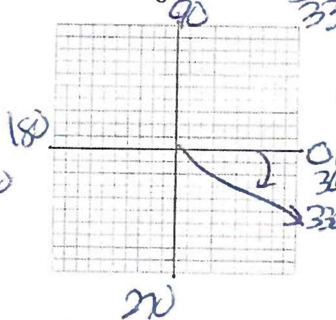
3. $\theta=2\cdot\frac{180}{\pi}\approx115°$



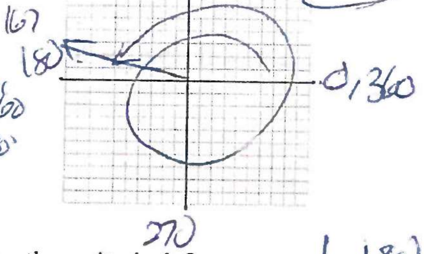
4. $\theta=4.1\cdot\frac{180}{\pi}\approx235°$



5. $\theta=-\frac{\pi}{6}\cdot\frac{180}{\pi}=\begin{array}{r}-30\\+360\\\hline330°\end{array}$



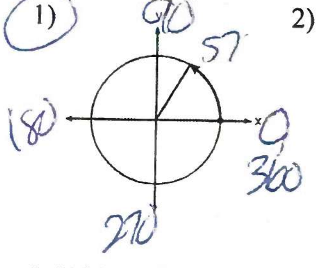
6. $\theta=9.2\cdot\frac{180}{\pi}\approx\begin{array}{r}527\\-360\\\hline\boxed{167°}\end{array}$



7. Which diagram shows an angle rotation of 1 radian on the unit circle?

$1\cdot\frac{180}{\pi}\approx57°$

[✓ 1)]



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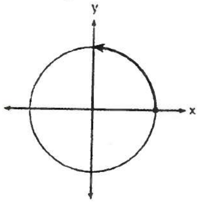
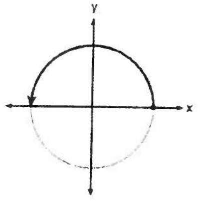


3)



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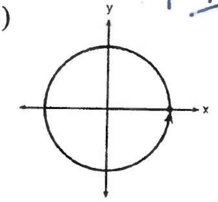
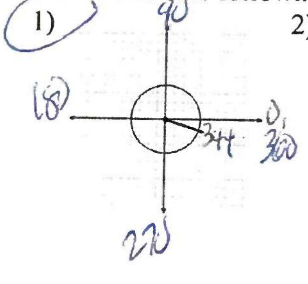


8. Which of the following sketches would represent 6 radians?

$6\cdot\frac{180}{\pi}\approx344°$

[✓ 1)]



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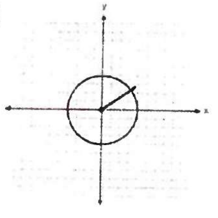

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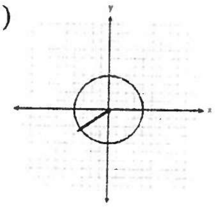

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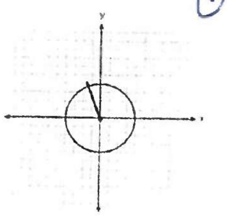


95

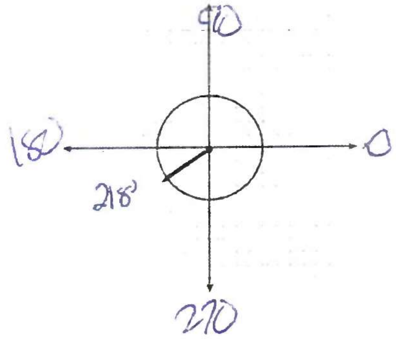
9. Which angle is sketched below? 218° $3.8 \cdot \frac{180}{\pi}$

1) 2.4 radians $2.4 \cdot \frac{180}{\pi} \approx 137$

[✓ 3) 3.8 radians]

2) 4.5 radians $4.5 \cdot \frac{180}{\pi} \approx 258$

4) 5.2 radians $5.2 \cdot \frac{180}{\pi}$ 300°



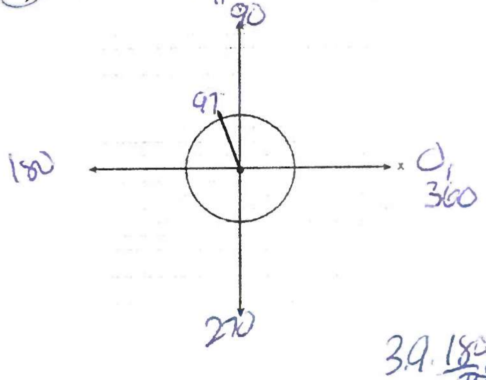
10. Which angle is sketched below? $3 \cdot \frac{180}{\pi} \approx 172$

1) 1 radian $1 \cdot \frac{180}{\pi} = 57$

[✓ 2) 1.7 radians] $1.7 \cdot \frac{180}{\pi} \approx 97$

3) 3 radians

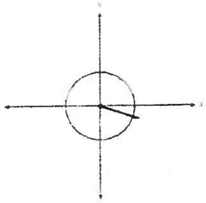
4) 4.1 radians $4.1 \cdot \frac{180}{\pi} \approx 235$



$3.9 \cdot \frac{180}{\pi} \approx 223$

11. Which of the following sketches would represent 3.9 radians?

1)



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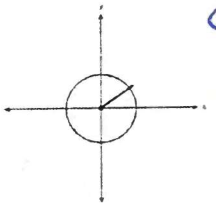
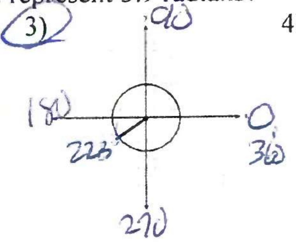
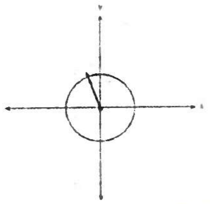


[✓ 3)]



4)



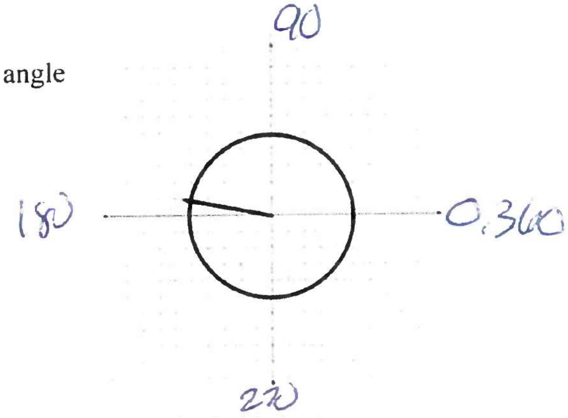
12. Which of the following can be the radian measure of the angle sketched below?

1) 1.5 $\cdot \frac{180}{\pi} \approx$ 86

[✓ 2) 3] $\cdot \frac{180}{\pi} \approx$ 172

3) 3.8 $\cdot \frac{180}{\pi} \approx$ 218

4) 5 $\cdot \frac{180}{\pi} \approx$ 286



13. An angle, θ, is rotated counterclockwise on the unit circle, with its terminal side in the second quadrant, as shown in the diagram below.

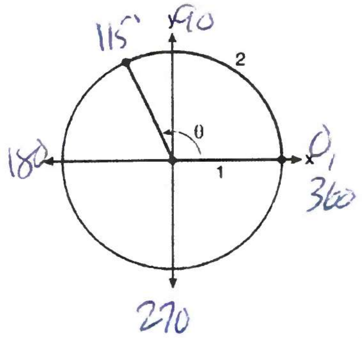
Which value represents the radian measure of angle θ?

1) 1 $\cdot \frac{180}{\pi} \approx 57°$

[✓ 2) 2] $\cdot \frac{180}{\pi} \approx 115°$

~~3) 65.4~~

~~4) 114.6~~ degrees



Evaluating Special Angles

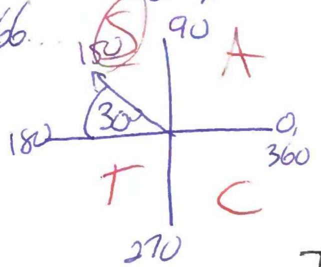
If multiple choice, type the problem in, type in each answer, see what matches up.

If open response, Q (quadrant), S (sign), F (trig function), R (reference angle). Match up to your table of special values.

	30	45	60
Sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
Cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

1. What is the exact value of $\cos\left(\frac{5\pi}{6}\right)$? − .866...

(or) $\frac{5\pi}{6}\cdot\frac{180}{\cancel{\pi}}=150$



Q S F R

II − cos 30

$-\frac{\sqrt{3}}{2}$

1) $\frac{\sqrt{3}}{2}$

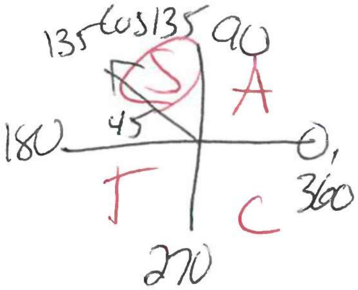
2) $\frac{1}{2}$

[✓ 3)] $-\frac{\sqrt{3}}{2}$ −.866...

4) $-\frac{1}{2}$

2. What is the exact value of $\cos\left(\frac{3\pi}{4}\right)$? −.707...

(or) $\frac{3\pi}{4}\cdot\frac{180}{\cancel{\pi}}=135$



Q S F R

II − cos 45

$-\frac{\sqrt{2}}{2}$

1) $\frac{\sqrt{3}}{2}$

2) $\frac{\sqrt{2}}{2}$

3) $-\frac{\sqrt{3}}{2}$

[✓ 4)] $-\frac{\sqrt{2}}{2}$ −.707...

3. The exact value of $\sin\left(\frac{8\pi}{3}\right)$ is .866...

1) $\frac{1}{2}$

2) $-\frac{1}{2}$

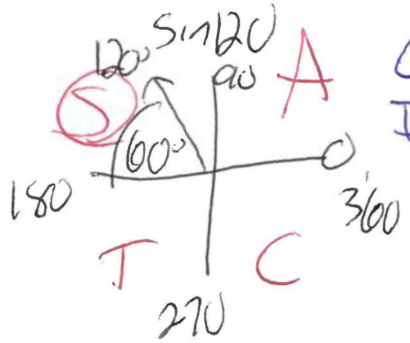
3) $-\frac{\sqrt{3}}{2}$

[✓ 4)] $\frac{\sqrt{3}}{2}$ = .866...

(or) $\frac{8\pi}{3}\cdot\frac{180}{\cancel{\pi}}=480$

-360

120



Q S F R

II + sin 60

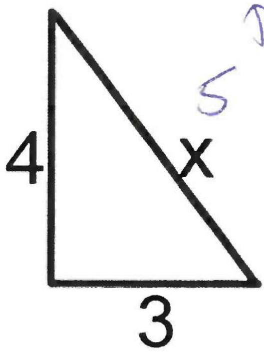
$\frac{\sqrt{3}}{2}$

Pythagorean Theorem
Look out for hidden right triangles where you may need to use $a^2 + b^2 = c^2$
a and b are the legs
c is the hypotenuse

Know your Pythagorean Triples!
3,4,5
5,12,13
7,24,25
8,15,17
9,40,41

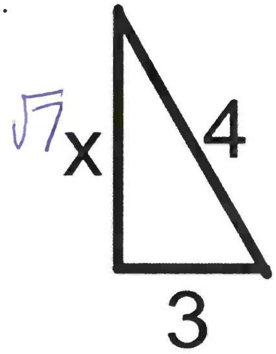
leaving your answer in radical form
Find the missing side of each right triangle ~~rounding to the nearest tenth~~

1.



Triple

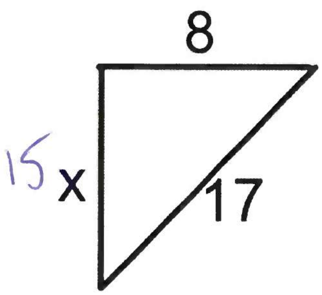
2.



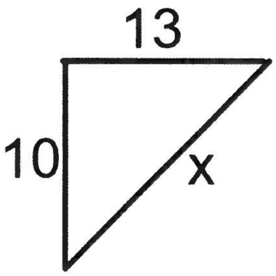
$a^2+b^2=c^2$
$3^2+x^2=4^2$
$9+x^2=16$
$-9 \quad -9$
$\sqrt{x^2}=\sqrt{7}$
$x=\sqrt{7}$

3.

Triple



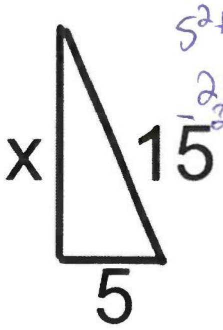
4.



$a^2+b^2=c^2$
$10^2+13^2=x^2$
$100+169=x^2$
$\sqrt{269}=\sqrt{x^2}$
$\sqrt{269}=X$

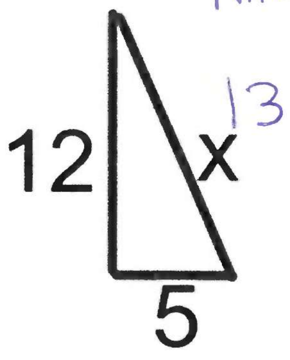
5.

$a^2+b^2=c^2$
$5^2+x^2=15^2$
$25+x^2=225$
$-25 \quad -25$
$\sqrt{x^2}=\sqrt{200}$
$x=\sqrt{200}$



6.

Triple



Rationalizing the Denominator

To rationalize the denominator, multiply top and bottom by the radical

When multiplying a radical by itself, the radical cancels out

Rationalize the following denominators

1. $\frac{2}{\sqrt{5}} \frac{\sqrt{5}}{\sqrt{5}} \quad \frac{2\sqrt{5}}{5}$

2. $\frac{-7}{\sqrt{11}} \frac{\sqrt{11}}{\sqrt{11}} \quad \frac{-7\sqrt{11}}{11}$

3. $\frac{3}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} \quad \frac{3\sqrt{2}}{2}$

4. $\frac{6}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} \quad \frac{{}^{2}\,\cancel{6}\sqrt{3}}{{}_{1}\,\cancel{3}} \quad 2\sqrt{3}$

5. $\frac{4}{\sqrt{6}} \frac{\sqrt{6}}{\sqrt{6}} \quad \frac{{}^{2}\cancel{4}\sqrt{6}}{{}_{3}\cancel{6}}$

$\frac{2\sqrt{6}}{3}$

6. $\frac{-5}{\sqrt{10}} \frac{\sqrt{10}}{\sqrt{10}} \quad \frac{-\overset{1}{\cancel{5}}\sqrt{10}}{{}_{2}\cancel{10}} \quad \frac{-\sqrt{10}}{2}$

Trig Ratios with Triangles

If an angle passes through a point or sin/cos/tan = $\frac{something}{something}$, make a right triangle and use

SOHCAHTOA

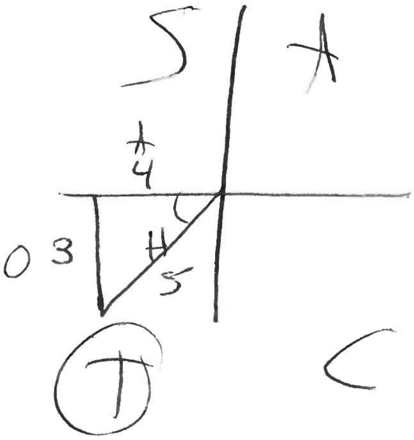
Any point on the unit circle is $(\cos\theta, \sin\theta)$

Know your Pythagorean triples: {3,4,5}, {5, 12, 13}, {8, 15, 17}, {7, 24, 25}

Reciprocal trig function pairs:

$\csc\theta$	$\sec\theta$	$\tan\theta$
$\sin\theta$	$\cos\theta$	$\cot\theta$

1. If $\sin\theta = -\frac{3\ O}{5\ H}$ and θ is in Quadrant III, find:



a) $\cos\theta$

$-\frac{4}{5}$

b) $\sin\theta$

$-\frac{3}{5}$

c) $\tan\theta$

$\frac{3}{4}$

d) $\sec\theta$

$-\frac{5}{4}$

e) $\csc\theta$

$-\frac{5}{3}$

f) $\cot\theta$

$\frac{4}{3}$

$3^2+b^2=5^2$

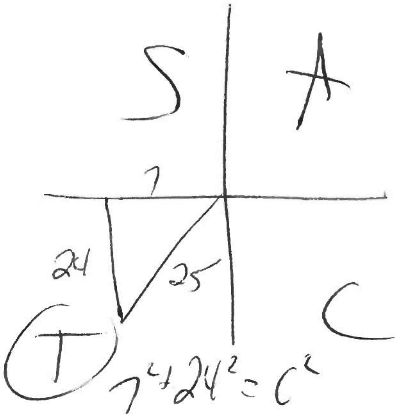
$9+b^2=25$

$-9 \quad -9$

$\sqrt{b^2}=\sqrt{16}$

$b=4$

2. If $\tan\theta = \frac{24\ O}{7\ A}$ and θ is in Quadrant III, find:



a) $\cos\theta$

$-\frac{7}{25}$

b) $\sin\theta$

$-\frac{24}{25}$

c) $\tan\theta$

$\frac{24}{7}$

d) $\sec\theta$

$-\frac{25}{7}$

e) $\csc\theta$

$-\frac{25}{24}$

f) $\cot\theta$

$\frac{7}{24}$

$7^2+24^2=c^2$

$49+576=c^2$

$\sqrt{625}=\sqrt{c^2}$

$25=c$

3. Angle θ is in standard position and $(4,-7)$ is a point on the terminal side of θ. Find:

a) $\cos\theta$

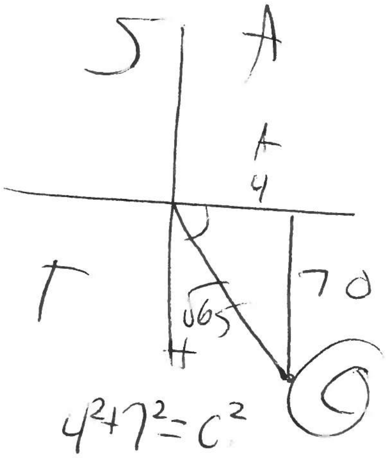
$\frac{4}{\sqrt{65}}\frac{\sqrt{65}}{\sqrt{65}}$ $\frac{4\sqrt{65}}{65}$

b) $\sin\theta$

$\frac{7}{\sqrt{65}}\frac{\sqrt{65}}{\sqrt{65}}$ $-\frac{7\sqrt{65}}{65}$

c) $\tan\theta$

$-\frac{7}{4}$



$4^2+7^2=c^2$

$16+49=c^2$

$\sqrt{65}=\sqrt{c^2}$

$\sqrt{65}=c$

d) $\sec\theta$

$\frac{\sqrt{65}}{4}$

e) $\csc\theta$

$-\frac{\sqrt{65}}{7}$

f) $\cot\theta$

$-\frac{4}{7}$

4. Angle θ is in standard position and $(-5,-12)$ is a point on the terminal side of θ. Find:

a) $\cos\theta$

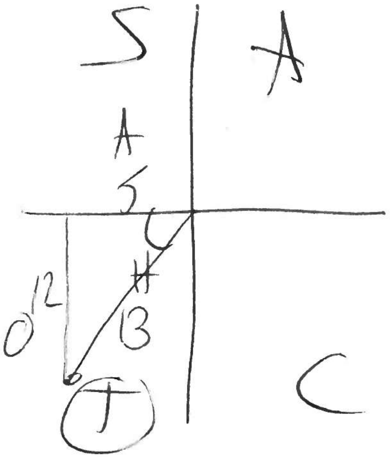
$-\frac{5}{13}$

b) $\sin\theta$

$-\frac{12}{13}$

c) $\tan\theta$

$\frac{12}{5}$



d) $\sec\theta$

$-\frac{13}{5}$

e) $\csc\theta$

$-\frac{13}{12}$

f) $\cot\theta$

$\frac{5}{12}$

$5^2+12^2=c^2$

$25+144=c^2$

$\sqrt{169}=\sqrt{c^2}$

$13=c$

5. A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ, intercepts the circle in Quadrant I at point *C*. The *y*-coordinate of point *C* is 8. Find:

a) $\cos\theta$

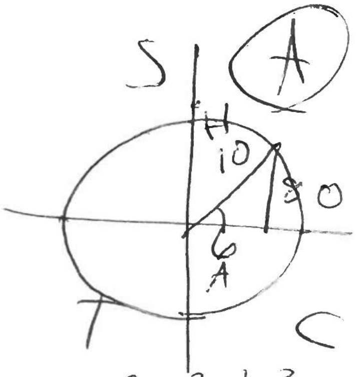
$\frac{6}{10}$

b) $\sin\theta$

$\frac{8}{10}$

c) $\tan\theta$

$\frac{8}{6}$



d) $\sec\theta$

$\frac{10}{6}$

e) $\csc\theta$

$\frac{10}{8}$

f) $\cot\theta$

$\frac{6}{8}$

$a^2+8^2=10^2$

$a^2+64=100$

$-64 \quad -64$

$\sqrt{a^2}=\sqrt{36}$

$a=6$

6. A circle centered at the origin has a radius of 4 units. The terminal side of an angle, θ, intercepts the circle in Quadrant II at point *P*. The *x*-coordinate of point *P* is 2. Find:

a) $\cos\theta$

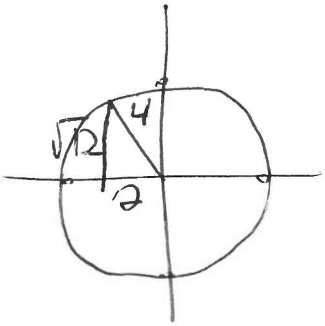
$\frac{2}{4}$

b) $\sin\theta$

$\frac{\sqrt{12}}{4}$

c) $\tan\theta$

$\frac{\sqrt{12}}{2}$



d) $\sec\theta$

$\frac{4}{2}=2$

e) $\csc\theta$

$\frac{4\sqrt{12}}{\sqrt{12}\sqrt{12}}=\frac{4\sqrt{12}}{12}$

f) $\cot\theta$

$\frac{2\sqrt{12}}{\sqrt{12}\sqrt{12}} \quad \frac{2\sqrt{12}}{12}$

$2^2+b^2=4^2$

$4+b^2=16$

$-4 \quad -4$

$\sqrt{b^2}=\sqrt{12}$

$b=\sqrt{12}$

$\cos\theta = \frac{3}{5}$ $\sin\theta = -\frac{4}{5}$

$\cos\theta, \sin\theta$

13. The point $\left(\frac{3}{5}, -\frac{4}{5}\right)$ lines on the unit circle. Find:

a) $\cos\theta$

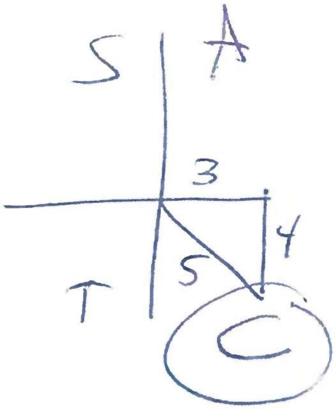
$\frac{3}{5}$

b) $\sin\theta$

$-\frac{4}{5}$

c) $\tan\theta$

$-\frac{4}{3}$



d) $\sec\theta$

$\frac{5}{3}$

e) $\csc\theta$

$-\frac{5}{4}$

f) $\cot\theta$

$-\frac{3}{4}$

$\sin\theta = -\frac{2}{3}$

$\cos\theta, \sin\theta$

14. The point $\left(x, -\frac{2}{3}\right)$ lines on the unit circle where $x > 0$. Find:

a) $\cos\theta$

$\frac{\sqrt{5}}{3}$

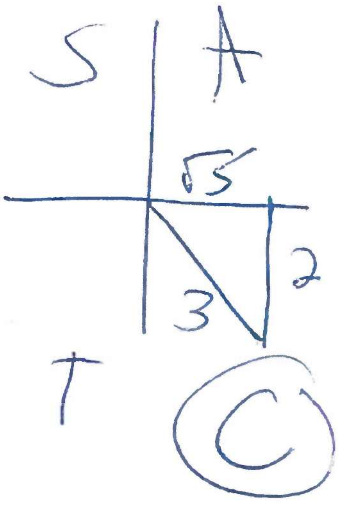
b) $\sin\theta$

$-\frac{2}{3}$

c) $\tan\theta$

$-\frac{2\sqrt{5}}{\sqrt{5}\sqrt{5}}$

$-\frac{2\sqrt{5}}{5}$



d) $\sec\theta$

$\frac{3\sqrt{5}}{\sqrt{5}\sqrt{5}}$

$\frac{3\sqrt{5}}{5}$

e) $\csc\theta$

$-\frac{3}{2}$

f) $\cot\theta$

$-\frac{\sqrt{5}}{2}$

$2^2+b^2=3^2$

$4+b^2=9$

$-4 \quad -4$

$\sqrt{b^2}=\sqrt{5}$

$b=\sqrt{5}$

Name Schlansky　　　　　　　　　　Date ____________
Mr. Schlansky　　　　　　　　　　Algebra II

Advanced Trig Ratios Regents Practice

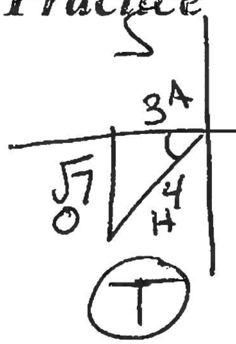
7. If $\cos\theta = -\frac{\overset{A}{3}}{\underset{H}{4}}$ and θ is in Quadrant III, then $\sin\theta$ is equivalent to

[✓ 1) $-\frac{\sqrt{7}}{4}$]　　3) $-\frac{5}{4}$

2) $\frac{\sqrt{7}}{4}$　　4) $\frac{5}{4}$

$\sin\theta = \frac{O}{H}$

$\sin\theta = -\frac{\sqrt{7}}{4}$



A
C

$a^2+b^2=c^2$

$3^2+b^2=4^2$

$9+b^2=16$

$-9 \quad -9$

$\sqrt{b^2}=\sqrt{7}$

$b=\sqrt{7}$

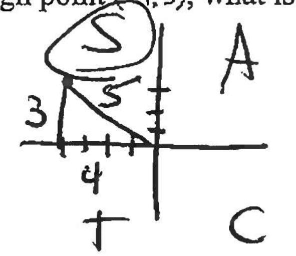
8. If the terminal side of angle θ, in standard position, passes through point (−4,3), what is the numerical value of $\sin\theta$?

[✓ 1) $\frac{3}{5}$]　　3) $-\frac{3}{5}$

2) $\frac{4}{5}$　　4) $-\frac{4}{5}$

$\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{3}{5}$



9. A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ, intercepts the circle in Quadrant II at point C. The y-coordinate of point C is 8. What is the value of $\cos\theta$?

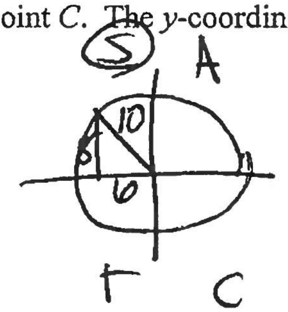
[✓ 1) $-\frac{3}{5}$]　　3) $\frac{3}{5}$

2) $-\frac{3}{4}$　　4) $\frac{4}{5}$

$\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{6}{10}$

$\cos\theta = -\frac{3}{5}$



$a^2+b^2=c^2$

$a^2+8^2=10^2$

$a^2+64=100$

$-64 \quad -64$

$\sqrt{a^2}=\sqrt{36}$

$a=6$

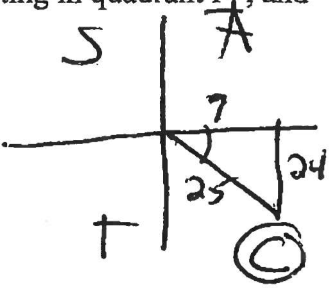
10 4. Given $\cos\theta = \frac{\overset{}{7}\,A}{25\,H}$, where θ is an angle in standard position terminating in quadrant IV, and $\sin^2\theta + \cos^2\theta = 1$, what is the value of $\tan\theta$?

1) $-\frac{24}{25}$　　3) $\frac{24}{25}$

[✓ 2) $-\frac{24}{7}$]　　4) $\frac{24}{7}$

$\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{-24}{7}$



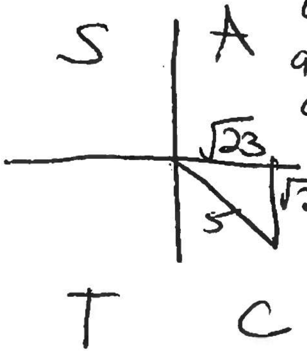
11 5. Given that $\sin^2\theta + \cos^2\theta = 1$ and $\sin\theta = -\frac{\overset{O}{\sqrt{2}}}{\underset{H}{5}}$, what is a possible value of $\cos\theta$? → Could be III or IV

1) $\frac{5+\sqrt{2}}{5}$　　3) $\frac{3\sqrt{3}}{5}$

[✓ 2) $\frac{\sqrt{23}}{5}$]　　4) $\frac{\sqrt{35}}{5}$

$\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{\sqrt{23}}{5}$



$a^2+b^2=c^2$

~~a^2+b^2~~

$a^2+\sqrt{2}^2=5^2$

$a^2+2=25$

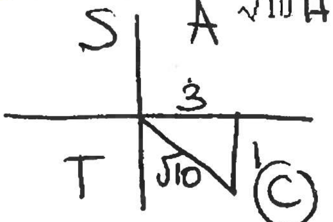
$-2 \quad -2$

$\sqrt{a^2}=\sqrt{23}$

cos+ tan −

12 6. Given $\cos A = \frac{3\ A}{\sqrt{10}\ H}$ and $\cot A = -3$, determine the value of $\sin A$ in radical form.

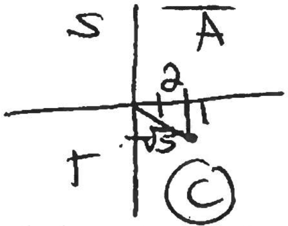
$\tan A = -\frac{1}{3}\ \frac{O}{A}$



$\sin A = \frac{O}{H}$

$\sin A = \frac{1}{\sqrt{10}} \frac{\sqrt{10}}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$

13 7. An angle, θ, is in standard position and its terminal side passes through the point $(2,-1)$. Find the *exact* value of $\sin\theta$.



$a^2+b^2=c^2$

$2^2+1^2=c^2$

$4+1=c^2$

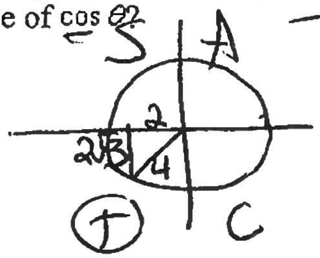
$\sqrt{5}=\sqrt{c^2}$

$\sqrt{5}=c$

$\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{1}{\sqrt{5}} \frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

14 8. A circle centered at the origin has a radius of 4 units. The terminal side of an angle, θ, intercepts the circle in Quadrant III at point P. The x-coordinate of point P is 2. What is the value of $\cos\theta$?



$a^2+b^2=c^2$

$2^2+b^2=4^2$

$4+b^2=16$

$-4 \quad -4$

$\sqrt{b^2}=\sqrt{12}$

$b=\sqrt{4}\sqrt{3}$

$b=2\sqrt{3}$

$\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{2}{4}$

$\cos\theta = -\frac{1}{2}$

15 9. The terminal side of θ, an angle in standard position, intersects the unit circle at $P\left(-\frac{1\ A}{3\ H}, \frac{\sqrt{8}}{3}\right)\frac{O}{H}$

cosθ, sinθ

What is the value of $\sec\theta$?

[✓ 1)] -3

2) $-\frac{3\sqrt{8}}{8}$

3) $-\frac{1}{3}$

4) $-\frac{\sqrt{8}}{3}$

If $\cos\theta = -\frac{1}{3}$

$\sec\theta = -3$

16 10. Point $M\left(t, \frac{4}{7}\right)$ is located in the second quadrant on the unit circle. Determine the exact value of t.

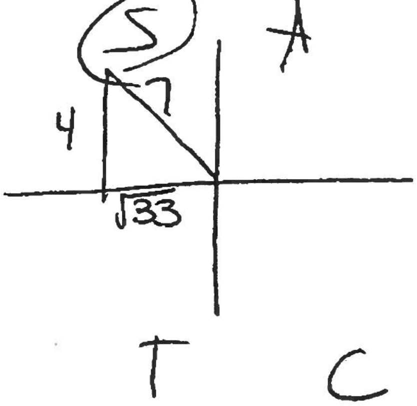
cosθ, sinθ

$\sin\theta = \frac{4}{7}\ \frac{O}{H}$

It is asking for cosθ.

$\cos\theta = \frac{A}{H}$

$\cos\theta = -\frac{\sqrt{33}}{7}$



$a^2+b^2=c^2$

$a^2+4^2=7^2$

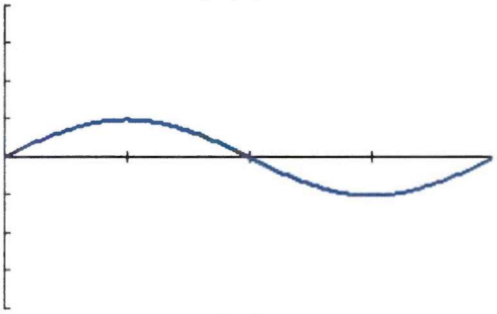
$a^2+16=49$

$-16 \quad -16$

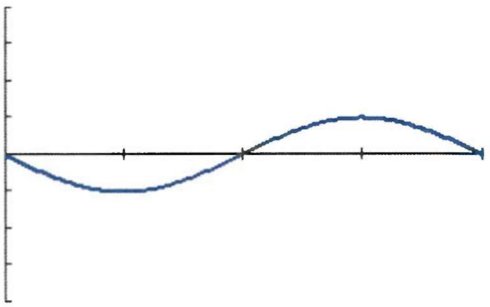
$\sqrt{a^2}=\sqrt{33}$

Trig Graphs:
Know what your waves look like!

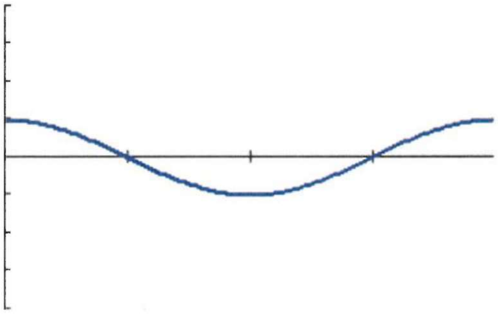
$f(x) = \sin x$



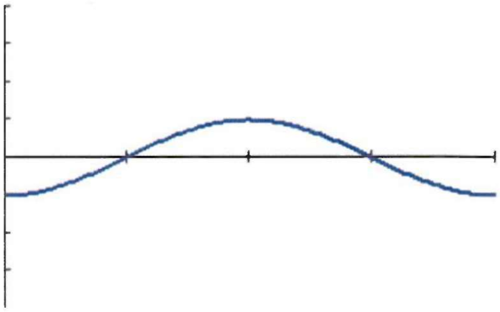
$f(x) = -\sin x$



$f(x) = \cos x$



$f(x) = -\cos x$



AMPSINFREQXSHIFT

Amplitude: Distance from the midline to minimum or maximum
Frequency: How many waves from 0 to 2π
Period: (Wavelength): How long it takes to make one full cycle
Shift/Midline: y value of the midline. The average value of the function.

To graph, list:
Amplitude
$\pm \sin/\cos$
Frequency
Shift/Midline

$$Period = \frac{2\pi}{frequency}$$

y-axis: Plot midline. Count amplitude above and below from the midline.
x-axis: Make 4 dashes on x-axis. Label the 4th dash with the period.
Plot the 5 points for the appropriate wave.

To write the equation, find:

$$Frequency = \frac{2\pi}{period} \text{ and } midline = \frac{\min + \max}{2}$$

Substitute components into $y = amp \sin freqxshift$

If multiple choice, cross out answers with incorrect components!

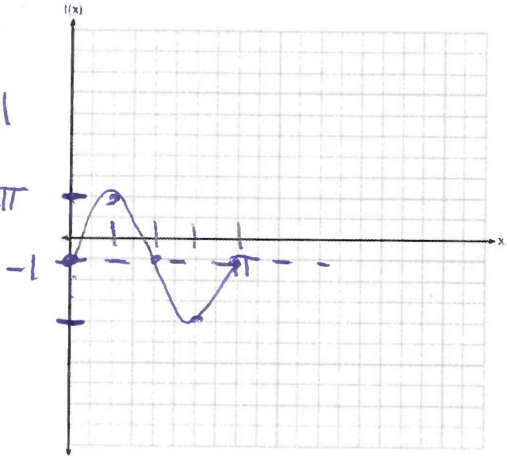
Graph one full wave of the following trigonometric functions

ampsinfreqxshift

1. $y = 3\sin 2x - 1$

amp=3
+sin
freq=2
shift=-1

$p=\frac{2\pi}{2}=\pi$

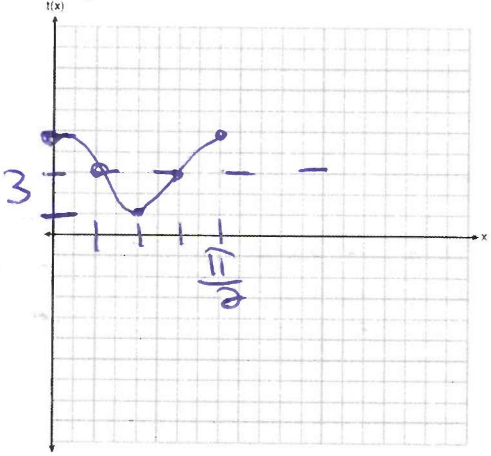


ampsinfreqxshift

2. $y = 2\cos 4x + 3$

amp=2
+cos
freq=4
shift=3

$p=\frac{2\pi}{4}=\frac{\pi}{2}$

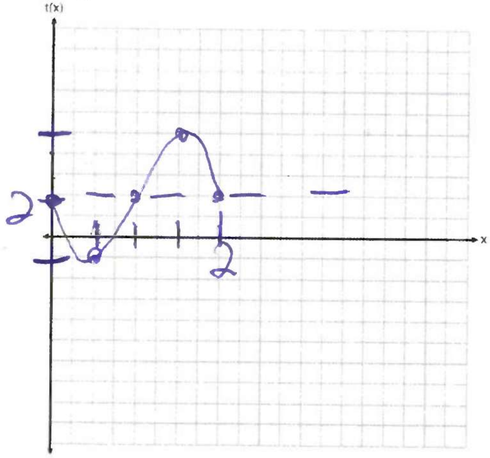


ampsinfreqxshift

3. $y = -3\sin \pi x + 2$

amp=3
-sin
freq=π
shift=2

$p=\frac{2\pi}{\pi}=2$

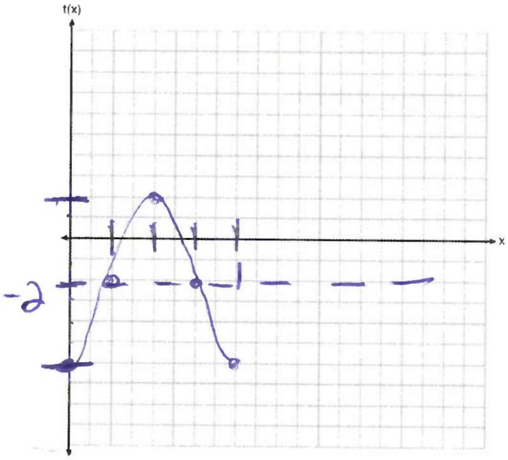


ampsinfreqxshift

4. $y = -4\cos 2\pi x - 2$

amp=4
-cos
freq=2π
shift=-2

$p=\frac{2\pi}{2\pi}=1$



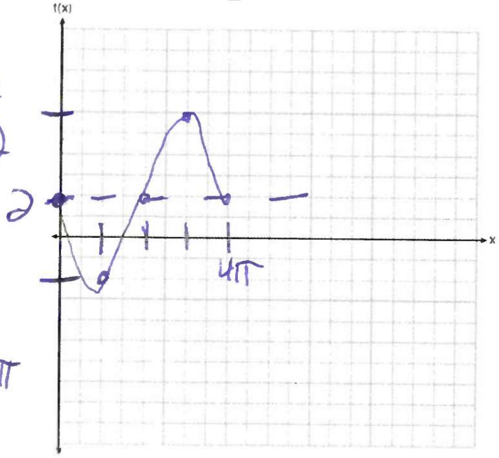
ampsinfreqxshift

5. $y = -4\sin\frac{1}{2}x + 2$

amp=4
-sin
freq=$\frac{1}{2}$
shift=2

$p=\frac{2\pi}{\frac{1}{2}}$

$2\pi \cdot \frac{2}{1}=4\pi$



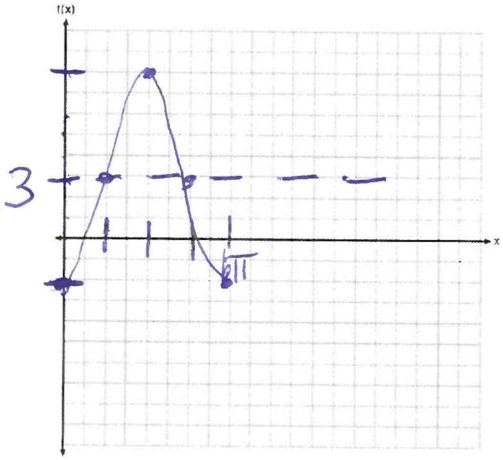
ampsinfreqxshift

6. $y = -5\cos\frac{1}{3}x + 3$

amp=5
-cos
freq=$\frac{1}{3}$
shift=3

$p=\frac{2\pi}{\frac{1}{3}}$

$2\pi \cdot \frac{3}{1}=6\pi$



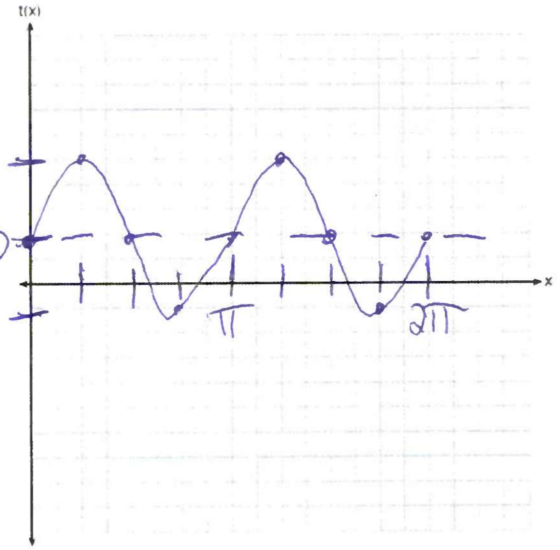
Graph the following two functions over the domain [0, 2π] on the set of axes below.

ampsinfreqxshift

7. $t(x) = 3\sin(2x) + 2$

amp=3
+sin
freq=2
shift=2

P=2π/2=π

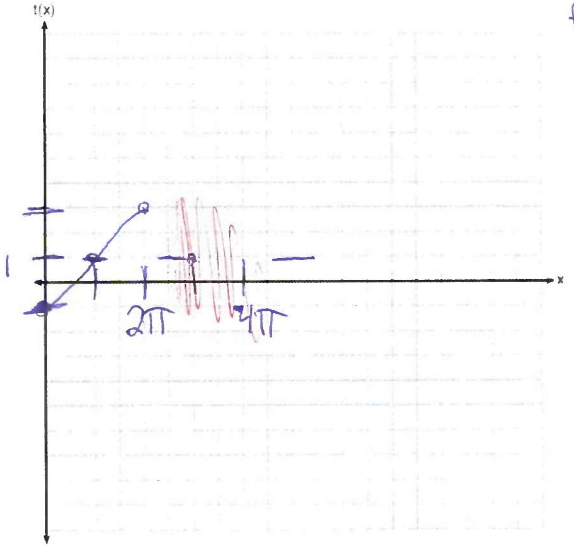


ampsinfreqxshift

8. $y = -2\cos\frac{1}{2}x + 1$

amp=2
-cos
freq=$\frac{1}{2}$
shift=1
$P=\frac{2\pi}{\frac{1}{2}}$

$2\pi \cdot \frac{2}{1} = 4\pi$



9. On the set of axes below, graph

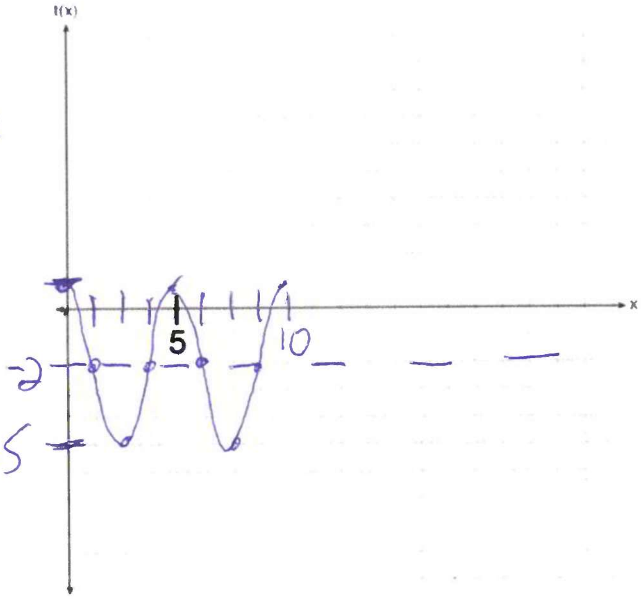
ampsinfreqx shift

$y = 3\cos\frac{2\pi}{5}x - 2$ over the domain [0,10]

amp=3
+cos
freq=$\frac{2\pi}{5}$
shift=-2

$P=\frac{2\pi}{\frac{2\pi}{5}}$

$2\pi \cdot \frac{5}{2\pi} = 5$



10. On the set of axes below, graph

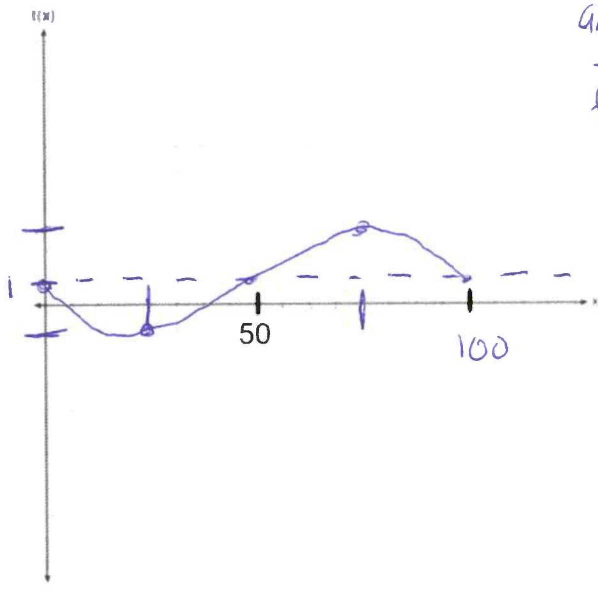
ampsinfreqx shift

$y = -2\sin\frac{\pi}{50}x + 1$ over the domain [0,100]

amp=2
-sin
freq=$\frac{\pi}{50}$
shift=1

$P=\frac{2\pi}{\frac{\pi}{50}}$

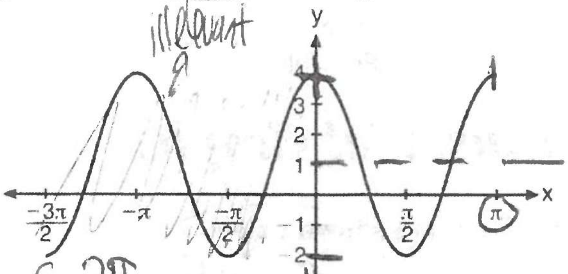
$2\pi \cdot \frac{50}{\pi} = 100$



11.

$y=3\cos 2x+1$

irrelevant 8



amp=3
+cos
freq=2
shift=1

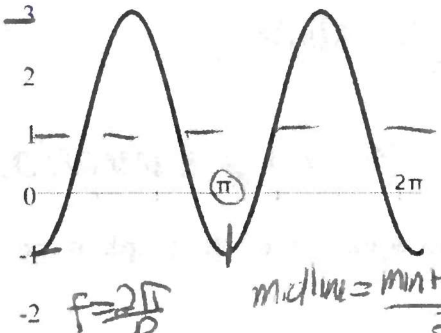
$f=\frac{2\pi}{p}$

$f=\frac{2\pi}{\pi}=2$

midline $=\frac{min+max}{2}$

$=\frac{-2+4}{2}=1$

12.



amp=2
-cos
freq=2
shift=1

$y=-2\cos 2x+1$

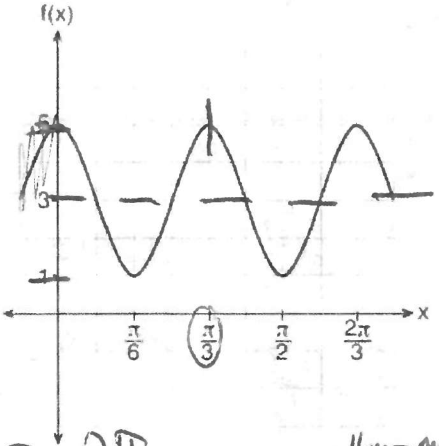
$f=\frac{2\pi}{p}$

$f=\frac{2\pi}{\pi}=2$

midline $=\frac{min+max}{2}$

$=\frac{-1+3}{2}=1$

13. ~~7.~~



amp=2
+cos
freq= ~~$\frac{\pi}{3}$~~ 6
shift=3

$y=2\cos 6x+3$

$f=\frac{2\pi}{p}$

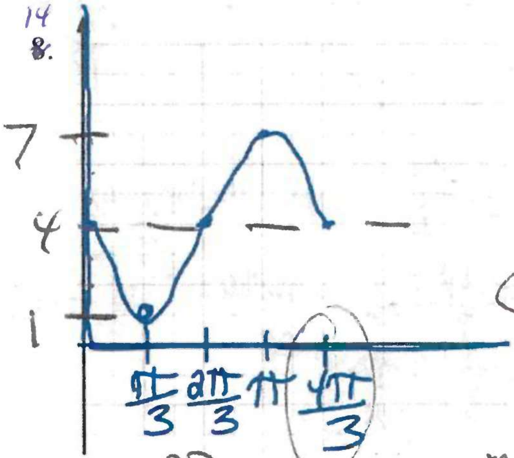
$f=\frac{2\pi}{\frac{\pi}{3}}$ $\frac{2\cancel{\pi}}{1}\cdot\frac{3}{\cancel{\pi}}$

f=6

midline $=\frac{min+max}{2}$

$=\frac{1+5}{2}=3$

14. ~~8.~~



amp=3
-sin
freq=$\frac{3}{2}$
shift=4

$y=-3\sin\frac{3}{2}x+4$

$f=\frac{2\pi}{p}$

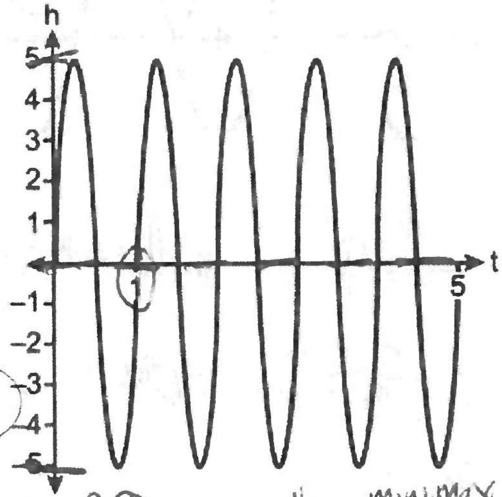
$f=\frac{2\pi}{\frac{4\pi}{3}}$

$\frac{\cancel{2\pi}}{1}\cdot\frac{3}{\cancel{4\pi}_{2}}=\frac{3}{2}$

midline $=\frac{min+max}{2}$

$=\frac{1+7}{2}=4$

15. ~~9.~~



amp=5
+sin
freq=2π
shift=0

$y=5\sin 2\pi x$

$f=\frac{2\pi}{p}$

$f=\frac{2\pi}{1}=2\pi$

midline $=\frac{min+max}{2}$

$=\frac{-5+5}{2}=0$

16. ~~10.~~

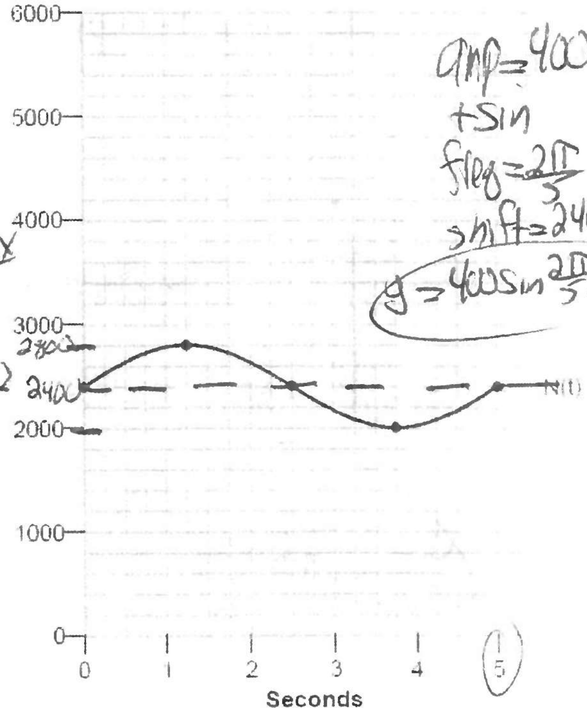
$f=\frac{2\pi}{p}$

$f=\frac{2\pi}{5}$

midline $=\frac{min+max}{2}$

midline $=\frac{2000+2800}{2}$

$=2400$



amp=400
+sin
freq=$\frac{2\pi}{5}$
shift=2400

$y=400\sin\frac{2\pi}{5}x+2400$

17 ~~11~~. The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.

If the depth, d, is measured in feet and time, t, is measured in hours since midnight, what is an equation for the depth of the water at the marker?

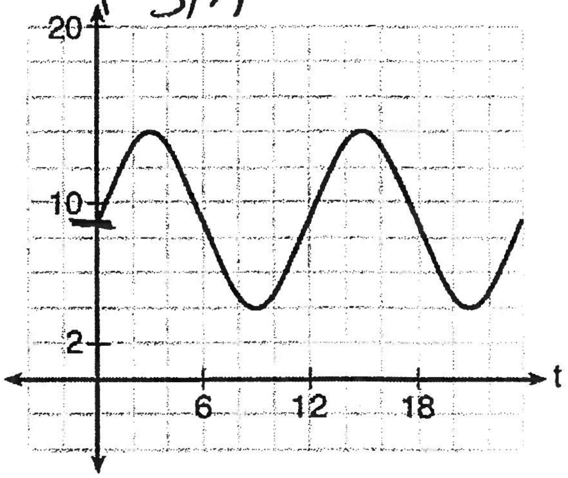
1) ~~$d = 5\cos\left(\frac{\pi}{6}t\right)+9$~~ not +sin

2) ~~$d = 9\cos\left(\frac{\pi}{6}t\right)+5$~~

3) ~~$d = 9\sin\left(\frac{\pi}{6}t\right)+5$~~ midline not 5

[✓ 4)] $d = 5\sin\left(\frac{\pi}{6}t\right)+9$

+sin



18 ~~12~~. Which equation is graphed in the diagram below?

not −cos

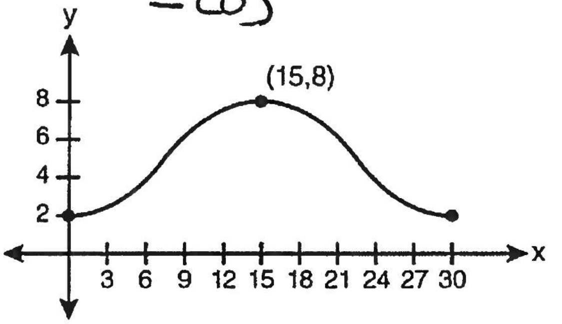
1) ~~$y = 3\cos\left(\frac{\pi}{30}x\right)+8$~~

2) ~~$y = 3\cos\left(\frac{\pi}{15}x\right)+5$~~

3) ~~$y = -3\cos\left(\frac{\pi}{30}x\right)+8$~~ midline not 8

4) $y = -3\cos\left(\frac{\pi}{15}x\right)+5$

−cos



19 ~~13~~. Which equation is represented by the graph below?

[✓ 1)] $y = 2\cos 3x$

2) ~~$y = 2\sin 3x$~~

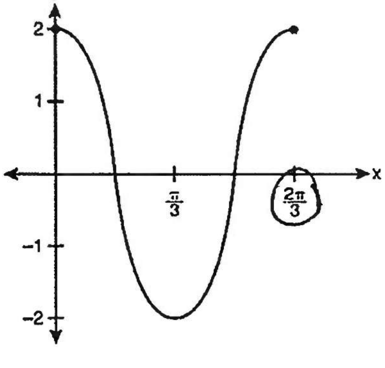
3) ~~$y = 2\cos\left(\frac{2\pi}{3}x\right)$~~

4) ~~$y = 2\sin\frac{2\pi}{3}x$~~

$p=\frac{2\pi}{3}$, not flip

not +cos

+cos

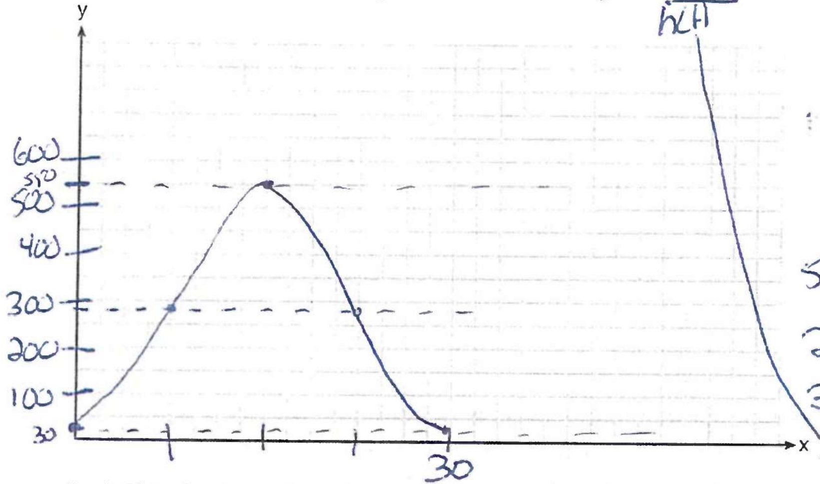


Name Schlansky
Mr. Schlansky

Date ______________
Algebra II

Graphing Sinusoidal Models

1. The High Roller, a Ferris wheel in Las Vegas, Nevada, opened in March 2014. A passenger's height, in feet, above the ground after t minutes can be modeled by the equation $h(t) = -260\cos\left(\frac{\pi}{15}t\right) + 290$. Graph one full cycle of $h(t)$ on the axes provided. Identify the period and state its meaning in the context of the problem. To the *nearest tenth of a* ~~*second*~~ minute, after how much time will the passenger first reach a height of 500 feet?



$h(t) = -260\cos(\frac{\pi}{15}t) + 290$

amp = 260

$p = \frac{2\pi}{\frac{\pi}{15}}$

−cos

freq = $\frac{\pi}{15}$

shift = 290

$\frac{2\pi}{1} \cdot \frac{15}{\pi} = 30$

550 ————
290 — — —
30 ————

Period = 30
It takes 30 minutes for the Ferris wheel to complete one full rotation

$500 = -260\cos(\frac{\pi}{15}t) + 290$

$y_1 = 500$
$y_2 = -260\cos\frac{\pi}{15}t + 290$

Intersect
*4 max = 550

12.0 minutes

2. Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function $f(t) = -13\cos(0.8\pi t) + 13$, where t represents the time (in seconds) since the nail first became caught in the tire. Determine the period of $f(t)$. Interpret what the period represents in this context. On the grid below, graph *at least one* cycle of $f(t)$ that includes the y-intercept of the function. Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

$f(t) = -13\cos(0.8\pi t) + 13$

amp = 13

$p = \frac{2\pi}{0.8\pi}$

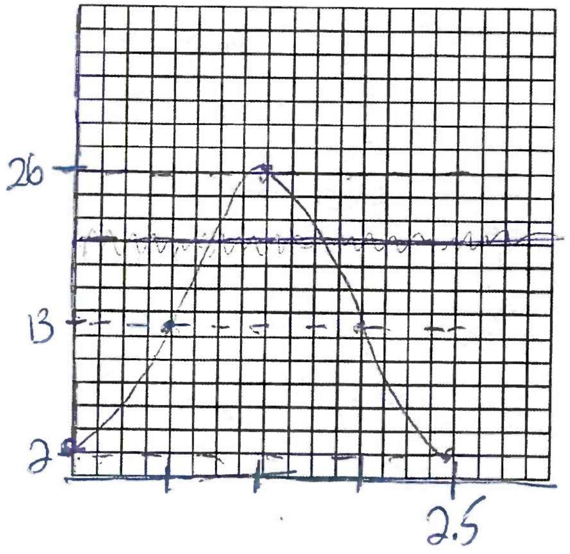
−cos

freq = 0.8π

p = 2.5

shift = 13

Period = 2.5
It takes 2.5 seconds for the bike wheel to make one full rotation



26 ————
13 — — —
0 ————

No, the nail never reaches 30 inches because the maximum height is 26 in

3. The ocean tides near Carter Beach follow a repeating pattern over time, which can be modeled by the equation $h(t) = -12\cos\left(\frac{2\pi}{13}t\right)$ where $h(t)$ represents height above sea level and t represents hours after 8:30 AM. On the grid below, graph one cycle of this function. Determine the period and state its meaning in the context of the problem. People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.

t=11
decreasing

t=14 increasing

$h(t) = -12\cos(\frac{2\pi}{13}t) = 0$

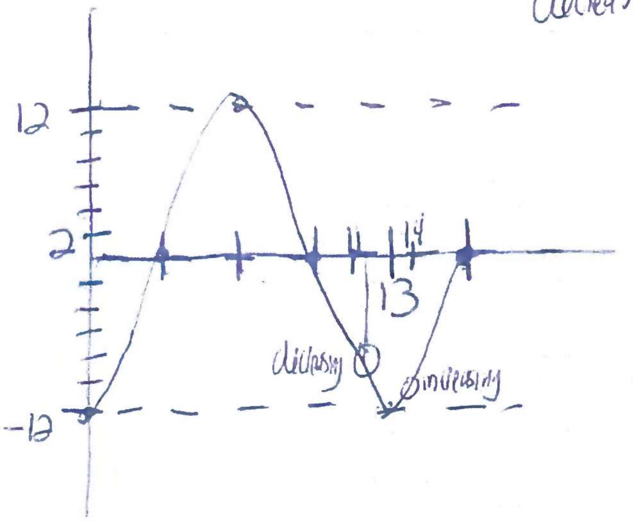
Period=13

It is 13 hours between low tides
or
it takes 13 hour for one complete cycle of the tides

Amp=12
-cos
freq=$\frac{2\pi}{13}$
shift=0

$P = \frac{2\pi}{\frac{2\pi}{13}}$

$\frac{2\pi}{1} \cdot \frac{13}{2\pi} = 13$



12 ———
0 - - -
-12 ———

10:30 (t=14) ~~the~~ the graph is increasing

4. The volume of air in an average lung during breathing can be modeled by the graph below. Using the graph, write an equation for $N(t)$, in the form $N(t) = A\sin(Bt) + C$. That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000\sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds. Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$. How many times during the 5-second interval will $N(t) = E(t)$?

intersect

molewaves!

midline = $\frac{\text{min}+\text{max}}{2}$

midline = $\frac{2000+2800}{2}$

midline = 2400

amp=400
+sin
freq=$\frac{2\pi}{5}$
shift=2400

$E(t) = 2000\sin(\pi t) + 3200$

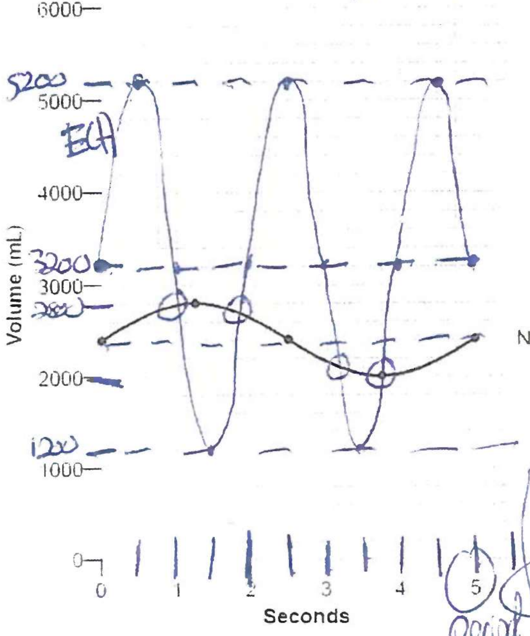
amp=2000
+sin
freq=π
shift=3200

$P = \frac{2\pi}{\pi}$

P=2

5200 ———
3200 - - -
1200 ———

4 intersections



period

$N(t) = 400\sin(\frac{2\pi}{5}t) + 2400$

5 7. The resting blood pressure of an adult patient can be modeled by the function P below, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \le t \le 2$.

ampsinfreqxShift

$24\cos(3\pi t)+120$

amp=24

+cos

freq=3π

shift=120

$P=\frac{2\pi}{f}$

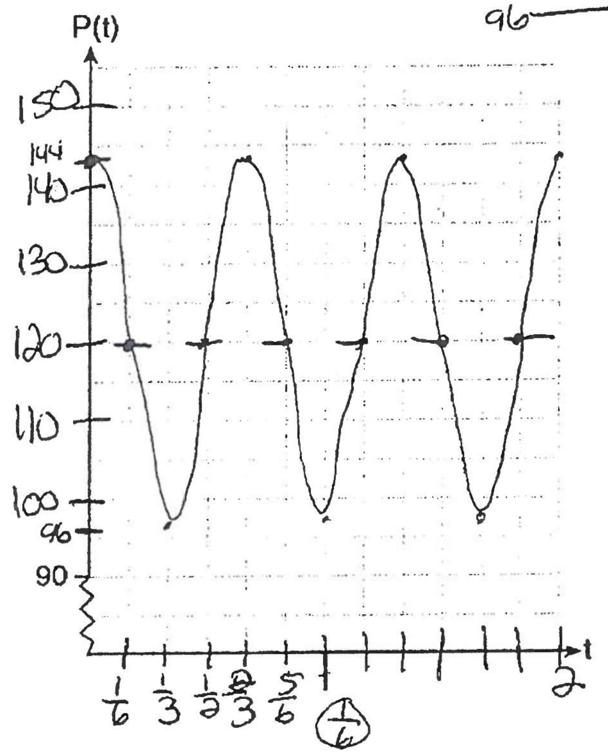
$P=\frac{2\cancel{\pi}}{3\cancel{\pi}}$

$P=\frac{2}{3}$

144 ———

120 – – –

96———



Determine the period of P. Explain what this value represents in the given context. Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.

$P=\frac{2}{3}$

It takes $\frac{2}{3}$ of a Second for blood Pressure to dropdown and Come back up

High Blood Pressure

$144>140$ and $96>90$

Name Schlansky
Mr. Schlansky

Date ____________
Algebra II

Sinusoidal Applications

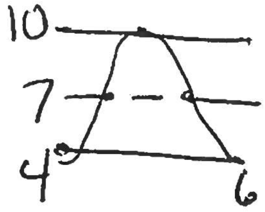
amp sin freq x shift

1. Which statement is *incorrect* for the graph of the function $y = -3\cos\left[\frac{\pi}{3}(x-4)\right]+7$?

1) The period is 6. ✓
2) The amplitude is 3. ✓
3) The range is [4,10]. ✓
[✓ 4)] The midline is $y = -4$. X

$y=7$

amp = 3
-cos
freq = $\frac{\pi}{3}$
shift = 7



$p=\frac{2\pi}{f}$

$p=\frac{2\pi}{\frac{\pi}{3}}$

$p=\frac{2\pi}{1}\cdot\frac{3}{\pi}$

$p=6$

2. Which function's graph has a period of 8 and reaches a maximum height of 1 if at least one full period is graphed?

[✓ 1)] $y = -4\cos\left(\frac{\pi}{4}x\right)-3$

1
-3
-7

2) $y = -4\cos\left(\frac{\pi}{4}x\right)+5$

9
5
1

~~3) $y = -4\cos(8x)-3$~~

~~4) $y = -4\cos(8x)+5$~~

1
8

$f=\frac{2\pi}{p}$

$f=\frac{2\pi}{8}$

$f=\frac{\pi}{4}$

3. The equation below can be used to model the height of a tide in feet, $H(t)$, on a beach at t hours.

amp sin freq x shift

$$H(t) = 4.8\sin\left[\frac{\pi}{6}(t+3)\right]+5.1$$

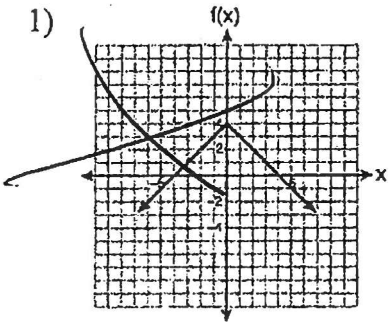
Using this function, the amplitude of the tide is

1) $\frac{\pi}{6}$
[✓ 2)] 4.8
3) 3
4) 5.1

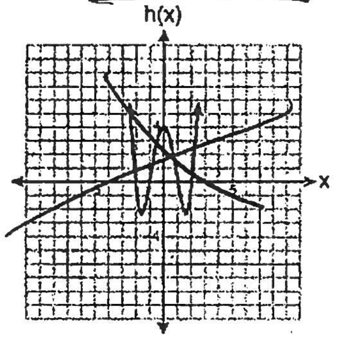
→ must be trig graph

4. Which function has a maximum y-value of 4 and a midline of $y = 1$?

1)



3)



[✓ 2)] $g(x) = -3\cos(x)+1$

4
1
-2

4) $j(x) = 4\sin(x)+1$

5
1
-3

5. The depth of the water, $d(t)$, in feet, on a given day at Thunder Bay, t hours after midnight is modeled by $d(t) = 5\sin\left(\frac{\pi}{6}(t-5)\right)+7$. Which statement about the Thunder Bay tide is *false*?

amp sin freq x shift

1) A low tide occurred at 2 a.m.
2) The maximum depth of the water was 12 feet.
3) The water depth at 9 a.m. was approximately 11 feet.
4) [✓ 4)] The difference in water depth between high tide and low tide is 14 feet. 10 feet

amp = 5
+sin
freq = $\frac{\pi}{6}$

shift = 7
$p = \frac{2\pi}{\frac{\pi}{6}}$

$\frac{2\pi}{1} \cdot \frac{6}{\pi} = 12$

12 ———
7 — — —
2 ———

6. A person's lung capacity can be modeled by the function $C(t) = 250\sin\left(\frac{2\pi}{5}t\right) + 2450$, where $C(t)$ represents the volume in mL present in the lungs after t seconds. State the maximum value of this function over one full cycle, and explain what this value represents.

amp sin freq x shift

2700 ———
2450 — — —
2200 ———

2700. the maximum volume present in the lungs is 2700 mL.

7. Based on climate data that have been collected in Bar Harbor, Maine, the average monthly temperature, in degrees F, can be modeled by the equation $B(x) = 23.914\sin(0.508x - 2.116) + 55.300$. The same governmental agency collected average monthly temperature data for Phoenix, Arizona, and found the temperatures could be modeled by the equation $P(x) = 20.238\sin(0.525x - 2.148) + 86.729$. Which statement can *not* be concluded based on the average monthly temperature models x months after starting data collection?

1) The average monthly temperature amp variation is more in Bar Harbor than in Phoenix. $23.914 > 20.238$
2) The midline average monthly temperature for Bar Harbor is lower than the midline temperature for Phoenix. $55.3 < 86.729$ ✓
3) ✓ The maximum average monthly temperature for Bar Harbor is 79° F, to the nearest degree.
4) [✓ 4)] The minimum average monthly temperature for Phoenix is 20° F, to the nearest degree. 66.441

B(x)
9.214 ———
55.3 — — —
31.386 ———

P(x)
106.967 ———
86.729 — — —
66.441 ———

8. The average monthly temperature of a city can be modeled by a cosine graph. Melissa has been living in Phoenix, Arizona, where the average annual temperature is 75°F. She would like to move, and live in a location where the average annual temperature is 62°F. When examining the graphs of the average monthly temperatures for various locations, Melissa should focus on the

average = midline

1) amplitude
2) horizontal shift
3) period
4) [✓ 4)] midline

9. Tides are a periodic rise and fall of ocean water. On a typical day at a seaport, to predict the ~~time of~~ the next high tide, the most important value to have would be the

[✓ 1)] time between consecutive low tides

2) time when the tide height is 20 feet

3) average depth of water over a 24-hour period

4) difference between the water heights at low and high tide

period

amp sin freq shift

10. Consider the function $h(x) = 2\sin(3x) + 1$ and the function q represented in the table below. Determine which function has the *smaller* minimum value for the domain $[-2, 2]$. Justify your answer.

3 ———

1 – – –

$\boxed{-1}$ ———

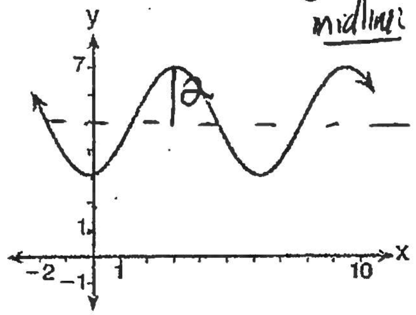
q(x) has the smaller minimum.

$-8 < -1$

x	$q(x)$
-2	$\boxed{-8}$
-1	0
0	0
1	-2
2	0

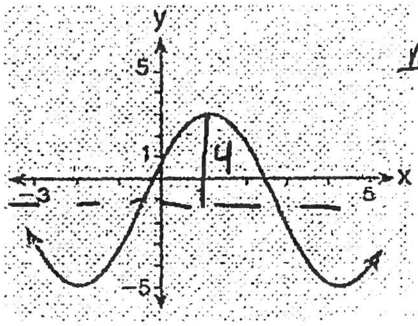
11. Which sinusoid has the greatest amplitude?

1)



midline: $\frac{3+7}{2} = 5$

3)



midline

$\frac{3+-5}{2} = -1$

2) $y = 3\sin(\theta - 3) + 5$

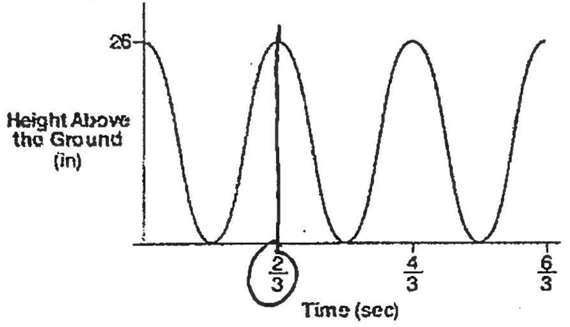
3

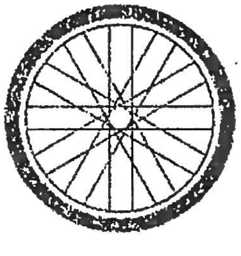
[✓ 4)] $y = -5\sin(\theta - 1) - 3$

5

12. The graph below represents the height above the ground, h, in inches, of a point on a triathlete's bike wheel during a training ride in terms of time, t, in seconds.

Identify the period of the graph and describe what the period represents in this context.

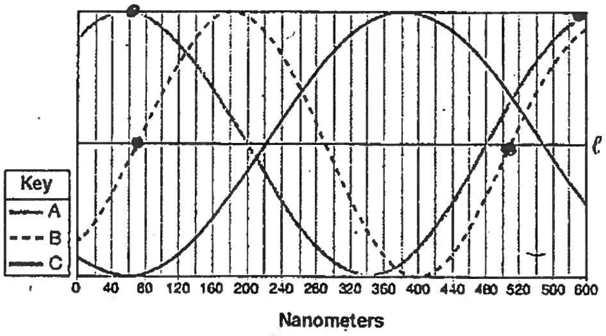




$\frac{2}{3}$. It takes the wheel $\frac{2}{3}$ of a second to make one complete rotation.

13. Visible light can be represented by sinusoidal waves. Three visible light waves are shown in the graph below. The midline of each wave is labeled ℓ. Based on the graph, which light wave has the longest period? Justify your answer.

C. It is the only one that can't fit one full cycle on the graph.



14. The Sea Dragon, a pendulum ride at an amusement park, moves from its central position at rest according to the trigonometric function $P(t) = -10\sin(\frac{\pi}{3}t)$, where t represents time, in seconds. How many seconds does it take the pendulum to complete one full cycle?

period

1) 5
[✓ 2) 6]
3) 3
4) 10

$P = \frac{2\pi}{f}$

$P = \frac{2\pi}{\frac{\pi}{3}}$ $\quad \frac{2\cancel{\pi}}{1} \cdot \frac{3}{\cancel{\pi}} = 6$

15. A wave displayed by an oscilloscope is represented by the equation $y = 3\sin x$. What is the period of this function?

[✓ 1) 2π]
2) 2
3) 3
4) 3π

$P = \frac{2\pi}{f}$

$P = \frac{2\pi}{1} = 2\pi$

16. The height above ground for a person riding a Ferris wheel after t seconds is modeled by $h(t) = 150\sin\left(\frac{\pi}{45}t + 67.5\right) + 160$ feet. How many seconds does it take to go from the bottom of the wheel to the top of the wheel? half of a full cycle

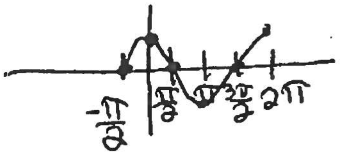
1) 10
[✓ 2) 45]
3) 90
4) 150

$P = \frac{2\pi}{f}$ $\quad P = \frac{2\pi}{\frac{\pi}{45}}$ $\quad \boxed{\frac{1}{2}(90) \\ 45}$

$P = \frac{2\cancel{\pi}}{1} \cdot \frac{45}{\cancel{\pi}} = 90$

17. As θ increases from $-\frac{\pi}{2}$ to 0 radians, the value of $\cos\theta$ will $\quad P = \frac{2\pi}{1} = 2\pi$

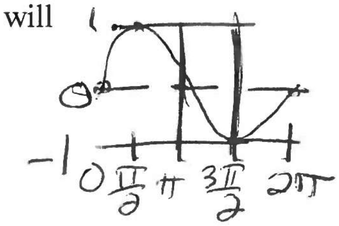
1) decrease from 1 to 0
2) decrease from 0 to −1
3) increase from −1 to 0
[✓ 4) increase from 0 to 1]



$\frac{2\pi}{4}, \frac{\pi}{2}$

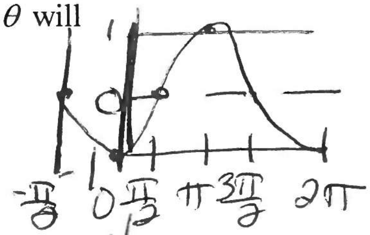
17. As θ increases from π to $\frac{3\pi}{2}$ radians, the graph of $y = \sin\theta$ will

1) Decrease from 1 to 0
[✓ 2) Decrease from 0 to -1]
3) Increase from -1 to 0
4) Increase from 0 to 1



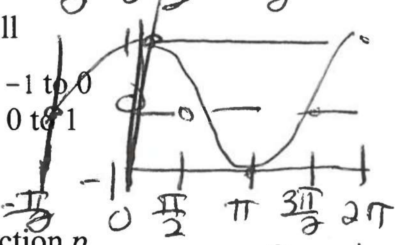
18. As θ increases from $-\frac{\pi}{2}$ to 0 radians, the graph of $y = -\cos\theta$ will

1) Decrease from 1 to 0
[✓ 2) Decrease from 0 to -1]
3) Increase from -1 to 0
4) Increase from 0 to 1



19. As θ increases from $-\frac{\pi}{2}$ to 0 radians, the value of $\cos\theta$ will

1) decrease from 1 to 0
2) decrease from 0 to -1
3) increase from -1 to 0
[✓ 4) increase from 0 to 1]



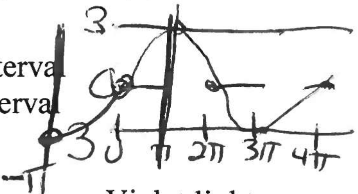
$\frac{4\pi}{4\pi}$

20. Given $p(\theta) = 3\sin\left(\frac{1}{2}\theta\right)$ on the interval $-\pi < \theta < \pi$, the function p

1) decreases, then increases
2) increases, then decreases
3) decreases throughout the interval
[✓ 4) increases throughout the interval]

$P = \frac{2\pi}{\frac{1}{2}}$

$P = 2\pi \cdot \frac{2}{1} = 4\pi$

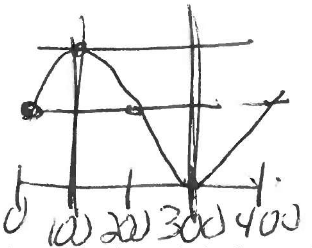


21. A sine function increasing through the origin can be used to model light waves. Violet light has a wavelength of 400 nanometers. Over which interval is the height of the wave *decreasing*, only?

P

$\frac{400}{4} = 100$

1) (0, 200)
[✓ 2) (100, 300)]
3) (200, 400)
4) (300, 400)



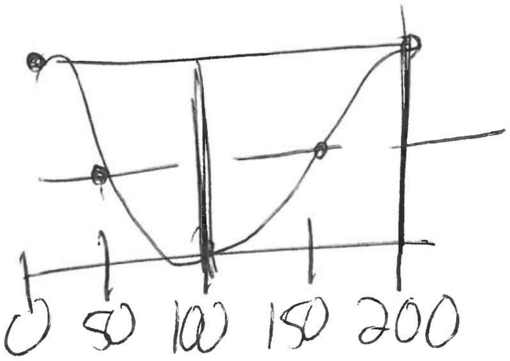
22. A cosine function decreasing through the origin has a frequency of $\frac{\pi}{100}$. What is the first positive interval where the wave is increasing?

$P = \frac{2\pi}{\frac{\pi}{100}}$

$\frac{200}{4} = 50$

$P = \cancel{2\pi} \cdot \frac{100}{\cancel{\pi}} = 200$

(100, 200)



1 ~~13~~. The probability that a student in Jacqua High School is in band is $\frac{127}{466}$ and the probability that a student is on the track team is $\frac{82}{466}$. If the probability that they are on the track team and in band is $\frac{74}{466}$, what is the probability that they are on the track ~~and~~ team or in band?

A= band
B= track

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$P(A\cup B)=\frac{127}{466}+\frac{82}{466}-\frac{74}{466}$$
$$P(A\cup B)=\frac{135}{466}$$

2 ~~14~~. The probability that a person files their tax return in March is $\frac{127}{165}$. The probability that a person watches College Basketball in March is $\frac{98}{123}$. If the probability that a person watches College Basketball and files their tax return in March is $\frac{62}{95}$, what is the probability that a person watches College Basketball or files their tax return? Round your answer to the nearest percent.

A= tax
B= basketball

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$P(A\cup B)=\frac{127}{165}+\frac{98}{123}-\frac{62}{95}$$
$$P(A\cup B)=.9138\_(100)\approx\boxed{91\%}$$

3 ~~15~~. On a given school day, the probability that Nick oversleeps is 48% and the probability he has a pop quiz is 25%. Assuming these two events are independent, what is the probability that Nick oversleeps and has a pop quiz on the same day?

A=oversleep
B=pop quiz

1) 73%
2) 36%
3) 23%
[✓ 4) 12%]

$$P(A\cap B)=P(A)\cdot P(B)$$
$$P(A\cap B)=.48\cdot .25$$
$$P(A\cap B)=.12$$

4 ~~18~~. In 2015 at Sabres Prep Academy, the probability that a student passed Algebra II was 78%. The probability that a student passed Chemistry was 86%. The probability they passed Algebra II or Chemistry was 88%. What is the probability that they *did not* pass Algebra II and Chemistry?

A= Algebra II
B= Chemistry

$$P(A\cap B)=P(A)+P(B)-P(A\cup B)\quad \text{not } P(\overline{A\cap B})=1-P(A\cap B)$$
$$P(A\cap B)=.78+.86-.88$$
$$P(A\cap B)=.76$$
$$\text{not } P(A\cap B)=1-.76=\boxed{.24}$$

5 ~~17~~. The probability that Chloe the cardinal shows up in the Schlansky's backyard is $\frac{12}{19}$.

The probability that Chloe shows up in the Silverman's backyard is $\frac{10}{17}$. If the probability that Chloe shows up in the Schlansky's backyard or the Silverman's backyard is $\frac{12}{16}$, what is the probability that Chloe shows up in both backyards?

and

A = Schlansky
B = Silverman

$P(A\cap B) = P(A)+P(B) - P(A\cup B)$

$P(A\cap B) = \frac{12}{19} + \frac{10}{17} - \frac{12}{16}$

$\boxed{P(A\cap B) = \frac{607}{1292}}$

6 ~~18~~. There are 24 students in a math class. 15 of them play a sport and 20 of them play an instrument. 22 play a sport or play an instrument. What is the probability that a student chosen at random will play a sport and play an instrument?

A = Sport
B = instrument
$P(A) = \frac{15}{24}$
$P(B) = \frac{20}{24}$
$P(A\cup B) = \frac{22}{24}$

$P(A\cap B) = P(A)+P(B) - P(A\cup B)$

$P(A\cap B) = \frac{15}{24} + \frac{20}{24} - \frac{22}{24}$

$\boxed{P(A\cap B) = \frac{13}{24}}$

7 ~~19~~. Over the past 30 nights, Baxter barked 8 nights and cried 15 nights. He barked or cried 11 nights. How many nights did he bark and cry?

A = bark
B = cry
$P(A) = \frac{8}{30}$
$P(B) = \frac{15}{30}$
$P(A\cup B) = \frac{11}{30}$

$P(A\cap B) = P(A)+P(B) - P\ A\cup B)$

$P(A\cap B) = \frac{8}{30} + \frac{15}{30} - \frac{11}{30}$

$\boxed{P(A\cap B) = \frac{12}{30}}$

8 ~~20~~. Suppose events A and B are independent and $P(A \text{ and } B)$ is 0.2. Which statement could be true?

$.4 \cdot .3 = .12$

1) $P(A) = 0.4$, $P(B) = 0.3$, $P(A \text{ or } B) = 0.5$

[✓ 2)] $P(A) = 0.8$, $P(B) = 0.25$ $.8 \cdot .25 = .2$ ✓

3) $P(A|B) = 0.2$, $P(B) = 0.2$ $P(A) = P(A|B)$

4) $P(A) = 0.15$, $P(B) = 0.05$ $.15 \cdot .05 = .0075$

$P(A\cap B) = P(A)\cdot P(B)$

$.2 = P(A)\cdot P(B)$

1. One-hundred employees of a company were asked their opinion on paying high salaries to the CEO. Their responses are summarized in the following contingency table. Express the following probabilities as fractions and rounded to the nearest percent.

	In Favor	Against	
Male	15	45	60.
Female	4	36	40
	14	81	100

a) Find the probability that they are male and in favor

$$\frac{15}{100}$$

b) Find the probability that they are female one thing

$$\frac{40}{100}$$

c) Find the probability that a male is in favor

$$\frac{15}{60}$$

d) Find the probability that they are against given that they are female

$$\frac{36}{40}$$

e) Find the probability that someone is in favor is a male

$$\frac{15}{19}$$

f) Find the probability that someone is female and against

$$\frac{36}{100}$$

g) Find the probability that a female is in favor

$$\frac{4}{40}$$

h) Find the probability that someone is male given that they are in favor

$$\frac{15}{19}$$

2 ~~11~~. A statistics class surveyed some students during one lunch period to obtain opinions about television programming preferences. The results of the survey are summarized in the table below.

Programming Preferences

	Comedy	Drama	
Male	70	35	105
Female	48	42	90
	118	77	195

What is the probability that a student is male and prefers comedy?

$\frac{70}{195}$

What is the probability that a male student would prefer comedy?

$\frac{70}{105}$

What is the probability that a student is male? 1 thing

$\frac{105}{195}$

What is the probability that a student is female given that they like drama?

$\frac{42}{77}$

3 ~~12~~. A public opinion poll was taken to explore the relationship between age and support for a candidate in an election. The results of the poll are summarized in the table below.

Age	For	Against	No Opinion	
21–40	30	12	8	50
41–60	20	40	15	75
Over 60	25	35	15	75
	75	87	38	200

What is the probability that someone has no opinion? 1 thing

$\frac{38}{200}$

What is the probability that someone is over 60 and against?

$\frac{35}{200}$

What is the probability that someone is for the candidate given that they are between 21-40?

$\frac{30}{50}$

4 14. A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

	Favorite Type of Program		
	Sports	**Reality Show**	**Comedy Series**
Senior	83	110	67
Freshmen	119	103	54

260
276
536

202 213 121

A student response is selected at random from the results. State the probability the student response is from a freshman, given the student prefers to watch reality shows on television.

$$\frac{103}{213}$$

5 15. At Berkeley Central High School, a survey was conducted to see if students preferred cheeseburgers, pizza, or hot dogs for lunch. The results of this survey are shown in the table below.

	Cheeseburgers	**Pizza**	**Hot Dogs**
Females	32	44	24
Males	36	30	34

100
100
200

68 74 58

Based on this survey, what percent of the students preferred pizza?

1) 30
[✓ 2) 37]
3) 44
4) 74

$$\frac{74}{200} = 37\%$$

6 16. A middle school conducted a survey of students to determine if they spent more of their time playing games or watching videos on their tablets. The results are shown in the table below.

	Playing Games	**Watching Videos**	**Total**
Boys	138	46	184
Girls	54	142	196
Total	192	188	380

Of the students who spent more time playing games on their tablets, approximately what percent were boys?

1) 41
2) 56
[✓ 3) 72]
4) 75

$$\frac{138}{192} = 71.875\%$$

7 16 17. A survey was given to 12th-grade students of West High School to determine the location for the senior class trip. The results are shown in the table below.

	Niagara Falls	Darien Lake	New York City	
Boys	56	74	103	233
Girls	71	92	88	251
	127	166	191	484

To the *nearest percent*, what percent of the boys chose Niagara Falls?

1) 12
[✓ 2)] 24
3) 44
4) 56

$\frac{56}{233} \approx 24\%$

8 17 18. Jenna took a survey of her senior class to see whether they preferred pizza or burgers. The results are summarized in the table below.

	Pizza	Burgers	
Male	23	42	65
Female	31	26	57
	54	68	122

Of the people who preferred burgers, approximately what percentage were female?

1) 21.3
[✓ 2)] 38.2
3) 45.6
4) 61.9

$\frac{26}{68} \approx 38.2$

9 18 19. Students were asked to name their favorite sport from a list of basketball, soccer, or tennis. The results are shown in the table below.

	Basketball	Soccer	Tennis	
Girls	42	58	20	120
Boys	84	41	5	130
	126	99	25	250

What percentage of the students chose soccer as their favorite sport?

[✓ 1)] 39.6%
2) 41.4%
3) 50.4%
4) 58.6%

$\frac{99}{250} = 39.6\%$

Independence

If events are independent:

$P(A \cap B) = P(A) \bullet P(B)$ → No condition, denominator is always total total. This formula is generally easier to use.

$P(A) = P(A/B)$

1. The results of a poll of 200 students are shown in the table below:

	Preferred Music Style			
	Techno	**Rap**	**Country**	
Female	54	25	27	106
Male	36	40	18	94
	90	65	45	200

A = male

B = techno

(doesn't matter which you pick)

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

$$P(A \cap B) = P(A) \circ P(B)$$

$$\frac{36}{200} \neq \frac{94}{200} \cdot \frac{90}{200}$$

Not Independent

2. At a local mall, 125 people were asked how they choose to pay for their merchandise. The data is shown in the table below:

	Credit Card	Cash	
Male	40	10	50
Female	60	15	75
	100	25	125

A = male

B = cash

(doesn't matter which you pick)

Does the data suggest that the gender and type of payment are independent of each other? Explain your answer.

$$P(A \cap B) = P(A) \cdot P(B)$$

$$\frac{10}{125} = \frac{50}{125} \cdot \frac{25}{125} \checkmark$$

Independent because $P(A \cap B) = P(A) \cdot P(B)$

125

total

3. One-hundred employees of a company were asked their opinion on paying high salaries to the CEO. Their responses are summarized in the following contingency table.

A=male
B=In Favor

	In Favor	Against	Total
Male	15	45	60
Female	4	36	40
Total	19	81	100

Based on the data, are gender an opinion on salaries independent of each other? Justify your answer.

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{15}{100} = \frac{60}{100} \cdot \frac{19}{100}$

$\frac{3}{20} = \frac{57}{500}$ X

Not independent

$P(A) = P(A|B)$

$\frac{60}{100} = \frac{15}{19}$

$\frac{3}{5} = \frac{15}{19}$ X

Not Independent

4. Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

A=Filipe wins
B=long practice time

	Juan Wins	**Felipe Wins**	Total
Short Practice Time	8	10	18
Long Practice Time	15	12	27
Total	23	22	45

Given that the practice time was long, determine the exact probability that Filipe wins the next match. Determine whether or not the two events "Filipe wins" and "long practice time" are independent. Justify your answer.

$P(A|B) = \boxed{\frac{12}{27}}$

Not Independent

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{12}{45} = \frac{22}{45} \cdot \frac{27}{45}$

$\frac{4}{15} \neq \frac{22}{75}$

$P(A) = P(A|B)$

$\frac{22}{45} = \frac{12}{27}$

$\frac{22}{45} \neq \frac{4}{9}$

Not Independent

5. The results of a survey of the student body at Central High School about television viewing preferences are shown below.

A=male
B=reality

	Comedy Series	**Drama Series**	**Reality Series**	**Total**
Males	95	65	70	230
Females	80	70	110	260
Total	175	135	180	490

Are the events "student is a male" and "student prefers reality series" independent of each other? Justify your answer.

Not Independent

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{70}{490} = \frac{230}{490} \cdot \frac{180}{490}$

$\frac{1}{7} \neq \frac{414}{2401}$

$P(A) = P(A|B)$

$\frac{230}{490} = \frac{70}{180}$

$\frac{23}{49} \neq \frac{7}{18}$

Not Independent

$P(\underline{A}) = P(\underline{A}|B)$

these letters must match

6. Given events A and B, such that $P(A) = 0.8$, $P(B) = 0.6$, and $P(A|B) = 0.6$. Determine whether A and B are independent. Explain your answer.

$P(A|B) \neq P(A)$ Not Independent

$0.6 \neq 0.8$

7. Given events A and B, such that $P(A) = 0.8$, $P(B) = 0.6$, and $P(B|A) = 0.6$. Determine whether A and B are independent. Explain your answer.

$P(B|A) = P(B)$ Independent

$0.6 = 0.6$ ✓

8. A fast-food restaurant analyzes data to better serve its customers. After its analysis, it discovers that the events *D*, that a customer uses the drive-thru, and *F*, that a customer orders French fries, are independent. The following data are given in a report:

$P(F) = 0.8$

$P(F \cap D) = 0.456$

Given this information, $P(F|D)$ is

1) 0.344
2) 0.3648
3) 0.57
[✓ 4) 0.8]

$P(F|D) = P(F)$

9. Given events T and K are independent of each other, if $P(T) = 0.35$, $P(K) = 0.48$, find $P(K|T)$ = .48

$P(K|T) = P(K)$

$P(P) = 0.5$

$P(R) = 0.4$

10. Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

1) independent
2) dependent
3) mutually exclusive
4) complements

$P(R|P) = .4$

$P(R|P) = P(R)$

$.4 = .4$ Independent

✓

Normal Distributions

2nd vars: 2:normal cdf

Lower = lower bound, Upper = upper bound, μ= mean, σ= standard devi

Less than 3:	More than 3:	Between 3 and 6
Lower: -99999999999 Upper 3	Lower: 3 Upper: 9999999999	Lower: 3 Upper: 6

If asked for:

Probability	Percent	Quantity
You're done!	Multiply by 100	Multiply by the total quantity

normalcdf

1. The weights of bags of Graseck's Chocolate Candies are normally distributed with a mean of 4.3 ounces and a standard deviation of 0.05 ounces. What is the probability that a bag of these chocolate candies weighs less than 4.27 ounces?

upper

1) 0.2257
[✓ 2)] 0.2743
3) 0.7257
4) 0.7757

lower = -999999
upper = 4.27
μ = 4.3
σ = .05
.2743

normalcdf

2. The weight of a bag of pears at the local market averages 8 pounds with a standard deviation of 0.5 pound. The weights of all the bags of pears at the market closely follow a normal distribution. Determine what percentage of bags, to the *nearest integer*, weighed *more* than 8.25 pounds.

lower

lower = 8.25
upper = 99999
μ = 8
σ = 0.5

.308.. (100) = 31%

total quantity

normalcdf

3. The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18. Determine the number of students who scored between 200 and 245.

lower = 200
upper = 245
μ = ~~[illegible]~~ 225
σ = ~~[illegible]~~ 18

.7843.. (1200)

941

normalcdf

4. The weights of students on the boys cross country team is normally distributed with a mean of 135.3 pounds and a standard deviation of 2.8 pounds. If the team has 32 members, how many of them, rounded to the *nearest person,* would be expected to weigh less than 132 pounds?

upper

lower = -99999
upper = 132
μ = 135.3
σ = 2.8

.119.. (32) = 4

normal cdf

5 ~~6~~. The lifespan of a 60-watt lightbulb produced by a company is normally distributed with a mean of 1450 hours and a standard deviation of 8.5 hours. If a 60-watt lightbulb produced by this company is selected at random, what is the probability that its lifespan will be between 1440 and 1465 hours?

1) 0.3803
2) 0.4612
[✓ 3)] 0.8415
4) 0.9612

lower = 1440
upper = 1465
$\mu = 1450$
$\sigma = 8.5$
.8415..

6 ~~7~~. The number of hours students spent studying for their Regents exam is normally distributed with a mean of 14 hours and a standard deviation of 3.2 hours. If a student is randomly selected, what is the probability that a student spent more than 22 hours studying? Round your answer to the nearest tenth of a percent.

normalcdf

.006..(100) ≈ 0.6%

lower = 22
upper = 99999
$\mu = 14$
$\sigma = 3.2$

normalcdf

7 ~~8~~. The scores on a math test are normally distributed with a mean of 76.2 and a standard deviation of 4.7. If 248 students took the exam, approximately how many students got between a 70 and an 80?

.647 (248) ≈ 173

lower = 70
upper = 80
$\mu = 76.2$
$\sigma = 4.7$

normalcdf

8 ~~9~~. The number of hours of sleep employees at a company get per night is normally distributed with a mean of 7.1 hours and a standard deviation of 1.4 hours. If there are 2500 employees at the company, approximately how many of them, to the nearest person, got less than 5 hours of sleep?

lower = -9999999
upper = 5
$\mu = 7.1$
$\sigma = 1.4$

.0668..(2500)

167

normalcdf

9 ~~10~~. The scores on a mathematics college-entry exam are normally distributed with a mean of 68 and standard deviation 7.2. Students scoring higher than one standard deviation above the mean will not be enrolled in the mathematics tutoring program. How many of the 750 incoming students can be expected to be enrolled in the tutoring program?

[✓ 1)] 631
2) 512
3) 238
4) 119

lower = -99999
upper = 75.2
$\mu = 68$
$\sigma = 7.2$

68+7.2 = 75.2

less than 75.2

.841..(750)

631

Statistical studies

1. Which scenario is best described as an observational study?

1) For a class project, students in Health class ask every tenth student entering the school if they eat breakfast in the morning. Survey (Sample)

2) A social researcher wants to learn whether or not there is a link between attendance and grades. She gathers data from 15 school districts.

Observational study

3) A researcher wants to learn whether or not there is a link between children's daily amount of physical activity and their overall energy level. During lunch at the local high school, she distributed a short questionnaire to students in the cafeteria. Survey

4) Sixty seniors taking a course in Advanced Algebra Concepts are randomly divided into two classes. One class uses a graphing calculator all the time, and the other class never uses graphing calculators. A guidance counselor wants to determine whether there is a link between graphing calculator use and students' final exam grades. Controlled experiment

2 3. A doctor wants to test the effectiveness of a new drug on her patients. She separates her sample of patients into two groups and administers the drug to only one of these groups. She then compares the results. Which type of study *best* describes this situation?

1) census
2) survey
3) observation
[✓ 4)] controlled experiment

Controlled experiment

3 4. A market research firm needs to collect data on viewer preferences for local news programming in Buffalo. Which method of data collection is most appropriate?

1) census
[✓ 2)] survey
3) observation
4) controlled experiment

ask people what they want to watch

4 5. A school cafeteria has five different lunch periods. The cafeteria staff wants to find out which items on the menu are most popular, so they give every student in the first lunch period a list of questions to answer in order to collect data to represent the school. Which type of study does this represent?

1) observation
2) controlled experiment
3) population survey
[✓ 4)] sample survey (not every student in the school was asked)

5 6. Determine whether each scenario is a survey, an observational study, or a controlled experiment. Explain your answer.

a) A study is done to see how high soda will erupt when mint candies are dropped into two-liter bottles of soda. You want to compare using one mint candy, five mint candies, and 10 mint candies. You design a cylindrical mechanism, which drops the desired number of mint candies all at once. You have 15 bottles of soda to use. You randomly assign five bottles into which you drop one candy, five into which you drop five candies, and five into which you drop 10 candies. For each bottle, you record the height of the eruption created after the candies are dropped into it.

Controlled experiment, you dropped the candies into the bottle (administered a treatment)

b) You want to see if fifth-grade boys or fifth-grade girls are faster at solving multiplication problems. You randomly select twenty fifth-grade boys and twenty fifth-grade girls from fifth graders in your school district. You time and record how long it takes each student to solve multiplication problems.

observational study, you're not giving one of the groups anything to make them faster and slower.

c) You want to determine if people would be interested in watching a video of you performing Mr. Schlansky's math songs. You ask every 5th student walking into Mr. Schlansky's math class if they would want to watch the video.

Survey (sample). You're asking questions to a sample.

6 7. Howard collected fish eggs from a pond behind his house so he could determine whether sunlight had an effect on how many of the eggs hatched. After he collected the eggs, he divided them into two tanks. He put both tanks outside near the pond, and he covered one of the tanks with a box to block out all sunlight. State whether Howard's investigation was an example of a controlled experiment, an observation, or a survey. Justify your response.

Controlled experiment. You covered one of the tanks (applied a treatment).

7 8. Darryl conducted a study comparing the statistics of baseball players in the steroid era compared to the non steroid era. Would this investigation be an example of a controlled experiment, an observation, or a survey? Justify your response.

Observation study. He's looking at data, not giving baseball players steroids.

Surveys (Choosing a sample)

A good sample is random. For example, every fifth student walking in the building.
A bad sample is bias.

1. Which statement(s) about statistical studies is true?

I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying. ✓

II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school. ✗ 9th graders don't drive

III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites. ✗ students is not a sample for adults

IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don't like math. ✗ not all students take calculus

1) I, only 2) II, only 3) I and III 4) III and IV

2. Which survey is *least* likely to contain bias?

[✓ 1)] surveying a sample of people leaving a movie theater to determine which flavor of ice cream is the most popular
2) surveying the members of a football team to determine the most watched TV sport football players like football
3) surveying a sample of people leaving a library to determine the average number of books a person reads in a year People at the library read books
4) surveying a sample of people leaving a gym to determine the average number of hours a person exercises per week People add the gym exercise more than most

3. A survey is to be conducted in a small upstate village to determine whether or not local residents should fund construction of a skateboard park by raising taxes. Which segment of the population would provide the most unbiased responses?

1) a club of local skateboard enthusiasts they will all say yes
2) senior citizens living on fixed incomes they will all say no
3) a group opposed to any increase in taxes they will all say no
[✓ 4)] every tenth person 18 years of age or older walking down Main St.

4. A survey is being conducted about American's favorite musicians. Which of the following survey methods would most likely produce a random sample?

1) Asking every 20th person at a Green Day concert → they all like rock music
2) Asking every 10th person at a vintage record store → they all like old music
3) Asking every 10th person at the Westbury Public Library → this is only one community in America
[✓ 4)] Sending out surveys to random households across the country.

8. Which method of collecting data would most likely result in an unbiased random sample?

(1) selecting every third teenager leaving a movie theater to answer a survey about entertainment

(2) placing a survey in a local newspaper to determine how people voted in the 2004 presidential election

[✓ (3) selecting students by the last digit of their school ID number to participate in a survey about cafeteria food]

(4) surveying honor students taking Trigonometry to determine the average amount of time students in a school spend doing homework each night

9. A survey completed at a large university asked 2,000 students to estimate the average number of hours they spend studying each week. Every tenth student entering the library was surveyed. The data showed that the mean number of hours that students spend studying was 15.7 per week. Which characteristic of the survey could create a bias in the results?

(1) the size of the sample

(2) the size of the population

(3) the method of analyzing the data

[✓ (4) the method of choosing the students who were surveyed]

there's bias asking students at the library how much they study

10. The yearbook staff has designed a survey to learn about student opinions on how the yearbook could be improved for this year. If they want to distribute this survey to 100 students and obtain the most reliable data, they should survey

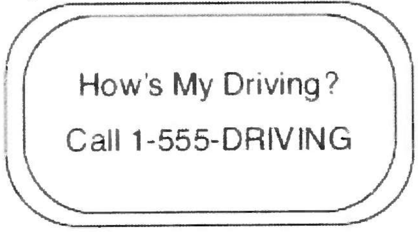
(1) Every third student sent to the office

(2) Every third student to enter the library

(3) Every third student to enter the gym for the basketball game

[✓ (4) Every third student arriving at school in the morning]

11. Chuck's Trucking Company has decided to initiate an Employee of the Month program. To determine the recipient, they put the following sign on the back of each truck. The driver who receives the highest number of positive comments will win the recognition. Explain *one* statistical bias in this data collection method.



Avoid volunteers.

Not every driver will see the sign

Not everyone makes calls while driving

Name Schlansky
Mr. Schlansky

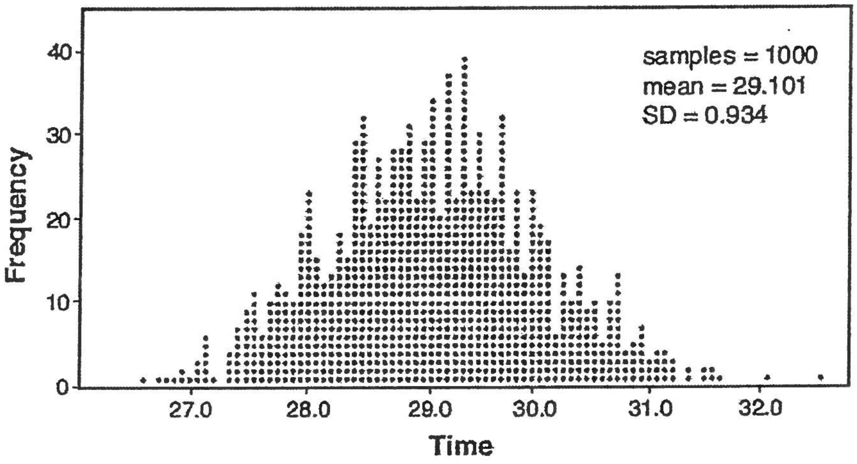
Date ______________
Algebra II

Sample Distributions Part III

1. A radio station claims to its advertisers that the mean number of minutes commuters listen to the station is 30. The station conducted a survey of 500 of their listeners who commute. The sample statistics are shown below.

$\bar{x}$	29.11
s_x	20.718

A simulation was run 1000 times based upon the results of the survey. The results of the simulation appear below.



Based on the simulation results, is the claim that commuters listen to the station on average 30 minutes plausible? Explain your response including an interval containing the middle 95% of the data, rounded to the *nearest hundredth*.

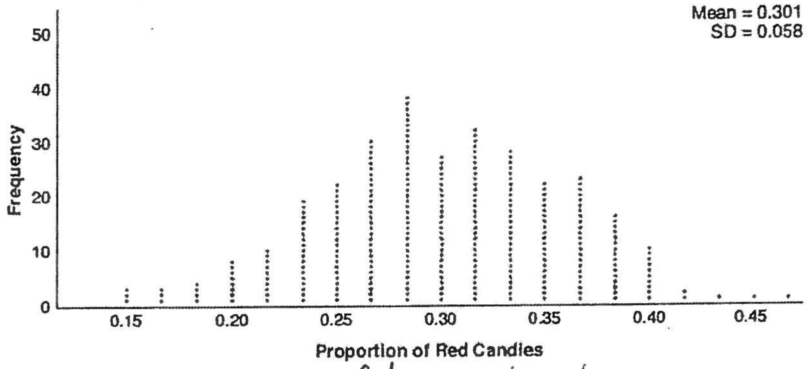
$CI = \bar{x} + 2S_x$

$= 29.101 + 2(.934) = 30.97$ $[27.23, 30.97]$

$= 29.101 - 2(.934) = 27.23$

Yes, 30 is inside the confidence interval.

2 3. Mary bought a pack of candy. The manufacturer claims that 30% of the candies manufactured are red. In her pack, 14 of the 60 candies are red. She ran a simulation of 300 samples, assuming the manufacturer is correct. The results are shown below.



confidence interval

Based on the simulation, determine the middle 95% of plausible values that the proportion of red candies in a pack is within. Based on the simulation, is it unusual that Mary's pack had 14 red candies out of a total of 60? Explain.

CI = mean ± 2(standard deviation)

CI = .301 ± 2(.058)

CI = .301 + 2(.058) = .417

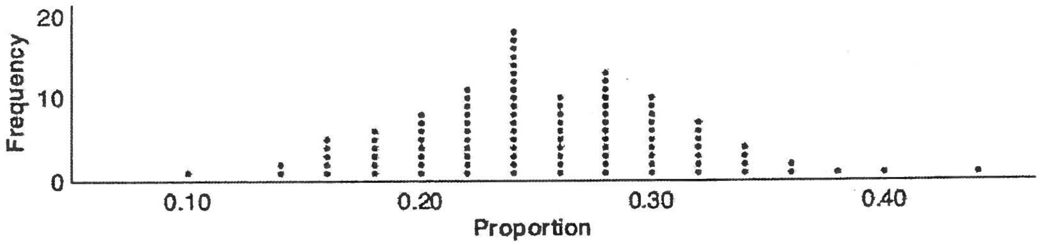
.301 − 2(.058) = .185

[.185, .417]

$\frac{14}{60} = .2\overline{3}$

No, it is usual because $.2\overline{3}$ is in the confidence interval.

3 4. A group of students was trying to determine the proportion of candies in a bag that are blue. The company claims that 24% of candies in bags are blue. A simulation was run 100 times with a sample size of 50, based on the premise that 24% of the candies are blue. The approximately normal results of the simulation are shown in the dot plot below.



The simulation results in a mean of 0.254 and a standard deviation of 0.060. Based on this simulation, what is a plausible interval containing the middle 95% of the data? A student found that 18 out of 50 of the candies were blue. Use statistical evidence to explain why this is an expected value.

CI = mean ± 2(standard deviation)

CI = .254 ± 2(.060)

.254 + 2(.060) = .374

.254 − 2(.060) = .134

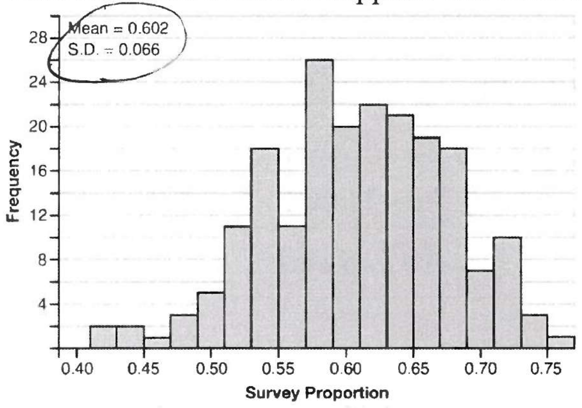
[.134, .374]

$\frac{18}{50} = .36$

.36 is inside the confidence interval.

4. Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band. A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.

Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.* Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50%-50% split. Explain what statistical evidence supports this concern.



$CI = mean \pm 2(SD)$

$CI = .602 + 2(.066) = .73$

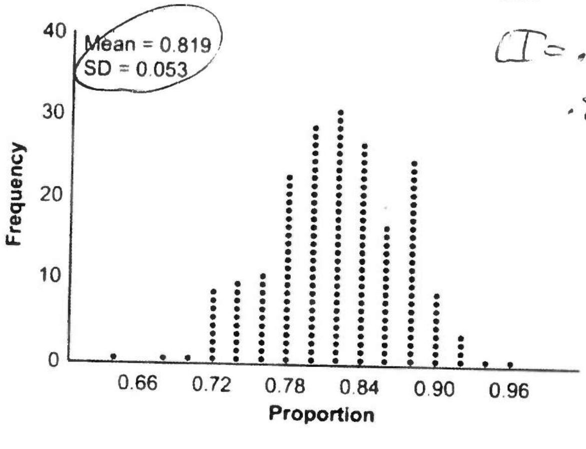
$.602 - 2(.066) = .47$

$[.47, .73]$

.5 is inside the confidence interval

5. State officials claim 82% of a community want to repeal the 30 mph speed limit on an expressway. A community organization devises a simulation based on the claim that 82% of the community supports the repeal. Each dot on the graph below represents the proportion of community members who support the repeal. The graph shows 200 simulated surveys, each of sample size 60.

$CI = mean \pm 2(SD)$



$CI = .819 + 2(.053) = .925$

$.819 - 2(.053) = .713$

$[.713, .925]$

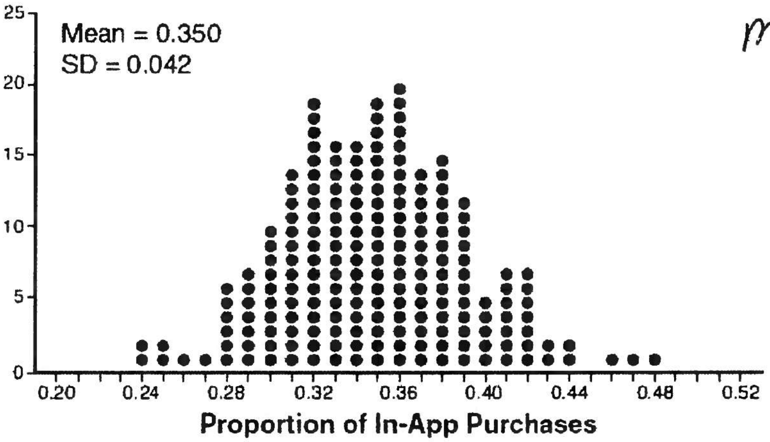
Based on the simulation, determine an interval containing the middle 95% of plausible proportions. Round your answer to the *nearest thousandth.* The community organization conducted its own sample survey of 60 people and found 70% supported the repeal. Based on the results of the simulation, explain why the organization should question the State officials' claim.

.7 is not inside the confidence interval.

Margin of Error

MOE=2(SD)

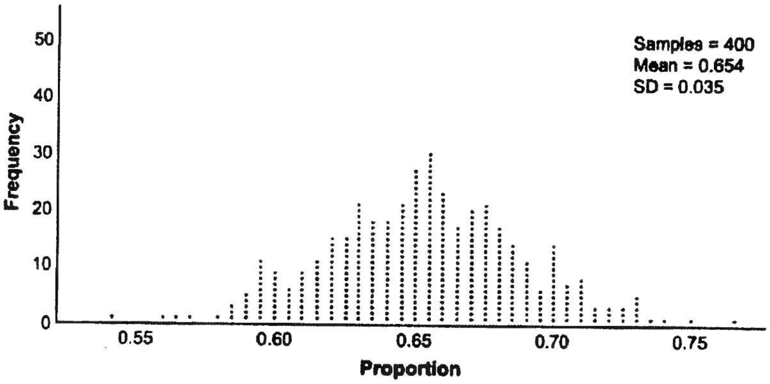
1 7. Some smart-phone applications contain "in-app" purchases, which allow users to purchase special content within the application. A random sample of 140 users found that 35 percent made in-app purchases. A simulation was conducted with 200 samples of 140 users assuming 35 percent of the samples make in-app purchases. The approximately normal results are shown below. Considering the middle 95% of the data, determine the margin of error, to the *nearest hundredth*, for the simulated results.



MOE=2(.042)

=.08

2 8. Betty conducted a survey of her class to see if they like pizza. She gathered 200 responses and 65% of the voters said they did like pizza. Betty then ran a simulation of 400 more surveys, each with 200 responses, assuming that 65% of the voters would like pizza. The output of the simulation is shown below. Considering the middle 95% of the data, what is the margin of error for the simulation?



MOE=2(.035)

=.07

Name Schlansky
Mr. Schlansky

Date ______________
Algebra II

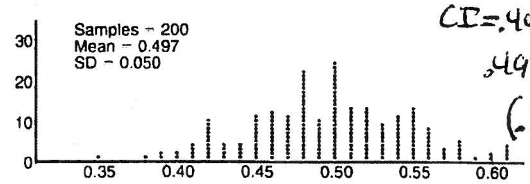
Confidence Interval (Fair)

1. Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

$\frac{73}{100}=.73$

Given the results of her coin flips and of her computer simulation, which statement is most accurate?

1) 73 of the computer's next 100 coin flips will be heads.
2) 50 of her next 100 coin flips will be heads.
[✓ 3)] Her coin is not fair.
4) Her coin is fair.



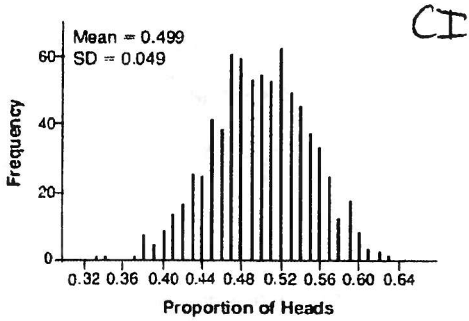
CI=.497+2(.050)

.497-2(.050)

(.397,.597)

Unfair, .73 is not an expected value of a fair coin.

2. Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below. Do the results of the simulation provide strong evidence that Robin's coin is unfair? Explain your answer.



CI= .499+2(.049) = .597

.499-2(.049) = .401

(.401,.597)

$\frac{43}{100}=.43$

No, the coin is fair because .43 is an expected value of a fair coin.

3. Juanita rolls a 6 sided die and recorded that it landed on 6 five times out of 50. She questioned whether the die was fair so she ran a computer simulation of 1000 samples of 50 rolls of a fair die. The mean of the simulation was .159 with a standard deviation of .102. Is her die fair? Explain your answer.

$\frac{6}{50}=.12$

CI=.159+2(.102) = .363

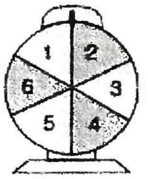
.159-2(.102) = -.045

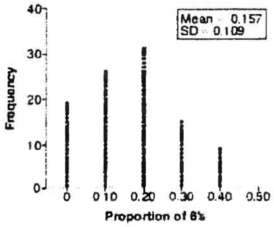
(-.045,.363)

Yes, .12 is an expected value of a fair die.

4. A game spinner is divided into 6 equally sized regions, as shown in the diagram below. For Miles to win, the spinner must land on the number 6. After spinning the spinner 10 times, and losing all 10 times, Miles complained that the spinner is unfair. At home, his dad ran 100 simulations of spinning the spinner 10 times, assuming the probability of winning each spin is $\frac{1}{6}$. The output of the simulation is shown in the diagram below.

$\frac{0}{10} = 0$





CI = .157 + 2(.109) = .375

.157 − 2(.109) = −.061

(−.061, .375)

Is there strong evidence to suggest that the spinner is unfair? Explain your answer.

No, the spinner is fair because 0 is an expected value of a fair spinner.

5. A spinner with 8 sectors labeled A, B, C, D, E, F, G, H was spun 100 times. The spinner landed on sector B 20 times out of 100. A computer simulation of 500 samples of 100 spins of a fair 8 sector spinner was run and it was found that the mean proportion of landing on sector B was .126 with a standard deviation of .027. Is the spinner fair? Explain your answer.

$\frac{20}{100} = .2$

CI = .126 + 2(.027) = .18

.126 − 2(.027) = .072

(.072, .18)

No, .2 is not an expected value of a fair 8 sector spinner.

6. Ally flipped a coin 100 times a got a proportion of .41 heads. She believed this coin was unfair so she ran a computer simulation of 200 samples of 100 coin flips of a fair coin. The mean of the simulation was .502 and the standard deviation was .024. Is Ally's coin fair? Explain your answer.

.41

CI = .502 + 2(.024) = .55

.502 − 2(.024) = .454

(.454, .55)

Her coin is not fair because .41 is not an expected value of a fair coin.

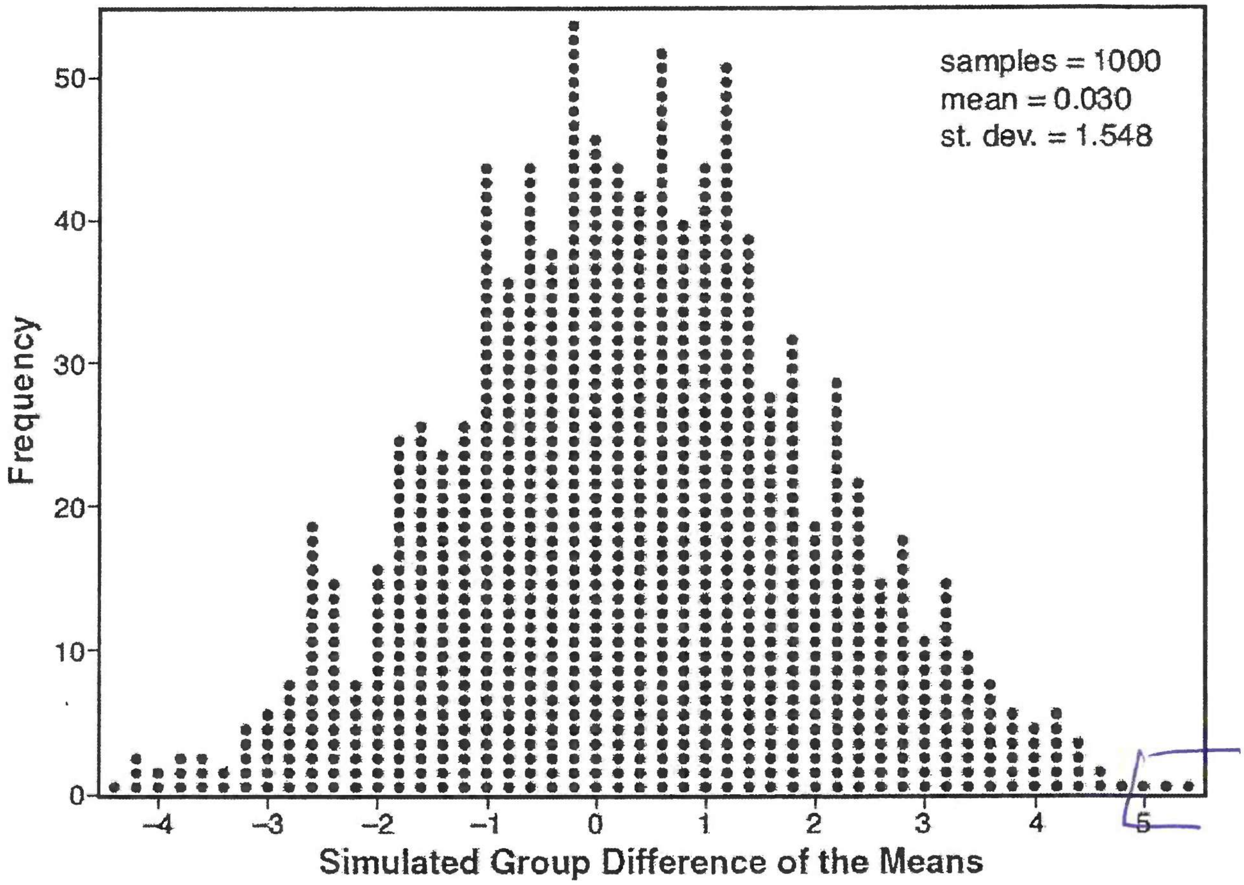
To Detelmine if a Tleatment is Effective (Mean Diffelences)

4. Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

	Scented Paper	**Unscented Paper**
$\bar{x}$	23	18
s_x	2.898	2.408

23-18=5

Calculate the difference in means in the experimental test grades (scented -unscented). A simulation was conducted in which the subjects' scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.



$\frac{3}{1000} \approx .3\%$

Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the *nearest hundredth*. Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.

$CI = \bar{x} \pm 2S_x$

$CI = .03 + 2(1.548) = 3.13$

$.03 - 2(1.548) = -3.07$

$[-3.07, 3.13]$

Yes, because 5 is outside the confidence interval

(OC) Yes, because 5 occulled less than 5% of the time

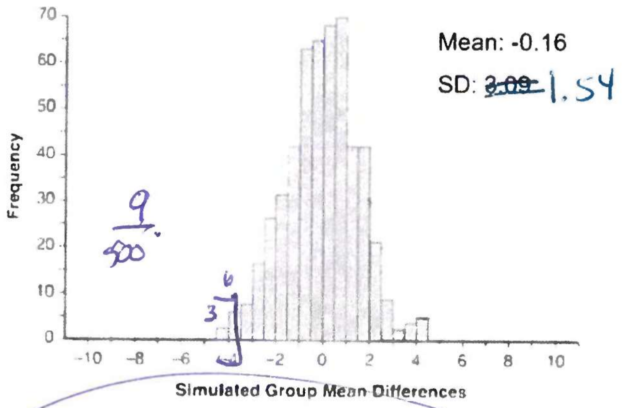
2. Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year. A summary of the two groups' final grades is shown below:

	Group 1	Group 2
$\bar{x}$	80.16	83.8
S_x	6.9	5.2

80.16 − 83.8 = −3.64

On average, students in group 1 scored 3.64 points less than group 2

Calculate the mean difference in the final grades (group 1 – group 2) and explain its meaning in the context of the problem. A simulation was conducted in which the students' final grades were rerandomized 500 times. The results are shown below.



Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.

$\frac{9}{500} = 1.8\%$

Yes, because -3.64 occurred less than 5% of the time

(or)

CI = $-.16 + 2(\overset{1.54}{\cancel{3.09}}) = 2.92$

$-.16 - 2(\underset{1.54}{\cancel{3.09}}) = -3.24$

(−3.24, 2.92)

Yes, −3.64 is not an expected value due to random chance.

Yes, −3.64 is not inside the confidence interval

2.

The effects of caffeine on the body have been extensively studied. In one experiment, researchers trained a sample of male college students to tap their fingers at a rapid rate. The sample was then divided at random into two groups of 10 students each. Each student drank the equivalent of about two cups of coffee, which included about 200 mg of caffeine for the students in one group but was decaffeinated coffee for the second group. After a 2-hour period, each student was tested to measure finger tapping rate (taps per minute). The students did not know whether or not their drinks included caffeine and the person measuring the tap rates was also unaware of the groups. The finger-tapping rates measured in this experiment are summarized in the table below.

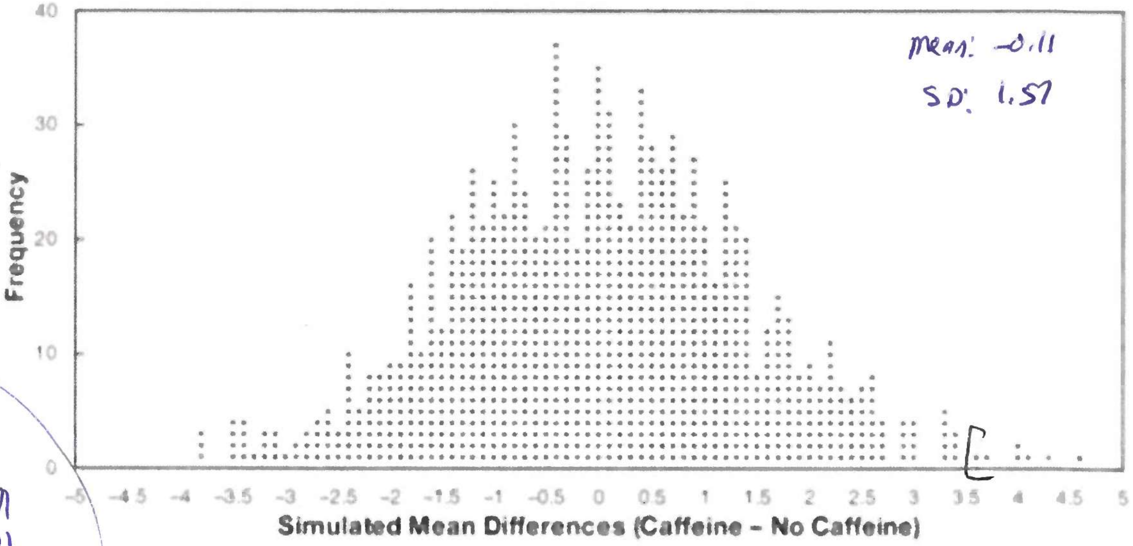
											Mean
Caffeine	246	248	250	252	248	250	246	248	245	250	248.3
No Caffeine	242	245	244	248	247	248	242	244	246	242	244.8

Calculate the mean difference (Caffeine – No Caffeine) and interpret your answer in the context of the problem.

248.3-244.8 = 3.5

On average, the students in the caffeine group tapped 3.5 more times per minute than the non-caffeine group.

The researchers then took the twenty finger-tapping rates and rerandomized them 1,000 times using simulation software. The output of the simulation results is shown in the dotplot below.



mean: -0.11

SD: 1.57

$\frac{7}{1000}$ = .7%

CI: -.11+2(1.57)
-.11-2(1.57)
(-3.25, 3.03)
Yes, ~~#~~3.5 is outside the confidence interval

Does the simulation data support the conclusion that caffeine causes an increase in average finger-tapping rate? Justify your answer.

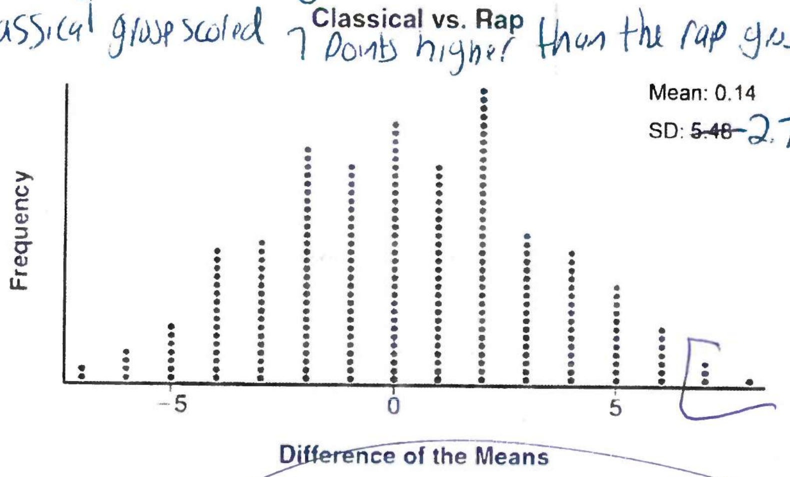
Yes, the mean difference occurred less than 5% of the time as it outside the confidence interval.

4. To determine if the type of music played while taking a quiz has a relationship to results, 16 students were randomly assigned to either a room softly playing classical music or a room softly playing rap music. The results on the quiz were as follows:

Classical: 74, 83, 77, 77, 84, 82, 90, 89
Rap: 77, 80, 78, 74, 69, 72, 78, 69

John correctly rounded the difference of the means of his experimental groups as 7. How did John obtain this value and what does it represent in the given context? Justify your answer. To determine if there is any significance in this value, John rerandomized the 16 scores into two groups of 8, calculated the difference of the means, and simulated this process 250 times as shown below.

He found the average of each grup and subtracted them to get 7. On average, the classical grup scored 7 points higher than the rap grp



$\frac{4}{250} = 1.6\%$

Does the simulation support the theory that there may be a significant difference in quiz scores? Explain.

$CI = .14 + 2(2.74) = 5.62$

$.14 - 2(2.74) = -5.34$

$(-5.34, 5.62)$

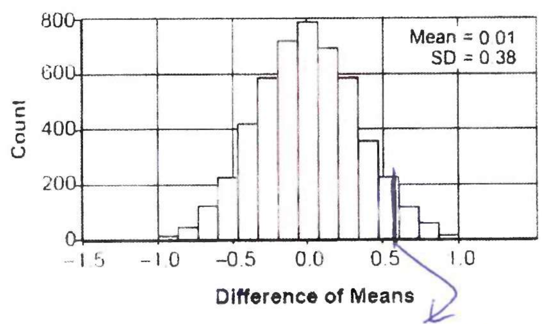
Yes, 7 is not an expected value due to random chance. 7 is not inside the confidence interval

$\frac{4}{250} = 1.6\%$ (or)

Yes, because 7 occurred less than 5% of the time.

5. Two classes of students were entered into an experiment to see whether using an interactive whiteboard leads to better grades. It was observed that the mean grade of students in the class with the interactive whiteboard was 0.6 points higher than the class without it. To determine if the observed difference is statistically significant, the classes were rerandomized 5000 times to study these random differences in the mean grades. The output of the simulation is summarized in the histogram below. Determine an interval containing the middle 95% of the simulation results. Round your answer to the *nearest hundredth*. Does the interval indicate that the difference between the classes' grades is significant? Explain.

mD=0.6



CI=.01+2(.38) = .77
.01-2(.38) =-.75

(-.75,.77)

No, 0.6 is an expected value due to random chance.

.6 is inside CI

0.6 is inside the confidence interval.

6. Gabriel performed an experiment to see if planting 13 tomato plants in black plastic mulch leads to larger tomatoes than if 13 plants are planted without mulch. He observed that the average weight of the tomatoes from tomato plants grown in black plastic mulch was 5 ounces greater than those from the plants planted without mulch. To determine if the observed difference is statistically significant, he rerandomized the tomato groups 100 times to study these random differences in the mean weights. The output of his simulation is summarized in the dotplot below. Do you believe that planting in black plastic mulch causes larger tomato size? Explain your answer.

mD = 5

CI=.9+2(8.75) = 18.4
.9-2(8.75) = -16.6

(-16.6,18.4)

5 is inside CI

No, 5 is an expected value due to random chance. 5 is inside the confidence interval.

Factoring:

Greatest Common Factor: GCF(   )

Difference of Two Squares: $(\sqrt{\ }_1 + \sqrt{\ }_2)(\sqrt{\ }_1 - \sqrt{\ }_2)$

Trinomials: (x   )(x   )

1) First sign comes down
2) The two signs must multiply for the last sign
3) Find two numbers that multiply to the last number and add/subtract to the middle number

Bridge Method: (Trinomial with a leading coefficient bigger than 1)

1) Build a bridge between the first and last numbers (Multiply)
2) Factor Trinomial Normally
3) Pay the toll (Divide by the leading coefficient)

*If possible, reduce the fraction
If they divide nicely, divide them
If not, put the denominator in front of the variable inside the parenthesis

Grouping: (4 Terms or More)

1) Look for a pattern in the exponents to determine the groups. **You cannot have two terms with the same exponent in the same group.**
2) Factor out the GCF in each group

*You should be left with the same factor. If signs are reversed, factor out a negative

3) Combine coefficients and keep like term.

***Factor further if necessary**

Sum/Difference of Two Cubes

SOAP for signs (Same, Opposite, Always Positive)

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Substitution Trinomials:

1) Replace binomial with y
2) Factor normally
3) Substitute back

*Factor further if possible

Factor each expression

1. $\frac{4x}{4} + \frac{8}{4}$

$4(x+2)$

2. $\frac{12x}{6} + \frac{18}{6}$

$6(2x+3)$

3. $\frac{x^2}{x} - \frac{7x}{x}$

$x(x-7)$

4. $\frac{2x^2}{2x} - \frac{4xy}{2x}$

$2x(x-2y)$

5. $\frac{5x^2y}{5x} - \frac{20x}{5x}$

$5x(xy-4)$

6. $\sqrt{x^2} - \sqrt{64}$ DOTS

$(x+8)(x-8)$

7. $\sqrt{y^2} - \sqrt{36}$ DOTS

$(y+6)(y-6)$

8. $\sqrt{4t^2} - \sqrt{25}$ DOTS

$(2t+5)(2t-5)$

9. $\sqrt{9x^2} - \sqrt{16y^4}$ DOTS

$(3x+4y^2)(3x-4y^2)$

10. $\sqrt{36} - \sqrt{25x^2}$ DOTS

$(6+5x)(6-5x)$

11. $\sqrt{100y^4} - \sqrt{49t^6}$ DOTS

$(10y^2+7t^3)(10y^2-7t^3)$

12. $1 - 9x^8y^4$ DOTS

$(1+3x^4y^2)(1-3x^4y^2)$

13. $x^2 + 4x - 12$

1,12
2,6
3,4

$(x+6)(x-2)$

14. $y^2 + 3y + 2$

1,2

$(y+2)(y+1)$

1,15
3,5

15. $m^2 - 8m + 15$

$(m-5)(m-3)$

1,20
2,10
4,5

16. $x^2 - 8x - 20$

$(x-10)(x+2)$

1,14
2,7

17. $y^2 + 5y - 14$

$(y+7)(y-2)$

1,12
2,6
3,4

18. $x^2 + 1x - 12$

$(x+4)(x-3)$

1,10
2,5

19. $x^2 - 3x - 10$

$(x-5)(x+2)$

1,12
2,6
3,4

20. $x^2 - 7x + 12$

$(x-4)(x-3)$

1,36 4,9
2,18 6,6
3,12

21. $x^2 - 9x - 36$

$(x-12)(x+3)$

1,110 10,11
2,55
5,22

22. $y^2 - 21y + 110$

$(y-11)(y-10)$

1,12
2,6
3,4

23. $x^4 + 4x^2 - 12$

$(x^2+6)(x^2-2)$

1,9
3,3

24. $x^6 - 6x^3 + 9$

$(x^3-3)(x^3-3)$

1,9
3,3

25. $x^4 - 8x^2 - 9$

$(x^2-9)(x^2+1)$

$(x+3)(x-3)$

1,2

26. $x^4 + x^2 - 2$

$(x^2+2)(x^2-1)$ DOTS

$(x^2+2)(x+1)(x-1)$

27. $\frac{2x^2}{2} \frac{-50}{2}$

$2(x^2-25)$ DOTS

$2(x+5)(x-5)$

28. $\frac{2x^2}{2} \frac{-8x}{2} \frac{-10}{2}$

$2(x^2-4x-5)$

$2(x-5)(x+1)$

29. $\frac{3x^2}{3} \frac{+9x}{3} \frac{-12}{3}$

$3(x^2+3x-4)$

$3(x+4)(x-1)$

30. $\frac{6x^2}{6} \frac{-54}{6}$

$6(x^2-9)$ DOTS

$6(x+3)(x-3)$

31. $2x^2+14x+24$

$\frac{2x^2}{2}\ \frac{+14x}{2}\ \frac{+24}{2}$

$2(x^2+7x+12)$

$2(x+4)(x+3)$

32. $5x^2-500$

$\frac{5x^2}{5}\ \frac{-500}{5}$

$5(x^2-100)$

$5(x+10)(x-10)$

33. $ax^2-2ax-8a$

$\frac{ax^2}{a}\ \frac{-2ax}{a}\ \frac{-8a}{a}$

$a(x^2-2x-8)$

$a(x-4)(x+2)$

34. yx^2-64y

$\frac{yx^2}{y}\ \frac{-64y}{y}$

$y(x^2-64)$

$y(x+8)(x-8)$

35. $12x^2-75$

$\frac{12x^2}{3}\ \frac{-75}{3}$

$3(4x^2-25)$

$3(2x+5)(2x-5)$

36. x^4-81

DOTS $(x^2-9)(x^2+9)$

DOTS $(x+3)(x-3)(x^2+9)$

37. $2y^2-5y-7$ PT

$y^2-5y-14$

$\left(y-\frac{7}{2}\right)\left(y+\frac{2}{2}\right) \rightarrow (2y-7)(y+1)$

38. $2x^2+15x-8$ PT

$x^2+15x-16$

$\left(x+\frac{16}{2}\right)\left(x-\frac{1}{2}\right)$

$(x+8)(2x-1)$

39. $2x^2+7x-4$ PT

x^2+7x-8

$\left(x+\frac{8}{2}\right)\left(x-\frac{1}{2}\right)$

$(x+4)(2x-1)$

40. $6x^2-11x-10$ PT

$x^2-11x-60$

$\left(x-\frac{15}{6}\right)\left(x+\frac{4}{6}\right)$ *reduce

$\left(x-\frac{5}{3}\right)\left(x+\frac{2}{3}\right) \rightarrow (2x-5)(3x+2)$

41. $2x^2-9x-18$ PT

$x^2-9x-36$

$\left(x-\frac{12}{2}\right)\left(x+\frac{3}{2}\right)$

42. $3x^2+2x-8$ PT

$x^2+2x-24$ ✓

$\left(x+\frac{6}{3}\right)\left(x-\frac{4}{3}\right)$

$(x+2)(3x-4)$

43. $8x^2+7x-1$ PT

x^2+7x-8

$\left(x+\frac{8}{8}\right)\left(x-\frac{1}{8}\right)$

$(x+1)(8x-1)$

44. $6x^2+x-12$ PT

x^2+x-72

$\left(x+\frac{9}{6}\right)\left(x-\frac{8}{6}\right)$ *reduce

$\left(x+\frac{3}{2}\right)\left(x-\frac{4}{3}\right)$

$(2x+3)(3x-4)$

45. $(x^3+6x^2)(-3x-18)$

$x^2 \quad x^2 \quad -3 \quad -3$

$x^2(x+6)-3(x+6)$

$(x^2-3)(x+6)$

46. $(x^3+10x^2)(-9x-90)$

$x^2 \quad x^2$

$x^2(x+10)-9(x+10)$

$(x^2-9)(x+10)$

$(x+3)(x-3)(x+10)$

47. $(x^3+3x^2)(-9x-27)$

$x^2 \quad x^2 \quad -9 \quad -9$

$x^2(x+3)-9(x+3)$

$(x^2-9)(x+3)$

$(x+3)(x-3)(x+3)$

48. $(8x^3+12x^2)(-2x-3)$

$4x^2 \quad 4x^2 \quad -1 \quad -1$

$4x^2(2x+3)-1(2x+3)$

$(4x^2-1)(2x+3)$

$(2x+1)(2x-1)(2x+3)$

49. $(x^3-3x^2+2x)(4x^2-12x+8)$

$x \quad x \quad x$

$x(x^2-3x+2)+4(x^2-3x+2)$

$(x+4)(x^2-3x+2)$

$(x+4)(x-2)(x-1)$

50. $(3x^3+x^2)(-12x^2-4x)(-63x-21)$

$x^2 \quad x^2 \quad -4x \quad -4x \quad -21 \quad -21$

$x^2(3x+1)-4x(3x+1)-21(3x+1)$

$(x^2-4x-21)(3x+1)$

$(x-7)(x+3)(3x+1)$

51. $(x^2+5x)^2-2(x^2+5x)-24$ $\quad y=x^2+5x$

$y^2-2y-24$

$(y-6)(y+4)$

$(x^2+5x-6)(x^2+5x+4)$

$(x+6)(x-1)(x+4)(x+1)$

52. $(x^2-2x)^2-11(x^2-2x)+24$ $\quad y=x^2-2x$

$y^2-11y+24$

$(y-8)(y-3)$

$(x^2-2x-8)(x^2-2x-3)$

$(x-4)(x+2)(x-3)(x+1)$

$a=y \quad b=5$

53. $\sqrt[3]{y^3} \; \sqrt[3]{125}$

$(a^3-b^3)=(a-b)(a^2+ab+b^2)$

$=(y-5)(y^2+5y+25)$

$a=z \quad b=4$

54. $\sqrt[3]{z^3}+\sqrt[3]{64}$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$z^3+64=(z+4)(z^2-4z+16)$

$2x \quad y^2$

55. $\sqrt[3]{8x^3} \; \sqrt[3]{y^6}$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$8x^3+y^6=(2x+y^2)(4x^2-2xy^2+y^4)$

$a=y^3 \quad b=6x$

56. $\sqrt[3]{y^9} \; \sqrt[3]{216x^3}$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$y^9-216x^3=(y^3-6x)(y^6+6xy^3+36x^2)$

Reducing Rational Expressions

1) Factor

2) Cancel Common Factors

*If a factor is written backwards with a minus sign, they cancel to -1.

Express each of the following in simplest form

GCF
1. $\frac{2x+6}{x^2-9}$
DOTS

$\frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \frac{2}{x-3}$

GCF
2. $\frac{10-5x}{x^2+2x-8}$
Tri

$\frac{5\cancel{(2-x)}(-1)}{(x+4)\cancel{(x-2)}}$

$-\frac{5}{x+4}$

GCF
3. $\frac{6x+18}{6x+12}$
GCF

$\frac{\cancel{6}(x+3)}{\cancel{6}(x+2)} = \frac{x+3}{x+2}$

Tricky Tri
4. $\frac{2x^2+x-6}{9-4x^2}$
DOTS

$2x^2+x-6$ pr

x^2+x-12

$(x+\frac{4}{2})(x-\frac{3}{2})$

$(x+2)(2x-3)$

$\frac{(x+2)\cancel{(2x-3)}(-1)}{(3+2x)\cancel{(3-2x)}}$

$-\frac{1(x+2)}{3+2x}$

Tri
5. $\frac{x^2+3x+2}{x^3+2x^2+8x+16}$
Grouping

$\left(\frac{x^3}{x^2}+\frac{2x^2}{x^2}\right)\left(\frac{+8x}{8}+\frac{16}{8}\right)$

$x^2(x+2)+8(x+2)$

$(x^2+8)(x+2)$

$\frac{\cancel{(x+2)}(x+1)}{(x^2+8)\cancel{(x+2)}}$

$\frac{x+1}{x^2+8}$

Tricky Tri
6. $\frac{3x^2+7x-6}{4-9x^2}$
DOTS

$3x^2+7x-6$ pr

$x^2+7x-18$

$(x+\frac{9}{3})(x-\frac{2}{3})$

$(x+3)(3x-2)$

$\frac{(x+3)\cancel{(3x-2)}(-1)}{(2+3x)\cancel{(2-3x)}}$

$\frac{-(x+3)}{2+3x}$

GCF/Tri
7. $\frac{2x^4+4x^3-6x^2}{4x^3-36x}$
GCF/DOTS

$\frac{1\cancel{2}x^{\cancel{2}}\cancel{(x+3)}(x-1)}{2\cancel{4}\cancel{x}\cancel{(x+3)}(x-3)}$

$\frac{2x^4}{2x^2}+\frac{4x^3}{2x^2}-\frac{6x^2}{2x^2}$

$2x^2(x^2+2x-3)$

$2x^2(x+3)(x-1)$

$\frac{4x^3}{4x}-\frac{36x}{4x}$

$4x(x^2-9)$

$4x(x+3)(x-3)$

$\frac{x(x-1)}{2(x-3)}$

Grouping
8. $\frac{2x^3+x^2-18x-9}{3x-x^2}$
GCF

$\left(\frac{2x^3}{x^2}+\frac{x^2}{x^2}\right)\left(\frac{-18x}{-9}\ \frac{-9}{-9}\right)$

$x^2(2x+1)-9(2x+1)$

$(x^2-9)(2x+1)$

$(x+3)(x-3)(2x+1)$

$\frac{(x+3)\overset{(-1)}{\cancel{(x-3)}}(2x+1)}{x\cancel{(3-x)}}$

$-\frac{1(x+3)(2x+1)}{x}$

Solving Quadratic Equations By Factoring

1) Bring everything to one side
2) Factor
3) Set each factor equal to zero

Divide away an integer GCF if possible.

A variable GCF would have to stay in front and would produce an answer of 0.

1. $y^2 - 5y - 6 = 0$

$(y-6)(y+1) = 0$

$y-\cancel{6}=0$ | $y+\cancel{1}=0$

$+\cancel{6}\ +6$ | $-\cancel{1}\ \ -1$

$y=6$ | $y=-1$

2. $x^2 + 4x = 0$

$x(x+4)=0$

$x=0$ | $x+\cancel{4}=0$

$-\cancel{4}\ -4$

$x=-4$

3. $a^2 - 8a = 20$

$-20\ \ -20$

$a^2-8a-20=0$

$(a-10)(a+2)=0$

$a-\cancel{10}=0$ | $a+\cancel{2}=0$

$+\cancel{10}\ +10$ | $-\cancel{2}\ -2$

$a=10$ | $a=-2$

4. $3x^2 = 48$

$-48\ \ -48$

$\frac{3x^2-48}{3}=\frac{0}{3}$

$x^2-16=0$

$(x+4)(x-4)=0$

$x+\cancel{4}=0$ | $x-\cancel{4}=0$

$-\cancel{4}\ -4$ | $+\cancel{4}\ +4$

$x=-4$ | $x=4$

5. $x^2 - 6x = -8$

$+8\ +8$

$x^2-6x+8=0$

$(x-4)(x-2)=0$

$x-\cancel{4}=0$ | $x-\cancel{2}=0$

$+\cancel{4}\ +4$ | $+\cancel{2}\ +2$

$x=4$ | $x=2$

6. $\frac{3x^2 + 3x - 6}{3} = \frac{0}{3}$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x+\cancel{2}=0$ | $x-\cancel{1}=0$

$-\cancel{2}\ -2$ | $+\cancel{1}\ +1$

$x=-2$ | $x=1$

7. $n^2 = 3n + 18$

$-3n-18\ \ -3n-18$

$n^2-3n-18=0$

$(n-6)(n+3)=0$

$n-\cancel{6}=0$ | $n+\cancel{3}=0$

$+\cancel{6}\ +6$ | $-\cancel{3}\ -3$

$n=6$ | $n=-3$

8. $2x^2 + 3x = 5$

$-5\ -5$ pt

$2x^2+3x-5=0$

$x^2+3x-10=0$

$(x+\frac{5}{2})(x-\frac{2}{2})=0$

$(2x+5)(x-1)=0$

$2x+\cancel{5}=0$ | $x-\cancel{1}=0$

$-\cancel{5}\ -5$ | $+\cancel{1}\ +1$

$\frac{2x}{2}=\frac{-5}{2}$ | $x=1$

$x=-\frac{5}{2}$

9. $x^2 - 6x = 2x + 20$

$-2x-20 \quad -2x-20$

$x^2-8x-20=0$

$(x-10)(x+2)=0$

$x-10=0 \quad x+2=0$

$+10 \quad +10 \quad -2 \quad -2$

$x=10 \quad x=-2$

10. $x^2 + 2(x-4) = 3x - 8$

$x^2+2x-8=3x-8$

$-3x+8 \quad -3x+8$

$x^2-1x=0$

$x(x-1)=0$

$x=0 \quad x-1=0$

$+1 \quad +1$

$x=1$

11. $4(x^2 + 2x) = 8x + 64$

$4x^2+8x=8x+64$

$-8x-64 \quad -8x-64$

$\frac{4x^2-64=0}{4 \quad 4}$

$x^2-16=0$

$(x+4)(x-4)=0$

$x+4=0 \quad x-4=0$

$-4 \quad -4 \quad +4 \quad +4$

$x=-4 \quad x=4$

12. $4x^2 + 4x = 3$

$-3 \quad -3$

PT

$4x^2+4x-3=0$

$x^2+4x-12$

$(x+6)(x-2)$

$4 \quad 4$

$(x+\frac{3}{2})(x-\frac{1}{2})$

$(2x+3)(2x-1)=0$

$2x+3=0 \quad 2x-1=0$

$-3 \quad -3 \quad +1 \quad +1$

$\frac{2x}{2}=\frac{-3}{2} \quad \frac{2x}{2}=\frac{1}{2}$

$x=-\frac{3}{2} \quad x=\frac{1}{2}$

13. $x^2 + 5x = -4(x+5)$

$x^2+5x=-4x-20$

$+4x+20 \quad +4x+20$

$x^2+9x+20=0$

$(x+5)(x+4)=0$

$x+5=0 \quad x+4=0$

$-5 \quad -5 \quad -4 \quad -4$

$x=-5 \quad x=-4$

14. $4x^2 = 25$

$-25 \quad -25$

$4x^2-25=0$

$(2x+5)(2x-5)=0$

$2x+5=0 \quad 2x-5=0$

$-5 \quad -5 \quad +5 \quad +5$

$\frac{2x}{2}=\frac{-5}{2} \quad \frac{2x}{2}=\frac{5}{2}$

$x=-\frac{5}{2} \quad x=\frac{5}{2}$

15. $3x^2 + 5x = 2x + 60$

$-2x-60 \quad -2x-60$

$\frac{3x^2+3x-60}{3}=\frac{0}{3}$

$x^2+x-20=0$

$(x+5)(x-4)=0$

$x+5=0 \quad x-4=0$

$-5 \quad -5 \quad +4 \quad +4$

$x=-5 \quad x=4$

16. $8m^2 + 20m = 12$

$-12 \quad -12$

$\frac{8m^2+20m-12}{4}=\frac{0}{4}$

PT

$2m^2+5m-3=0$

m^2+5m-6

$(m+6)(m-1)$

$2 \quad 2$

$(m+3)(2m-1)=0$

$m+3=0 \quad 2m-1=0$

$-3 \quad -3 \quad +1 \quad +1$

$\frac{2m}{2}=\frac{1}{2}$

$m=-3 \quad m=\frac{1}{2}$

Reducing Radicals

1) Separate into two radicals (perfect squares and non-perfect squares)
2) Take the square root of the perfect square

*If there is a negative inside the radical, bring it outside and make it an *i*

Perfect Squares

1
4
9
16
25
36
49
64
81
100

Reduce the following radicals

1. $\sqrt{12}$

$\sqrt{4}\sqrt{3}$

$2\sqrt{3}$

2. $\sqrt{-50}$

$i\sqrt{50}$

$i\sqrt{25}\sqrt{2}$

$5i\sqrt{2}$

3. $\sqrt{-45}$

$i\sqrt{45}$

$i\sqrt{9}\sqrt{5}$

$3i\sqrt{5}$

4. $\sqrt{75}$

$\sqrt{25}\sqrt{3}$

$5\sqrt{3}$

5. $\sqrt{-20}$

$i\sqrt{20}$

$i\sqrt{4}\sqrt{5}$

$2i\sqrt{5}$

6. $\sqrt{-54}$

$i\sqrt{54}$

$i\sqrt{9}\sqrt{6}$

$3i\sqrt{6}$

7. $\sqrt{162}$

$\sqrt{81}\sqrt{2}$

$9\sqrt{2}$

8. $\sqrt{-32}$

$i\sqrt{32}$

$i\sqrt{16}\sqrt{2}$

$4i\sqrt{2}$

Solving Quadratic Equations Using the Quadratic Formula

IF MULTIPLE CHOICE:

APPS, PLYSMLT2, 1: POLY ROOT FINDER
Type each choice in to match up the decimal.

Algebraically:

1) Bring everything to one side. Keep the leading coefficient positive.
If you cannot factor, USE QUADRATIC FORMULA!

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) $ax^2 + bx + c = 0$
2) List a, b, and c values
3) Substitute values into quadratic formula
4) Type what's inside the radical into the calculator
5) REDUCE THE RADICAL off to the side (If possible)
6) Draw your heart, break the fraction apart.

1. What are the solutions to $4x^2 - 7x - 2 = -10$

$+10$ $+10$

$4x^2 - 7x + 8 = 0$

1) $-\frac{1}{4}, 2$
[✓ 2)] $\frac{7}{8} \pm \frac{\sqrt{79}}{8}i$
3) $\frac{7}{8} \pm \frac{\sqrt{241}}{8}$
4) $\frac{7}{8} \pm \frac{\sqrt{143}}{8}i$

$1.1110243002i$

plySmlt2
1
$x = \frac{7}{8} \pm 1.1110243002i$

2. The solutions to the equation $3x^2 - 4x + 2 = 2x - 3$ are

$-2x + 3$ $\cancel{-2x}$ $+3$

$3x^2 - 6x + 5 = 0$

1) $\frac{2}{3} \pm \frac{\sqrt{2}}{3}i$
[✓ 2)] $1 \pm \frac{\sqrt{6}}{3}i$
3) $1 \pm \frac{\sqrt{12}}{3}$
4) $1 \pm 2\sqrt{6}i$

$.8164965809i$

plySmlt2
1
$x = 1 \pm .8164965809i$

3. The roots of the equation $0 = x^2 + 6x + 10$ in simplest $a + bi$ form are

1) $-3 \pm 2i$
2) $-6 \pm i$
[✓ 3)] $-3 \pm i$
4) $-3 \pm i\sqrt{2}$

plySmlt2
1
$x = -3 \pm i$

4. The roots of the equation $x^2 - 4x = -13$ are

$+13$ $+13$

$x^2 - 4x + 13 = 0$

[✓ 1)] $2 \pm 3i$
2) $2 \pm 6i$
3) $2 \pm \sqrt{17}$
4) $2 \pm \sqrt{13}$

plySmlt2
1
$x = 2 \pm 3i$

5. A solution of the equation $2x^2 + 3x + 2 = 0$ is

$.6614378278i$ [✓ 1)] $-\frac{3}{4} + \frac{1}{4}i\sqrt{7}$
2) $-\frac{3}{4} + \frac{1}{4}i$
3) $-\frac{3}{4} + \frac{1}{4}\sqrt{7}$
4) $\frac{1}{2}$

plySmlt2
1
$x = -\frac{3}{4} \pm .6614378278i$

6. The solutions to the equation $-\frac{1}{2}x^2 = -6x + 20$ are

1) $-6 \pm 2i$
2) $-6 \pm 2\sqrt{19}$
[✓ 3) $6 \pm 2i$]
4) $6 \pm 2\sqrt{19}$

$+\frac{1}{2}x^2 \quad +\frac{1}{2}x^2$

$0 = \frac{1}{2}x^2 - 6x + 20$

PlySmlt 2

1

$x = 6 \pm 2i$

$x = 3 \pm i$

7. Which equation has roots of $3 + i$ and $3 - i$?

[✓ 1) $x^2 - 6x + 10 = 0$]
2) $x^2 + 6x - 10 = 0$
3) $x^2 - 10x + 6 = 0$
4) $x^2 + 10x - 6 = 0$

PlySmlt 2 each choice

8. If a solution of $2(2x - 1) = 5x^2$ is expressed in simplest $a + bi$ form, the value of b is

b is what's in front of i

[✓ 1) $\frac{\sqrt{6}}{5}i$]
2) $\frac{\sqrt{6}}{5}$
3) $\frac{1}{5}i$
4) $\frac{1}{5}$

b is what's in front of i

$4x - 2 = 5x^2$

$-4x + 2 \quad -4x + 2$

$0 = 5x^2 - 4x + 2$

PlySmlt 2

L

$\frac{2}{5} \pm .4898i$

9. Solve the equation $x^2 + 3x + 11 = 0$ algebraically. Express the answer in $a + bi$ form.

a=1
b=3
c=11

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(11)}}{2(1)}$

$x = \frac{-3 \pm \sqrt{-35}}{2}$

$\sqrt{-35}$

$i\sqrt{35}$

$x = \frac{-3 \pm i\sqrt{35}}{2}$

$x = \frac{-3}{2} \pm \frac{i\sqrt{35}}{2}$

Quadratic Formula
Use PlySmlt 2 to check

10. Solve the equation $3x^2 + 5x + 8 = 0$. Write your solution in $a + bi$ form.

a=3
b=5
c=8

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(8)}}{2(3)}$

$x = \frac{-5 \pm \sqrt{-71}}{6}$

$\sqrt{-71}$

$i\sqrt{71}$

$x = \frac{-5 \pm i\sqrt{71}}{6}$

$x = -\frac{5}{6} \pm \frac{i\sqrt{71}}{6}$

11. Algebraically determine the roots, in simplest $a + bi$ form, to the equation below.

$x^2 - 2x + 7 = 4x - 10$

$-4x + 10 \quad -4x + 10$

$x^2 - 6x + 17 = 0$

a=1
b=-6
c=17

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(17)}}{2(1)}$

$x = \frac{6 \pm \sqrt{-32}}{2}$

$x = \frac{6 \pm 4i\sqrt{2}}{2}$

$x = 3 \pm 2i\sqrt{2}$

$\sqrt{-32}$

$i\sqrt{32}$

$\sqrt{16}\sqrt{2}$

$4i\sqrt{2}$

Solving Quadratic Equations Using the Quadratic Formula

1) Bring everything to one side. Keep the leading coefficient positive.

If you cannot factor, USE QUADRATIC FORMULA!

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) $ax^2 + bx + c = 0$
2) List a, b, and c values
3) Substitute values into quadratic formula
4) Type what's inside the radical into the calculator
5) REDUCE THE RADICAL off to the side (If possible)
6) Break the fraction apart into separate fractions

Solve the following equations and express your answer in *simplest a+bi form.*

12. $x^2 + 4x = -8$

$+8 \quad +8$

$a=1$, $b=4$, $c=8$

$x^2+4x+8=0$

$x = \frac{-4 \pm \sqrt{(4)^2-4(1)(8)}}{2(1)}$

$x = \frac{-4 \pm \sqrt{-32}}{2}$

$\sqrt{-32}$, $i\sqrt{32}$, $i\sqrt{16}\sqrt{2}$, $4i\sqrt{2}$

$x = \frac{\overset{\div 2}{-4} \pm \overset{\div 2}{4i}\sqrt{2}}{2_{\div 2}}$

$x = -2 \pm 2i\sqrt{2}$

13. $4x^2 + 2x = -1$

$+1 \quad +1$

$a=4$, $b=2$, $c=1$

$4x^2+2x+1=0$

$x = \frac{-2 \pm \sqrt{(2)^2-4(4)(1)}}{2(4)}$

$x = \frac{-2 \pm \sqrt{-12}}{8}$

$\sqrt{-12}$, $i\sqrt{12}$, $i\sqrt{4}\sqrt{3}$, $2i\sqrt{3}$

$x = \frac{\overset{\div 2}{-2} \pm \overset{\div 2}{2}i\sqrt{3}}{8_{\div 2}}$

$x = \frac{-1}{4} \pm \frac{1}{4}i\sqrt{3}$

14. $2x^2 - 6x = -5$

$+5 \quad +5$

$a=2$, $b=-6$, $c=5$

$2x^2-6x+5=0$

$x = \frac{6 \pm \sqrt{(-6)^2-4(2)(5)}}{2(2)}$

$x = \frac{6 \pm \sqrt{-4}}{4}$

$\sqrt{-4}$, $i\sqrt{4}$, $2i$

$x = \frac{6 \pm 2i}{4}$

$x = \frac{3}{2} \pm \frac{1}{2}i$

14. $3x^2 = 4x - 2$

$-4x+2 \quad -4x+2$

$3x^2-4x+2=0$

$a=3$, $b=-4$, $c=2$

$x = \frac{4 \pm \sqrt{(-4)^2-4(3)(2)}}{2(3)}$

$x = \frac{4 \pm \sqrt{-8}}{6}$

$\sqrt{-8}$, $i\sqrt{8}$, $i\sqrt{4}\sqrt{2}$, $2i\sqrt{2}$

$x = \frac{4 \pm 2i\sqrt{2}}{6}$

$x = \frac{2}{3} \pm \frac{1}{3}i\sqrt{2}$

15. $x^2 + 2x = -8$

$+8 \quad +8$

$a=1$, $b=2$, $c=8$

$x^2+2x+8=0$

$x = \frac{-2 \pm \sqrt{(2)^2-4(1)(8)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{-28}}{2}$

$\sqrt{-28}$, $i\sqrt{28}$, $i\sqrt{4}\sqrt{7}$, $2i\sqrt{7}$

$x = \frac{-2 \pm 2i\sqrt{7}}{2}$

$x = -1 \pm i\sqrt{7}$

16. $3x^2 + 6 = 5x$

$-5x \quad -5x$

$3x^2-5x+6=0$

$a=3$, $b=-5$, $c=6$

$x = \frac{5 \pm \sqrt{(-5)^2-4(3)(6)}}{2(3)}$

$x = \frac{5 \pm \sqrt{-47}}{6}$

$\sqrt{-47}$, $i\sqrt{47}$

$x = \frac{5 \pm i\sqrt{47}}{6}$

$x = \frac{5}{6} \pm \frac{i\sqrt{47}}{6}$

Polynomial Equations

1) Bring everything to one side. Keep the leading coefficient positive.
2) Factor
3) Set each factor equal to zero

If you end up with (x^2+a), use isolate/square root method.

To find the roots/zeros algebraically, replace $f(x)$ with 0.

1. Solve $x^3+5x^2=4x+20$ algebraically.

$-4x-20 \quad -4x-20$

$x^3+5x^2-4x-20=0$

$x^2 \quad x^2 \quad -4 \quad -4$

$x^2(x+5)-4(x+5)=0$

$(x^2-4)(x+5)=0$

$(x+2)(x-2)(x+5)=0$

$x+\cancel{2}=0 \quad x-\cancel{2}=0 \quad x+\cancel{5}=0$

$-\cancel{2} \; -2 \quad +\cancel{2} \; +2 \quad -\cancel{5} \; -5$

$x=-2 \quad x=2 \quad x=-5$

2. Algebraically determine the zeros of the function below.

$r(x)=3x^3+12x^2-3x-12$

$0=(3x^3+12x^2-3x-12)$

$3x^2 \quad 3x^2 \; -3 \quad -3$

~~$0=(3x$~~ $0=3x^2(x+4)-3(x+4)$

$0=(3x^2-3)(x+4)$

$0=3(x^2-1)(x+4)$

$0=3(x+1)(x-1)(x-4)$

$\cancel{3}\neq 0 \quad x+1=0 \quad x-1=0 \quad x-4=0$

$-1 \; -1 \quad +1 \; +1 \quad +4 \; +4$

$x=-1 \quad x=1 \quad x=4$

3. Solve for all values of x:

$x^4-6x^2=-8$

$+8 \quad +8$

$x^4-6x^2+8=0$

$(x^2-4)(x^2-2)=0$

$(x+2)(x-2)(x^2-2)=0$

$x+\cancel{2}=0 \quad x-\cancel{2}=0 \quad x^2-\cancel{2}=0$

$-\cancel{2} \; -2 \quad +\cancel{2} \; +2 \quad +\cancel{2} \; +2$

$\boxed{x=-2} \quad \boxed{x=2} \quad \sqrt{x^2=2}$

$\boxed{x=\pm\sqrt{2}}$

4. Find algebraically the zeros for $p(x)=x^3+x^2-4x-4$.

$0=(x^3+x^2-4x-4)$

$x^2 \quad x^2 \quad -4 \quad -4$

$0=x^2(x+1)-4(x+1)$

$(x^2-4)(x+1)=0$

$(x+2)(x-2)(x+1)=0$

$x+\cancel{2}=0 \quad x-\cancel{2}=0 \quad x+\cancel{1}=0$

$-\cancel{2} \; -2 \quad +\cancel{2} \; +2 \quad -\cancel{1} \; -1$

$x=-2 \quad x=2 \quad x=-1$

5. Solve the equation $2x^3-x^2-8x+4=0$ algebraically for all values of x.

$x^2 \quad x^2 \quad -4 \quad -4$

$x^2(2x-1)-4(2x-1)=0$

$(x^2-4)(2x-1)=0$

$(x+2)(x-2)(2x-1)=0$

$x+2=0 \quad x-2=0 \quad 2x-\cancel{1}=0$

$-2 \; -2 \quad +2 \; +2 \quad +\cancel{1} \; +1$

$x=-2 \quad x=2 \quad \frac{2x}{2}=\frac{1}{2}$

$x=\frac{1}{2}$

6. Solve for all values of x:

$x^4 - 5x^2 - 36 = 0$

$(x^2-9)(x^2+4)=0$

$x^2-9=0$ | $x^2+4=0$

$+9$ $+9$ | -4 -4

$\sqrt{x^2}=\sqrt{9}$ $\sqrt{x^2}=\sqrt{-4}$

$x=\pm3$ $x=\pm2i$

7. Find algebraically the zeros of $p(x) = x^3 - 3x^2 + 4x - 12$

$0=(x^3-3x^2)(+4x-12)$

x^2 x^2 4 4

$x^2(x-3)+4(x-3)=0$

$(x^2+4)(x-3)=0$

$x^2+4=0$ | $x-3=0$

-4 -4 | $+3$ $+3$

$\sqrt{x^2}=\sqrt{-4}$ $x=3$

$x=\pm2i$

8. What are the zeros of $P(m) = (m^2 - 4)(m^2 + 1)$?

$0=(m^2-4)(m^2+1)$

$m^2-4=0$ | $m^2+1=0$

$+4$ $+4$ | -1 -1

$\sqrt{m^2}=\sqrt{4}$ $\sqrt{m^2}=\sqrt{-1}$

$m=\pm2$ $m=\pm i$

9. Algebraically find the zeros for $f(x) = x^4 - 4x^3 - 9x^2 + 36x$

$0=x^4-4x^3-9x^2+36x$

$0=x[(x^3-4x^2)(-9x+36)]$

x^2 x^2 -9 -9

$0=x[x^2(x-4)-9(x-4)]$

$0=x[(x^2-9)(x-4)]$

$0=x(x+3)(x-3)(x-4)$

$x=0$ | $x+3=0$ | $x-3=0$ | $x-4=0$

-3 -3 | $+3$ $+3$ | $+4$ $+4$

$x=-3$ $x=3$ $x=4$

10. Solve algebraically for all values of x: $x^4 + 4x^3 + 4x^2 = -16x$

$+16x$ $+16x$

$x^4+4x^3+4x^2+16x=0$

$x[(x^3+4x^2)(4x+16)]$

x^2 x^2 4 4

$x[x^2(x+4)+4(x+4)]$

$x(x^2+4)(x+4)=0$

$x=0$ | $x^2+4=0$ | $x+4=0$

-4 -4 | -4 -4

$\sqrt{x^2}=\sqrt{-4}$ $x=-4$

$x=\pm2i$

Radical Equations

1) Isolate
2) Square both sides
3) Check

1. $\sqrt{2x+1}+\cancel{4}=8$

$-\cancel{4}\ -4$

$\left(\sqrt{2x+1}\right)^{\cancel{2}}=(4)^2$

$2x+\cancel{1}=16$

$-\cancel{1}\ -1$

$2x=15$

$\frac{\cancel{2}x}{\cancel{2}}=\frac{15}{2}$

$\boxed{x=7.5}$ ✓

2. $\sqrt{x-5}+\cancel{x}=7$

$-\cancel{x}\ -x$

$\left(\sqrt{x-5}\right)^{\cancel{2}}=(7-x)^2$

	7	$-x$
7	49	$-7x$
$-x$	$-7x$	x^2

$x^2-14x+49$

$x-5=(7-x)^2$

$\cancel{x}-\cancel{5}=x^2-14x+49$

$-\cancel{x}+5\quad -x+5$

$0=x^2-15x+54$

$0=(x-9)(x-6)$

$\cancel{x=9}$ ✗ $\boxed{x=6}$ ✓

3. $\left(\sqrt{56-x}\right)^{\cancel{2}}=(x)^2$

$\cancel{56}-\cancel{x}=x^2$

$-\cancel{56}+x\ -56+x$

$0=x^2+x-56$

$0=(x+8)(x-7)$

$\cancel{x=-8}$ ✗ $\boxed{x=7}$ ✓

4. $\sqrt{2x-7}+\cancel{x}=5$

$-\cancel{x}\ -x$

$\left(\sqrt{2x-7}\right)^{\cancel{2}}=(5-x)^2$

	5	$-x$
5	25	$-5x$
$-x$	$-5x$	x^2

$x^2-10x+25$

$2x-7=(5-x)^2$

$\cancel{2x}-\cancel{7}=x^2-10x+25$

$-\cancel{2x}+7\quad -2x+7$

$0=x^2-12x+32$

$0=(x-8)(x-4)$

$\cancel{x=8}$ ✗ $\boxed{x=4}$ ✓

5. $\left(\sqrt{5x+29}\right)^{\cancel{2}}=(x+3)^2$

	x	$+3$
x	x^2	$+3x$
$+3$	$+3x$	$+9$

x^2+6x+9

$5x+29=(x+3)^2$

$\cancel{5x}+\cancel{29}=x^2+6x+9$

$-\cancel{5x}+29\quad -5x-29$

$0=x^2+x-20$

$0=(x+5)(x-4)$

$\cancel{x=-5}$ ✗ $\boxed{x=4}$ ✓

6. $\left(\sqrt{2x-4}\right)^{\cancel{2}}=(x-2)^2$

	x	-2
x	x^2	$-2x$
-2	$-2x$	$+4$

x^2-4x+4

$2x-4=(x-2)^2$

$\cancel{2x}-\cancel{4}=x^2-4x+4$

$-2x+4\quad -2x+4$

$0=x^2-6x+8$

$(x-4)(x-2)$

$\boxed{x=4}$ ✓ $\boxed{x=2}$ ✓

7. $\sqrt{x^2+x-1}+\cancel{11x}=7x+3$

$-\cancel{11x}\ -11x$

$\left(\sqrt{x^2+x-1}\right)^{\cancel{2}}=(-4x+3)^2$

	$-4x$	$+3$
$-4x$	$16x^2$	$-12x$
$+3$	$-12x$	$+9$

$x^2-24x+9$

$\cancel{x^2}+\cancel{x}-\cancel{1}=16x^2-24x+9$

$-\cancel{x^2}-\cancel{x}+\cancel{1}\quad -x^2-x+1$

$\frac{0}{5}=\frac{15x^2}{5}-\frac{25x}{5}+\frac{10}{5}$

$0=3x^2-5x+2$ AT

x^2-5x+6

$\left(x-\frac{3}{3}\right)\left(x-\frac{2}{3}\right)$

$(x-1)(3x-2)=0$

$\cancel{x=1}$ ✗

$3x-\cancel{2}=0$

$+2\ +2$

$\frac{\cancel{3}x}{\cancel{3}}=\frac{2}{3}$

$\boxed{x=\frac{2}{3}}$ ✓

8. $3\sqrt{x}-\cancel{2x}=-5$

$+\cancel{2x}\ +2x$

$\frac{\cancel{3}\sqrt{x}}{\cancel{3}}=\frac{(2x-5)}{3}$

$\left(\sqrt{x}\right)^{\cancel{2}}=\left(\frac{2x-5}{3}\right)^2$

	$2x$	-5
$2x$	$4x^2$	$-10x$
-5	$-10x$	$+25$

$4x^2-20x+25$

$9(x)=\left(\frac{4x^2-20x+25}{\cancel{9}}\right)\cancel{9}$

$9x=4x^2-20x+25$

$-9x\quad -9x$

$0=4x^2-29x+25$ AT

$0=x^2-29x+100$

$0=\left(x-\frac{25}{4}\right)\left(x-\frac{4}{4}\right)$

$0=(4x-25)(x-1)$

$4x-25=0$

$+25+25$

$\frac{4x}{4}=\frac{25}{4}$

✓ $\boxed{x=\frac{25}{4}}$

$\cancel{x=1}$ ✗

5. $\sqrt{5x+29} = x+3$

6. $\sqrt{2x-4} = x-2$

7. $\sqrt{x^2+x-1} + 11x = 7x+3$

8. $3\sqrt{x} - 2x = -5$

	$2x$	-5
$2x$	$4x^2$	$-10x$
-5	$-10x$	$+25$

$4x^2-20x+25$

	11	$-x$
11	121	$-11x$
$-x$	$-11x$	$+x^2$

$x^2-22x+121$

9. $\sqrt{49-10x} + \cancel{5} = 2x$

$-5 \quad -5$

$\left(\sqrt{49-10x}\right)^{\cancel{2}} = (2x-5)^2$

$49-10x = (2x-5)(2x-5)$

$\cancel{49}-\cancel{10x} = 4x^2-20x+25$

$-49+10x \qquad +10x-49$

$\frac{0}{2} = \frac{4x^2-10x-24}{2}$

PT

$0 = 2x^2-5x-12$

$x^2-5x-24$

$(x-\frac{8}{2})(x+\frac{3}{2})$

$(x-4)(2x+3)=0$

$x-4=0$

$+4 \; +4$

$x=4$ ✓

$2x+3=0$

$-3 \quad -3$

$\frac{2x}{2} = \frac{-3}{2}$

~~$x=-\frac{3}{2}$~~

10. $\left(\sqrt{4x+1}\right)^{\cancel{2}} = (11-x)^2$

$4x+1 = (11-x)(11-x)$

$\cancel{4x+1} = x^2-22x+121$

$-4x-1 \qquad -4x-1$

$0 = x^2-26x+120$

$0 = (x-20)(x-6)$

$x-20=0$

$+20 \; +20$

~~$x=20$~~

$x-6=0$

$+6 \; +6$

$x=6$ ✓

Fractional Equations: MULTIPLY BY THE LCD

To find a common denominator:

1) Factor (if necessary)
2) Put all of your factors together

3x ~~3x~~ ~~3x~~ LCD: 3x

1. $\left(\frac{1}{\cancel{x}}\right)-\left(\frac{1}{\cancel{3}}\right)=-\left(\frac{1}{\cancel{3x}}\right)$

$3-x=-1$
$-3 \qquad -3$

$\frac{-x}{-1}=\frac{-4}{-1}$

$\boxed{x=4}$ ✓

2x ~~4x~~ 4~~x~~ 4x LCD: 4x

2. $\left(\frac{5x}{\cancel{2}}\right)=\left(\frac{1}{\cancel{x}}\right)+\left(\frac{x}{\cancel{4}}\right)$

$2x(5x)=4+x^2$

$10x^2=4+x^2$
$-x^2-4 \quad -4-x^2$

DOTS $9x^2-4=0$

$(3x+2)(3x-2)=0$

$3x+2=0 \qquad 3x-2=0$
$-2 \;\; -2 \qquad +2 \;\; +2$

$\frac{3x}{3}=\frac{-2}{3} \qquad \frac{3x}{3}=\frac{2}{3}$

$\boxed{x=\frac{-2}{3}}$ ✓ $\boxed{x=\frac{2}{3}}$ ✓

3x 3x 3x LCD: 3x

3. $\left(\frac{x+2}{\cancel{x}}\right)+\left(\frac{x}{\cancel{3}}\right)=\left(\frac{2x^2+6}{\cancel{3x}}\right)$

$3(x+2)+x^2=2x^2+6$

$\cancel{3x}+\cancel{6}+\cancel{x^2}=2x^2+6$
$-3x-6 \;\; -x^2 \quad -x^2-3x-6$

GCF $0=x^2-3x$

$0=x(x-3)$

~~x=0~~ reject $\qquad x-3=0$
$+3 \;\; +3$

$\boxed{x=3}$ ✓

3 ~~6x~~ ~~6x~~ 6x LCD: 6x

4. $\left(\frac{1}{\cancel{2x}}\right)-\left(\frac{5}{\cancel{6}}\right)=\left(\frac{3}{\cancel{x}}\right)$

$3-5x=18$
$-3 \qquad -3$

$\frac{-5x}{-5}=\frac{15}{-5}$

$\boxed{x=-3}$ ✓

LCD: x(x+2)

5. $\frac{3}{x}+\frac{x}{x+2}=-\frac{2}{x+2}$

~~x~~(x+2) x~~(x+2)~~ x(x+2)

$\left(\frac{3}{\cancel{x}}\right)+\left(\frac{x}{\cancel{x+2}}\right)=-\left(\frac{2}{\cancel{x+2}}\right)$

$3(x+2)+x^2=-2x$

$3x+6+x^2=-2x$
$+2x \qquad +2x$

$x^2+5x+6=0$

$(x+3)(x+2)=0$

$x+3=0 \qquad x+2=0$
$-3 \;\; -3 \qquad -2 \;\; -2$

✓ $\boxed{x=-3}$ ~~x=-2~~ reject

x(x-1) x(x-1) x(x-1) LCD: x(x-1)

6. $\frac{x}{\cancel{x-1}}=\frac{2}{\cancel{x}}+\frac{1}{\cancel{x-1}}$

$x^2=2(x-1)+x$

$x^2=2x-2+x$

$x^2=3x-2$
$-3x+2 \quad -3x+2$

$x^2-3x+2=0$

$(x-2)(x-1)=0$

$x-2=0 \qquad x-1=0$
$+2 \;\; +2 \qquad +1 \;\; +1$

$\boxed{x=2}$ ✓ ~~x=1~~ reject

LCD: $x(x+7)$

$\cancel{x(x+7)}$ $x(x+7)$ $\cancel{x}(x+7)$

7. $\left(\frac{3x+25}{\cancel{x+7}}\right)-(5)=\left(\frac{3}{\cancel{x}}\right)$

$x(3x+25)-5x(x+7)=3(x+7)$

$3x^2+25x-5x^2-35x=3x+21$

$-2x^2-10x=3x+21$

$+2x^2+10x \quad +2x^2+10x$

$0=2x^2+13x+21$

$x^2+13x+42 \rightarrow \frac{(x+7)}{2}\frac{(x+6)}{2}$

$0=(2x+7)(x+3)$

$2x+7=0 \quad x+3=0$

$-7\ -7 \quad -3\ -3$

$\frac{2x}{2}=\frac{-7}{2} \quad x=-3$

$x=-\frac{7}{2}$

LCD: $x(x+5)$

$\cancel{x(x+5)}$ $\cancel{x}(x+5)$ $x(x+5)$

8. $\left(\frac{8}{\cancel{x+5}}\right)-\left(\frac{3}{\cancel{x}}\right)=(5)$

$8x-3(x+5)=5x(x+5)$

$8x-3x-15=5x^2+25x$

$5x-15=5x^2+25x$

$-5x+15 \quad -5x+15$

$\frac{0}{5}=\frac{5x^2+20x+15}{5}$

$0=x^2+4x+3$

$0=(x+3)(x+1)$

$x+3=0 \quad x+1=0$

$-3\ -3 \quad -1\ -1$

$x=-3$ ✓ $\quad x=-1$ ✓

LCD: $4x(x+1)$

$\overset{2}{\cancel{4}}\cancel{x}(x+1)$ $4x\cancel{(x+1)}$ $\cancel{4}x(x+1)$

9. $\left(\frac{7}{\cancel{2x}}\right)-\left(\frac{2}{\cancel{x+1}}\right)=\left(\frac{1}{\cancel{4}}\right)$

$14(x+1)-8x=x(x+1)$

$14x+14-8x=x^2+1x$

$6x+14=x^2+1x$

$-6x-14 \quad -6x-14$

$0=x^2-5x-14$

$0=(x-7)(x+2)$

$x-7=0 \quad x+2=0$

$+7\ +7 \quad -2\ -2$

$x=7 \quad x=-2$ ✓

n^2 n n^2

10. $\left(\frac{2}{\cancel{n^2}}\right)+\left(\frac{3}{\cancel{n}}\right)=\left(\frac{4}{\cancel{n^2}}\right)$

$\cancel{2}+3n=4$

$-\cancel{2} \quad -2$

$\frac{3n}{3}=\frac{2}{3}$

$n=\frac{2}{3}$ ✓

	x	$+3$
x	x^2	$+3x$
$+2$	$+2x$	$+6$

	x	$+3$
x	x^2	$+3x$
-3	$-3x$	-9

LCD: $(x-5)(x+2)$

11. $\frac{x+3}{x-5}+\frac{6}{x+2}=\frac{6+10x}{(x-5)(x+2)}$

$\cancel{(x-5)}(x+2)$ $(x-5)\cancel{(x+2)}$ $\cancel{(x-5)(x+2)}$

$\left(\frac{x+3}{\cancel{x-5}}\right)+\left(\frac{6}{\cancel{x+2}}\right)=\left(\frac{6+10x}{\cancel{(x-5)(x+2)}}\right)$

$(x+3)(x+2)+6(x-5)=6+10x$

$x^2+5x+6+6x-30=6+10x$

$x^2+11x-24=\cancel{6}+\cancel{10x}$

$-10x-6 \quad -6-10x$

$x^2+x-30=0$

$(x+6)(x-5)=0$

$x+6=0 \quad x-5=0$

$-6\ -6 \quad +5\ +5$

✓ $x=-6 \quad \cancel{x=5}$ reject

LCD: $(x+3)(x-3)$

12. $\frac{30}{x^2-9}+1=\frac{5}{x-3}$

$\cancel{(x+3)(x-3)}$ $(x+3)(x-3)$ $(x+3)\cancel{(x-3)}$

$\left(\frac{30}{\cancel{(x+3)(x-3)}}\right)+(1)=\left(\frac{5}{\cancel{x-3}}\right)$

$30+(x+3)(x-3)=5(x+3)$

$30+x^2-9=5x+15$

$x^2+21=5x+15$

$-5x-15 \quad -5x-15$

$x^2-5x+6=0$

$(x-3)(x-2)=0$

$x-3=0 \quad x-2=0$

$+3\ +3 \quad +2\ +2$

$\cancel{x=3}$ reject $\quad x=2$ ✓

LCD: (x+5)(x-2)

LCD: (x-4)(x+3)

13. $\frac{1}{x-2}+\frac{4}{x+5}=\frac{7}{x^2+3x-10}$

$(x+5)\cancel{(x-2)}\left(\frac{1}{\cancel{x-2}}\right)+\cancel{(x+5)}(x-2)\left(\frac{4}{\cancel{x+5}}\right)=\left(\frac{7}{\cancel{(x+5)(x-2)}}\right)\cancel{(x+5)(x-2)}$

$1(x+5)+4(x-2)=7$

$1x+5+4x-8=7$

$5x-3=7$

$+3 \quad +3$

$5x=10$

$\frac{5x}{5}=\frac{10}{5}$

~~$x=2$~~ reject

No solution

14. $\frac{x}{x+3}+\frac{2}{x-4}=\frac{2x+27}{x^2-x-12}$

$(x-4)\cancel{(x+3)}\left(\frac{x}{\cancel{x+3}}\right)+\cancel{(x-4)}(x+3)\left(\frac{2}{\cancel{x-4}}\right)=\left(\frac{2x+27}{\cancel{(x-4)(x+3)}}\right)\cancel{(x-4)(x+3)}$

$x(x-4)+2(x+3)=2x+27$

$x^2-4x+2x+6=2x+27$

$x^2-2x+6=2x+27$

$-2x-27 \quad -2x-27$

$x^2-4x-21=0$

$(x-7)(x+3)=0$

$x-7=0$ | $x+3=0$

$+7 \ +7$ | $-3 \ -3$

$x=7$ ✓ | ~~$x=-3$~~ reject

LCD

(x-6)(x-2)

15. $\frac{1}{x-6}+\frac{x}{x-2}=\frac{4}{x^2-8x+12}$

$\cancel{(x-6)}(x-2)\left(\frac{1}{\cancel{x-6}}\right)+(x-6)\cancel{(x-2)}\left(\frac{x}{\cancel{x-2}}\right)=\left(\frac{4}{\cancel{(x-6)(x-2)}}\right)\cancel{(x-6)(x-2)}$

$1(x-2)+x(x-6)=4$

$1x-2+x^2-6x=4$

$x^2-5x-2=4$

$-4 \quad -4$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$x-6=0$ | $x+1=0$

$+6 \ +6$ | $-1 \ -1$

~~$x=6$~~ reject | $x=-1$ ✓

16. $\frac{4}{k^2-8k+12}=\frac{k}{k-2}+\frac{1}{k-6}$

$\frac{4}{(k-6)(k-2)}=\frac{k}{k-2}+\frac{1}{k-6}$

It's the same question!

LCD

(x+2)(x-2)

17. $\frac{x}{x+2}+\frac{1}{x^2-4}=\frac{4}{x-2}$

$\cancel{(x+2)}(x-2)\left(\frac{x}{\cancel{x+2}}\right)+\cancel{(x+2)(x-2)}\left(\frac{1}{\cancel{(x+2)(x-2)}}\right)=\left(\frac{4}{\cancel{x-2}}\right)(x+2)\cancel{(x-2)}$

$x(x-2)+1=4(x+2)$

$x^2-2x+1=4x+8$

$-4x-8 \quad -4x-8$

$x^2-6x-7=0$

$(x-7)(x+1)=0$

$x-7=0$ | $x+1=0$

$+7 \ +7$ | $-1 \ -1$

$x=7$ ✓ | $x=-1$ ✓

LCD

(x+3)(x-4)

18. $\frac{2}{x+3}-\frac{3}{4-x}=\frac{2x-2}{x^2-x-12}$

(-1)

$\cancel{(x+3)}(x-4)\left(\frac{2}{\cancel{x+3}}\right) - (x+3)\cancel{(x-4)}\left(\frac{3}{\cancel{4-x}}\right)=\frac{2x-2}{\cancel{x^2-x-12}\ \cancel{(x-4)(x+3)}}\cancel{(x+3)(x-4)}$

$2(x-4)+3(x+3)=2x-2$

$2x-8+3x+9=2x-2$

$5x+1=2x-2$

$-2x \quad -2x$

$3x+1=-2$

$-1 \quad -1$

$\frac{3x}{3}=\frac{-3}{3}$

✓ $x=-1$

19. Markus is a long-distance walker. In one race, he walked 55 miles in t hours and in another race walked 65 miles in $t + 3$ hours. His rates are shown in the equations below.

$$r = \frac{55}{t} \quad r = \frac{65}{t+3}$$

Markus walked at an equivalent rate, r, for each race. Determine the number of hours that each of the two races took.

$$\text{LCD: } \frac{}{t(t+3)}$$

$$t\cancel{(t+3)}\left(\frac{65}{\cancel{t+3}}\right) = \left(\frac{55}{\cancel{t}}\right)\cancel{t}(t+3)$$

$$65t = 55(t+3)$$

$$65t = 55t + 165$$

$$-55t \quad -55t$$

$$\frac{10t}{10} = \frac{165}{10}$$

$$t = 16.5$$

$$t = 16.5$$

$$t+3 = 16.5+3 = 19.5$$

$$\boxed{16.5 \text{ and } 19.5}$$

20. Sarah is fighting a sinus infection. Her doctor prescribed a nasal spray and an antibiotic to fight the infection. The active ingredients, in milligrams, remaining in the bloodstream from the nasal spray, $n(t)$, and the antibiotic, $a(t)$, are modeled in the functions below, where t is the time in hours since the medications were taken.

$$n(t) = \frac{t+1}{t+5} + \frac{18}{t^2 + 8t + 15}$$

$$a(t) = \frac{9}{t+3}$$

Determine which drug is made with a greater initial amount of active ingredient. Justify your answer. Sarah's doctor told her to take both drugs at the same time. Determine algebraically the number of hours after taking the medications when both medications will have the same amount of active ingredient remaining in her bloodstream.

t=0

set them equal

$$n(0) = \frac{0+1}{0+5} + \frac{18}{(0)^2+8(0)+15} = \frac{7}{5}$$

$$\boxed{a(0) = \frac{9}{0+3} = 3}$$

antibiotic has greater initial amount of active ingredient

$$\frac{t+1}{t+5} + \frac{18}{t^2+8t+15} = \frac{9}{t+3} \quad \text{LCD } \frac{}{(t+5)(t+3)}$$

$$\cancel{(t+5)}(t+3)\left(\frac{t+1}{\cancel{t+5}}\right) + \overset{\cancel{(t+5)}\cancel{(t+3)}}{\left(\frac{18}{\cancel{(t+5)}\cancel{(t+3)}}\right)} = \left(\frac{9}{\cancel{t+3}}\right)(t+5)\cancel{(t+3)}$$

$$(t+3)(t+1) + 18 = 9(t+5)$$

$$t^2+4t+3+18 = 9t+45$$

$$t^2+4t+21 = 9t+45$$

$$-9t-45 \quad -9t-45$$

	t	+3
t	t^2	+3t
+1	+1t	+3

$$t^2+4t+3$$

$$t^2-5t-24=0$$

$$(t-8)(t+3)=0$$

$$t-8=0 \qquad t+3=0$$

$$+8 \ +8 \qquad -3 \ -3$$

$$t=8 \qquad \cancel{t=-3} \text{ reject}$$

$$\boxed{8 \text{ hours}}$$

extraneous solution means it's a rejects

LCD. (x+7)(x-7)

$(\cancel{x+7})(x-7)$ $(x+7)(\cancel{x-7})$ $(x+7)(\cancel{x-7})$

21. To solve the equation $\frac{7}{\cancel{x+7}}+\frac{4x}{\cancel{x-7}}=\frac{3x+7}{\cancel{x-7}}$, Joan's first step is to multiply both sides by the least common denominator. Which statement is true?

1) -14 is an extraneous solution.
2) 7 and -7 are extraneous solutions.
[✓ 3) 7 is an extraneous solution.]
4) There are no extraneous solutions.

$7(x-7)+4x(x+7)=(3x+7)(x+7)$ → at this point, you can use y1, y2, intersect

	3x	+7
x	3x²	+7x
+7	+21x	+49

$3x^2+28x+49$

$7x-49+4x^2+28x=3x^2+28x+49$

$4x^2+35x-49=3x^2+28x+49$

$-3x^2-28x-49 \quad -3x^2-28x-49$

$x^2+7x-98=0$

$(x+14)(x-7)=0$

$(x+14)=0 \quad x-7=0$

$-14 \; -14 \quad +7 \; +7$

$x=-14$ ✓ $\quad$ ~~$x=7$~~ reject

LCD
x(x-2)

22. To solve $\frac{2x}{x-2}-\frac{11}{x}=\frac{8}{x^2-2x}$, Ren multiplied both sides by the least common denominator. Which statement is true?

[✓ 1) 2 is an extraneous solution.]
2) $\frac{7}{2}$ is an extraneous so1ution.
3) 0 and 2 are extraneous solutions.
4) This equation does not contain any extraneous solutions.

$\left(\frac{2x}{\cancel{x-2}}\right)x(\cancel{x-2})-\left(\frac{11}{\cancel{x}}\right)\cancel{x}(x-2)=\left(\frac{8}{\cancel{x}(\cancel{x-2})}\right)\cancel{x}(\cancel{x-2})$

$2x^2-11(x-2)=8$

$2x^2-11x+22=8$

~~$-8 \quad -8$~~

~~$2x^2-19x+22=0$~~

~~$x^2-19x+44=0$~~

$2x^2-11x+14=0$

$x^2-11x+28$

$(x-\frac{7}{2})(x-\frac{4}{2})$

$(2x-7)(x-2)=0$

$2x-7=0 \quad x-2=0$

$+7 \; +7 \quad +2 \; +2$

$\frac{\cancel{2}x}{\cancel{2}}=\frac{7}{2}$ $\quad$ ~~$x=2$~~ reject

$x=\frac{7}{2}$ ✓

23. Jin solved the equation $(\sqrt{4-x})^2=(x+8)^2$ by squaring both sides. What extraneous solution did he find?

1) -5
[✓ 2) -12]
3) 3
4) 4

$4-x=(x+8)(x+8)$

$\cancel{4}\cancel{-x}=x^2+16x+64$

$-4+x \qquad +x-4$

	x	+8
x	x²	+8x
+8	+8x	+64

$x^2+16x+64$

$0=x^2+17x+60$

$0=(x+12)(x+5)$

$x+12=0 \quad x+5=0$

$-12 \; -12 \quad -5 \; -5$

~~$x=-12$~~ reject $\quad x=-5$ ✓

Quadratic Systems of Equations Algebraically

1) Isolate at least one variable in one of the equations
2) Substitute one equation into the other (set them equal if you solved both equations for the same variables).
3) Solve equation (Mr. x^2 /Polynomial Equations)
4) Substitute answers into one of the original equations to find the second variable

1. $y = x^2 - 5$
$y = 3x - 1$

$x^2-5=3x-1$
$-3x+1 \quad -3x+1$
$x^2-3x-4=0$
$(x-4)(x+1)=0$
$x-4=0 \quad x+1=0$
$+4 \; +4 \quad -1 \; -1$
$x=4 \quad x=-1$

$x=4$
$y=3x-1$
$y=3(4)-1$
$y=11$
$(4,11)$

$x=-1$
$y=3x-1$
$y=3(-1)-1$
$y=-4$
$(-1,-4)$

2. $x^2 + y^2 = 2$
$y + 2 = x$

$(y+2)^2+y^2=2$
$y^2+4y+4+y^2=2$
$2y^2+4y+4=2$
$-2 \quad -2$
$2y^2+4y+2=0$
$\frac{2y^2+4y+2}{2}=\frac{0}{2}$
$y^2+2y+1=0$
$(y+1)(y+1)=0$
$y=-1$

	y	$+2$
y	y^2	$+2y$
$+2$	$+2y$	$+4$

y^2+4y+4
$y=-1$
$x=y+2$
$x=-1+2$
$x=1$
$(1,-1)$

3. $x^2 + y^2 = 25$
$y + 5 = 2x$
$-5 \quad -5$
$y=2x-5$

	$2x$	-5
$2x$	$4x^2$	$-10x$
-5	$-10x$	$+25$

$4x^2-20x+25$

$x^2+(2x-5)^2=25$ *
$x^2+4x^2-20x+25=25$
$-25 \quad -25$
$5x^2-20x=0$
$5x(x-4)=0$
$\frac{5x}{5}=\frac{0}{5} \quad x-4=0$
$+4 \; +4$
$x=0 \quad x=4$
$y=2x-5 \quad y=2x-5$
$y=2(0)-5 \quad y=2(4)-5$
$y=-5 \quad y=3$

$(0,-5)$
$(4,3)$

4. $y = 2x^2 - 7x + 4$
$y = 11 - 2x$

$11-2x=2x^2-7x+4$
$-11+2x \quad +2x-11$
$0=2x^2-5x-7$ pt
$0=x^2-5x-14$
$(x-7)(x+2)$
$\frac{}{2} \quad \frac{}{2}$
$0=(2x-7)(x+1)$
$2x-7=0 \quad x+1=0$
$+7 \; +7 \quad -1 \; -1$
$\frac{2x}{2}=\frac{7}{2} \quad x=-4$
$x=3.5$

$x=3.5 \quad x=-4$
$y=11-2x \quad y=11-2x$
$y=11-2(3.5) \quad y=11-2(-4)$
$y=4 \quad y=19$
$(3.5,4) \quad (-4,19)$

	x	$+2$
x	x^2	$+2x$
$+2$	$+2x$	$+4$

x^2+4x+4

5. $(x+2)^2+(y-4)^2=40$

$y=x+2$

	x	-2
x	x^2	$-2x$
-2	$-2x$	$+4$

x^2-4x+4

$(x+2)^2+(x+2-4)^2=40$

$(x+2)^2+(x-2)^2=40$

$(x+2)(x+2)+(x-2)(x-2)=40$

$x^2\cancel{+4x}+4+x^2\cancel{-4x}+4=40$

$2x^2+8=40$

$-8 \quad -8$

$\frac{2x^2}{2}=\frac{32}{2}$

$\sqrt{x^2}=\sqrt{16}$

$x=\pm4$

$x=4 \qquad x=-4$

$y=x+2 \qquad y=x+2$

$y=4+2 \qquad y=-4+2$

$y=6 \qquad y=-2$

$(4,6) \qquad (-4,-2)$

6. $(x-2)^2+(y-3)^2=16$

$x+y-1=0$

$-x \quad +1 \quad -x+1$

$y=-x+1$

	x	-2
x	x^2	$-2x$
-2	$-2x$	$+4$

x^2-4x+4

	$-x$	-2
$-x$	x^2	$+2x$
-2	$+2x$	$+4$

x^2+4x+4

$(x-2)^2+(-x+1-3)^2=16$

$(x-2)^2+(-x-2)^2=16$

$x^2\cancel{-4x}+4+x^2\cancel{+4x}+4=16$

$2x^2+8=16$

$-8 \quad -8$

$\frac{2x^2}{2}=\frac{8}{2}$

$\sqrt{x^2}=\sqrt{4}$

$x=\pm2$

$x=2 \qquad x=-2$

$y=-x+1 \qquad y=-x+1$

$y=-2+1 \qquad y=2+1$

$y=-1 \qquad y=3$

$(2,-1) \qquad (-2,3)$

7. $x+y=5$ $\quad y=5-x$

$-x \quad -x$

$(x+3)^2+(y-3)^2=53$

	x	$+3$
x	x^2	$+3x$
$+3$	$+3x$	$+9$

x^2+6x+9

	2	$-x$
2	4	$-2x$
$-x$	$-2x$	$+x^2$

x^2-4x+4

$(x+3)^2+(5-x-3)^2=53$

$(x+3)^2+(2-x)^2=53$

$x^2+6x+9+x^2-4x+4=53$

$2x^2+2x+13=53$

$-53 \quad -53$

$\frac{2x^2+2x-40}{2}=\frac{0}{2}$

$x^2+x-20=0$

$(x+5)(x-4)=0$

$x+5=0 \qquad x-4=0$

$-5 \; -5 \qquad +4 \; +4$

$x=-5 \qquad x=4$

$x=-5 \qquad x=4$

$y=5-x \qquad y=5-x$

$y=5-5 \qquad y=5-4$

$y=10 \qquad y=1$

$(-5,10) \qquad (4,1)$

8. $(x-3)^2+(y+2)^2=16$

$2x+2y=10$

$-2x \quad -2x$

$\frac{2y}{2}=\frac{-2x+10}{2}$

$y=-x+5$

	x	-3
x	x^2	$-3x$
-3	$-3x$	$+9$

x^2-6x+9

	$-x$	$+7$
$-x$	x^2	$-7x$
$+7$	$-7x$	$+49$

$x^2-14x+49$

$(x-3)^2+(-x+5+2)^2=16$

$(x-3)^2+(-x+7)^2=16$

$x^2-6x+9+x^2-14x+49=16$

$2x^2-20x+58=16$

$-16 \quad -16$

$\frac{2x^2-20x+42}{2}=\frac{0}{2}$

$x^2-10x+21=0$

$(x-7)(x-3)=0$

$x-7=0 \qquad x-3=0$

$+7 \; +7 \qquad +3 \; +3$

$x=7 \qquad x=3$

$x=7 \qquad x=3$

$y=-x+5 \qquad y=-x+5$

$y=-7+5 \qquad y=-3+5$

$y=-2 \qquad y=2$

$(7,-2) \qquad (3,2)$

Linear Systems In Three Variables
Elimination Method:

1) Choose two pairs of equations and get the same variable to cancel
2) Use Addition Method to solve the system with your two new equations
3) Substitute those two answers into one of the original equations to find your third variables

*Make sure all variables are in order on the left hand side and all constants are on the right hand side.

1. Solve the following system of equations algebraically for all values of x, y, and z:

A $x + 3y + 5z = 45$
B $6x - 3y + 2z = -10$
C $-2x + 3y + 8z = 72$

A and B

$x+\cancel{3y}+5z=45$
$+\ 6x-\cancel{3y}+2z=-10$
D $7x+7z=35$

B and C

$6x-3y+2z=-10$
$+\ -2x+\cancel{3y}+8z=72$
E $4x+10z=62$

D and E

$-4(7x+7z=35)$
$7(4x+10z=62)$
$-\cancel{28x}-28z=-140$
$\cancel{28x}+70z=434$
$42z=294$
$\frac{42z}{42}=\frac{294}{42}$
$z=7$

$7x+7z=35$
$7x+7(7)=35$
$7x+49=35$
$-49\quad -49$
$\frac{7x}{7}=\frac{-14}{7}$
$x=-2$

$x+3y+5z=45$
$-2+3y+5(7)=45$
$-2+3y+35=45$
$3y+33=45$
$-33\quad -33$
$\frac{3y}{3}=\frac{12}{3}$
$y=4$

2. Solve the following system of equations algebraically for all values of x, y, and z:

A $x + 2y - 3z = -2$
B $2x - 2y + z = 7$
C $x + y + 2z = -4$

A and B

$x+\cancel{2y}-3z=-2$
$+\ 2x-\cancel{2y}+z=7$
D $3x-2z=5$

B and C

$1(2x-2y+z=7)$
$2(x+y+2z=-4)$
$2x-\cancel{2y}+z=7$
$+\ 2x+\cancel{2y}+4z=-8$
E $4x+5z=-1$

D and E

$5(3x-2z=5)$
$2(4x+5z=-1)$
$15x-\cancel{10z}=25$
$+\ 8x+\cancel{10z}=-2$
$\frac{23x}{23}=\frac{23}{23}$
$x=1$

$4x+5z=-1$
$4(1)+5z=-1$
$4+5z=-1$
$-4\quad -4$
$\frac{5z}{5}=\frac{-5}{5}$
$z=-1$

$x+2y-3z=-2$
$1+2y-3(-1)=-2$
$1+2y+3=-2$
$2y+4=-2$
$-4\quad -4$
$\frac{2y}{2}=\frac{-6}{2}$
$y=-3$

3. Solve the following system of equations algebraically for all values of x, y, and z:

A $2x + 3y - 4z = -1$

B $x - 2y + 5z = 3$

C $-4x + y + z = 16$

A and B

$1(2x+3y-4z=-1)$

$-2(x-2y+5z=3)$

$\cancel{2x}+3y-4z=-1$

$+\ \cancel{-2x}+4y-10z=-6$

D $7y-14z=-7$

A and C

$2(2x+3y-4z=-1)$

$1(-4x+y+z=16)$

$\cancel{4x}+6y-8z=-2$

$+\ \cancel{-4x}+y+z=16$

E $7y-7z=14$

D and E

$-1(7y-14z=-7)$

$7y-7z=14$

$\cancel{-7y}+14z=7$

$+\ \cancel{7y}-7z=14$

$\frac{7z}{7}=\frac{21}{7}$

$z=3$

$7y-7z=14$

$7y-7(3)=14$

$7y-\cancel{21}=14$

$+\cancel{21}\ +21$

$\frac{7y}{7}=\frac{35}{7}$

$y=5$

$x-2y+5z=3$

$x-2(5)+5(3)=3$

$x-10+15=3$

$x+\cancel{5}=3$

$\cancel{-5}\ -5$

$x=-2$

4. Solve the following system of equations algebraically for all values of a, b, and c.

A $a + 4b + 6c = 23$

B $a + 2b + c = 2$

$6b + 2c = a + 14$

$-a \qquad -a$

C $-a+6b+2c=14$

A and C

$\cancel{a}+4b+6c=23$

$+\ \cancel{-a}+6b+2c=14$

D $10b+8c=37$

B and C

$+\ \cancel{a}+2b+c=2$

$\cancel{-a}+6b+2c=14$

E $8b+3c=16$

D and E

$-3(10b+8c=37)$

$8(8b+3c=16)$

$-30b-\cancel{24c}=-111$

$64b+\cancel{24c}=128$

$\frac{34b}{34}=\frac{17}{34}$

$b=.5$

$10b+8c=37$

$10(.5)+8c=37$

$\cancel{5}+8c=37$

$\cancel{-5}\qquad -5$

$\frac{8c}{8}=\frac{32}{8}$

$c=4$

$a+4b+6c=23$

$a+4(.5)+6(4)=23$

$a+2+24=23$

$a+\cancel{26}=23$

$\cancel{-26}\ -26$

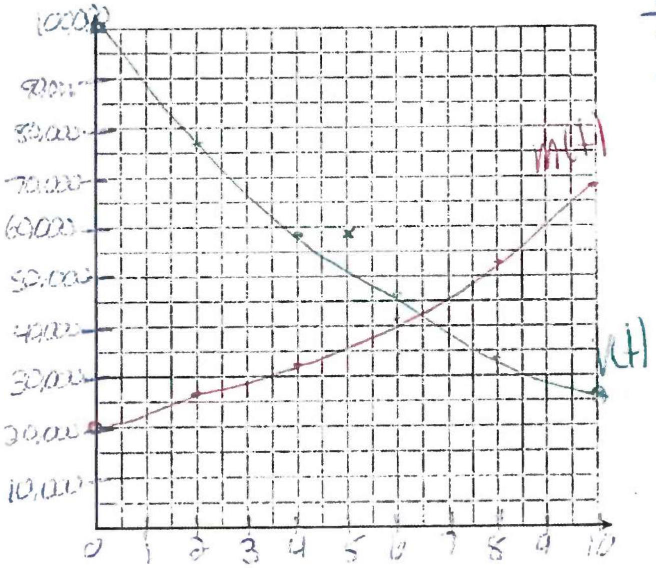
$a=-3$

Name Schlansky
Mr. Schlansky

Date ____________
Algebra II

Exponential Graphs (Part IV)

1. The value of Tom's bank account is currently 100000 and is decreasing according to the equation $V(t) = 100000(.876)^t$. The amount of money he has paid for his mortgage can be represented by the equation $M(t) = 20000(1.1304)^t$. Graph and label $V(t)$ and $M(t)$ over the interval [0,10]. No axis



V(t)

x	y
0	100,000
2	76738
4	58887
6	45188
8	34676
10	26610

M(t)

x	y
0	20,000
2	25556
4	32656
6	41728
8	53320
10	68132

V(t)=M(t) Intersect

After how many years will the value of Tom's bank account be equal to the amount of money he paid for his mortgage? Round your answer to the *nearest tenth of a year*. Tom will open a new bank account when the value of his account is $20,000. After how many years, to the *nearest hundredth of a year*, will that happen?

V(t)

2nd Graph: Intersect

t=6.3 years.

$y_1 = 100000(.876)^t$

$y_2 = 20000(1.1304)^t$

intersect

$$\frac{20{,}000}{100{,}000} = \frac{100{,}000(.876)^t}{100{,}000}$$

$$\log .2 = \log .876^t$$

$$\frac{\log .2}{\log .876} = \frac{t \log .876}{\log .876}$$

$$12.16 = t$$

or

$y_1 = 20000$

$y_2 = 100000(.876)^t$

intersect

t=12.16

~~25~~ 170

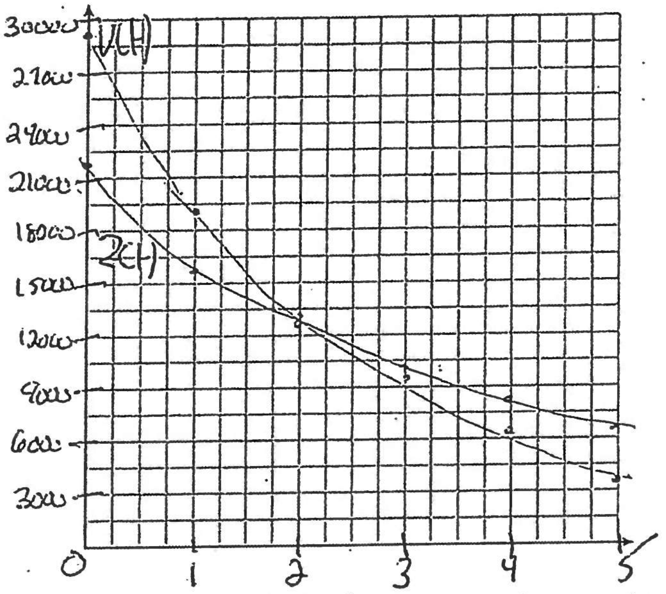
2 ~~14~~. The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time. Graph $V(t)$ and $Z(t)$ over the interval $0 \le t \le 5$, on the set of axes below.

V(t)

X	y
0	28483
1	19482
2	13326
3	9114.8
4	6234.6
5	4264.4

Z(t).

X	y
0	22151
1	17234
2	13408
3	10431
4	8115.6
5	6313.4



Scale

$x \geq \frac{5}{20}$

$x \geq .25$

$y \geq \frac{28483}{20}$

$y \geq$ ~~14~~ 1424.15

$y = 1500$

State when $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem. Zach takes out an insurance policy that requires him to pay a \$3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the *nearest year*, how long will it take Zach to cancel this policy? Justify your answer.

3000

V(t)=3000

Y1= 28482.698(.684)^t

Y2= 22151.327(.778)^t

intersect

t=1.95

After 1.95 years, the value of the loans will be the same ($13569.24)

~~Z(t) = 22151.327(0.778)~~^t

3000= ~~22151.327~~(0.778)^t

~~22151.327~~ ~~22151.327~~

or Y1=3000 ~~22151~~ 28482.698(.684)^t

Y2= ~~22151.327(.778)~~^t

intersect t=6

3000 = 28482.698(.684)^t

28482.698 28482.698

log .105 = log .684^t

log .105 = t log .684

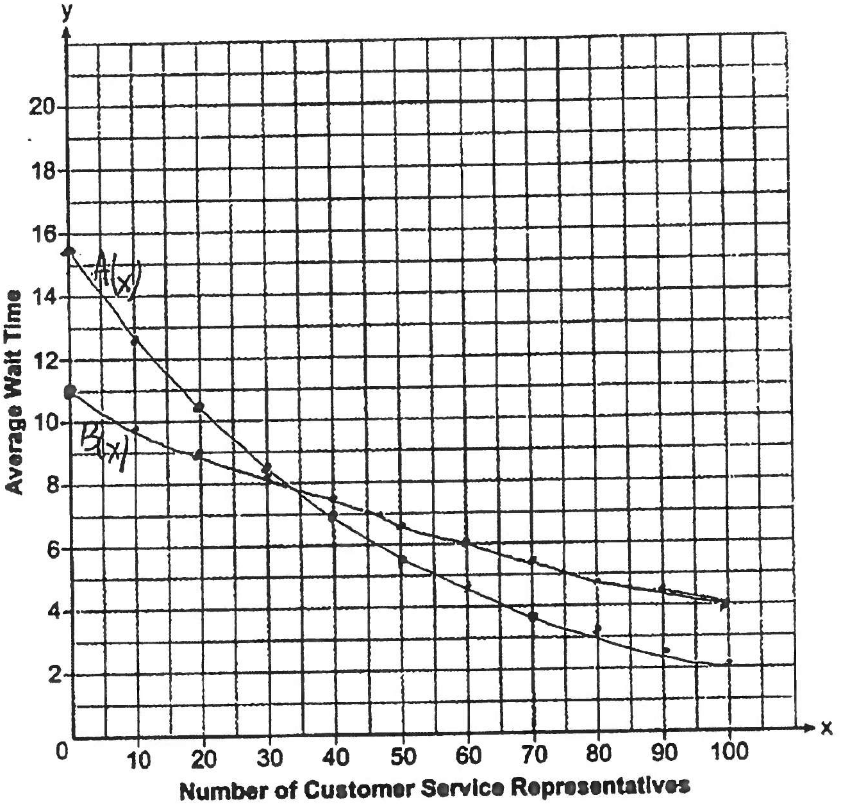
log .684 log .684

6 = t

3. A technology company is comparing two plans for speeding up its technical support time. Plan *A* can be modeled by the function $A(x) = 15.7(\underset{41}{0.98})^x$ and plan *B* can be modeled by the function $B(x) = \underset{42}{11(0.99)^x}$ where *x* is the number of customer service representatives employed by the company and $A(x)$ and $B(x)$ represent the average wait time, in minutes, of each customer. Graph $A(x)$ and $B(x)$ in the interval $0 \le x \le 100$ on the set of axes below.

A(x)

x	y
0	15.7
10	12.8
20	10.5
30	8.6
40	7.0
50	5.7
60	4.7
70	3.8
80	3.1
90	2.5
100	2.1



B(x)

x	y
0	11
10	9.9
20	9.0
30	8.1
40	7.4
50	6.7
60	6.0
70	5.4
80	4.9
90	4.5
100	4.0

To the *nearest integer*, solve the equation $A(x) = B(x)$ Determine~~, to the *nearest minute*,~~ ~~$B(100) - A(100)$. Explain what this value represents in the given context.~~ How many customer service representatives would company B need in order for the average wait time to be 3 minutes?

$\underset{B(x)}{3}$

41 42 intersect

~~B(100) − A(100)~~

$41 = 15.7(0.98)^x$

$42 = 11(0.99)^x$

intersect

$x = 35$

~~4.0 − 2.1~~

~~1.9~~

$B(x) = 11(0.99)^x$

$\frac{3}{11} = \frac{11(0.99)^x}{11}$

$\log \frac{3}{11} = \log 0.99^x$

~~If they hire 100 customer service representatives, the average wait time time would be 1.9 minutes longer with Plan B.~~

$\frac{\log \frac{3}{11}}{\log 0.99} = \frac{x \log 0.99}{\log 0.99}$

$\boxed{129 = x}$

46. Tony is evaluating his retirement savings. He currently has $318,000 in his account, which earns an interest rate of 7% compounded annually. He wants to determine how much he will have in the account in the future, even if he makes no additional contributions to the account. Write a function, $A(t)$, to represent the amount of money that will be in his account in t years. Graph $A(t)$ where $0 \le t \le 20$ on the set of axes below.

P

$A=P(1+r)^t$

$A=A(t)$

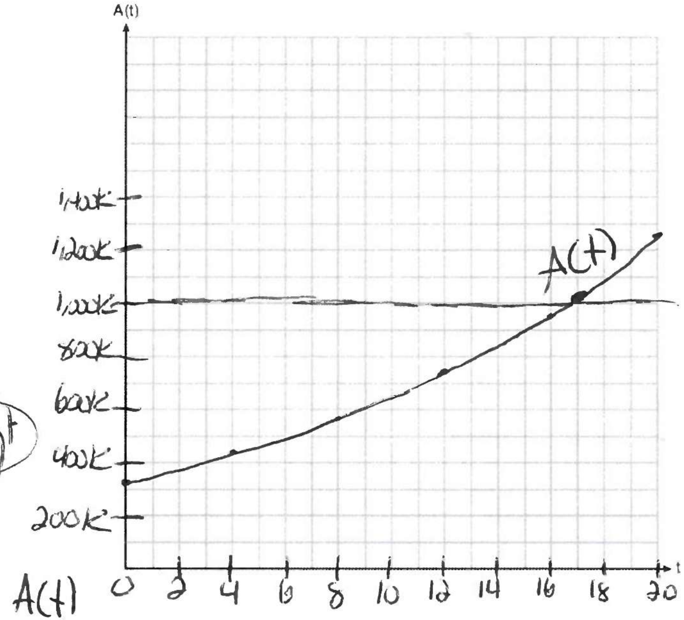
$P=318{,}000$

$r=.07$

$t=t$

$A=318{,}000(1+.07)^t$

$\boxed{A(t)=318{,}000(1.07)^t}$



A(t)

x	y
0	318,000
4	416833
8	546383
12	716197
16	938788
20	1230000

$\text{Scale} \ge \frac{\text{max}}{\text{\# of boxes}} \ge \frac{1230000}{20} \ge 61500$

Tony's goal is to save $1,000,000. Determine algebraically, to the *nearest year*, how many years it will take for him to achieve his goal. Explain how your graph of $A(t)$ confirms your answer. Scale = 100,000

$$\frac{1{,}000{,}000}{318{,}000} = \frac{318{,}000(1.07)^t}{318{,}000}$$

$$\log \frac{500}{159} \log = 1.07^t$$

$$\frac{\log \frac{500}{159}}{\log 1.07} = \frac{t \log 1.07}{\log 1.07}$$

$$\boxed{17 = t}$$

17 is the first year the graph crosses 1,000,000.

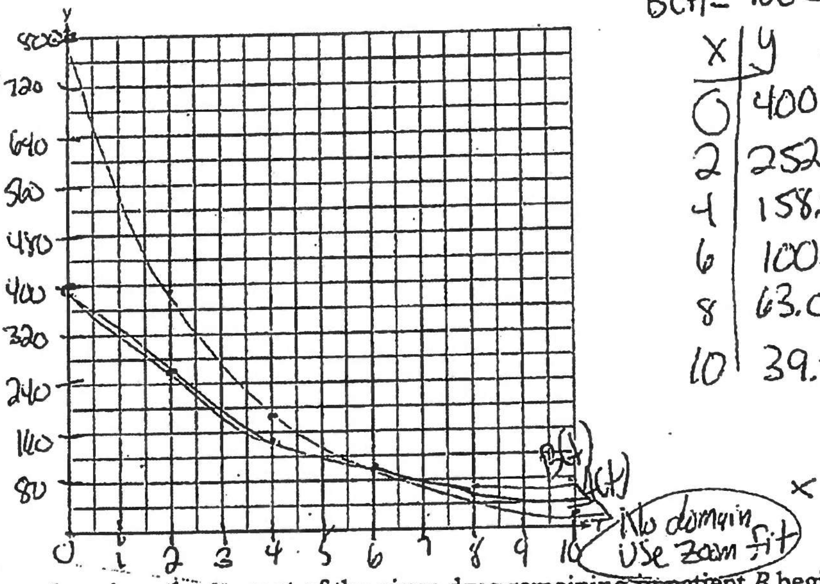
173

5.

3 ~~12.~~ Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 N_0 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231. Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.

$A(t) = 800e^{-.347t}$

x	y
0	800
2	399.66
4	199.66
6	99.744
8	49.83
10	24.894



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x and y min = 0

$B(t) = 400e^{-.231t}$

x	y
0	400
2	252.01
4	158.77
6	100.03
8	63.021
10	39.105

To the *nearest hour*, t, when does the amount of the given drug remaining ~~in~~ patient B begin to exceed the amount of the given drug remaining in patient A? The doctor will allow patient A to take another 800 milligram dose of the drug once only ~~15%~~ 120 mg of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another ~~800 milligram~~ dose of the drug.

ame
Table
Intersect

(5.98, 100.60)
hours

Scale

$x \geq \frac{10}{20}$

$x \geq .5$

$x = .5$

$y \geq \frac{800}{20}$

$y \geq 40$

$y = 40$

$y1 = 800e^{-.347t}$

$y2 = 400e^{-.231t}$

intersect

$t = 5.98$

$A(t) = ~~.15(800)~~$ 120

120 $~~.15(800)~~ = 800e^{-.347t}$

y_1 y_2 Find intersection

or

120 $\frac{~~.15(800)~~}{800} = \frac{800e^{-.347t}}{800}$

$\ln .15 = \ln e^{-.347t}$

$\frac{\ln .15}{-.347 \ln e} = \frac{-.347t \ln e}{-.347 \ln e}$

$5.5 = t$

Complex Formulas

List what each variable represents and CAREFULLY substitute into the given formula. Solve the equation using the appropriate Algebra skills

S_o V_o

1. A baseball is hit straight up from a height of 6 feet with an initial velocity of 90 feet per second. The equation that models the height of the ball, s, as a function of time, t, is $s = -16t^2 + v_0 t + s_0$ where v_0 is the initial velocity and s_0 is the initial height. How high is the ball after $\overset{t}{4}$ seconds?

S = height of ball = S
t = time = 4
V_o = initial velocity = 90
S_o = initial height = 6

$S = -16(4)^2 + 90(4) + 6$

$S =$ 110 feet

2. A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, t, in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation $t = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum in meters and g is a constant of 9.81 m/s². The first Foucault pendulum was constructed in 1851 and has a pendulum $\overset{L}{\text{length of 67 m}}$. Determine, to the *nearest tenth of a second*, the time it takes this pendulum to complete one swing.

t = time = t
L = length = 67
g = constant = 9.81

$t = 2\pi\sqrt{\frac{67}{9.81}}$

$\boxed{t = 16.4}$

3. The loudness of sound is measured in units called decibels (dB). These units are measured by first assigning an intensity I_0 to a very soft sound that is called the threshold sound. The sound to be measured is assigned an intensity, I, and the decibel rating, d, of this sound is found using $d = 10\log\frac{I}{I_0}$. The threshold sound audible to the average person is $\underset{I_o}{1.0 \times 10^{-12}}$ W/m² (watts per square meter). Consider the following sound level classifications. How would a sound with intensity $\overset{I}{6.3 \times 10^{-3}}$ W/m² be classified?

1) moderate
2) loud
[✓ 3)] very loud
4) deafening

Moderate	45-69 dB
Loud	70-89 dB
Very loud	90-109 dB
Deafening	>110 dB

I_o = threshold sound = 1.0×10^{-12}
I = intensity = 6.3×10^{-3}
d = decibel rating = d

$d = 10\log\frac{6.3\times10^{-3}}{1.0\times10^{-12}}$

$d = 97..$

4. The speed of a tidal wave, s, in hundreds of miles per hour, can be modeled by the equation $s = \sqrt{t} - 2t + 6$, where t represents the time from its origin in hours. Algebraically determine the time when $s = 0$.

S = Speed = 0

t = time = t

	$2t$	-6
$2t$	$4t^2$	$-12t$
-6	$-12t$	$+36$

$4t^2 - 24t + 36$

$0 = \sqrt{t} - 2t + 6$

$+2t - 6 \quad +2t - 6$

$(2t-6)^2 = (\sqrt{t})^2$

$4t^2 - 24t + 36 = t$

$-t \quad -t$

$4t^2 - 25t + 36 = 0$

$t^2 - 25t + 144 = 0$

$(t - \frac{16}{4})(t - \frac{9}{4})$

$(t-4)(4t-9) = 0$

$t - 4 = 0$

$+4 \quad +4$

$t = 4$ ✓

$4t - 9 = 0$

$+9 \quad +9$

$\frac{4t}{4} = \frac{9}{4}$

~~$t = \frac{9}{4}$~~

reject

5. A formula for work problems involving two people is shown below.

$$\frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{t_b}$$

t_1 = the time taken by the first person to complete the job $= 8$

t_2 = the time taken by the second person to complete the job $= 6$

t_b = the time it takes for them working together to complete the job

Fred and Barney are carpenters who build the same model desk. It takes Fred eight hours to build the desk while it only takes Barney six hours. Write an equation that can be used to find the time it would take both carpenters working together to build a desk. Determine, to the *nearest tenth of an hour*, how long it would take Fred and Barney working together to build a desk.

LCD

48x

$\frac{1}{8} + \frac{1}{6} = \frac{1}{t_b}$

$\overset{6x}{\cancel{48x}}\left(\frac{1}{\cancel{8}}\right) + \overset{8x}{\cancel{48x}}\left(\frac{1}{\cancel{6}}\right) = \left(\frac{1}{\cancel{x}}\right)48\cancel{x}$

$6x + 8x = 48$

$\frac{14x}{14} = \frac{48}{14}$

$x = 3.4$

$t_b = 3.4$

6. A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, t, in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation $t = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum in meters and g is a constant of 9.81 m/s^2. The first Foucault pendulum was constructed in 1851 and has a pendulum illelevant ~~length of 67 m~~. Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the *nearest tenth of a meter*, the length of this pendulum.

t=time = 9.6
L=Length = L
g=constant=9.81

$$\frac{9.6}{2\pi} = \frac{\cancel{2\pi}\sqrt{\frac{L}{9.81}}}{\cancel{2\pi}}$$

$$(1.52)^2 = \left(\cancel{\sqrt{\frac{L}{9.81}}}\right)^{\cancel{2}}$$

$$9.81(2.3) = \left(\frac{L}{\cancel{9.81}}\right)\cancel{9.81}$$

$$\boxed{22.9 = L}$$

7. The Beaufort Wind Scale was devised by British Rear Admiral Sir Francis Beaufort, in 1805 based upon observations of the effects of the wind. Beaufort numbers, B, are determined by the equation $B = 1.69\sqrt{s + 4.45} - 3.49$, where s is the speed of the wind in mph, and B is rounded to the nearest integer from 0 to 12.

Beaufort Wind Scale	
Beaufort Number	**Force of Wind**
0	Calm
1	Light air
2	Light breeze
3	Gentle breeze
4	Moderate breeze
5	Fresh breeze
6	Steady breeze
7	Moderate gale
8	Fresh gale
9	Strong gale
10	Whole gale
11	Storm
12	Hurricane

B=Beaufort numbers = B
S=Speed = 30
$B = 1.69\sqrt{30+4.45} - 3.49$
B=3.4..
B=3 Gentle Breeze

Using the table above, classify the force of wind at a speed of 30 mph. Justify your answer. In 1946, the scale was extended to accommodate strong hurricanes. A strong hurricane received a B value of exactly 15. Algebraically determine the value of s, to the *nearest mph*.

S=30

B=15
S=S

$$15 = 1.69\sqrt{S+4.45} - 3.49$$
$$+3.49 \qquad\qquad +3.49$$
$$\frac{18.49}{1.69} = \frac{\cancel{1.69}\sqrt{S+4.45}}{\cancel{1.69}}$$
$$(10.9..)^2 = \left(\cancel{\sqrt{S+4.45}}\right)^2$$

$$119.. = S + \cancel{4.45}$$
$$-4.45 \qquad -\cancel{4.45}$$
$$115.25 = S$$
$$\boxed{115 = S}$$

THE UNIVERSITY OF THE STATE OF NEW YORK

Common Core High School Math Reference Sheet
(Algebra I, Geometry, Algebra II)

CONVERSIONS

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilograms
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

FORMULAS

Triangle	$A = \frac{1}{2}bh$	Pythagorean Theorem	$a^2 + b^2 = c^2$
Parallelogram	$A = bh$	Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Circle	$A = \pi r^2$	Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Circle	$C = \pi d$ or $C = 2\pi r$	Geometric Sequence	$a_n = a_1 r^{n-1}$
General Prisms	$V = Bh$	Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Cylinder	$V = \pi r^2 h$	Radians	1 radian = $\frac{180}{\pi}$ degrees
Sphere	$V = \frac{4}{3}\pi r^3$	Degrees	1 degree = $\frac{\pi}{180}$ radians
Cone	$V = \frac{1}{3}\pi r^2 h$	Exponential Growth/Decay	$A = A_0 e^{k(t-t_0)} + B_0$
Pyramid	$V = \frac{1}{3}Bh$		