Name:

Common Core Algebra II Regents Review Packet!

Mr. Schlansky

2026

Multiple Choice Strategy with Variables
If variables in the problems and answers:
10 STO $\rightarrow$ X, 15 STO $\rightarrow$ Y
Type in original problem, write down the value.
Type in each choice, write down the value.
If they match up, they are equivalent.
Check all four choices as more than one may be equivalent!

1. The expression $\frac{6x^3 + 17x^2 + 10x + 2}{2x + 3}$ equals

1) $3x^2 + 4x - 1 + \frac{5}{2x+3}$
2) $6x^2 + 8x - 2 + \frac{5}{2x+3}$
3) $6x^2 - x + 13 - \frac{37}{2x+3}$
4) $3x^2 + 13x + \frac{49}{2} + \frac{151}{2x+3}$

2. The expression $\frac{4x^3 + 5x + 10}{2x + 3}$ is equivalent to

1) $2x^2 + 3x - 7 + \frac{31}{2x+3}$
2) $2x^2 - 3x + 7 - \frac{11}{2x+3}$
3) $2x^2 + 2.5x + 5 + \frac{15}{2x+3}$
4) $2x^2 - 2.5x - 5 - \frac{20}{2x+3}$

3. What is the completely factored form of $k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48$?

1) $(k-2)(k-2)(k+3)(k+4)$
2) $(k-2)(k-2)(k+6)(k+2)$
3) $(k+2)(k-2)(k+3)(k+4)$
4) $(k+2)(k-2)(k+6)(k+2)$

4. When factored completely, the expression $3x^3 - 5x^2 - 48x + 80$ is equivalent to

1) $(x^2 - 16)(3x - 5)$
2) $(x^2 + 16)(3x - 5)(3x + 5)$
3) $(x + 4)(x - 4)(3x - 5)$
4) $(x + 4)(x - 4)(3x - 5)(3x - 5)$

5. Given i is the imaginary unit, $(2 - yi)^2$ in simplest form is

1) $y^2 - 4yi + 4$
2) $-y^2 - 4yi + 4$
3) $-y^2 + 4$
4) $y^2 + 4$

6. The expression $(x + i)^2 - (x - i)^2$ is equivalent to

1) 0
2) $-2 + 4xi$
3) -2
4) $4xi$

7. The expression $6xi^3(-4xi + 5)$ is equivalent to

1) $2x - 5i$
2) $-24x^2 - 30xi$
3) $-24x^2 + 30x - i$
4) $26x - 24x^2i - 5i$

8. The expression $\frac{a^2b^{-3}}{a^{-4}b^2}$ is equivalent to

1) $\frac{a^6}{b^5}$
2) $\frac{b^5}{a^6}$
3) $\frac{a^2}{b}$
4) $a^{-2}b^{-1}$

9. Which expression is equivalent to $\frac{x^{-1}y^2}{x^2y^{-4}}$?

1) $\frac{x}{y^2}$
2) $\frac{x^3}{y^6}$
3) $\frac{y^2}{x}$
4) $\frac{y^6}{x^3}$

10. What is the product of $\sqrt[3]{4a^2b^4}$ and $\sqrt[3]{16a^3b^2}$?

1) $4ab^2\sqrt[3]{a^2}$
2) $4a^2b^3\sqrt[3]{a}$
3) $8ab^2\sqrt[3]{a^2}$
4) $8a^2b^3\sqrt[3]{a}$

11. The expression $\sqrt[4]{16x^2y^7}$ is equivalent to

1) $2x^{\frac{1}{2}}y^{\frac{7}{4}}$
2) $2x^8y^{28}$
3) $4x^{\frac{1}{2}}y^{\frac{7}{4}}$
4) $4x^8y^{28}$

12. For positive values of x, which expression is equivalent to $\sqrt{16x^2} \bullet x^{\frac{2}{3}} + \sqrt[3]{8x^5}$

1) $6\sqrt[3]{x^5}$
2) $6\sqrt[5]{x^3}$
3) $4\sqrt[3]{x^2} + 2\sqrt[3]{x^5}$
4) $4\sqrt{x^3} + 2\sqrt[5]{x^3}$

13. Written in simplest form, $\frac{c^2 - d^2}{d^2 + cd - 2c^2}$ where $c \neq d$, is equivalent to

1) $\frac{c+d}{d+2c}$
2) $\frac{c-d}{d+2c}$
3) $\frac{-c-d}{d+2c}$
4) $\frac{-c+d}{d+2c}$

14. The expression $\frac{-3x^2 - 5x + 2}{x^3 + 2x^2}$ can be rewritten as

1) $\frac{-3x-3}{x^2+2x}$
2) $\frac{-3x-1}{x^2}$
3) $-3x^{-1} + 1$
4) $-3x^{-1} + x^{-2}$

Multiple Choice Strategy with Equations

-Store each potential answer (_____ STO $\rightarrow$ X)

-Type in left hand side, type in right hand side. If they match up, it is a solution.

*Be sure to check all potential answers as most equations have multiple answers

1. The solution set of the equation $\sqrt{x+3} = 3 - x$ is

1) $\{1\}$
2) $\{0\}$
3) $\{1, 6\}$
4) $\{2, 3\}$

2. What is the solution set for the equation $\sqrt{5x+29} = x+3$?

1) $\{4\}$
2) $\{-5\}$
3) $\{4, 5\}$
4) $\{-5, 4\}$

3. The solution set of $\sqrt{3x+16} = x+2$ is

1) $\{-3, 4\}$
2) $\{-4, 3\}$
3) $\{3\}$
4) $\{-4\}$

4. The solution set of the equation $\sqrt{2x-4} = x-2$ is

1) $\{-2, -4\}$
2) $\{2, 4\}$
3) $\{4\}$
4) $\{\ \}$

5. What is the solution set of the equation $\frac{30}{x^2-9} + 1 = \frac{5}{x-3}$?

1) $\{2, 3\}$
2) $\{2\}$
3) $\{3\}$
4) $\{\ \}$

6. What is the solution set of the equation $\frac{3x+25}{x+7} - 5 = \frac{3}{x}$?

1) $\left\{\frac{3}{2}, 7\right\}$
2) $\left\{\frac{7}{2}, -3\right\}$
3) $\left\{-\frac{3}{2}, 7\right\}$
4) $\left\{-\frac{7}{2}, -3\right\}$

7. The solution set for the equation $\sqrt{56-x} = x$ is

1) $\{-8, 7\}$
2) $\{-7, 8\}$
3) $\{7\}$
4) $\{\ \}$

8. Which is the solution to: $2(3)^{4x} + 1 = 11$?

1) $\frac{\log 5}{4\log 3}$
2) $\frac{4\log 5}{\log 3}$
3) $\frac{\log 3}{4\log 5}$
4) $\frac{4\log 3}{\log 5}$

9. Which is the solution to: $256 + 4(2)^{6x} = 2700$?

1) $\frac{\ln 4}{6\ln 2}$
2) $\frac{6\ln 423}{\ln 4}$
3) $\frac{\ln 611}{6\ln 2}$
4) $\frac{6\ln 2444}{\ln 4}$

10. Which is the solution to: $1 - 2(5)^{2x} = -5$?

1) $\frac{\ln 6}{2\ln 3}$
2) $\frac{2\ln 5}{\ln 1}$
3) $\frac{2\ln 4}{\ln 3}$
4) $\frac{\ln 3}{2\ln 5}$

Open Response Equations

1) Type in left hand side into Y1
2) Type in right hand side into Y2
3) Adjust window (if necessary)
4) 2^{nd} Trace (Calc), 5: Intersect
5) The solution is the x value of the intersection

*You may want to divide both sides at the beginning to make the values smaller

1. Solve for all values of x: $\sqrt{x-5}+x=7$

2. What is the solution set for the equation $\sqrt{56-x}=x$?

3. What is the solution set for the equation $\sqrt{5x+29}=x+3$?

4. Solve algebraically for x: $\sqrt{x^2+x-1}+11x=7x+3$

5. What is the solution set of the equation $\frac{30}{x^2-9}+1=\frac{5}{x-3}$?

6. What is the solution set of the equation $\frac{3x+25}{x+7}-5=\frac{3}{x}$?

7. What is the solution, if any, of the equation $\frac{2}{x+3}-\frac{3}{4-x}=\frac{2x-2}{x^2-x-12}$?

8. Solve for x: $\frac{1}{x}-\frac{1}{3}=-\frac{1}{3x}$

9. Solve the equation $2x^3 - x^2 - 8x = 4$ for all values of x.

10. Solve for x: $x^3 + x^2 = 4x + 4$

11. Solve for x: $x^3 - 2x^2 = x - 2$

12. Solve for x and round your answer to the nearest thousandth: $\frac{1}{2}(1.8)^x = 7.5$

13. Solve for x and round your answer to the nearest thousandth: $2\left(\frac{1}{3}\right)^x = 4$

14. Solve for x and round your answer to the nearest thousandth: $1 - 2(3)^{2x} = -5$

15. Solve $x^3 + 5x^2 = 4x + 20$.

16. Solve for all values of x: $x^4 + 4x^3 + 4x^2 = -16x$

17. Solve for x and round your answer to the nearest hundredth: $4^x - 5 = 12$

18. Solve for x and round your answer to the nearest hundredth: $8 + 2(4)^{x-5} = 14$

Profit
Profit = revenue – cost, $p(x) = r(x) - c(x)$
Net worth = value of accounts - debt
*Keep, change, change when subtracting polynomials

1. Mr. Schlansky's tutoring revenue can be represented by $r(x) = 25x^2 - 90x + 14$ and his costs can be represented by $c(x) = 12x^2 + 21x + 10$. If his profit can be determined using $p(x) = r(x) - c(x)$, write a polynomial function what would represent $p(x)$.

2. Stone Manufacturing has developed a cost model, $C(x) = 0.18x^3 + 0.02x^2 + 4x + 180$, where x is the number of sprockets sold, in thousands. The sales price can be modeled by $S(x) = 95.4 - 6x$ and the company's revenue by $R(x) = x \bullet S(x)$. The company's profits, $R(x) - C(x)$, could be modeled by

1) $0.18x^3 + 6.02x^2 + 91.4x + 180$
2) $0.18x^3 - 5.98x^2 - 91.4x + 180$
3) $-0.18x^3 - 6.02x^2 + 91.4x - 180$
4) $0.18x^3 + 5.98x^2 + 99.4x + 180$

3. Chet has $1200 invested in a bank account modeled by the function $P(n) = 1200(1.002)^n$, where $P(n)$ is the value of his account, in dollars, after n months. Chet's debt is modeled by the function $Q(n) = 100n$, where $Q(n)$ is the value of debt, in dollars, after n months. After n months, which function represents Chet's net worth, $R(n)$?

1) $R(n) = 1200(1.002)^n + 100n$
2) $R(n) = 1200(1.002)^{12n} + 100n$
3) $R(n) = 1200(1.002)^n - 100n$
4) $R(n) = 1200(1.002)^{12n} - 100n$

4. A manufacturing company has developed a cost model, $C(x) = 0.15x^3 + 0.01x^2 + 2x + 120$, where x is the number of items sold, in thousands. The sales price can be modeled by $S(x) = 30 - 0.01x$. Therefore, revenue is modeled by $R(x) = x \bullet S(x)$. The company's profit, $P(x) = R(x) - C(x)$, could be modeled by

1) $0.15x^3 + 0.02x^2 - 28x + 120$
2) $-0.15x^3 - 0.02x^2 + 28x - 120$
3) $-0.15x^3 + 0.01x^2 - 2.01x - 120$
4) $-0.15x^3 + 32x + 120$

5. A major car company analyzes its revenue, $R(x)$, and costs $C(x)$, in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years, x, using the given functions.

$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$
$$C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$$

The company's profits can be represented as the difference between its revenue and costs. Write the profit function, $P(x)$, as a polynomial in standard form.

6. The profit function, $p(x)$, for a company is the cost function, $c(x)$, subtracted from the revenue function, $r(x)$. The profit function for the Acme Corporation is $p(x) = -0.5x^2 + 250x - 300$ and the revenue function is $r(x) = -0.3x^2 + 150x$. The cost function for the Acme Corporation is

1) $c(x) = 0.2x^2 - 100x + 300$
2) $c(x) = 0.2x^2 + 100x + 300$
3) $c(x) = -0.2x^2 + 100x - 300$
4) $c(x) = -0.8x^2 + 400x - 300$

Dividing Polynomials: (Synthetic Division)
Negative the value of what you are dividing by and put it outside
Bring the first number down
Multiply, Add, Multiply, Add, etc.
Decrease the first terms exponent by 1, the last number is the remainder. The remainder goes over the divisor.
(Put 0 as a placeholder if necessary)

Divide each of the following polynomials

1. $\dfrac{2x^3 + 5x^2 - 31x - 84}{x + 3}$

2. $\dfrac{x^4 - 2x^2 - 7x + 12}{x + 6}$

3. $\dfrac{x^3 + 5x^2 - 1}{x + 2}$

4. $\dfrac{4x^3 + 12x^2 - 5}{x + 5}$

5. $\dfrac{6x^3 - 5x + 3}{x - 3}$

6. $\dfrac{5x^3 - 60}{x - 2}$

7. $\dfrac{x^2 + x - 4}{x - 3}$

8. $\dfrac{-3x^2 + 10x - 6}{x + 1}$

Long Division
Divide first term by the first term
Multiply (Distribute top to outside)
Subtract (Distribute the negative)
Bring Down

1. $\dfrac{6x^3+19x^2+11x-6}{3x-1}$

2. $\dfrac{15x^3+29x^2-23x-21}{5x+3}$

3. $\dfrac{2x^3-3x^2+2x+5}{x-5}$

4. $\dfrac{9x^2-2}{3x+1}$

5. $\frac{3x^3+x^2+2x+5}{x^2+2x+1}$

6. $\frac{20x^3-9x^2-56x+52}{5x^2+4x-9}$

7. $\frac{x^4-2x^3+3x^2-4x+5}{x^2-2}$

8. $\frac{x^4+2x^3-4x^2+x+2}{x^2-x-2}$

Operations with Polynomials
Substitute the expression in for *f(x)* or *g(x)*.
Perform the operations with polynomials (expect to use box method).
Combine like terms and express in simplest terms.

Express each of the following in polynomial standard form

1. Given $f(x) = 3x^2 - 5x + 1$ and $g(x) = x + 1$.
Express $[g(x)]^2 - 2f(x)$

2. Given $p(x) = x^2 + 2x - 3$ and $g(x) = 2x - 3$.
Express $[p(x)][g(x)] - 3p(x)$

3. Given $f(x) = 2x^2 + 4x - 2$ and $g(x) = x - 2$.
Express $f(x) - [g(x)]^3$

4. For $c(x) = 3x^2 - 4x + 7$ and $d(x) = x - 2$, determine $c(x) \bullet d(x) - [d(x)]^3$ as a polynomial in standard form.

To determine if a binomial is a factor:
Find the remainder! (Use remainder theorem)
If the remainder is 0, it is a factor
If the remainder is not 0, it is not a factor

1. Is $x-6$ a factor of $p(x)=x^3-6x^2+4x-1$? Explain your answer.

2. Is $x+2$ a factor of $p(x)=x^3-3x^2-8x+4$? Explain your answer.

3. Is $2x+1$ a factor of $p(x)=2x^2+5x+2$? Explain your answer.

4. Is $3x-2$ a factor of $p(x)=3x^3-2x^2-27x+18$? Explain your answer.

5. Determine if $x-5$ is a factor of $2x^3-4x^2-7x-10$. Explain your answer.

6. Which binomial is a factor of x^4-4x^2-4x+8?

1) $x-2$	3) $x-4$
2) $x+2$	4) $x+4$

7. Which binomial is *not* a factor of the expression $x^3-11x^2+16x+84$?

1) $x+2$	3) $x-6$
2) $x+4$	4) $x-7$

8. Which binomial is *not* a factor of the expression $x^3-6x^2-49x-66$?

1) $x-11$
2) $x+2$
3) $x+6$
4) $x+3$

9. Which binomial is a factor of the expression x^3-7x-6?

1) $x+3$
2) $x-1$
3) $x-2$
4) $x+2$

10. Which binomial is *not* a factor of the expression $x^3-4x^2-25x+28$?

1) $x+6$
2) $x-7$
3) $x-1$
4) $x+4$

11. Which binomial is not a factor of $p(x)=2x^3+7x^2-5x-4$?

1) $x+4$
2) $x+1$
3) $x-1$
4) $2x+1$

12. Which binomial is not a factor of $p(x)=2x^3-5x^2+6x-2$?

1) $x-1$
2) $x-2$
3) $2x-1$
4) $2x+1$

13. Given $P(x)=x^3-3x^2-2x+4$, which statement is true?

1) $(x-1)$ is a factor because $P(-1)=2$.
2) $(x+1)$ is a factor because $P(-1)=2$.
3) $(x+1)$ is a factor because $P(1)=0$.
4) $(x-1)$ is a factor because $P(1)=0$.

14. If $f(x)=2x^4-x^3-16x+8$, then $f\left(\frac{1}{2}\right)$

1) equals 0 and $2x+1$ is a factor of $f(x)$
2) equals 0 and $2x-1$ is a factor of $f(x)$
3) does not equal 0 and $2x+1$ is not a factor of $f(x)$
4) does not equal 0 and $2x-1$ is a factor of $f(x)$

15. Consider the function $f(x)=2x^3+x^2-18x-9$. Which statement is true?

1) $2x-1$ is a factor of $f(x)$.
2) $x-3$ is a factor of $f(x)$.
3) $f(3)\neq f\left(-\frac{1}{2}\right)$
4) $f\left(\frac{1}{2}\right)=0$

Finding k in a Polynomial Equation
If $x+a$ is a factor then a is a zero. Replace $p(x)$ with 0 and x with a.

1. Consider the polynomial $p(x)=x^3+kx^2+x+6$. Find a value of k so that $x+1$ is a factor of P.

2. Consider the polynomial $p(x)=x^3+kx-30$. Find a value of k so that $x+3$ is a factor of P.

3. If $x-1$ is a factor of x^3-kx^2+2x, what is the value of k?

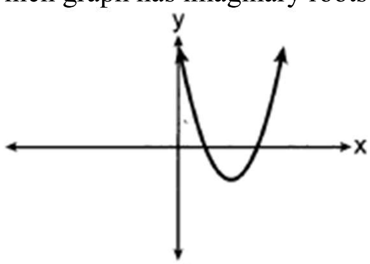
4. The polynomial function $g(x)=x^3+ax^2-5x+6$ has a factor of $(x-3)$. Determine the value of a.

Imaginary Solutions

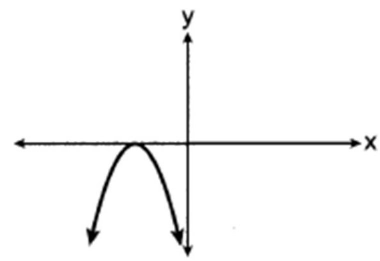
Imaginary solutions do not touch the x-axis

1. Which graph has imaginary roots?

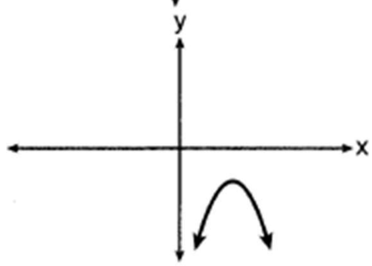
1)



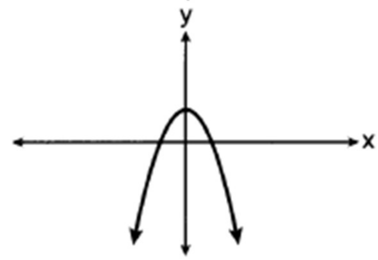
3)



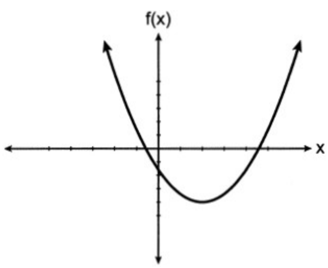
2)



4)



2. If $f(x)$ is represented by the graph below, Does $f(x)$ have imaginary roots? Explain your answer.



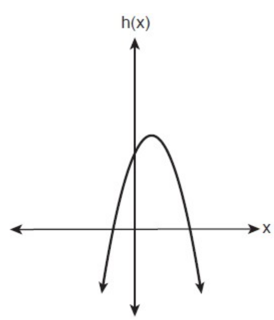
3. Which quadratic functions have imaginary roots?

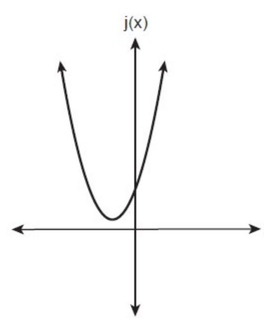
1) $h(x)$ only

2) $j(x)$ only

3) Both $j(x)$ and $h(x)$

4) Neither $j(x)$ or $h(x)$

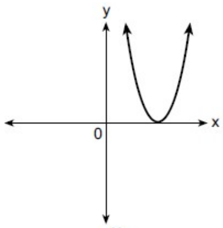




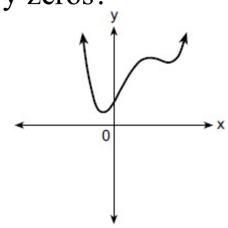
4. Does the equation $x^2 - 4x + 13 = 0$ have imaginary solutions? Justify your answer.

5. Which of the following graphs have imaginary zeros?

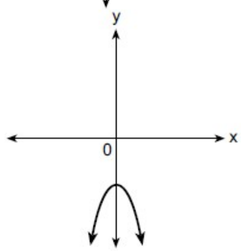
I



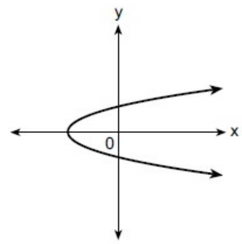
II



III



IV



1) I and IV
2) II and III
3) II only
4) III and IV

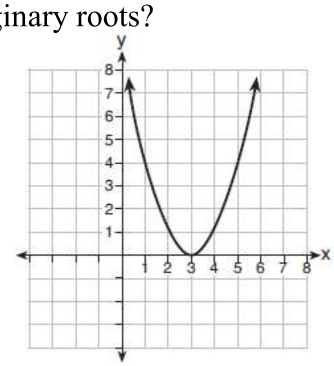
6. Which representation of a quadratic has imaginary roots?

1)

x	y
−2.5	2
−2.0	0
−1.5	−1
−1.0	−1
−0.5	0
0.0	2

2) $2(x+3)^2 = 64$

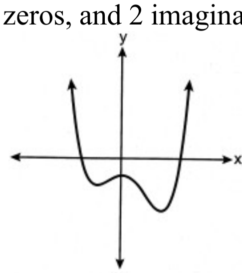
3)



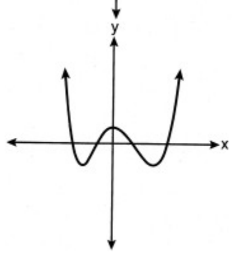
4) $2x^2 + 32 = 0$

7. Which graph could represent a 4th degree polynomial function with a positive leading coefficient, 2 real zeros, and 2 imaginary zeros?

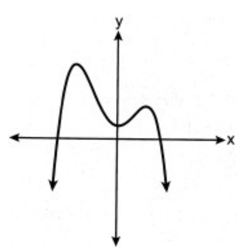
1)



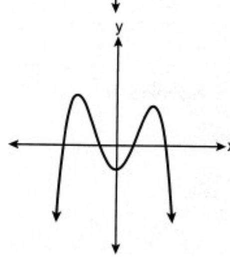
2)



3)



4)



Writing Equations of Polynomial Functions

List the factors (change the sign of the zeros. Factors have an x).

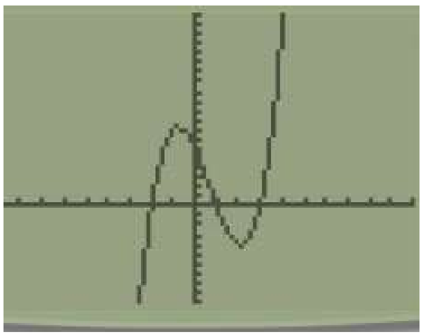
*Don't forget y =

*Check if positive (opens up) or negative (opens down)

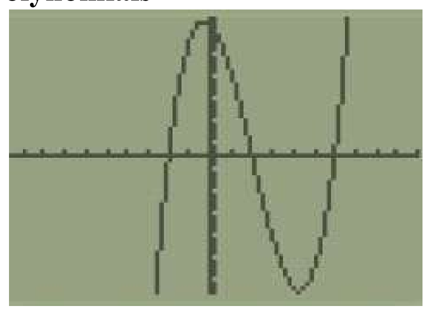
*Check for double roots (bounces off the x-axis)

Write a possible equation for each of the following polynomials

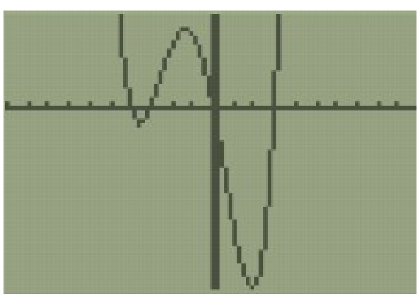
1.



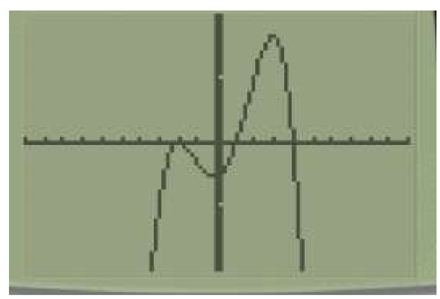
2.



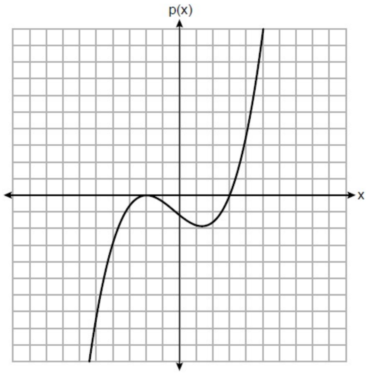
3.



4.

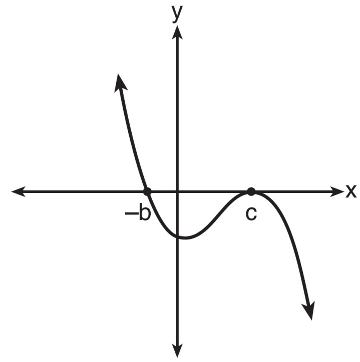


5.



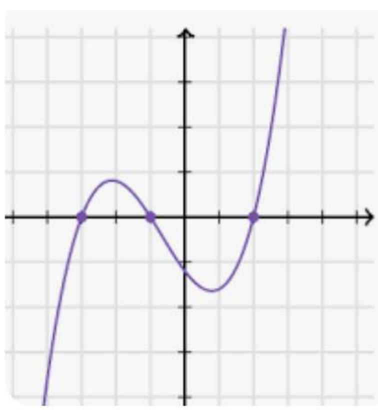
1) $p(x)=(x-2)^2(x+3)$
2) $p(x)=(x-2)(x+3)^2$
3) $p(x)=(x+2)^2(x-3)$
4) $p(x)=(x+2)(x-3)^2$

6.



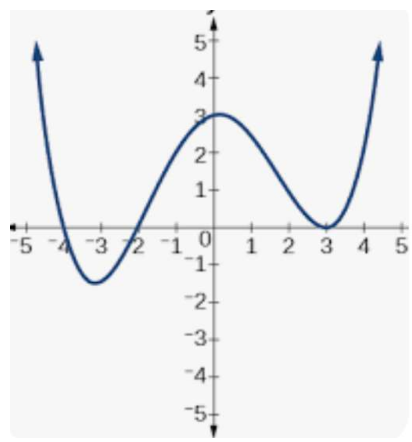
1) $y=-(x+b)^2(x-c)$
2) $y=-(x+b)(x-c)^2$
3) $y=-(x-b)^2(x+c)$
4) $y=-(x-b)(x+c)^2$

7.



1) $p(x)=(x-3)(x-1)(x+2)$
2) $p(x)=-(x-3)(x-1)(x+2)$
3) $p(x)=(x+3)(x+1)(x-2)$
4) $p(x)=-(x+3)(x+1)(x-2)$

8.



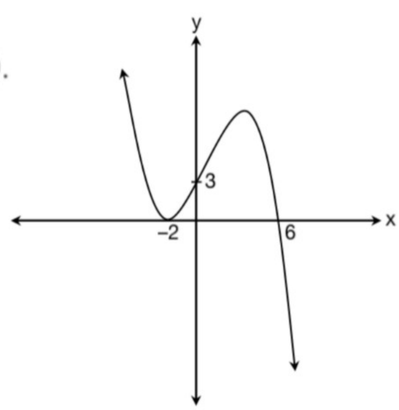
1) $y=(x-4)(x-2)(x+3)^2$
2) $y=(x+4)(x+2)(x-3)^2$
3) $y=(x+4)^2(x+2)^2(x-3)$
4) $y=(x-4)^2(x-2)^2(x+3)$

9.

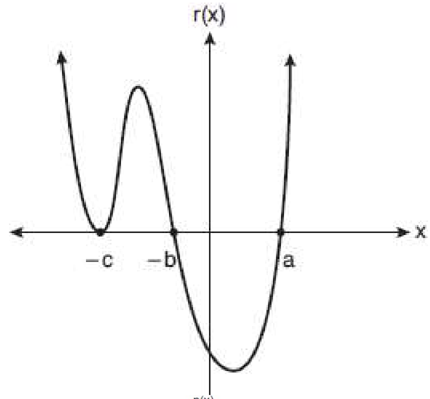
The graph below shows the polynomial $y=p(x)$.

The factors of $p(x)$ are

(1) $(x+2)$, $(x-3)$, and $(x+6)$
(2) $(x-2)$, $(x+3)$, and $(x+6)$
(3) $(x-2)$, $(x-2)$, and $(x+6)$
(4) $(x+2)$, $(x+2)$, and $(x-6)$



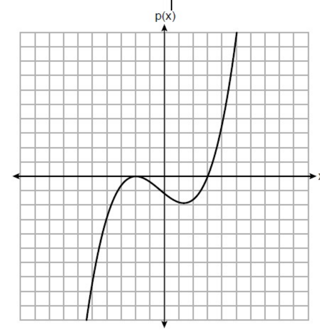
10. A sketch of $r(x)$ is shown below.



An equation for $r(x)$ could be

1) $r(x) = (x-a)(x+b)(x+c)$
2) $r(x) = (x+a)(x-b)(x-c)^2$
3) $r(x) = (x+a)(x-b)(x-c)$
4) $r(x) = (x-a)(x+b)(x+c)^2$

11. The graph of a cubic polynomial function $p(x)$ is shown below.

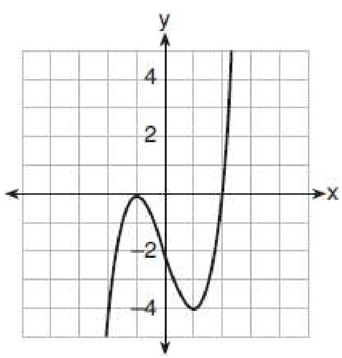


If $p(x)$ is written as a product of linear factors, which factor would appear twice?

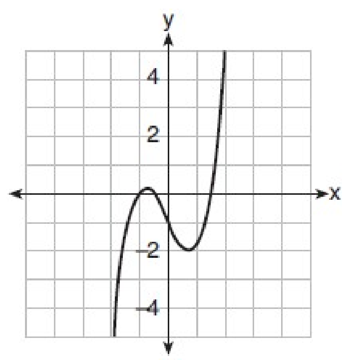
1) $x-2$
2) $x+2$
3) $x-3$
4) $x+3$

12. Which graph represents a polynomial function that contains $x^2 + 2x + 1$ as a factor?

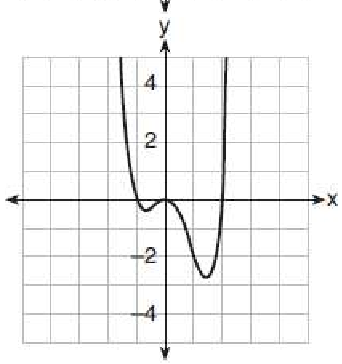
1)



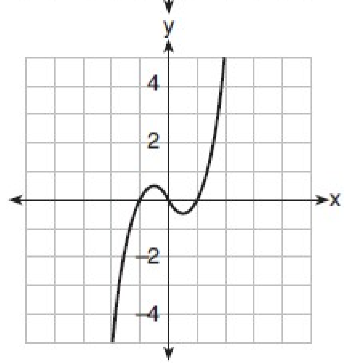
3)



2)

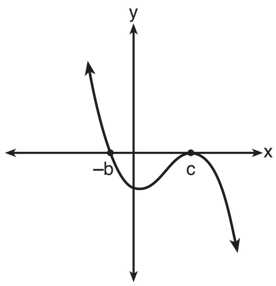


4)

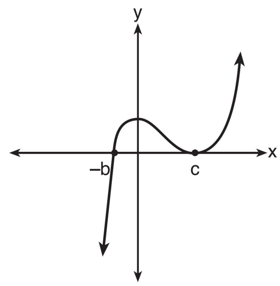


13. If a, b, and c are all positive real numbers, which graph could represent the sketch of the graph of $p(x) = -a(x+b)\left(x^2 - 2cx + c^2\right)$?

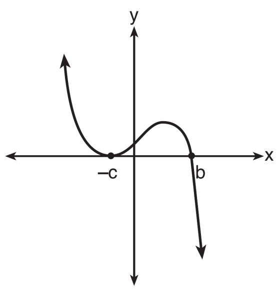
1)



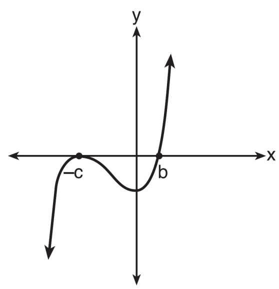
3)



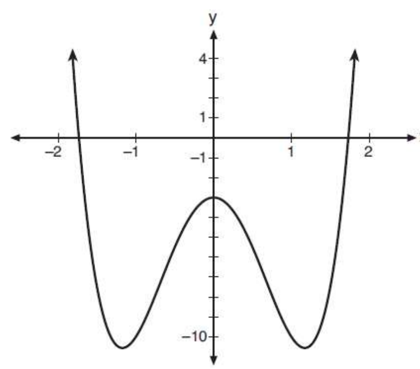
2)



4)



14. Consider the function $p(x) = 3x^3 + x^2 - 5x$ and the graph of $y = m(x)$ below.

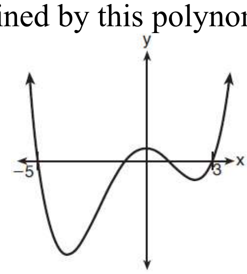


Which statement is true?

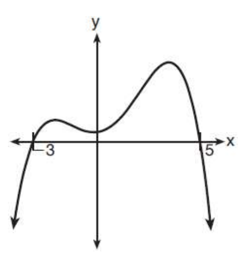
1) $p(x)$ has three real roots and $m(x)$ has two real roots.
2) $p(x)$ has one real root and $m(x)$ has two real roots.
3) $p(x)$ has two real roots and $m(x)$ has three real roots.
4) $p(x)$ has three real roots and $m(x)$ has four real roots.

15. A 4th degree polynomial has zeros -5, 3, i, and $-i$. Which graph could represent the function defined by this polynomial?

1)



3)



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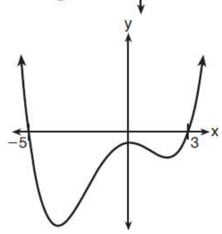
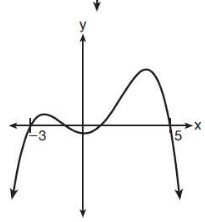
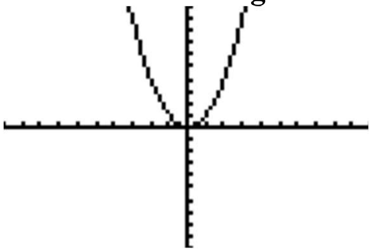


4)



End Behavior

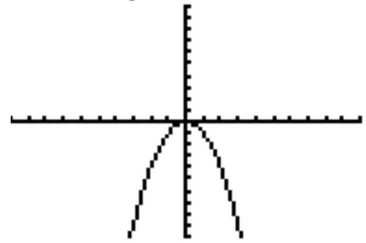
Positive leading coefficient, Even Degree



$x \to -\infty, f(x) \to \infty$

$x \to \infty, f(x) \to \infty$

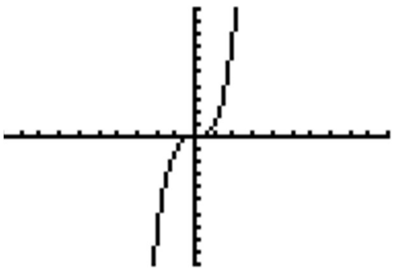
Negative leading coefficient, Even Degree



$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to -\infty$

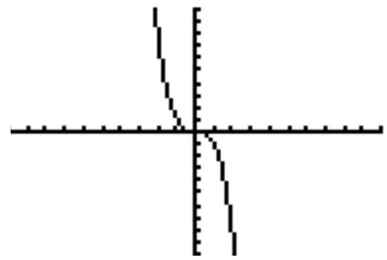
Positive leading coefficient, Odd Degree



$x \to -\infty, f(x) \to -\infty$

$x \to \infty, f(x) \to \infty$

Negative leading coefficient, Odd Degree



$x \to -\infty, f(x) \to \infty$

$x \to \infty, f(x) \to -\infty$

Sketch the shape and fill in the end behavior for each of the following polynomial equations

1. $f(x) = x^3 + 2x^2 - 9x - 18$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

2. $f(x) = x^4 - 10x^2 + 9$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

3. $p(x) = -x^3 - 3x^2 + 4x + 12$

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

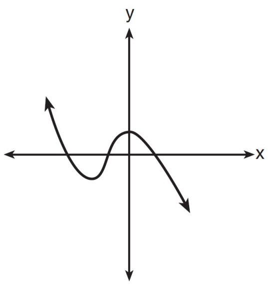
4. $f(x) = -x^4 + 3x^3 + 10x^2$

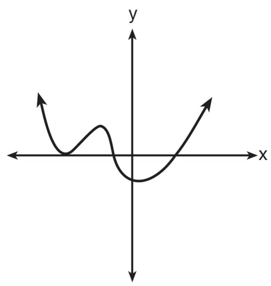
$x \to -\infty,\ f(x) \to$

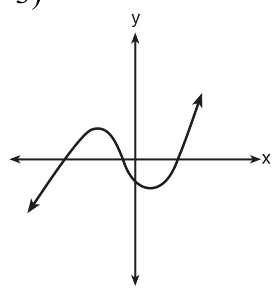
$x \to \infty,\ f(x) \to$

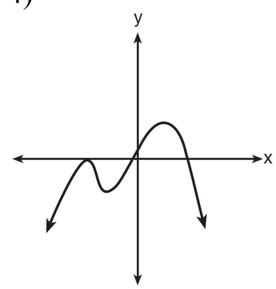
5. Which graph has the following characteristics?

- as $x \to -\infty,\ f(x) \to -\infty$
- as $x \to \infty,\ f(x) \to \infty$

1)


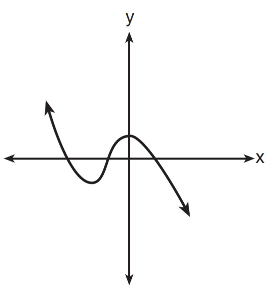
2)


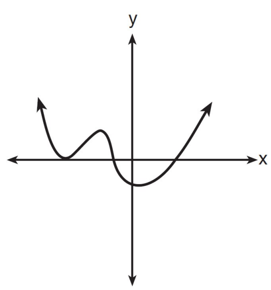
3)


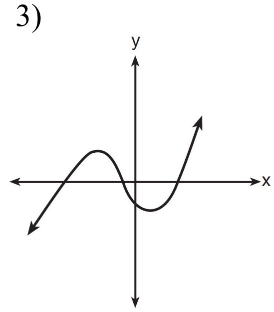
4)


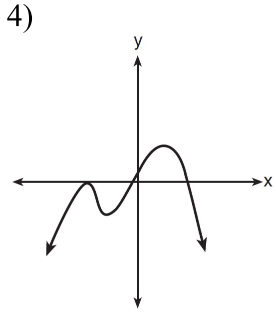
6. Which graph has the following characteristics?

- $x \to -\infty, f(x) \to \infty$
- $x \to \infty, f(x) \to \infty$

1)


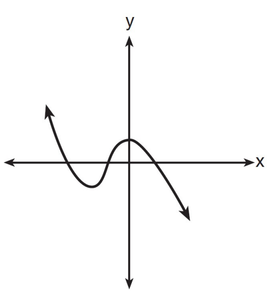
2)


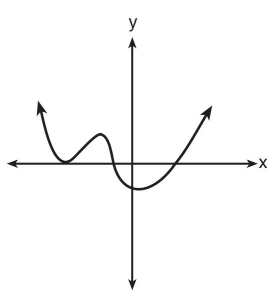
3)


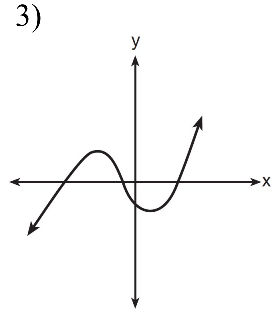
4)


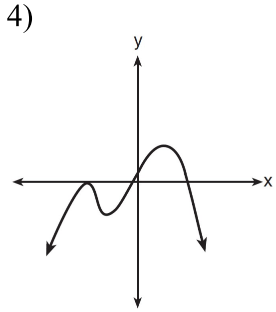
7. Which graph has the following characteristics?

- $x \to -\infty, f(x) \to -\infty$
- $x \to \infty, f(x) \to -\infty$

1)


2)


3)


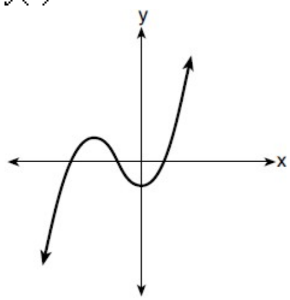
4)


8. Consider the end behavior description below.

- as $x \to -\infty, f(x) \to \infty$
- as $x \to \infty, f(x) \to -\infty$

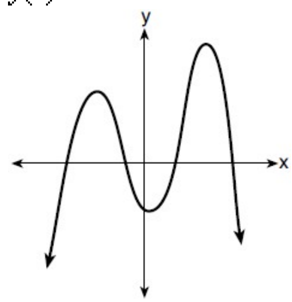
1) $f(x) = x^4 + 2x^2 + 1$

2)



3) $f(x) = -x^3 + 2x - 6$

4)

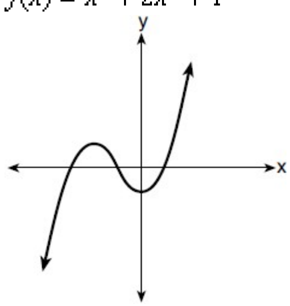


9. Consider the end behavior description below.

- as $x \to -\infty, f(x) \to \infty$
- as $x \to \infty, f(x) \to \infty$

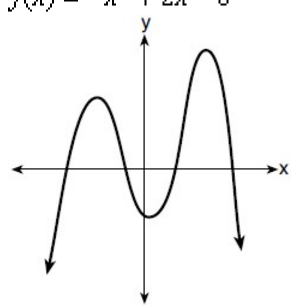
1) $f(x) = x^4 + 2x^2 + 1$

2)



3) $f(x) = -x^3 + 2x - 6$

4)



10. Consider the end behavior description below.

- as $x \to -\infty, f(x) \to \infty$
- as $x \to \infty, f(x) \to -\infty$

Which function satisfies the given conditions?

1) $f(x) = -x^4 + 3x^3 + 2x^2 - 1$

2) $f(x) = 2x^3 - 7x + 5$

3) $f(x) = -7x^5 + 5x^4 + 8x^2 - 6$

4) $f(x) = -8x^7 + 5x^5 - 11x^2 + 2x - 7$

Complex Numbers

Treat *i* like a normal variable except know that $i^2 = -1$ and $i^3 = -i$

a + b*i* form simply means there will be an *i* in the answer

Express the following in simplest $a + bi$ form

1. $(3k - 2i)^2$

2. $(4x - 3yi)^2$

3. $3xi(3 - 2i)$

4. $5i + 4i(2 + 3i)$

5. $2xi(i - 4i^2)$

6. $6xi^3(-4xi + 5)$

7. $2i(\sqrt{-4} - 4)$

8. $-\frac{1}{2}i^3\left(\sqrt{-9} - 4\right) - 3i^2$

Solving Systems of Equations Graphically Using TI-84+ ($f(x) = g(x)$)

1) Type equations into Y_1 and Y_2
2) Zoom 6 (Standard) is your standard window. Adjust window OR try Zoom 0(Fit) if you don't see what you want to see.
3) 2nd Trace (Calc), 5 (Intersect)
4) Place cursor over point of intersection, hit enter, enter, enter. Repeat the process for any other points of intersection.

***The solutions to the system of equations are the x values of the intersections.**

1. To the *nearest tenth*, the value of x that satisfies $2^x = -2x + 11$ is

1) 2.5	3) 5.8
2) 2.6	4) 5.9

2. For which values of x, rounded to the *nearest hundredth*, will $\left|x^2 - 9\right| - 3 = \log_3 x$?

1) 2.29 and 3.63	3) 2.84 and 3.17
2) 2.37 and 3.54	4) 2.92 and 3.06

3. For which approximate value(s) of x will $\log(x + 5) = |x - 1| - 3$?

1) $5, 1$	3) $-2.41, 5$
2) $-2.41, 0.41$	4) 5, only

4. Which value, to the *nearest tenth*, is *not* a solution of $p(x) = q(x)$ if $p(x) = x^3 + 3x^2 - 3x - 1$ and $q(x) = 3x + 8$?

1) -3.9	3) 2.1
2) -1.1	4) 4.7

5. If $f(x) = 3|x| - 1$ and $g(x) = 0.03x^3 - x + 1$, an approximate solution for the equation $f(x) = g(x)$ is

1) 1.96	3) $(-0.99, 1.96)$
2) 11.29	4) $(11.29, 32.87)$

6. If $p(x) = 2\ln(x) - 1$ and $m(x) = \ln(x + 6)$, then what is the solution for $p(x) = m(x)$?

1) 1.65	3) 5.62
2) 3.14	4) no solution

7. If $f(x) = \frac{1}{2}x^3 + 3x^2 - 4x$ and $g(x) = 5\log_3(x + 10)$, then which value, rounded to the *nearest tenth*, is *not* a solution to $f(x) = g(x)$?

1) -6.9	3) 2.2
2) -1.4	4) 9.8

8. Given: $h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2$

$k(x) = -|0.7x| + 5$

State the solutions to the equation $h(x) = k(x)$, rounded to the *nearest hundredth*.

9. If $f(t) = 325e^{-.0735t} + 75$ and $g(t) = 375e^{-.0817t} + 75$, for what value of t does $f(t) = g(t)$ rounded to the *nearest tenth*?

10. A technology company is comparing two plans for speeding up its technical support time. Plan *A* can be modeled by the function $A(x) = 15.7(0.98)^x$ and plan *B* can be modeled by the function $B(x) = 11(0.99)^x$ where x is the number of customer service representatives employed by the company and $A(x)$ and $B(x)$ represent the average wait time, in minutes, of each customer. To the *nearest integer*, solve the equation $A(x) = B(x)$.

11. Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is $P(x) = \log(x - 4)$, where x is the number of visits per week in thousands and $P(x)$ is the website's popularity rating. An alternative rating model is represented by $R(x) = \frac{1}{2}x - 6$, where x is the number of visits per week in thousands. For what number of weekly visits will the two models provide the same rating?

12. The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time. State when $V(t) = Z(t)$, to the *nearest hundredth*,

Systems of Inequalities with TI

Find the intersection points (x only!)

List the interval where the first function is above/below the other

1. Given $f(x) = x^2$ and $g(x) = -\frac{1}{2}x + 5$, over what interval is $f(x) < g(x)$?

2. Given $f(x) = -|x|$ and $g(x) = -\sqrt{x+4}$, over what interval is $f(x) \geq g(x)$?

3. Given $m(x) = \log(x)$ and $n(x) = (x-5)^2$, over what interval is $m(x) \geq n(x)$?

4. Given $a(x) = e^x - 9$ and $b(x) = -|x-3| - 2$, over what interval is $a(x) < b(x)$?

5. If $f(x) = \frac{1}{2}x^3 + 3x^2 - 4x$ and $g(x) = 5\log_3(x+10)$, then which value, rounded to the *nearest tenth*, is a solution to $f(x) > g(x)$?

1) -7.0
2) -6.8
3) -1.1
4) 2.1

6. For which value of *x* will $log(x+5) \geq |x-1| - 3$?

1) -6
2) -4
3) 4
4) 6

7. For which value of *x* will $\sqrt[3]{x-1} > -\frac{1}{2}|x| + 3$?

1) -3.1
2) 1.1
3) 2.7
4) 3.9

8. The function $r(x) = \frac{1}{12}x$ represents the revenue from Carla's business and $c(x) = 2\log(x)$ represents her cost for selling *x* units of merchandise. To the *nearest tenth*, over what interval will *c(x)* > *r(x)*? Explain the meaning of this interval in the context of the problem.

9. The height of a ball thrown in the air can be modeled by $b(t) = -16t^2 + 32t$ and the height of an eagle can be modeled by $e(t) = -\frac{1}{2}t + 14$ after *t* seconds. To the *nearest hundredth*, over what interval is *e(t)*<*b(t)*? Explain the meaning of this interval in the context of the problem.

10. The height of object A can be represented by $A(x) = 2\sqrt[3]{x} + 15$ and the height of object B can be represented by $B(x) = 20(0.8)^x$ after *x* seconds. Over what interval is $A(x) > B(x)$? Explain its meaning in the context of the problem.

11. The value of stock A can be modeled by $A(t) = 2\sqrt{t+10} + 1$ and the value of stock B can be represented by $B(t) = t^3 - 3t^2 - 3t + 10$, where *t* represents time in days. Over what positive interval, rounded to the *nearest tenth*, is *A(x)*>*B(x)*? Explain the meaning of this interval in the context of the problem.

Finding Key Points of Polynomial Functions Using TI
-Type equation into Y=
-2nd Trace (Calc)

1. Given the function $f(x) = x^3 + 3x^2 - x - 2$, find the zeros and relative extrema to the *nearest tenth*.

2. Given the function $f(x) = -x^3 - 2x^2 + 2x + 3$, find the zeros and relative extrema to the *nearest tenth*.

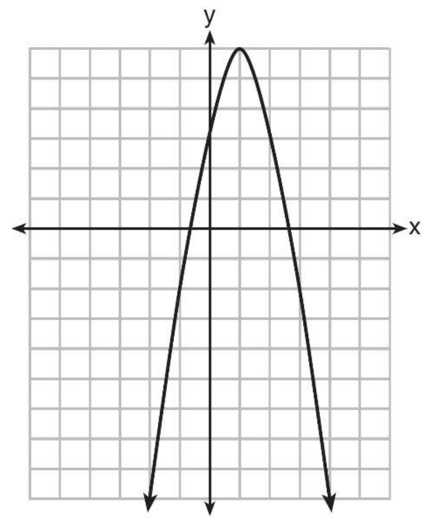
3. Over what intervals are $f(x) = -x^4 + 15x^2 - 7$:

Increasing Decreasing Positive Negative

4. Over what intervals are $f(x) = x^3 + 8x^2 + 3x - 8$:

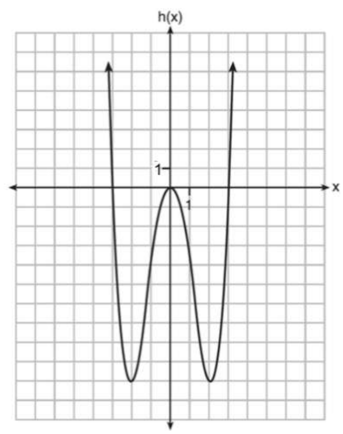
Increasing Decreasing Positive Negative

5. Let *f* be the function represented by the graph below.
Let *g* be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$.
Determine which function has the larger maximum value.
Justify your answer.



6. Which graph has a smaller relative minimum?

$g(x) = x^3 + 4x^2 - 2x - 10$



7. Which quadratic function has the largest maximum?

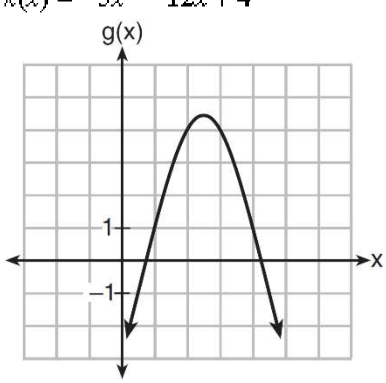
1) $h(x) = (3 - x)(2 + x)$

2)

x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

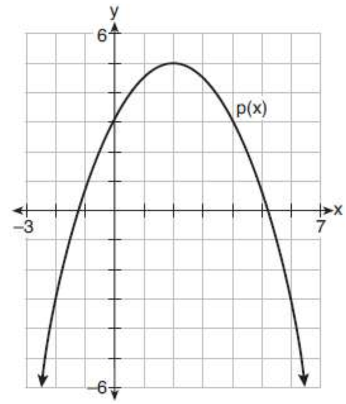
3) $k(x) = -5x^2 - 12x + 4$

4)



8. Consider $f(x) = 4x^2 + 6x - 3$, and $p(x)$ defined by the graph below. The difference between the values of the maximum of p and minimum of f is

1) 0.25 3) 3.25
2) 1.25 4) 10.25



9. The function $v(x) = x(3 - x)(x + 4)$ models the volume, in cubic inches, of a rectangular solid for $0 \le x \le 3$. To the *nearest tenth of a cubic inch*, what is the maximum volume of the rectangular solid?

10. Given $f(x) = x^4 - x^3 - 6x^2$, for what values of x will $f(x) > 0$?

1) $x < -2$, only
2) $x < -2$ or $x > 3$
3) $x < -2$ or $0 \le x \le 3$
4) $x > 3$, only

11. At which x value is the graph of $f(x) = 2x^3 - 11x^2 - 14x + 26$ *increasing*?

1) -.5 3) 1.7
2) 3.9 4) 4.3

12. The graph of $y = 2^x - 4$ is positive on which interval?

1) $(-\infty, \infty)$
2) $(2, \infty)$
3) $(0, \infty)$
4) $(-4, \infty)$

Graphing Polynomial Functions

1) Type equation into $Y=$
2) 2nd Graph (Table)
   *Plot points in given domain or that fit on the given graph

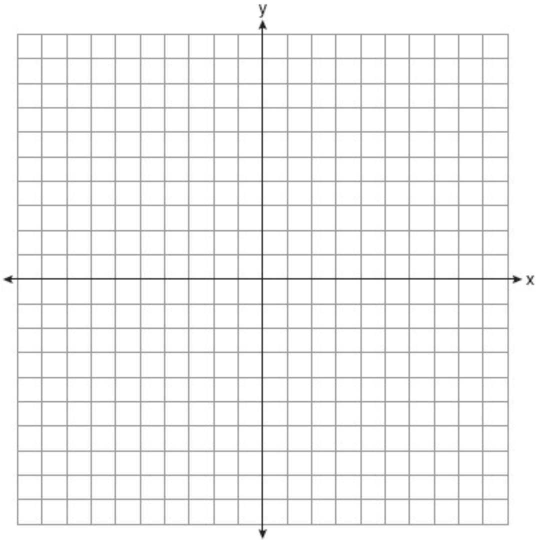
- Domain: no arrows. No domain: arrows.

Exponential: $y = \textit{vertical shift}$ or y value that is repeated in the table

Logarithmic: $x = \textit{horizontal shift}$ or the x value that contains the last error

Graph the following equations (Include domain and asymptotes if necessary)

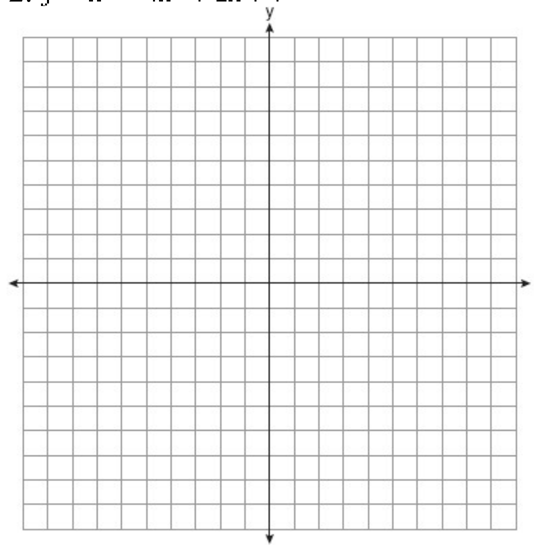
1. $f(x) = x^3 - 6x^2 + 9x + 6$ on the domain $-1 \le x \le 4$.



$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

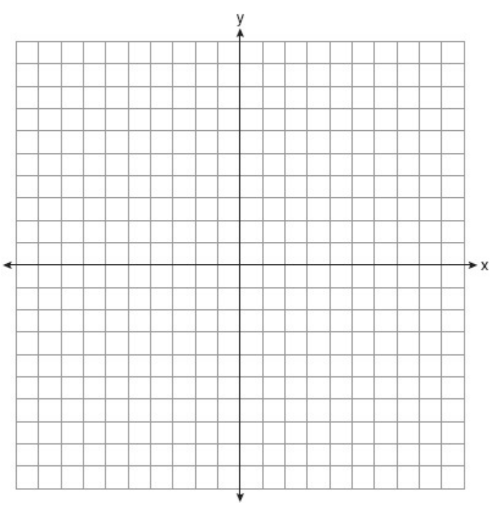
2. $y = x^3 - 4x^2 + 2x + 7$



$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$

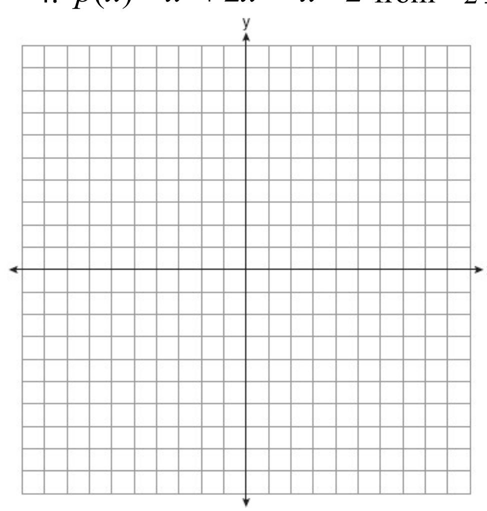
3. $p(x) = x^3 + x^2 - 4x - 4$



$x \to -\infty,\ f(x) \to$

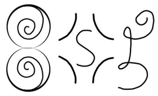
$x \to \infty,\ f(x) \to$

4. $p(x) = x^3 + 2x^2 - x - 2$ from $-2 \le x \le 1$



$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$



Inverse of a function $f^{-1}(x)$ **Algebraically**:
Switch x and y, solve for y
*If multiple choice:
-type original function into Y = and write down three nice points.
-switch x and y
-type all four choices into Y1, Y2, Y3, and Y4 and see which table matches.

1. What is the inverse of the function $y=2x-3$?

1) $y=\frac{x+3}{2}$ 3) $y=-2x+3$

2) $y=\frac{x}{2}+3$ 4) $y=\frac{1}{2x-3}$

2. If a function is defined by the equation y = 3x + 2, which equation defines the inverse of this function?

1) x = $\frac{1}{3}y+\frac{1}{2}$ 3) y = $\frac{1}{3}x-\frac{2}{3}$

2) y = $\frac{1}{3}x+\frac{1}{2}$ 4) y = -3x – 2

3. If $f(x)=5x-7$, find $f^{-1}(x)$

4. What is $g^{-1}(x)$ if $g(x)=3x+6$

5. What is the inverse of $y=\frac{1}{2}x+2$?

6. What is $h^{-1}(x)$ if $h(x) = x^2 + 2$

7. What is the inverse of the function $y = 4x + 5$?

1) $x = \frac{1}{4}y - \frac{5}{4}$

2) $y = \frac{1}{4}x - \frac{5}{4}$

3) $y = 4x - 5$

4) $y = \frac{1}{4x + 5}$

8. What is the inverse of $f(x) = -6(x - 2)$?

1) $f^{-1}(x) = -2 - \frac{x}{6}$

2) $f^{-1}(x) = 2 - \frac{x}{6}$

3) $f^{-1}(x) = \frac{1}{-6(x - 2)}$

4) $f^{-1}(x) = 6(x + 2)$

9. Given $f(x) = \frac{1}{2}x + 8$, which equation represents the inverse, $g(x)$?

1) $g(x) = 2x - 8$

2) $g(x) = 2x - 16$

3) $g(x) = -\frac{1}{2}x + 8$

4) $g(x) = -\frac{1}{2}x - 16$

10. The inverse of the function $f(x) = \frac{x + 1}{x - 2}$ is

1) $f^{-1}(x) = \frac{x + 1}{x + 2}$

2) $f^{-1}(x) = \frac{2x + 1}{x - 1}$

3) $f^{-1}(x) = \frac{x + 1}{x - 2}$

4) $f^{-1}(x) = \frac{x - 1}{x + 1}$

11. What is the inverse of $f(x) = \frac{x}{x + 2}$, where $x \neq -2$?

1) $f^{-1}(x) = \frac{2x}{x - 1}$

2) $f^{-1}(x) = \frac{-2x}{x - 1}$

3) $f^{-1}(x) = \frac{x}{x - 2}$

4) $f^{-1}(x) = \frac{-x}{x - 2}$

Inverse of a function $f^{-1}(x)$ Graphically:

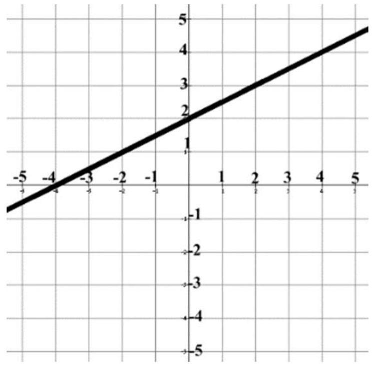
-write down as many nice points as you can from the graph

-switch x and y

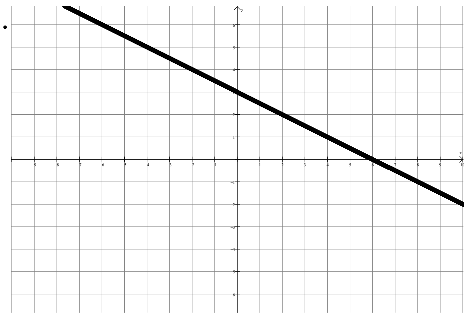
-plot new points on the same set of axes

Graph the inverse of the functions below on the same axes

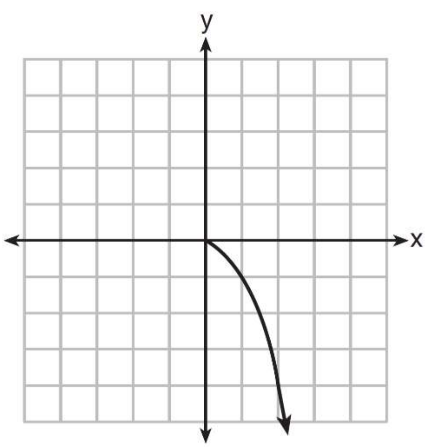
1.



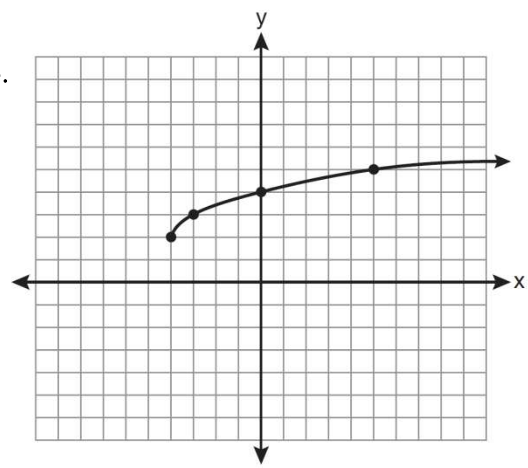
2.



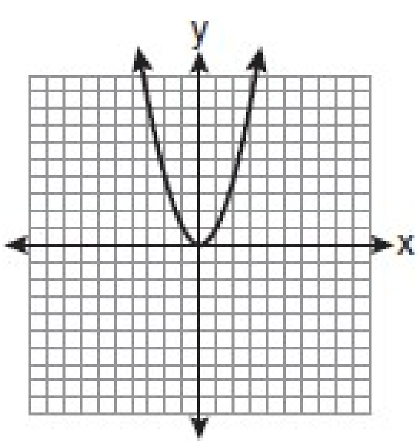
3.



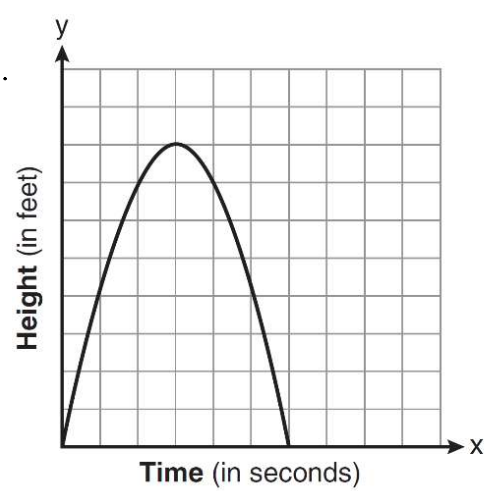
4.



5.



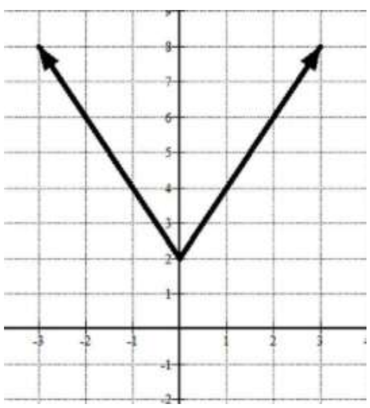
6.

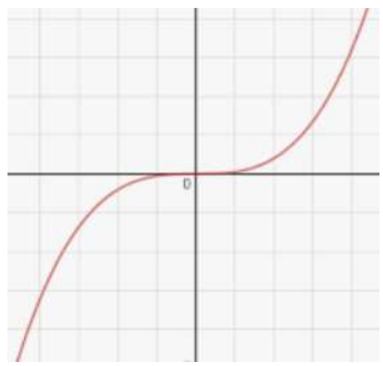


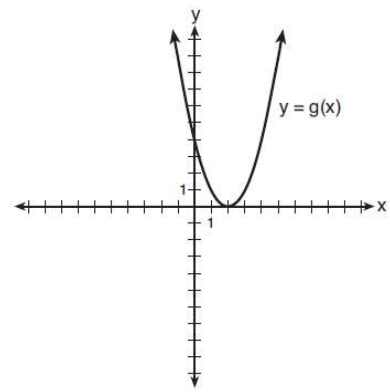
Even and Odd Functions
Even Functions are symmetric to the y-axis
Odd Functions are symmetric to the origin (turn the paper upside down)

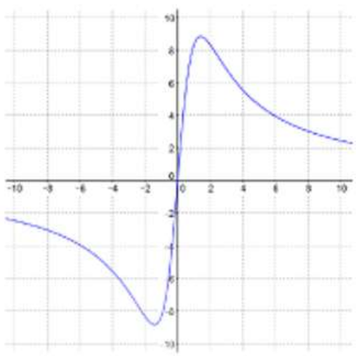
Algebraically:
Substitute -x in for x.
If the new functions is exactly the same, it is even $f(x) = f(-x)$
If the new function's signs are all opposite of the originals, it is odd $f(-x) = -f(x)$

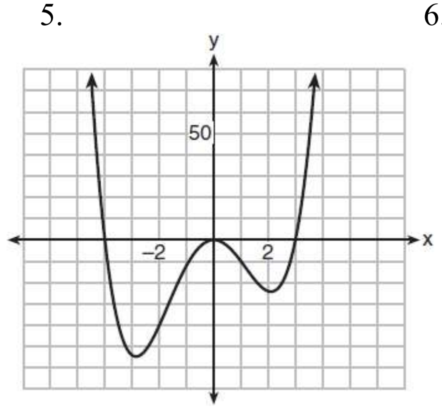
Determine graphically whether the following functions are even, odd, or neither

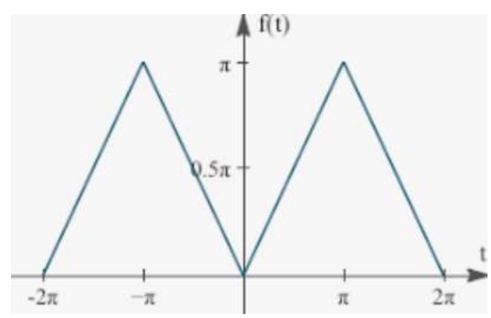
1.


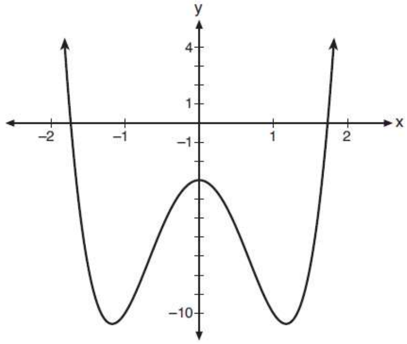
2.


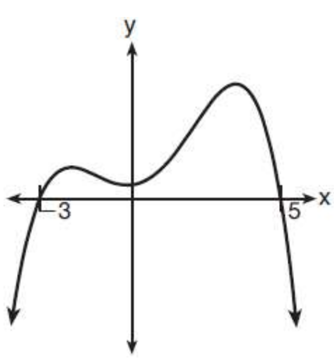
3.


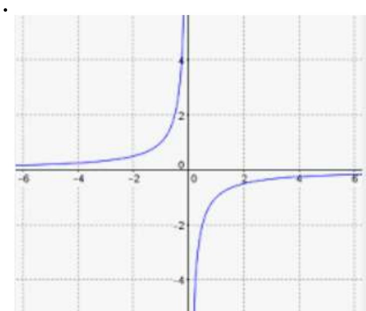
4.


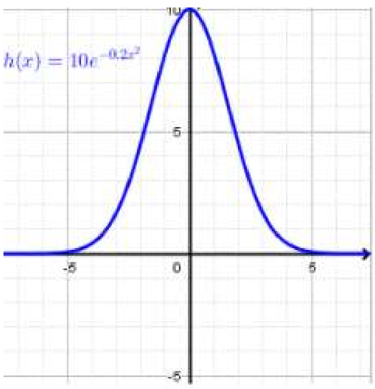
5.


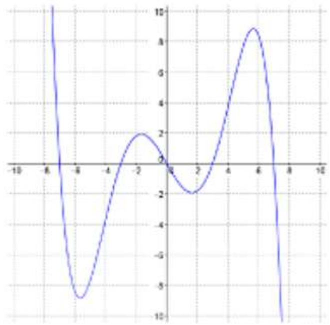
6.


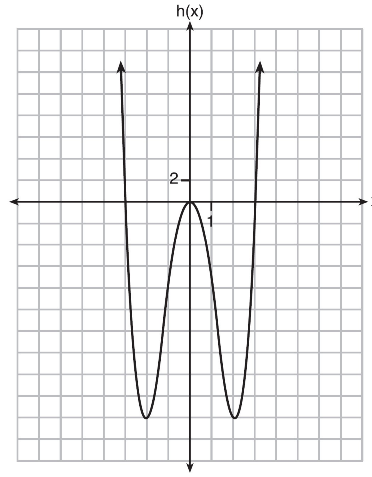
7.


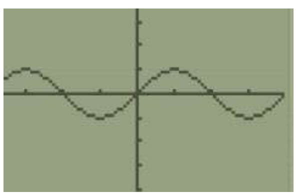
8.


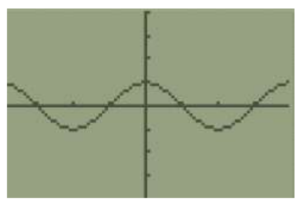
9.


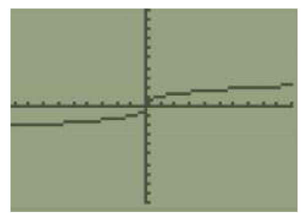
10.


11.


12.


13.


14.


15.


16. $f(x) = -x^4 + 4$

17. $f(x) = \frac{1}{2}x^5 - 2x$

18. $f(x) = 4x^3 - 6$

19. $f(x) = |x| + 4$

20. $f(x) = |x + 4|$

21. $f(x) = \frac{10}{x}$

Transforming Functions

Translations (+ or -)

If adding to f(x), the graph moves up or down

If adding to x, the graph moves left or right (the opposite direction in which you would think)

$y = f(x) + a$ moves UP a units

$y = f(x) - a$ moves DOWN a units

$y = f(x + a)$ moves LEFT a units

$y = f(x - a)$ moves RIGHT a units

Reflections (Negative)

$y = -f(x)$ is a reflection over the x axis (negate the y)

$y = f(-x)$ is a reflection over the y axis (negate the x)

Dilations (Multiply)

$y = af(x)$, Vertical Dilation

If $|a| > 1$, vertical stretch by a scale factor of a

If $|a| < 1$, vertical shrink/compression by a scale factor of a

$y = f(ax)$, Horizontal Dilation

If $|a| > 1$, horizontal shrink/compression by a scale factor of $\frac{1}{a}$

If $|a| < 1$, horizontal stretch by a scale factor of $\frac{1}{a}$

*Horizontal transformations are always the "opposite" of what you would expect.

1. If $g(x) = f(x-4) + 2$, how is the graph of $f(x)$ translated to form the graph of $g(x)$?

2. If $h(x) = f(x+1) - 3$, how is the graph of $f(x)$ translated to form the graph of $g(x)$?

3. How is the parent function transformed to create $f(x) = |x+3| - 2$?

4. How is the parent function transformed to create $f(x) = (x-4)^2 + 3$?

5. Relative to the graph of $y = 3\sin x$, what is the shift of the graph of $y = 3\sin\left(x + \frac{\pi}{3}\right)$?

1) $\frac{\pi}{3}$ right 2) $\frac{\pi}{3}$ left 3) $\frac{\pi}{3}$ up 4) $\frac{\pi}{3}$ down

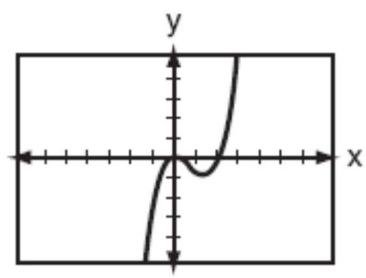
6. Given the parent function $p(x) = \cos x$, which phrase best describes the transformation used to obtain the graph of $g(x) = \cos(x + a) - b$, if *a* and *b* are positive constants?

1) right *a* units, up *b* units
2) right *a* units, down *b* units
3) left *a* units, up *b* units
4) left *a* units, down *b* units

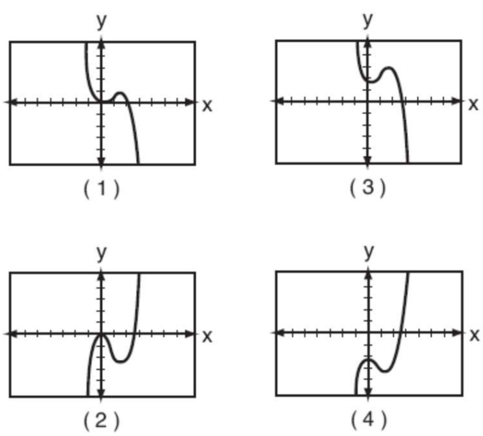
7. If $f(x) = \log_3 x$ and $g(x)$ is the image of $f(x)$ after a translation five units to the left, which equation represents $g(x)$?

1) $g(x) = \log_3(x + 5)$
2) $g(x) = \log_3 x + 5$
3) $g(x) = \log_3(x - 5)$
4) $g(x) = \log_3 x - 5$

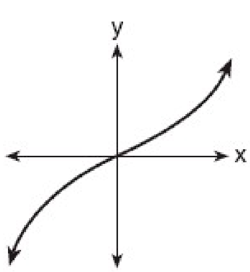
8. The accompanying graph represents the equation $y = f(x)$.



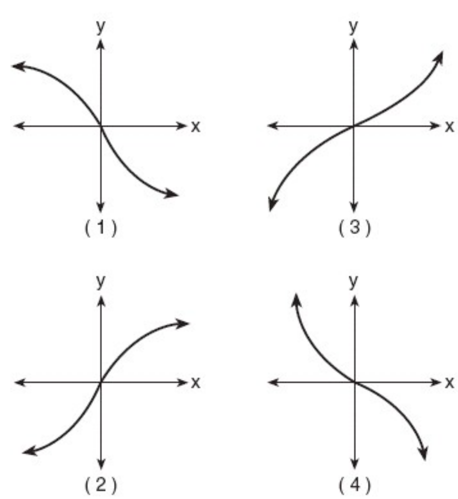
Which graph represents $g(x)$, if $g(x) = -f(x)$?



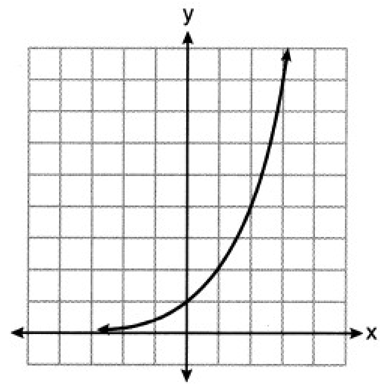
9. The graph below represents f(*x*).



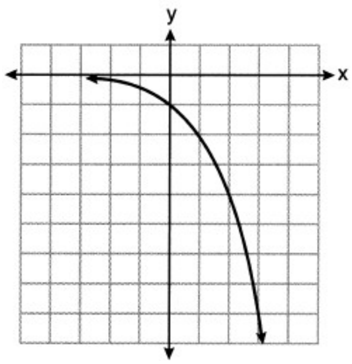
Which graph best represents f(-*x*)?



10. Consider the function $y = h(x)$, defined by the graph to the right. Which equation could be used to represent the graph shown below?



1) $y = h(x) - 2$
2) $y = h(x - 2)$
3) $y = -h(x)$
4) $y = h(-x)$



State the transformations that were performed on $f(x)$ to produce $g(x)$

11. $g(x) = 2f(x)$

12. $g(x) = f(2x)$

13. $g(x) = \frac{1}{3}f(x)$

14. $g(x) = f\left(\frac{1}{3}x\right)$

15. $g(x) = f(4x)$

16. $g(x) = \frac{2}{5}f(x)$

17. If (-2,4) is included in $f(x)$, what point must be included in $g(x)$ if $g(x) = 2f(x)$.

18. If (-2,4) is included in $f(x)$, what point must be included in $g(x)$ if $g(x) = f(2x)$.

19. If (4,-8) is included in $f(x)$, what point must be included in $g(x)$ if $g(x) = \frac{1}{2}f(x)$.

20. If (4,-8) is included in $f(x)$, what point must be included in $g(x)$ if $g(x) = f\left(\frac{1}{2}x\right)$.

21. If (-3,2) is included in $f(x)$, what point must be included in $g(x)$ if $g(x) = f\left(\frac{1}{3}x\right)$.

22. The function f(x) is given by the following table of values. Which table of values would represent g(x) if g(x) = f(x + 5)?

x	f(x)
1	2
2	4
3	8

1)

x	g(x)
5	2
6	4
7	8

2)

x	g(x)
1	7
2	9
3	13

3)

x	g(x)
1	-3
2	-1
3	3

4)

x	g(x)
-4	2
-3	4
-2	8

23. The function f(x) is given by the following table of values. Which table of values would represent g(x) if g(x) = f(2x)?

x	f(x)
2	18
4	10
8	2

1)

x	g(x)
2	36
4	20
8	4

2)

x	g(x)
1	18
2	10
4	2

3)

x	g(x)
2	9
4	5
8	1

4)

x	g(x)
4	18
8	10
16	2

Average rate of change: $\frac{y_2 - y_1}{x_2 - x_1}$

Always create a table!

1) If given table, circle values in the table.
2) If given a graph, pull y values from the graph.
3) If given an equation, type into y= and pull the values from the table.

Context: "On average, from "x" to "x", the "y topic" is "increasing/decreasing" by "AROC" "y units" per "x unit."

Intervals:

If given graph, the steepest slope is the greatest average rate of change. The flattest slope is the smallest average rate of change. If you cannot tell, find the average rate of change for each interval.

If given table, calculate the average rate of change for each interval.

1. The function $h(x)$ is given in the table below. Which of the following gives its average rate of change over the interval $2 \le x \le 6$?

(1) $-\frac{3}{2}$

(2) $\frac{6}{4}$

(3) $-\frac{7}{6}$

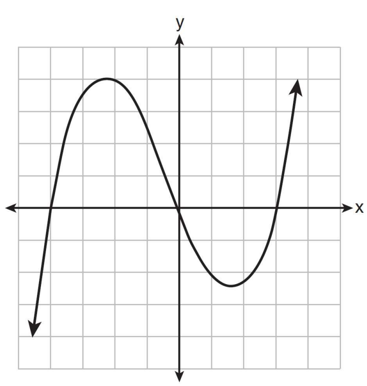
(4) -1

x	$h(x)$
0	10
2	9
4	6
6	3

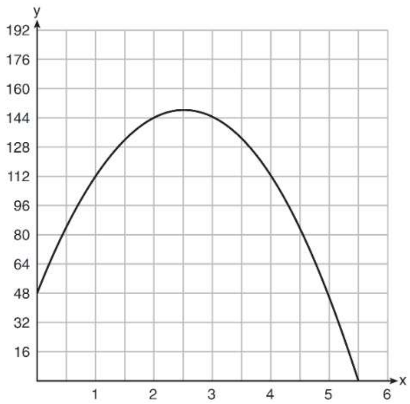
2. What is the average rate of change from 0 to 2?

x	f(x)
0	1
1	2
2	5
3	7

3. The graph of $p(x)$ is shown below. What is the average rate of change over the interval $-4 \le x \le 1$?



4. A ball is thrown into the air from the edge of a 48-foot-high cliff so that it eventually lands on the ground. The graph below shows the height, y, of the ball from the ground after x seconds. What is the average rate of change of the ball between 1 and 5 seconds?



5. For the function $f(x) = 3^x$, find the average rate of change over the interval -5 to -1 rounded to the nearest thousandth.

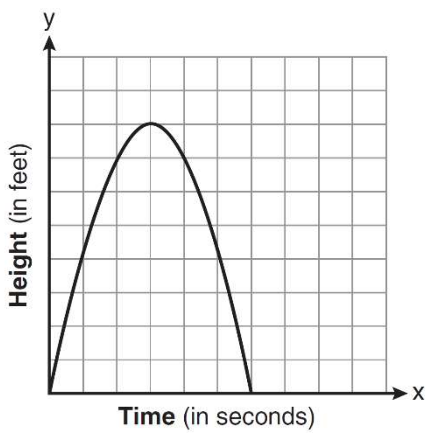
6. Find the average rate of change of the function $f(t) = 2500(0.97)^{4t}$ over the interval $10 \leq t \leq 15$ rounded to the nearest tenth.

7. The distance needed to stop a car after applying the brakes varies directly with the square of the car's speed. The table below shows stopping distances for various speeds.
Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.

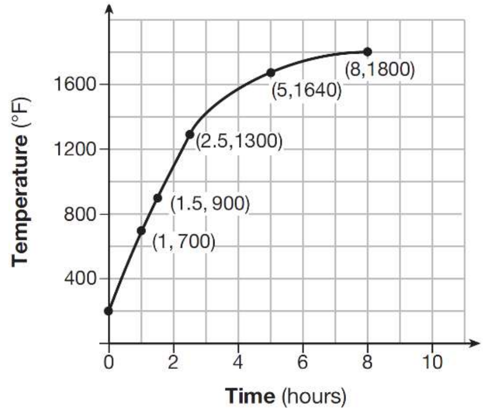
Speed (mph)	10	20	30	40	50	60	70
Distance (ft)	6.25	25	56.25	100	156.25	225	306.25

8. The average monthly high temperature in Buffalo, in degrees Fahrenheit, can be modeled by the function $B(t) = 25.29\sin(0.4895t - 1.9752) + 55.2877$, where t is the month number (January = 1). State, to the *nearest tenth*, the average monthly rate of temperature change between August and November. Explain its meaning in the given context.

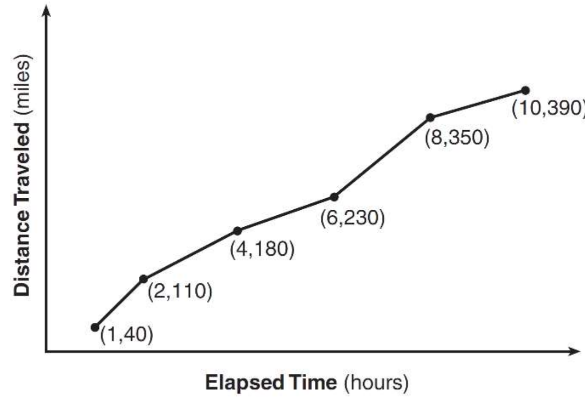
9. The graph below represents the parabolic path of a ball kicked by a young child. Find the average rate of change from 3 to 6 seconds. Explain its meaning in the context of the problem.



10. Firing a piece of pottery in a kiln takes place at different temperatures for different amounts of time. The graph below shows the temperatures in a kiln while firing a piece of pottery after the kiln is preheated to 200ºF.
During which time interval did the temperature in the kiln show the greatest average rate of change?
1) 0 to 1 hour
2) 1 hour to 1.5 hours
3) 2.5 hours to 5 hours
4) 5 hours to 8 hours



11. The Jamison family kept a log of the distance they traveled during a trip, as represented by the graph below.



During which interval was their average speed the greatest?

1) the first hour to the second hour
2) the second hour to the fourth hour
3) the sixth hour to the eighth hour
4) the eighth hour to the tenth hour

12. The table below shows the year and the number of households in a building that had high-speed broadband internet access.

Number of Households	11	16	23	33	42	47
Year	2002	2003	2004	2005	2006	2007

For which interval of time was the average rate of change the *smallest*?

1) 2002 - 2004
2) 2003 - 2005
3) 2004 - 2006
4) 2005 - 2007

13. Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of *B* dollars after *m* months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after *m* months. Over which interval of time is her average rate of change for the balance on her credit card account the greatest?

m	B
0	1000.00
10	1172.00
19	1352.00
36	1770.80
60	2591.90
69	2990.00
72	3135.80
73	3186.00

1) month 10 to month 60
2) month 19 to month 69
3) month 36 to month 72
4) month 60 to month 73

Exponents
FOLLOW THE FOLLOWING ORDER! STRUCTURE IS IMPERATIVE!!!

1) Radicals are fractional exponents (Fractional exponent = $\frac{power}{root}$)

2) Get rid of parenthesis (exponent outside parenthesis goes to everything inside)
Negative exponents are fractions (Move whatever is being raised to the negative power)
Clean it up (Multiply, divide/reduce, evaluate/put into radical)
*Add exponents when multiplying. Subtract exponents when dividing. Use a calculator for fractions.

Negative exponents are fractions! $x^{-2} = \frac{1}{x^2}$

If exponent is outside parenthesis, everything gets it $\left(\frac{xy}{z}\right)^3 = \frac{x^3y^3}{z^3}$

Rewrite the following as radicals

1. $x^{\frac{2}{3}}$
2. $x^{\frac{3}{4}}$
3. $x^{\frac{5}{6}}$
4. $x^{\frac{1}{3}}$
5. $x^{\frac{3}{2}}$
6. $x^{\frac{1}{2}}$

Rewrite the following using fractional exponents

7. $\sqrt[3]{x^4}$
8. $\sqrt[5]{x^3}$
9. $\sqrt[4]{x^7}$
10. $\sqrt{x^3}$
11. $\sqrt[6]{x^5}$
12. $\sqrt{x}$

Express with a rational exponent

13. $\sqrt[4]{x^3} \bullet \sqrt{x^5}$

14. $\sqrt[3]{b^5} \bullet \sqrt[4]{b}$

15. $\dfrac{\sqrt[6]{x^5}}{\sqrt[3]{x^2}}$

16. $\dfrac{\sqrt{m^7}}{\sqrt[5]{m^2}}$

17. $\dfrac{\sqrt[3]{x^2} \bullet \sqrt{x^5}}{\sqrt[6]{x}}$

18. $\dfrac{x\sqrt{x^3}}{\sqrt[3]{x^5}}$

19. $a\sqrt[5]{a^4}$

20. $2xy^2\sqrt[3]{x^2y}$

21. Kenzie believes that for $x \geq 0$, the expression $\left(\sqrt[7]{x^2}\right)\left(\sqrt[5]{x^3}\right)$ is equivalent to $\sqrt[35]{x^6}$. Is she correct? Justify your response algebraically.

22. Justify why $\dfrac{\sqrt[3]{x^2y^5}}{\sqrt[4]{x^3y^4}}$ is equivalent to $x^{\frac{-1}{12}}y^{\frac{2}{3}}$ using properties of rational exponents, where $x \neq 0$ and $y \neq 0$.

23. For n and $p > 0$, is the expression $\left(p^2 n^{\frac{1}{2}}\right)^8 \sqrt{p^5 n^4}$ equivalent to $p^{18}n^6\sqrt{p}$? Justify your answer.

24. Use the properties of rational exponents to determine the value of y for the equation:

$$\frac{\sqrt[3]{x^8}}{\left(x^4\right)^{\frac{1}{3}}} = x^y,\ x > 1$$

25. Given that $\left(\dfrac{y^{\frac{17}{8}}}{y^{\frac{5}{4}}}\right)^{-4} = y^n$, where $y > 0$, determine the value of n.

Express in simplest form:

26. $\dfrac{2x^{-2}y^{-2}}{4y^{-5}}$

27. $(5^{-2}a^{3}b^{-4})^{-1}$

28. $\dfrac{(3x^{-2}y^{2})^{2}}{9x^{-3}y^{-3}}$

29. $\dfrac{3x^{-4}y^{5}}{(2x^{3}y^{-7})^{-2}}$

Graphing Exponential and Logarithmic Functions

1) Type into y = and plot the points
2) Asymptote of exponential is y = vertical shift OR repeated value in the table
   Asymptote of logarithmic is x = horizontal shift OR the last error in the table

For the following equations, graph the equation and the asymptote. State the domain, range, equation of the asymptote, and end behavior.

1. $y = 2^{x} - 3$

Domain:

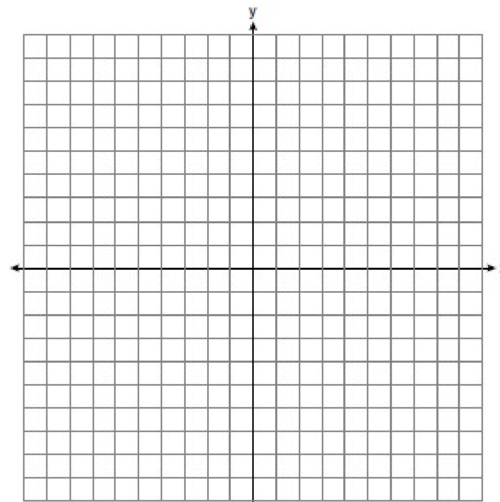
Range:

Asymptote:

End Behavior:

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$



2. $y = \left(\frac{1}{2}\right)^{x-3} + 1$

Domain:

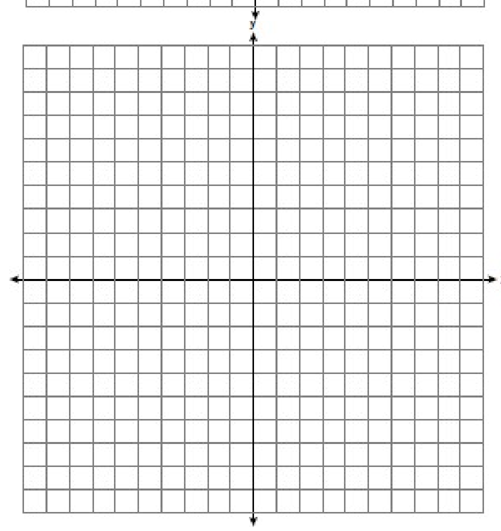
Range:

Asymptote:

End Behavior:

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$



3. $y = -3^{x-2} + 4$

Domain:

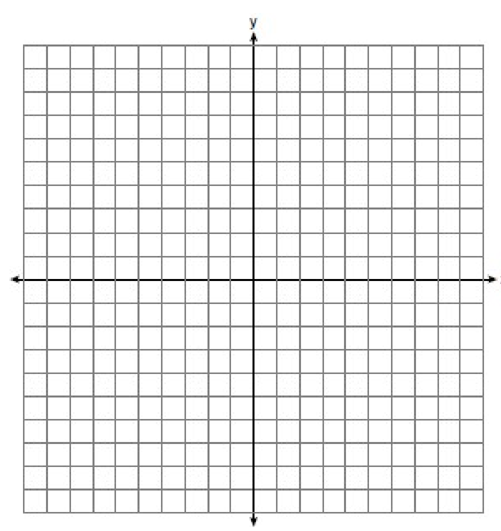
Range:

Asymptote:

End Behavior:

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$



4. $y = 4\log_{\frac{1}{2}}(x-3)+1$

Domain:

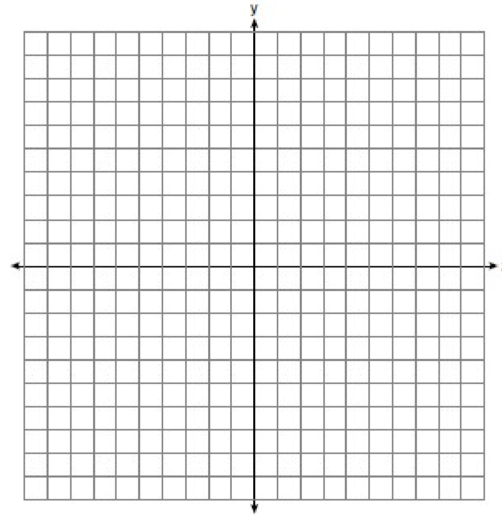
Range:

Asymptote:

End Behavior:

$x \to 3,\ f(x) \to$

$x \to \infty,\ f(x) \to$



5. $y = 3\log_4(x+1)-8$

Domain:

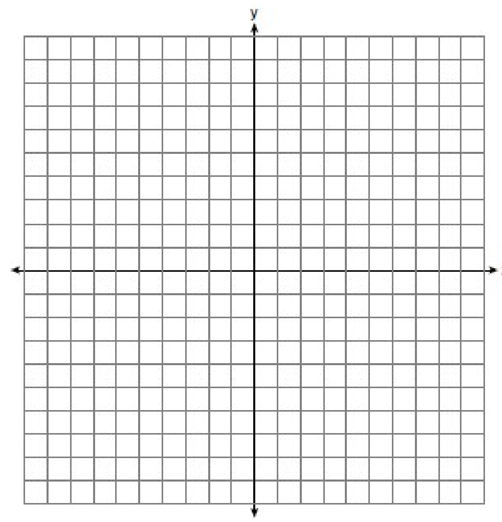
Range:

Asymptote:

End Behavior:

$x \to -1,\ f(x) \to$

$x \to \infty,\ f(x) \to$



6. $y = -4\log_2(x+9)+4$

Domain:

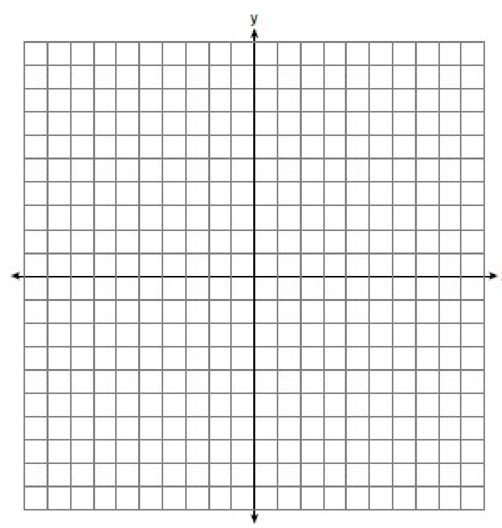
Range:

Asymptote:

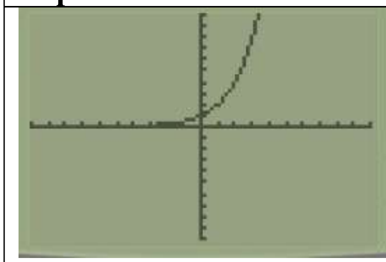
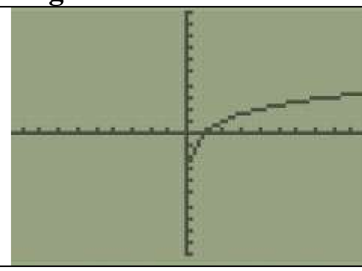
End Behavior:

$x \to -9,\ f(x) \to$

$x \to \infty,\ f(x) \to$



Sketching Exponential and Logarithmic Functions

Exponential	**Logarithmic**
	
Horizontal Asymptote at $y = 0$	Vertical Asymptote at $x = 0$
Passes through $(0,1)$	Passes through $(1,0)$
Domain is all real numbers	Domain is all positive real numbers
Range is all positive real numbers	Range is all real numbers
Exponents and logarithms are inverses of each other!!!!!!!!!!!	

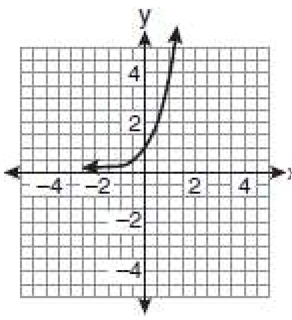
1. Which statement about the graph of $c(x) = \log_6 x$ is *false*?
 1) The asymptote has equation $y = 0$.
 2) The graph has no y-intercept.
 3) The domain is the set of positive reals.
 4) The range is the set of all real numbers.

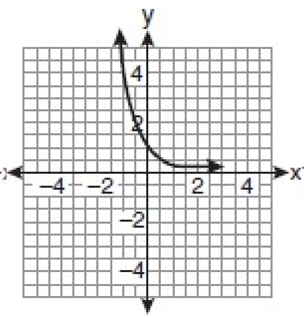
2. Which statement about the graph of the equation $y = e^x$ is *not* true?
 1) It is asymptotic to the x-axis.
 2) The domain is the set of all real numbers.
 3) It lies in Quadrants I and II.
 4) It passes through the point $(e, 1)$.

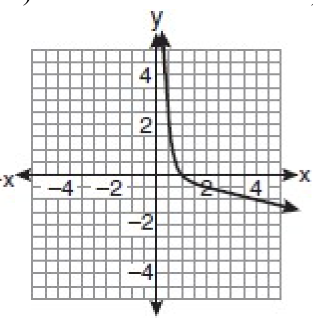
3. Given the equation $f(x) = \pi^x$, which of the following statements is true?
1) The graph passes through $(\pi, 1)$
2) The domain is $[0, \infty)$
3) The graph passes through $(0,1)$
4) The range is all real numbers

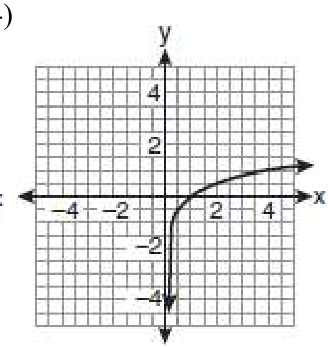
4. Which statement is false regarding the equation $f(x) = \log_a x$?
1) The range is $[0, \infty)$
2) The graph passes through $(0,1)$
3) The domain is all real numbers
4) The equation of the asymptote is $x=0$

5. If a function is defined by the equation $f(x) = 4^x$, which graph represents the inverse of this function?

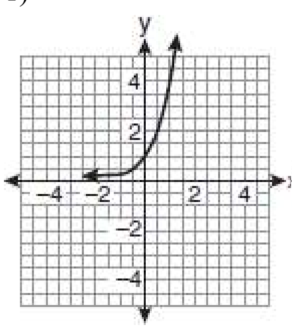
1) 

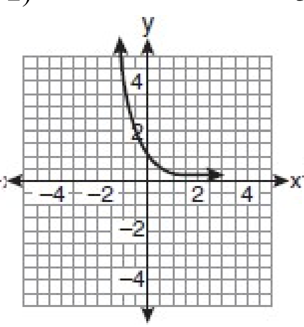
2) 

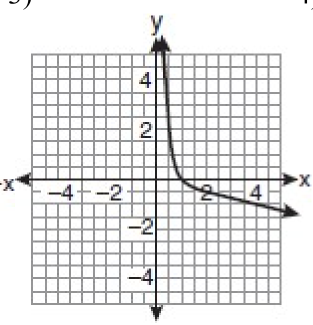
3) 

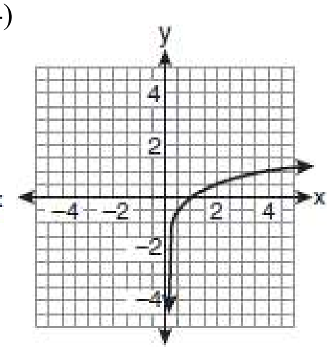
4) 

6. If a function is defined by the equation $f(x) = \log_4 x$, which graph represents the inverse of this function?

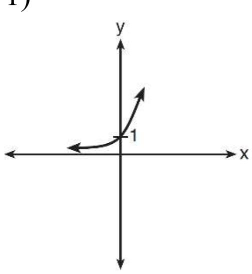
1) 

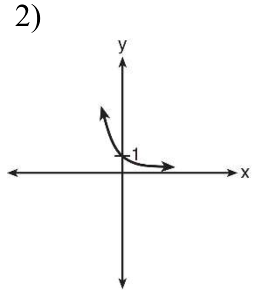
2) 

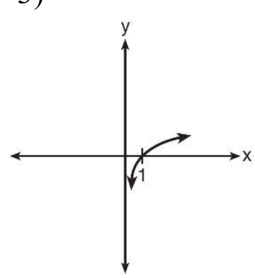
3) 

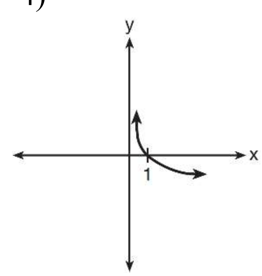
4) 

7. Which sketch shows the inverse of $y = a^x$, where $a > 1$?

1) 

2) 

3) 

4) 

8. What is the inverse of the function $y = \log_3 x$?

1) $y = x^3$ 2) $y = \log_x 3$ 3) $y = 3^x$ 4) $x = 3^y$

9. If $f(x) = a^x$ where $a > 1$, then the inverse of the function is

1) $f^{-1}(x) = \log_x a$

3) $f^{-1}(x) = \log_a x$

2) $f^{-1}(x) = a \log x$

4) $f^{-1}(x) = x \log a$

10. The asymptote of the graph of $f(x) = 5\log(x+4)$ is

1) $y = 6$
2) $x = -4$
3) $x = 4$
4) $y = 5$

11. The asymptote of the graph of $j(x) = 2e^{x-4} - 1$ is

1) $x = 4$
2) $x = -4$
3) $y = -1$
4) $y = 2$

12. The asymptote of the graph of $e(x) = \log_3(x-5) + 1$ is

1) $y = 1$
2) $x = 1$
3) $y = 5$
4) $x = 1$

13. The asymptote of the graph of $m(x) = -3(2)^{x+1} - 4$ is

1) $x = -1$
2) $x = 3$
3) $y = -4$
4) $y = -3$

14. For the equation $f(x) = 2^{x-3} + 1$, as $x \to -\infty$

1) $f(x) \to -\infty$
2) $f(x) \to 1$
3) $f(x) \to \infty$
4) $f(x) \to 3$

15. For the equation $f(x) = \log_2(x-4) + 3$, as $x \to 4$

1) $f(x) \to -\infty$
2) $f(x) \to 3$
3) $f(x) \to \infty$
4) $f(x) \to 4$

16. For the equation $f(x) = -\log_3(x+1) - 2$, as $x \to \infty$

1) $f(x) \to -\infty$
2) $f(x) \to -1$
3) $f(x) \to \infty$
4) $f(x) \to -2$

17. Given $f(x) = 3^{x-1} + 2$, as $x \to -\infty$

1) $f(x) \to -1$
2) $f(x) \to 0$
3) $f(x) \to 2$
4) $f(x) \to -\infty$

18. For the equation $f(x) = 3\ln(x-4) + 1$, $f(x) \to -\infty$ as

1) $x \to 4$
2) $x \to 1$
3) $x \to \infty$
4) $x \to -\infty$

Variable Exponential Equations

1) Isolate the base
2) Take log of both sides
   *multiply by LCD is fraction comes to the front
3) Divide to isolate x

1. $3^{2x} = 182$

2. $e^{2n} = 245$

3. $3(5)^{2x} = 60$

4. $20e^{4x} = 120$

5. $250(1.04)^{4x} = 500$

6. $48e^{.12x} = 60$

7. $1.2(4)^{2x} = 20$

8. $400(.987)^{2.5x} = 300$

9. $2(3)^{2x} + 8 = 18$

10. $4(2)^{3x} + 3 = 15$

11. $8 + 2e^{-5x} = 14$

12. $12 + 2(5)^{8x} = 2000$

13. $500e^{\frac{x}{2}} = 200$

14. $2000(2)^{\frac{x}{4.2}} = 1500$

15. $1.2(3)^{\frac{x}{4.1}} + 15 = 195$

16. $18 - 4(6)^{\frac{x}{3}} = 16$

Regression Equations

1) Stat, Edit
2) Input x column into L1 and y column into L2
3) Stat, Calc, 0: ExpReg or 5: QuadReg or A: PwrReg
4) READ AND ROUND CAREFULLY
***Use logs or Y1 Y2 Intersect to find x given y.**

1. The table below gives air pressures in kPa at selected altitudes above sea level measured in kilometers.

x	**Altitude** (km)	0	1	2	3	4	5
y	**Air Pressure** (kPa)	101	90	79	70	62	54

Write an exponential regression equation that models these data rounding all values to the *nearest thousandth*.

Use this equation to find the air pressure to the *nearest tenth* when the altitude is 10 km?

Use this equation to determine the altitude, to the *nearest hundredth* of a kilometer, when the air pressure is 29 kPa.

2. The stopping distances for Jim's car while driving at various speeds are shown in the table below.

x	**Speed** (mph)	10	15	20	25	30	40
y	**Stopping Distance** (ft)	12	22	39	58	84	150

Based on these data, find the power regression equation for the set of data. Round all coefficients to the *nearest hundredth*.

Jim is driving along a main street in his town and sees traffic building up ahead and needs to stop. Use the equation to find the distance needed for Jim's car to stop if it is traveling at a speed of 55 mph, rounded to the *nearest foot*.

If Jim's car needs 100 feet of stopping distance, determine how fast Jim is driving, to the *nearest mile per hour*.

3. The concentration, y, in milligrams per liter, of a medication in a patient's bloodstream x hours after taking the medication is listed for specified values in the table below.

Time (hours) **(x)**	**Concentration** (mg/l) **(y)**
0	0
0.5	78.1
1	99.8
1.5	84.4
2	50.1
2.5	15.6

Write the equation of the quadratic regression that models these data, rounding all values to the *nearest tenth*.

Based on your regression equation from above, determine the concentration in the patient's bloodstream 1.75 hours after the medication was taken, rounded to the *nearest tenth of a milligram per liter*.

Determine to the *nearest tenth of an hour*, the number of hours after taking the medication it would take for the concentration to be 35 milligrams per liter.

4. Water is draining from a tank maintained by the Yorkville Fire Department. Students measured the depth of the water in 15-second intervals and recorded the results in the accompanying table.

Time (x) (in seconds)	**Depth of Water (y)** (in feet)
15	11.8
30	9.9
45	8.2
60	6.3
75	5.9

Write the power regression equation for this set of data, rounding all values to the *nearest ten-thousandth*.

Using this equation, predict the depth of the water at 120 seconds, to the *nearest tenth of a foot*.

Using this equation, find the number of seconds it will take for the tank to have only 3 feet of water remaining. Round your answer to the *nearest second*.

5. The following data table shows a car's speed in miles per hour and the car's fuel efficiency in miles per gallon for each speed.

Speed (mph)	**Fuel Efficiency** (mpg)
18.6	26.1
24.9	29.4
31.1	31.4
37.3	33.1
43.5	33.2
49.7	31.4
55.9	29.5
62.1	26.0

Write the quadratic regression equation for these data, rounding all coefficients to the *nearest thousandth*.

Use the equation to determine, to the *nearest mile per gallon*, the fuel efficiency of the car when it is driven at a speed of 70 miles per hour.

Using the equation above, algebraically determine, to the *nearest tenth of a mile per hour*, the fastest speed the car can be driven so that its fuel efficiency is 30 miles per gallon.

6. Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

Hours (x)	**Average Number of Spores** (y)
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*.

Using this equation, how many spores will there be, to the *nearest spore*, after 7.5 hours?

The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest hundredth*, that the meat can be kept at room temperature safely.

Newton's Law of Heating and Cooling

The formula will be given to you. Write out what each variable represents and carefully substitute in. There may be multiple questions within the problem so make sure you read only one sentence at a time.

If solving for T, type the entire right hand side in.

If solving for *k* or *t*, solve the exponential equation by ISOLATING and taking the log/ln of both sides.

*Once you find *k*, you will need to use that *k* value to answer the next question. THE VALUES YOU USED IN THE FIRST QUESTION DO NOT APPLY TO THE SECOND QUESTION.

1. The Fahrenheit temperature of a heated object can be modeled by the function below.

$$F(t) = F_S + (F_0 - F_S)e^{-kt}$$

$F(t)$ = the temperature of the object after *t* minutes

t = time in minutes

F_S = the surrounding temperature

F_0 = the initial temperature of the object

k = a constant

Hot chocolate at a temperature of 200°F is poured into a container. The room temperature is kept at a constant 68°F and $k = 0.05$.

After how much time, to the *nearest minute*, will the temperature of the hot chocolate be 150°F?

After how much time, to the *nearest tenth of a minute*, will the temperature of the hot chocolate be 120°F?

2. The Fahrenheit temperature, $F(t)$, of a heated object at time *t*, in minutes, can be modeled by the function below. F_s is the surrounding temperature, F_0 is the initial temperature of the object, and *k* is a constant.

$$F(t) = F_S + (F_0 - F_S)e^{-kt}$$

Coffee at a temperature of 195°F is poured into a container. The room temperature is kept at a constant 68°F and $k = 0.05$. Coffee is safe to drink when its temperature is, at most, 120°F. To the *nearest minute*, how long will it take until the coffee is safe to drink?

3. After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + \left(T_0 - T_a\right)e^{-kt}$$

T_a = the temperature surrounding the object

T_0 = the initial temperature of the object

t = the time in hours

T = the temperature of the object after t hours

k = decay constant

The turkey reaches the temperature of approximately 100° F after 2 hours. Find the value of *k*, to the *nearest thousandth*. Determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m.

4. Empanadas are taken out of an oven when they reached a temperature of 168°F and put on the kitchen table at room temperature (68°F). After 8 minutes, the temperature of the empanadas is 125°F. The temperature of a cooled object can be given by the formula below:

$T = T_a + \left(T_0 - T_a\right)e^{-kt}$

T = the temperature of the object after t minutes

t = time in minutes

T_a = the surrounding temperature

T_0 = the initial temperature of the object

k = decay constant

Find the value of *k*, rounded to the *nearest thousandth*. Using your value of *k*, to the *nearest minute*, how long will it take for the empanadas to reach 100°F?

5. Megan is performing an experiment in a lab where the air temperature is a constant 73ºF and the liquid is 237ºF. One and a half hours later, the temperature of the liquid is 112ºF. Newton's law of cooling states $T(t) = T_a + (T_0 - T_a)e^{-kt}$ where:

$T(t)$: temperature, °F, of the liquid at t hours

T_a: air temperature

T_0: initial temperature of the liquid

k: constant

Determine the value of *k*, to the *nearest thousandth*, for this liquid. Determine the temperature of the liquid using your value for *k*, to the *nearest degree*, after two and a half hours. Megan needs the temperature of the liquid to be 80ºF to perform the next step in her experiment. Use your value for *k* to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

6. Objects cool at different rates based on the formula below.

$T = (T_0 - T_R)e^{-rt} + T_R$

T_0: initial temperature

T_R: room temperature

r: rate of cooling of the object

t: time in minutes that the object cools to a temperature, T

Mark makes T-shirts using a hot press to transfer designs to the shirts. He removes a shirt from a press that heats the shirt to 400°F. The rate of cooling for the shirt is 0.0735 and the room temperature is 75°F. Find the temperature of the shirt, to the *nearest degree*, after five minutes. At the same time, Mark's friend Jeanine removes a hoodie from a press that heats the hoodie to 450°F. After eight minutes, the hoodie measured 270°F. The room temperature is still 75°F. Determine the rate of cooling of the hoodie, to the *nearest ten thousandth*. The T-shirt and hoodie were removed at the same time. Determine when the temperature will be the same, to the *nearest minute*.

Modeling Exponential Functions:

$A = P(1 \pm r)^{t}$	Nothing Below!
$A = P\left(1+\frac{r}{n}\right)^{nt}$	Compounding (Not Continuous)
$A = Pe^{rt}$	Compounding Continuously
$A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$	Half Life
$A = P(1 \pm r)^{\frac{t}{h}}$	Irregular Time

	n	
A = after amount	Annually	1
P = principal (initial/starting) amount	Quarterly	4
r = rate (as a decimal)	Monthly	12
n = number of times compounded per year	Weekly	52
t = time (that is passing)	Daily	365
h = half life or time it takes for the percent to be applied		

*To find *t,* solve the exponential equation using logs!

1. Jackie deposits $26,000 into a savings account with interest compounded monthly at a rate of 4.6% each year. Write an equation for $A(t)$, the value of her account after *t* years. Use your equation to determine how much money will be in her account after 4 years?

2. The population of Schlansky, Arizona increases by 18% every 3.2 years. If the population is currently 2750, write an equation for $p(t)$, the population after *t* years. Using your equation, what will be the population, to the *nearest person*, 12 years from now?

3. A bank account is opened with $2700 and interest is compounded continuously at a rate of 3.76% per year. Write an equation for $b(t)$, the balance of the account after *t* years. Using your equation, what will be the balance of the account after 8.1 years?

4. A certain car depreciates at a rate of 14% each year. If the car was initially worth $22,500, write an equation for $v(t)$, the value of the account after t years. Using your equation, what is the value of the car, rounded to the *nearest cent*, 12 years later?

5. The half life of an element is 73 minutes. If there were initially 7.4 kg of the substance, write an equation for $a(t)$, the amount of the substance remaining after t minutes. Using your equation, to the *nearest hundredth of a kg*, how much will remain after 110 minutes?

6. Skylar bought an antique mirror for $800. If the value of her mirror increases 6% annually, write an equation for $v(t)$, the value of her mirror after t years. Using your equation, determine the value of Skylar's mirror at the end of 4 years to the *nearest dollar*?

7. A bank account is opened with $1500 and interest is compounded quarterly at an interest rate of 3.1%. Write an equation for $b(t)$, the balance of the account after t years. Using your equation, how much money will be in the account after 7 years?

8. Megan opens a savings account with $5,000 in it. If interest is compounded quarterly at a rate of 4.3%, write an equation for $b(t)$, the balance of her account after t years. Using your equation, how long will it take, to the *nearest tenth of a year*, for Megan's money to reach $8,000?

9. One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Create an equation for $a(t)$, the amount of Iodine-131 remaining after t days. Determine, to the *nearest day*, the amount of time needed before the amount of I–131 in the patient's body is approximately 7 milligrams.

10. Tyler opens a bank account with $5,450 with an annual interest rate of 5.3% compounded continuously. Write an equation for $b(t)$, the balance of Tyler's account after t years. Using your equation, to the *nearest hundredth of a year*, how long will it take for Tyler's account to triple?

11. Jessica deposits $2000 into a bank account where 4% interest is given every 2.4 years. Write an equation for $v(t)$, the value of Jessica's account after t years. Using your equation, to the *nearest tenth of a year*, how long will it take for Jessica's investment to reach $5000?

12. Manny opens a savings account with $6,400.00 with a 5.2% interest rate that is compounded quarterly. Write an equation for $b(t)$, the balance of the account after t years. Using your equation, to the nearest *tenth of a year,* how long will it take for Manny's balance to double?

13. Christopher is preparing for the Nassau County Spelling Bee. Currently, Christopher knows 1200 words and will learn 20% more words every 4 days. Write an equation, $A(t)$, to represent how many words Christopher will be able to spell after t days. After how many days, to the *nearest day,* will Christopher be able to spell 5000 words?

14. If a bank account was opened with $3000 and interest is compounded continuously at 5.2%. Write an equation for $v(t)$, the value of the account after t years. To the *nearest hundredth of a year*, how long will it take for the value of the account to reach $4000?

15. Danielle bought a basketball card for $2125 its value is increasing by 4.1% each year. Create an equation for $v(t)$, the value of the basketball card after t years. Using your equation, how long, to the *nearest year*, will it take for the value of the basketball card to reach $10000?

16. Miguel opened a bank account with $1000 and interest is compounded monthly at a rate of 8.1%. Write an equation to represent $b(t)$, the balance of Miguel's account after t years. Using your equation, how much time, to the *nearest year*, will it take for Miguel's money to triple?

Modeling Exponential Functions with Inequalities
At least/minimum: $\geq$
At most/maximum: $\leq$

1. Tage deposits \$2500 into an account that earns 2.7% interest compounded continuously. Which inequality can be used to determine how long it will take for his account to have at least \$4000?
1) $2500(1.027)^t \leq 4000$
2) $2500(1.027)^t \geq 4000$
3) $2500e^{.027t} \leq 4000$
4) $2500e^{.027t} \geq 4000$

2. Lamar has 2000 ants in an ant colony and the population is doubling every 4 days. His tank can hold a maximum amount of 325000 ants. Which inequality can be used to determine how many days, *d*, can pass before he will need to buy a bigger tank?
1) $2000(2)^{\frac{d}{4}} \leq 325000$
2) $2000(2)^{\frac{d}{4}} \geq 325000$
3) $2000(4)^{\frac{d}{2}} \leq 325000$
4) $2000(4)^{\frac{d}{2}} \geq 325000$

3. Caleb bought a car for \$19,100 and its value is decreasing by 12% each year. He wants to sell his car while its value is greater than \$5000. Which inequality can be used to find the maximum number of years, *t*¸ he can keep his car while its value is greater than \$5000?
1) $19100(.12)^t \geq 5000$
2) $19100(.12)^t \leq 5000$
3) $19100(.88)^t \geq 5000$
4) $19100(.88)^t \leq 5000$

4. A sample of 2000 grams of Fluorine-18 has a half life of 109.734 minutes. Which inequality can be used to represent how many minutes, *m*, can pass for there to be a minimum of 67 grams remaining?
1) $2000(\frac{1}{2})^{\frac{t}{109.734}} \geq 67$
2) $2000(\frac{1}{2})^{\frac{t}{109.734}} \leq 67$
3) $2000(2)^{\frac{t}{109.734}} \geq 67$
4) $2000(2)^{\frac{t}{109.734}} \leq 67$

Finding the Exponential Rate
To find exponential rate:

1) Substitute values into $A = P\left(1 \pm r\right)^t$
2) Isolate the parenthesis
3) Root both sides to get rid of the constant exponent
4) Solve for r (divide by -1 if decay)
5) Multiply by 100 to find the percent

1. A bank account opened up 3 years ago with an initial balance of $12000 now has a balance of $12824. Find the annual growth rate, to the *nearest tenth of a percent*.

2. Jack bought a new car in 2010 for $16100. In 2018, the car is now worth $6125. What is the annual rate of decrease to the *nearest percent*?

3. A collectible toy was bought 15 years ago for $5 and is now worth $42. Find the annual growth rate to the *nearest tenth of a percent*.

4. A colony of 120 timberwolves increased to 245 over a 6 year span. Assuming exponential growth, what was the annual growth rate to the *nearest percent*?

Equivalent Exponential Forms (Absorbing the Exponent)
If you have a value in the exponent, absorb it into the parenthesis.
To interpret an exponential function, the initial value is in front of the parenthesis and $(1 \pm rate)$ is what is inside the parenthesis. If it is less than 1, it is decreasing. If it is more than 1, it is increasing.

Express each of the following functions with an exponent of *t* rounding values to the *nearest hundredth.* Express the percent of increase/decrease to the nearest percent.

1. $A = 12{,}000(1.025)^{12t}$

2. $A = 17{,}000(.889)^{9.4t}$

3. $A = 11{,}185(.764)^{\frac{t}{12}}$

4. $A = 125{,}000(.785)^{\frac{t}{4}}$

5. lridium-192 is an isotope of iridium and has a half-life of 73.83 days. If a laboratory experiment begins with 100 grams of Iridium-192, the number of grams, A, of Iridium-192 present after t days would be $A = 100\left(\frac{1}{2}\right)^{\frac{t}{73.83}}$. Which equation approximates the amount of Iridium-192 present after t days?

1) $A = 100\left(\frac{73.83}{2}\right)^{t}$

2) $A = 100\left(\frac{1}{147.66}\right)^{t}$

3) $A = 100(0.990656)^{t}$

4) $A = 100(0.116381)^{t}$

6. The amount of a substance, $A(t)$, that remains after t days can be given by the equation $A(t) = A_0(0.5)^{\frac{t}{0.0803}}$, where A_0 represents the initial amount of the substance. An equivalent form of this equation is

1) $A(t) = A_0(0.000178)^{t}$

2) $A(t) = A_0(0.945861)^{t}$

3) $A(t) = A_0(0.04015)^{t}$

4) $A(t) = A_0(1.08361)^{t}$

7. A certain pain reliever is taken in 220 mg dosages and has a half-life of 12 hours. The function $A = 220\left(\frac{1}{2}\right)^{\frac{t}{12}}$ can be used to model this situation, where *A* is the amount of pain reliever in milligrams remaining in the body after *t* hours. According to this function, which statement is true?

1) Every hour, the amount of pain reliever remaining is cut in half.
2) In 12 hours, there is no pain reliever remaining in the body.
3) In 24 hours, there is no pain reliever remaining in the body.
4) In 12 hours, 110 mg of pain reliever is remaining.

8. An equation to represent the value of a car after *t* months of ownership is $v = 32,000(0.81)^{\frac{t}{12}}$. Which statement is *not* correct?

1) The car lost approximately 19% of its value each month.
2) The car maintained approximately 98% of its value each month.
3) The value of the car when it was purchased was $32,000.
4) The value of the car 1 year after it was purchased was $25,920.

9. The value of an investment account, $v(t)$, can be modeled by the equation $v(t) = 500(1.15)^{3.2t}$ after *t* years. Which of the following statements must be true?

1) The account is increasing approximately 15% each year.
2) The account is increasing approximately 56% each year
3) There will be $1216.80 in the account after two years
4) It will take 3.68 years for the account to double

10. The amount of a substance, $A(t)$, in grams, remaining after *t* days is modeled by $A(t) = 50(0.5)^{\frac{t}{3}}$. Which statement is false?

1) In 20 days, there is no substance remaining.
2) After two half-lives, there is 25% of the substance remaining.
3) The amount of the substance remaining can also be modeled by $A(t) = 50(2)^{\frac{-t}{3}}$.
4) After one week, there is less than 10g of the substance remaining.

11. If $f(t) = 50(.5)^{\frac{t}{5715}}$ represents a mass, in grams, of carbon-14 remaining after *t* years, which statement(s) must be true?

I. The mass of the carbon-14 is decreasing by half each year.
II. The mass of the original sample is 50 g.

1) I, only
2) II, only
3) I and II
4) neither I nor II

Converting Rates (For example, from annual to monthly)

Phil step 1: Raise what's inside the parenthesis to the $\frac{1}{n}$ power if the timeframe is decreasing. If the timeframe is increasing, raise to the *n* power.

Phil step 2: Read carefully for what the variable represents.

How many times per year do you get the monthly rate? 12y

How many times per month do you get the monthly rate? 1m

How many times per year do you get the yearly rate? 1y

How many times per month do you get the yearly rate? $\frac{m}{12}$

1. Stephanie found that the number of white-winged cross bills in an area can be represented by the formula $C = 550(1.08)^t$, where *t* represents the number of years since 2010. Which equation correctly represents the number of white-winged cross bills in terms of the monthly rate of population growth?

1) $C = 550(1.00643)^t$

2) $C = 550(1.00643)^{12t}$

3) $C = 550(1.08)^{12t}$

4) $C = 550(1.00643)^{\frac{t}{12}}$

2. The value of a stock after t years can be modeled by the function $V = 2500(1.14)^t$ after *t* years. Which function would represent the weekly rate of increase after *w* weeks?

1) $V = 2500(1.14)^w$

2) $V = 2500(1.14)^{52w}$

3) $V = 2500(1.0025)^w$

4) $V = 2500(1.0025)^{52w}$

3. The value of a home after t years can be modeled by the function $A = 525000(1.36)^t$ after *t* years. Which function would represent the monthly rate of increase after *m* months?

2) $A = 525000(1.36)^m$

2) $A = 525000(1.36)^{12m}$

3) $A = 525000(1.026)^m$

4) $A = 525000(1.026)^{12m}$

4. A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, $B(t)$, can be represented by the function $B(t) = 750(1.16)^t$, where the *t* represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function

1) $B(t) = 750(1.012)^t$

2) $B(t) = 750(1.16)^{12t}$

3) $B(t) = 750(1.012)^{12t}$

4) $B(t) = 750(1.16)^{\frac{t}{12}}$

5. Mia has a student loan that is in deferment, meaning that she does not need to make payments right now. The balance of her loan account during her deferment can be represented by the function $f(x) = 35,000(1.0325)^x$, where x is the number of years since the deferment began. If the bank decides to calculate her balance showing a monthly growth rate, an approximately equivalent function would be

1) $f(x) = 35,000(1.0027)^{12x}$
2) $f(x) = 35,000(1.0027)^{\frac{x}{12}}$
3) $f(x) = 35,000(1.0325)^{12x}$
4) $f(x) = 35,000(1.0325)^{\frac{x}{12}}$

6. The value of Kiara's stock can be modeled by $A = 2500(1.016)^m$ where m represents the number of months her stock has been invested. Which equation would represent the yearly growth rate of her stock after y years?

1) $A = 2500(1.0013)^y$
2) $A = 2500(1.2098)^y$
3) $A = 2500(1.0013)^{\frac{y}{12}}$
4) $A = 2500(1.2098)^{\frac{y}{12}}$

7. The value of Kiara's stock can be modeled by $A = 2500(1.016)^m$ where m represents the number of months her stock has been invested. Which equation would represent the yearly growth rate of her stock after m months?

1) $A = 2500(1.0013)^m$
2) $A = 2500(1.2098)^m$
3) $A = 2500(1.2098)^{12m}$
4) $A = 2500(1.2098)^{\frac{m}{12}}$

8. Nvidia stock has been increasing by 1.7% each day according to the formula $v(t) = 1000(1.017)^t$ where t represents days. Which of the following equations can be used to find the weekly growth rate after w weeks?

1) $v(w) = 1000(1.0024)^w$
2) $v(w) = 1000(1.1252)^w$
3) $v(w) = 1000(1.0024)^{\frac{w}{7}}$
4) $v(w) = 1000(1.1252)^{7w}$

9. Rose's bank account has been increasing according to the equation $A = 7500(1.0098)^w$ where w represents weeks. Which of the following equations can be used to find the yearly growth rate after w weeks?

1) $A = 7500(1.0002)^{52w}$
2) $A = 7500(1.0002)^{\frac{w}{52}}$
3) $A = 7500(1.6605)^{\frac{w}{52}}$
4) $A = 7500(1.6605)^w$

10. The population of Schlansky, Utah is increasing according to the formula $p(t) = 10421(1.23)^t$ after t years. Which expression can represent the weekly growth rate, after w weeks?

1) $10421(1.23)^{52w}$
2) $10421(1.004)^{52w}$
3) $10421(1.23)^w$
4) $10421(1.004)^w$

11. To prepare for lacrosse tryouts, Kole is increasing the amount of time he spends at the gym. This week he is spending 150 minutes there and he plans to increase this amount by 2% each week. The amount of time, in minutes, that he plans to spend at the gym t weeks from now is given by the function $A(t) = 150(1.02)^t$. In terms of a daily growth rate, the amount of time Kole is planning to spend at the gym can best be modeled by the function

1) $A(t) = 150(1.14869)^{\frac{t}{7}}$
2) $A(t) = 150(1.14869)^{7t}$
3) $A(t) = 150(1.00283)^{\frac{t}{7}}$
4) $A(t) = 150(1.00283)^{7t}$

12. Cameron's YouTube video currently has 1200 views and can be modeled by the expression $1200(1.102)^d$ where d represents days. Which expression represents the weekly rate after t weeks.

1) $1200(1.9737)^t$
2) $1200(1.9737)^{\frac{t}{7}}$
3) $1200(1.0140)^t$
4) $1200(1.0140)^{\frac{t}{7}}$

13. The number of people who have read an article grows exponentially throughout the day and can be modeled by the function $N(t) = 2(1.0098)^t$, where t represents the number of minutes since the article has been posted. Which equation best represents the number of people who have read the article in terms of the growth rate per second?

1) $N(t) = 2(1.000163)^{\frac{t}{60}}$
2) $N(t) = 2(1.000163)^{60t}$
3) $N(t) = 2(1.79524)^{\frac{t}{60}}$
4) $N(t) = 2(1.79524)^{60t}$

14. A student studying public policy created a model for the population of Detroit, where the population decreased 25% over a decade. He used the model $P = 714(0.75)^d$, where P is the population, in thousands, d decades after 2010. Another student, Suzanne, wants to use a model that would predict the population after y years. Suzanne's model is best represented by

1) $P = 714(0.6500)^y$
2) $P = 714(0.8500)^y$
3) $P = 714(0.9716)^y$
4) $P = 714(0.9750)^y$

15. The black bear population for a certain area of the Adirondacks can be modeled by the equation $B = 5835.943(1.026)^t$, where t is measured in years since 2010. Kieran would like to rewrite this model in terms of a 5-year growth rate. Kieran's model is best represented by

1) $B = 5835.943(1.005147)^{\frac{t}{5}}$
2) $B = 5835.943(1.005147)^{5t}$
3) $B = 5835.943(1.136938)^{\frac{t}{5}}$
4) $B = 5835.943(1.136938)^{5t}$

16. According to the USGS, an agency within the Department of Interior of the United States, the frog population in the U.S. is decreasing at the rate of 3.79% per year. A student created a model, $P = 12,150(0.962)^t$, to estimate the population in a pond after t years. The student then created a model that would predict the population after d decades. This model is best represented by

1) $P = 12,150(0.461)^d$
2) $P = 12,150(0.679)^d$
3) $P = 12,150(0.996)^d$
4) $P = 12,150(0.998)^d$

Sequences:

Arithmetic: add a constant difference, Geometric: multiply by a common ratio

Explicit Formulas (From Reference Sheet)

Arithmetic: $a_n = a_1 + (n-1)d$ Geometric: $a_n = a_1(r)^{n-1}$

If initial or a_0 is given, $(n-1)$ becomes *n*. Same formulas as Algebra I modeling.

Arithmetic: $a_n = a_0 + nd$ Geometric: $a_n = a_0(r)^n$

Recursive Formula

$a_1 =$

$a_n = a_{n-1}$

Write an explicit AND recursive equation for the following sequences and find the tenth term. Round to the nearest tenth if necessary

1. 19, 16, 13, 10 …

2. 2,8,32,128,…

3. 3, -12, 48, -192, ...

4. 63, 57, 51, 45, ...

1. 329.6, 376.8, 424, 471.2,…

6. 120, 192, 307.2, 491.52

7. 5400, 4050, 3037.5, 2278.125

8. 5205.20, 4208.15, 3211.1, 2214.05

9. The formula below can be used to model which scenario?

$$a_1 = 3000$$

$$a_n = 0.80a_{n-1}$$

1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
3) A bank account starts with a deposit of $3000, and each year it grows by 80%.
4) The initial value of a specialty toy is $3000, and its value each of the following years is 20% less.

10. The formula below can be used to model which scenario?

$a_0 = 92.2$

$a_n = 1.015a_{n-1}$

1) The initial population of a county is 92.2 thousand and it is increasing by 15% each year.
2) The initial population of a county is 92.2 thousand and it is increasing by 1.5% each year.
3) The population after one year is 92.2 thousand and it is increasing by 15% each year.
4) The population after one year is 92.2 thousand and it is increasing by 1.5% each year.

11. The sequence defined by $r_1 = 15$ and $r_n = 0.75r_{n-1}$ best models which scenario?

1) Gerry's $15 allowance is increased by $0.75 each week.
2) A store that has not sold a $15 item reduces the price by $0.25 each week until someone purchases it.
3) A 15-gram sample of a chemical compound decays at a rate of 75% per hour.
4) A picture with an area of 15 square inches is reduced by 25% over and over again to make a proportionally smaller picture.

12. The sequence defined by $a_1 = 20$ and $a_n = 1.05a_{n-1}$ best models which scenario?

1) Jamal scored 20 baskets the first week and scores 5 more baskets each week.
2) Julie made $20 her first month working and earns 5% more each month.
3) Samantha creates 20 paintings the first year and makes 50% more paintings each year.
4) Jennifer's flower is 20 inches tall on day 1 and increases by .05 inches each day.

13. Which situation *cannot* be modeled by the formula $a_n = a_{n-1} + 20$ with $a_1 = 10$?

1) Nancy put $10 in her piggy bank on the first day and then added $20 daily to her piggy bank.
2) Jay has a box of ten crayons and his teacher gives him twenty new crayons each month for good behavior.
3) Buzz has ten apples and that number increases by 20% per week.
4) Teresa has a block of metal that is 10°F and she heats it up at a rate of 20°F per minute.

14. Which situation *can* be modeled by the formula $a_n = 1.025a_{n-1}$ with $a_0 = 100$?

1) Devin has $100 saved and he will increase that amount by $2.50 each week.
2) Catherine has 100 Pokemon cards and gets 25% more each week.
3) Lucas has 100 points and each week increases by 2.5%.
4) Olivia's plant is 100 cm tall and it grows .025 cm each week.

15. Which situation *cannot* be modeled by the formula $a_n = a_{n-1} - 6$ with $a_0 = 1000$?

1) A bank account with an initial balance of $1000 increases by 6% each year.
2) Taylor is assigned 1000 SAT problems and completes 6 each day.
3) The starting population of fish in a pond is 1000 and the population decreases by 6% each day.
4) Jessica has $1000 saved and saves an additional $6 each week.

16. The height of Jenny's sunflower when she planted it was 6 inches. The sunflower grows by 0.25 inches per day. Which formula can be used to determine the height, in inches, of Jenny's sunflower on day *n*?

(1) $h_0 = 6$, $h_n = 0.25a_{n-1}$

(2) $h_0 = 6$, $h_n = 6 + 0.25h_{n-1}$

(3) $h_0 = 6$, $h_n = h_{n-1} + 0.25$

(4) $h_0 = 6$, $h_n = 6h_{n-1} + 0.25$

17. A population of bacteria triples every day. If on the first day there are 300 bacteria in a Petri dish, which recursive sequence can be used to determine the population on day *n*?

1) $b_1 = 300$, $b_n = 3b_{n-1}$

2) $b_1 = 300$, $b_n = b_{n-1} + 3$

3) $b_1 = 300$, $b_n = 300(3b_{n-1})$

4) $b_1 = 300$, $b_n = \frac{1}{3}b_{n-1}$

18. A lumber yard has 1500 2" by 4" pieces of wood that need to be transported to a construction site. A truck can take 100 pieces of wood per trip. Which sequence can be used to determine the number of pieces of wood left at the lumberyard after *n* trips?

(1) $a_0 = 1500$, $a_n = a_{n-1} - 100$

(3) $a_0 = 1500$, $a_n = 1500 - 100a_{n-1}$

(2) $a_0 = 1500$, $a_n = 100 - a_{n-1}$

(4) $a_0 = 1500$, $a_n = 100 - 1500a_{n-1}$

19. Daniela invested $2000 in a stock that increases by 1.6% each week. Which of the following recursive sequences represents the value of her stock after n weeks?

1) $a_0 = 2000$, $a_n = a_{n-1} + 1.6$

3) $a_0 = 2000$, $a_n = 1.6a_{n-1}$

2) $a_0 = 2000$, $a_n = a_{n-1} + 1.016$

4) $a_0 = 2000$, $a_n = 1.016a_{n-1}$

20. At her job, Pat earns $25,000 the first year and receives a raise of $1000 each year. The explicit formula for the *n*th term of this sequence is $a_n = 25,000 + (n-1)1000$. Which rule best represents the equivalent recursive formula?

1) $a_n = 24,000 + 1000n$

3) $a_1 = 25,000,\ a_n = a_{n-1} + 1000$

2) $a_n = 25,000 + 1000n$

4) $a_1 = 25,000,\ a_n = a_{n+1} + 1000$

21. The average depreciation rate of a new boat is approximately 8% per year. If a new boat is purchased at a price of $75,000, which model is a recursive formula representing the value of the boat *n* years after it was purchased?

1) $a_n = 75,000(0.08)^n$

3) $a_n = 75,000(1.08)^n$

2) $a_0 = 75,000$, $a_n = (0.92)^n$

4) $a_0 = 75,000$, $a_n = 0.92(a_{n-1})$

22. An initial investment of $5000 in an account earns 3.5% annual interest. Which function correctly represents a recursive model of the investment after *n* years?

1) $A = 5000(0.035)^n$

3) $A = 5000(1.035)^n$

2) $a_0 = 5000$, $a_n = a_{n-1}(0.035)$

4) $a_0 = 5000$, $a_n = a_{n-1}(1.035)$

23. MathSchlansky posts a video to his YouTube channel and it receives 4 views on the first day. Each day after that, the number of views increases by 7%. Which sequence can be used to determine the number of views his video receives after *n* days?

1) $a_1 = 4$
$a_n = a_{n-1} + 7$

2) $a_1 = 4$
$a_n = a_{n-1} + 1.07$

3) $a_1 = 4$
$a_n = .07a_{n-1}$

4) $a_1 = 4$
$a_n = 1.07a_{n-1}$

24. A tree farm initially has 150 trees. Each year, 20% of the trees are cut down and 80 seedlings are planted. Which recursive formula models the number of trees, a_n, after *n* years?

1) $a_1 = 150$
$a_n = a_{n-1}(0.2) + 80$

2) $a_1 = 150$
$a_n = a_{n-1}(0.8) + 80$

3) $a_n = 150(0.2)^n + 80$

4) $a_n = 150(0.8)^n + 80$

25. A recursive formula for the sequence $18, 9, 4.5, \ldots$ is

1) $g_1 = 18$
$g_n = \frac{1}{2}g_{n-1}$

2) $g_n = 18\left(\frac{1}{2}\right)^{n-1}$

3) $g_1 = 18$
$g_n = 2g_{n-1}$

4) $g_n = 18(2)^{n-1}$

26. A recursive formula for the sequence $40, 30, 22.5, \ldots$ is

1) $g_n = 40\left(\frac{3}{4}\right)^n$

2) $g_1 = 40$
$g_n = g_{n-1} - 10$

3) $g_n = 40\left(\frac{3}{4}\right)^{n-1}$

4) $g_1 = 40$
$g_n = \frac{3}{4}g_{n-1}$

27. A recursive formula for the sequence $64, 48, 36, \ldots$ is

1) $a_n = 64(0.75)^{n-1}$

2) $a_1 = 64$
$a_n = a_{n-1} - 16$

3) $a_n = 64 + (n-1)(-16)$

4) $a_1 = 64$
$a_n = 0.75a_{n-1}$

28. After Roger's surgery, his doctor administered pain medication in the following amounts in milligrams over four days.

Day (n)	1	2	3	4
Dosage (m)	2000	1680	1411.2	1185.4

How can this sequence best be modeled recursively?

1) $m_1 = 2000$
$m_n = m_{n-1} - 320$

2) $m_n = 2000(0.84)^{n-1}$

3) $m_1 = 2000$
$m_n = (0.84)m_{n-1}$

4) $m_n = 2000(0.84)^{n+1}$

21. The population of Jamesburg for the years 2010-2013, respectively, was reported as follows:
250,000 250,937 251,878 252,822
How can this sequence be recursively modeled?

1) $j_n = 250,000(1.00375)^{n-1}$

2) $j_1 = 250,000$
$j_n = 1.00375j_{n-1}$

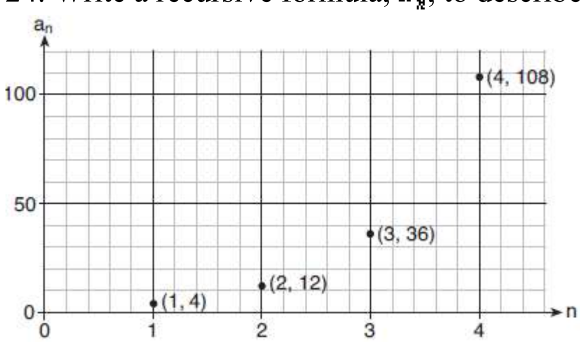
3) $j_n = 250,000 + 937^{(n-1)}$

4) $j_1 = 250,000$
$j_n = j_{n-1} + 937$

22. Write a recursive formula for the sequence $6, 9, 13.5, 20.25, \ldots$

23. Write a recursive formula for the sequence $189, 63, 21, 7, \ldots$.

24. Write a recursive formula, a_n, to describe the sequence graphed below.



25. The explicit formula $a_n = 6 + 6n$ represents the number of seats in each row in a movie theater, where *n* represents the row number. Rewrite this formula in recursive form.

Evaluating Recursive Sequences

a_{n-1} means the previous term!

1) Start with the term after the one they give you
2) Substitute the previous term in for a_{n-1}

n is the term that you are finding

1. Find the first 4 terms of the sequence $a_n = a_{n-1} + 4$ where a $a_1 = -1$.

2. Find the first 4 terms of the sequence $a_n = 4a_{n-1}$ where $a_1 = 12$.

3. Find the first 4 terms of the recursive sequence $\begin{array}{l} a_1 = -3 \\ a_n = 4 - 3a_{n-1} \end{array}$

4. If $a_n = 3a_{n-1} - 4$ and $a_1 = 9$, find a_5

5. Find the 8th term for the sequence where $a_n = 5a_{n-1} + 2n$ where $a_5 = 3$

6. Find the first four terms of the recursive sequence defined below.

$a_1 = -3$

$a_n = a_{(n-1)} - n$

7. A sequence is defined recursively by $f(1) = 16$ and $f(n) = f(n-1) + 2n$. Find $f(4)$.

(1) 32 (2) 30 (3) 28 (4) 34

8. Find the third term in the recursive sequence $a_{k+1} = 2a_k - 1$, where $a_1 = 3$.

9. Which recursively defined function represents the sequence $3, 7, 15, 31, \ldots$?

1) $f(1) = 3,\ f(n+1) = 2^{f(n)} + 3$
2) $f(1) = 3,\ f(n+1) = 2^{f(n)} - 1$
3) $f(1) = 3,\ f(n+1) = 2f(n) + 1$
4) $f(1) = 3,\ f(n+1) = 3f(n) - 2$

10. What is the fourth term of the sequence defined by $a_1 = 3xy^5$

$$a_n = \left(\frac{2x}{y}\right)a_{n-1}?$$

1) $12x^3y^3$
2) $24x^2y^4$
3) $24x^4y^2$
4) $48x^5y$

Series
Formulas are on Reference Sheet!

To write a geometric series (percents) explicitly: $S_n = \frac{a_1(1-r^n)}{1-r}$

Increasing by 5%: r = 1.05
Decreasing by 5%: r = .95

To write a geometry series using summations: $\sum_{k=1}^{n} a_1(r)^{k-1}$

Always use explicit unless it says otherwise

Arithmetic Series: $S_n = \frac{n(a_1 + a_n)}{2}$. You must find a_n using $a_n = a_1 + d(n-1)$ first.

1. Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year. Write a geometric series formula, S_n, for Alexa's total earnings over *n* years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

2. Ross has a hobby of collecting comic books. He currently has 50 comic books and each year, he will increase his collection by 15%. Write a geometric series formula, S_n, for Ross' total amount of comic books after *n* years. Use this formula to find the total number of comic books Ross will have 12 years from now.

3. Dee is planning on decreasing the amount of time she eats fast food per month. After the first month, she ate fast food 42 times. Each month, she eats at fast food restaurants 10% less than the previous month. Write a geometric series formula, S_n, for the total amount of fast food Dee eats after *n* months. Using your formula, how many total times does she eat fast food in the first four months? Round your answer to the nearest integer.

4. Kina earns a $27,000 salary for the first year of work at her job. She earns annual increases of 2.5%. What is the total amount, to the *nearest cent*, that Kina will earn for the first eight years at this job?

5. Rowan is training to run in a race. He runs 15 miles in the first week, and each week following, he runs 3% more than the week before. Using a geometric series formula, find the total number of miles Rowan runs over the first ten weeks of training, rounded to the *nearest thousandth*.

6. A 7-year lease for office space states that the annual rent is $85,000 for the first year and will increase by 6% each additional year of the lease. What will the total rent expense be for the entire 7-year lease?

7. A fisherman harvests 350 kilograms of crab on Monday. From Monday to Friday, the fisherman harvests 8% less kilograms of crab per day. To the *nearest tenth of a kilogram*, what is the total amount of crab harvested between Monday and Friday?

8. A ball is dropped from a height of 32 feet. It bounces and rebounds 80% of the height from which it was falling. What is the total downward distance, in feet, the ball traveled up to the 12th bounce?

9. Your parents want you to do some work around the house. You get them to agree to pay you \$.01 on the first day, \$.02 on the second day, \$.04 on the third day, and so on.
At the end of the 30-day month, what is the total amount of money your parents have paid you, to the *nearest cent*?

10. On Sunday, the first day of the week, Natasha does 5 pushups. Each day, she doubles the amount of pushups she does. How many total pushups will Tasha complete at the end of the 7 day week?

11. Samantha logged her weekly running distances in the table below. If the continues increasing her distance at this rate, what is the total amount of miles Samantha will have ran after 10 weeks to the nearest tenth of a mile?

Week	Distance (In Miles)
1	12
2	14.4
3	17.28
4	20.736

12. Brian deposited 1 cent into an empty non-interest bearing bank account on the first day of the month. He then additionally deposited 3 cents on the second day, 9 cents on the third day, and 27 cents on the fourth day. What would be the total amount of money in the account at the end of the 20th day if the pattern continued?

13. Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

1) $\sum_{n=1}^{6} 8(1.10)^{n-1}$

2) $\sum_{n=1}^{6} 8(1.10)^{n}$

3) $\frac{8-8(1.10)^6}{0.90}$

4) $\frac{8-8(0.10)^n}{1.10}$

14. In his first year running track, Brendon earned 8 medals. He increases his amount of medals by 25% each year. Which of the following expressions can be used to determine how many total medals Brendon will have after four years of high school?

1) $\frac{8-8(0.25)^4}{-.25}$

2) $\sum_{n=1}^{4} 8(0.25)^{n-1}$

3) $\frac{8-8(1.25)^4}{1-.25}$

4) $\sum_{n=1}^{4} 8(1.25)^{n-1}$

15. A company fired several employees in order to save money. The amount of money the company saved per year over five years following the loss of employees is shown in the table below.

Year	**Amount Saved** (in dollars)
1	59,000
2	64,900
3	71,390
4	78,529
5	86,381.9

Which expression determines the total amount of money saved by the company over 5 years?

1) $\frac{59,000-59,000(1.1)^5}{1-1.1}$

2) $\frac{59,000-59,000(0.1)^5}{1-0.1}$

3) $\sum_{n=1}^{5} 59,000(1.1)^{n}$

4) $\sum_{n=1}^{5} 59,000(0.1)^{n-1}$

Mortgage/Annuities
The formulas will be given to you for each problem!
Amount of loan (P) = total cost – down payment
Number of monthly payments (n) = 12(# of years)

1. Jim is looking to buy a vacation home for $172,600 near his favorite southern beach. The formula to compute a mortgage payment, *M*, is $M = P \bullet \frac{r(1+r)^N}{(1+r)^N - 1}$ where *P* is the principal amount of the loan, *r* is the monthly interest rate, and *N* is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage. With a $20,000 down payment, determine Jim's mortgage payment, rounded to the *nearest dollar*.

2. Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the *nearest cent*.

$$P_n = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

P_n = present amount borrowed
n = number of monthly pay periods
PMT = monthly payment
i = interest rate per month

3. Monthly mortgage payments can be found using the formula below:

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{n}}{\left(1+\frac{r}{12}\right)^{n}-1}$$

M = monthly payment
P = amount borrowed
r = annual interest rate
n = number of monthly payments

The Banks family would like to purchase a home for $220,000. They qualified for an annual interest rate of 4.8%. If they put make a down payment of $100,000 and plan to spend 15 years to repay the loan, what will be the monthly payment rounded to the *nearest* cent?

4. The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is $152,500 and they will make a $15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the *nearest dollar*.

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{n}}{\left(1+\frac{r}{12}\right)^{n}-1}$$

M = monthly payment
P = amount borrowed
r = annual interest rate
n = total number of monthly payments

5. Monthly mortgage payments can be found using the formula below, where M is the monthly payment, P is the amount borrowed, r is the annual interest rate, and n is the total number of monthly payments. If Adam takes out a 15-year mortgage, borrowing \$240,000 at an annual interest rate of 4.5%, What will his monthly payment be?

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^{n}}{\left(1 + \frac{r}{12}\right)^{n} - 1}$$

6. Robert is buying a car that costs \$22,000. After a down payment of \$4000, he borrows the remainder from a bank, a six year loan at 6.24% annual interest rate. The following formula can be used to calculate his monthly loan payment. What will Robert's monthly payment be?

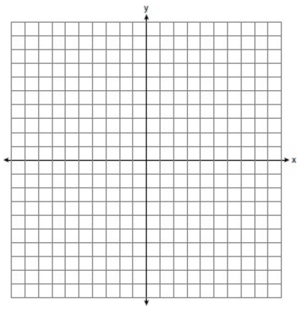
$$R = \frac{(P)(i)}{1 - (1 + i)^{-t}}$$

R = monthly payment
P = loan amount
i = monthly interest rate
t = time, in months

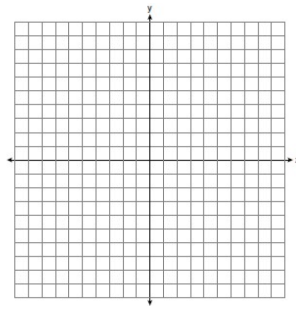
Sketching Radian Angles

Radians to degrees: Multiply by $\dfrac{180}{\pi}$

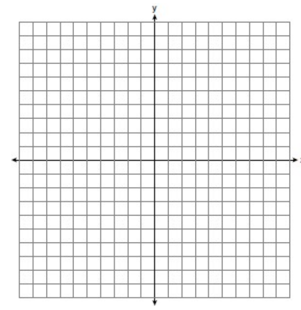
1. $\theta = \dfrac{5\pi}{3}$



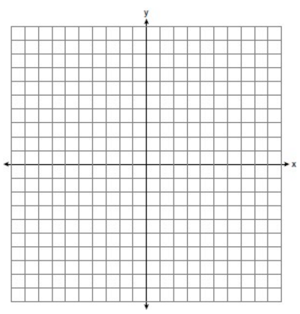
2. $\theta = \dfrac{7\pi}{4}$



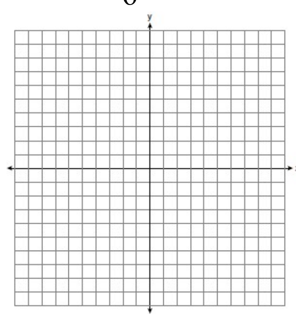
3. $\theta = 2$



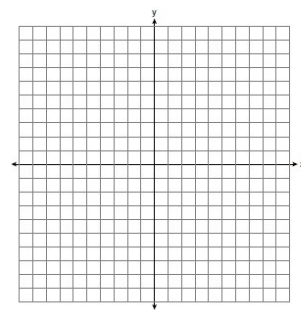
4. $\theta = 4.1$



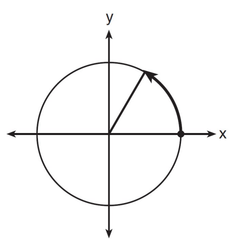
5. $\theta = -\dfrac{\pi}{6}$

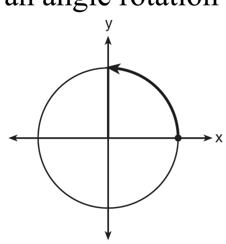


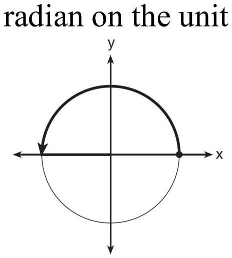
6. $\theta = 9.2$

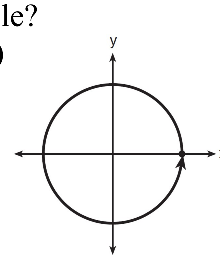


7. Which diagram shows an angle rotation of 1 radian on the unit circle?

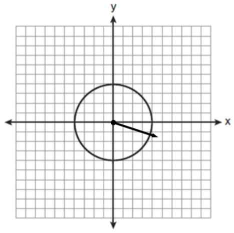
1)


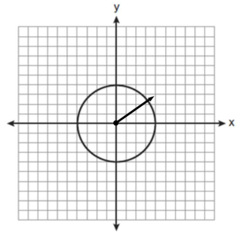
2)


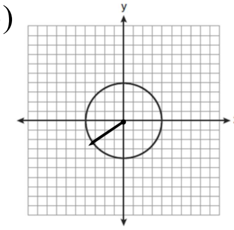
3)


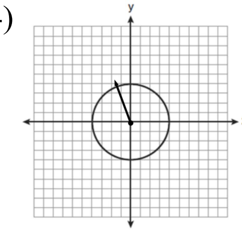
4)


8. Which of the following sketches would represent 6 radians?

1)


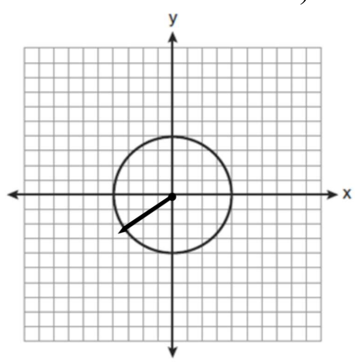
2)


3)


4)


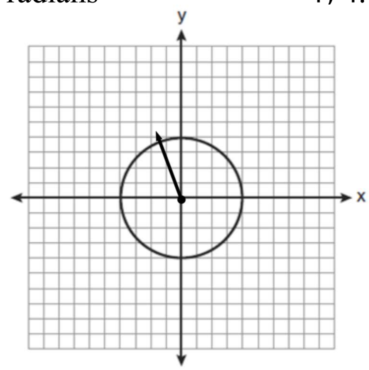
9. Which angle is sketched below?

1) 2.4 radians
2) 4.5 radians
3) 3.8 radians
4) 5.2 radians

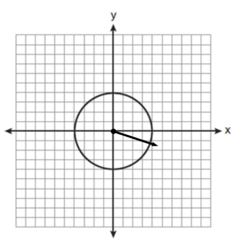


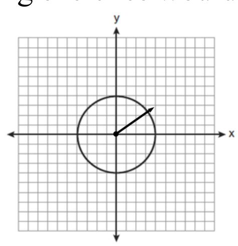
10. Which angle is sketched below?

1) 1 radian
2) 1.7 radians
3) 3 radians
4) 4.1 radians



11. Which of the following sketches would represent 3.9 radians?

1)


2)


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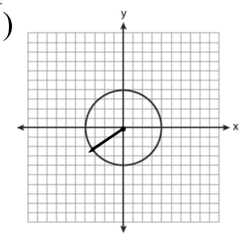

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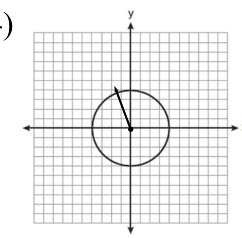
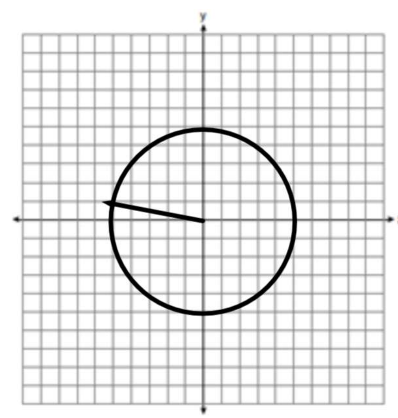


12. Which of the following can be the radian measure of the angle sketched below?

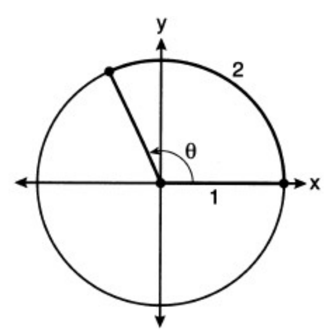
1) 1.5
2) 3
3) 3.8
4) 5



13. An angle, θ, is rotated counterclockwise on the unit circle, with its terminal side in the second quadrant, as shown in the diagram below.

Which value represents the radian measure of angle θ?

1) 1
2) 2
3) 65.4
4) 114.6



Evaluating Special Angles

If multiple choice, type the problem in, type in each answer, see what matches up.

If open response, Q (quadrant), S (sign), F (trig function), R (reference angle). Match up to your table of special values.

	30	45	60
Sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
Cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

1. What is the exact value of $\cos\left(\frac{5\pi}{6}\right)$?

1) $\frac{\sqrt{3}}{2}$
2) $\frac{1}{2}$
3) $-\frac{\sqrt{3}}{2}$
4) $-\frac{1}{2}$

2. What is the exact value of $\tan\left(\frac{3\pi}{4}\right)$?

1) 1
2) -1
3) $\sqrt{3}$
4) $-\sqrt{3}$

3. What is the exact value of $\cos\left(-\frac{5\pi}{6}\right)$?

1) $\frac{\sqrt{3}}{2}$
2) $\frac{1}{2}$
3) $-\frac{\sqrt{3}}{2}$
4) $-\frac{\sqrt{2}}{2}$

4. The exact value of $\sin\left(\frac{8\pi}{3}\right)$ is

1) $\frac{1}{2}$
2) $-\frac{1}{2}$
3) $-\frac{\sqrt{3}}{2}$
4) $\frac{\sqrt{3}}{2}$

5. What is the exact value of $\sec\left(\frac{2\pi}{3}\right)$?

1) -2

2) $\frac{-2\sqrt{3}}{3}$

3) 2

4) $\frac{2\sqrt{3}}{3}$

6. What is the exact value of $\cot\left(\frac{5\pi}{3}\right)$?

1) $\frac{1}{\sqrt{3}}$

2) $-\frac{1}{\sqrt{3}}$

3) $\sqrt{3}$

4) $-\sqrt{3}$

7. The value of $\cos\left(\frac{5\pi}{6}\right)$

1) $\sec\left(\frac{5\pi}{6}\right)$

2) $\sin\left(\frac{4\pi}{3}\right)$

3) $\tan\left(\frac{5\pi}{4}\right)$

4) $\sin\left(\frac{2\pi}{3}\right)$

8. The value of $\sin\left(\frac{2\pi}{3}\right)$ is equivalent to

1) $\sin\left(\frac{4\pi}{3}\right)$

2) $\cos\left(\frac{\pi}{6}\right)$

3) $\tan\left(\frac{\pi}{3}\right)$

4) $\csc\left(\frac{2\pi}{3}\right)$

Pythagorean Theorem

Look out for hidden right triangles where you may need to use $a^2 + b^2 =$

a and *b* are the legs

c is the hypotenuse

Find the missing side of each right triangle rounding to the nearest

Know your Pythagorean Triples!

3,4,5

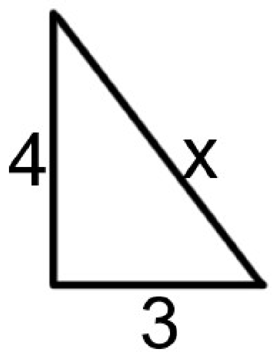
5,12,13

7,24,25

8,15,17

9,40,41

1.



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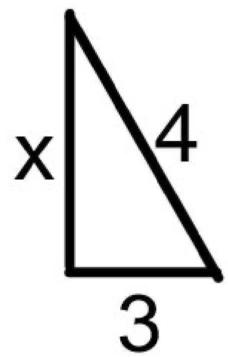

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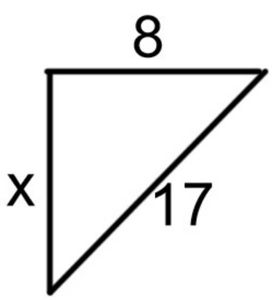

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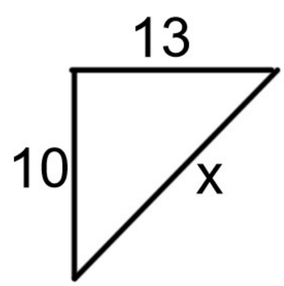

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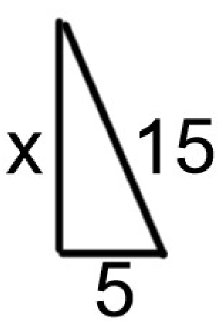
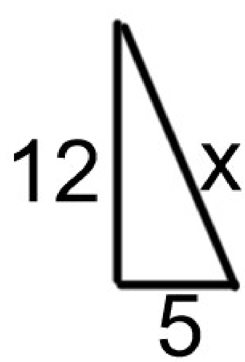


6.



Rationalizing the Denominator

To rationalize the denominator, multiply top and bottom by the radical

When multiplying a radical by itself, the radical cancels out

Rationalize the following denominators

1. $\frac{2}{\sqrt{5}}$

2. $\frac{-7}{\sqrt{11}}$

3. $\frac{3}{\sqrt{2}}$

4. $\frac{6}{\sqrt{3}}$

5. $\frac{4}{\sqrt{6}}$

6. $\frac{-5}{\sqrt{10}}$

Trig Ratios with Triangles

If an angle passes through a point or sin/cos/tan = $\frac{something}{something}$, make a right triangle and use SOHCAHTOA

Any point on the unit circle is $(\cos\theta, \sin\theta)$

Know your Pythagorean triples: {3,4,5}, {5, 12, 13}, {8, 15, 17}, {7, 24, 25}

Reciprocal trig function pairs:

$\csc\theta$	$\sec\theta$	$\tan\theta$
$\sin\theta$	$\cos\theta$	$\cot\theta$

1. If $\sin\theta = -\frac{3}{5}$ and θ is in Quadrant III, find:

a) $\cos\theta$ b) $\sin\theta$ c) $\tan\theta$

d) $\sec\theta$ e) $\csc\theta$ f) $\cot\theta$

2. If $\tan\theta = \frac{24}{7}$ and θ is in Quadrant III, find:

a) $\cos\theta$ b) $\sin\theta$ c) $\tan\theta$

d) $\sec\theta$ e) $\csc\theta$ f) $\cot\theta$

3. Angle θ is in standard position and $(4,-7)$ is a point on the terminal side of θ. Find:

a) $\cos\theta$ b) $\sin\theta$ c) $\tan\theta$

d) $\sec\theta$ e) $\csc\theta$ f) $\cot\theta$

4. Angle θ is in standard position and $(-5,-12)$ is a point on the terminal side of θ. Find:

a) $\cos\theta$ b) $\sin\theta$ c) $\tan\theta$

d) $\sec\theta$ e) $\csc\theta$ f) $\cot\theta$

5. A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ, intercepts the circle in Quadrant I at point C. The y-coordinate of point C is 8. Find:

a) $\cos\theta$ b) $\sin\theta$ c) $\tan\theta$

d) $\sec\theta$ e) $\csc\theta$ f) $\cot\theta$

6. A circle centered at the origin has a radius of 4 units. The terminal side of an angle, θ, intercepts the circle in Quadrant II at point P. The x-coordinate of point P is 2. Find:

a) $\cos\theta$ b) $\sin\theta$ c) $\tan\theta$

d) $\sec\theta$ e) $\csc\theta$ f) $\cot\theta$

7. The point $\left(\frac{3}{5},-\frac{4}{5}\right)$ lines on the unit circle. Find:

a) $\cos\theta$

b) $\sin\theta$

c) $\tan\theta$

d) $\sec\theta$

e) $\csc\theta$

f) $\cot\theta$

8. The point $\left(x,-\frac{2}{3}\right)$ lines on the unit circle where $x > 0$. Find:

a) $\cos\theta$

b) $\sin\theta$

c) $\tan\theta$

d) $\sec\theta$

e) $\csc\theta$

f) $\cot\theta$

9. If $\cos\theta = -\frac{3}{4}$ and θ is in Quadrant III, then $\sin\theta$ is equivalent to

1) $-\frac{\sqrt{7}}{4}$
2) $\frac{\sqrt{7}}{4}$
3) $-\frac{5}{4}$
4) $\frac{5}{4}$

10. If the terminal side of angle θ, in standard position, passes through point $(-4, 3)$, what is the numerical value of $\sin\theta$?

1) $\frac{3}{5}$
2) $\frac{4}{5}$
3) $-\frac{3}{5}$
4) $-\frac{4}{5}$

11. A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ, intercepts the circle in Quadrant II at point *C*. The *y*-coordinate of point *C* is 8. What is the value of $\cos\theta$?

1) $-\frac{3}{5}$
2) $-\frac{3}{4}$
3) $\frac{3}{5}$
4) $\frac{4}{5}$

12. Given $\cos\theta = \frac{7}{25}$, where θ is an angle in standard position terminating in quadrant IV, and $\sin^2\theta + \cos^2\theta = 1$, what is the value of $\tan\theta$?

1) $-\frac{24}{25}$
2) $-\frac{24}{7}$
3) $\frac{24}{25}$
4) $\frac{24}{7}$

13. Given that $\sin^2\theta + \cos^2\theta = 1$ and $\sin\theta = -\frac{\sqrt{2}}{5}$, what is a possible value of $\cos\theta$?

1) $\frac{5+\sqrt{2}}{5}$
2) $\frac{\sqrt{23}}{5}$
3) $\frac{3\sqrt{3}}{5}$
4) $\frac{\sqrt{35}}{5}$

14. Given $\cos A = \frac{3}{\sqrt{10}}$ and $\cot A = -3$, determine the value of $\sin A$ in radical form.

15. An angle, θ, is in standard position and its terminal side passes through the point $(2,-1)$. Find the *exact* value of $\sin\theta$.

16. A circle centered at the origin has a radius of 4 units. The terminal side of an angle, θ, intercepts the circle in Quadrant III at point *P*. The *x*-coordinate of point *P* is 2. What is the value of $\cos\theta$?

17. The terminal side of θ, an angle in standard position, intersects the unit circle at $P\left(-\frac{1}{3},-\frac{\sqrt{8}}{3}\right)$. What is the value of $\sec\theta$?

1) -3

2) $-\frac{3\sqrt{8}}{8}$

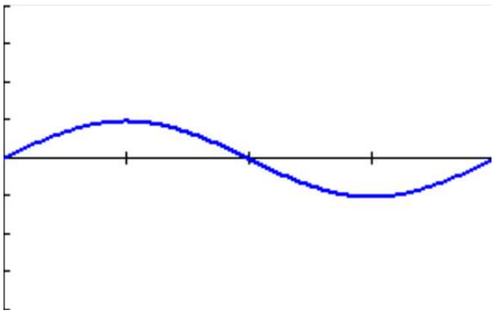
3) $-\frac{1}{3}$

4) $-\frac{\sqrt{8}}{3}$

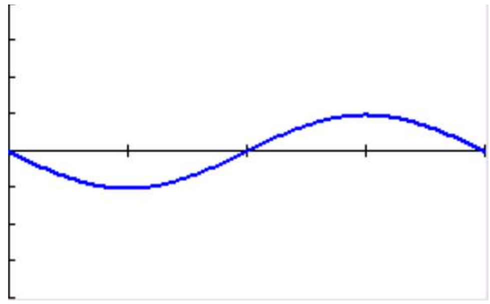
18. Point $M\left(t,\frac{4}{7}\right)$ is located in the second quadrant on the unit circle. Determine the exact value of *t*.

Trig Graphs:
Know what your waves look like!

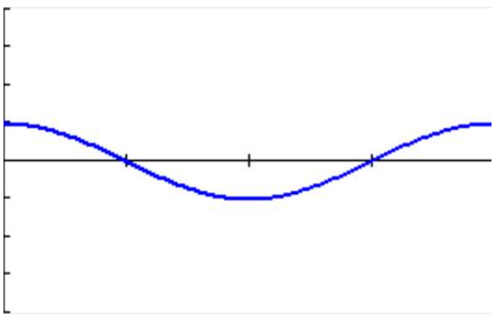
$f(x) = \sin x$



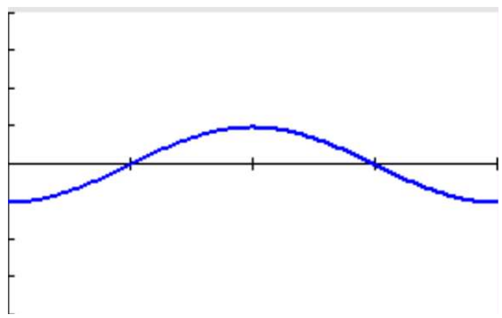
$f(x) = -\sin x$



$f(x) = \cos x$



$f(x) = -\cos x$



AMPSINFREQXSHIFT
Amplitude: Distance from the midline to minimum or maximum
Frequency: How many waves from 0 to 2π
Period: (Wavelength): How long it takes to make one full cycle
Shift/Midline: y value of the midline. The average value of the function.

To graph, list:
Amplitude
$\pm \sin / \cos$
Frequency
Shift/Midline

$$Period = \frac{2\pi}{frequency}$$

y-axis: Plot midline. Count amplitude above and below from the midline.
x-axis: Make 4 dashes on x-axis. Label the 4^{th} dash with the period.
Plot the 5 points for the appropriate wave.

To write the equation, find:

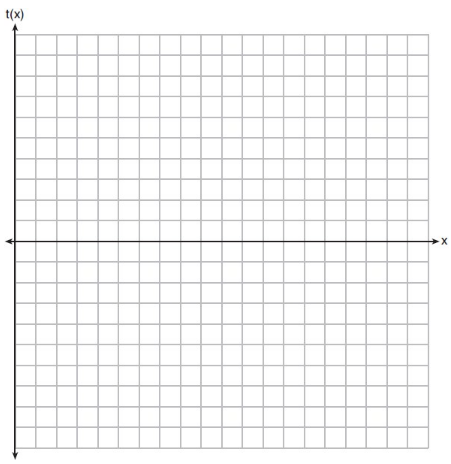
$Frequency = \dfrac{2\pi}{period}$ and $midline = \dfrac{\min + \max}{2}$

Substitute components into $y = amp \sin freqxshift$

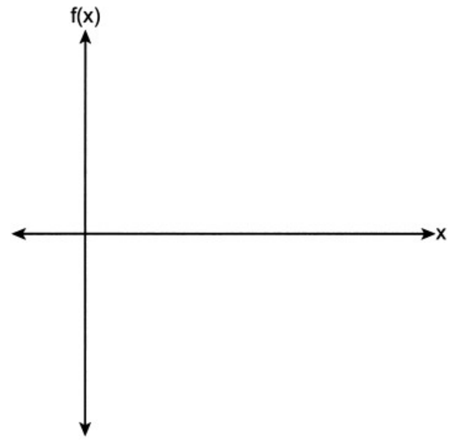
If multiple choice, cross out answers with incorrect components!

Graph one full wave of the following trigonometric functions

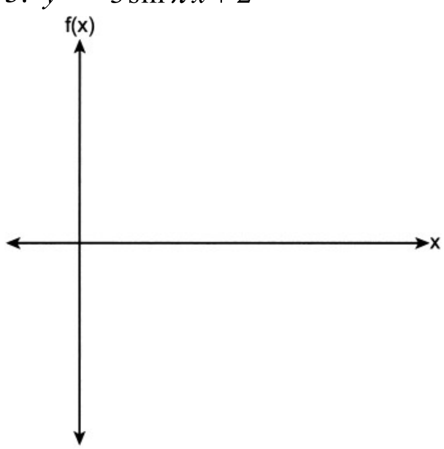
2. $y = 3\sin 2x - 1$



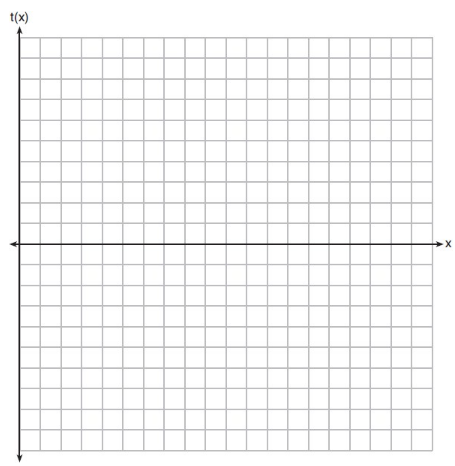
2. $y = 2\cos 4x + 3$



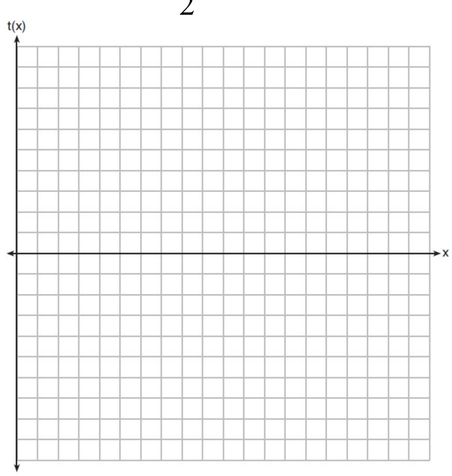
3. $y = -3\sin \pi x + 2$



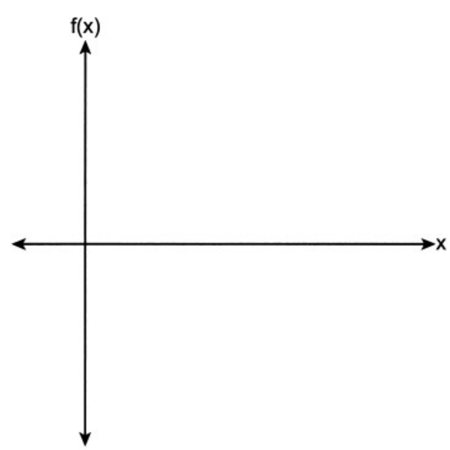
4. $y = -4\cos 2\pi x - 2$



5. $y = -4\sin \frac{1}{2}x + 2$

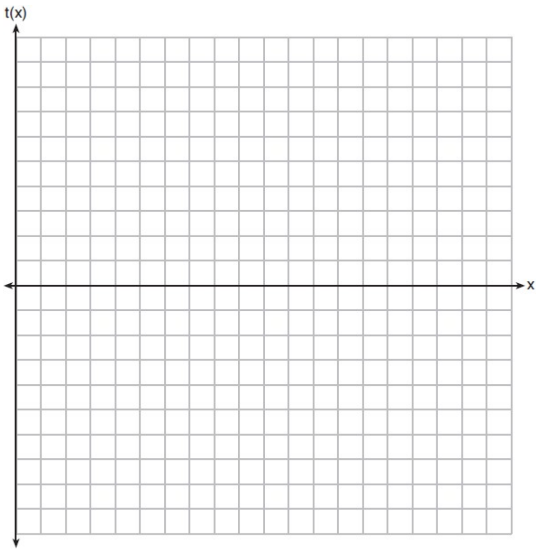


6. $y = -5\cos \frac{1}{3}x + 3$

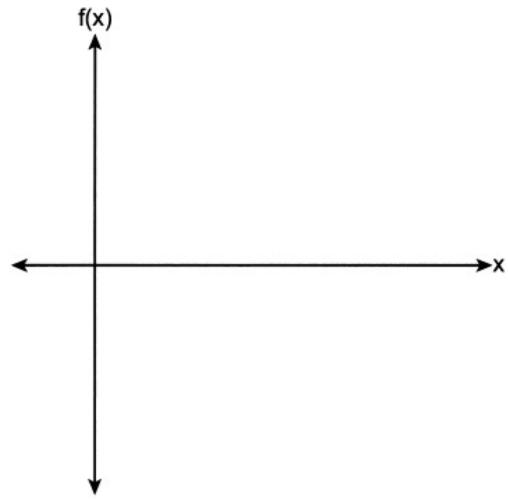


Graph the following two functions over the domain $[0, 2\pi]$ on the set of axes below.

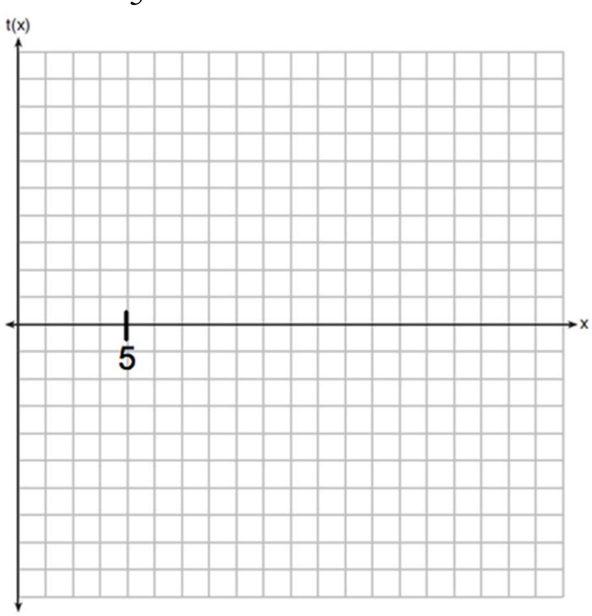
7. $t(x) = 3\sin(2x) + 2$



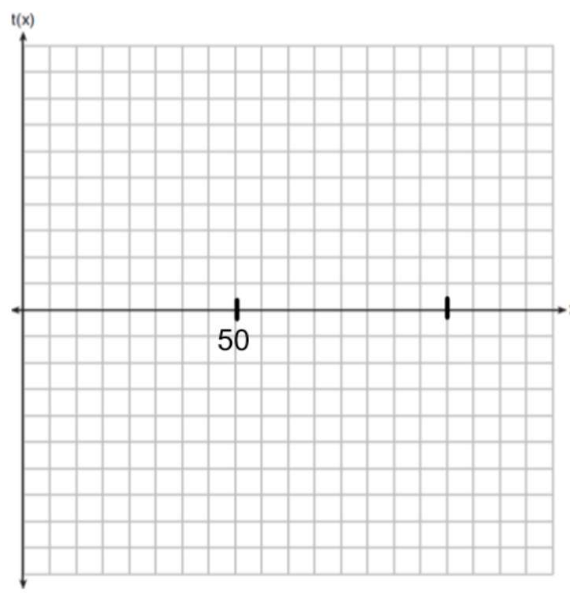
8. $y = -2\cos\frac{1}{2}x + 1$



9. On the set of axes below, graph $y = 3\cos\frac{2\pi}{5}x - 2$ over the domain $[0,10]$

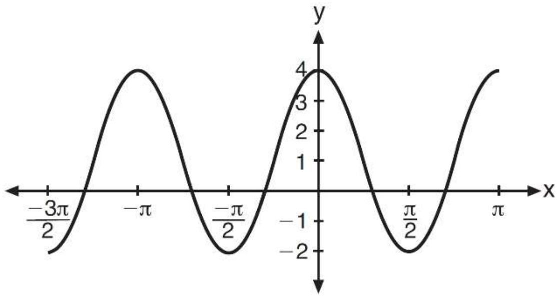


10. On the set of axes below, graph $y = -2\sin\frac{\pi}{50}x + 1$ over the domain $[0,100]$

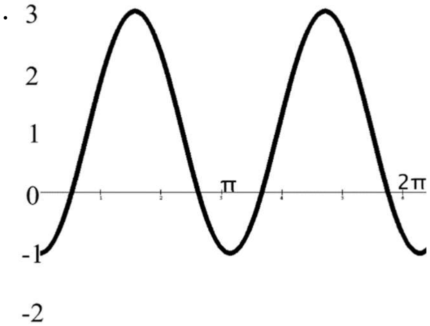


Write the equation of the following graphs

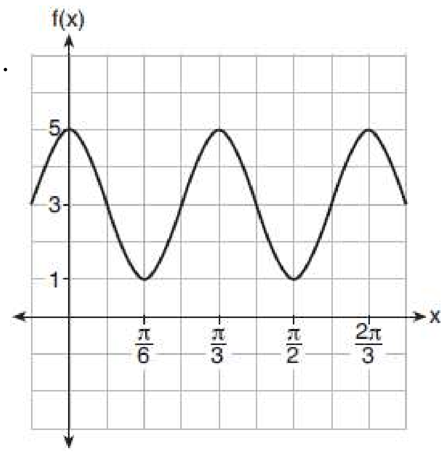
11.



12.



13.



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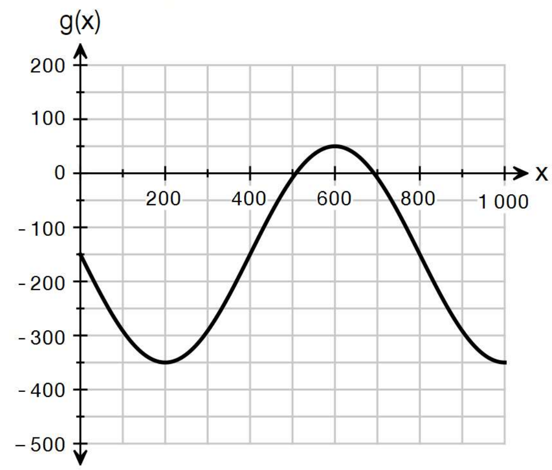
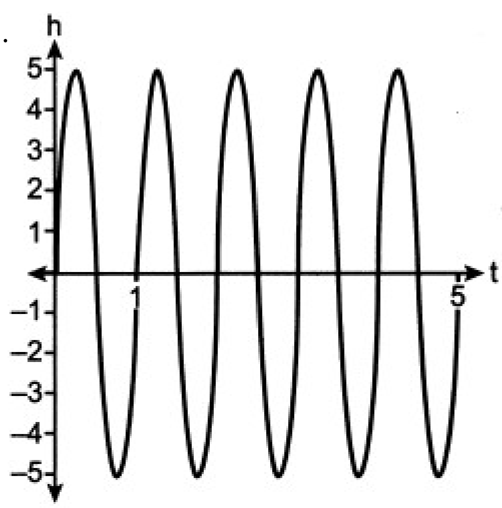
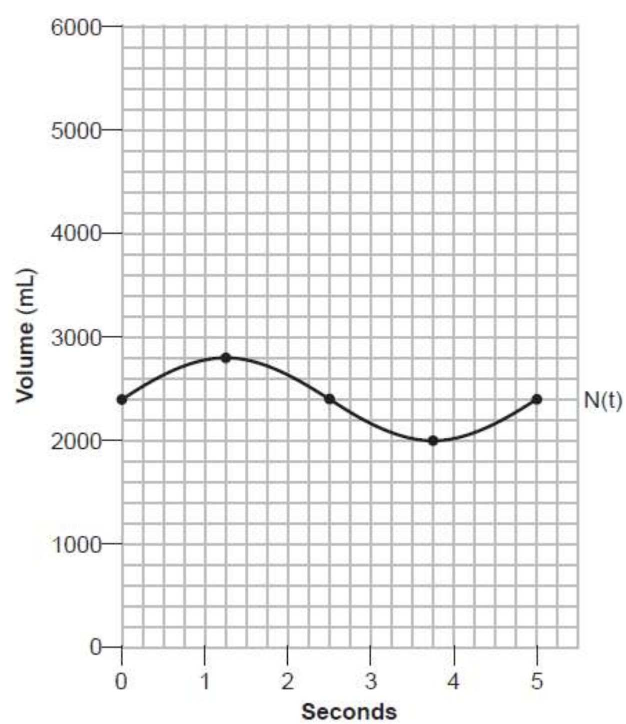


15.

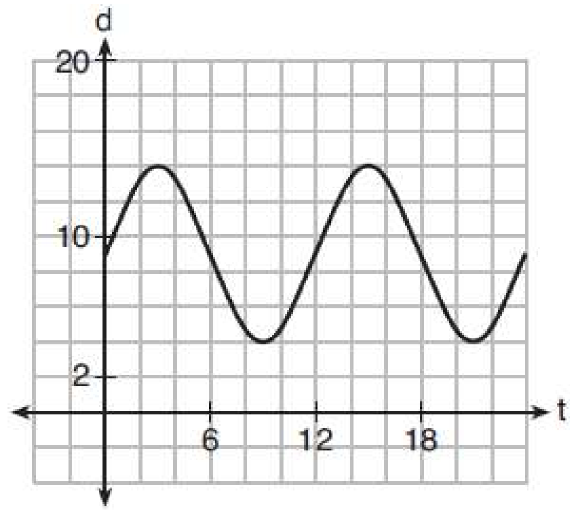


16.



17. The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.

If the depth, d, is measured in feet and time, t, is measured in hours since midnight, what is an equation for the depth of the water at the marker?



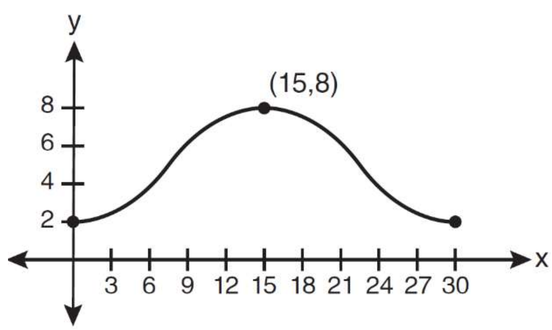
1) $d = 5\cos\left(\frac{\pi}{6}t\right) + 9$

2) $d = 9\cos\left(\frac{\pi}{6}t\right) + 5$

3) $d = 9\sin\left(\frac{\pi}{6}t\right) + 5$

4) $d = 5\sin\left(\frac{\pi}{6}t\right) + 9$

18. Which equation is graphed in the diagram below?



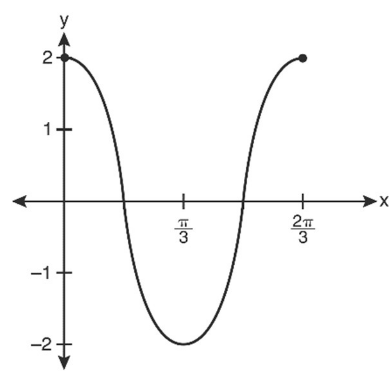
1) $y = 3\cos\left(\frac{\pi}{30}x\right) + 8$

2) $y = 3\cos\left(\frac{\pi}{15}x\right) + 5$

3) $y = -3\cos\left(\frac{\pi}{30}x\right) + 8$

4) $y = -3\cos\left(\frac{\pi}{15}x\right) + 5$

19. Which equation is represented by the graph below?



1) $y = 2\cos 3x$

2) $y = 2\sin 3x$

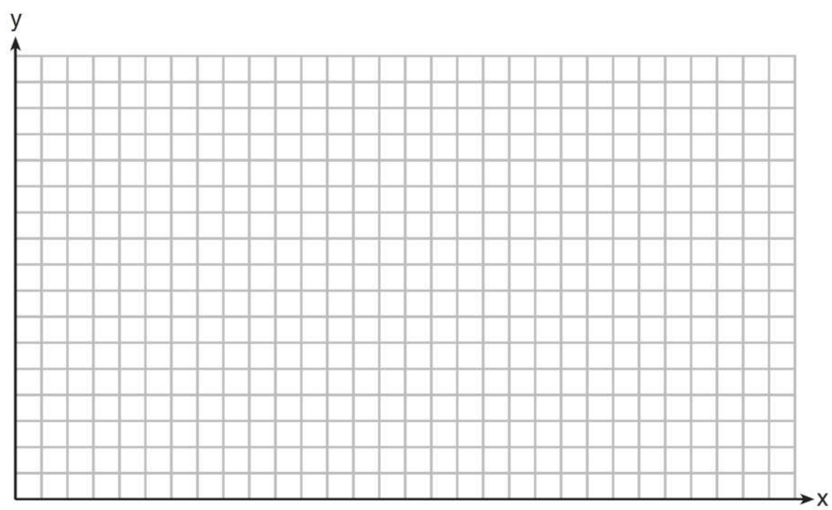
3) $y = 2\cos\frac{2\pi}{3}x$

4) $y = 2\sin\frac{2\pi}{3}x$

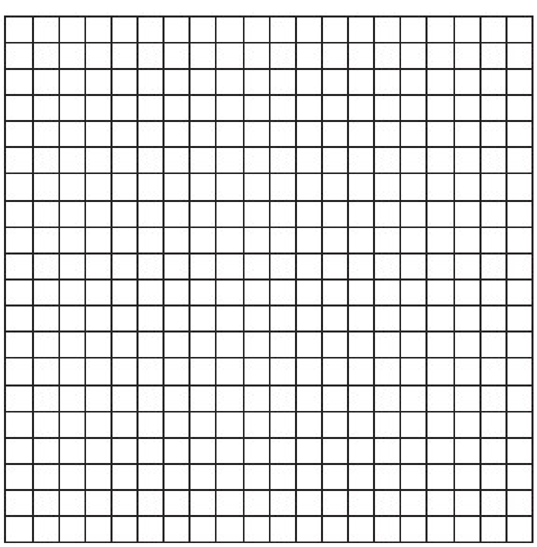
Modeling Trig Graphs

-Graph your equation like you normally would. Guess and check a y-axis scale.
-The first follow up question is usually period and its meaning in context. "It takes "period" for the "blank" to complete one full "blank.""
-The second follow up question can be anything!
*Draw a little picture to help organize your information.

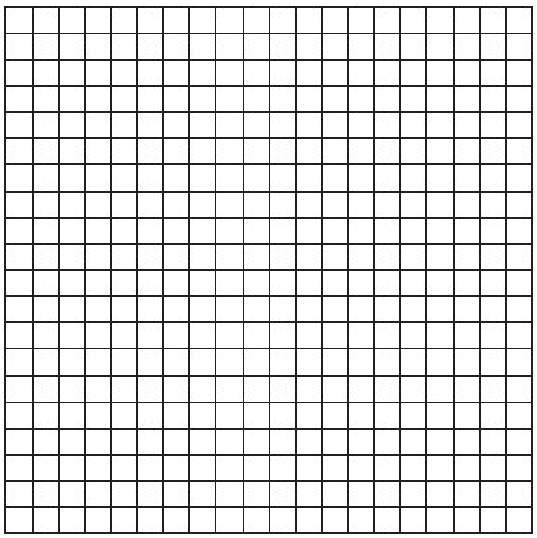
1. The High Roller, a Ferris wheel in Las Vegas, Nevada, opened in March 2014. A passenger's height, in feet, above the ground after t minutes can be modeled by the equation $h(t) = -260\cos\left(\frac{\pi}{15}t\right) + 290$. Graph one full cycle of $h(t)$ on the axes provided. Identify the period and state its meaning in the context of the problem. To the *nearest tenth of a second*, after how much time will the passenger first reach a height of 500 feet?



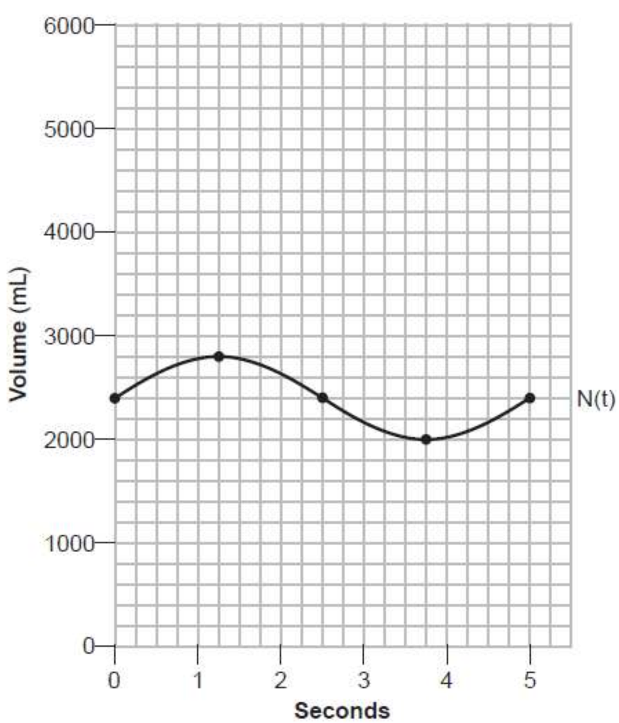
2. Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function $f(t) = -13\cos(0.8\pi t) + 13$, where t represents the time (in seconds) since the nail first became caught in the tire. Determine the period of $f(t)$. Interpret what the period represents in this context. On the grid below, graph *at least one* cycle of $f(t)$ that includes the y-intercept of the function. Does the height of the nail ever reach 30 inches above the ground? Justify your answer.



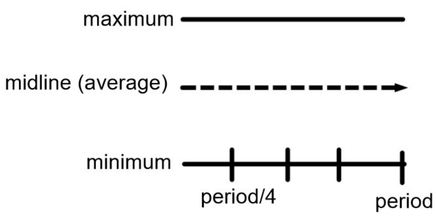
3. The ocean tides near Carter Beach follow a repeating pattern over time, which can be modeled by the equation $h(t) = -12\cos\left(\frac{2\pi}{13}t\right)$ where $h(t)$ represents height above sea level and t represents hours after 8:30 AM. On the grid below, graph one cycle of this function. Determine the period and state its meaning in the context of the problem. People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.



4. The volume of air in an average lung during breathing can be modeled by the graph below. Using the graph, write an equation for $N(t)$, in the form $N(t) = A\sin(Bt) + C$. That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000\sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds. Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$. How many times during the 5-second interval will $N(t) = E(t)$?



Trig Graphs Applications
DRAW A LITTLE PICTURE!



AMPSINFREQXSHIFT
Period is the time is takes to complete one full cycle

$$Period = \frac{2\pi}{frequency}, Frequency = \frac{2\pi}{period}$$

1. Which statement is *incorrect* for the graph of the function $y = -3\cos\left[\frac{\pi}{3}(x-4)\right] + 7$?

1) The period is 6.
2) The amplitude is 3.
3) The range is [4,10].
4) The midline is $y = -4$.

2. Which function's graph has a period of 8 and reaches a maximum height of 1 if at least one full period is graphed?

1) $y = -4\cos\left(\frac{\pi}{4}x\right) - 3$
2) $y = -4\cos\left(\frac{\pi}{4}x\right) + 5$
3) $y = -4\cos(8x) - 3$
4) $y = -4\cos(8x) + 5$

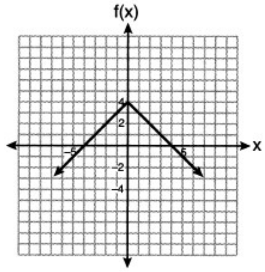
3. The equation below can be used to model the height of a tide in feet, $H(t)$, on a beach at t hours.

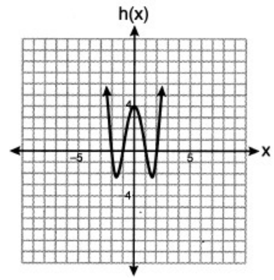
$$H(t) = 4.8\sin\left(\frac{\pi}{6}(t+3)\right) + 5.1$$

Using this function, the amplitude of the tide is

1) $\frac{\pi}{6}$
2) 4.8
3) 3
4) 5.1

4. Which function has a maximum y-value of 4 and a midline of $y = 1$?

1) 

3) 

2) $g(x) = -3\cos(x) + 1$

4) $j(x) = 4\sin(x) + 1$

5. The depth of the water, $d(t)$, in feet, on a given day at Thunder Bay, t hours after midnight is modeled by $d(t) = 5\sin\left(\frac{\pi}{6}(t-5)\right) + 7$. Which statement about the Thunder Bay tide is *false*?

1) A low tide occurred at 2 a.m.

2) The maximum depth of the water was 12 feet.

3) The water depth at 9 a.m. was approximately 11 feet.

4) The difference in water depth between high tide and low tide is 14 feet.

6. A person's lung capacity can be modeled by the function $C(t) = 250\sin\left(\frac{2\pi}{5}t\right) + 2450$, where $C(t)$ represents the volume in mL present in the lungs after t seconds. State the maximum value of this function over one full cycle, and explain what this value represents.

7. Based on climate data that have been collected in Bar Harbor, Maine, the average monthly temperature, in degrees F, can be modeled by the equation $B(x) = 23.914\sin(0.508x - 2.116) + 55.300$. The same governmental agency collected average monthly temperature data for Phoenix, Arizona, and found the temperatures could be modeled by the equation $P(x) = 20.238\sin(0.525x - 2.148) + 86.729$. Which statement can *not* be concluded based on the average monthly temperature models x months after starting data collection?

1) The average monthly temperature variation is more in Bar Harbor than in Phoenix.

2) The midline average monthly temperature for Bar Harbor is lower than the midline temperature for Phoenix.

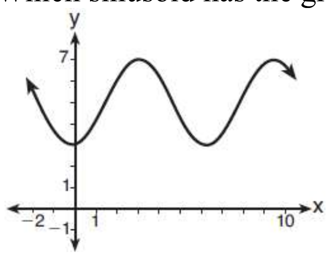
3) The maximum average monthly temperature for Bar Harbor is 79° F, to the nearest degree.

4) The minimum average monthly temperature for Phoenix is 20° F, to the nearest degree.

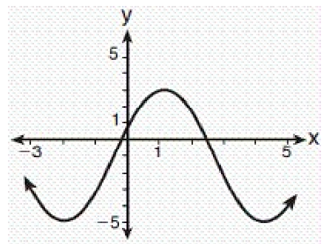
9. Consider the function $h(x) = 2\sin(3x) + 1$ and the function q represented in the table below. Determine which function has the *smaller* minimum value for the domain $[-2, 2]$. Justify your answer.

x	$q(x)$
-2	-8
-1	0
0	0
1	-2
2	0

10. Which sinusoid has the greatest amplitude?

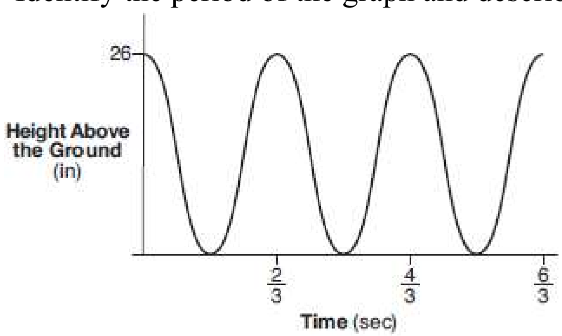
1)


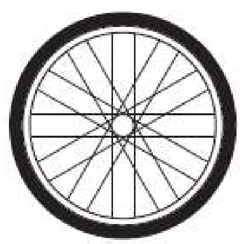
2) $y = 3\sin(\theta - 3) + 5$

3)


4) $y = -5\sin(\theta - 1) - 3$

11. The graph below represents the height above the ground, h, in inches, of a point on a triathlete's bike wheel during a training ride in terms of time, t, in seconds.
Identify the period of the graph and describe what the period represents in this context.





12. The Sea Dragon, a pendulum ride at an amusement park, moves from its central position at rest according to the trigonometric function $P(t) = -10\sin(\frac{\pi}{3}t)$, where t represents time, in seconds. Identify the period and explain its meaning in the context of the problem.

13. A wave displayed by an oscilloscope is represented by the equation $y = 3\sin\frac{\pi}{3}(x-4)+1$. What is the period of this function?

14. The depth of the water, $d(t)$, in feet, on a given day at Thunder Bay, t hours after midnight is modeled by $d(t) = 5\sin\left(\frac{\pi}{6}(t-5)\right)+7$. What is the frequency of this function?

1) 12

2) $\frac{1}{12}$

3) $\frac{\pi}{6}$

4) $\frac{6}{\pi}$

15. What is the frequency of $\mathrm{P}(t) = -2\sin(6\pi(t-4))+3$?

16. Which statement is true of the function $y = -2\cos[3(x-4)]+7$?

1) The midline is $y = -4$

2) The amplitude is -2

3) The range is $[-2, 2]$

4) The frequency is $\frac{3}{2\pi}$

17. The height above ground for a person riding a Ferris wheel after t seconds is modeled by $h(t) = 150\sin\left(\frac{\pi}{45}t + 67.5\right)+160$ feet. How many seconds does it take to go from the bottom of the wheel to the top of the wheel?

1) 10

2) 45

3) 90

4) 150

18. As θ increases from π to $\frac{3\pi}{2}$ radians, the graph of $y = \sin\theta$ will

1) Decrease from 1 to 0
2) Decrease from 0 to -1
3) Increase from -1 to 0
4) Increase from 0 to 1

19. As θ increases from $-\frac{\pi}{2}$ to 0 radians, the graph of $y = -\cos\theta$ will

1) Decrease from 1 to 0
2) Decrease from 0 to -1
3) Increase from -1 to 0
4) Increase from 0 to 1

20. As θ increases from $-\frac{\pi}{2}$ to 0 radians, the value of $\cos\theta$ will

1) decrease from 1 to 0
2) decrease from 0 to -1
3) increase from -1 to 0
4) increase from 0 to 1

21. Given $p(\theta) = 3\sin\left(\frac{1}{2}\theta\right)$ on the interval $-\pi < \theta < \pi$, the function p

1) decreases, then increases
2) increases, then decreases
3) decreases throughout the interval
4) increases throughout the interval

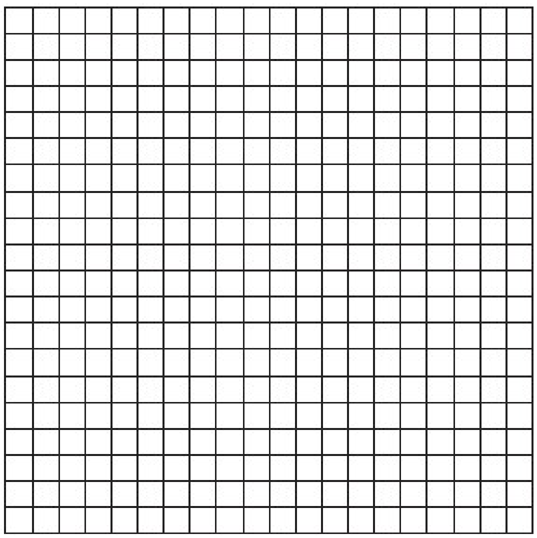
22. As x increases from 0 to $\frac{\pi}{2}$, the graph of the equation $y = 2\tan x$ will

1) increase from 0 to 2
2) decrease from 0 to -2
3) increase without limit
4) decrease without limit

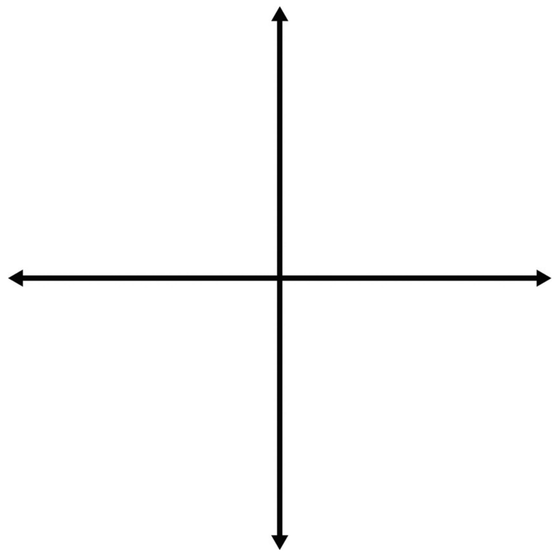
Tangent Graphs

1) Graph Asymptotes ($-\frac{\pi}{2}$ and $\frac{\pi}{2}$)
2) Plot y-intercept (0, vertical shift) or use table
3) $+\tan$: Draw a positively sloped curve guided by the asymptotes
   $-\tan$: Draw a negatively sloped curve guided by the asymptotes

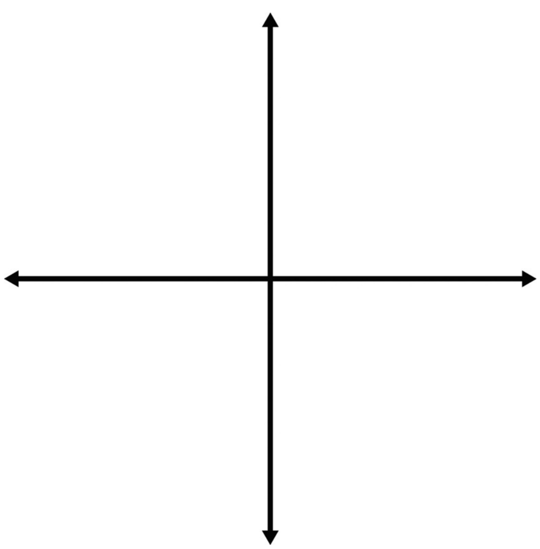
1. $y = 2\tan x - 1$



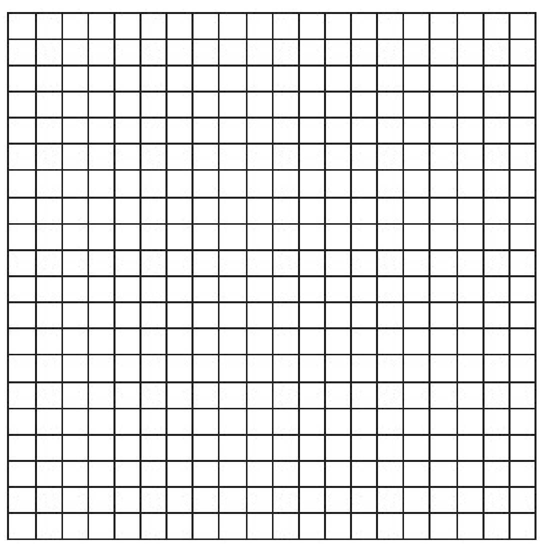
2. $y = -3\tan x + 2$



3. $y = -3\tan x - 4$



4. $y = \frac{1}{2}\tan x + 3$



Sets: (Intersection, Union, and Complement)

Intersection($\cap$) is both (List everything in both (all) sets) $\cap$

Union($\cup$) is all together (List everything that you see in either one. List once if it appears twice)

Complement is everything else (Cross out what is in the subset and list everything else)

1. Given: A = {2, 3, 5, 6, 8, 11, 15, 17, 19}
B = {4, 5, 6, 9, 15, 18}

a) What is $A \cap B$?

b) What is $A \cup B$?

2. Given: A = {11, 14, 16, 21, 27, 33, 35}
B = {4, 8, 12, 16}

a) What is $A \cap B$?

b) What is $A \cup B$?

3. Given: Set $U = \{S, O, P, H, I, A\}$
Set $B = \{A, I, O\}$

If set *B* is a subset of set *U*, what is the complement of set *B*?

1) $\{O, P, S\}$
2) $\{I, P, S\}$
3) $\{A, H, P\}$
4) $\{H, P, S\}$

4. Given: $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$B = \{2, 3, 5, 6\}$

Set *B* is a subset of set *U*. What is the complement of set *B*?

1) $\{\ \}$
2) $\{2, 3, 5, 6\}$
3) $\{1, 4, 7, 8\}$
4) $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Probability with $\cap$ (and) and $\cup$ (or)
Formulas are on Reference Sheet!
Or: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
And: $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
And (Independent) $P(A \cap B) = P(A) \bullet P(B)$

1. The probability that a student in Jacqua High School is in band is $\frac{127}{466}$ and the probability that a student is on the track team is $\frac{82}{466}$. If the probability that they are on the track team and in band is $\frac{74}{466}$, what is the probability that they are on the track team or in band?

2. The probability that a person files their tax return in March is $\frac{127}{165}$. The probability that a person watches College Basketball in March is $\frac{98}{123}$. If the probability that a person watches College Basketball and files their tax return in March is $\frac{62}{95}$, what is the probability that a person watches College Basketball or files their tax return? Round your answer to the nearest percent.

3. On a given school day, the probability that Nick oversleeps is 48% and the probability he has a pop quiz is 25%. Assuming these two events are independent, what is the probability that Nick oversleeps and has a pop quiz on the same day?

1) 73%	3) 23%
2) 36%	4) 12%

4. In 2015 at Sabres Prep Academy, the probability that a student passed Algebra II was 78%. The probability that a student passed Chemistry was 86%. The probability they passed Algebra II or Chemistry was 88%. What is the probability that they passed Algebra II and Chemistry?

5. The probability that Chloe the cardinal shows up in the Schlansky's backyard is $\frac{12}{19}$. The probability that Chloe shows up in the Silverman's backyard is $\frac{10}{17}$. If the probability that Chloe shows up in the Schlansky's backyard or the Silverman's backyard is $\frac{12}{16}$, what is the probability that Chloe shows up in both backyards?

6. There are 24 students in a math class. 15 of them play a sport and 20 of them play an instrument. 22 play a sport or play an instrument. What is the probability that a student chosen at random will play a sport and play an instrument?

7. Over the past 30 nights, Baxter barked 8 nights and cried 15 nights. He barked or cried 11 nights. How many nights did he bark and cry?

8. Suppose events A and B are independent and $P(A \text{ and } B)$ is 0.2. Which statement could be true?

1) $P(A) = 0.4$, $P(B) = 0.3$, $P(A \text{ or } B) = 0.5$
2) $P(A) = 0.8$, $P(B) = 0.25$
3) $P(A|B) = 0.2$, $P(B) = 0.2$
4) $P(A) = 0.15$, $P(B) = 0.05$

Probability with Two Way Tables

2 Things "and"	1 Thing	2 Things "given"	2 Things No key words
$\frac{\qquad}{total\ total}$	$\frac{\qquad}{total\ total}$	$\frac{\qquad}{condition\ (last)}$	$\frac{\qquad}{condition\ (first)}$

1. One-hundred employees of a company were asked their opinion on paying high salaries to the CEO. Their responses are summarized in the following contingency table. Express the following probabilities as fractions and rounded to the nearest percent.

	In Favor	Against
Male	15	45
Female	4	36

a) Find the probability that they are male and in favor

b) Find the probability that they are female

c) Find the probability that a male is in favor

d) Find the probability that they are against given that they are female

e) Find the probability that someone is in favor is a male

f) Find the probability that someone is female and against

g) Find the probability that a female is in favor

h) Find the probability that someone is male given that they are in favor

2. A statistics class surveyed some students during one lunch period to obtain opinions about television programming preferences. The results of the survey are summarized in the table below.

Programming Preferences

	Comedy	Drama
Male	70	35
Female	48	42

What is the probability that a student is male and prefers comedy?

What is the probability that a male student would prefer comedy?

What is the probability that a student is male?

What is the probability that a student is female given that they like drama?

3. A public opinion poll was taken to explore the relationship between age and support for a candidate in an election. The results of the poll are summarized in the table below.

Age	For	Against	No Opinion
21–40	30	12	8
41–60	20	40	15
Over 60	25	35	15

What is the probability that someone has no opinion?

What is the probability that someone is over 60 and against?

What is the probability that someone is for the candidate given that they are between 21-40?

4. A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

Favorite Type of Program			
	Sports	**Reality Show**	**Comedy Series**
Senior	83	110	67
Freshmen	119	103	54

A student response is selected at random from the results. State the *exact* probability the student response is from a freshman, given the student prefers to watch reality shows on television.

5. At Berkeley Central High School, a survey was conducted to see if students preferred cheeseburgers, pizza, or hot dogs for lunch. The results of this survey are shown in the table below.

	Cheeseburgers	**Pizza**	**Hot Dogs**
Females	32	44	24
Males	36	30	34

Based on this survey, what percent of the students preferred pizza?

1) 30
2) 37
3) 44
4) 74

6. A middle school conducted a survey of students to determine if they spent more of their time playing games or watching videos on their tablets. The results are shown in the table below.

	Playing Games	**Watching Videos**	**Total**
Boys	138	46	184
Girls	54	142	196
Total	192	188	380

Of the students who spent more time playing games on their tablets, approximately what percent were boys?

1) 41
2) 56
3) 72
4) 75

7. A survey was given to 12th-grade students of West High School to determine the location for the senior class trip. The results are shown in the table below.

	Niagara Falls	Darien Lake	New York City
Boys	56	74	103
Girls	71	92	88

To the *nearest percent*, what percent of the boys chose Niagara Falls?

1) 12
2) 24
3) 44
4) 56

8. Jenna took a survey of her senior class to see whether they preferred pizza or burgers. The results are summarized in the table below.

	Pizza	Burgers
Male	23	42
Female	31	26

Of the people who preferred burgers, approximately what percentage were female?

1) 21.3
2) 38.2
3) 45.6
4) 61.9

9. Students were asked to name their favorite sport from a list of basketball, soccer, or tennis. The results are shown in the table below.

	Basketball	Soccer	Tennis
Girls	42	58	20
Boys	84	41	5

What percentage of the students chose soccer as their favorite sport?

1) 39.6%
2) 41.4%
3) 50.4%
4) 58.6%

Independence

If events are independent:

$P(A \cap B) = P(A) \bullet P(B)$

The denominator is the total total for each!

$P(A) = P(A \mid B)$

1. The results of a poll of 200 students are shown in the table below:

	Preferred Music Style		
	Techno	**Rap**	**Country**
Female	54	25	27
Male	36	40	18

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

2. At a local mall, 125 people were asked how they choose to pay for their merchandise. The data is shown in the table below:

	Credit Card	Cash
Male	40	10
Female	60	15

Does the data suggest that the gender and type of payment are independent of each other? Explain your answer.

3. One-hundred employees of a company were asked their opinion on paying high salaries to the CEO. Their responses are summarized in the following contingency table.

	In Favor	Against
Male	15	45
Female	4	36

Based on the data, are gender an opinion on salaries independent of each other? Justify your answer.

4. Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

	Juan Wins	**Felipe Wins**
Short Practice Time	8	10
Long Practice Time	15	12

Determine whether or not the two events "Filipe wins" and "long practice time" are independent. Justify your answer.

5. The results of a survey of the student body at Central High School about television viewing preferences are shown below.

	Comedy Series	**Drama Series**	**Reality Series**	**Total**
Males	95	65	70	230
Females	80	70	110	260
Total	175	135	180	490

Are the events "student is a male" and "student prefers reality series" independent of each other? Justify your answer.

6. Given events A and B, such that $P(A) = 0.8$, $P(B) = 0.6$, and $P(A \mid B) = 0.6$. Determine whether A and B are independent. Explain your answer.

7. Given events A and B, such that $P(A) = 0.8$, $P(B) = 0.6$, and $P(B \mid A) = 0.6$. Determine whether A and B are independent. Explain your answer.

8. A fast-food restaurant analyzes data to better serve its customers. After its analysis, it discovers that the events *D*, that a customer uses the drive-thru, and *F*, that a customer orders French fries, are independent. The following data are given in a report:

$$P(F) = 0.8$$

$$P(F \cap D) = 0.456$$

Given this information, $P(F|D)$ is

1) 0.344
2) 0.3648
3) 0.57
4) 0.8

9. Given events T and K are independent of each other, if $P(T) = 0.35$, $P(K) = 0.48$, find $P(K \mid T)$

10. Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

1) independent
2) dependent
3) mutually exclusive
4) complements

Normal Distributions

2nd vars: 2:normal cdf

Lower = lower bound, Upper = upper bound, μ= mean, σ= standard deviation

Less than 3:	More than 3:	Between 3 and 6
Lower: -99999999999 Upper 3	Lower: 3 Upper: 9999999999	Lower: 3 Upper: 6

If asked for:

Probability	Percent	Quantity
You’re done!	Multiply by 100	Multiply by the total quantity

1. The weights of bags of Graseck's Chocolate Candies are normally distributed with a mean of 4.3 ounces and a standard deviation of 0.05 ounces. What is the probability that a bag of these chocolate candies weighs less than 4.27 ounces?

1) 0.2257
2) 0.2743
3) 0.7257
4) 0.7757

2. The weight of a bag of pears at the local market averages 8 pounds with a standard deviation of 0.5 pound. The weights of all the bags of pears at the market closely follow a normal distribution. Determine what percentage of bags, to the *nearest integer*, weighed *more* than 8.25 pounds.

3. The scores of a recent test taken by 1200 students had an approximately normal distribution with a mean of 225 and a standard deviation of 18. Determine the number of students who scored between 200 and 245.

4. The weights of students on the boys cross country team is normally distributed with a mean of 135.3 pounds and a standard deviation of 2.8 pounds. If the team has 32 members, how many of them, rounded to the *nearest person,* would be expected to weigh less than 132 pounds?

5. The lifespan of a 60-watt lightbulb produced by a company is normally distributed with a mean of 1450 hours and a standard deviation of 8.5 hours. If a 60-watt lightbulb produced by this company is selected at random, what is the probability that its lifespan will be between 1440 and 1465 hours?
1) 0.3803
2) 0.4612
3) 0.8415
4) 0.9612

6. The number of hours students spent studying for their Regents exam is normally distributed with a mean of 14 hours and a standard deviation of 3.2 hours. If a student is randomly selected, what percent, to the nearest integer, of the students spent more than 22 hours studying?

7. The scores on a math test are normally distributed with a mean of 76.2 and a standard deviation of 4.7. If 248 students took the exam, approximately how many students got between a 70 and an 80?

8. The number of hours of sleep employees at a company get per night is normally distributed with a mean of 7.1 hours and a standard deviation of 1.4 hours. If there are 2500 employees at the company, approximately how many of them, to the nearest person, got less than 5 hours of sleep?

9. The scores on a mathematics college-entry exam are normally distributed with a mean of 68 and standard deviation 7.2. Students scoring higher than one standard deviation above the mean will not be enrolled in the mathematics tutoring program. How many of the 750 incoming students can be expected to be enrolled in the tutoring program?
1) 631
2) 512
3) 238
4) 119

Statistical Studies

A *survey (sample survey)* is ***asking questions*.**

*A census (population survey) is asking every member of the population (RARELY practical).

An *observational study* ***observes*** data WITHOUT ADMINISTERING A TREATMENT.

A *controlled experiment* ADMINISTERS A TREATMENT.

1. Which scenario is best described as an observational study?

1) For a class project, students in Health class ask every tenth student entering the school if they eat breakfast in the morning.
2) A social researcher wants to learn whether or not there is a link between attendance and grades. She gathers data from 15 school districts.
3) A researcher wants to learn whether or not there is a link between children's daily amount of physical activity and their overall energy level. During lunch at the local high school, she distributed a short questionnaire to students in the cafeteria.
4) Sixty seniors taking a course in Advanced Algebra Concepts are randomly divided into two classes. One class uses a graphing calculator all the time, and the other class never uses graphing calculators. A guidance counselor wants to determine whether there is a link between graphing calculator use and students' final exam grades.

2. A doctor wants to test the effectiveness of a new drug on her patients. She separates her sample of patients into two groups and administers the drug to only one of these groups. She then compares the results. Which type of study *best* describes this situation?

1) census
2) survey
3) observation
4) controlled experiment

3. A market research firm needs to collect data on viewer preferences for local news programming in Buffalo. Which method of data collection is most appropriate?

1) census
2) survey
3) observation
4) controlled experiment

4. A school cafeteria has five different lunch periods. The cafeteria staff wants to find out which items on the menu are most popular, so they give every student in the first lunch period a list of questions to answer in order to collect data to represent the school. Which type of study does this represent?

1) observation
2) controlled experiment
3) population survey
4) sample survey

5. Determine whether each scenario is a survey, an observational study, or a controlled experiment. Explain your answer.

a) A study is done to see how high soda will erupt when mint candies are dropped into two-liter bottles of soda. You want to compare using one mint candy, five mint candies, and 10 mint candies. You design a cylindrical mechanism, which drops the desired number of mint candies all at once. You have 15 bottles of soda to use. You randomly assign five bottles into which you drop one candy, five into which you drop five candies, and five into which you drop 10 candies. For each bottle, you record the height of the eruption created after the candies are dropped into it.

b) You want to see if fifth-grade boys or fifth-grade girls are faster at solving multiplication problems. You randomly select twenty fifth-grade boys and twenty fifth-grade girls from fifth graders in your school district. You time and record how long it takes each student to solve multiplication problems.

c) You want to determine if people would be interested in watching a video of you performing Mr. Schlansky's math songs. You ask every 5^{th} student walking into Mr. Schlansky's math class if they would want to watch the video.

6. Howard collected fish eggs from a pond behind his house so he could determine whether sunlight had an effect on how many of the eggs hatched. After he collected the eggs, he divided them into two tanks. He put both tanks outside near the pond, and he covered one of the tanks with a box to block out all sunlight. State whether Howard's investigation was an example of a controlled experiment, an observation, or a survey. Justify your response.

6. Darryl conducted a study comparing the statistics of baseball players in the steroid era compared to the non steroid era. Would this investigation be an example of a controlled experiment, an observation, or a survey? Justify your response.

Surveys (Choosing a sample)
A good sample is random. For example, every fifth student walking into the building.
A bad sample is bias. Don't ask the soccer team if they like soccer.

1. Which statement(s) about statistical studies is true?
I. A survey of all English classes in a high school would be a good sample to determine the number of hours students throughout the school spend studying.
II. A survey of all ninth graders in a high school would be a good sample to determine the number of student parking spaces needed at that high school.
III. A survey of all students in one lunch period in a high school would be a good sample to determine the number of hours adults spend on social media websites.
IV. A survey of all Calculus students in a high school would be a good sample to determine the number of students throughout the school who don't like math.
1) I, only 2) II, only 3) I and III 4) III and IV

2. Which survey is *least* likely to contain bias?
1) surveying a sample of people leaving a movie theater to determine which flavor of ice cream is the most popular
2) surveying the members of a football team to determine the most watched TV sport
3) surveying a sample of people leaving a library to determine the average number of books a person reads in a year
4) surveying a sample of people leaving a gym to determine the average number of hours a person exercises per week

3. A survey is to be conducted in a small upstate village to determine whether or not local residents should fund construction of a skateboard park by raising taxes. Which segment of the population would provide the most unbiased responses?
1) a club of local skateboard enthusiasts
2) senior citizens living on fixed incomes
3) a group opposed to any increase in taxes
4) every tenth person 18 years of age or older walking down Main St.

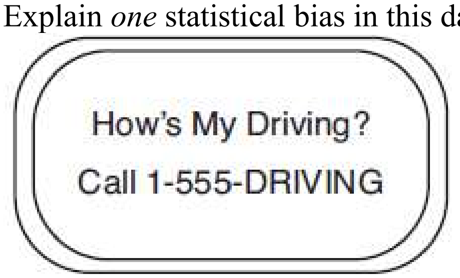
4. A survey is being conducted about American's favorite musicians. Which of the following survey methods would most likely produce a random sample?
(1) Asking every 20th person at a Green Day concert
(2) Asking every 10th person at a vintage record store
(3) Asking every 10th person at the Westbury Public Library
(4) Sending out surveys to random households across the country.

5. Which method of collecting data would most likely result in an unbiased random sample?
(1) selecting every third teenager leaving a movie theater to answer a survey about entertainment
(2) placing a survey in a local newspaper to determine how people voted in the 2004 presidential election
(3) selecting students by the last digit of their school ID number to participate in a survey about cafeteria food
(4) surveying honor students taking Trigonometry to determine the average amount of time students in a school spend doing homework each night

6. A survey completed at a large university asked 2,000 students to estimate the average number of hours they spend studying each week. Every tenth student entering the library was surveyed. The data showed that the mean number of hours that students spend studying was 15.7 per week. Which characteristic of the survey could create a bias in the results?
(1) the size of the sample (3) the method of analyzing the data
(2) the size of the population (4) the method of choosing the students who were surveyed

7. The yearbook staff has designed a survey to learn about student opinions on how the yearbook could be improved for this year. If they want to distribute this survey to 100 students and obtain the most reliable data, they should survey
1) Every third student sent to the office
2) Every third student to enter the library
3) Every third student to enter the gym for the basketball game
4) Every third student arriving at school in the morning

8. Chuck's Trucking Company has decided to initiate an Employee of the Month program. To determine the recipient, they put the following sign on the back of each truck.
The driver who receives the highest number of positive comments will win the recognition.
Explain *one* statistical bias in this data collection method.



Sample Distributions

To determine if something is usual or unusual, expected or unexpected:

1) Find the confidence interval!!

$Confidence\ Interval = mean \pm 2(Standard\ Deviation)$

2) Determine if the given value is inside or outside the confidence interval.

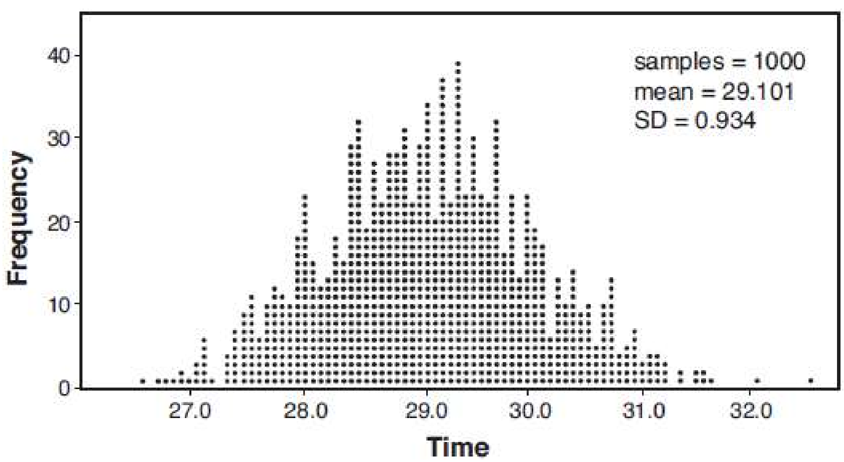
If inside, it is an expected value

If outside, it is not an expected value

1. A radio station claims to its advertisers that the mean number of minutes commuters listen to the station is 30. The station conducted a survey of 500 of their listeners who commute. The sample statistics are shown below.

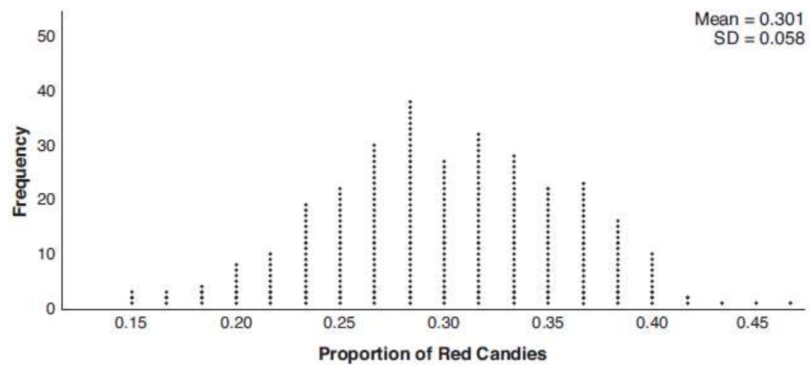
$\bar{x}$	29.11
s_x	20.718

A simulation was run 1000 times based upon the results of the survey. The results of the simulation appear below.



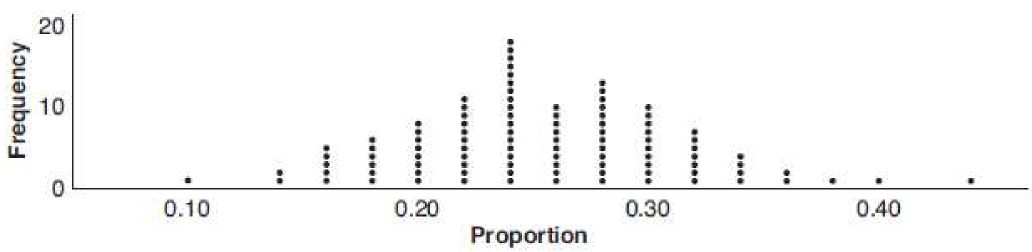
Based on the simulation results, is the claim that commuters listen to the station on average 30 minutes plausible? Explain your response including an interval containing the middle 95% of the data, rounded to the *nearest hundredth*.

2. Mary bought a pack of candy. The manufacturer claims that 30% of the candies manufactured are red. In her pack, 14 of the 60 candies are red. She ran a simulation of 300 samples, assuming the manufacturer is correct. The results are shown below.



Based on the simulation, determine the middle 95% of plausible values that the proportion of red candies in a pack is within. Based on the simulation, is it unusual that Mary's pack had 14 red candies out of a total of 60? Explain.

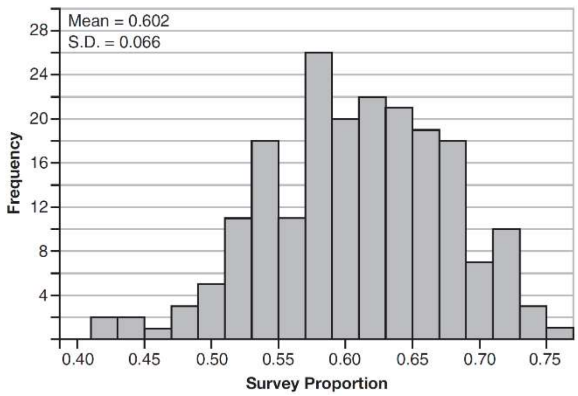
3. A group of students was trying to determine the proportion of candies in a bag that are blue. The company claims that 24% of candies in bags are blue. A simulation was run 100 times with a sample size of 50, based on the premise that 24% of the candies are blue. The approximately normal results of the simulation are shown in the dot plot below.



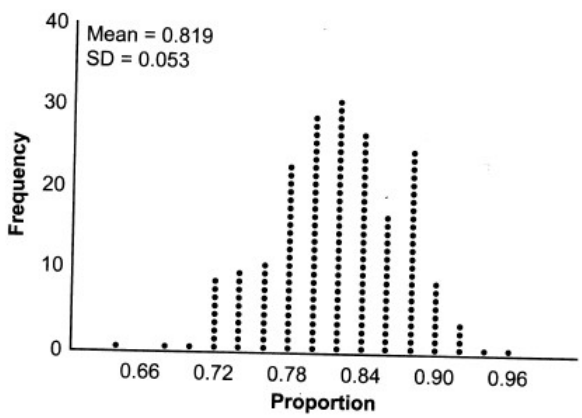
The simulation results in a mean of 0.254 and a standard deviation of 0.060. Based on this simulation, what is a plausible interval containing the middle 95% of the data? A student found that 18 out of 50 of the candies were blue. Use statistical evidence to explain why this is an expected value.

4. Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band. A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.

Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*. Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50%-50% split. Explain what statistical evidence supports this concern.



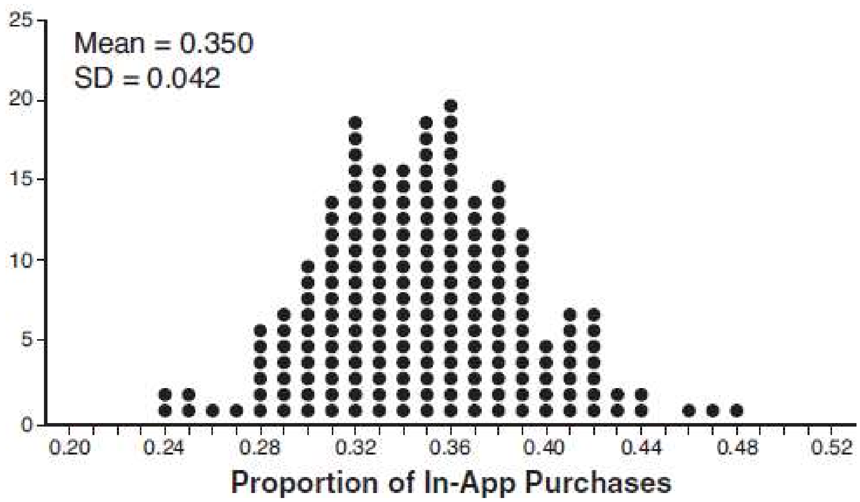
5. State officials claim 82% of a community want to repeal the 30 mph speed limit on an expressway. A community organization devises a simulation based on the claim that 82% of the community supports the repeal. Each dot on the graph below represents the proportion of community members who support the repeal. The graph shows 200 simulated surveys, each of sample size 60.



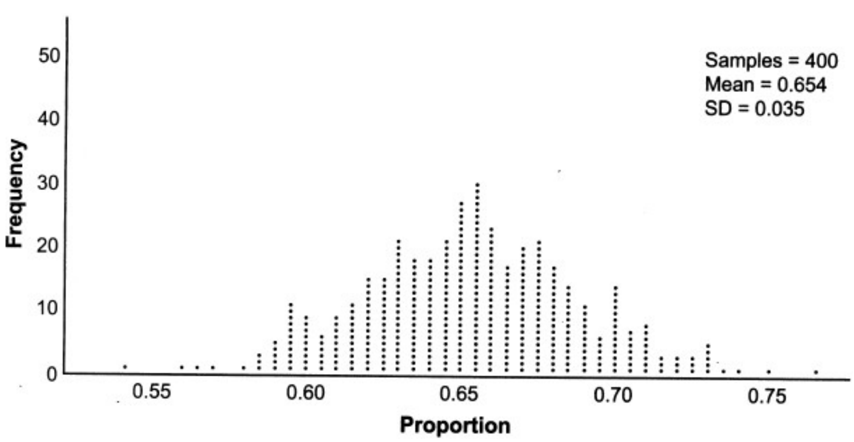
Based on the simulation, determine an interval containing the middle 95% of plausible proportions. Round your answer to the *nearest thousandth*. The community organization conducted its own sample survey of 60 people and found 70% supported the repeal. Based on the results of the simulation, explain why the organization should question the State officials' claim.

Margin Of Error
MOE=2(SD)

1. Some smart-phone applications contain "in-app" purchases, which allow users to purchase special content within the application. A random sample of 140 users found that 35 percent made in-app purchases. A simulation was conducted with 200 samples of 140 users assuming 35 percent of the samples make in-app purchases. The approximately normal results are shown below. Considering the middle 95% of the data, determine the margin of error, to the *nearest hundredth*, for the simulated results.



2. Betty conducted a survey of her class to see if they like pizza. She gathered 200 responses and 65% of the voters said they did like pizza. Betty then ran a simulation of 400 more surveys, each with 200 responses, assuming that 65% of the voters would like pizza. The output of the simulation is shown below. Considering the middle 95% of the data, what is the margin of error for the simulation?

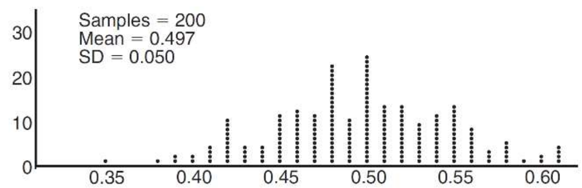


Fair

A coin (or other object) is fair if the actual outcome is in the confidence interval of a *fair coin/object.*

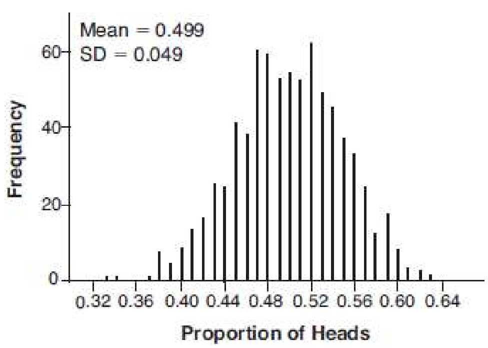
1. Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

Given the results of her coin flips and of her computer simulation, which statement is most accurate?



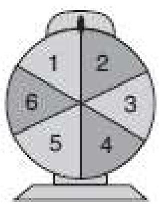
1) 73 of the computer's next 100 coin flips will be heads.
2) 50 of her next 100 coin flips will be heads.
3) Her coin is not fair.
4) Her coin is fair.

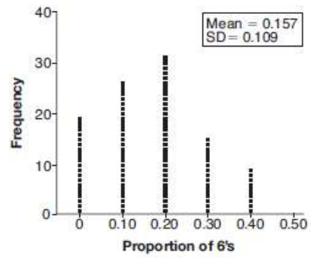
2. Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below. Do the results of the simulation provide strong evidence that Robin's coin is unfair? Explain your answer.



3. Juanita rolls a 6 sided die and recorded that it landed on 6 five times out of 50. She questioned whether the die was fair so she ran a computer simulation of 1000 samples of 50 rolls of a fair die. The mean of the simulation was .159 with a standard deviation of .102. Is her die fair? Explain your answer.

4. A game spinner is divided into 6 equally sized regions, as shown in the diagram below. For Miles to win, the spinner must land on the number 6. After spinning the spinner 10 times, and losing all 10 times, Miles complained that the spinner is unfair. At home, his dad ran 100 simulations of spinning the spinner 10 times, assuming the probability of winning each spin is $\frac{1}{6}$. The output of the simulation is shown in the diagram below.





Is there strong evidence to suggest that the spinner is unfair? Explain your answer.

5. A spinner with 8 sectors labeled A, B, C, D, E, F, G, H was spun 100 times. The spinner landed on sector B 20 times out of 100. A computer simulation of 500 samples of 100 spins of a fair 8 sector spinner was run and it was found that the mean proportion of landing on sector B was .126 with a standard deviation of .027. Is the spinner fair? Explain your answer.

6. Ally flipped a coin 100 times a got a proportion of .41 heads. She believed this coin was unfair so she ran a computer simulation of 200 samples of 100 coin flips of a fair coin. The mean of the simulation was .502 and the standard deviation was .024. Is Ally's coin fair? Explain your answer.

Factoring:

Is there a Greatest Common Factor? GCF(   )

2 Terms: Difference of Two Squares: $(\sqrt{_1}+\sqrt{_2})(\sqrt{_1}-\sqrt{_2})$ **OR Cubes**

3 Terms: Trinomials: (x   )(x   )

1) First sign comes down
2) The two signs must multiply for the last sign
3) Find two numbers that multiply to the last number and add/subtract to the middle number

***Bridge Method: (Trinomial with a leading coefficient bigger than 1)**

1) Build a bridge between the first and last numbers (Multiply)
2) Factor Trinomial Normally
3) Pay the toll (Divide by the leading coefficient)

*If possible, reduce the fraction
If they divide nicely, divide them
If not, put the denominator in front of the variable inside the parenthesis

4 or More Terms: Grouping

1) Look for a pattern in the exponents to determine the groups. **You cannot have two terms with the same exponent in the same group.**
2) Factor out the GCF in each group
*You should be left with the same factor. If signs are reversed, factor out a negative
3) Combine coefficients and keep like term.

***Factor further if necessary**

Sum/Difference of Two Cubes

SOAP for signs (Same, Opposite, Always Positive)

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

Substitution Trinomials:

1) Replace binomial with y
2) Factor normally
3) Substitute back

*Factor further if possible

Factor each expression

1. $4x + 8$

2. $12x + 18$

3. $x^2 - 7x$

4. $2x^2 - 4xy$

5. $5x^2 y - 20x$

6. $x^2 - 64$

7. $y^2 - 36$

8. $4t^2 - 25$

9. $9x^2 - 16y^4$

10. $36 - 25x^2$

11. $100y^4 - 49t^6$

12. $1 - 9x^8y^4$

13. $x^2 + 4x - 12$

14. $y^2 + 3y + 2$

15. $m^2 - 8m + 15$

16. $x^2 - 8x - 20$

17. $y^2 + 5y - 14$

18. $x^2 + x - 12$

19. $x^2 - 3x - 10$

20. $x^2 - 7x + 12$

21. $x^2 - 9x - 36$

22. $y^2 - 21y + 110$

23. $x^4 + 4x^2 - 12$

24. $x^6 - 6x^3 + 9$

25. $x^4 - 8x^2 - 9$

26. $x^4 + x^2 - 2$

27. $2x^2 - 50$

28. $2x^2 - 8x - 10$

29. $3x^2 + 9x - 12$

30. $6x^2 - 54$

31. $2x^2 + 14x + 24$

32. $5x^2 - 500$

33. $ax^2 - 2ax - 8a$

34. $yx^2 - 64y$

35. $12x^2 - 75$

36. $x^4 - 81$

37. $2y^2 - 5y - 7$

38. $2x^2 + 15x - 8$

39. $2x^2 + 7x - 4$

40. $6x^2 - 11x - 10$

41. $2x^2 - 9x - 18$

42. $3x^2 + 2x - 8$

45. $x^3 + 6x^2 - 3x - 18$

46. $x^3 + 10x^2 - 9x - 90$

47. $x^3 + 3x^2 - 9x - 27$

48. $8x^3 + 12x^2 - 2x - 3$

49. $x^3 - 3x^2 + 2x + 4x^2 - 12x + 8$

50. $3x^3 + x^2 - 12x^2 - 4x - 63x - 21$

51. $(x^2 + 5x)^2 - 2(x^2 + 5x) - 24$

52. $(x - 4)^2 - 3(x - 4) - 10$

53. $(x^2 - 2x)^2 - 11(x^2 - 2x) + 24$

54. $(x + 7)^2 - 7(x + 7) + 12$

55. $y^3 - 125$

56. $z^3 + 64$

57. $8x^3 + y^6$

58. $y^9 - 216x^3$

Reducing Rational Expressions

1) Factor
2) Cancel Common Factors
*If a factor is written backwards with a minus sign, they cancel to -1.

Express each of the following in simplest form

1. $\frac{2x+6}{x^2-9}$

2. $\frac{10-5x}{x^2+2x-8}$

3. $\frac{6x+18}{6x+12}$

4. $\frac{2x^2+x-6}{9-4x^2}$

5. $\frac{x^2+3x+2}{x^3+2x^2+8x+16}$

6. $\frac{3x^2+7x-6}{4-9x^2}$

7. $\frac{2x^4+4x^3-6x^2}{4x^3-36x}$

8. $\frac{2x^3+x^2-18x-9}{3x-x^2}$

Solving Quadratic Equations By Factoring

1) Bring everything to one side
2) Factor
3) Set each factor equal to zero

Divide away an integer GCF if possible.
A variable GCF would have to stay in front and would produce an answer of 0.

1. $y^2 - 5y - 6 = 0$

2. $x^2 + 4x = 0$

3. $a^2 - 8a = 20$

4. $3x^2 = 48$

5. $x^2 - 6x = -8$

6. $3x^2 + 3x - 6 = 0$

7. $n^2 = 3n + 18$

8. $2x^2 + 3x = 5$

9. $x^2 - 6x = 2x + 20$

10. $x^2 + 2(x - 4) = 3x - 8$

11. $4(x^2 + 2x) = 8x + 64$

12. $4x^2 + 4x = 3$

13. $x^2 + 5x = -4(x + 5)$

14. $4x^2 = 25$

15. $3x^2 + 5x = 2x + 60$

16. $8m^2 + 20m = 12$

Reducing Radicals

1) Separate into two radicals (perfect squares and non-perfect squares)
2) Take the square root of the perfect square

*If there is a negative inside the radical, bring it outside and make it an *i*

Reduce the following radicals

1. $\sqrt{12}$

2. $\sqrt{-50}$

3. $\sqrt{-45}$

4. $\sqrt{75}$

5. $\sqrt{-20}$

6. $\sqrt{-54}$

7. $\sqrt{162}$

8. $\sqrt{-32}$

Perfect Squares
1
4
9
16
25
36
49
64
81
100

Solving Quadratic Equations Using the Quadratic Formula
IF MULTIPLE CHOICE:
APPS, PLYSMLT2, 1: POLY ROOT FINDER
Type each choice in to match up the decimal.
Algebraically:
1) Bring everything to one side. Keep the leading coefficient positive.
If you cannot factor, USE QUADRATIC FORMULA!
Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) $ax^2 + bx + c = 0$
2) List a, b, and c values
3) Substitute values into quadratic formula
4) Type what's inside the radical into the calculator
5) REDUCE THE RADICAL off to the side (If possible)
6) Draw your heart, break the fraction apart.

1. What are the solutions to $4x^2 - 7x - 2 = -10$

1) $-\frac{1}{4}, 2$
2) $\frac{7}{8} \pm \frac{\sqrt{79}}{8} i$
3) $\frac{7}{8} \pm \frac{\sqrt{241}}{8}$
4) $\frac{7}{8} \pm \frac{\sqrt{143}}{8} i$

2. The solutions to the equation $3x^2 - 4x + 2 = 2x - 3$ are

1) $\frac{2}{3} \pm \frac{\sqrt{2}}{3} i$
2) $1 \pm \frac{\sqrt{6}}{3} i$
3) $1 \pm \frac{\sqrt{12}}{3}$
4) $1 \pm 2\sqrt{6} i$

3. The roots of the equation $0 = x^2 + 6x + 10$ in simplest $a + bi$ form are

1) $-3 \pm 2i$
2) $-6 \pm i$
3) $-3 \pm i$
4) $-3 \pm i\sqrt{2}$

4. The roots of the equation $x^2 - 4x = -13$ are

1) $2 \pm 3i$
2) $2 \pm 6i$
3) $2 \pm \sqrt{17}$
4) $2 \pm \sqrt{13}$

5. A solution of the equation $2x^2 + 3x + 2 = 0$ is

1) $-\frac{3}{4} + \frac{1}{4} i\sqrt{7}$
2) $-\frac{3}{4} + \frac{1}{4} i$
3) $-\frac{3}{4} + \frac{1}{4} \sqrt{7}$
4) $\frac{1}{2}$

6. The solutions to the equation $-\frac{1}{2}x^2 = -6x + 20$ are

1) $-6 \pm 2i$
2) $-6 \pm 2\sqrt{19}$
3) $6 \pm 2i$
4) $6 \pm 2\sqrt{19}$

7. Which equation has roots of $3 + i$ and $3 - i$?

1) $x^2 - 6x + 10 = 0$
2) $x^2 + 6x - 10 = 0$
3) $x^2 - 10x + 6 = 0$
4) $x^2 + 10x - 6 = 0$

8. If a solution of $2(2x - 1) = 5x^2$ is expressed in simplest $a + bi$ form, the value of b is

1) $\frac{\sqrt{6}}{5}i$
2) $\frac{\sqrt{6}}{5}$
3) $\frac{1}{5}i$
4) $\frac{1}{5}$

9. Solve the equation $x^2 + 3x + 11 = 0$ algebraically. Express the answer in $a + bi$ form.

10. Solve the equation $3x^2 + 5x + 8 = 0$. Write your solution in $a + bi$ form.

11. Algebraically determine the roots, in simplest $a + bi$ form, to the equation below.

$x^2 - 2x + 7 = 4x - 10$

Solve the following equations and express your answer in *simplest a+bi form*.

12. $x^2 + 4x = -8$

13. $4x^2 + 2x = -1$

14. $2x^2 - 6x = -5$

15. $3x^2 = 4x - 2$

16. $x^2 + 2x = -8$

17. $3x^2 + 6 = 5x$

Polynomial Equations
*Use PLYSMLT2
1) Bring everything to one side. Keep the leading coefficient positive.
2) Factor
3) Set each factor equal to zero
If you end up with $(x^2 + a)$, use isolate/square root method.
To find the roots/zeros algebraically, replace $f(x)$ with 0.

1. Solve $x^3 + 5x^2 = 4x + 20$ algebraically.

2. Algebraically determine the zeros of the function below.
$r(x) = 3x^3 + 12x^2 - 3x - 12$

3. Solve for all values of x:
$x^4 - 6x^2 = -8$

4. Find algebraically the zeros for $p(x) = x^3 + x^2 - 4x - 4$.

5. Solve the equation $2x^3 - x^2 - 8x + 4 = 0$ algebraically for all values of x.

6. Solve for all values of x:
$x^4 - 5x^2 - 36 = 0$

7. Find algebraically the zeros of $p(x) = x^3 - 3x^2 + 4x - 12$

8. What are the zeros of $P(m) = (m^2 - 4)(m^2 + 1)$?

3. Algebraically find the zeros for $f(x) = x^4 - 4x^3 - 9x^2 + 36x$

10. Solve algebraically for all values of x: $x^4 + 4x^3 + 4x^2 = -16x$

Radical Equations

1) Isolate the radical
2) Square both sides
3) Check

1. $\sqrt{56-x}=x$

2. $\sqrt{2x-7}+x=5$

3. $\sqrt{5x+29}=x+3$

4. $\sqrt{2x-4}=x-2$

5. $\sqrt{49-10x}+5=2x$

6. $\sqrt{4x+1}=11-x$

7. $\sqrt{x^2 + x - 1} + 11x = 7x + 3$

8. $3\sqrt{x} - 2x = -5$

9. $\sqrt{9x} + 2 = \sqrt{7x + 18}$

10. $\sqrt{2x} - 3 = \sqrt{10x + 11}$

Fractional Equations: MULTIPLY BY THE LCD
To find a common denominator:
1) Factor (if necessary)
2) Put all of your factors together

1. $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

2. $\frac{5x}{2} = \frac{1}{x} + \frac{x}{4}$

3. $\frac{x+2}{x} + \frac{x}{3} = \frac{2x^2+6}{3x}$

4. $\frac{1}{2x} - \frac{5}{6} = \frac{3}{x}$

5. $\frac{3}{x} + \frac{x}{x+2} = -\frac{2}{x+2}$

6. $\frac{x}{x-1} = \frac{2}{x} + \frac{1}{x-1}$

7. $\frac{3x+25}{x+7} - 5 = \frac{3}{x}$

8. $\frac{8}{x+5} - \frac{3}{x} = 5$

9. $\frac{7}{2x} - \frac{2}{x+1} = \frac{1}{4}$

10. $\frac{2}{n^2} + \frac{3}{n} = \frac{4}{n^2}$

11. $\frac{x+3}{x-5} + \frac{6}{x+2} = \frac{6+10x}{(x-5)(x+2)}$

12. $\frac{\mathbf{30}}{x^2 - \mathbf{9}} + \mathbf{1} = \frac{\mathbf{5}}{x - \mathbf{3}}$

13. $\frac{1}{x-2}+\frac{4}{x+5}=\frac{7}{x^2+3x-10}$

14. $\frac{x}{x+3}+\frac{2}{x-4}=\frac{2x+27}{x^2-x-12}$

15. $\frac{1}{x-6}+\frac{x}{x-2}=\frac{4}{x^2-8x+12}$

16. $\frac{4}{k^2-8k+12}=\frac{k}{k-2}+\frac{1}{k-6}$

17. $\frac{x}{x+2}+\frac{1}{x^2-4}=\frac{4}{x-2}$

18. $\frac{2}{x+3}-\frac{3}{4-x}=\frac{2x-2}{x^2-x-12}$

19. Markus is a long-distance walker. In one race, he walked 55 miles in t hours and in another race walked 65 miles in $t + 3$ hours. His rates are shown in the equations below.

$$r = \frac{55}{t} \quad r = \frac{65}{t+3}$$

Markus walked at an equivalent rate, r, for each race. Determine the number of hours that each of the two races took.

20. Sarah is fighting a sinus infection. Her doctor prescribed a nasal spray and an antibiotic to fight the infection. The active ingredients, in milligrams, remaining in the bloodstream from the nasal spray, $n(t)$, and the antibiotic, $a(t)$, are modeled in the functions below, where t is the time in hours since the medications were taken.

$$n(t) = \frac{t+1}{t+5} + \frac{18}{t^2 + 8t + 15}$$

$$a(t) = \frac{9}{t+3}$$

Determine which drug is made with a greater initial amount of active ingredient. Justify your answer. Sarah's doctor told her to take both drugs at the same time. Determine algebraically the number of hours after taking the medications when both medications will have the same amount of active ingredient remaining in her bloodstream.

21. To solve the equation $\frac{7}{x+7}+\frac{4x}{x-7}=\frac{3x+7}{x-7}$, Joan's first step is to multiply both sides by the least common denominator. Which statement is true?

1) -14 is an extraneous solution.
2) 7 and -7 are extraneous solutions.
3) 7 is an extraneous solution.
4) There are no extraneous solutions.

22. To solve $\frac{2x}{x-2}-\frac{11}{x}=\frac{8}{x^2-2x}$, Ren multiplied both sides by the least common denominator. Which statement is true?

1) 2 is an extraneous solution.
2) $\frac{7}{2}$ is an extraneous so1ution.
3) 0 and 2 are extraneous solutions.
4) This equation does not contain any extraneous solutions.

23. Jin solved the equation $\sqrt{4-x}=x+8$ by squaring both sides. What extraneous solution did he find?

1) -5
2) -12
3) 3
4) 4

Quadratic Systems of Equations Algebraically

1) Isolate at least one variable in one of the equations
2) Substitute one equation into the other (set them equal if you solved both equations for the same variables).
3) Solve equation (Mr. x^2 /Polynomial Equations)
4) Substitute answers into one of the original equations to find the second variable

1. $y = x^2 - 5$
   $y = 3x - 1$

2. $x^2 + y^2 = 2$
   $y + 2 = x$

3. $x^2 + y^2 = 25$
   $y + 5 = 2x$

4. $y = 2x^2 - 7x + 4$
   $y = 11 - 2x$

5. $(x+2)^2+(y-4)^2=40$
$y=x+2$

6. $(x-2)^2+(y-3)^2=16$
$x+y-1=0$

7. $x+y=5$
$(x+3)^2+(y-3)^2=53$

8. $(x-3)^2+(y+2)^2=16$
$2x+2y=10$

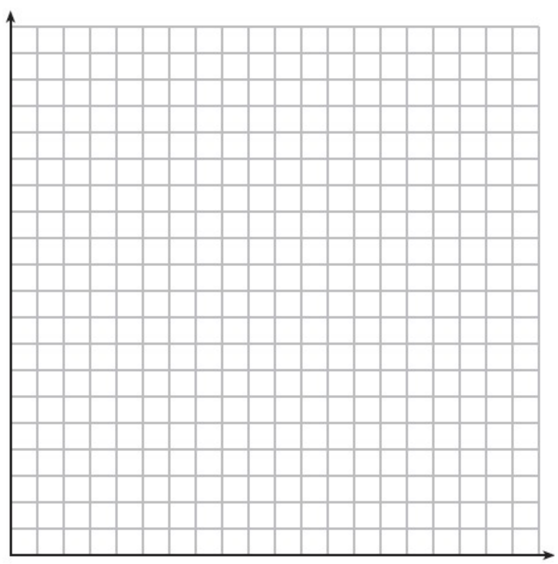
Graphing Functions Part IVs

1) Type equation(s) into Y= in calculator
2) The domain (what you're graphing between) will either be given in the problem or on the graph. If not, you must find an appropriate window in your calculator and use that as your domain.
3) Determine your scale. Guess and check or $scale \geq \frac{\max}{\#\ of\ boxes}$
4) Plot Points

-First follow up question is usually $A(x) = B(x)$ (Y1,Y2,Intersect)

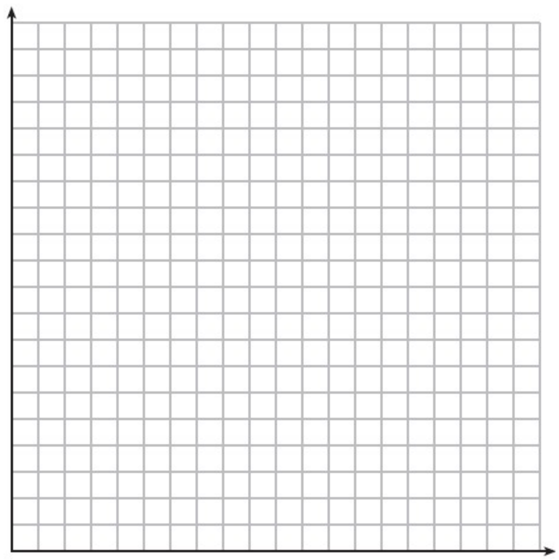
-Second follow up question is usually substituting a given value into the left hand side of one of the equations and solving algebraically with logs or graphically with Y1,Y2,Intersect.

1. The value of Tom's bank account is currently 100000 and is decreasing according to the equation $V(t) = 100000(.876)^t$. The amount of money he has paid for his mortgage can be represented by the equation $M(t) = 20000(1.1304)^t$. Graph and label $V(t)$ and $M(t)$ over the interval $[0,10]$.



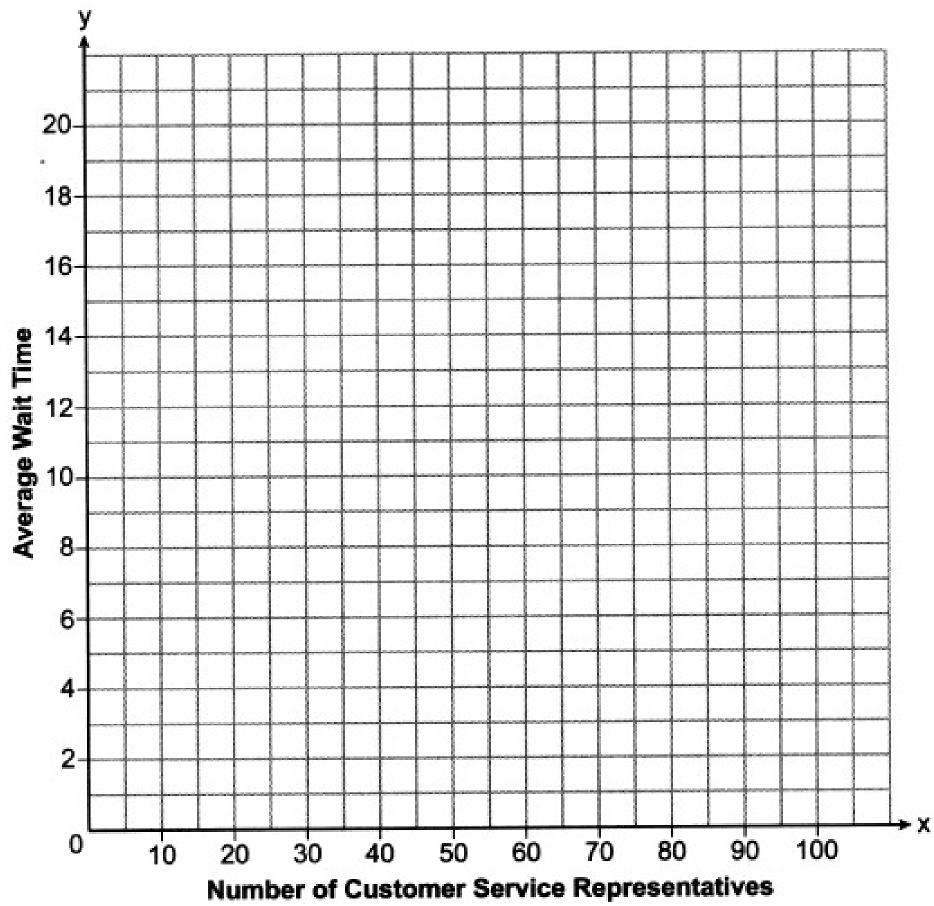
After how many years will the value of Tom's bank account be equal to the amount of money he paid for his mortgage? Round your answer to the *nearest tenth of a year*. Tom will open a new bank account when the value of his account is $20,000. After how many years, to the *nearest hundredth of a year*, will that happen?

2. The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time. Graph $V(t)$ and $Z(t)$ over the interval $0 \le t \le 5$, on the set of axes below.



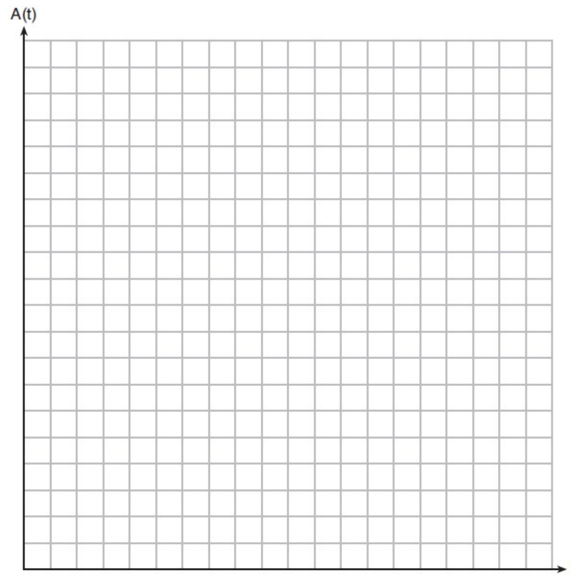
State when $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem. Zach will cancel the collision policy when the value of his car equals $3000. To the *nearest tenth of a year*, how long will it take Zach to cancel this policy? Justify your answer.

3. A technology company is comparing two plans for speeding up its technical support time. Plan A can be modeled by the function $A(x) = 15.7(0.98)^x$ and plan B can be modeled by the function $B(x) = 11(0.99)^x$ where x is the number of customer service representatives employed by the company and $A(x)$ and $B(x)$ represent the average wait time, in minutes, of each customer. Graph $A(x)$ and $B(x)$ in the interval $0 \leq x \leq 100$ on the set of axes below.



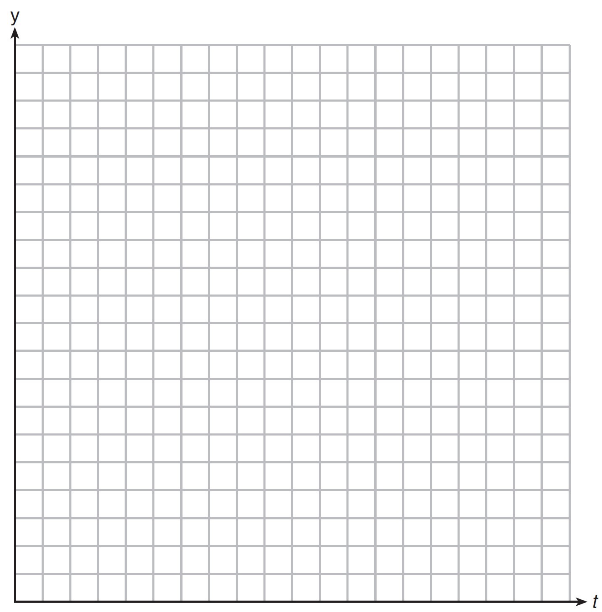
To the *nearest integer*, solve the equation $A(x) = B(x)$. How many Customer Service Representatives would the Company B need in order to the average wait time to be 3 minutes? Round to the *nearest representative*.

4. Tony is evaluating his retirement savings. The value of his account can be represented by $A(t)=318000(1.07)^t$. Graph $A(t)$ where $0 \le t \le 20$ on the set of axes below.



Tony's goal is to save \$1,000,000. Determine algebraically, to the *nearest year*, how many years it will take for him to achieve his goal. Explain how your graph of $A(t)$ confirms your answer.

5. Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, *r* is the decay rate, and *t* is time in hours. Patient *A*, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient *B*, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231. Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.



To the *nearest tenth of an hour*, *t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*? The doctor will allow patient *A* to take another dose of the drug once 120 milligrams is left in the body. Determine, to the *nearest tenth of an hour*, how long patient *A* will have to wait to take another dose of the drug.

Complex Formulas
List what each variable represents and CAREFULLY substitute into the given formula.
Solve the equation using the appropriate Algebra skills

1. A baseball is hit straight up from a height of 6 feet with an initial velocity of 90 feet per second. The equation that models the height of the ball, *s*, as a function of time, *t,* is $s = -16t^2 + v_0 t + s_0$ where v_0 is the initial velocity and s_0 is the initial height. How high is the ball after 4 seconds?

2. A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, *t*, in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation $t = 2\pi\sqrt{\frac{L}{g}}$ where *L* is the length of the pendulum in meters and *g* is a constant of 9.81 m/s^2. The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the *nearest tenth of a second*, the time it takes this pendulum to complete one swing.

3. The loudness of sound is measured in units called decibels (dB). These units are measured by first assigning an intensity I_0 to a very soft sound that is called the threshold sound. The sound to be measured is assigned an intensity, *I*, and the decibel rating, *d*, of this sound is found using $d = 10\log\frac{I}{I_0}$. The threshold sound audible to the average person is 1.0×10^{-12} W/m^2 (watts per square meter). Consider the following sound level classifications. How would a sound with intensity 6.3×10^{-3} W/m^2 be classified?

1) moderate
2) loud
3) very loud
4) deafening

Moderate	45-69 dB
Loud	70-89 dB
Very loud	90-109 dB
Deafening	>110 dB

4. The speed of a tidal wave, s, in hundreds of miles per hour, can be modeled by the equation $s = \sqrt{t} - 2t + 6$, where t represents the time from its origin in hours. Algebraically determine the time when $s = 0$.

5. A formula for work problems involving two people is shown below.

$$\frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{t_b}$$

t_1 = the time taken by the first person to complete the job

t_2 = the time taken by the second person to complete the job

t_b = the time it takes for them working together to complete the job

Fred and Barney are carpenters who build the same model desk. It takes Fred eight hours to build the desk while it only takes Barney six hours. Write an equation that can be used to find the time it would take both carpenters working together to build a desk. Determine, to the *nearest tenth of an hour*, how long it would take Fred and Barney working together to build a desk.

6. A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, t, in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation $t = 2\pi\sqrt{\frac{L}{g}}$ where L is the length of the pendulum in meters and g is a constant of 9.81 m/s^2. The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the *nearest tenth of a meter*, the length of this pendulum.

7. The Beaufort Wind Scale was devised by British Rear Admiral Sir Francis Beaufort, in 1805 based upon observations of the effects of the wind. Beaufort numbers, B, are determined by the equation $B = 1.69\sqrt{s + 4.45} - 3.49$, where s is the speed of the wind in mph, and B is rounded to the nearest integer from 0 to 12.

Beaufort Wind Scale	
Beaufort Number	**Force of Wind**
0	Calm
1	Light air
2	Light breeze
3	Gentle breeze
4	Moderate breeze
5	Fresh breeze
6	Steady breeze
7	Moderate gale
8	Fresh gale
9	Strong gale
10	Whole gale
11	Storm
12	Hurricane

Using the table above, classify the force of wind at a speed of 30 mph. Justify your answer. In 1946, the scale was extended to accommodate strong hurricanes. A strong hurricane received a B value of exactly 15. Algebraically determine the value of s, to the *nearest mph*.

Rounding

1) Identify and underline the digit that is in the spot that you are rounding to:
    - a) Thousand: 1234000
    - b) Hundred: 1234500
    - c) Ten: 1234560
    - d) Unit: 1234567
    - e) Tenth:1234.5
    - f) Hundredth: 1234.56
    - g) Thousandth: 1234.567
    - h) Ten-Thousandth: 1234.5678
    - i) Quarter: 1234.75
2) Draw a line after it
    - a) If the *next* digit is 5 or higher, round up.
    - b) If the *next* digit is 4 or lower, keep it as is.

*When rounding up a 9, look at it as one big number and increase it by 1.

Round 104.9437 to the nearest:

1. Unit:
2. Tenth:
3. Hundredth:
4. Thousandth:

Round 28.3518 to the nearest:

5. Degree:
6. Tenth:
7. Hundredth:
8. Thousandth:

Round 54.8561 to the nearest:

9. Meter:
10. Tenth:
11. Hundredth:
12. Thousandth:

13. Round 59.61 to the nearest inch

14. Round 124.95 to the nearest tenth

15. Round 91.8995 to the nearest hundredth

16. Round 2.1999 to the nearest thousandth

Round the following numbers to the nearest unit

17. 12.92 18. 102.4 19. 47.251 20. 49.75

Round the following numbers to the nearest tenth

21. 15.718 22. 105.519 23. 89.253 24. 235.983

Round the following numbers to the nearest hundredth

25. 29.6901 26. 328.297 27. 181.406 28. 2.4951

Round the following numbers to the nearest thousandth

29. 209.6749 30. 0.57813 31. 111.1142 32. 3.1499

Round 218632.432 to the nearest:

33. Thousand 34. Hundred 35. Ten 36. Quarter

Round 8917521.79 to the nearest:

37. Ten-Thousand 38. Thousand 39. Hundred 40. Quarter

Round 19278132.598271 to the nearest:

41. Hundred-Thousand 42. Thousand 43. Ten 44. Quarter

Round 918361277.9214 to the nearest:

41. Million 42. Ten-Thousand 43. Hundred 44. Quarter

Algebra II Reference Sheet (NGLS)

Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Trigonometric Identities	$\sin^2(\theta) + \cos^2(\theta) = 1$ $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$ $\csc(\theta) = \frac{1}{\sin(\theta)}$ $\sec(\theta) = \frac{1}{\cos(\theta)}$
Cubic Factorizations	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
Probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$
Independence	$P(A \cap B) = P(A) \cdot P(B)$ $P(A \mid B) = P(A)$

Arithmetic Sequence	$a_n = a_1 + d(n-1)$
Arithmetic Series	$S_n = \frac{n(a_1 + a_n)}{2}$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1(1 - r^n)}{1 - r}, r \neq 1$ $S_n = \sum_{k=1}^{n} a_1 r^{k-1}, r \neq 1$
Exponential Growth and Decay	$A = P\left(1 + \frac{r}{n}\right)^{nt}$ $A = Pe^{rt}$ $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$

Normal Curve