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Mr. Schlansky

Date _____
Algebra 2

CCA2 Common Regents Test

Part I: (2 Points Each)

1. What is the completely factored form of $k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48$? 18432 mc strategy

- 1) $(k-2)(k-2)(k+3)(k+4)$ 11648
2) $(k-2)(k-2)(k+6)(k+2)$ 12288
3) $(k+2)(k-2)(k+3)(k+4)$ 17472
④ $(k+2)(k-2)(k+6)(k+2)$ 18432

2. What is the solution set of the equation $\frac{3x+25}{x+7} - 5 = \frac{3}{x}$? mc strategy

- 1) $\left\{\frac{3}{2}, 7\right\}$
2) $\left\{\frac{7}{2}, -3\right\}$
3) $\left\{-\frac{3}{2}, 7\right\}$
④ $\left\{-\frac{7}{2}, -3\right\}$

3. Solve graphically for x : $\sqrt{x^2 + x - 1} + 11x = 7x + 3$ Intersect
 $x = .6$

4. Which factorizations are correct?

- I. $a^3 + 27b^3 = (a+3b)(a^2 - 3ab + 9b^2)$ ✓ 91082.125
II. $c^3 - 6c^2 + 8c + 5c^2 - 30c + 40 = (c-2)(c-4)(c+5)$ ✓ 61.875
III. $1 - x^4 = (1+x)^2(1-x)^2$ ✗ 126

- 1) I, only
② I and II only
3) II and III only
4) I, II, and III

5. Given $f(x) = 3x^2 + 7x - 20$ and $g(x) = x - 2$, state the quotient and remainder of $\frac{f(x)}{g(x)}$, in the form $q(x) + \frac{r(x)}{g(x)}$.

$$\begin{array}{r|rrrr} 2 & 3 & 7 & -20 & \\ & \downarrow & 6 & 26 & \\ \hline & 3 & 13 & 6 & \end{array} \quad 3x+13+\frac{6}{x-2}$$

6. Is $x+2$ a factor of $p(x) = x^3 - 3x^2 - 8x + 4$? Justify your answer.

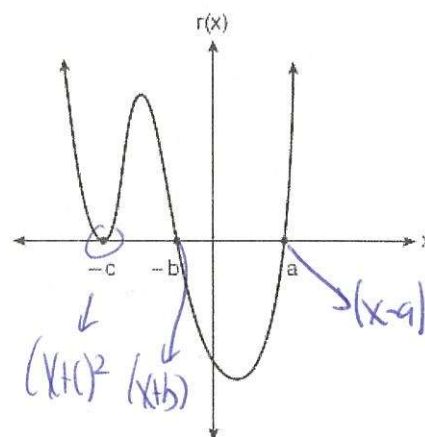
$x+2=0$
 $-2=0$
 $x=-2$

$p(-2) = (-2)^3 - 3(-2)^2 - 8(-2) + 4$
 $p(-2) = 0$
 Yes, the remainder is 0

7. A sketch of $r(x)$ is shown below.

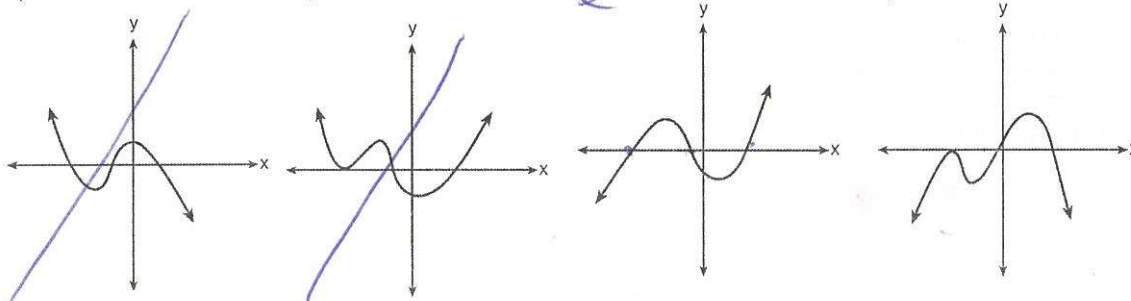
An equation for $r(x)$ could be

- 1) $r(x) = (x-a)(x+b)(x+c)$ 3) $r(x) = (x+a)(x-b)(x-c)$
 2) $r(x) = (x+a)(x-b)(x-c)^2$ 4) $r(x) = (x-a)(x+b)(x+c)^2$



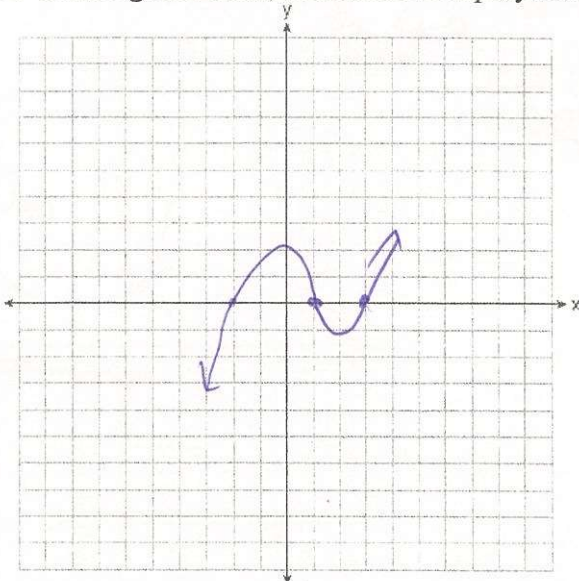
8. Which graph has the following characteristics?

- three real zeros
 - as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$ (with a handwritten 'left' above the arrow)
 - as $x \rightarrow \infty$, $f(x) \rightarrow \infty$ (with a handwritten 'right' above the arrow)
- 1) 2) 3) 4)



The zeros hit the x-axis, not the factors

9. On the grid below, sketch a cubic polynomial whose factors are $x-1$, $x-3$, and $x+2$



Zeros: 1, 3, -2

MC strategy -4-80i

10. Which expression is equivalent to $(2x-i)^2 - (2x-i)(2x+3i)$ where i is the imaginary unit and x is a real number?

- ① $-4 - 8xi$ ~~-4-80i~~
 2) $-4 - 4xi$

- 3) 2
 4) $8x - 4i$

11. If x is a real number, express $2xi(i - 4i^2)$ in simplest $a + bi$ form.

$$\begin{aligned} & 2xi^2 - 8xi^3 \\ & 2x(-1) - 8x(-i) \\ & -2x + 8xi \end{aligned}$$

$$\begin{aligned} i^2 &= -1 \\ i^3 &= -i \end{aligned}$$

12. Which equation represents the equation of the parabola with focus $(-3, 3)$ and directrix $y = 7$?

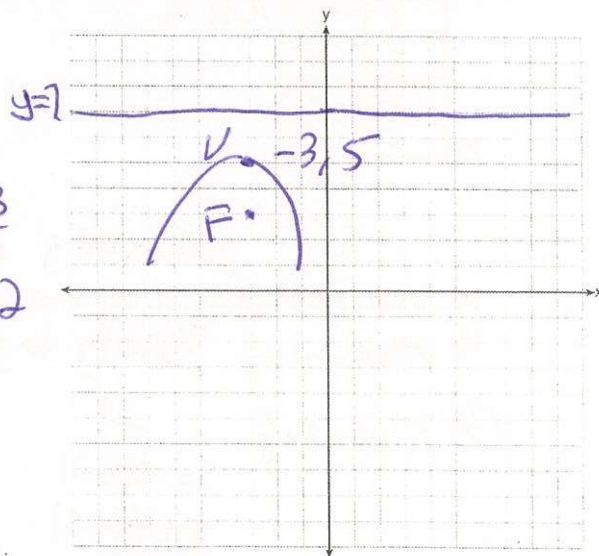
- 1) $y = \frac{1}{8}(x+3)^2 - 5$ ③ $y = -\frac{1}{8}(x+3)^2 + 5$
 2) $y = \frac{1}{8}(x-3)^2 + 5$ 4) $y = -\frac{1}{8}(x-3)^2 + 5$

$$y = \frac{1}{4p}(x-h)^2 + k$$

$$y = \frac{1}{4(-2)}(x+3)^2 + 5$$

$$y = -\frac{1}{8}(x+3)^2 + 5$$

$$\begin{aligned} h &= -3 \\ k &= 5 \\ p &= -2 \end{aligned}$$

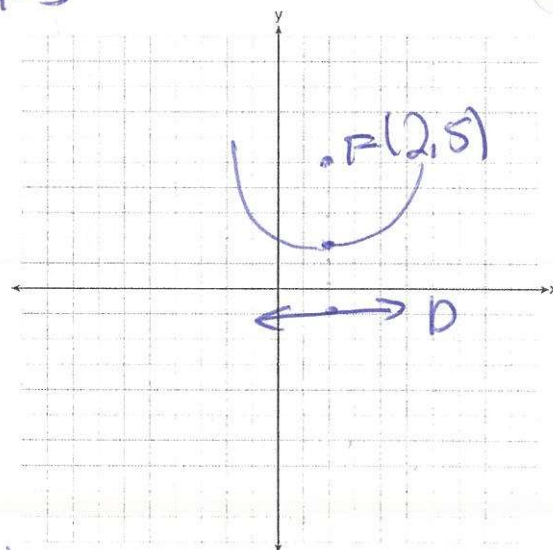


13. The parabola described by the equation $y = \frac{1}{12}(x-2)^2 + 2$ has the directrix at $y = -1$.

The focus of the parabola is

- 1) (2, -1)
- 2) (2, 2)

- 3) (2, 3)
- 4) (2, 5)



14. If $f(x) = 3|x| - 1$ and $g(x) = 0.03x^3 - x + 1$, an approximate solution for the equation $f(x) = g(x)$ is $x =$

- 1) 1.96
- 2) 11.29

- 3) (-0.99, 1.96)
- 4) (11.29, 32.87)

adjust x-max and y-max

never a coordinate

15. Which quadratic function has the largest maximum?

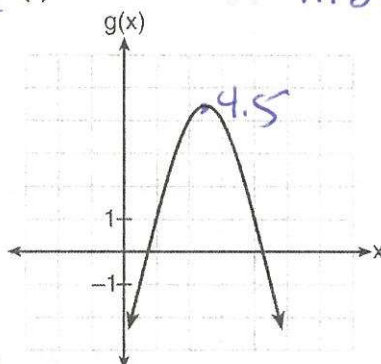
1) $h(x) = (3-x)(2+x)$ 6.25

3) $k(x) = -5x^2 - 12x + 4$

11.2 2nd Trial, 1/5 Maximum

x	f(x)
-1	-3
0	5
1	9
2	9
3	5
4	-3

~9.5



2)

4)

16. What is the inverse of $f(x) = -6(x-2)$?

1) $f^{-1}(x) = -2 - \frac{x}{6}$

2) $f^{-1}(x) = 2 - \frac{x}{6}$

3) $f^{-1}(x) = \frac{1}{-6(x-2)}$

4) $f^{-1}(x) = 6(x+2)$

Switch x and y

Symmetry to y=x

or
 $y = -6(x-2)$
 $x = -6(y-2)$
 $-\frac{x}{6} = y-2$
 $-\frac{x}{6} + 2 = y$

$2 - \frac{x}{6} = y$

$y_1 = -6(x-2)$

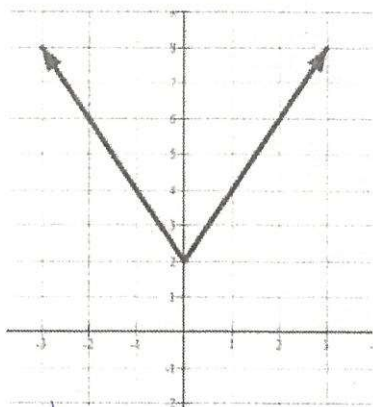
$y_2 = 2 - \frac{x}{6}$

$y_3 = x$

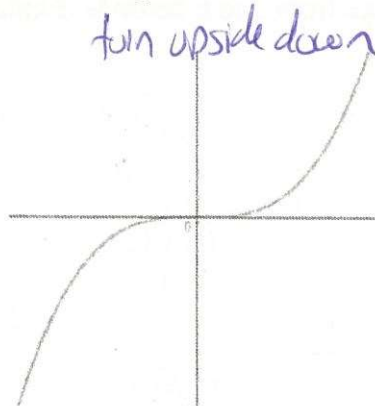
look for symmetry

even symmetric to y-axis
odd symmetric to origin

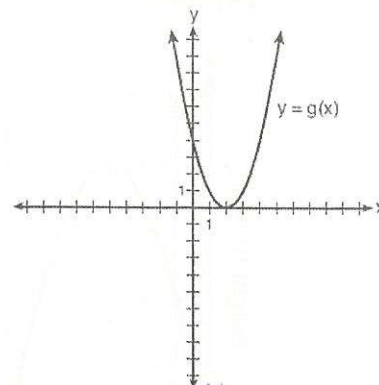
17. Determine graphically whether the following functions are even, odd, or neither



even because symmetric to the y-axis



odd because symmetric to the origin



neither

18. Given the parent function $p(x) = \cos x$, which phrase best describes the transformation used to obtain the graph of $g(x) = \cos(x+a) - b$, if a and b are positive constants?

- 1) right a units, up b units
2) right a units, down b units
3) left a units, up b units
4) left a units, down b units

19. Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
-4	0.3125
-3	0.625
-2	1.25
-1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$$\frac{f(b) - f(a)}{b - a}$$

$$\frac{80 - 1.25}{4 - (-2)} = 13.125$$

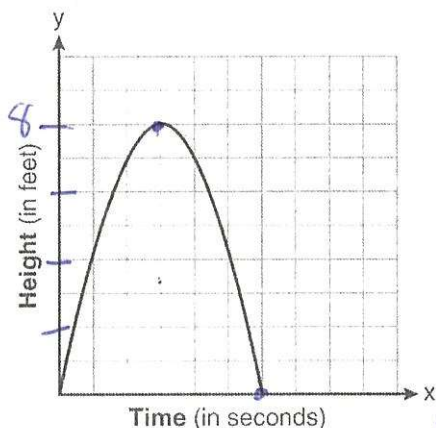
$$\boxed{9/4}$$

$$g(x) = 4x^3 - 5x^2 + 3$$

$$\begin{array}{r} x/y \\ -2 \overline{) -49} \\ 4 \overline{) 179} \end{array}$$

$$\frac{179 - 49}{4 - (-2)} = 38$$

20. The graph below represents the parabolic path of a ball kicked by a young child. Find the average rate of change from 3 to 6 seconds. Explain its meaning in the context of the problem.



$$\begin{array}{r|l} x & y \\ \hline 3 & 8 \\ 6 & 0 \end{array} \quad \frac{f(b) - f(a)}{b - a}$$

from 3 to 6 seconds $\frac{0 - 8}{6 - 3}$

On average, the height of the ball decreases by $\frac{8}{3}$ ft per second.

21. Which value is *not* contained in the solution of the system shown below?

1) -2 ✓

2) 2

3) 3

4) -3 ✓

$$a + 5b - c = -20$$

$$4a - 5b + 4c = 19$$

$$-a - 5b - 5c = 2$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix}$$

MC strategy

22. For positive values of x , which expression is equivalent to $\sqrt{16x^2} \cdot x^{\frac{2}{3}} + \sqrt[3]{8x^5}$

1) $6\sqrt[3]{x^5}$ 278..

2) $6\sqrt{x^3}$ 23..

3) $4\sqrt[3]{x^2} + 2\sqrt[3]{x^5}$

4) $4\sqrt{x^3} + 2\sqrt[3]{x^5}$ 134..

278..

23. Justify why $\frac{\sqrt[3]{x^2y^5}}{\sqrt[4]{x^3y^4}}$ is equivalent to $x^{-\frac{1}{12}}y^{\frac{2}{3}}$ using properties of rational exponents,

where $x \neq 0$ and $y \neq 0$.

Radicals are fractional exponents
Get rid of parenthesis
Negative exponents are fractions
Clean it up ← multiply, divide, evaluate/rational

$$\frac{(x^2y^5)^{\frac{1}{3}}}{(x^3y^4)^{\frac{1}{4}}} = x^{-\frac{1}{12}}y^{\frac{2}{3}}$$

$$\frac{x^{\frac{2}{3}}y^{\frac{5}{3}}}{x^{\frac{3}{4}}y^1} = x^{-\frac{1}{12}}y^{\frac{2}{3}}$$

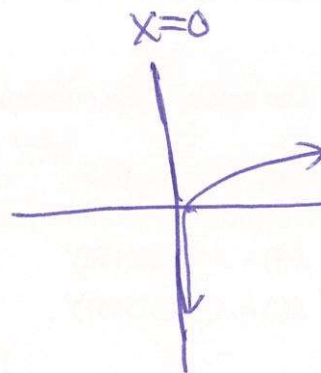
$$\frac{2}{3} - \frac{3}{4} = -\frac{1}{12}$$

$$\frac{5}{3} - 1 = \frac{2}{3}$$

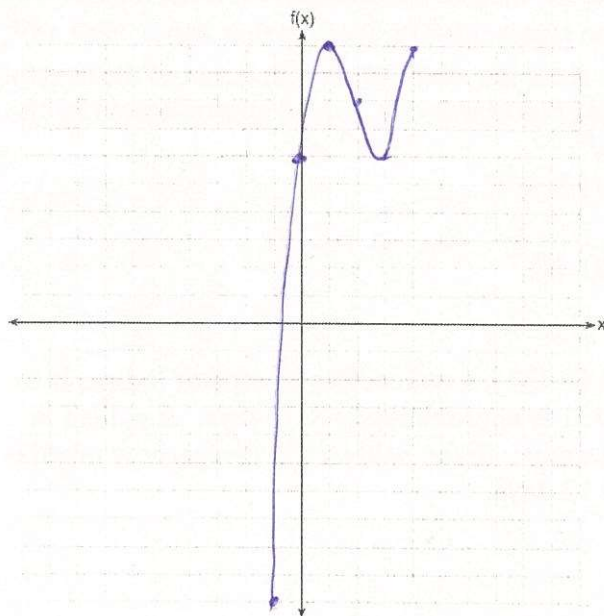
$$x^{-\frac{1}{12}}y^{\frac{2}{3}} = x^{-\frac{1}{12}}y^{\frac{2}{3}}$$

24. Which statement about the graph of $c(x) = \log_6 x$ is false?

- 1) The asymptote has equation $y = 0$. ~~X~~ $x=0$
- 2) The graph has no y -intercept.
- 3) The domain is the set of positive reals.
- 4) The range is the set of all real numbers.



25. On the grid below, graph the function $f(x) = x^3 - 6x^2 + 9x + 6$ on the domain $-1 \leq x \leq 4$ below.



x	y
-1	-10
0	6
1	10
2	8
3	6
4	10

26. The table below shows three different investment options in which Lauren can invest \$3,200.

Option	Annual Interest Rate	Frequency of Compounding
A	4.9%	Annually
B	4.81%	Continuously
C	4.85%	Weekly

$A = P(1 + \frac{r}{n})^{nt}$
 $A = Pe^{rt}$
Option C

Which option will allow Lauren to earn the most money over the course of a four-year period? Justify your answer.

Option A
 $A = P(1 + \frac{r}{n})^{nt}$
 $A = 3200$
 $P = 3200$
 $r = .049$
 $n = 1$
 $t = 4$
 $A = 3200(1 + \frac{.049}{1})^{1(4)}$
 $A = 3671.82$

Option B
 $A = Pe^{rt}$
 $A = 3200$
 $P = 3200$
 $r = .0481$
 $t = 4$
 $A = 3200e^{.0481(4)}$
 $A = 3678.90$

Option C
 $A = P(1 + \frac{r}{n})^{nt}$
 $A = 3200$
 $P = 3200$
 $r = .0485$
 $n = 52$
 $t = 4$
 $A = 3200(1 + \frac{.0485}{52})^{52(4)}$
 $A = 3684.76$

27. The amount of a substance, $A(t)$, that remains after t days can be given by the

equation $A(t) = A_0(0.5)^{\frac{t}{0.0803}}$, where A_0 represents the initial amount of the substance. An equivalent form of this equation is

1) $A(t) = A_0(0.000178)^t$

2) $A(t) = A_0(0.945861)^t$

3) $A(t) = A_0(0.04015)^t$

4) $A(t) = A_0(1.08361)^t$

$1.78 \cdot E-4 = .000178$

$A(t) = A_0(0.5)^{\frac{t}{0.0803}}$
 $A(t) = A_0(0.000178)^t$

28. A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, $B(t)$, can be represented by the function $B(t) = 750(1.16)^t$, where the t represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function

1) $B(t) = 750(1.012)^t$

3) $B(t) = 750(1.012)^{12t}$

2) $B(t) = 750(1.16)^{12t}$

4) $B(t) = 750(1.16)^{\frac{t}{12}}$

$1.16^{\frac{1}{12}} = 1.012$

you get the monthly rate 12 times per year

29. One of the medical uses of Iodine-131 ($I-131$), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of $I-131$ is approximately 8.02 days. A patient is injected with 20 milligrams of $I-131$. Determine, to the nearest milligram, how much $I-131$ will remain in the patient's body after 12 days.

$A = A$
 $P = 20$
 $t = 12$
 $h = 8.02$
 $A = P(\frac{1}{2})^{\frac{t}{h}}$
 $A = 20(\frac{1}{2})^{\frac{12}{8.02}}$
 $A = 7$

30. Biologists are studying a new bacterium. They create a culture with 100 of the bacteria and anticipate that the number of bacteria will double every 30 hours. Write an equation for the number of bacteria, B , in terms of the number of hours, t , since the experiment began. Determine how many bacteria will be present after 20 hours rounded to the nearest tenth of a bacteria.

$A = B$
 $P = 100$
 $t = t$
 $h = 30$
 $A = P(2)^{\frac{t}{h}}$
 $B = 100(2)^{\frac{t}{30}}$
 $B = 100(2)^{\frac{20}{30}}$
 $B = 158.1$

31 27. Write an explicit AND recursive equation for the following sequences

19, 16, 13, 10 ...

$$a_n = a_1 + (n-1)d$$

$$a_n = 19 + (n-1)(-3)$$

$$a_1 = 19$$

$$a_n = a_{n-1} - 3$$

2, 8, 32, 128, ...

2, 8, 32, 128, ...

$$a_n = a_1(r)^{n-1}$$

$$a_n = 2(4)^{n-1}$$

$$a_1 = 2$$

$$a_n = 4a_{n-1}$$

32 28. Find the first 4 terms of the recursive sequence $a_1 = -3$
 $a_n = 4 - 3a_{n-1}$

-3, 13, -35, 109

$$a_2 = 4 - 3(-3) = 13$$

$$a_3 = 4 - 3(13) = -35$$

$$a_4 = 4 - 3(-35) = 109$$

33 29. The population of Jamesburg for the years 2010-2013, respectively, was reported as follows:

250,000 250,937 251,878 252,822

How can this sequence be recursively modeled?

~~1) $j_n = 250,000(1.00375)^{n-1}$~~ ~~2) $j_n = 250,000 + 937^{(n-1)}$~~

3) $j_1 = 250,000$

4) $j_1 = 250,000$

$$j_n = 1.00375j_{n-1}$$

$$j_n = j_{n-1} + 937$$

$$\frac{250937}{250000}$$

$$\frac{251878}{250937}$$

$$1.00375$$

$$1.00375$$

34 30. The sequence defined by $r_1 = 15$ and $r_n = 0.75r_{n-1}$ best models which scenario?

→ decreasing by 25%

- 1) Gerry's \$15 allowance is increased by \$0.75 each week.
- 2) A store that has not sold a \$15 item reduces the price by \$0.25 each week until someone purchases it.
- 3) A 15-gram sample of a chemical compound decays at a rate of 75% per hour.
- 4) A picture with an area of 15 square inches is reduced by 25% over and over again to make a proportionally smaller picture.

3531. Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year. Write a geometric series formula, S_n , for Alexa's total earnings over n years. Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the nearest cent.

$$S_n = a_1 - a_1 (r)^n$$

$$S_n = \frac{33,000 - 33,000(1.04)^n}{1 - 1.04}$$

$$S_{15} = \frac{33,000 - 33,000(1.04)^{15}}{1 - 1.04}$$

$$S_{15} = 660,778.39$$

3632. Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of \$21,000 and a \$1000 down payment, to the nearest cent.

$$P_n = PMT \left(\frac{1 - (1+i)^{-n}}{i} \right)$$

P_n = present amount borrowed T-D $21,000 - 1,000 = 20,000$

n = number of monthly pay periods $5(12) = 60$

PMT = monthly payment x

i = interest rate per month $.00625$

$$20,000 = x \left(\frac{1 - (1 + .00625)^{-60}}{.00625} \right)$$

$$\frac{20,000}{49.9...} = \frac{x(49.9...)}{49.9...}$$

$$x = 400.76$$

The affordable monthly payment is \$300 for the same time period. Determine an appropriate down payment, to the nearest dollar.

$$P_n = x$$

$$n = 60$$

$$PMT = 300$$

$$i = .00625$$

find P

$$x = 300 \left(\frac{1 - (1 + .00625)^{-60}}{.00625} \right)$$

$$x = 14971...$$

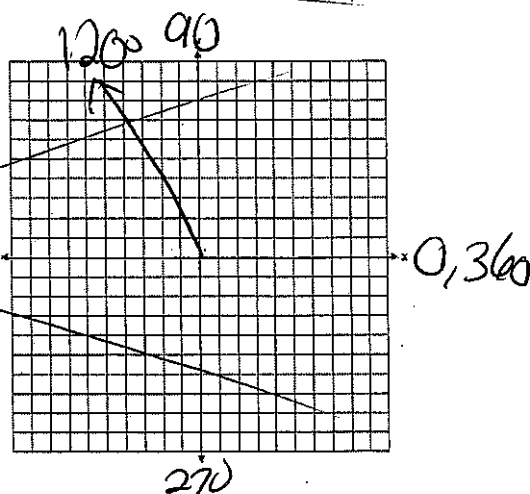
$$D = T - P$$

$$D = 21,000 - 14,971...$$

$$D = 6028$$

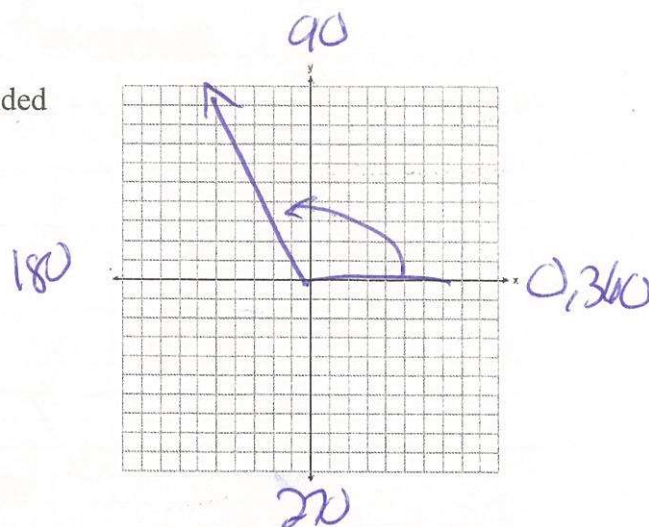
34. Sketch the following angle on the grid provided

$$\theta = \frac{2\pi}{3} \cdot \frac{180}{\pi} = 120^\circ$$



37. Sketch the following angle on the grid provided

$$\theta = \frac{2\pi}{3} \cdot \frac{180}{\pi} = 120^\circ$$



38. Angle θ is in standard position and $(-2, 3)$ is a point on the terminal side of θ . Find:

a) $\cos \theta$

$$-\frac{2\sqrt{13}}{\sqrt{13}\sqrt{13}} = -\frac{2\sqrt{13}}{13}$$

b) $\sin \theta$

$$\frac{3\sqrt{13}}{\sqrt{13}\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

c) $\tan \theta$

$$-\frac{3}{2}$$

d) $\sec \theta$

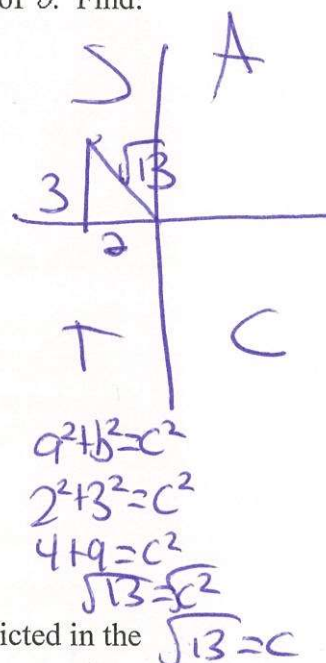
$$-\frac{\sqrt{13}}{2}$$

e) $\csc \theta$

$$\frac{\sqrt{13}}{3}$$

f) $\cot \theta$

$$-\frac{2}{3}$$



39. The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.

If the depth, d , is measured in feet and time, t , is measured in hours since midnight, what is an equation for the depth of the water at the marker?

1) $d = 5 \cos\left(\frac{\pi}{6}t\right) + 9$

2) $d = 9 \cos\left(\frac{\pi}{6}t\right) + 5$

3) $d = 9 \sin\left(\frac{\pi}{6}t\right) + 5$

④ $d = 5 \sin\left(\frac{\pi}{6}t\right) + 9$

$$\text{midline} = \frac{\text{min} + \text{max}}{2}$$

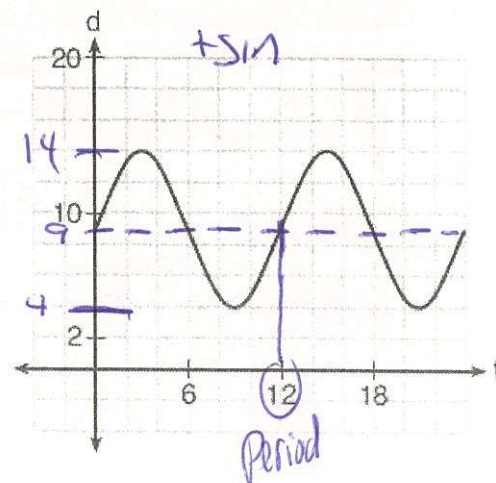
$$\text{midline} = \frac{4 + 14}{2} = 9$$

$$\text{freq} = \frac{2\pi}{\text{Period}}$$

$$\text{freq} = \frac{2\pi}{12} = \frac{\pi}{6}$$

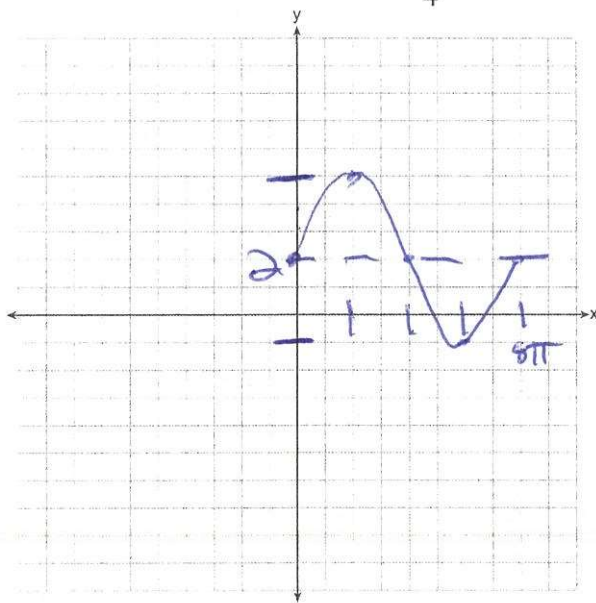
$$y = \text{amp} \sin \text{freq} \times \text{shift}$$

$$y = 5 \sin \frac{\pi}{6}x + 9$$



amp sin freq shift

40. Graph one cycle of $y = 3\sin\frac{1}{4}x + 2$ on the accompanying set of axes



amp = 3

+ sin

freq = $\frac{1}{4}$

shift = 2

$P = \frac{2\pi}{f}$

$P = \frac{2\pi}{\frac{1}{4}}$

$\frac{2\pi}{1} \cdot \frac{4}{1} = 8\pi$

41. Which statement is *incorrect* for the graph of the function $y = -3\cos\left[\frac{\pi}{3}(x-4)\right] + 7$?

- 1) The period is 6. ✓
- 2) The amplitude is 3. ✓
- 3) The range is [4, 10]. ✓
- 4) The midline is $y = -4$. ✗

$y = 7$

$P = \frac{2\pi}{f}$

$2\pi \cdot \frac{3}{\pi} = 6$

$P = \frac{2\pi}{\frac{\pi}{3}}$

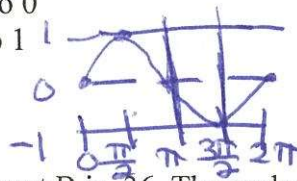
10

7
4

42. As θ increases from π to $\frac{3\pi}{2}$ radians, the graph of $y = \sin\theta$ will

- 1) Decrease from 1 to 0
- 2) Decrease from 0 to -1

- 3) Increase from -1 to 0
- 4) Increase from 0 to 1



$P = \frac{2\pi}{1} = 2\pi$

$IL = \frac{P}{4}$

$IL = \frac{2\pi}{4} = \frac{\pi}{2}$

43. The probability of event A is .27. The probability of event B is .36. The probability of both events happening is .11. What is the probability that event A or event B happens?

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = .27 + .36 - .11$

$P(A \cup B) = .52$

38. The probability of event A is .27. The probability of event B is .36. The probability of both events happening is .11. What is the probability that event A or event B happens?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = .27 + .36 - .11$$

$$P(A \cup B) = .52$$

44. The probability of event A happening is 14% and the probability of event B happening is 18%. The probability that event A or event B happens is 20%. What is the probability that event A and event B happens?

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$P(A \cap B) = .14 + .18 - .2$$

$$P(A \cap B) = .12$$

45. On a given school day, the probability that Nick oversleeps is 48% and the probability he has a pop quiz is 25%. Assuming these two events are independent, what is the probability that Nick oversleeps and has a pop quiz on the same day?

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap B) = .48 \cdot .25$$

$$P(A \cap B) = .12$$

46. A public opinion poll was taken to explore the relationship between age and support for a candidate in an election. The results of the poll are summarized in the table below.

Age	For	Against	No Opinion
21-40	30	12	8
41-60	20	40	15
Over 60	25	35	15

What is the probability that someone is over 60 and against the candidate?

$$\frac{35}{150}$$

What percent of the 21-40 age group was for the candidate?

$$\frac{30}{50} = .6(100) = 60\%$$

47 42. The results of a poll of 200 students are shown in the table below:

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

	Preferred Music Style		
	Techno	Rap	Country
Female	54	25	27
Male	36	40	18

90 65 45 200

A = male
B = techno

$$P(A \cap B) = P(A) \cdot P(B)$$

$$\frac{36}{200} = \frac{94}{200} \cdot \frac{90}{200}$$

9/50 ≠ 423/2000 Not Independent

48 43. The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. What is the percent of women whose heights are less than 60 inches rounded to the nearest whole percent? Out of 250 women, to the nearest woman, how many would be expected to be taller than 69 inches?

lower = 0
upper = 60
 $\mu = 64$
 $\sigma = 2.75$
 $.072 \cdot (100) = 7.2$

lower = 69
upper = 999999
 $\mu = 64$
 $\sigma = 2.75$
 $.0345 \cdot (250) = 8.6$

49 44. A doctor wants to test the effectiveness of a new drug on her patients. She separates her sample of patients into two groups and administers the drug to only one of these groups. She then compares the results. Which type of study best describes this situation?

- 1) census
- 2) survey
- 3) observation
- 4) controlled experiment

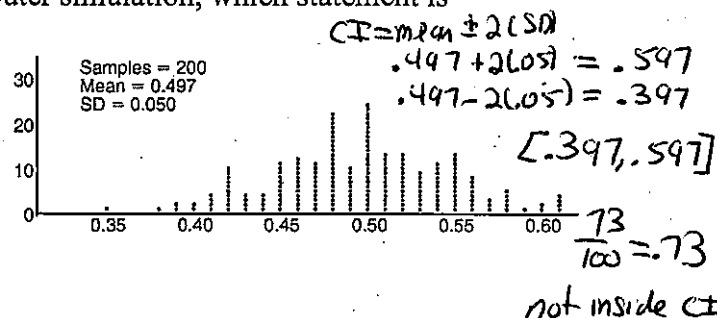
50 45. A survey is being conducted about American's favorite musicians. Which of the following survey methods would most likely produce a random sample?

- 1) Asking every 20th person at a Green Day concert
- 2) Asking every 10th person at a vintage record store
- 3) Asking every 10th person at the Westbury Public Library
- 4) Sending out surveys to random households across the country.

448. Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.

Given the results of her coin flips and of her computer simulation, which statement is most accurate?

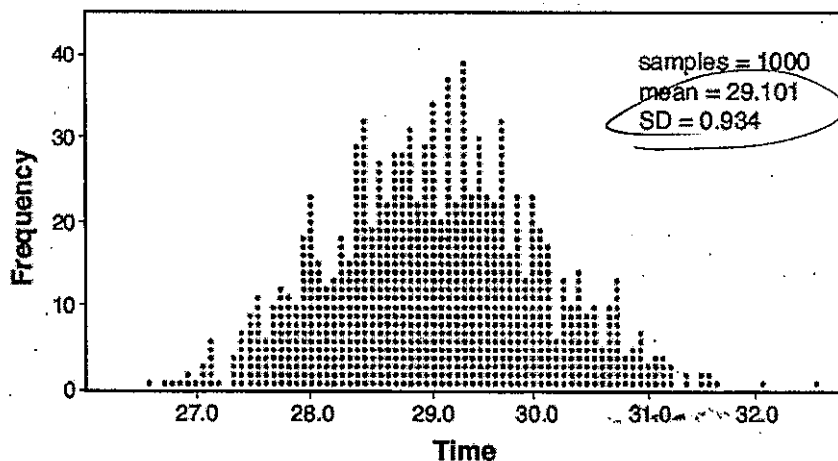
- 1) 73 of the computer's next 100 coin flips will be heads.
- 2) 50 of her next 100 coin flips will be heads.
- 3) Her coin is not fair.
- 4) Her coin is fair.



547. A radio station claims to its advertisers that the mean number of minutes commuters listen to the station is 30. The station conducted a survey of 500 of their listeners who commute. The sample statistics are shown below.

\bar{x}	29.11
s_x	20.718

A simulation was run 1000 times based upon the results of the survey. The results of the simulation appear below.



Based on the simulation results, is the claim that commuters listen to the station on average 30 minutes plausible? Explain your response including an interval containing the middle 95% of the data, rounded to the nearest hundredth.

$$CI = \text{mean} \pm 2(SD)$$

$$CI = 29.101 + 2(.934) = 30.969$$

$$29.101 - 2(.934) = 27.233$$

$$[27.23, 30.97]$$

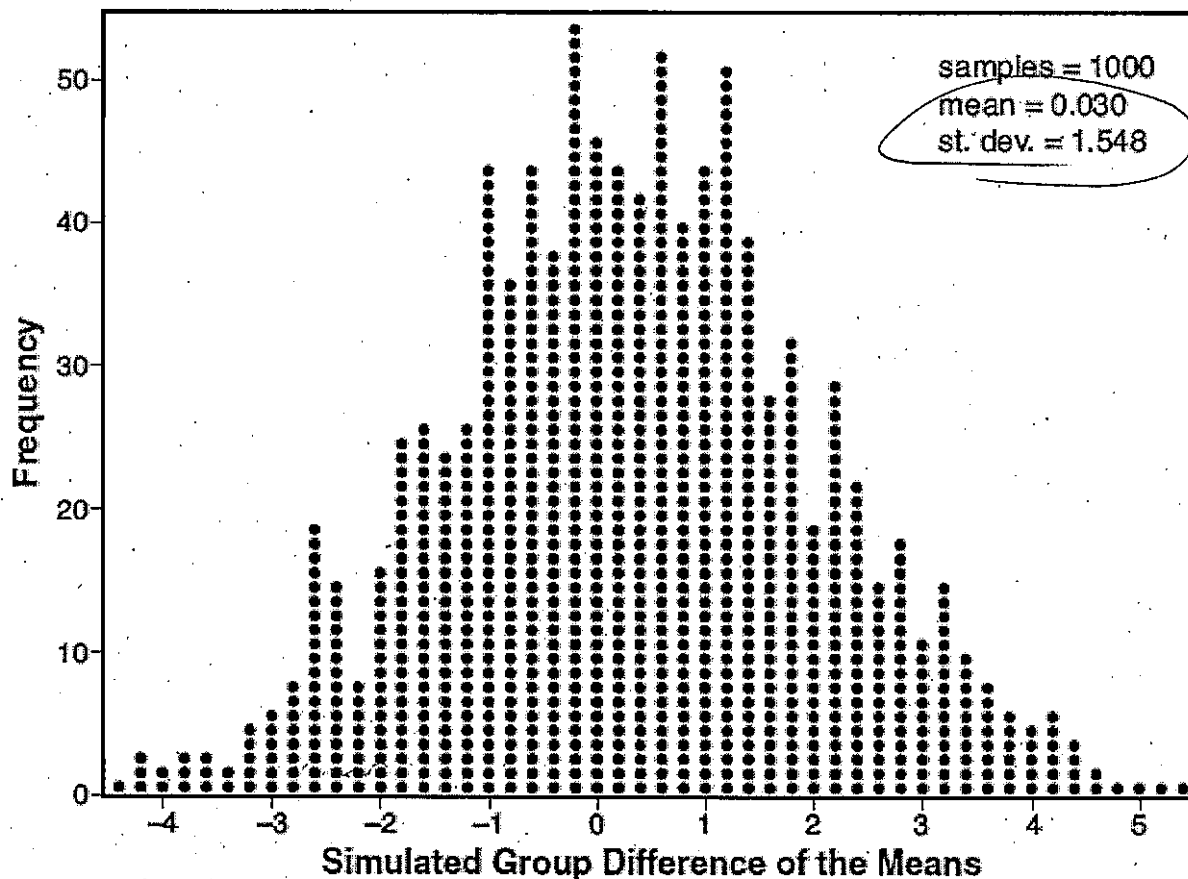
Yes, 30 is inside the confidence interval.

- 53 48. Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

	Scented Paper	Unscented Paper
\bar{x}	23	18
s_x	2.898	2.408

$$23 - 18 = 5$$

Calculate the difference in means in the experimental test grades (scented - unscented). A simulation was conducted in which the subjects' scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.



Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the nearest hundredth. Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.

$$CI = 0.030 + 2(1.548) = 3.126$$

$$0.030 - 2(1.548) = -3.066$$

$$[-3.07, 3.13]$$

Yes, 5 is not inside the confidence interval.

54. Jean invested \$380 in stocks. Over the next 5 years, the value of her investment grew, as shown in the accompanying table.

Years Since Investment (x)	Value of Stock, in Dollars (y)
0	380
1	395
2	411
3	427
4	445
5	462

Write the exponential regression equation for this set of data, rounding all values to two decimal places. Using this equation, find the value of her stock, to the nearest dollar, 10 years after her initial purchase.

Exp Reg

$$y = a(b)^x$$

$$y = 379.92(1.04)^x$$

$$y = 379.92(1.04)^{10}$$

$$y = 562$$

55. After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F . Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below. If the value of k is $.066$, determine the Fahrenheit temperature of the turkey, to the nearest degree, at 3 p.m.

$$T = T_a + (T_0 - T_a)e^{-kt}$$

325 T_a = the temperature surrounding the object

68 T_0 = the initial temperature of the object

7 t = the time in hours

T = the temperature of the object after t hours

.066 k = decay constant

T_a

$$t = 7 \text{ (8 AM - 3 PM)}$$

$$T = 325 + (68 - 325)e^{-.066(7)}$$

$$T = 163^\circ$$

56 ~~52~~ Factor the following

a) $36 - 25x^2$ DOTS
 $(6+5x)(6-5x)$

b) $x^2 - 7x + 12$ Trinomial
 $(x-4)(x-3)$

c) $\frac{3x^2}{3} + \frac{9x}{3} - \frac{12}{3}$ GCF
 $3(x^2+3x-4)$ Trinomial
 $3(x+4)(x-1)$

d) $\frac{6x^2}{6} - \frac{54}{6}$ GCF
 $6(x^2-9)$ DOTS
 $6(x+3)(x-3)$

e) $2x^2 + 7x - 4$ Tricky Tri
 $x^2 + 7x - 8$
 $(x+8)(x-1)$
 $(x+4)(2x-1)$

f) $(x^3 + 3x^2) - (9x - 27)$ Grouping
 $x^2(x+3) - 9(x-3)$
 $(x^2-9)(x+3)$
 $(x+3)(x-3)(x+3)$

g) $\frac{3x^3 + x^2}{x^2} - \frac{12x^2 - 4x}{x^2} - \frac{63x - 21}{x^2}$ degree stepped decreasing Grouping
 $x^2(3x+1) - 4x(3x+1) - 21(3x+1)$

Trinomial $(x^2 - 4x - 21)(3x+1)$
 $(x-7)(x+3)(3x+1)$

h) $(x^2 - 2x)^2 - 11(x^2 - 2x) + 24$ Substitution Trinomial
 $y = x^2 - 2x$
 $y^2 - 11y + 24$
 $(y-8)(y-3)$
 $(x^2 - 2x - 8)(x^2 - 2x - 3)$
 $(x-4)(x+2)(x-3)(x+1)$

i) $y^3 - 125$ cubes
 $a=y$
 $b=5$
 $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
 $y^3 - 125 = (y-5)(y^2 + 5y + 25)$

53. Solve $x^3 + 5x^2 = 4x + 20$ algebraically.
 $-4x-20 \quad -4x-20$

57. $x^2 + 5x = 2x + 40$
 $-2x-40 \quad -2x-40$

Grouping $(x^3 + 5x^2 - 4x - 20) = 0$
 $x^2(x+5) - 4(x+5) = 0$
 $(x^2-4)(x+5) = 0$
 $(x+2)(x-2)(x+5) = 0$
 $x = -2 \quad x = 2 \quad x = -5$

$x^2 + 3x - 40 = 0$
 $(x+8)(x-5) = 0$
 $x+8=0 \quad x-5=0$
 $x=-8 \quad x=5$

58. Solve the equation $x^2 + 2x = -8$ algebraically and express the answer in simplest $a+bi$ form.

quadratic formula

$$\begin{aligned} a &= 1 \\ b &= 2 \\ c &= 8 \end{aligned}$$

$$+8 +8$$

$$x^2 + 2x + 8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(8)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{-28}}{2}$$

$$x = \frac{-2 \pm 2i\sqrt{7}}{2}$$

$$x = -1 \pm i\sqrt{7}$$

$$\sqrt{-28}$$

$$i\sqrt{28}$$

$$i\sqrt{4}\sqrt{7}$$

$$2i\sqrt{7}$$

59. Solve $x^3 + 5x^2 = 4x + 20$ algebraically.

$$-4x - 20 - 4x - 20$$

$$(x^3 + 5x^2 - 4x - 20) = 0$$

$$x^2(x+5) - 4(x+5) = 0$$

$$(x^2 - 4)(x+5) = 0$$

$$(x+2)(x-2)(x+5) = 0$$

$$x+2=0 \quad x-2=0 \quad x+5=0$$

$$x=-2 \quad x=2 \quad x=-5$$

60. Solve the following equation algebraically:

$$\sqrt{2x-7} + x = 5$$

$$-x -x$$

$$(\sqrt{2x-7})^2 = (5-x)^2$$

$$2x-7 = (5-x)(5-x)$$

$$2x-7 = x^2 - 10x + 25$$

$$-2x+7 \quad -2x+7$$

$$0 = x^2 - 12x + 32$$

	5	-x
5	25	-5x
-x	-5x	x^2

$$x^2 - 10x + 25$$

$$0 = (x-8)(x-4)$$

$$x-8=0$$

$$x=8$$

$$x-4=0$$

$$x=4$$

doesn't check

$$\frac{3}{x} + \frac{x}{x+2} = -\frac{2}{x+2}$$

61. Solve algebraically for x:

$$F1. x$$

$$F2. x+2$$

$$L.O. x(x+2)$$

$$3(x+2) + x^2 = -2x$$

$$3x+6+x^2 = -2x$$

$$+2x \quad +2x$$

$$x^2 + 5x + 6 = 0$$

$$(x+3)(x+2) = 0$$

$$x+3=0 \quad x+2=0$$

$$x=-3 \quad x=-2$$

doesn't check

62. Solve algebraically for x:

$$12 + 3(1.2)^{\frac{x}{3}} = 100$$

$$\begin{array}{r} -12 \\ 3(1.2)^{\frac{x}{3}} = \frac{88}{3} \end{array}$$

$$\log(1.2)^{\frac{x}{3}} = \frac{\log 88}{3}$$

$$\frac{x}{3} \log 1.2 = \log \frac{88}{3}$$

$$\frac{x \log 1.2}{\log 1.2} = \frac{2 \log \frac{88}{3}}{\log 1.2}$$

$$x = 37.1$$

63. Solve algebraically for x:

$$2x^{\frac{2}{3}} = 8$$

$$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = 4^{\frac{3}{2}}$$

$$x = 8$$

64. Solve the following system of equations algebraically for x and y

$$(x+2)^2 + (y-4)^2 = 40$$

$$y = x+2$$

$$(x+2)^2 + (x+2-4)^2 = 40$$

$$(x+2)^2 + (x-2)^2 = 40$$

$$x^2 + 4x + 4 + x^2 - 4x + 4 = 40$$

$$\begin{array}{r} 2x^2 + 8 = 40 \\ 40 - 40 \end{array}$$

$$\frac{2x^2 - 32}{2} = \frac{0}{2}$$

$$x^2 - 16 = 0$$

$$(x+4)(x-4) = 0$$

$$\begin{array}{r} x^2 + 2x \\ + 2x + 4 \\ \hline x^2 + 4x + 4 \end{array}$$

$$\begin{array}{r} x + y = 0 \\ -4 - 4 \\ \hline x = -4 \end{array}$$

$$\begin{array}{l} y = x + 2 \\ y = -4 + 2 \\ y = -2 \\ (-4, -2) \end{array}$$

$$\begin{array}{r} x^2 - 2x \\ - 2x + 4 \\ \hline x^2 - 4x + 4 \end{array}$$

$$\begin{array}{r} x - y = 0 \\ +4 +4 \\ \hline x = 4 \end{array}$$

$$\begin{array}{l} y = x + 2 \\ y = 4 + 2 \\ y = 6 \\ (4, 6) \end{array}$$

65. Solve the following system of equations algebraically for all values of x , y , and z :

$$\begin{array}{l} A \quad x + 2y - 3z = -2 \\ B \quad 2x - 2y + z = 7 \\ C \quad x + y + 2z = -4 \end{array}$$

A and B

$$\begin{array}{r} x + 2y - 3z = -2 \\ + \quad 2x - 2y + z = 7 \\ \hline 3x - 2z = 5 \end{array}$$

B and C

$$\begin{array}{r} 2x - 2y + z = 7 \\ + \quad x + y + 2z = -4 \\ \hline 3x - y + 3z = 3 \\ \hline 2x - 2y + z = 7 \\ + \quad 2x + 2y + 4z = -8 \\ \hline 4x + 5z = -1 \end{array}$$

D and E

$$\begin{array}{r} 3x - 2z = 5 \\ 2(4x + 5z = -1) \\ \hline 15x - 10z = 25 \\ + \quad 8x + 10z = -2 \\ \hline 23x = 23 \\ \hline x = 1 \end{array}$$

$$\begin{array}{r} 15x - 10z = 25 \\ + \quad 8x + 10z = -2 \\ \hline 23x = 23 \\ \hline x = 1 \end{array}$$

$$\begin{array}{r} 4x + 5z = -1 \\ 4(1) + 5z = -1 \\ 4 + 5z = -1 \\ -4 \quad -4 \\ \hline 5z = -5 \\ \hline z = -1 \end{array}$$

$$\begin{array}{r} x + 2y - 3z = -2 \\ 1 + 2y - 3(-1) = -2 \\ 1 + 2y + 3 = -2 \\ 2y + 4 = -2 \\ -4 \quad -4 \\ \hline 2y = -6 \\ \hline y = -3 \end{array}$$

$$\begin{array}{r} 2y = -6 \\ \hline y = -3 \end{array}$$

66. A car that was bought for \$24,320 is worth \$9,200 after 7 years. To the nearest percent, what is the annual rate of depreciation?

$$\begin{array}{l} A = 9200 \\ P = 24320 \\ r = r \\ t = 7 \end{array} \quad \begin{array}{l} A = P(1-r)^t \\ 9200 = 24320(1-r)^7 \\ \frac{9200}{24320} = (1-r)^7 \\ (.378) = (1-r)^7 \end{array}$$

$$\begin{array}{r} .81 = 1-r \\ -1 \quad -1 \\ \hline -.19 = -r \\ \hline r = .19 \end{array} \quad \begin{array}{r} 100(.19) = 19\% \end{array}$$

67. Susie invests \$500 in an account that is compounded continuously at an annual interest rate of 5%. Approximately how many years will it take for Susie's money to double?

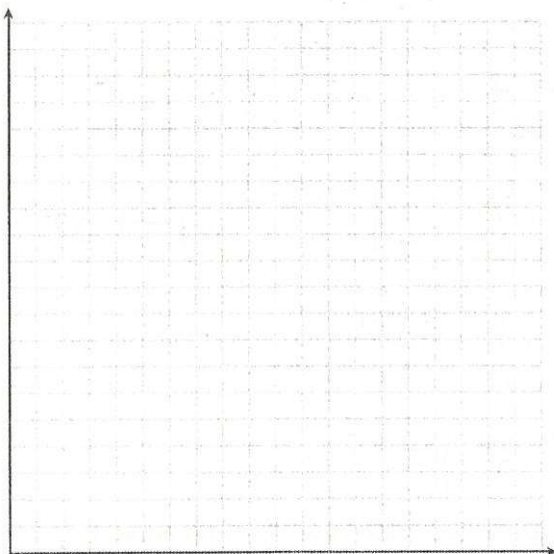
$$\begin{array}{l} A = 2P \\ P = 500 \\ r = .05 \\ t = t \end{array} \quad \begin{array}{l} A = Pe^{rt} \\ 2(500) = 500e^{.05t} \\ \frac{2(500)}{500} = \frac{500e^{.05t}}{500} \\ \ln 2 = \ln e^{.05t} \end{array}$$

$$\begin{array}{r} \ln 2 = .05t \\ \hline .05 \quad .05 \\ \hline 14 = t \end{array}$$

68. One of the medical uses of Iodine-131 (I-131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I-131 is approximately 8.02 days. A patient is injected with 20 milligrams of I-131. Determine, to the nearest day, the amount of time needed before the amount of I-131 in the patient's body is approximately 7 milligrams.

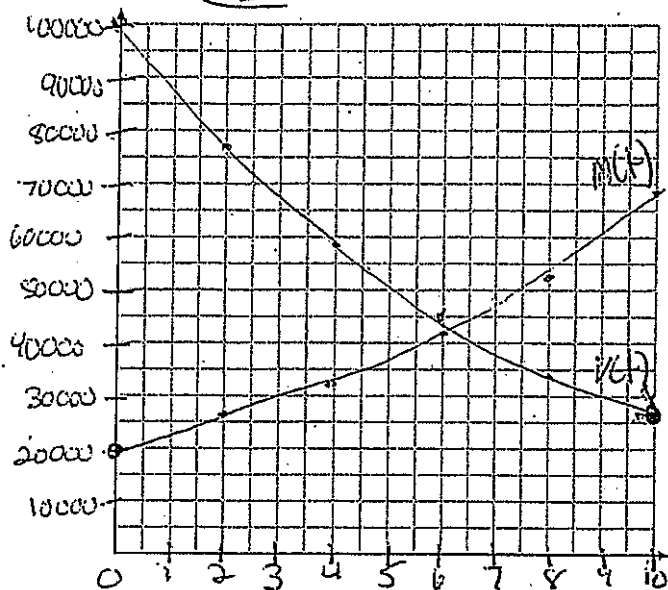
$$\begin{aligned}
 A &= 7 \\
 P &= 20 \\
 t &= ? \\
 h &= 8.02
 \end{aligned}
 \quad
 \begin{aligned}
 A &= P \left(\frac{1}{2} \right)^{\frac{t}{h}} \\
 \frac{7}{20} &= \frac{20}{20} \left(\frac{1}{2} \right)^{\frac{t}{8.02}} \\
 \log \frac{7}{20} &= \log \frac{1}{2} \cdot \frac{t}{8.02} \\
 8.02 \log \frac{7}{20} &= t \log \frac{1}{2} \\
 \frac{8.02 \log \frac{7}{20}}{\log \frac{1}{2}} &= t
 \end{aligned}
 \quad
 \begin{aligned}
 &\overline{A} \\
 &\textcircled{12 = t}
 \end{aligned}$$

69. The value of Tom's bank account is currently 100000 and is decreasing according to the equation $V(t) = 100000(.876)^t$. The amount of money he has paid for his mortgage can be represented by the equation $M(t) = 20000(1.1304)^t$. Graph and label $V(t)$ and $M(t)$ over the interval $[0, 10]$.



After how many years will the value of Tom's bank account be equal to the amount of money he paid for his mortgage? Round your answer to the nearest tenth of a year. Tom will open a new bank account when the value of his account is \$30,000. After how many years, to the nearest hundredth of a year, will that happen?

68. The value of Tom's bank account is currently 100000 and is decreasing according to the equation $V(t) = 100000(.876)^t$. The amount of money he has paid for his mortgage can be represented by the equation $M(t) = 20000(1.1304)^t$. Graph and label $V(t)$ and $M(t)$ over the interval $[0, 10]$.



X	y
0	100000
2	76738
4	58887
6	45188
8	34676
10	26610

X	y
0	20000
2	25556
4	32656
6	41728
8	53320
10	68132

Scale

$$x \geq \frac{10}{20}$$

$$x \geq 0.5$$

$$y \geq \frac{100000}{20}$$

$$y \geq 5000$$

$$y = 5000$$

After how many years will the value of Tom's bank account be equal to the amount of money he paid for his mortgage? Round your answer to the nearest tenth of a year. Tom will open a new bank account when the value of his account has decreased by 72%. After how many years, to the nearest hundredth of a year, will that happen?

$$1 - .72$$

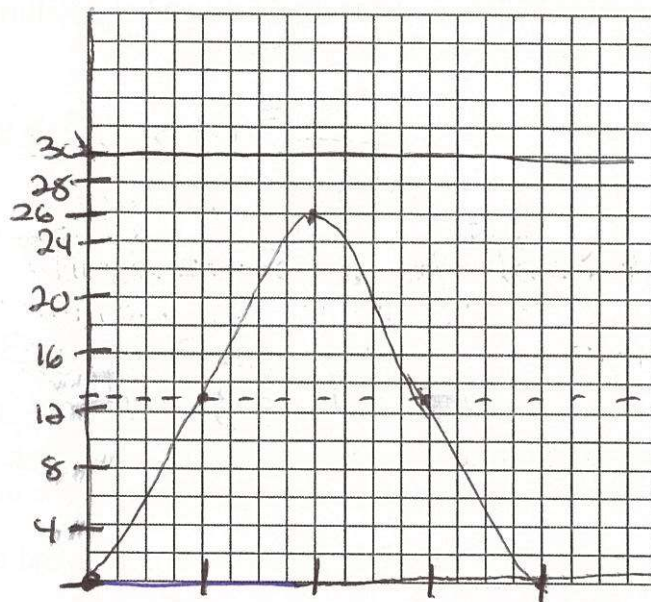
$$.28$$

2nd Trace, Intersect

$$(6.3, 43356.8)$$

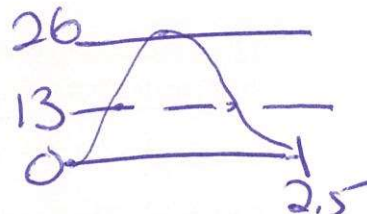
6.3 years

70. Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function $f(t) = -13 \cos(0.8\pi t) + 13$, where t represents the time (in seconds) since the nail first became caught in the tire. Determine the period of $f(t)$. Interpret what the period represents in this context. On the grid below, graph at least one cycle of $f(t)$ that includes the y-intercept of the function.



amp sin / cos shift
 $f(t) = -13 \cos(0.8\pi t) + 13$

amp = 13 $P = \frac{2\pi}{0.8\pi} = 2.5$
 -cos
 freq = 0.8π $P = 2.5$
 shift = 13



Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

No, its maximum value/height is 26.

71. The table below gives air pressures in kPa at selected altitudes above sea level measured in kilometers.

x	Altitude (km)	0	1	2	3	4	5
y	Air Pressure (kPa)	101	90	79	70	62	54

Write an exponential regression equation that models these data rounding all values to the nearest thousandth. Use this equation to algebraically determine the altitude, to the nearest hundredth of a kilometer, when the air pressure is 29 kPa.

Exp Reg
 $y = a(b)^x$

$$y = 101.523(.883)^x$$

$$\frac{29}{101.523} = \frac{101.523}{101.523} (.883)^x$$

$$\log .2856 = \log .883^x$$

$$\frac{\log .2856}{\log .883} = \frac{x \log .883}{\log .883}$$

$$10.07 = x$$

72. A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, t , in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation $t = 2\pi \sqrt{\frac{L}{g}}$ where L is the length of the pendulum in meters and g is a constant of

9.81 m/s^2 . The first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m. Determine, to the nearest tenth of a second, the time it takes this pendulum to complete one swing. Another Foucault pendulum at the United Nations building takes 9.6 seconds to complete one swing. Determine, to the nearest tenth of a meter, the length of this pendulum.

$t =$ time for one swing
 $67 = L =$ Length of Pendulum
 $9.81 = g =$ Constant

$$t = 2\pi \sqrt{\frac{67}{9.81}}$$

$$t = 16.4 \text{ seconds}$$

$t = 9.6$
 $L = L$
 $g = 9.81$

$$22.9 = L$$

$$\frac{9.6}{2\pi} = \frac{2\pi \sqrt{\frac{L}{9.81}}}{2\pi}$$

$$(1.527)^2 = \left(\sqrt{\frac{L}{9.81}}\right)^2$$

$$9.81(2.33) = \frac{L}{9.81} \cdot 9.81$$