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Date ____________
Algebra II

Applications of Equations Regents Review

Modeling Exponential Functions

Basic Exponential Growth/Decay Formula: $A = P(1 \pm r)^t$

COMPOUNDING Interest: $A = P\left(1 \pm \frac{r}{n}\right)^{nt}$, where A is the current amount, P is the initial amount, r is the rate as a decimal (divide by 100), n is the number of times compounded (yearly =1, semiannually = 2, quarterly = 4, monthly = 12, weekly = 52, daily = 365) and t is time.

COMPOUNDING CONTINUOUSLY: $A = Pe^{rt}$

Half Life/Irregular Time (Given percent every x unit of time)

$A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$ where h is the amount of time for the half life

$A = P(1 \pm r)^{\frac{t}{h}}$ where h is the amount of time the rate is applied. For example, if the rate increases by 15% every 5 years, r = .15 and h = 5.

1. A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

$A = 135{,}000$
$P = 100{,}000$
$r = r$
$t = 5$

$A = P(1 \pm r)^t$

$\frac{135{,}000}{100{,}000} = \frac{100{,}000(1+r)^5}{100{,}000}$

$(1.35)^{\frac{1}{5}} = ((1+r)^5)^{\frac{1}{5}}$

$1.06... = 1 + r$
$-1 \quad -1$

$.06185...(100)$
$6.1858...$
6%

2. A car that was bought for \$24,320 is worth \$9,200 after 7 years. To the *nearest percent*, what is the annual rate of depreciation?

$A = 9200$
$P = 24320$
$r = r$
$t = 7$

$A = P(1 \pm r)^t$

$\frac{9200}{24320} = \frac{24320(1-r)^7}{24320}$

$.378... = (1-r)^7$

$(.378...)^{\frac{1}{7}} = ((1-r)^7)^{\frac{1}{7}}$

$.870... = 1 - r$
$-1 \quad -1$

$\frac{-.129...}{-1} = \frac{-r}{-1}$

$r = .129...(100)$
$r = 12.96595275$
$r = 13\%$

3. Determine, to the *nearest year*, how long it will take \$750 invested at an annual rate of 3% to triple.

$A = 3(750)$
$P = 750$
$r = .03$
$t = t$

$A = P(1 \pm r)^t$

$\frac{3(750)}{750} = \frac{750(1+.03)^t}{750}$

$\log 3 = \log(1.03)^t$

$\frac{\log 3}{\log 1.03} = \frac{t \log 1.03}{\log 1.03}$

$37.167... = t$

$37 = t$

4. Susie invests $500 in an account that is compounded continuously at an annual interest rate of 5%. Approximately how many years will it take for Susie's money to double?

P $A=Pe^{rt}$

$A=2(500)$
$P=500$
$r=.05$
$t=t$

$A=Pe^{rt}$

$\frac{2(500)}{500}=\frac{500e^{.05t}}{500}$

$\ln 2=\ln e^{.05t}$

$\frac{\ln 2}{.05\ln e}=\frac{.05t\ln e}{.05\ln e}$

$14=t$

5. The number of bacteria present in a Petri dish can be modeled by the function $N=50e^{3t}$, where N is the number of bacteria present in the Petri dish after t hours. Using this model, determine, to the *nearest hundredth*, the number of hours it will take for N to reach 30,700.

$\frac{30,700}{50}=\frac{50e^{3t}}{50}$

$\ln 614 = \ln e^{3t}$

$\frac{\ln 614}{3\ln e}=\frac{3t\ln e}{3\ln e}$

$2.14=t$

6. One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the *nearest day*, the amount of time needed before the amount of I–131 in the patient's body is approximately 7 milligrams.

$A=7$
$P=20$
$t=t$
$h=8.02$

$A=P(\frac{1}{2})^{\frac{t}{h}}$

$\frac{7}{20}=\frac{20(\frac{1}{2})^{\frac{t}{8.02}}}{20}$

$\log\frac{7}{20}=\log(\frac{1}{2})^{\frac{t}{8.02}}$

$8.02(\log\frac{7}{20})=(\frac{t}{8.02}\log(\frac{1}{2}))8.02$

$8.02\log(\frac{7}{20})=t\log\frac{1}{2}$

$\frac{8.02\log(\frac{7}{20})}{\log(\frac{1}{2})}=\frac{t\log\frac{1}{2}}{\log\frac{1}{2}}$

$12=t$

7. Seth's parents gave him $5000 to invest for his 16th birthday. He is considering two investment options. Option A will pay him 4.5% interest compounded annually. Option B will pay him 4.6% compounded quarterly. Write a function of option A and option B that calculates the value of each account after n years. Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option B will earn than option A to the *nearest cent*. Algebraically determine, to the *nearest tenth of a year*, how long it would take for option B to double Seth's initial investment.

A

$A=P(1+\frac{r}{n})^{nt}$

$A=A(n)$
$P=5000$
$r=.045$
$n=1$
$t=n$

$A(n)=5000(1+\frac{.045}{1})^{1n}$

$A(n)=5000(1.045)^{n}$

$A(6)=5000(1.045)^{6}$

$A(6)=6511.30$

B

$A=P(1+\frac{r}{n})^{nt}$

$A=B(n)$
$P=5000$
$r=.046$
$n=4$
$t=n$

$B(n)=5000(1+\frac{.046}{4})^{4n}$

$B(n)=5000(1.0115)^{4n}$

$B(6)=5000(1.0115)^{4(6)}$

$B(6)=6578.87$

$6578.87-6511.30$

$\$67.57$

$\frac{2(5000)}{5000}=\frac{5000(1.0115)^{4n}}{5000}$

$\log 2=\log 1.0115^{4n}$

$\frac{\log 2}{4\log 1.0115}=\frac{4n\log 1.0115}{4\log 1.0115}$

$15.1548...=n$

$15.2=n$

Polynomial Equations Given a Factor

1) Use Remainder Theorem with the given factor to set up equation.
2) Solve for k.
3) Divide out the given factor using synthetic division.
4) Factor completely

*To find zeros, set each factor equal to zero.

$P(-1)=0$

1. Consider the polynomial $p(x) = x^3 + kx^2 + x + 6$. Find a value of k so that $x+1$ is a factor of P. Find all the zeros of P.

$0=(-1)^3+k(-1)^2+(-1)+6$

$0=-1+k-1+6$

$0=k+4$

$-4 \quad -4$

$-4=k$

$p(x)=\dfrac{x^3-4x^2+x+6}{x+1}$

$$\begin{array}{r|rrrr} -1 & 1 & -4 & 1 & 6 \\ & & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$0=(x+1)(x^2-5x+6)$

$0=(x+1)(x-3)(x-2)$

$x=-1 \quad x=3 \quad x=2$

2. Consider the polynomial $p(x) = x^3 + kx - 30$. Find a value of k so that $x+3$ is a factor of P. Find all the zeros of P.

$P(-3)=0$

$0=(-3)^3+k(-3)-30$

$0=-27-3k-30$

$0=-3k-57$

$+57 \quad +57$

$\dfrac{57}{-3}=\dfrac{-3k}{-3}$

$-19=k$

$p(x)=\dfrac{x^3-19x-30}{x+3}$

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -19 & -30 \\ & & -3 & 9 & 30 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

$(x+3)(x^2-3x-10)=0$

$(x+3)(x-5)(x+2)=0$

$x=-3 \quad x=5 \quad x=-2$

$x+4$ is a factor

3. Given $p(x) = 6x^3 + 31x^2 + kx - 12$, and $p(-4) = 0$, algebraically determine all the zeros of $p(x)$.

$0 = 6(-4)^3 + 31(-4)^2 + k(-4) - 12$

$0 = -384 + 496 - 4k - 12$

$0 = -4k + 100$

$-100 \qquad -100$

$\frac{-100}{-4} = \frac{-4k}{-4}$

$25 = k$

$p(x) = 6x^3 + 31x^2 + 25x - 12$

$\frac{6x^3 + 31x^2 + 25x - 12}{x+4}$

$$\begin{array}{r|rrrr} -4 & 6 & 31 & 25 & -12 \\ & & -24 & -28 & 12 \\ \hline & 6 & 7 & -3 & 0 \end{array}$$

$p(x) = (x+4)(6x^2+7x-3)$

PT

$= (x+4)(6x^2+7x-3)$

$= (x+4)(x^2+7x-18)$

$(x+\frac{9}{6})(x-\frac{2}{6})$

$(x+\frac{3}{2})(x-\frac{1}{3})$

$= (x+4)(2x+3)(3x-1)$

$x+4=0$, $-4 \quad -4$, $x=-4$

$2x+3=0$, $-3 \quad -3$, $\frac{2x}{2} = \frac{-3}{2}$, $x = \frac{-3}{2}$

$3x-1=0$, $+1 \quad +1$, $\frac{3x}{3} = \frac{1}{3}$, $x = \frac{1}{3}$

4. Given $z(x) = 6x^3 + bx^2 - 52x + 15$, $z(2) = 35$, and $z(-5) = 0$, algebraically determine all the zeros of $z(x)$.

PT

$x+5$ is a factor

$0 = 6(-5)^3 + b(-5)^2 - 52(-5) + 15$

$0 = -750 + 25b + 260 + 15$

$0 = 25b - 475$

$+475 \qquad +475$

$\frac{475}{25} = \frac{25b}{25}$

$19 = b$

$z(x) = 6x^3 + 19x^2 - 52x + 15$

$\frac{6x^3 + 19x^2 - 52x + 15}{x+5}$

$$\begin{array}{r|rrrr} -5 & 6 & 19 & -52 & 15 \\ & & -30 & 55 & -15 \\ \hline & 6 & -11 & 3 & 0 \end{array}$$

$z(x) = (x+5)(6x^2 - 11x + 3)$

$z(x) = (x+5)(x^2 - 11x + 18)$

$z(x) = (x+5)(x - \frac{9}{6})(x - \frac{2}{6})$

$z(x) = (x+5)(x - \frac{3}{2})(x - \frac{1}{3})$

$z(x) = (x+5)(2x-3)(3x-1)$

$x+5=0$, $-5 \quad -5$, $x=-5$

$2x-3=0$, $+3 \quad +3$, $\frac{2x}{2} = \frac{3}{2}$, $x = \frac{3}{2}$

$3x-1=0$, $+1 \quad +1$, $\frac{3x}{3} = \frac{1}{3}$, $x = \frac{1}{3}$

5. Given $p(x) = x^3 + 5x^2 + kx - 24$, and $x+3$ is a factor, algebraically determine all the zeros of $p(x)$.

$p(-3) = 0$

$0 = (-3)^3 + 5(-3)^2 + k(-3) - 24$

$0 = -27 + 45 - 3k - 24$

$0 = -3k - 6$

$+6 \qquad +6$

$\frac{6}{-3} = \frac{-3k}{-3}$

$-2 = k$

$p(x) = x^3 + 5x^2 - 2x - 24$

$\frac{x^3 + 5x^2 - 2x - 24}{x+3}$

$$\begin{array}{r|rrrr} -3 & 1 & 5 & -2 & -24 \\ & & -3 & -6 & 24 \\ \hline & 1 & 2 & -8 & 0 \end{array}$$

$p(x) = (x+3)(x^2 + 2x - 8)$

$0 = (x+3)(x+4)(x-2)$

$x=3 \quad x=-4 \quad x=2$

Graphing Functions

1) Type equation(s) into Y= in calculator
2) The domain (what you're graphing between) will either be given in the problem or on the graph. If not, you must find an appropriate window in your calculator and use that as your domain.
3) Determine your scale. $scale \geq \dfrac{\max}{\#\ of\ boxes}$
4) Plot Points

*For follow up questions, you may need to solve equations graphical

1. Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is $P(x) = \log(x - 4)$, where x is the number of visits per week in thousands and $P(x)$ is the website's popularity rating. According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the *nearest tenth*? Graph $y = P(x)$ on the axes below.

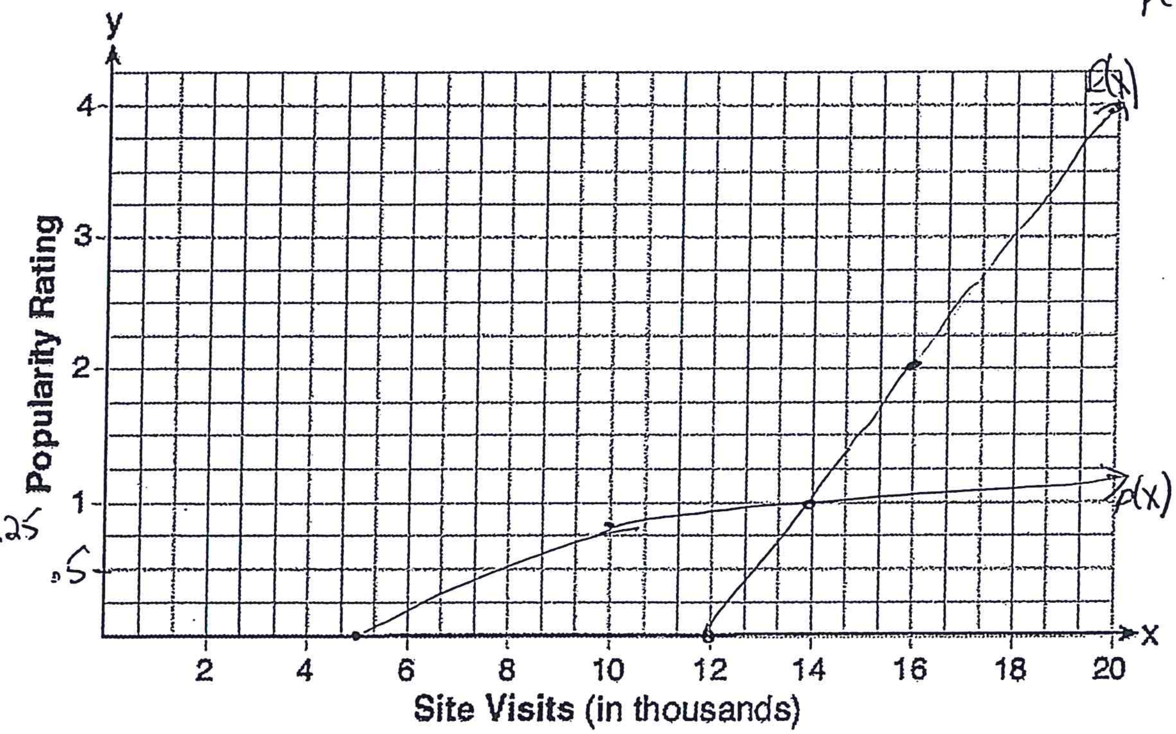
$P(16) = \log(16-4)$

$P(16) = 1.1$

$P(x)$

x	y
5	0
10	.77815
14	1
20	1.2

Scale $= \frac{1}{4} = .25$



An alternative rating model is represented by $R(x) = \frac{1}{2}x - 6$, where x is the number of visits per week in thousands. Graph $R(x)$ on the same set of axes. For what number of weekly visits will the two models provide the same rating?

$P(x) = R(x)$

2nd Trace, Intersect

$(14, 1)$

14,000 weekly visits

$R(x)$

x	y
0	-6
4	-4
8	-2
12	0
16	2
20	4

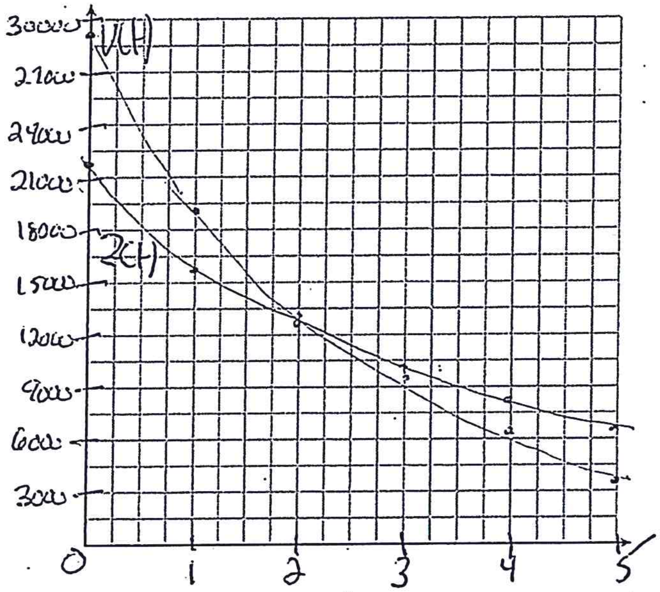
2 ~~11~~. The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time. Graph $V(t)$ and $Z(t)$ over the interval $0 \leq t \leq 5$, on the set of axes below.

V(t)

X	y
0	28483
1	19482
2	13326
3	9114.8
4	6234.6
5	4264.4

Z(t)

X	y
0	22151
1	17234
2	13408
3	10431
4	8115.6
5	6313.4



Scale

$x \geq \frac{5}{20}$ $\quad$ $y \geq \frac{28483}{20}$

$x \geq .25$ $\quad$ $y \geq 1424.15$

$y = 1500$

State when $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem. Zach takes out an insurance policy that requires him to pay a \$3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the *nearest year*, how long will it take Zach to cancel this policy? Justify your answer.

3000

$t = 1.95$

After 1.95 years, the value of the loans will be the same (\$13569.24)

$Z(t) = 22151.327(0.778)^t$

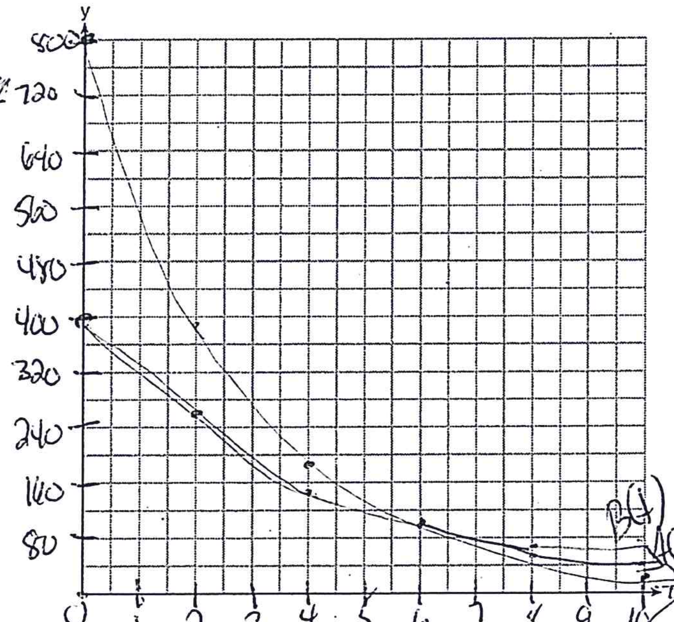
$3000 = 22151.327(0.778)^t$

3 ~~12~~. Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 N_0 milligrams of a drug with a decay rate of 0.347 r. Patient B, $B(t)$, is given 400 N_0 milligrams of another drug with a decay rate of 0.231 r. Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.

$A(t) = 800e^{-.347t}$

x	y
0	800
2	399.66
4	199.66
6	99.744
8	49.83
10	24.894

Scale



$B(t) = 400e^{-.231t}$

x	y
0	400
2	252.01
4	158.77
6	100.03
8	63.021
10	39.105

x and y min = 0

No domain Use Zoom fit

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A? The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

same
1 Trace
intersect
(5.98, 100.10)
hours

Scale

$x \geq \frac{10}{20}$ $\quad y \geq \frac{800}{20}$

$x \geq .5$ $\quad y \geq 40$

$x = .5$ $\quad y = 40$

$A(t) = .15(800)$

$.15(800) = 800e^{-.347t}$

$Y_1 \quad Y_2 \quad$ Find intersection

or

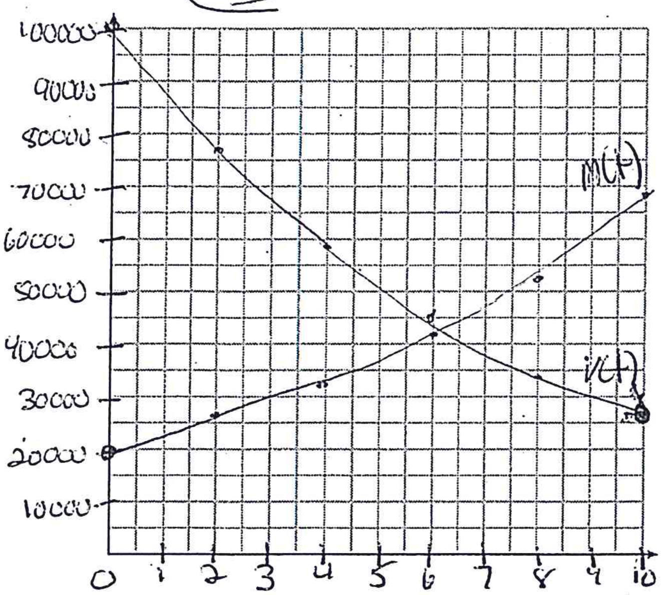
$\frac{.15(800)}{800} = \frac{800e^{-.347t}}{800}$

$\ln .15 = \ln e^{-.347t}$

$\frac{\ln .15}{-.347 \ln e} = \frac{-.347t \ln e}{-.347 \ln e}$

$5.5 = t$

4 ~~13~~. The value of Tom's bank account is currently 100000 and is decreasing according to the equation $V(t)=100000(.876)^t$. The amount of money he has paid for his mortgage can be represented by the equation $M(t)=20000(1.1304)^t$. Graph and label $V(t)$ and $M(t)$ over the interval [0,10].



x	y
0	100000
2	76738
4	58887
6	45188
8	34676
10	26610

x	y
0	20000
2	25556
4	32656
6	41728
8	53320
10	68132

Scale

$x \geq \frac{10}{20}$

$x \geq .5$

$x = .5$

$y \geq \frac{100000}{20}$

$y \geq 5000$

$y = 5000$

After how many years will the value of Tom's bank account be equal to the amount of money he paid for his mortgage? Round your answer to the *nearest tenth of a year*. Tom will open a new bank account when the value of his account has decreased by 72%. After how many years, to the *nearest hundredth of a year*, will that happen?

$1-.72$

$.28$

2nd Trace, Intersect

(6.3, ~~44~~ 43356.8)

6.3 years

5 ~~14~~. A major car company analyzes its revenue, $R(x)$, and costs $C(x)$, in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years, x, using the given functions.

$$R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000$$

$$C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000$$

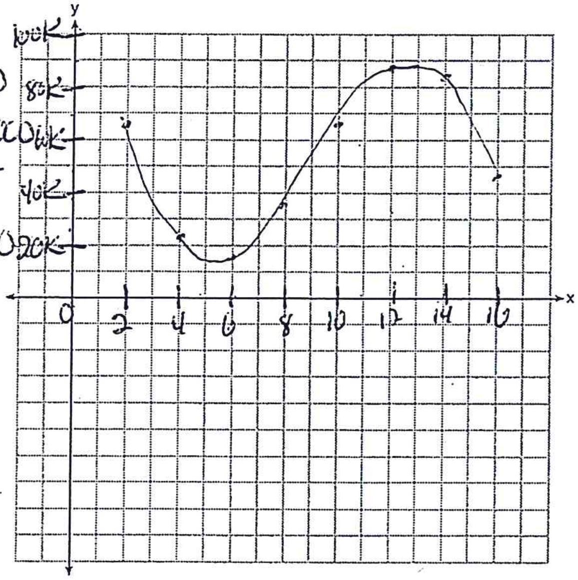
The company's profits can be represented as the difference between its revenue and costs. Write the profit function, $P(x)$, as a polynomial in standard form. Graph $y = P(x)$ on the set of axes below over the domain $2 \le x \le 16$.

$(Y) = R(x) - C(x)$

$(Y) = 550x^3 - 12000x^2 + 83,000x + 7000$

$+ \; -880x^3 + 21000x^2 - 150,000x + 160000$

$Y = -330x^3 + 9000x^2 - 67000x + 167000$



X	y
2	66360
4	21880
6	17720
8	38040
10	67000
12	88760
14	87840
16	47320

Over the given domain, state when the company was the least profitable and the most profitable, to the *nearest year*. Explain how you determined your answer.

Least profitable is the relative minimum.

2nd Trace, minimum

(5, 15557)

5 years

Scale

$x \ge \frac{16}{18}$ $\quad y \ge \frac{88760}{10}$

$x \ge .8$ $\quad y \ge 8876$

$x = 1$ $\quad y = 10000$

Most Profitable is relative maximum

2nd Trace, Maximum

(13, 91996)

13 years

Linear Systems In Three Variables
Elimination Method:

1) Choose two pairs of equations and get the same variable to cancel
2) Use Addition Method to solve the system with your two new equations
3) Substitute those two answers into one of the original equations to find your third variables

*Make sure all variables are in order on the left hand side and all constants are on the right hand side.

1. Solve the following system of equations algebraically for all values of x, y, and z:

A $x + 3y + 5z = 45$

B $6x - 3y + 2z = -10$

C $-2x + 3y + 8z = 72$

A and B

$x+3y+5z=45$
$+\ 6x-3y+2z=-10$
D $7x+7z=35$

B and C

$6x-3y+2z=-10$
$+\ -2x+3y+8z=72$
E $4x+10z=62$

D and E

$-4(7x+7z=35)$
$7(4x+10z=62)$
$-28x-28z=-140$
$28x+70z=434$
$\frac{42z}{42}=\frac{294}{42}$
$z=7$

$7x+7z=35$
$7x+7(7)=35$
$7x+49=35$
$-49\ \ -49$
$\frac{7x}{7}=\frac{-14}{7}$
$x=-2$

$x+3y+5z=45$
$-2+3y+5(7)=45$
$-2+3y+35=45$
$3y+33=45$
$-33\ \ -33$
$\frac{3y}{3}=\frac{12}{3}$
$y=4$

2. Solve the following system of equations algebraically for all values of x, y, and z:

A $x+2y-3z=-2$

B $2x-2y+z=7$

C $x+y+2z=-4$

A and B

$x+2y-3z=-2$
$+\ 2x-2y+z=7$
D $3x-2z=5$

B and C

$1(2x-2y+z=7)$
$2(x+y+2z=-4)$
$2x-2y+z=7$
$+\ 2x+2y+4z=-8$
E $4x+5z=-1$

D and E

$5(3x-2z=5)$
$2(4x+5z=-1)$
$15x-10z=25$
$+\ 8x+10z=-2$
$\frac{23x}{23}=\frac{23}{23}$
$x=1$

$4x+5z=-1$
$4(1)+5z=-1$
$4+5z=-1$
$-4\ \ \ -4$
$\frac{5z}{5}=\frac{-5}{5}$
$z=-1$

$x+2y-3z=-2$
$1+2y-3(-1)=-2$
$1+2y+3=-2$
$2y+4=-2$
$-4\ \ -4$
$\frac{2y}{2}=\frac{-6}{2}$
$y=-3$

3. Solve the following system of equations algebraically for all values of x, y, and z:

A $2x + 3y - 4z = -1$

B $x - 2y + 5z = 3$

C $-4x + y + z = 16$

A and B

$1(2x+3y-4z=-1)$

$-2(x-2y+5z=3)$

$2x+3y-4z=-1$

$+\ -2x+4y-10z=-6$

D $7y-14z=-7$

A and C

$2(2x+3y-4z=-1)$

$1(-4x+y+z=16)$

$4x+6y-8z=-2$

$+\ -4x+y+z=16$

E $7y-7z=14$

D and E

$-1(7y-14z=-7)$

$7y-7z=14$

$-7y+14z=7$

$+\ 7y-7z=14$

$7z=21$

$\frac{7z}{7}=\frac{21}{7}$

$z=3$

$7y-7z=14$

$7y-7(3)=14$

$7y-21=14$

$+21 \quad +21$

$7y=35$

$\frac{7y}{7}=\frac{35}{7}$

$y=5$

$x-2y+5z=3$

$x-2(5)+5(3)=3$

$x-10+15=3$

$x+5=3$

$-5 \quad -5$

$x=-2$

4. Solve the following system of equations algebraically for all values of a, b, and c.

A $a + 4b + 6c = 23$

B $a + 2b + c = 2$

$6b + 2c = a + 14$

$-a \qquad -a$

C $-a+6b+2c=14$

A and C

$a+4b+6c=23$

$+\ -a+6b+2c=14$

D $10b+8c=37$

B and C

$a+2b+c=2$

$+\ -a+6b+2c=14$

E $8b+3c=16$

D and E

$-3(10b+8c=37)$

$8(8b+3c=16)$

$-30b-24c=-111$

$64b+24c=128$

$34b=17$

$\frac{34b}{34}=\frac{17}{34}$

$b=.5$

$10b+8c=37$

$10(.5)+8c=37$

$5+8c=37$

$-5 \qquad -5$

$8c=32$

$\frac{8c}{8}=\frac{32}{8}$

$c=4$

$a+4b+6c=23$

$a+4(.5)+6(4)=23$

$a+2+24=23$

$a+26=23$

$-26 \quad -26$

$a=-3$