

Equations Review

① $0 = x^4 - 4x^3 - 9x^2 + 36x$ polynomial equation

$0 = x(x^3 - 4x^2 - 9x + 36)$ Factor grouping

$0 = x((x^2(x-4) - 9(x-4)))$ Dot

$0 = x(x^2 - 9)(x - 4)$

$0 = x(x+3)(x-3)(x-4)$

$x=0$	$x+3=0$ $x=-3$ $x=-3$	$x-3=0$ $x=3$ $x=3$	$x-4=0$ $x=4$ $x=4$
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② $0 = \frac{x^3}{x^2} + \frac{x^2}{x^2} - \frac{4x}{-4} - \frac{4}{-4}$ polynomial equation
Factor grouping

$0 = x^2(x+1) - 4(x+1)$ Dot

$0 = (x^2 - 4)(x+1)$

$0 = (x+2)(x-2)(x+1)$

$x+2=0$ $x=-2$ $x=-2$	$x-2=0$ $x=2$ $x=2$	$x+1=0$ $x=-1$ $x=-1$
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③ $x^3 + 5x^2 = 4x + 20$ polynomial equation
Bring everything to 1 side

$\frac{x^3}{x^2} + \frac{5x^2}{x^2} - \frac{4x}{-4} - \frac{20}{-4} = 0$

Factor grouping

$x^2(x+5) - 4(x+5) = 0$ Dot

$(x^2 - 4)(x+5) = 0$

$(x+2)(x-2)(x+5) = 0$

$x+2=0$ $x=-2$ $x=-2$	$x-2=0$ $x=2$ $x=2$	$x+5=0$ $x=-5$ $x=-5$
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$$(4) \quad \underbrace{(x^3 - x)}_{x^2 - 4} \underbrace{(8x + 4)}_{-4} = 0$$

polynomial Equation
Factor
Grouping

$$x^2(2x-1) - 4(2x-1) = 0$$

$$(x^2 - 4)(2x - 1) = 0 \quad \rightarrow \text{DOTS}$$

$$(x+2)(x-2)(2x-1) = 0$$

$x+2=0$ -2 -2 $x=-2$	$x-2=0$ +2 -2 $x=2$	$2x-1=0$ +1 -1 $2x=1$ $\frac{2x}{2} = \frac{1}{2}$ $x = \frac{1}{2}$
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$$(5) \quad 6x^2 - 2x - 3 = 0$$

quadratic equation
quadratic formula

$$a=6 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b=-2$$

$$c=-3$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{2 \pm \sqrt{76}}{12}$$

$$x = \frac{2 \pm 2\sqrt{19}}{12 \div 2}$$

$$\begin{array}{c} \sqrt{76} \\ \swarrow \quad \searrow \\ \sqrt{4} \quad \sqrt{19} \\ 2\sqrt{19} \end{array}$$

$$x = \frac{1 \pm \sqrt{19}}{6}$$

⑥ $X^2 + 2X = -8$ Quadratic Equation
 $\begin{matrix} +8 & +8 \end{matrix} \rightarrow$ Bring everything to 1 side

$$X^2 + 2X + 8 = 0$$

quadratic formula or completing the square

quadratic formula

$a=1$
 $b=2$
 $c=8$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(8)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{-28}}{2}$$

$$x = \frac{-2 \pm 2i\sqrt{7}}{2}$$

$$x = -1 \pm i\sqrt{7}$$

$\sqrt{-28}$
 $i\sqrt{28}$
 $\sqrt{4} \sqrt{7}$
 $2i\sqrt{7}$

Completing the square

$$X^2 + 2X + 8 = 0$$

$$\begin{matrix} -8 & -8 \end{matrix}$$

$$X^2 + 2X = -8$$

$$\left(\frac{b}{2}\right)^2$$

$$\left(\frac{2}{2}\right)^2$$

$$1$$

$$X^2 + 2X + 1 = -8 + 1$$

$$(X+1)(X+1) = -7$$

$$\sqrt{(X+1)^2} = \sqrt{-7}$$

$$X+1 = \pm i\sqrt{7}$$

$$\begin{matrix} -1 & -1 \end{matrix}$$

$$x = -1 \pm i\sqrt{7}$$

⑦ $3X^2 + 6 = 5X$
 $-5X - 5X$

Quadratic Equation
 Bring everything to 1 side
 Quadratic formula

$$3X^2 - 5X + 6 = 0$$

$a=3$
 $b=-5$
 $c=6$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(6)}}{2(3)}$$

$$X = \frac{5 \pm \sqrt{-47}}{6}$$

$$X = \frac{5 \pm i\sqrt{47}}{6}$$

$\sqrt{-47}$
 $i\sqrt{47}$

$$⑧ \quad 2x^2 + 3x + 2 = 0$$

quadratic equation
quadratic formula

$$a=2 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b=3$$

$$c=2$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(2)}}{2(2)}$$

$$\sqrt{-7}$$

$$x = \frac{-3 \pm \sqrt{-7}}{4}$$

$$⑩ \quad -\frac{1}{2}x^2 = -6x + 20$$

$$+\frac{1}{2}x^2 \quad +\frac{1}{2}x^2$$

$$0 = \frac{1}{2}x^2 - 6x + 20$$

$$a = \frac{1}{2} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b = -6$$

$$c = 20$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(\frac{1}{2})(20)}}{2(\frac{1}{2})}$$

quadratic equation
bring everything to 1 side
quadratic formula

$$\sqrt{-4}$$

$$x = \frac{6 \pm \sqrt{-4}}{1}$$

$$⑪$$

$$6^2 = (\frac{1}{2}x^2 - 6x + 20)^2$$

$$⑨ \quad 18x^2 - 24x + 87 = 0$$

quadratic equation
quadratic formula

$$a=18 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b=-24$$

$$c=87$$

$$x = \frac{24 \pm \sqrt{(-24)^2 - 4(18)(87)}}{2(18)}$$

$$\sqrt{-5688}$$

$$x = \frac{24 \pm \sqrt{-5688}}{36}$$

$$i\sqrt{5688}$$

$$x = \frac{24 \pm 6i\sqrt{158}}{36 \div 6}$$

$$x = \frac{4 \pm i\sqrt{158}}{6}$$

$$4) \frac{2}{3} \pm \frac{1}{6}i\sqrt{158}$$

$$0 = x^2 - 12x + 40 \quad \text{completing the square}$$

$$x^2 - 12x = -40$$

$$x^2 - 12x + 36 = -40 + 36$$

$$(x-6)(x-6) = -4$$

$$\sqrt{(x-6)^2} = \sqrt{-4}$$

$$x-6 = \pm 2i$$

$$x = 6 \pm 2i$$

$$3) 6 \pm 2i$$

$$(-\frac{12}{2})^2 = 36$$

$$\sqrt{-4}$$

$$i\sqrt{4}$$

$$2i$$

⑪ $x^4 + 4x^3 + 4x^2 = -16x$
^{4 solutions}
 $+16x +16x$

polynomial equation
 Bring everything to 1 side

⑬ $\sqrt{x-5} + x = 7$
 $-x -x$

$x^4 + 4x^3 + 4x^2 + 16x = 0$

Factor Radical Equation
 Isolate

$(x-5)^2 = (7-x)^2$

$x(x^3 + 4x^2 + 4x + 16) = 0$

Grouping

square both sides

$x-5 = 49-14x+x^2$
 $-x+5 +5-x$

$x((x^2(x+4) + 4(x+4))) = 0$

square binomial theorem

$0 = x^2 - 15x + 54$

$x(x^2+4)(x+4) = 0$

or Box method

$0 = (x-9)(x-6)$

$x=0$
 $x^2+4=0$
 $-4 -4$
 $x^2 = -4$
 $x = \pm 2i$

Quadratic Equation
 Bring everything to 1 side

$x-9=0$
 $+9 +9$
 $x=9$
 $x-6=0$
 $+6 +6$
 $x=6$

⑫ $\sqrt{2x+1} + 4 = 8$
 $-4 -4$

Radical equation
 Isolate

Factor
 check

$\sqrt{9-5} + 9 = 7$ $\sqrt{6-5} + 6 = 7$

$(\sqrt{2x+1}) = (4)^2$
 $2x+1 = 16$
 $-1 -1$

square both sides

⑭ $(\sqrt{56-x}) = (x)^2$

Radical Equation
 isolated already
 square both sides

$2x = 15$
 $\frac{2}{2} \frac{15}{2}$
 $x = 7.5$

$56-x = x^2$
 $-56+x +x-56$

Quadratic Equation
 Bring everything to 1 side
 Factor

check!

$0 = x^2 + x - 56$

$0 = (x+8)(x-7)$

$x+8=0$
 $-8 -8$
 $x=-8$
 $x-7=0$
 $+7 +7$
 $x=7$

check

$\sqrt{2(7.5)+1} + 4 = 8$ ✓

$\sqrt{56-8} = -8$ $\sqrt{56-7} = 7$
 $\times$ ✓

⑮ $\sqrt{2x-7} + x = 5$
 $-x -x$

Radical Equation
 Isolate

$0 = x^2 - 12x + 32$

Factor

$(\sqrt{2x-7}) = (5-x)^2$

square both sides

$0 = (x-8)(x-4)$

$2x-7 = x^2 - 10x + 25$
 $-2x+7 -2x+7$

square binomial theorem

$x-8=0$
 $+8 +8$
 $x=-8$
 $x-4=0$
 $+4 +4$
 $x=4$

Quadratic Equation
 Bring everything to 1 side

$\sqrt{2(8)-7} + 8 = 5$

check

$\times$

$\sqrt{2(4)-7} + 4 = 5$ ✓

$$(16) \sqrt{5x+29} = (x+3)^2$$

$$\begin{array}{r} 5x+29 = x^2+6x+9 \\ -5x-29 \quad -5x-29 \\ \hline \end{array}$$

$$0 = x^2+x-20$$

$$0 = (x+5)(x-4)$$

$$\begin{array}{cc} x+5=0 & x-4=0 \\ -5 & +4 \end{array}$$

$$x = -5 \quad x = 4$$

$$\sqrt{5(-5)+29} = -5+3$$

$$\sqrt{5(4)+29} = 4+3$$

Radical Equation
Isolated Already

Square both sides

Square binomial theorem

Quadratic Equation

Bring everything to 1 side

Factor

check

$$(17) \sqrt{2x-4} = (x-2)^2$$

$$\begin{array}{r} 2x-4 = x^2-4x+4 \\ -2x+4 \quad -2x+4 \\ \hline \end{array}$$

$$0 = x^2-6x+8$$

$$0 = (x-4)(x-2)$$

$$\begin{array}{cc} x-4=0 & x-2=0 \\ +4 & +2 \end{array}$$

$$x = 4 \quad x = 2$$

Radical Equation

Isolated Already

Square both sides

Square binomial theorem

Quadratic Equation

Bring everything to 1 side

Factor

check

$$(18) \sqrt{x^2+x-1} + 11x = 7x+3$$

Radical Equation
Isolate

$$\sqrt{x^2+x-1} = (-4x+3)^2$$

$$\begin{array}{r} x^2+x-1 = 16x^2-24x+9 \\ -x^2-x+1 \quad -x^2-x+1 \\ \hline \end{array}$$

$$0 = 15x^2-25x+10$$

$$0 = 3x^2-5x+2$$

$$0 = x^2-5x+6$$

$$(x-3)(x-2)$$

$$0 = (x-1)(3x-2)$$

$$\begin{array}{cc} x-1=0 & 3x-2=0 \\ +1 & +2 \end{array}$$

$$x = 1 \quad \frac{3x}{3} = \frac{2}{3}$$

$$x = \frac{2}{3}$$

Square both sides

Square binomial theorem

Quadratic Equation

Bring everything to 1 side

Factor

Bridge Trinomials

check

$$\sqrt{(1)^2+1-1} + 11(1) = 7(1)+3$$

$$\sqrt{(\frac{2}{3})^2+\frac{2}{3}-1} + 11(\frac{2}{3}) = 7(\frac{2}{3})+3$$

$$(19) \frac{1}{x} - \frac{1}{3} = \frac{1}{3x}$$

Fractional Equations
multiply by the LCD

$$\frac{1}{3} - x = -\frac{1}{3}$$

$$-x = -\frac{4}{3}$$

$$x = \frac{4}{3}$$

$$\frac{1}{4} - \frac{1}{3} = -\frac{1}{12}$$

check

$$(20) \frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$$

Fractional Equation
multiply by LCD

$$x^3+9 = x^3+8+1$$

$$x^3+9 = x^3+9$$

② $2x \left(\frac{5x}{2} \right) = \left(\frac{1}{x} \right) + \left(\frac{x}{4} \right)$ Fractional Equations
multiply by LCD

$$10x^2 = 4 + x^2$$

$$-x^2 - 4 - x^2$$

Quadratic Equation
Bring everything to 1 side

Factor
(DOTS)

$$ax^2 - 4 = 0$$

$$(3x+2)(3x-2) = 0$$

$$\begin{array}{l|l} 3x+2=0 & 3x-2=0 \\ -2 & +2 \end{array}$$

$$\frac{3x}{3} = \frac{-2}{3} \quad \frac{3x}{3} = \frac{2}{3}$$

$$x = -\frac{2}{3} \quad x = \frac{2}{3}$$

$$\frac{5(-\frac{2}{3})}{2} = -\frac{2}{3} + \frac{-2}{4}$$

$$\frac{5(\frac{2}{3})}{2} = \frac{1}{3} + \frac{2}{4}$$

check

② $\frac{3x+25}{x+7} - 5 = \frac{3}{x}$ Fractional Equations
multiply by LCD

$$x(3x+25) - 5x(x+7) = 3(x+7)$$

$$3x^2 + 25x - 5x^2 - 35x = 3x + 21$$

combine like terms
Quadratic Equation
Bring everything to 1 side

$$-2x^2 - 10x = 3x + 21$$

$$0 = 2x^2 + 13x + 21$$

$$0 = x^2 + 13x + 42$$

$$(x+7)(x+6)$$

Factor
(Bridge Trinomial)

$$0 = (2x+7)(x+3)$$

$$\begin{array}{l|l} 2x+7=0 & x+3=0 \\ -7 & -3 \end{array}$$

$$\frac{2x}{2} = \frac{-7}{2} \quad x = -3$$

$$x = -\frac{7}{2}$$

check

② $\frac{30}{x^2-9} + 1 = \frac{5}{x-3}$

Fractional Equations
multiply by LCD
(Factor first)

$$\left(\frac{30}{(x+3)(x-3)} + 1 \right) = \left(\frac{5}{x-3} \right)$$

$$30 + x^2 - 9 = 5(x+3)$$

$$x^2 + 21 = 5x + 15$$

$$-5x - 15 - 5x - 15$$

Quadratic Equation
Bring everything to 1 side

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$\begin{array}{l|l} x-3=0 & x-2=0 \\ +3 & +2 \end{array}$$

$$x = 3 \quad x = 2$$

check

$$\frac{30}{3^2-9} + 1 = \frac{5}{3-3} \quad \frac{30}{2^2-9} + 1 = \frac{5}{2-3}$$

$$\frac{3(-\frac{7}{2}) + 25}{-\frac{7}{2} + 7} - 5 = \frac{3}{-\frac{7}{2}}$$

$$\frac{3(-3) + 25}{-3 + 7} - 5 = \frac{3}{-3}$$

$$(24) \quad \frac{2}{x+3} - \frac{3}{4-x} = \frac{2x-2}{x^2-x-12}$$

Fractional Equation
multiply by LCD
(Factor first)

$$\frac{2}{x+3} - \frac{3}{4-x} = \frac{2x-2}{(x-4)(x+3)}$$

$$2(x-4) + 3(x+3) = 2x-2$$

$$2x-8+3x+9=2x-2$$

$$5x+1=2x-2$$

$$3x+1=-2$$

$$\frac{3x}{3} = \frac{-3}{3}$$

$$x=-1$$

check

$$\frac{2}{-1+3} - \frac{3}{4-1} = \frac{2(-1)-2}{(-1)^2(-1)-12}$$

$$(25) \quad \frac{3}{x} + \frac{x}{x+2} = -\frac{2}{x+2}$$

Fractional Equation

multiply by LCD

Quadratic Equation

Bring everything to 1 side

Factor

$$3(x+2) + x^2 = -2x$$

$$3x+6+x^2=-2x$$

$$x^2+5x+6=0$$

$$(x+3)(x+2)=0$$

$$x+3=0 \quad x+2=0$$

$$-3-3 \quad -2-2$$

$$x=-3 \quad x=-2$$

check!

$$\frac{3}{-3} + \frac{-3}{-3+2} = \frac{-2}{-3+2}$$

$$\frac{3}{-2} + \frac{-2}{-2+2} = \frac{-2}{-2+2}$$

(26)

$$\frac{1}{x+3} - \frac{2}{3-x} = \frac{4}{x^2-9}$$

Fractional Equation
multiply by LCD
(Factor first)

$$\frac{1}{x+3} - \frac{2}{3-x} = \frac{4}{(x+3)(x-3)}$$

$$x-3+2(x+3)=4$$

$$x-3+2x+6=4$$

$$3x+3=4$$

$$\frac{3x}{3} = \frac{1}{3}$$

$$x = \frac{1}{3}$$

check

$$\frac{1}{\frac{1}{3}+3} - \frac{2}{3-\frac{1}{3}} = \frac{4}{(\frac{1}{3})^2-9}$$

(27)

$$\frac{13x}{x} = \frac{10x}{x} - \frac{x^2}{x}$$

Fractional Equation
multiply by LCD

$$13 = 10x - x^2$$

Quadratic Equation
Bring everything to 1 side

$$x^2-10x+13=0$$

Quadratic Formula

or

completing the square

$$x^2-10x+13=0$$

$$a=1$$

$$b=-10$$

$$c=13$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{10 \pm \sqrt{(-10)^2-4(1)(13)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{48}}{2}$$

$$x = \frac{10 \pm 4\sqrt{3}}{2}$$

$$x = 5 \pm 2\sqrt{3}$$

$$x^2-10x+13=0$$

$$\left(\frac{-10}{2}\right)^2$$

$$(-5)^2$$

$$25$$

$$x^2-10x+13=-13+26$$

$$(x-5)(x-5)=12$$

$$\sqrt{(x-5)^2} = \sqrt{12}$$

$$x-5 = \pm\sqrt{12}$$

$$x = 5 \pm \sqrt{12}$$

$$x = 5 \pm 2\sqrt{3}$$

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$$\frac{5}{x-3} - \frac{2}{x} = 1$$

Fractional Equation
multiply by LCD

$$5x - 2(x-3) = x(x-3)$$

$$5x - 2x + 6 = x^2 - 3x$$

$$3x + 6 = x^2 - 3x$$

$$-3x + 6 = -3x - 6$$

$$0 = x^2 - 6x - 6$$

Quadratic Equation
Bring everything to 1 side
Quadratic Formula
or
completing the square

$$0 = x^2 - 6x - 6$$

$$a = 1 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b = -6$$

$$c = -6$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-6)}}{2(1)}$$

$$\sqrt{60} \quad x = \frac{6 \pm \sqrt{60}}{2}$$

$$\frac{6 \pm \sqrt{60}}{2} \quad x = \frac{6 \pm 2\sqrt{15}}{2}$$

$$2\sqrt{15} \quad x = 3 \pm \sqrt{15}$$

$$0 = x^2 - 6x - 6$$

$$b = x^2 - 6x$$

$$x^2 - 6x + 9 = 6 + 9$$

$$(x-3)(x-3) = 15$$

$$\sqrt{(x-3)^2} = \sqrt{15}$$

$$x-3 = \pm \sqrt{15}$$

$$x = 3 \pm \sqrt{15}$$

$$\left(-\frac{6}{2}\right)^2$$

30

$$\frac{3(5)^{2x}}{3} = \frac{60}{3}$$

$$5^{2x} = 20$$

$$\log 5^{2x} = \log 20$$

$$\frac{2x \log 5}{2 \log 5} = \frac{\log 20}{2 \log 5}$$

$$x = .98$$

Exponential Equation
Isolate
Log of both sides

$$4^x - 5 = 12$$

$$4^x = 17$$

$$\log 4^x = \log 17$$

$$\frac{x \log 4}{\log 4} = \frac{\log 17}{\log 4}$$

$$x = 2.04$$

$$8 + 2(4)^{x-5} = 14$$

$$\frac{2(4)^{x-5}}{2} = \frac{6}{2}$$

$$4^{x-5} = 3$$

$$\log 4^{x-5} = \log 3$$

$$\frac{(x-5) \log 4}{\log 4} = \frac{\log 3}{\log 4}$$

$$x-5 = .79$$

$$x = 5.79$$

Exponential Equation
Isolate
log of both sides

Exponential Equation
Isolate!

log of both sides

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$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

Fractional Equation
multiply by LCD

$$WF + JF = JW$$

$$-JF -JF$$

$$WF = \frac{JW - JF}{J} \quad \text{GCF}$$

$$\frac{WF}{W-F} = \frac{J(W-F)}{W-F}$$

$$\frac{WF}{W-F} = J$$

$$3) \frac{FW}{W-F}$$

(33) $1 - 2(3)^{2x} = -5$ Exponential Equation
Isolate

$$\frac{-2(3)^{2x}}{-2} = \frac{-6}{-2}$$

$$3^{2x} = 3$$

log of both sides

$$\log 3^{2x} = \log 3$$

$$\frac{2 \times \log 3}{2 \log 3} = \frac{\log 3}{2 \log 3}$$

$$x = 0.000$$

(34) $e^{4x} = 12$

Exponential Equation
Already Isolated

$$\log e^{4x} = \log 12$$

log of both sides

$$\frac{4 \times \log e}{4 \log e} = \frac{\log 12}{4 \log e}$$

$$x = .62$$

(35) $x^2 - y = 5$ $y = 3x - 1$

Quadratic System
Isolate a variable

$$x^2 = y + 5$$

$$y = 3(4) - 1$$

$$x^2 - 5 = y$$

$$(4, 11)$$

$$x^2 - 5 = 3x - 1$$

$$x = -1$$

$$x^2 - 3x - 4 = 0$$

$$y = 3(-1) - 1$$

$$(x-4)(x+1) = 0$$

$$y = -4$$

Substitute one equation into the other
 Quadratic Equation
 Bring everything to 1 side
 Factor

Substitute into 1 of the original equations to find the other variable

(36) $x^2 + y^2 = 2$ $y + 2 = x$

Quadratic System
 Variable already isolated

$$(y+2)^2 + y^2 = 2$$

Substitute one equation into the other
 Square binomial theorem

$$y^2 + y + 1 + y^2 = 2$$

Quadratic equation
 Bring everything to 1 side

$$2y^2 + y + 1 = 2$$

Factor

$$\frac{2y^2}{2} + \frac{y}{2} + \frac{1}{2} = \frac{2}{2}$$

$$y^2 + 2y + 1 = 0$$

$$(y+1)(y+1) = 0$$

$$\begin{matrix} y+1=0 & y+1=0 \\ -1-1 & -1-1 \end{matrix}$$

$$y = -1 \quad y = -1$$

substitute into original equation to find the other variable

$$y + 2 = x$$

$$-1 + 2 = x \quad (1, 1)$$

(37) $y^2 - x^2 + 32 = 0$

$$3y - x = 0$$

Quadratic System
Isolate a variable
 substitute 1 equation into the other

$$y^2 - (3y)^2 + 32 = 0$$

Quadratic equation
 Bring everything to 1 side

$$y^2 - 9y^2 + 32 = 0$$

Factor

$$\frac{-8y^2 + 32}{-8} = \frac{0}{-8}$$

$$y^2 - 4 = 0$$

$$(y+2)(y-2) = 0$$

$$\begin{matrix} y+2=0 & y-2=0 \\ -2-2 & -2+2 \end{matrix}$$

$$y = -2 \quad y = 2$$

Substitute into 1 of the other original equations to find the other variable

$$x = 3y \quad x = 3y$$

$$x = 3(-2) \quad x = 3(2)$$

$$x = -6 \quad x = 6$$

$$(-6, -2) \quad (6, 2)$$

38) $x+y=5$
 $-x \quad -x$
 $y=-x+5$

$(x+3)^2 + (-x+5-3)^2 = 53$

$(x+3)^2 + (-x+2)^2 = 53$

$x^2+6x+9+x^2-4x+4=53$

$2x^2+2x+13 = 53$
 $-53 \quad -53$

$2x^2+2x-40=0$
 $\frac{2}{2} \quad \frac{2}{2} \quad \frac{-40}{2} \quad \frac{0}{2}$

$x^2+x-20=0$

$(x+5)(x-4)=0$

$x+5=0 \quad x-4=0$
 $-5-5 \quad +4+4$
 $x=-5 \quad x=4$

$y=-x+5 \quad y=-x+5$
 $y=-5+5 \quad y=-4+5$
 $y=0 \quad y=1$
 $(-5,0) \quad (4,1)$

Quadratic system
 isolate a variable
 substitute one equation
 into the other

Square binomial theorem

quadratic equation
 Bring everything to 1 side
 Factor

Substitute into original equation
 to find other variable

39) $(x-3)^2 + (y+2)^2 = 16$
 $2x+y=10$
 $-2x \quad -2x$
 $2y=-2x+10$
 $y=-x+5$

$(x-3)^2 + (-x+5+2)^2 = 16$

$(x-3)^2 + (-x+7)^2 = 16$

$x^2-6x+9+x^2-14x+49=16$

$2x^2-20x+58=16$
 $-16 \quad -16$

$2x^2-20x+42=0$
 $\frac{2}{2} \quad \frac{-20}{2} \quad \frac{42}{2} \quad \frac{0}{2}$

$x^2-10x+21=0$

$(x-7)(x-3)=0$

$x-7=0 \quad x-3=0$
 $+7+7 \quad +3+3$
 $x=7 \quad x=3$

$y=-x+5 \quad y=-x+5$
 $y=-7+5 \quad y=-3+5$
 $y=-2 \quad y=2$
 $(7,-2) \quad (3,2)$

Quadratic system
 isolate a variable
 substitute 1
 equation into the
 other

Square binomial
 theorem

quadratic equation
 Bring everything to
 1 side

Factor

Substitute into
 1 of original equations
 to find the other
 variable

40) $x^5=7$

constant exponential
 equation

$(x^5)^{\frac{1}{5}} = 7^{\frac{1}{5}}$

Already isolated
 Radical 6) reciprocal
 power

$x=7^{\frac{1}{5}}$

$x=1.47..$

41) $(x^{\frac{2}{3}})^{\frac{3}{2}} = (4)^{\frac{3}{2}}$

constant exponential equation

Already isolated

Reciprocal power

$x=4^{\frac{3}{2}}$

$x=8$

$x^5=7$

$x=\sqrt[5]{7}$

$x=1.47..$

42) $x^{\frac{4}{3}} + 1 = 5$
 $(x^{\frac{4}{3}})^{\frac{3}{4}} = (6)^{\frac{3}{4}}$
 $x = 6^{\frac{3}{4}}$
 $x = 3.83...$

constant exponential
equation
Isolate
Reciprocal power

43) $4x^{\frac{2}{3}} - 5 = 20$
 $\frac{4x^{\frac{2}{3}}}{4} = \frac{25}{4}$
 $(x^{\frac{2}{3}})^{\frac{3}{2}} = (\frac{25}{4})^{\frac{3}{2}}$
 $x = \frac{125}{8}$

constant exponential
equation
Isolate
Reciprocal power

44) $\sqrt[3]{x^5} = y^{\frac{5}{6}}$
 $(x^{\frac{5}{3}})^{\frac{6}{5}} = (y^{\frac{5}{6}})^{\frac{6}{5}}$
 $x^2 = y$

constant exponential
equation
Radicals are fractional
exponents
Reciprocal power

45) $z(2) = 35$ $z(-5) = 0$
 $z(x) = 6x^3 + bx^2 - 52x + 15$
 $0 = 6(-5)^3 + b(-5)^2 - 52(-5) + 15$
 $0 = -750 + 25b + 260 + 15$
 $0 = 25b - 475$
 $475 = 25b$
 $19 = b$

If $x+5$ is
a factor, the polynomial
is divisible by $x+5$

$z(x) = 6x^3 + 19x^2 - 52x + 15$
 $\frac{6x^3 + 19x^2 - 52x + 15}{x+5}$
 $\begin{array}{r} 6x^3 + 19x^2 - 52x + 15 \\ - (6x^3 + 30x^2) \\ \hline -11x^2 - 52x + 15 \\ + (11x^2 + 55x) \\ \hline 3x + 15 \\ - (3x + 15) \\ \hline 0 \end{array}$

$z(x) = (6x^2 - 11x + 3)(x+5)$
 $x^2 - 11x + 18$
 $(x-9)(x-2)$

$(x - \frac{3}{2})(x - \frac{1}{3})$
 $0 = (2x-3)(3x-1)(x+5)$
 $\begin{array}{c|c|c} 2x-3=0 & 3x-1=0 & x+5=0 \\ \hline x=\frac{3}{2} & x=\frac{1}{3} & x=-5 \end{array}$