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Date \_\_\_\_\_  
Algebra 2

## Algebra II Factoring Review

### Factoring:

Greatest Common Factor: GCF( )

Difference of Two Squares:  $(\sqrt{1} + \sqrt{2})(\sqrt{1} - \sqrt{2})$

Trinomials:  $(x \quad)(x \quad)$

1) First sign comes down

2) The two signs must multiply for the last sign

3) Find two numbers that multiply to the last number and add/subtract to the middle number

**Bridge Method: (Trinomial with a leading coefficient bigger than 1)**

1) Build a bridge between the first and last numbers (Multiply)

2) Factor Trinomial Normally

3) Pay the toll (Divide by the leading coefficient)

\*If possible, reduce the fraction

If they divide nicely, divide them

If not, put the denominator in front of the variable inside the parenthesis

**Grouping: (4 Terms or More)**

1) Look for a pattern in the exponents to determine the groups. **You cannot have two terms with the same exponent in the same group.**

2) Factor out the GCF in each group

\*You should be left with the same factor. If signs are reversed, factor out a negative

3) Combine coefficients and keep like term.

\*Factor further if necessary

**Sum/Difference of Two Cubes**

SOAP for signs (Same, Opposite, Always Positive)

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

### Substitution Trinomials:

1) Replace binomial with y

2) Factor normally

3) Substitute back

\*Factor further if possible

Factor each expression

1. 
$$\frac{4x+8}{4} \\ 4(x+2)$$

2. 
$$\frac{12x+18}{6} \\ 6(2x+3)$$

3. 
$$\frac{x^2 - 7x}{x} \\ x(x-7)$$

4. 
$$\frac{2x^2 - 4xy}{2x} \\ 2x(x-2y)$$

5. 
$$\frac{5x^2 y - 20x}{5x} \\ 5x(xy-4)$$

6. 
$$\sqrt{x^2 - 64} \text{ DOTS} \\ (x+8)(x-8)$$

7. 
$$\sqrt{y^2 - 36} \text{ DOTS} \\ (y+6)(y-6)$$

8. 
$$\sqrt{4t^2 - 25} \text{ DOTS} \\ (2t+5)(2t-5)$$

9. 
$$\sqrt{9x^2 - 16y^4} \text{ DOTS} \\ (3x+4y^2)(3x-4y^2)$$

10. 
$$\sqrt{36 - 25x^2} \text{ DOTS} \\ (6+5x)(6-5x)$$

11. 
$$\sqrt{100y^4 - 49t^6} \text{ DOTS} \\ (10y^2 + 7t^3)(10y^2 - 7t^3)$$

12. 
$$1 - 9x^8 y^4 \text{ DOTS} \\ (1 + 3x^4 y^2)(1 - 3x^4 y^2)$$

13. 
$$x^2 + 4x - 12 \\ \frac{4+12}{3,4} \\ (x+6)(x-2)$$

14. 
$$y^2 + 3y + 2 \\ (y+2)(y+1)$$

$$15. m^2 - 8m + 15$$

$$(m-5)(m-3)$$

$$17. y^2 + 5y - 14$$

$$(y+7)(y-2)$$

$$19. x^2 - 3x - 10$$

$$(x-5)(x+2)$$

$$21. x^2 - 9x - 36$$

$$(x-12)(x+3)$$

$$23. x^4 + 4x^2 - 12$$

$$(x^2+6)(x^2-2)$$

$$25. x^4 - 8x^2 - 9$$

$$(x^2-9)(x^2+1)$$

$$(x+3)(x-3)$$

$$27. \frac{2x^2}{2} - \frac{50}{2}$$

$$2(x^2-25) \text{ DOTS}$$

$$2(x+5)(x-5)$$

$$29. \frac{3x^2}{3} + \frac{9x}{3} - \frac{12}{3}$$

$$3(x^2+3x-4)$$

$$3(x+4)(x-1)$$

$$16. x^2 - 8x - 20$$

$$(x-10)(x+2)$$

$$18. x^2 + x - 12$$

$$(x+4)(x-3)$$

$$20. x^2 - 7x + 12$$

$$(x-4)(x-3)$$

$$22. y^2 - 21y + 110$$

$$(y-11)(y-10)$$

$$26. x^4 + x^2 - 2$$

$$(x^2+2)(x^2-1) \text{ DOTS}$$

$$(x^2+2)(x+1)(x-1)$$

$$28. \frac{2x^2}{2} - \frac{8x}{2} - \frac{10}{2}$$

$$2(x^2-4x-5)$$

$$2(x-5)(x+1)$$

$$30. \frac{6x^2}{6} - \frac{54}{6}$$

$$6(x^2-9) \text{ DOTS}$$

$$6(x+3)(x-3)$$

$$31. \frac{2x^2 + 14x + 24}{2} \quad \frac{2}{2} \quad \frac{2}{2}$$

$$2(x^2 + 7x + 12)$$

$$2(x+4)(x+3)$$

$$33. \frac{ax^2 - 2ax - 8a}{a} \quad \frac{a}{a} \quad \frac{a}{a}$$

$$a(x^2 - 2x - 8)$$

$$a(x-4)(x+2)$$

$$35. \frac{12x^2 - 75}{3} \quad \frac{3}{3}$$

$$3(4x^2 - 25)$$

$$3(2x+5)(2x-5)$$

$$37. \frac{2y^2 - 5y - 7}{2} \quad PT$$

$$y^2 - 5y - 14$$

$$\frac{(y-7)(y+2)}{2}$$

$$39. \frac{2x^2 + 7x - 4}{2} \quad PT$$

$$x^2 + 7x - 8$$

$$\frac{(x+8)(x-1)}{2}$$

$$(x+4)(2x-1)$$

$$41. \frac{2x^2 - 9x - 18}{2} \quad PT$$

$$x^2 - 9x - 36$$

$$\frac{(x-12)(x+3)}{2}$$

$$43. \frac{8x^2 + 7x - 1}{8} \quad PT$$

$$x^2 + 7x - 8$$

$$\frac{(x+8)(x-1)}{8}$$

$$(x+1)(8x-1)$$

$$32. \frac{5x^2 - 500}{5} \quad \frac{5}{5}$$

$$5(x^2 - 100)$$

$$5(x+10)(x-10)$$

$$34. \frac{yx^2 - 64y}{y} \quad \frac{y}{y}$$

$$y(x^2 - 64)$$

$$y(x+8)(x-8)$$

$$36. x^4 - 81$$

$$Dots (x^2 - 9)(x^2 + 9)$$

$$Dots (x+3)(x-3)(x^2 + 9)$$

$$38. \frac{2x^2 + 15x - 8}{2} \quad PT$$

$$x^2 + 5x - 16$$

$$\frac{(x+16)(x-1)}{2}$$

$$(x+8)(2x-1)$$

$$40. \frac{6x^2 - 11x - 10}{6} \quad PT$$

$$x^2 - \frac{11}{6}x - \frac{10}{6}$$

$$\frac{(x-15)(x+4)}{6} * reduce$$

$$(x-\frac{5}{3})(x+\frac{2}{3})$$

$$42. \frac{3x^2 + 2x - 8}{3} \quad PT$$

$$x^2 + \frac{2}{3}x - \frac{8}{3}$$

$$\frac{(x+6)(x-4)}{3}$$

$$(x+2)(3x-4)$$

$$44. \frac{6x^2 + x - 12}{6} \quad PT$$

$$x^2 + x - 12$$

$$\frac{(x+9)(x-8)}{6} * reduce$$

$$(x + \frac{3}{2})(x - \frac{4}{3})$$

$$(2x+3)(3x-4)$$

$$45. \frac{(x^3 + 6x^2)}{x^2} - \frac{-3x - 18}{x^2}$$

$$x^2(x+6) - 3(x+6)$$

$$(x^2 - 3)(x+6)$$

$$47. \frac{(x^3 + 3x^2)}{x^2} - \frac{-9x - 27}{x^2}$$

$$x^2(x+3) - 9(x+3)$$

$$(x^2 - 9)(x+3)$$

$$(x+3)(x-3)(x+3)$$

$$49. \frac{(x^3 - 3x^2 + 2x)}{x} - \frac{(4x^2 - 12x + 8)}{x}$$

$$x(x^2 - 3x + 2) + 4(x^2 - 3x + 2)$$

$$(x+4)(x^2 - 3x + 2)$$

$$(x+4)(x-2)(x-1)$$

$$51. (x^2 + 5x)^2 - 2(x^2 + 5x) - 24 \quad y = x^2 + 5x$$

$$y^2 = 2y - 24 \quad (x^2 + 5x - 6)(x^2 + 5x + 4)$$

$$(y - 6)(y + 4) \quad (x+6)(x-1)(x+4)(x+1)$$

$$53. \frac{a=y}{5^3} \quad \frac{b=5}{y^3} \quad \sqrt[3]{125}$$

$$(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$$

$$=(y-5)(y^2 + 5y + 25)$$

$$55. \frac{2x}{\sqrt[3]{8x^3}} \quad \frac{y^2}{\sqrt[3]{y^6}}$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$8x^3 + y^6 = (2x + y^2)(4x^2 - 2xy^2 + y^4)$$

$$46. \frac{(x^3 + 10x^2)}{x^2} - \frac{-9x - 90}{x^2}$$

$$x^2(x+10) - 9(x+10)$$

$$(x^2 - 9)(x+10)$$

$$(x+3)(x-3)(x+10)$$

$$48. \frac{(8x^3 + 12x^2)}{4x^2} - \frac{-2x - 3}{4x^2}$$

$$4x^2(2x+3) - 1(2x+3)$$

$$(4x^2 - 1)(2x+3)$$

$$(2x+1)(2x-1)(2x+3)$$

$$50. \frac{(3x^3 + x^2)}{x^2} - \frac{-12x^2 - 4x}{x^2} - \frac{-63x^2 - 21}{x^2}$$

$$x^2(3x+1) - 4x(3x+1) - 21(3x+1)$$

$$(x^2 - 4x - 21)(3x+1)$$

$$(x-7)(x+3)(3x+1)$$

$$52. (x^2 - 2x)^2 - 11(x^2 - 2x) + 24 \quad y = x^2 - 2x$$

$$y^2 = 11y + 24 \quad (y-8)(y-3)$$

$$(x^2 - 2x - 8)(x^2 - 2x - 3)$$

$$(x-4)(x+2)(x-3)(x+1)$$

$$54. \frac{a=z}{z^3} \quad \frac{b=4}{\sqrt[3]{64}}$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\underline{\underline{z^3 + 64 = (z+4)(z^2 - 4z + 16)}}$$

$$56. \frac{a=y^3}{y^9} \quad \frac{b=6x}{\sqrt[3]{216x^3}}$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$y^9 - 216x^3 = (y^3 - 6x)(y^6 + 6xy^3 + 36x^2)$$



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## CCA2 Equations Review

### Polynomial Equations

1) Bring everything to one side. Keep the leading coefficient positive.

2) Factor

3) Set each factor equal to zero

If you cannot factor a quadratic, use quadratic formula or square root of both sides

### Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1)  $ax^2 + bx + c = 0$

2) List a, b, and c values

3) Substitute values into quadratic formula

4) Type discriminant into the calculator (what is underneath the radical)

5) REDUCE THE RADICAL off to the side (If possible)

6) Reduce from all three terms (If possible)

1. Find algebraically the zeros for  $p(x) = x^3 + x^2 - 4x - 4$ .

$$\begin{aligned} 0 &= \cancel{x^3 + x^2} \cancel{(x-4)} \\ &\quad \cancel{x^2} \cancel{x^2 - 4} \cancel{-4} \\ &\quad x^2(x+1) - 4(x+1) \\ &\quad (x^2 - 4)(x+1) \end{aligned}$$

$\cancel{(x+2)(x-2)(x+1)} = 0$   
 $x = -2 \quad x = 2 \quad x = -1$

2. Solve  $x^3 + 5x^2 = 4x + 20$  algebraically.

$$\begin{aligned} 0 &= \cancel{x^3 + 5x^2} \cancel{(4x+20)} \\ &\quad \cancel{x^2} \cancel{x^2 - 4} \cancel{-4} \\ &\quad x^2(x+5) - 4(x+5) = 0 \end{aligned}$$

$\cancel{(x-4)(x+5)} = 0$   
 $\cancel{(x+2)(x-2)(x+5)} = 0$   
 $x = -2 \quad x = 2 \quad x = -5$

3. Solve the equation  $\cancel{\frac{2x^3 - x^2}{x^2}} \cancel{\left(\frac{-8x + 4}{-4}\right)} = 0$  algebraically for all values of  $x$ .

$$\begin{aligned} 0 &= \cancel{x^2(2x-1)} - 4(2x-1) = 0 \end{aligned}$$

$$(x^2 - 4)(2x-1) = 0$$

$$\cancel{(x+2)(x-2)}(2x-1) = 0$$

$$\begin{aligned} x &= -2 \quad x = 2 \quad 2x-1 = 0 \\ x &= \frac{1}{2} \end{aligned}$$

## quadratic formula

4. Solve the equation  $6x^2 - 2x - 3 = 0$  and express the answer in simplest radical form.

$$\begin{aligned} a &= 6 \\ b &= -2 \\ c &= -3 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{2 \pm \sqrt{76}}{12}$$

$$x = \frac{2 \pm 2\sqrt{19}}{12}$$

$$x = \frac{1 \pm \sqrt{19}}{6}$$

$$\begin{array}{c} \sqrt{76} \\ \downarrow \\ \sqrt{4} \quad \sqrt{19} \\ \hline 2\sqrt{19} \end{array}$$

5. Solve the equation  $x^2 + 2x = -8$  and express the answer in simplest  $a + bi$  form.

$$\begin{aligned} a &= 1 \\ b &= 2 \\ c &= 8 \end{aligned}$$

$$x^2 + 2x + 8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(8)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{-28}}{2}$$

$$x = \frac{-2 \pm 2i\sqrt{7}}{2}$$

$$x = -1 \pm i\sqrt{7}$$

$$\begin{array}{c} \sqrt{-28} \\ \downarrow \\ i\sqrt{28} \\ \downarrow \\ i\sqrt{4} \quad i\sqrt{7} \\ \hline 2i\sqrt{7} \end{array}$$

6. Solve the equation  $3x^2 + 6 = 5x$  and express the answer in simplest  $a + bi$  form.

$$\begin{aligned} 3x^2 - 5x + 6 &= 0 \\ a &= 3 \\ b &= -5 \\ c &= 6 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(6)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{-47}}{6}$$

$$x = \frac{5 \pm i\sqrt{47}}{6}$$

$$x = \frac{5}{6} \pm \frac{i\sqrt{47}}{6}$$

7. A solution of the equation  $2x^2 + 3x + 2 = 0$  is

- |  |   |         |
|--|---|---------|
| 1) $-\frac{3}{4} + \frac{1}{4}i\sqrt{7}$ | 3) $-\frac{3}{4} + \frac{1}{4}\sqrt{7}$ | $a = 2$ |
| 2) $-\frac{3}{4} + \frac{1}{4}i$         | 4) $\frac{1}{2}$                        | $b = 3$ |
|  |   | $c = 2$ |

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(2)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{-7}}{4}$$

$$x = \frac{-3 \pm i\sqrt{7}}{4}$$

8. The solutions to the equation  $-\frac{1}{2}x^2 = -6x + 20$  are

- 1)  $-6 \pm 2i$
- 2)  $-6 \pm 2\sqrt{19}$
- 3)  ~~$6 \pm 2i$~~
- 4)  $6 \pm 2\sqrt{19}$

$$+\frac{1}{2}x^2 + \frac{1}{2}x^2$$

$$0 = \frac{1}{2}x^2 - 6x + 20$$

$$\begin{aligned} a &= \frac{1}{2} \\ b &= -6 \\ c &= 20 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{6 \pm \sqrt{-4}}{1}$$

$$x = 6 \pm 2i$$

9. Solve algebraically for all values of  $x$ :  $x^4 + 4x^3 + 4x^2 = -16x$

$$x^4 + 4x^3 + 4x^2 + 16x = 0$$

$$x((x^3 + 4x^2) + 4(x + 4)) = 0$$

$$x(x^2(x+4) + 4(x+4)) = 0$$

$$x(x^2 + 4)(x+4) = 0$$

$$x(x^2 + 4)(x+4) = 0$$

$$x = 0 \quad x^2 + 4 = 0 \quad x+4 = 0$$

$$x = 0 \quad x = -2i \quad x = -4$$

## Radical Equations

- 1) Isolate
- 2) Square both sides
- 3) Check

1.  $\sqrt{2x+1} + 4 = 8$

$$\begin{aligned} (\cancel{\sqrt{2x+1}} + 4)^2 &= 8^2 \\ 2x+1+16 &= 64 \\ 2x &= 47 \\ x &= 23.5 \end{aligned}$$

$$\begin{array}{c} 2x = 15 \\ \hline x^2 \\ x = 7.5 \end{array}$$

3.  $\sqrt{56-x} = x^2$

$$\begin{aligned} 56-x &= x^4 \\ -56+x &= -56+x \end{aligned}$$

$$0 = x^4 + x - 56$$

$$0 = (x+8)(x-7)$$

$$\begin{array}{c} x = 8 \\ x = 7 \end{array}$$

5.  $\sqrt{5x+29} = (x+3)^2$

$$\begin{aligned} 5x+29 &= x^2 + 6x + 9 \\ -5x+29 &= -5x+29 \end{aligned}$$

$$0 = x^2 + x - 20$$

$$0 = (x+5)(x-4)$$

7.  $\sqrt{x^2+x-1} + 11x = 7x+3$

$$\begin{aligned} -11x+11x &= -11x+11x \\ \cancel{x^2+x-1} &= \cancel{-4x+3}^2 \end{aligned}$$

$$\begin{aligned} x^2+x-1 &= 16x^2-24x+9 \\ -x^2-x+1 &= -x^2-x+1 \end{aligned}$$

$$\begin{aligned} 0 &= 15x^2-25x+10 \\ \frac{0}{5} &= \frac{3x^2-5x+2}{5} \\ 0 &= 3x^2-5x+2 \end{aligned}$$

$$\begin{array}{c} 7-x \\ \hline 7|49-7x \\ -x |7x-7x \\ \hline 0 \end{array} \quad x^2-14x+49$$

2.  $\sqrt{x-5} + x = 7$

$$\begin{array}{c} x-x \\ \hline (x-5)(7-x) \\ x-5 = (7-x)^2 \end{array}$$

$$\begin{array}{c} x-5 \\ \hline x-5-x+5 \\ \hline 0 \end{array} \quad x^2-14x+49$$

$$\begin{array}{c} 70 = x^2-15x+54 \\ 0 = (x-9)(x-6) \\ x=9 \quad x=6 \end{array}$$

4.  $\sqrt{2x-7} + x = 5$

$$\begin{array}{c} x-x \\ \hline (2x-7) = (5-x)^2 \end{array}$$

$$2x-7 = (5-x)^2$$

$$\begin{array}{c} 2x-7 = x^2-10x+25 \\ -2x+7 = -2x+7 \\ 0 = x^2-12x+32 \end{array}$$

6.  $\sqrt{2x-4} = (x-2)^2$

$$2x-4 = (x-2)^2$$

$$\begin{array}{c} x-2 \\ \hline x-2-2x+4 \\ \hline x^2-4x+4 \end{array}$$

$$x^2-4x+4$$

$$0 = x^2-4x+4$$

$$0 = (x-2)(x-2)$$

$$x = 2 \quad x = 2$$

## Fractional Equations

Multiply by the LCD

To find a common denominator:

1) Factor (if necessary)

2) Put all of your factors together

$$1. \frac{3x}{x} - \frac{1}{2} = -\frac{1}{3x}$$

F1: 3      F2: x      LCD:  $3x$

$$\begin{aligned} x - x &= -1 \\ 3 & \\ -x &= -4 \\ -1 & \\ x &= 4 \end{aligned}$$

$$3. \frac{3}{x} + \frac{x}{x+2} = -\frac{2}{x+2}$$

F1: x      F2:  $x+2$       LCD:  $x(x+2)$

$$\begin{aligned} 3(x+2) + x^2 &= -2x \\ 3x+6+x^2 &= -2x \\ +2x & \\ x^2+5x+6 &= 0 \end{aligned}$$

F1:  $x+3$       F2:  $x-3$       LCD:  $(x+3)(x-3)$

$$5. \frac{30}{x^2-9} + 1 = \frac{5(x+3)(x-3)}{(x+3)(x-3)}$$

F1:  $x+3$       F2:  $x-3$       LCD:  $(x+3)(x-3)$

$$\begin{aligned} 30+x^2-9 &= 5(x+3) \\ x^2+21 &= 5x+15 \\ -5x+15 & \\ x^2-5x+6 &= 0 \end{aligned}$$

Solve the equations below algebraically, and express the result in simplest radical form:

$$7. \frac{13}{x} = 10 - (x)^2$$

F1: x      F2:  $x^2$       LCD:  $x(x^2)$

$$\begin{aligned} 13 &= 10x - x^2 \\ -10x + x^2 & \\ -10x + x^2 & \end{aligned}$$

$$x^2 - 10x + 13 = 0$$

$$a=1$$

$$b=-10$$

$$c=13$$

$$\begin{aligned} \sqrt{48} & \\ \cancel{\sqrt{16}} \cancel{\sqrt{3}} & \\ 4\sqrt{3} & \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{10 \pm \sqrt{(-10)^2 - 4(1)(13)}}{2(1)} \\ x &= \frac{10 \pm \sqrt{48}}{2} \end{aligned}$$

$$2. \frac{4x}{x^2} = \frac{1}{x} + \frac{x}{4}$$

F1: 4      F2: x      LCD:  $4x$

$$\begin{aligned} 9x^2 - 4 &= 0 \\ 10x^2 &= 4 + x^2 \\ -x^2 & \\ 9x^2 &= 4 \\ -4 & \end{aligned}$$

$(3x+2)(3x-2) = 0$

$$3x+2 = 0$$

$\cancel{2-2}$

$$3x = -2$$

$\cancel{+2+2}$

$$3x = 2$$

$\cancel{3-3}$

$$x = \frac{2}{3}$$

$$4. \frac{3x+25}{x+7} - 5 = \frac{3x+25}{x}$$

F1:  $x+7$       F2:  $x$

$$x(3x+25) - 5x(x+7) = 3(x+7)$$

F1: x      F2:  $x+7$       LCD:  $x(x+7)$

$$3x^2 + 25x - 5x^2 - 35x = 3x+21$$

F1: x+7      F2:  $x+7$       PT

$$-2x^2 - 10x = 3x+21$$

$$+2x^2 + 10x + 2x^2 + 10x = 3x+21$$

F1:  $x+7$       F2:  $x+7$       PT

$$2x^2 + 20x = 3x+21$$

$$2x^2 + 17x = 3x+21$$

$$2x^2 + 17x = 3x+21$$

$$2x^2 + 14x = 3x+21$$

$$2x^2 + 14x = 3x+21$$

$$2x^2 + 11x = 3x+21$$

$$2x^2 + 11x = 3x+21$$

$$2x^2 + 8x = 3x+21$$

$$2x^2 + 8x = 3x+21$$

$$2x^2 + 5x = 3x+21$$

$$2x^2 + 5x = 3x+21$$

$$2x^2 + 2x = 3x+21$$

## Exponential Equations

Isolate the base

- a) Constants: Take the appropriate root of both sides or raise each side to the reciprocal power
- b) Variables: Take the log of both sides

Solve for x and round your answers to the nearest tenth:

$$1. 4^x - 5 = 12$$

$$\log 4^x = \log 17$$

$$\frac{\log 4}{\log 4} = \frac{\log 17}{\log 4}$$

$$3. 8 + 2(4)^{5x} = 14$$

$$\frac{8}{8} + 2(4)^{5x} = \frac{14}{8}$$

$$\frac{2(4)^{5x}}{2} = \frac{6}{2}$$

$$4^{5x} = 3$$

$$\ln 4^{5x} = \ln 12$$

$$\frac{\ln 4^x}{\ln 4} = \frac{\ln 12}{\ln 4}$$

$$x = .6$$

$$7. \left(\frac{x}{5}\right)^3 = 7$$

$$x = 7^{\frac{1}{3}}$$

$$x = 1.5$$

$$9. x^{\frac{4}{3}} - 1 = 5$$

$$+1 +1$$

$$\left(\frac{x^{\frac{4}{3}}}{1}\right)^3 = \left(\frac{6}{1}\right)^3$$

$$x = 6^{\frac{3}{4}}$$

$$x = 3.8$$

$$2. \frac{3(5)^{2x}}{3} = \frac{60}{3}$$

$$\text{by } 5^{2x} = 20$$

$$\frac{2x \log 5}{2 \log 5} = \frac{\log 20}{2 \log 5}$$

$$4. 1 - 2(3)^{2x} = -5$$

$$-1 -1$$

$$\frac{-2(3)^{2x}}{2} = \frac{-6}{2}$$

$$\text{by } 3^{2x} = 3$$

$$6. 12 + 3(1.2)^{\frac{x}{2}} = 100$$

$$-12 -12$$

$$\frac{3(1.2)^{\frac{x}{2}}}{3} = \frac{88}{3}$$

$$\log 1.2^{\frac{x}{2}} = \log \frac{88}{3}$$

$$8. \left(\frac{3}{2}\right)^{\frac{x}{2}} = \frac{88}{3}$$

$$x = 8$$

$$10. 4x^{\frac{2}{3}} - 5 = 20$$

$$+5 +5$$

$$4x^{\frac{2}{3}} = 25$$

$$\frac{4x^{\frac{2}{3}}}{4} = \frac{25}{4}$$

$$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(\frac{25}{4}\right)^{\frac{3}{2}}$$

$$x = \sqrt[3]{\frac{125}{8}}$$

$$x = 15.6$$

## Quadratic Systems of Equations Algebraically

- 1) Isolate at least one variable in one of the equations
- 2) Substitute one equation into the other (set them equal if you solved both equations for the same variables).
- 3) Solve equation (Mr.  $x^2$ /Polynomial Equations)
- 4) Substitute answers into one of the original equations to find the second variable

1.  $x^2 - y = 5$      $x^2 = y + 5$   
 $y = 3x - 1$      $x^2 - 5 = y$

$$\begin{aligned} x^2 - 5 &= 3x - 1 & x = 4 & x = -1 \\ -3x + 1 &\rightarrow 3x + 1 & y = 3x - 1 & y = 3x - 1 \\ x^2 - 3x - 4 &= 0 & y = 3(4) - 1 & y = 3(-1) - 1 \\ (x-4)(x+1) &= 0 & y = 11 & y = -4 \\ x = 4 & \quad x = -1 & (4, 11) & (-1, -4) \end{aligned}$$

3.  $(x+2)^2 + (y-4)^2 = 40$      $x+2$   
 $y = x+2$      $x+2$      $x-2$   
 $(y+2)^2 + (x+2-4)^2 = 40$      $x+2$      $x-2$   
 $(x+2)^2 + (x-2)^2 = 40$      $x+2$      $x-2$

$$\begin{aligned} x^2 + 4x + 4 + x^2 - 4x + 4 &= 40 & x^2 - 16 &= 0 & (-4, -2) \\ 2x^2 + 8 &= 40 & (x+4)(x-4) &= 0 & (4, 6) \\ -40 &-40 & x = -4 & x = 4 & \\ 2x^2 - 32 &= 0 & y = x+2 & y = x+2 & \\ \frac{2}{2} & \frac{-32}{2} & y = -4 & y = 4+2 & \\ & & y = -2 & y = 6 & \end{aligned}$$

5.  $x + y = 5$      $y = 5 - x$   
 $(x+3)^2 + (5-x-3)^2 = 53$      $x+3$      $2-x$   
 $(x+3)^2 + (2-x)^2 = 53$      $x+3$      $2-x$

$$\begin{aligned} x^2 + 6x + 9 + x^2 - 4x + 4 &= 53 & x = -5 & x = 4 \\ 2x^2 + 2x + 13 &= 53 & y = 5 - x & y = 5 - x \\ -53 &-53 & y = 5 - 5 & y = 5 - 4 \\ 2x^2 + 2x - 40 &= 0 & y = 0 & y = 1 \\ \frac{2}{2} & \frac{-40}{2} & (-5, 0) & (4, 1) \\ x^2 + x - 20 &= 0 & & \\ (x+5)(x-4) &= 0 & & \end{aligned}$$

2.  $x^2 + y^2 = 2$   
 $y+2 = x$

$$\begin{aligned} (y+2)^2 + y^2 &= 2 & y^2 + 4y + 4 + y^2 &= 2 \\ 2y^2 + 4y + 4 &= 2 & 2y^2 + 4y + 4 &= 2 \\ \frac{2}{2} & \frac{4}{2} & \frac{2}{2} & \frac{4}{2} \\ y^2 + 2y + 2 &= 0 & y^2 + 2y + 2 &= 0 \\ (y+1)(y+1) &= 0 & y = -1 & y = -1 \\ y = -1 & \quad y = -1 & x = y+2 & x = -1+2 \\ & & x = 1 & x = 1 \end{aligned}$$

$$\begin{array}{|c|c|c|} \hline y & y+2 & \\ \hline y & y^2 & 2y \\ \hline y^2 & 2y & 4 \\ \hline \end{array} \quad y^2 + 4y + 4$$

4.  $x^2 + (y+4)^2 = 109$   
 $y = x+3$

$$\begin{aligned} x^2 + (x+3+4)^2 &= 109 & x^2 + (x+7)^2 &= 109 \\ x^2 + (x+7)^2 &= 109 & x^2 + x^2 + 14x + 49 &= 109 \\ x^2 + x^2 + 14x + 49 &= 109 & 2x^2 + 14x + 49 &= 109 \\ -109 &-109 & \frac{2}{2} & \frac{-109}{2} \\ 2x^2 + 14x - 60 &= 0 & x = -10 & x = 3 \\ \frac{2}{2} & \frac{-60}{2} & y = -10+3 & y = 3+3 \\ & & y = -7 & y = 6 \end{aligned}$$

$$\begin{array}{|c|c|c|} \hline x & x+7 & \\ \hline x & x^2 & 7x \\ \hline x^2 & 7x & 49 \\ \hline \end{array} \quad x^2 + 14x + 49$$

6.  $(x-3)^2 + (y+2)^2 = 16$

$$\begin{aligned} 2x + 2y &= 10 & x &= -3 \\ -2x &-2x & y &= -2x+5 \\ 2y &= -2x+10 & \frac{2}{2} & \frac{-2x+10}{2} \\ y &= -x+5 & x = 7 & x = 3 \\ & & y = -7+5 & y = 3+5 \\ & & y = -2 & y = 8 \end{aligned}$$

$$\begin{array}{|c|c|c|} \hline x & -x+7 & \\ \hline x & x^2 & 3x \\ \hline x^2 & 3x & 9 \\ \hline \end{array} \quad x^2 - 14x + 49$$

$(x-3)^2 + (-x+5+2)^2 = 16$

$$\begin{aligned} (x-3)^2 + (-x+7)^2 &= 16 & (x-7)(x-3) &= 0 \\ x^2 - 6x + 9 + x^2 - 14x + 49 &= 16 & x = 7 & x = 3 \\ 2x^2 - 20x + 58 &= 16 & y = -x+5 & y = -x+5 \\ -16 &-16 & y = -7+5 & y = 3+5 \\ 2x^2 - 20x + 42 &= 0 & y = -2 & y = 2 \\ \frac{2}{2} & \frac{-20x}{2} & (7, -2) & (3, 2) \\ x^2 - 10x + 21 &= 0 & & \end{aligned}$$