

Name Schlansky
Mr. Schlansky

Date _____
Algebra II

Algebra II Guide to 65 Test

Part I: (2 Points Each)

1. What is the inverse of $f(x) = \frac{x}{x+2}$, where $x \neq -2$?

- 1) $f^{-1}(x) = \frac{2x}{x-1}$ 2) $f^{-1}(x) = \frac{x}{x-2}$
3) $f^{-1}(x) = \frac{x}{x+2}$ 4) $f^{-1}(x) = \frac{-x}{x-2}$
② $f^{-1}(x) = \frac{-2x}{x-1}$ Symmetry to $y=x$

2. Which value is *not* contained in the solution of the system shown below?

$$4x - 5y + 2z = 130$$

$$3x + 2y - 7z = -99$$

$$10x - 6y - 4z = 112$$

1) 8 2) 10

3) -12 4) 15

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 & -5 & 2 \\ 3 & 2 & -7 \\ 10 & -6 & -4 \end{pmatrix}^{-1} \begin{pmatrix} 130 \\ -99 \\ 112 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ -12 \\ 15 \end{pmatrix}$$

3. Given the parent function $p(x) = \cos x$, which phrase best describes the transformation used to obtain the graph of $g(x) = \cos(x+a) - b$, if a and b are positive constants?

- 1) right a units, up b units 2) right a units, down b units
3) left a units, up b units 4) left a units, down b units

left a down b

4. The expression $\left(\frac{m^2}{\frac{1}{m^{\frac{1}{2}}}} \right)^{-\frac{1}{2}}$ is equivalent to

1) $-\sqrt[6]{m^5}$ 2) $-\sqrt[5]{m^6}$

3) $\frac{1}{\sqrt[6]{m^5}}$ 4) $\frac{1}{\sqrt[5]{m^6}}$

5. Where i is the imaginary unit, the expression $(x+3i)^2 - (2x-3i)^2$ is equivalent to

1) $-3x^2$

2) $-3x^2 - 18$

3) $-3x^2 + 18xi$

4) $-3x^2 - 6xi - 18$

$-300 + 180i$

$-300 + 180i$

6. Which factorizations are correct?

I. $a^3 + 27b^3 = (a + 3b)(a^2 - 3ab + 9b^2)$

$92125 = 92125$ ✓

II. $c^3 - 6c^2 + 8c + 5c^2 - 30c + 40 = (c - 2)(c - 4)(c + 5)$

$720 = 720$ ✓

III. $1 - x^4 = (1 + x)^2(1 - x)^2$

$-9999 = 9801$ ✗

1) I, only

3) II and III only

2) I and II only

4) I, II, and III

7. For which values of x , rounded to the nearest hundredth, will $|x^2 - 9| - 3 = \log_2 x$?

1) 2.29 and 3.63

3) 2.84 and 3.17

2) 2.37 and 3.54

4) 2.92 and 3.06

$41 \mid 42$ intersect

8. The solution set of the equation $\sqrt{2x - 4} = x - 2$ is

1) $\{-2, -4\}$

2) $\{2, 4\}$ substitute each value in for x

3) $\{4\}$

4) $\{ \}$

9. Which scenario is best described as a controlled experiment?

1) For a class project, students in Health class ask every tenth student entering the school if they eat breakfast in the morning. Survey

3) A researcher wants to learn whether or not there is a link between children's daily amount of physical activity and their overall energy level. During lunch at the local high school, she distributed a short questionnaire to students in the cafeteria. Survey

2) A social researcher wants to learn whether or not there is a link between attendance and grades. She gathers data from 15 school districts. observational study

4) Sixty seniors taking a course in Advanced Algebra Concepts are randomly divided into two classes. One class uses a graphing calculator all the time, and the other class never uses graphing calculators. A guidance counselor wants to determine whether there is a link between graphing calculator use and students' final exam grades. controlled experiment

10. The yearbook staff has designed a survey to learn about student opinions on how the yearbook could be improved for this year. If they want to distribute this survey to 100 students and obtain the most reliable data, they should survey

1) Every third student sent to the office

2) Every third student to enter the library

3) Every third student to enter the gym for the basketball game

4) Every third student arriving at school in the morning

random

Part II: (2 Points Each)

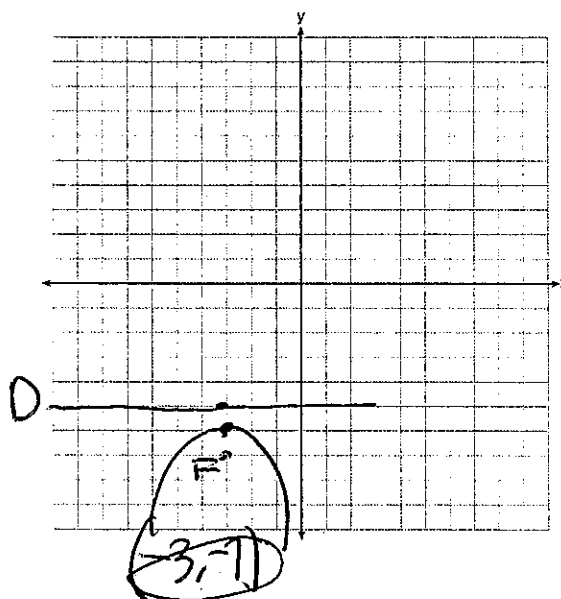
11. Determine if $x+3$ is a factor of $p(x) = x^4 + 7x^3 + 9x^2 - 21x - 36$. Explain your answer.

$$p(-3) = (-3)^4 + 7(-3)^3 + 9(-3)^2 - 21(-3) - 36$$
$$p(-3) = 0 \quad \text{Yes, the remainder is 0}$$

12. The parabola $y = -\frac{1}{4}(x+3)^2 - 6$ has a directrix at $y = -5$. What is the focus?

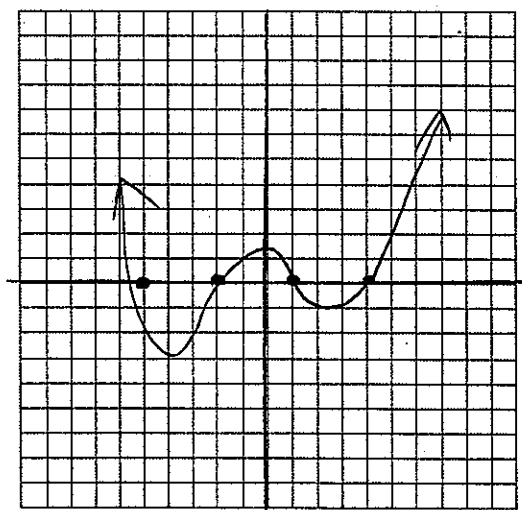
$$\text{Vertex: } (-3, -6)$$

$$p = -\frac{1}{4} = -1$$

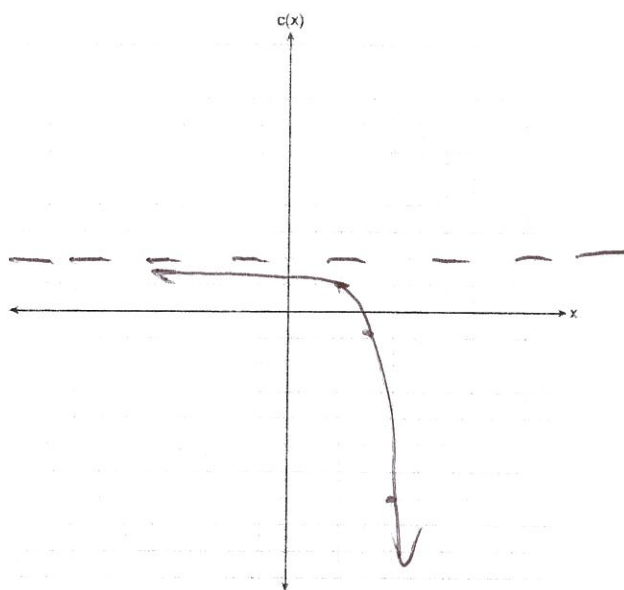


13. On the grid below, sketch a quartic polynomial whose factors are $x+5$, $x+2$, $x-1$, and $x-4$.

$$\text{Zeros: } -5, -2, 1, 4$$

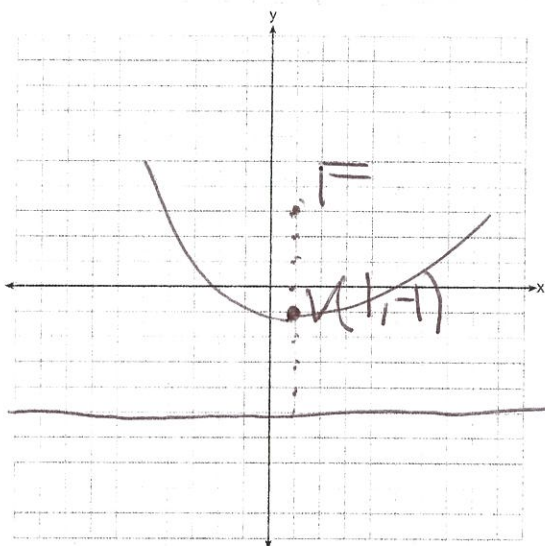


14. Graph $c(x) = -9(3)^{x-4} + 2$ on the axes below.



x	y
2	1
3	-1
4	-7

15. Write the equation of the parabola with a focus of $(1,3)$ and a directrix of $y = -5$



$$\begin{aligned}
 v &= 1 & y &= \frac{1}{4p}(x-v)^2 + t \\
 t &= -1 & & \\
 p &= 4 & y &= \frac{1}{4(4)}(x-1)^2 - 1 \\
 & & y &= \frac{1}{16}(x-1)^2 - 1
 \end{aligned}$$

16. Kina earns a \$27,000 salary for the first year of work at her job. She earns annual increases of 2.5%. What is the total amount, to the nearest cent, that Kina will earn for the first eight years at this job?

$$1.025$$

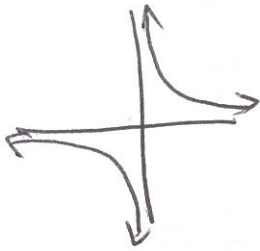
$$S_n = \frac{a_1 - a_1(r)^n}{1-r}$$

$$S_8 = \frac{27,000 - 27,000(1.025)^8}{1-1.025}$$

$$S_8 = 235815.13$$

4=

17. Is $f(x) = \frac{10}{x}$ even, odd, or neither? Justify your answer.



odd because it's symmetric to the origin.

18. For the function $f(x) = 3^x$, find the average rate of change over the interval -5 to -1 rounded to the nearest thousandth.

$$\begin{array}{r|l} x & y \\ \hline -5 & .00412 \\ -1 & .33333 \end{array}$$

$$\frac{f(b) - f(a)}{b - a} = \frac{.33333 - .00412}{-1 - -5} = .082$$

19. A middle school conducted a survey of students to determine if they spent more of their time playing games or watching videos on their tablets. The results are shown in the table below. Of the students who spent more time playing games on their tablets, to the *nearest* percent, what percent were boys?

	Playing Games	Watching Videos	Total
Boys	138	46	184
Girls	54	142	196
Total	192	188	380

$$\frac{138}{192} = 72\%$$

20. The weights of students on the boys cross country team is normally distributed with a mean of 135.3 pounds and a standard deviation of 2.8 pounds. If the team has 32 members, how many of them, rounded to the *nearest person*, would be expected to weigh less than 130 pounds?

normal cdf

$$\begin{aligned} \text{lower} &= -9999 \\ \text{upper} &= 130 \\ \mu &= 135.3 \\ \sigma &= 2.8 \end{aligned}$$

$$.029(32) = .928$$

1

21. The results of a survey of the student body at Central High School about television viewing preferences are shown below.

	Comedy Series	Drama Series	Reality Series	Total
Males	95	65	70	230
Females	80	70	110	260
Total	175	135	180	490

Are the events "student is a male" and "student prefers reality series" independent of each other? Justify your answer.

$$P(A \cap B) = P(A) \cdot P(B)$$

Not Independent

$$\frac{70}{490} = \frac{180}{490} \cdot \frac{230}{490}$$

$$\frac{1}{7} \neq \frac{414}{2401}$$

22. A Foucault pendulum can be used to demonstrate that the Earth rotates. The time, t , in seconds, that it takes for one swing or period of the pendulum can be modeled by the equation

$t = 2\pi \sqrt{\frac{L}{g}}$ where L is the length of the pendulum in meters and g is a constant of 9.81 m/s^2 . The

first Foucault pendulum was constructed in 1851 and has a pendulum length of 67 m.

Determine, to the nearest tenth of a second, the time it takes this pendulum to complete one swing.

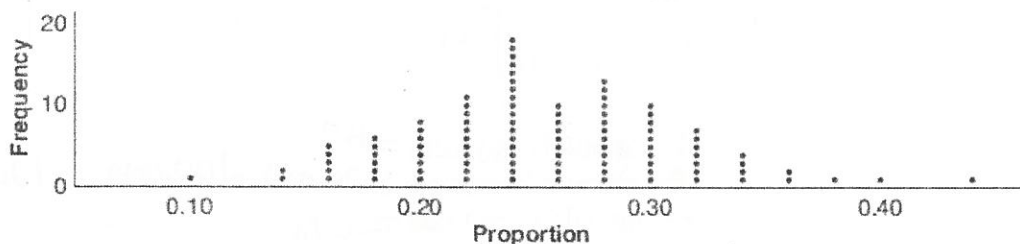
t = time for one swing
 67 L = Length of Pendulum
 9.81 g = constant

$$t = 2\pi \sqrt{\frac{67}{9.81}}$$

$$t = 16.4$$

Part III: (4 Points Each)

23. A group of students was trying to determine the proportion of candies in a bag that are blue. The company claims that 24% of candies in bags are blue. A simulation was run 100 times with a sample size of 50, based on the premise that 24% of the candies are blue. The approximately normal results of the simulation are shown in the dot plot below.



The simulation results in a mean of 0.254 and a standard deviation of 0.060. Based on this simulation, what is a plausible interval containing the middle 95% of the data? A student found that 18 out of 50 of the candies were blue. Use statistical evidence to explain why this is an expected value.

$$CI = \text{mean} \pm 2(SD)$$

$$CI = .254 \pm 2(.060) = .374$$

$$.254 - 2(.060) = .134$$

$$[.134, .374]$$

$$\frac{18}{50} = .36$$

.36 is inside the confidence interval.

24. The accompanying table shows the number of bacteria present in a certain culture over a 5-hour period, where x is the time, in hours, and y is the number of bacteria.

x	y
0	1,000
1	1,049
2	1,100
3	1,157
4	1,212
5	1,271

Write an exponential regression equation for this set of data, rounding all values to four decimal places. Using this equation, determine the number of whole bacteria present after 6.5 hours.

$$y = 999.9725(1.0493)^x$$

$$y = 999.9725(1.0493)^{6.5}$$

$$y = 1367$$

25. Monthly mortgage payments can be found using the formula below:

$$M = \frac{P \left(\frac{r}{12} \right) \left(1 + \frac{r}{12} \right)^n}{\left(1 + \frac{r}{12} \right)^n - 1}$$

M = monthly payment = m

P = amount borrowed $220,000 - 100,000 = 120,000$

r = annual interest rate = $.048$

n = number of monthly payments $15(12) = 180$

The Banks family would like to purchase a home for \$220,000. They qualified for an annual interest rate of 4.8%. If they put make a down payment of \$100,000 and plan to spend 15 years to repay the loan, what will be the monthly payment rounded to the *nearest cent*? $\frac{.048}{12}$
 n

$$M = \frac{120,000 \left(\frac{.048}{12} \right) \left(1 + \frac{.048}{12} \right)^{180}}{\left(1 + \frac{.048}{12} \right)^{180} - 1}$$

$$M = 936.50$$

If they want their monthly payment to be \$1500, what would their down payment have to be?

$$M = 1500$$

$$P = P$$

$$r = .048$$

$$n = 180$$

$$1500 = \frac{P \left(\frac{.048}{12} \right) \left(1 + \frac{.048}{12} \right)^{180}}{\left(1 + \frac{.048}{12} \right)^{180} - 1} - \frac{220,000}{192,205} = \frac{27,794.43}{27,794.43}$$

$$\frac{1500}{.0078} = \frac{P(1.0078 \dots)}{.0078 \dots} \quad 192,205 = P$$



his $300 = V(t)$

$$\log \cdot 105 \dots = \log \cdot 684^+$$

$$G = t$$

After 1.95 years, the Value of the Car will be the Same as the unpaid amount of the loan.

