Algebra II Guide to 85 Review

Express the following in simplest $a+bi$ form

1. $2xi(i^2 - 4i^2)$

$2xi^2 - 8xi^3$

$2x(-1) - 8x(-i)$

$-2x + 8xi$

2. $6xi^3(-4xi + 5)$

$6x(-i)(-4xi + 5)$

$-6xi(-4xi + 5)$

$24x^2i^2 - 30xi$

$24x^2(-1) - 30xi$

$-24x^2 - 30xi$

$i^2 = -1$

$i^3 = -i$

3. Kenzie believes that for $x \geq 0$, the expression $\left(\sqrt[7]{x^2}\right)\left(\sqrt[5]{x^3}\right)$ is equivalent to $\sqrt[35]{x^6}$. Is she correct? Justify your response algebraically.

$\left(x^{\frac{2}{7}}\right)\left(x^{\frac{3}{5}}\right) = \cancel{x}\, x^{\frac{6}{35}}$

$\frac{2}{7} + \frac{3}{5} = \frac{31}{35}$

$x^{\frac{31}{35}} \neq x^{\frac{6}{35}}$

No!

Radicals are fractional exponents
Get rid of parenthesis
Negative exponents are fraction
Clean it up — multiply/divide radical/exponents

4. For n and $p > 0$, is the expression $\left[p^2 n^{\frac{1}{2}}\right]^8 \sqrt{p^5 n^4}$ equivalent to $p^{18} n^6 \sqrt{p}$? Justify your answer.

$p^{16} n^4 \left(p^5 n^4\right)^{\frac{1}{2}} = p^{18} n^6 p^{\frac{1}{2}}$ Yes!

$p^{16} n^4 p^{\frac{5}{2}} n^2 = p^{\frac{37}{2}} n^6$

$p^{\frac{37}{2}} n^6 = p^{\frac{37}{2}} n^6$

5. A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, $B(t)$, can be represented by the function $B(t) = 750(1.16)^t$, where the t represents the number of years since the study began. In terms of the <u>monthly rate</u> of growth, the population of red-winged blackbirds can be best approximated by the function

1) $B(t) = 750(1.012)^t$ ③ $B(t) = 750(1.012)^{12t}$

2) $B(t) = 750(1.16)^{12t}$ 4) $B(t) = 750(1.16)^{\frac{t}{12}}$

Converting rates

$1.16^{\frac{1}{12}}$

1.012

6. Mia has a student loan that is in deferment, meaning that she does not need to make payments right now. The balance of her loan account during her deferment can be represented by the function $f(x) = 35,000(1.0325)^x$, where x is the number of years since the deferment began. If the bank decides to calculate her balance showing a <u>monthly</u> growth rate, an approximately equivalent function would be

① $f(x) = 35,000(1.0027)^{12x}$ 3) $f(x) = 35,000(1.0325)^{12x}$

2) $f(x) = 35,000(1.0027)^{\frac{x}{12}}$ 4) $f(x) = 35,000(1.0325)^{\frac{x}{12}}$

$1.0325^{\frac{1}{12}}$

1.0027

$1-.12 = .88$

7. A population of bunnies decreases by 12% every year. If initially there are 3000 bunnies, which recursive sequence can <u>be used to determine</u> the population on year n?

1) $b_1 = 3000$
   $b_n = .12b_{n-1}$

3) $b_1 = 3000$
   $b_n = b_{n-1} - 12$

② $b_1 = 3000$
   $b_n = .88b_{n-1}$

4) $b_1 = 3000$
   $b_n = 1.12b_{n-1}$

8. Daniela invested \$2000 in a stock that increases by 1.6% each week. Which of the following recursive sequences represents the value of her stock after n weeks?

1) $a_0 = 2000$
   $a_n = a_{n-1} + 1.6$

3) $a_0 = 2000$
   $a_n = 1.6a_{n-1}$

$1 + .016 = 1.016$

2) $a_0 = 2000$
   $a_n = a_{n-1} + 1.016$

④ $a_0 = 2000$
   $a_n = 1.016a_{n-1}$

9. If $\tan\theta = \dfrac{24}{7}$ and θ is in Quadrant III, find:

a) $\cos\theta$

$$\frac{7}{25}$$

b) $\sin\theta$

$$\frac{24}{25}$$
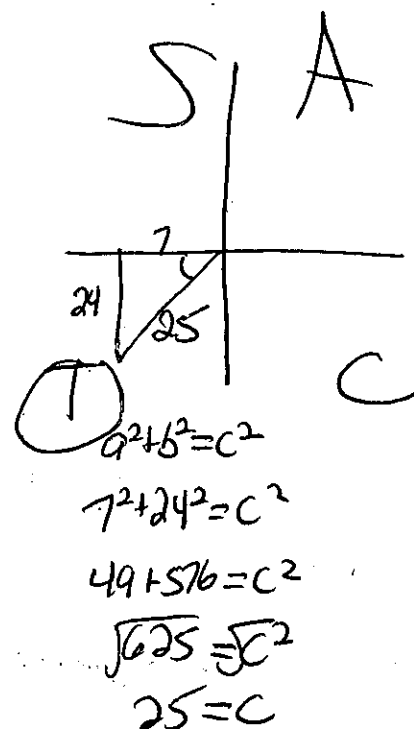
c) $\tan\theta$

$$\frac{24}{7}$$

d) $\sec\theta$

$$\frac{25}{7}$$

e) $\csc\theta$

$$\frac{25}{24}$$

f) $\cot\theta$

$$\frac{7}{24}$$

$$S \quad | \quad A$$

$$\frac{7}{24 \quad 25}$$

$$C$$

$$a^2+b^2=c^2$$
$$7^2+24^2=c^2$$
$$49+576=c^2$$
$$\sqrt{625}=\sqrt{c^2}$$
$$25=c$$

10. Angle θ is in standard position and $(4,-7)$ is a point on the terminal side of θ. Find:

a) $\cos\theta$

$$\frac{4\sqrt{65}}{\sqrt{65}\sqrt{65}}$$
$$\frac{4\sqrt{65}}{65}$$

b) $\sin\theta$

$$\frac{7\sqrt{65}}{\sqrt{65}\sqrt{65}}$$
$$-\frac{7\sqrt{65}}{65}$$
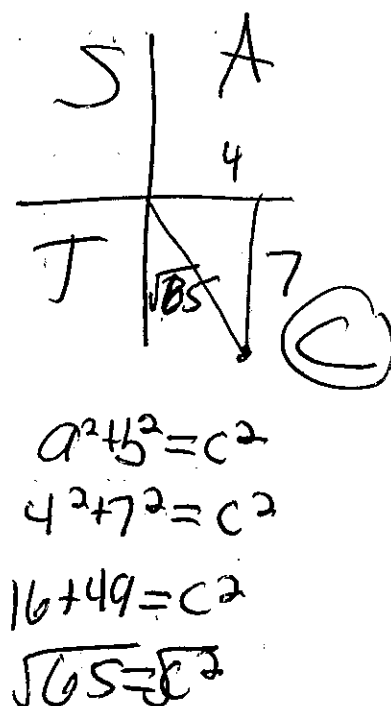
c) $\tan\theta$

$$-\frac{7}{4}$$

d) $\sec\theta$

$$\frac{\sqrt{65}}{4}$$

e) $\csc\theta$

$$-\frac{\sqrt{65}}{7}$$
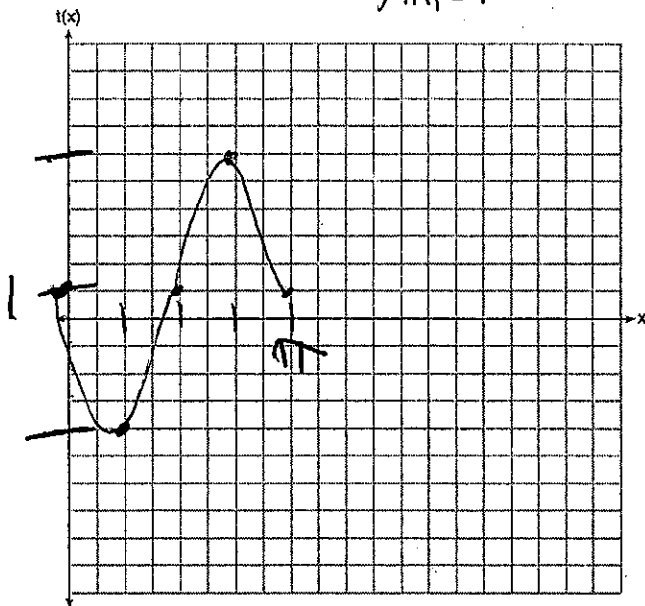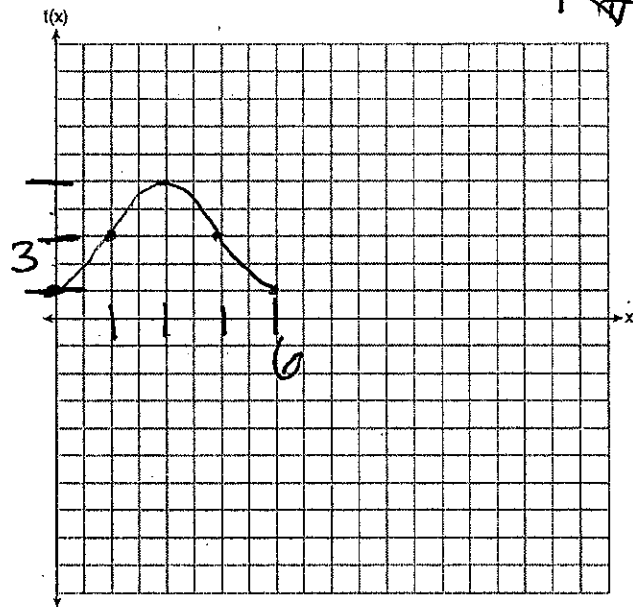
f) $\cot\theta$

$$-\frac{4}{7}$$

$$S \quad | \quad A$$

$$\frac{4}{T \quad \sqrt{65} \quad 7}$$

$$C$$

$$a^2+b^2=c^2$$
$$4^2+7^2=c^2$$
$$16+49=c^2$$
$$\sqrt{65}=\sqrt{c^2}$$

Graph one full cycle of the following waves

amp=5 p=2π/b
ampsinfrqshift -sin p=π
  freq=2
11. $y = -5\sin 2x + 1$ shift=1

amp=2
ampsinfrqshift -cos freq=π/3 p=2π/(π/3)
12. $y = -2\cos\frac{\pi}{3}x + 3$ shift=3
  2π/1 · 3/π = 6

13. The solutions to the equation $-\frac{1}{2}x^2 = -6x + 20$ are

1) $-6 \pm 2i$
2) $-6 \pm 2\sqrt{19}$
3) $6 \pm 2i$
4) $6 \pm 2\sqrt{19}$

$+\frac{1}{2}x^2 \quad +\frac{1}{2}x^2$

$0 = \frac{1}{2}x^2 - 6x + 20$

$a = \frac{1}{2}$

$b = -6$

$c = 20$

$b = -$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(\frac{1}{2})(20)}}{2(\frac{1}{2})}$

$x = \frac{6 \pm \sqrt{-4}}{1}$

$x = 6 \pm 2i$

$\sqrt{-4}$
$i\sqrt{4}$
$2i$

14. Which equation has roots of $3 + i$ and $3 - i$?

1) $x^2 - 6x + 10 = 0$ 3) $x^2 - 10x + 6 = 0$
2) $x^2 + 6x - 10 = 0$ 4) $x^2 + 10x - 6 = 0$

1) ※ $x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$
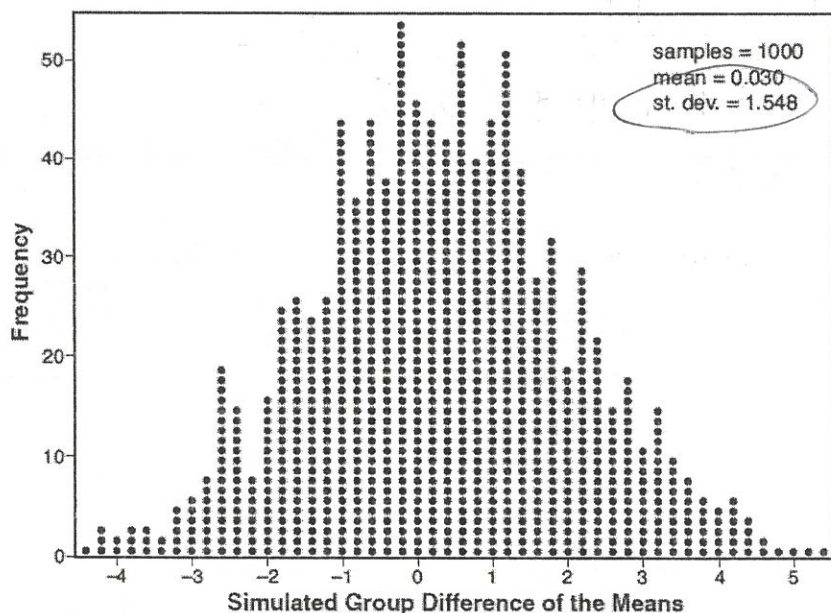
$a = 1$

$b = -6$ $x = \frac{6 \pm \sqrt{-4}}{2}$

$c = 10$

$\sqrt{-4}$ $x = \frac{6 \pm 2i}{2}$
$i\sqrt{4}$
$2i$ $x = 3 \pm i$

15. Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

	Scented Paper	Unscented Paper	
$\bar{x}$	23	18	=5
S_x	2.898	2.408	

Calculate the difference in means in the experimental test grades (scented -unscented). A simulation was conducted in which the subjects' scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.

samples = 1000
mean = 0.030
st. dev. = 1.548

Simulated Group Difference of the Means

Confidence interval

Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the *nearest hundredth*. Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.

CI=mean ± 2(SD)

CI= .030 + 2(1.548) = 3.13

.030 - 2(1.548) = -3.07

(-3.07, 3.13)

Yes, 5 is outside the confidence interval so it is not an expected value due to random chance.

16.

The effects of caffeine on the body have been extensively studied. In one experiment, researchers trained a sample of male college students to tap their fingers at a rapid rate. The sample was then divided at random into two groups of 10 students each. Each student drank the equivalent of about two cups of coffee, which included about 200 mg of caffeine for the students in one group but was decaffeinated coffee for the second group. After a 2-hour period, each student was tested to measure finger tapping rate (taps per minute). The students did not know whether or not their drinks included caffeine and the person measuring the tap rates was also unaware of the groups. The finger-tapping rates measured in this experiment are summarized in the table below.
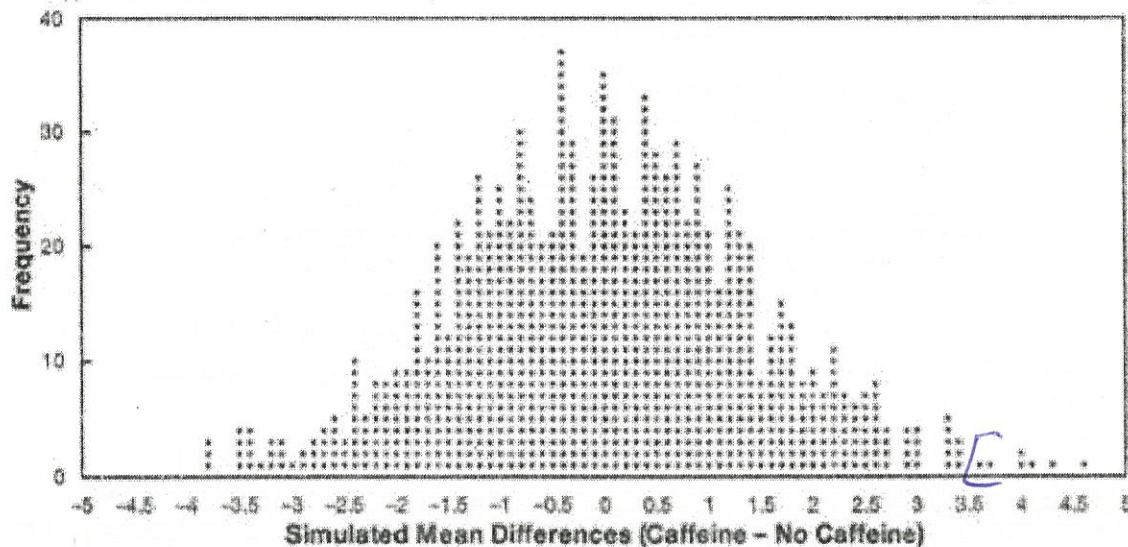
											Mean
Caffeine	246	248	250	252	248	250	246	248	245	250	248.3
No Caffeine	242	245	244	248	247	248	242	244	246	242	244.8

Calculate the mean difference (Caffeine – No Caffeine) and interpret your answer in the context of the problem. $248.3 - 244.8 = 3.5$

On average, the caffeine group tapped 3.5 times more than the non-caffeine group.

The researchers then took the twenty finger-tapping rates and rerandomized them 1,000 times using simulation software. The output of the simulation results is shown in the dotplot below.

Does the simulation data support the conclusion that caffeine causes an increase in average finger-tapping rate? Justify your answer.

Yes, the mean difference is outside the confidence interval because it occured less than 5% of the time.

$\frac{7}{1000} = .7\%$

Factor the following expressions

17. $y = (2x^3 - 3x^2)(-4x + 6)$

$\dfrac{}{x^2}\ \dfrac{}{x^2}\ \dfrac{}{-2}\ \dfrac{}{-2}$

$x^2(2x-3) - 2(2x-3)$

$y = (x^2 - 2)(2x - 3)$

18. $y = (4x^3 + 12x^2)(-9x - 27)$

$\dfrac{}{4x^2}\ \dfrac{}{4x^2}\ \dfrac{}{-9}\ \dfrac{}{-9}$

$4x^2(x+3) - 9(x+3)$

$(4x^2 - 9)(x+3)$

$y = (2x+3)(2x-3)(x+3)$

Solve the following equations algebraically for all values of x

19. $\sqrt{49 - 10x} + 5 = 2x$

$-5\ -5$

$(\sqrt{49-10x})^2 = (2x-5)^2$

$2x$	-5
$2x$	$4x^2$	$-10x$
-5	$-10x$	$+25$

$4x^2 - 20x + 25$

$49 - 10x = 4x^2 - 20x + 25$

$-49 + 10x +10x - 49$

$0 = 4x^2 - 10x - 24$

$\dfrac{0}{2} = \dfrac{4x^2 - 10x - 24}{2}$

$0 = 2x^2 - 5x - 12$

$0 = x^2 - 5x - 24$

$(x-8)(x+3)$

33

$0 = (x-4)(2x+3)$

$x-4=0 \quad 2x+3=0$

$+4\ +4 \quad 2x = -3$

$x=4 \quad \dfrac{2x}{2} = \dfrac{-3}{2}$

$x = -\dfrac{3}{2}$

20. $\sqrt{4x+1} = 11 - x$

$(\sqrt{4x+1})^2 = (11-x)^2$

$4x+1 = (11-x)^2$

	11	$-x$
11	121	$-11x$
$-x$	$-11x$	$+x^2$

$x^2 - 22x + 121$

$4x+1 = x^2 - 22x + 121$

$-4x - 1 -4x - 1$

$0 = x^2 - 26x + 120$

$0 = (x-20)(x-6)$

$x - 20 = 0 \quad x - 6 = 0$

$+20\ +20 \quad +6\ +6$

$x = 20 \quad \boxed{x = 6}$

21. $\dfrac{3}{x} + \dfrac{x}{x+2} = -\dfrac{2}{x+2}$

$x(x+2) \qquad x(x+2) \qquad x(x+2)$

$3(x+2) + x^2 = -2x$

$\text{LCD: } x(x+2)$

$3x + 6 + x^2 = -2x$

$+2x \qquad +2x$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x+3 = 0 \quad x+2 = 0$

$-3\ -3 \quad -2\ -2$

$\boxed{x = -3} \quad x = -2$

22. $\dfrac{x}{x-1} = \dfrac{2}{x} + \dfrac{1}{x-1}$ $\qquad x(x-1)$

$x^2 = 2(x-1) + x$

$x^2 = 2x - 2 + x$

$x^2 = 3x - 2$

$-3x + 2$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x-2 = 0 \quad x-1 = 0$

$+2\ +2 \quad +1\ +1$

$\boxed{x = 2} \quad x = 1$

23. $x^2 + y^2 = 25$

$y + 5 = 2x$

$-5\ -5$

$y = 2x - 5$

$2x$	-5
$2x$	$4x^2$	$-10x$
-5	$-10x$	$+25$

$4x^2 - 20x + 25$

$x^2 + (2x-5)^2 = 25$

$x^2 + 4x^2 - 20x + 25 = 25$

$5x^2 - 20x + 25 = 25$

$ -25\ -25$

$5x^2 - 20x = 0$

$5x(x-4) = 0$

$5x = 0 \quad x-4 = 0$

$\dfrac{5x}{5} = \dfrac{0}{5} \quad +4\ +4$

$x = 0 \quad x = 4$

$y = 2x - 5 \quad y = 2x - 5$

$y = 2(0) - 5 \quad y = 2(4) - 5$

$y = -5 \quad y = 3$

$(0, -5) \quad (4, 3)$

24. $y = 2x^2 - 7x + 4$

$y = 11 - 2x$

$11 - 2x = 2x^2 - 7x + 4$

$-11 + 2x +2x - 11$

$0 = 2x^2 - 5x - 7 \qquad \text{or}$

$0 = x^2 - 5x - 14$

$(x-7)(x+2)$

$2 2$

$0 = (2x-7)(x+1)$

$2x - 7 = 0 \quad x+1 = 0$

$+7\ +7 \quad -1\ -1$

$\dfrac{2x}{2} = \dfrac{7}{2} \quad x = -1$

$x = \dfrac{7}{2}$

$x = \dfrac{7}{2} \quad x = -1$

$y = 11 - 2x \quad y = 11 - 2x$

$y = 11 - \dfrac{7}{} \quad y = 11 - (-1)$

$y = \dfrac{15}{2} \quad y = 12$

$\left(\dfrac{7}{2}, \dfrac{15}{2}\right) \quad (-1, 12)$

25. Carla wants to start a college fund for her daughter Lila. She puts $63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, $C(t)$, that represents the amount of money in the account t years after the account is opened, given that no more money is deposited into or withdrawn from the account. Calculate algebraically the number of years it will take for the account to reach $100,000, to the *nearest hundredth of a year*.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A = C(t)$
$P = 63,000$
$r = .0255$
$n = 12$
$t = t$

$$C(t) = 63,000\left(1 + \frac{.0255}{12}\right)^{12t}$$

$$\frac{100,000}{63,000} = \frac{63,000(1.0021\overline{25})^{12t}}{63,000}$$

$$\boxed{18.14 = t}$$

$$\log\frac{100}{63} = (1.002125)^{12t}$$

$$\log\frac{100}{63} = 12t\log 1.002125$$

$$\frac{\log\frac{100}{63}}{12\log 1.002125} = \frac{12t\log 1.002125}{12\log 1.002125}$$

26. When observed by researchers under a microscope, a smartphone screen contained approximately 11,000 bacteria per square inch. Bacteria, under normal conditions, double in population every 20 minutes.

a) Assuming an initial value of 11,000 bacteria, write a function, $p(t)$, that can be used to model the population of bacteria, p, on a smartphone screen, where t represents the time in minutes after it is first observed under a microscope.

b) Using $p(t)$ from part a, determine algebraically, to the *nearest hundredth of a minute*, the amount of time it would take for a smartphone screen that was not touched or cleaned to have a population of 1,000,000 bacteria per square inch.

$$A = P(2)^{\frac{t}{h}}$$

$$p(t) = 11,000(2)^{\frac{t}{20}}$$

$A = P(t)$
$P = 11,000$
$t = t$
$h = 20$

$$\frac{1,000,000}{11,000} = \frac{11,000(2)^{\frac{t}{20}}}{11,000}$$

$$\log(90.909\ldots) = \log\left(2^{\frac{t}{20}}\right)$$

$$20\left(\log(90.909\ldots)\right) = \left(\frac{t}{20}\log 2\right)20$$

$$\frac{20\log(90.909\ldots)}{\log 2} = \frac{t\log 2}{\log 2}$$

$$130.13 = t$$