Name _____ Date _____
Mr. Schlansky Algebra II

Common Core Algebra II Key Understandings

Polynomials

Subtracting: From comes first, keep, change, change.
Keep the first polynomial
Change subtraction to addition
Change EVERY sign in the second polynomial
Multiplying Binomials: Box Method
Dividing Polynomials:
Divide (first term by first term)
Multiply (Distribute)
Subtract (Keep, change, change)
Bring Down
*Put the remainder over the divisor
(Put 0 as a placeholder if necessary)
Squaring Binomials:
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$

Factoring:

Greatest Common Factor: GCF( )
Difference of Two Squares: $(\sqrt{\ }_1 + \sqrt{\ }_2)(\sqrt{\ }_1 - \sqrt{\ }_2)$
Trinomials: (x)(x)
1) First sign comes down
2) The two signs must multiply for the last sign
3) Find two numbers that multiply to the last number and add/subtract to the middle number
Bridge Method: (Trinomial with a leading coefficient bigger than 1)
1) Build a bridge between the first and last numbers (Multiply)
2) Factor Trinomial Normally
3) Pay the toll (Divide by the leading coefficient)
*If possible, reduce the fraction
If they divide nicely, divide them
If not, put the denominator in front of the variable inside the parenthesis
Grouping: (4 Terms or More)
1) Look for a pattern in the exponents to determine the groups. **You cannot have two terms with the same exponent in the same group.**
2) Factor out the GCF in each group
*You should be left with the same factor. If signs are reversed, factor out a negative
3) Combine coefficients and keep like term.
***Factor further if necessary**

Sum/Difference of Two Cubes
SOAP for signs (Same, Opposite, Always Positive)

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Substitution Trinomials:
1) Replace binomial with y
2) Factor normally
3) Substitute back

*Factor further if possible

A binomial is a factor of a polynomial if it divides nicely into the polynomial

Remainder Theorem: When a polynomial is divided by x - a, the remainder is the value of the polynomial evaluated at a.

Conclusion: If the value of the polynomial evaluated at a is 0, x - a must be a factor

If a is a zero, $p(a) = 0$, $x - a$ is a factor, and the polynomial is divisible by $x - a$. Once you have one of the four pieces of information, you have all four.

To determine if $x - a$ is a factor:
1) **Use remainder theorem and see if** $p(a) = 0$ or
2) **Divide the polynomial by** $x - a$ and see if there is a remainder

Polynomial Equations

1) Bring everything to one side. Keep the leading coefficient positive.
2) Factor (Refer to above section)
3) Set each factor equal to zero

If you cannot factor a quadratic, use quadratic formula or completing the square.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) $ax^2 + bx + c = 0$

2) List a, b, and c values
3) Substitute values into quadratic formula
4) Type discriminant into the calculator (what is underneath the radical)
5) REDUCE THE RADICAL off to the side (If possible)
6) Reduce from all three terms (If possible)

Completing the Square: $(x-a)^2$

*Divide out a GCF if possible

1) Bring both variable terms to one side and the constant term to the other side

2) Add $\left(\frac{b}{2}\right)^2$ to both sides

3) Factor the trinomial (Both factors must be the same)

4) Rewrite the factors as a binomial squared

5) Take the square root of both sides (The right hand side should have a $\pm$)

6) Add or subtract to isolate x

Polynomial Equations Given a Factor

1) Use Remainder Theorem with the given factor to set up equation.
2) Solve for k.
3) Divide out the given factor using synthetic division.
4) Factor completely

*To find zeros, set each factor equal to zero.

Complex Numbers

A negative inside a radical becomes an *i* and comes outside

$i = \sqrt{-1}$, $i^2 = -1$

a + b*i* form simply means there will be an *i* in the answer

Parabolas

Definition of a Parabola: A parabola is the set of all points equidistant between a point (focus) and a line (directrix).

The vertex is directly in between the focus and the directrix. USE GRAPH PAPER AND COUNT!

$$\frac{(x-v)^2}{4p} = y - t$$

$(v,t) = vertex$

$p = dis\tan ce\ \ from\ \ vertex\ \ to\ \ focus\ /\ directrix$

Vertex Form of a Parabola: $y = a(x-v)^2 + t$,where (v,t) is the vertex.

Quadratic Systems of Equations Algebraically

1) Isolate at least one variable in one of the equations
2) Substitute one equation into the other (set them equal if you solved both equations for the same variables).
3) Solve equation (Mr. x^2 /Polynomial Equations)
4) Substitute answers into one of the original equations to find the second variable

Quadratic Systems of Equations Graphically

1) Graph each equation
2) Find the point(s) of intersection

Systems of Equations Graphically Using TI-84+ ($f(x) = g(x)$)

1) Type equations into Y_1 and Y_2
2) Zoom 6 (Standard) is your standard window. Adjust window OR try Zoom 0(Fit) if you don't see what you want to see.
3) 2^{nd} Trace (Calc), 5 (Intersect)
4) Place cursor over point of intersection, hit enter, enter, enter. Repeat the process for any other points of intersection.

*The solutions to the system of equations are the x values of the intersections.

Rational Expressions:

-Reducing/Multiplying: **FACTOR MARRIED TERMS**, Reduce single terms, write out exponents

-If a binomial is written backwards with a minus sign, it cancels to negative one

-Dividing: Keep/Change/Flip and then follow the rules of multiplication

-Adding and Subtracting: Find a common denominator

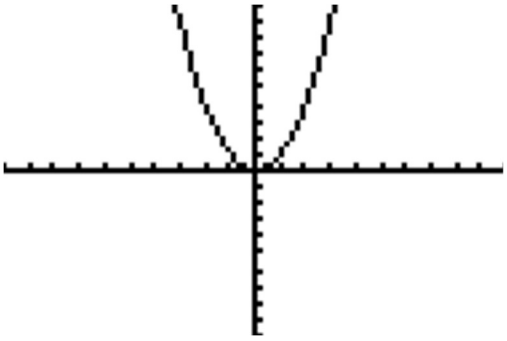
-To find a common denominator: Put all of your factors together

-To get rid of fractions in a complex fraction or equation: Multiply by the LCD.

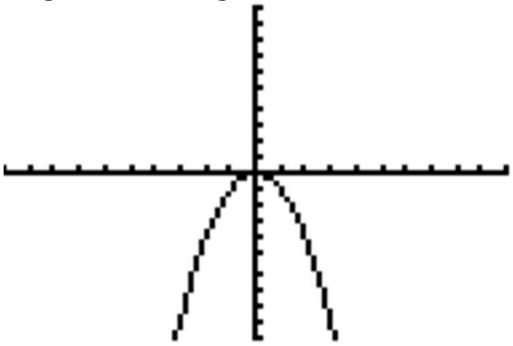
Graphing Polynomials

End Behavior

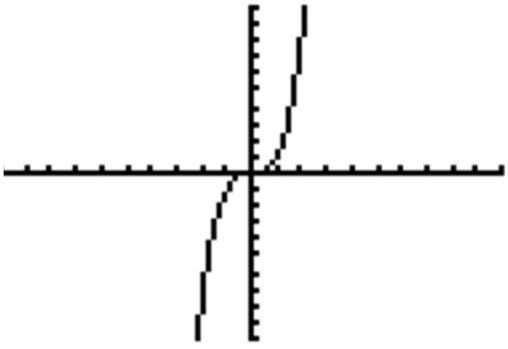
Positive leading coefficient, Even Degree



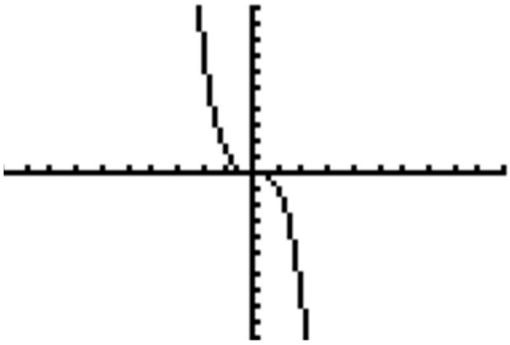
Negative leading coefficient, Even Degree



Positive leading coefficient, Odd Degree



Negative leading coefficient, Odd Degree



Even degree begins and ends pointing in same direction
Odd degree begins and ends point in opposite directions
Positive leading coefficient slopes up at the end
Negative leading coefficient slopes down at the end
To find degree:
If in standard form, it is the largest exponent.
If in factored form, add up all of the powers of the x's.

To find zeros:
Set equations equal to zero and solve polynomial equation.
If there is an odd amount of multiple roots, the graph passes through the x-axis
If there is an even amount of multiple roots, the graph bounces off the x-axis.

To find y-intercept:
If in standard form, it is the constant term
If in factored form, multiply together all of the constant terms

There is a maximum of $n-1$ relative minima/maxima for a polynomial of degree n

Functions

A function is when each guy talks to one girl (each x corresponds to one y) and passes the vertical line test. (The x values do not repeat.)
Domain: Hold pen vertically and travel left to right to find where graph starts and ends.
Range: Hold pen horizontally and travel bottom to top to find where graph starts and ends.

Inverse of a function $f^{-1}(x)$:
Switch x and y, solve for y
Reflect over the line $y = x$

Even Functions:
$f(x) = f(-x)$
The graph is symmetric to the y axis
If it is a polynomial function, all of the exponents are even.
Odd Functions:
$f(-x) = -f(x)$
The graph is symmetric to the origin
If it is a polynomial function, all of the exponents are odd.
Constant terms have an exponent of 0. ($5 = 5x^0$)

Average rate of change: $\frac{f(b)-f(a)}{b-a}$, you may have to find $f(a)$ and $f(b)$ by typing into the calculator and using your table or using a graph.
"On average, the function is increasing/decreasing x units per unit of time."

Transforming Functions:

Translations

If adding to f(x), the graph moves up or down

If adding to x, the graph moves left or right (the opposite direction in which you would think)

$y = f(x) + a$ moves UP a units

$y = f(x) - a$ moves DOWN a units

$y = f(x + a)$ moves LEFT a units

$y = f(x - a)$ moves RIGHT a units

Reflections

If the x is negated, the graph is reflected over the y axis

If the f(x) (aka y) is negated, the graph is reflected over the x axis

$y = f(-x)$ reflect over y axis

$y = -f(x)$ reflect over x axis

If a is positive, the vertex is a minimum and the graph opens upward

If a is negative, the vertex is a maximum and the graph opens downward

Dilations

$y = af(x)$ Vertical Dilation

If $|a| > 1$, vertical stretch, narrower

If $|a| < 1$, vertical shrink, wider

$y = f(ax)$ Horizontal Dilation

If $|a| > 1$, Horizontal shrink

If $|a| < 1$, Horizontal stretch

Exponents/Logarithms

Multiplying: Add exponents $x^2 \bullet x^3 = x^{2+3} = x^5$

Dividing: Subtract exponents: $\frac{x^8}{x^5} = x^{8-5} = x^3$

When raising a power to a power, multiply exponents: $(x^2)^3 = x^{2\bullet 3} = x^6$

Anything to the zero power is equal to 1 $x^0 = 1$

Negative exponents are fractions! $x^{-2} = \frac{1}{x^2}$

If exponent is outside parenthesis, everything gets it $\left(\frac{xy}{z}\right)^3 = \frac{x^3 y^3}{z^3}$

Radicals are fractional exponents (Fractional exponent = $\frac{power}{root}$**)**

Get rid of parenthesis

Negative exponents are fractions (Move whatever is being raised to the negative power)

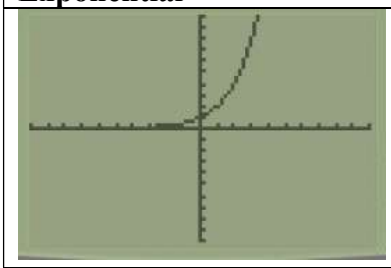
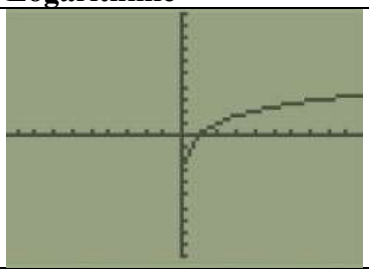
Clean it up for me

Exponential Equations:

Isolate the base

a) **Constants:** Take the appropriate root of both sides or raise each side to the reciprocal power
b) **Variables:** Take the log of both sides

Graphing Exponential and Logarithmic Functions

Exponential	Logarithmic
	
Horizontal Asymptote at $y = 0$	Vertical Asymptote at $x = 0$
Passes through $(0,1)$	Passes through $(1,0)$
Domain is all real numbers	Domain is all positive real numbers
Range is all positive real numbers	Range is all real numbers
Exponents and logarithms are inverses of each other!!!!!!!!!!	

Modeling Exponential Functions

Basic Exponential Growth/Decay Formula: $A = P(1 \pm r)^t$

COMPOUNDING Interest: $A = P\left(1 \pm \frac{r}{n}\right)^{nt}$, where A is the current amount, P is the initial amount, r is the rate as a decimal (divide by 100), n is the number of times compounded (yearly =1, semiannually = 2, quarterly = 4, monthly = 12, weekly = 52, daily = 365) and t is time.

COMPOUNDING CONTINUOUSLY: $A = Pe^{rt}$

Given an exponential function: What is in front of the parenthesis is the INITIAL amount, what is inside the parenthesis is 1 + the rate or 1 – the rate.

Example: $A = 500(1.2)^t$: 500 is initial amount, rate is .2 or 20% growth (1 + .2)

$A = 500(0.8)^t$: 500 is initial amount, rate is .2 or 20% decay (1 - .2)

To convert from an annual rate to a monthly rate or other:

Start with $A = P(1 \pm r)^t$

$A = P\left((1 \pm r)^{\frac{1}{n}}\right)^{nt}$ if t is years or $A = P\left((1 \pm r)^{\frac{1}{n}}\right)^{m}$ if m is the rate unit. n is number of new units in old units. For example: yearly to monthly: $n = 12$, weekly to daily: $n = 7$

Half Life (Or a given percent every x unit of time)

$A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$ where h is the amount of time for the half life

$A = P(1 \pm r)^{\frac{t}{h}}$ where h is the amount of time the rate is applied. For example, if the rate increases by 15% every 5 years, r = .15 and h = 5.

Sequences/Series

Sequences:

Arithmetic: add a constant difference, Geometric: multiply by a common ratio

Explicit Formulas (From Reference Sheet)

Arithmetic: $a_n = a_1 + (n-1)d$ Geometric: $a_n = a_1(r)^{n-1}$

If initial or a_0 is given, $(n-1)$ becomes *n.* Same formulas as Algebra I modeling.

Arithmetic: $a_n = a_0 + nd$ Geometric: $a_n = a_0(r)^n$

Recursive Formulas

Arithmetic: $a_1 =$
 $a_n = a_{n-1} + d$ Geometric: $a_1 =$
 $a_n = ra_{n-1}$

***r* is the common ratio (what you're multiplying by). If you're increasing or decreasing by a percent,** $common\ ratio = 1 \pm rate$ **.**

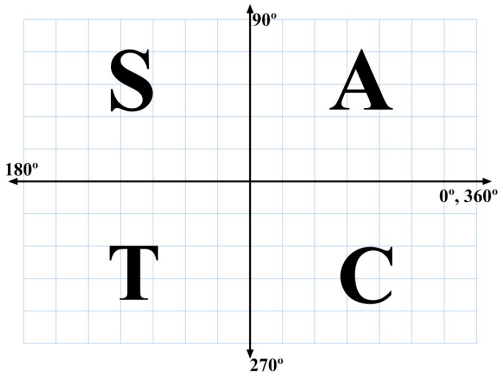
For example: Increases by 12% each year, common ratio is $1 + .12 = 1.12$
Decreases by 20% each year, common ratio is $1 - .20 = .80$

Series is the sum of a sequence

To write a series explicitly: $S_n = \frac{a_1 - a_1(r)^n}{1-r}$ where *r* is the common ratio $(1 \pm rate)$

To write a series using summations: $\sum_{n=1}^{n} a_1(r)^{n-1}$ or $\sum_{n=0}^{n} a_0(r)^n$

Trigonometry



Unit Circle: $(x, y) = (\cos\theta, \sin\theta)$

Degrees to radians: Multiply by $\frac{\pi}{180}$

Radians to degrees: Multiply by $\frac{180}{\pi}$ OR replace π with 180

To find trig ratios:

Reciprocal trig function pairs:

$\csc\theta$	$\sec\theta$	$\tan\theta$
$\sin\theta$	$\cos\theta$	$\cot\theta$

Triangle Method:

If an angle passes through a point or sin/cos/tan = $\frac{something}{something}$, make a right triangle and use SOHCAHTOA

Any point on the unit circle is $(\cos\theta, \sin\theta)$

Know your Pythagorean triples: {3,4,5}, {5, 12, 13}, {8, 15, 17}, {7, 24, 25}

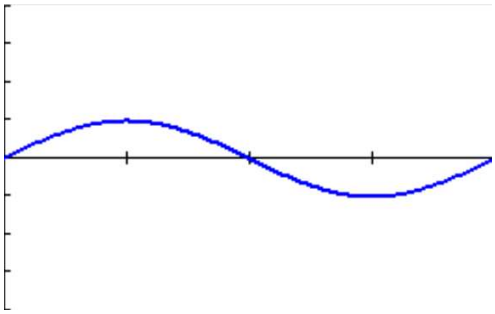
Pythagorean Identity Method:
Trig Ratios with Pythagorean Identity

1) Substitute sine or cosine into the Pythagorean identity to find the other
2) Use $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to find tangent
3) Put one over each of them to find their reciprocal functions
4) Use ASTC to determine the signs depending on the quadrant

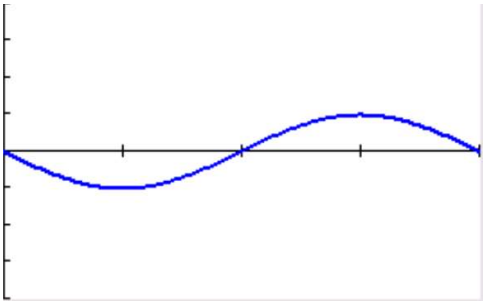
Trig Graphs:

Know what your waves look like!

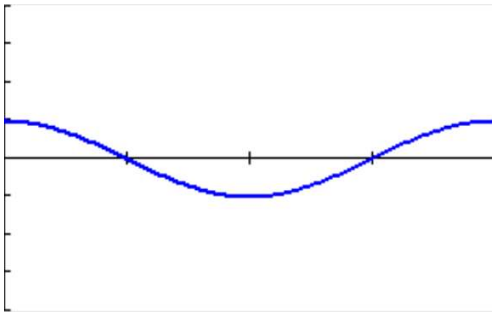
$f(x) = \sin x$



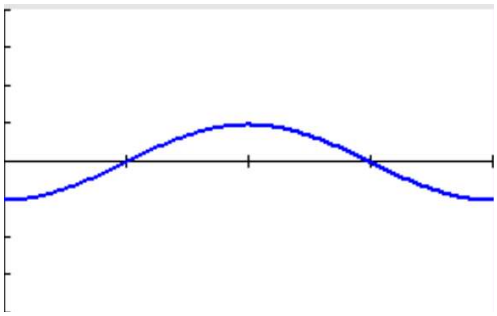
$f(x) = -\sin x$



$f(x) = \cos x$



$f(x) = -\cos x$



AMPSINFREQXSHIFT

Amplitude: Distance from the midline to minimum or maximum
Frequency: How many waves from 0 to 2π
Period: (Wavelength): How long it takes to make one full cycle
Shift: y value of the midline. The average value of the function.

$$Period = \frac{2\pi}{frequency}, Frequency = \frac{2\pi}{period}$$

To graph: Draw a little picture! Find midline ($\frac{\min+\max}{2}$) and period! Make 4 dashes on x-axis and put period at the 4th dash. Divide that value by 4 to find the scale.

Probability

$\cup = or$, $\cap = and$, $|= given\ that$ (The condition comes after given that)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

If given the two events are independent, use $P(A \cap B) = P(A) \bullet P(B)$

To express as a percent, divide and multiply by 100.

To determine if events are independent:

$P(A \cap B) = P(A) \bullet P(B)$

$P(A) = P(A / B)$

To create two way tables:

One axis has yes and no for one variable, the other has yes and no for the other variable.

If given probabilities, create a hypothetical table where the total total is 100.

For conditional probabilities, circle the row or column that is the condition.

Normally distributed: Use *normalcdf* (2^{nd} VARS (Distr), 2:normalcdf)

To compare scores from different distributions, find the z score (number of standard deviations from the mean)

$z = \frac{x - \mu}{\sigma}$ where z = z score, x = data point, μ= mean, and σ= standard deviation.

Statistics

A *survey* is a type of observational study that gathers data by asking people a number of questions.

A good sample should be randomly selected where every member of the population has a chance of being chosen.

An *observational study* records the values of variables for members of a sample. NO TREATMENT IS ADMINISTERED.

A *controlled experiment* **randomly selects** a sample and **randomly assigns** members of the sample to a treatment and control group for the purpose of seeing what effect the treatments have on some response. THE SAMPLE RECEIVES A TREATMENT.

To determine if a treatment is effective:

1) Find the mean difference between the treatment and control group
2) Rerandomize the sample many times and record the mean differences on a dot plot

If the mean difference falls within the confidence interval, the treatment is not effective.

If the mean difference falls outside the confidence interval (less than 5%), the treatment is effective.

A sample statistic leads to a population characteristic. For example, if the sample has a mean of 10, then the population's mean should be relatively close to 10.

As sample size increases:

The mean remains relatively the same (it may differ due to random chance).

The spread/variability decreases.

The standard deviation decreases.

$\sigma = \sqrt{\frac{p(1-p)}{n}}$ where σ = standard deviation, p = population proportion (probability), n = sample size

Expect a specific result if it falls within the confidence interval!

$Confidence\ Interval = mean \pm 2(S\tan dard\ Deviation)$

$M\arg in\ of\ Error = 2(S\tan dard\ Deviation)$

To calculate mean and standard deviation:

Stat Edit, Stat, Calc, 1-Var stats. $\bar{x} = mean, S_x = sample\ s\tan dard\ deviation$

Equations

Fractional Equations

Multiply by the LCD

Polynomial Equations

1) Bring everything to one side. Keep the leading coefficient positive.
2) Factor (Refer to above section)
3) Set each factor equal to zero

If you cannot factor a quadratic, use quadratic formula or completing the square.

Radical Equations

1) Isolate
2) Square both sides
3) Check

Exponential Equations

Isolate the base

a) Constants: Take the appropriate root of both sides or raise each side to the reciprocal power
b) Variables: Take the log of both sides

Logarithmic equations

1) Isolate (Convert to a single log. Use log rules)
2) Put in exponential form ($\log_x 64 = 2 \rightarrow x^2 = 64$)

Miscellaneous:

Multiple Choice Strategy with Variables
If variables in the problems and answers:
Type in original problem, 2nd Math (Test), =, type in each solution. 1 is equivalent, 0 is not equivalent. Make sure to try all four choices.
Multiple Choice Strategy with Equations
Substitute each answer in for each variable

Linear Systems In Three Variables
Elimination Method:
1) Choose two pairs of equations and get the same variable to cancel
2) Use Addition Method to solve the system with your two new equations
3) Substitute those two answers into one of the original equations to find your third variables

Matrix Method:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$$

1) 2nd Matrix, Edit, A, 3X3 (Coefficient of the left hand side)
2) 2nd Matrix, Edit, B, 3X1 (Right hand side)
3) $A^{-1}B$

Complex Formulas
List what each variable represents and carefully substitute the appropriate values in