Name Schlansky

Algebra II CC – Midterm Review #1

Date ____________

Period ____

UNIT 1: FACTORING AND SOLVING POLYNOMIAL EQUATIONS

1. If $p(x)=(5x^2+2x-1)(x^2-x-4)$ and $q(x)=7x^4-x^3+5x-9$, find $p(x)-q(x)$ in simplest form.

	$5x^2$	$+2x$	-1
x^2	$5x^4$	$+2x^3$	$-1x^2$
$-x$	$-5x^3$	$-2x^2$	$+1x$
-4	$-20x^2$	$-8x$	$+4$

$(5x^4-3x^3-23x^2-7x+4)-(7x^4-x^3+5x-9)$

Keep, change, change

$$\begin{array}{rl} & 5x^4-3x^3-23x^2-7x+4 \\ + & -7x^4+x^3\qquad-5x+9 \\ \hline & -2x^4-2x^3-23x^2-12x+13 \end{array}$$

2. Prove that $(2x)^2+(x^2-1)^2=(x^2+1)^2$ is an identity for all real numbers, x. $(a+b)^2=a^2+2ab+b^2$

$4x^2+(x^4-2x^2+1)=(x^4+2x^2+1)$

$4x^2+x^4-2x^2+1=x^4+2x^2+1$

$x^4+2x^2+1=x^4+2x^2+1$ ✓

$2x+\cancel{1}=0$ $\frac{2x}{2}=\frac{-1}{2}$ $x=-\frac{1}{2}$

$\ \ \ -1\ -1$

3. Emily thinks that $2x+1$ is a factor of $2x^3-13x^2-x+3$. Is Emily correct? Explain your answer.

$P(-\frac{1}{2})=2(-\frac{1}{2})^3-13(-\frac{1}{2})^2-(-\frac{1}{2})+3$

$P(-\frac{1}{2})=0$

Yes

There is no remainder

$$\begin{array}{r} x^2-7x+3 \\ 2x+1\overline{)\,2x^3-13x^2-x+3} \\ +\ -2x^3+{}^-x^2\qquad\quad \\ \hline -14x^2-x\quad \\ +\ +14x^2+7x\quad \\ \hline 6x+3 \\ +\ -6x+{}^-3 \\ \hline 0 \end{array}$$

Divide
Multiply
Subtract
Bring Down

Yes

4. Factor each expression completely:

a $4x^2-20x-96$

$\overline{4} \quad \overline{4} \quad \overline{4}$

$4(x^2-5x-24)$

$4(x-8)(x+3)$

b a^4+4a^2-32

$(a^2+8)(a^2-4)$

$(a^2+8)(a+2)(a-2)$

c $(9x^3+45x^2)(-4x-20)$

$\overline{9x^2} \quad \overline{9x^2} \quad \overline{-4} \quad \overline{-4}$

$9x^2(x+5)-4(x+5)$

$(9x^2-4)(x+5)$

$(3x+2)(3x-2)(x+5)$

$a=5c \quad b=4$

d $125c^3-64$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$125c^3-64=(5c-4)(25c^2+20c+16)$

e $2x^2-11x+14$

$x^2-11x+28$

$(x-\frac{7}{2})(x-\frac{4}{2})$

$(2x-7)(x-2)$

f $(x^3+3x^2-18x)(+2x^2+6x-36)$

$\overline{x} \quad \overline{x} \quad \overline{x} \quad \overline{2} \quad \overline{2} \quad \overline{2}$

$x(x^2+3x-18)+2(x^2+3x-18)$

$(x+2)(x^2+3x-18)$

$(x+2)(x+6)(x-3)$

Remember Types of Factoring:
GCF
DOTS
Trinomials/Bridge
Grouping
***Always factor completely!!**
SOAP Cubes!

5. It is known that $x-5$ is a factor of $p(x)=x^4-12x^3+35x^2-9x^2+108x-315$. Determine and state the zeroes of the equation $p(x)=0$.

$\overline{x^2} \quad \overline{x^2} \quad \overline{x^2} \quad \overline{-9} \quad \overline{-9} \quad \overline{-9}$

$p(5)=(5)^4-12(5)^3+35(5)^2-9(5)^2+108(5)-315$

$p(5)=0$

$(x^2-9)(x^2-12x+35)$

$(x+3)(x-3)(x-7)(x-5)=0$

$x+3=0 \quad x-3=0 \quad x-7=0 \quad x-5=0$

$-3 \; -3 \quad +3 \; +3 \quad +7 \; +7 \quad +5 \; +5$

$x=-3 \quad x=3 \quad x=7 \quad x=5$

6. Find the zeroes of each equation:

a $x^4 - 25x^2 + 144 = 0$

$(x^2-16)(x^2-9)=0$

$(x+4)(x-4)(x+3)(x-3)=0$

$x+4=0$ $\quad x-4=0$ $\quad x+3=0$ $\quad x-3=0$

$-4\ -4$ $\quad +4\ +4$ $\quad -3\ -3$ $\quad +3\ +3$

$x=-4$ $\quad x=4$ $\quad x=3$ $\quad x=3$

b $4x^2 = 15 - 4x$

$-15+4x$

PT

$4x^2+4x-15=0$

$x^2+4x-60=0$

$(x+\frac{10}{4})(x-\frac{6}{4})=0$

$(x+\frac{5}{2})(x-\frac{3}{2})=0$

$2x+5=0$ $\quad 2x-3=0$

$-5\ -5$ $\quad +3\ +3$

$\frac{2x}{2}=\frac{-5}{2}$ $\quad \frac{2x}{2}=\frac{3}{2}$

$x=\frac{-5}{2}$ $\quad x=\frac{3}{2}$

c $(6x^3 - 5x^2 - 4x) + (36x^2 - 30x - 24) = 0$

$\frac{6x^3}{x}\ \frac{5x^2}{x}\ \frac{4x}{x}\ \frac{36x^2}{6}\ \frac{30x}{6}\ \frac{24}{6}$

$x(6x^2-5x-4)+6(6x^2-5x-4)=0$

$(x+6)(6x^2-5x-4)=0$

$x^2-5x-24$

$(x-\frac{8}{6})(x+\frac{3}{6})$

$(x+6)(x-\frac{4}{3})(x+\frac{1}{2})$

$(x+6)(3x-4)(2x+1)=0$

$x+6=0$ $\quad 3x-4=0$ $\quad 2x+1=0$

$-6\ -6$ $\quad +4\ +4$ $\quad -1\ -1$

$\boxed{x=-6}$ $\quad \frac{3x}{3}=\frac{4}{3}$ $\quad \frac{2x}{2}=\frac{-1}{2}$

$\boxed{x=\frac{4}{3}}$ $\quad \boxed{x=\frac{-1}{2}}$

d $(2x^2 + 3x)^2 - 4(2x^2 + 3x) - 5 = 0$ Let $y=2x^2+3x$

$y^2-4y-5=0$

$(y-5)(y+1)=0$

$(2x^2+3x-5)(2x^2+3x+1)=0$

$(x^2+3x-10)(x^2+3x+2)$

$(x+\frac{5}{2})(x-\frac{2}{2})(x+\frac{2}{2})(x+\frac{1}{2})$

$(2x+5)(x-1)(x+1)(2x+1)=0$

$2x+5=0$ $\quad x-1=0$

$-5\ -5$ $\quad +1\ +1$

$\frac{2x}{2}=\frac{-5}{2}$ $\quad \boxed{x=1}$

$\boxed{x=\frac{-5}{2}}$

$x+1=0$ $\quad 2x+1=0$

$-1\ -1$ $\quad -1\ -1$

$\boxed{x=-1}$ $\quad \frac{2x}{2}=\frac{-1}{2}$

$\boxed{x=-\frac{1}{2}}$

UNIT 2: POLYNOMIAL GRAPHS & THE REMAINDER THEOREM

7. Determine algebraically the zeroes of $q(x) = x^3 + x^2 - 4x - 4$.

$0=x^3+x^2-4x-4$

$\frac{x^3}{x^2}\ \frac{x^2}{x^2}\ \frac{-4x}{-4}\ \frac{-4}{-4}$

$0=$

$0=x^2(x+1)-4(x+1)$

$0=(x^2-4)(x+1)$

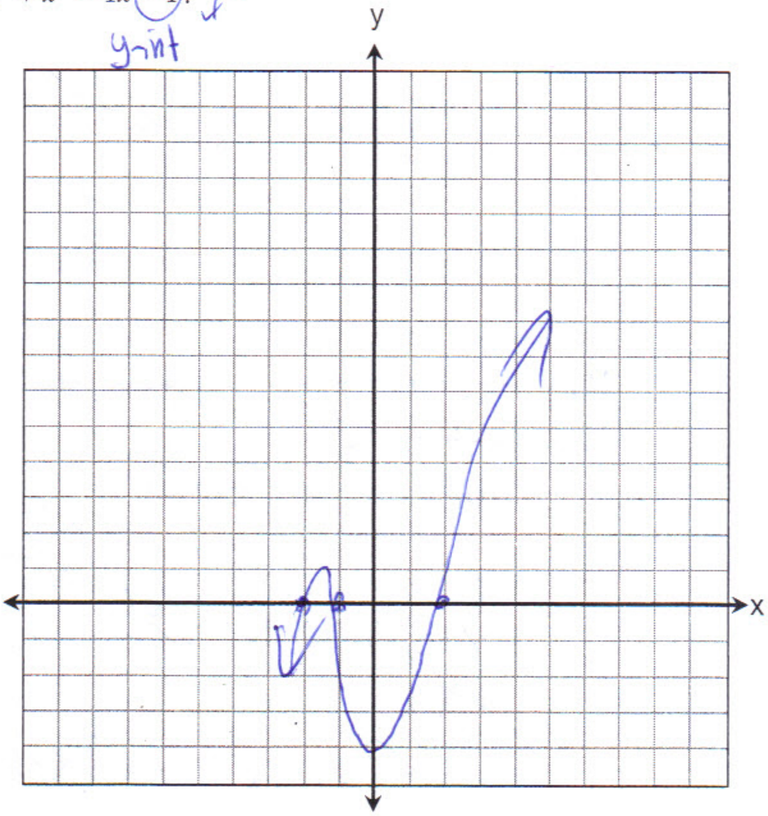
$0=(x+2)(x-2)(x+1)$

$x+2=0$ $\quad x-2=0$ $\quad x+1=0$

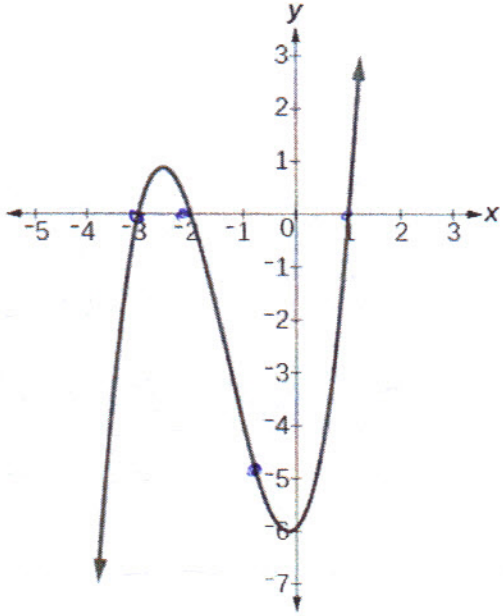
$-2\ -2$ $\quad +2\ +2$ $\quad -1\ -1$

$\boxed{x=-2}$ $\quad \boxed{x=2}$ $\quad \boxed{x=-1}$

Graph $q(x)$ on the set of axes below.



8. The graph to the right shows $y = p(x)$.



State the zeroes of $p(x)$.

$-3, -2, 1$

State the factors of $p(x)$.

$(x+3)(x+2)(x-1)$

What is the degree of $p(x)$?

3 (3 zeros, 2 relative extrema)

How many relative maxima does $p(x)$ have?

1

How many relative minima does $p(x)$ have?

1

Describe the end-behavior of $p(x)$ using proper notation.

$x \to -\infty,\ p(x) \to -\infty$

$x \to \infty,\ p(x) \to \infty$

State the interval(s) over which $p(x)$ is increasing using a proper notation.

$(-\infty, -2.5)\ (-.5, \infty)$

State the interval(s) over which $p(x)$ is decreasing using a proper notation.

$(-2.5, -.5)$

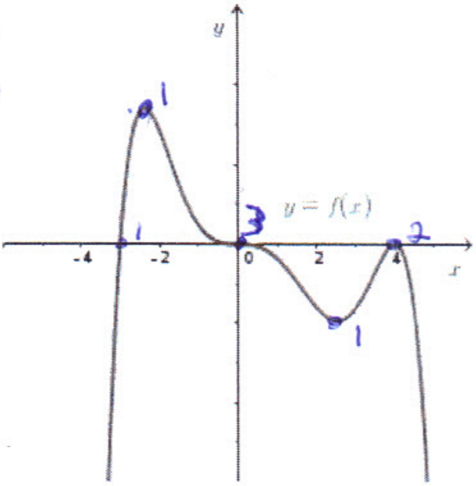
Would the remainder when $p(x)$ is divided by $x-1$ be positive, negative, or zero?

0 $x-1$ is a factor

Would the remainder when $p(x)$ is divided by $x+1$ be positive, negative, or zero?

Negative $p(-1) = -5$

9. The diagram to the right is the graph of the polynomial $y = f(x)$.



State the least degree that $f(x)$ could be. Justify your answer. 6

~~There are 4 relative extrema so it must be at least 5~~

there are 6 zeros.

~~$x^2 - x - 12$~~

Write a possible equation for $f(x)$.

$(x+3)(x^3)(x-4)^2 = f(x)$

$f(x) = x^3(x-4)^2(x+3)$

10. State the Remainder Theorem for a polynomial $p(x)$.

the remainder when a polynomial is divided by x-a is P(a)

Explain how the Remainder Theorem can be used to determine factors (and then zeroes) of a polynomial.

If P(a) equals 0, then x-a is not a factor

11. If the polynomial $b(x) = x^4 + 2x^3 + kx^2 - 2x + 8$, determine the value of k such that $x - 2$ is a factor of $b(x)$.

2 is a zero
(2,0)

$0=(2)^4+2(2)^3+k(2)^2-2(2)+8$

$0=16+16+4k-4+8$

$0=4k+36$

$-36 \quad -36$

$\frac{-36}{4}=\frac{4k}{4}$ $\boxed{k=-9}$

Determine all the zeroes of $b(x)$.

x^3+4x^2-x-4

$x-2 \overline{\big) x^4+2x^3-9x^2-2x+8}$

$+\ {}^-x^4+2x^3$

$4x^3-9x^2$

$+\ {}^-4x^3+8x^2$

$-1x^2-2x$

$+\ +1x^2+2x$

$-4x+8$

$+\ +4x-8$

0

$(x-2)(x^3+4x^2-x-4)=0$

$(x-2)(x^2(x+4)-1(x+4))=0$

$(x-2)(x^2-1)(x+4)=0$

$(x-2)(x+1)(x-1)(x+4)=0$

$x-2=0$, $+2\ +2$, $x=2$

$x+1=0$, $-1\ -1$, $x=-1$

$x-1=0$, $+1\ +1$, $x=1$

$x+4=0$, $-4\ -4$, $x=-4$

12. Find the quotient: $\dfrac{3x^4 - 8x^3 + 2x + 7}{x - 2}$

~~$3x^3-2x^2$~~

~~$x-2 \overline{\big) 3x^4-8x^3+2x+7}$~~

~~$+\ {}^-3x^4+6x^3$~~

~~$-2x^3+2x^2$~~

~~$-2x^3+4x^2$~~

$\boxed{3x^3-2x^2-4x-6-\frac{5}{x-2}}$

$x-2 \overline{\big) 3x^4-8x^3+0x^2+2x+7}$

$+\ {}^-3x^4+6x^3$

$-2x^3+0x^2$

$+\ +2x^3-4x^2$

$-4x^2+2x$

$+\ +4x^2-8x$

$-6x+7$

$+\ +6x-12$

-5