

UNIT 3: RATIONAL EXPRESSIONS & EQUATIONS

Factor each expression!

1. Simplify: $\frac{3x - 4x^2}{8x^2 - 2x - 3}$

$$-\frac{x}{2x+1}$$

$$\frac{x(3-4x)(-1)}{(4x-3)(2x+1)}$$

$$\begin{aligned} &8x^2 - 2x - 3 \\ &x^2 - 2x - 24 \\ &(x-6)(x+4) \\ &\frac{8}{8} \quad \frac{1}{8} \\ &(x-\frac{3}{4})(x+\frac{1}{2}) \\ &(4x-3)(2x+1) \end{aligned}$$

2. Perform the indicated operation and express in simplest form:

$$\frac{c^3 - 9c}{2c^2 + 7c + 3} \div \frac{5c - 15}{4c^2 - 1}$$

$$\frac{c^3 - 9c}{2c^2 + 7c + 3} \cdot \frac{4c^2 - 1}{5c - 15}$$

Keep, change, flip

$$\frac{c(c+3)(c-3)}{(c+3)(2c+1)} \cdot \frac{(2c+1)(2c-1)}{5(c-3)}$$

$$\frac{c(2c-1)}{5}$$

$$\begin{aligned} &2c^2 + 7c + 3 \quad \left| \begin{array}{l} c^3 - 9c \\ c^2 + 7c + 6 \\ c + 6 \end{array} \right. \begin{array}{l} c^3 - 9c \\ c(c^2 - 9) \\ c(c+3)(c-3) \end{array} \\ &\frac{c+6}{2} \quad \frac{c+1}{2} \\ &(c+3)(2c+1) \end{aligned}$$

3. Perform the indicated operation and express in simplest form:

$$\frac{3}{x^2 + 5x - 14} - \frac{2}{x^2 + 8x + 7}$$

$$\frac{3}{(x+7)(x-2)} - \frac{2}{(x+7)(x+1)}$$

$$\frac{3(x+1)}{(x+7)(x-2)(x+1)} - \frac{2(x-2)}{(x+7)(x-2)(x+1)}$$

*Factor denominators
Find LCD
multiply top and bottom
by what's missing.*

$$\frac{3x+3}{(x+7)(x-2)(x+1)} - \frac{2x-4}{(x+7)(x-2)(x+1)}$$

$$\frac{3x+3-2x+4}{(x+7)(x-2)(x+1)} \quad (RCC)$$

$$\frac{x+7}{(x+7)(x-2)(x+1)} \cdot \frac{1}{(x-2)(x+1)}$$

4. Simplify:

multiply by LCD

$$\frac{\frac{x^2}{16} - 1}{\frac{x}{8} - \frac{1}{2}}$$

$$\frac{x^2 - 16}{2x - 8}$$

$$\frac{(x+4)(x-4)}{2(x-2)}$$

$$\frac{x+4}{2}$$

multiply by LCD $3/5r$ $5/3r$ $15r$

5. Solve for r algebraically:

$$\frac{3}{5r} - \frac{1}{3r} = \frac{1}{15}$$

$$9 - 5 = r$$

$$4 = r$$

check ✓

x	-4
x^2	$-4x$
$-2x$	$+8$

$x^2 - 6x + 8$

6. Solve for all values of x : $\frac{2}{x-2} + \frac{x-4}{x-1} = \frac{2x+1}{x^2-3x+2}$

$$\frac{2}{x-2} + \frac{x-4}{x-1} = \frac{2x+1}{(x-2)(x-1)}$$

$$2(x-1) + (x-4)(x-2) = 2x+1$$

$$2x-2 + x^2-6x+8 = 2x+1$$

$$x^2-4x+6 = 2x+1$$

$$-2x-1 \quad -2x-1$$

$$x^2-6x+5=0$$

$$(x-5)(x-1)=0$$

$$x-5=0$$

$$+5 \quad +5$$

$$x=5$$

check ✓

$$x-1=0$$

$$+1 \quad +1$$

$$x=1$$

check ✓

7. Solve for x : $x = \sqrt{3x+16} - 2$

Isolate
square both sides
check

$$(x+2)^2 = 3x+16$$

$$x^2+4x+4 = 3x+16$$

$$-3x-16 \quad -3x-16$$

$$x^2+x-12=0$$

$$(x+4)(x-3)=0$$

$$x+4=0$$

$$-4 \quad -4$$

$$x=-4$$

check ✓

$$x-3=0$$

$$+3 \quad +3$$

$$x=3$$

check ✓

8. Solve for x and express the roots in simplest $a+bi$ form: $3x^2+3=5x$

$$3x^2-5x+3=0$$

$$a=3$$

$$b=-5$$

$$c=3$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{5 \pm \sqrt{(-5)^2-4(3)(3)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{-11}}{6}$$

$$x = \frac{5 \pm i\sqrt{11}}{6}$$

$$x = \frac{5}{6} \pm \frac{\sqrt{11}}{6}i$$

If possible, reduce the radical
 $\sqrt{11}$ doesn't reduce

UNIT 4: SYSTEMS OF EQUATIONS & COMPLEX NUMBERS

9. Determine the solution set of the system of equations:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & -2 \\ 1 & -2 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ 9 \\ 16 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

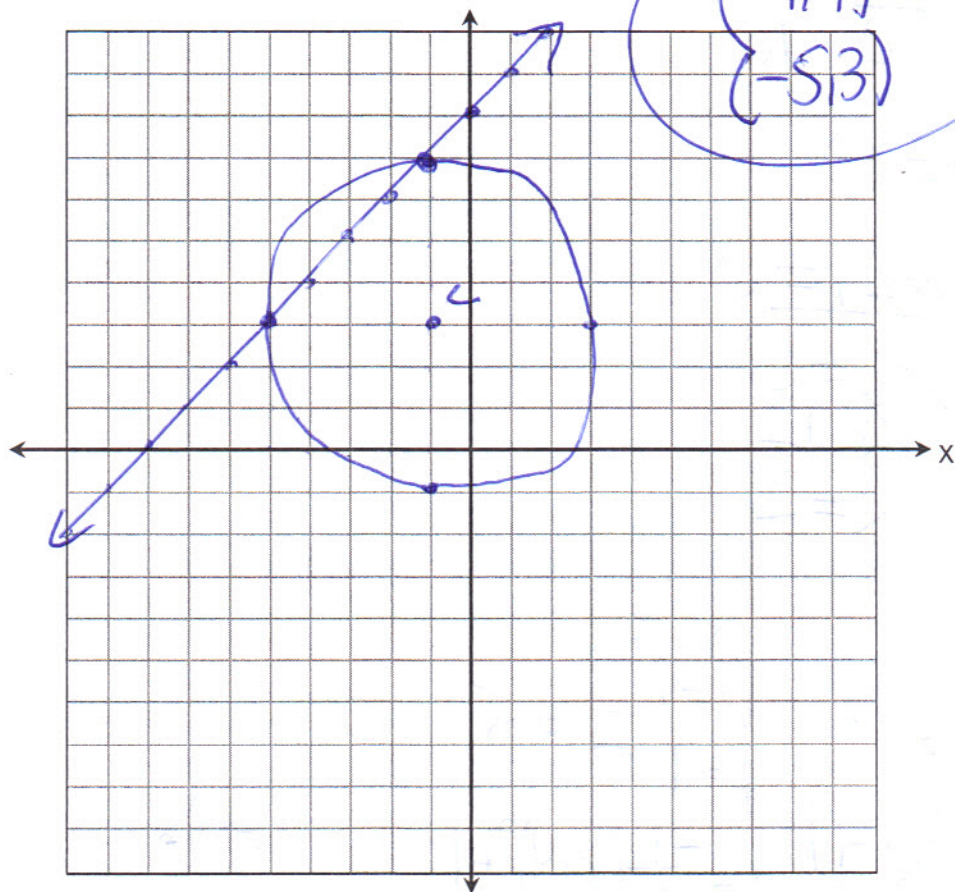
$$\begin{aligned} x + y + z &= 5 \\ 2x - y + z &= 9 \\ x - 2y + 3z &= 16 \end{aligned}$$

$$\begin{aligned} x &= 2 \\ y &= -1 \\ z &= 4 \end{aligned}$$

10. On the set of axes below, solve the following system of equations for x and y .

$$(x+1)^2 + (y-3)^2 = 16 \quad \text{Circle } (-1, 3), r=4$$

$$\begin{aligned} y - x &= 8 \\ \text{line} \\ y &= x + 8 \end{aligned}$$



11. What is the solution set of the following system of equations:

Type into calculator

$$y = (x+2)^2 - 4$$

$$y - 3x = 2$$

$$+3x \quad +3x \quad y = 3x + 2$$

x	y ₁	y ₂
-5	5	-13
-4	0	-10
-3	-3	-7
-2	-4	-4
-1	-3	-1
0	0	2
1	5	5

(1) (-2,1)

(2) (5,-4)

~~(3) (-2,-4) and (1,5)~~

(4) (-4,-4) and (5,5)

12. Solve the following system of equations for all values of x and y:

$$x^2 + (3x)^2 - 40 = 0$$

$$x^2 + y^2 - 40 = 0$$

$$y - 3x = 0$$

$$+3x \quad +3x$$

$$y = 3x$$

- Isolate a variable
- Sub one equation into the other

$$x^2 + 9x^2 - 40 = 0$$

$$\frac{10x^2 - 40 = 0}{10 \quad 10 \quad 10}$$

$$x^2 - 4 = 0$$

$$(x+2)(x-2) = 0$$

$$\begin{array}{l|l} x+2=0 & x-2=0 \\ -2 & +2 \\ \hline x=-2 & x=2 \end{array}$$

13. Write the equation of the parabola whose focus has the coordinates (-4,-4) and whose directrix has the equation y = 6.

$$(x-h)^2 = 4p(y-k)$$

$$\begin{aligned} p &= 5 \\ v &= \frac{4-10}{2} = -3 \\ h &= -4 \end{aligned}$$

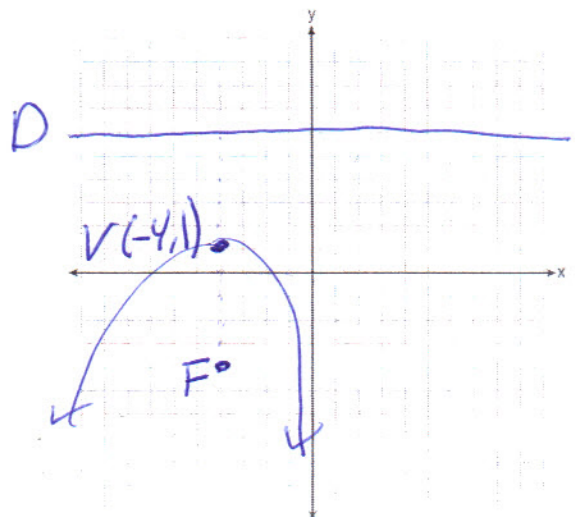
$$(x+4)^2 = 4(-3)(y-1)$$

$$\frac{(x+4)^2}{80} = \frac{80(y-1)}{80}$$

$$\frac{1}{80}(x+4)^2 = y-1$$

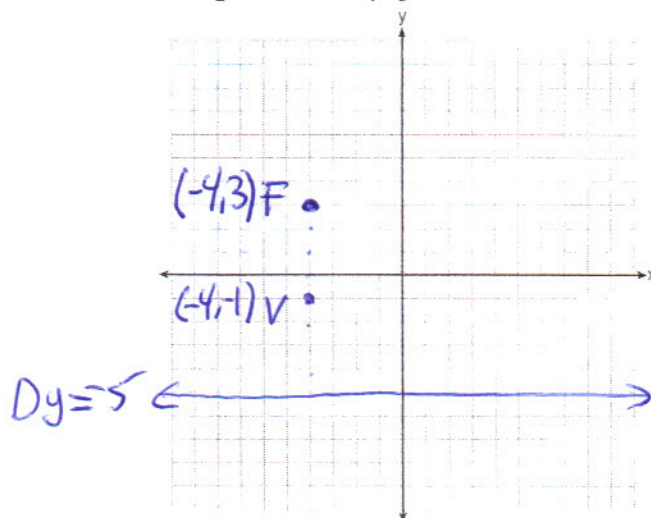
$$y = -\frac{1}{80}(x+4)^2 + 1$$

(negative because it opens downward)



14. The equation of a parabola $16(y+1) = (x+4)^2$ has a directrix represented by $y = -5$. Determine the coordinates of the focus of the parabola.

$(-4, 3)$



15. Express in simplest $a + bi$ form: $-3i^6(4x - 2i^{13}) - (x - 7i)$

	$3x$	$-i$
$2x$	$6x^2$	$-2xi$
$-4i$	$-12xi$	$+4i^2$

16. Multiply $(3x - i)(2x - 4i)$ and express in simplest $a + bi$ form.

$6x^2 - 14xi + 4i^2$
 $6x^2 - 14xi + 4(-1)$
 $6x^2 - 14xi - 4$

$6x^2 - 4 - 14xi$

17. Determine all solutions to the equation $x^4 + 13x^2 + 12 = 0$ and express the solutions in simplest form in terms of i .

$(x^2 + 12)(x^2 + 1) = 0$
 $x^2 + 12 = 0 \quad | \quad x^2 + 1 = 0$
 $-12 \quad -12 \quad | \quad 1 \quad -1$
 $\sqrt{x^2} = \sqrt{-12} \quad \sqrt{x^2} = \sqrt{-1}$
 $x = \pm\sqrt{-12} \quad x = \pm i$
 $x = \pm i\sqrt{12}$
 $\pm i\sqrt{4} \sqrt{3}$
 $x = \pm 2i\sqrt{3}$