Name: Schlansky

Common Core Algebra I Common Regents Questions!

Mr. Schlansky

Multiple Choice Strategy with Variables

If variables in the problems and answers:

10 STO → X, 15 STO → Y

See which matches up.

Type in original problem, ~~2nd Math (Test), =~~, type in each solution. ~~1 is equivalent, 0 is not equivalent.~~ Make sure to try all four choices.

1. Which expression represents $\frac{(2x^3)(8x^5)}{4x^6}$ in simplest form? 400

1) x^2 100
2) x^9 1,000,000,000
3) $4x^2$ 400
4) $4x^9$ 4,000,000,000

2. Factored, the expression $16x^2 - 25y^2$ is equivalent to −4025

1) $(4x-5y)(4x+5y)$ −4025
2) $(4x-5y)(4x-5y)$ 1225
3) $(8x-5y)(8x+5y)$ 775
4) $(8x-5y)(8x-5y)$ 25

3. Factored completely, the expression $2x^2+10x-12$ is equivalent to 288

1) $2(x-6)(x+1)$ 88
2) $2(x+6)(x-1)$ 288
3) $2(x+2)(x+3)$ 312
4) $2(x-2)(x-3)$ 112

4. The expression $9x^2-100$ is equivalent to 800

1) $(9x-10)(x+10)$ 1600
2) $(3x-10)(3x+10)$ 800
3) $(3x-100)(3x-1)$ −2030
4) $(9x-100)(x+1)$ −110

5. What is $\frac{6}{5x}-\frac{2}{3x}$ in simplest form? $\frac{4}{75}$

1) $\frac{8}{15x^2}$ $\frac{2}{375}$
2) $\frac{8}{15x}$ $\frac{4}{75}$
3) $\frac{4}{15x}$ $\frac{2}{75}$
4) $\frac{4}{2x}$ $\frac{1}{5}$

6. Which expression represents $\frac{12x^3-6x^2+2x}{2x}$ in simplest form? 571

1) $6x^2-3x$ 570
2) $10x^2-4x$ 960
3) $6x^2-3x+1$ 571
4) $10x^2-4x+1$ 961

7. The sum of $4x^3+6x^2+2x-3$ + and $3x^3+3x^2-5x-5$ is 7862

1) $7x^3+3x^2-3x-8$ 7262
2) $7x^3+3x^2+7x+2$ 7372
3) $7x^3+9x^2-3x-8$ 7862
4) $7x^6+9x^4-3x^2-8$ 7089692

8. What is the sum of $\frac{-x+7}{2x+4}$ + and $\frac{2x+5}{2x+4}$? $\frac{11}{12}$

1) $\frac{x+12}{2x+4}$ $\frac{11}{12}$

2) $\frac{3x+12}{2x+4}$ $\frac{7}{4}$

3) $\frac{x+12}{4x+8}$ $\frac{11}{24}$

4) $\frac{3x+12}{4x+8}$ $\frac{7}{8}$

9. Factored completely, the expression $3x^2 - 3x - 18$ is equivalent to 252

1) $3(x^2 - x - 6)$ 252

2) $3(x-3)(x+2)$ 252
The only choice factored completely

3) $(3x-9)(x+2)$ 252

4) $(3x+6)(x-3)$ 252

10. Four expressions are shown below.

I $2(2x^2 - 2x - 60)$ 240

II $4(x^2 - x - 30)$ 240

III $4(x+6)(x-5)$ 320

IV $4x(x-1) - 120$ 240

The expression $4x^2 - 4x - 120$ 240 is equivalent to

1) I and II, only

2) II and IV, only

3) I, II, and IV

4) II, III, and IV

11. Which trinomial is equivalent to $3(x-2)^2 - 2(x-1)$? 174

1) $3x^2 - 2x - 10$ 270

2) $3x^2 - 2x - 14$ 266

3) $3x^2 - 14x + 10$ 170

4) $3x^2 - 14x + 14$ 174

12. When factored completely, $x^3 - 13x^2 - 30x$ is -600

1) $x(x+3)(x-10)$ 0

2) $x(x-3)(x-10)$ 0

3) $x(x+2)(x-15)$ -600

4) $x(x-2)(x+15)$ 2000

13. The expression $x^4 - 16$ 9984 is equivalent to

1) $(x^2+8)(x^2-8)$ 9936

2) $(x^2-8)(x^2-8)$ 8464

3) $(x^2+4)(x^2-4)$ 9984

4) $(x^2-4)(x^2-4)$ 9216

14. The expression $3(x^2 - 1) - (x^2 - 7x + 10)$ is equivalent to 257

1) $2x^2 - 7x + 7$ 137

2) $2x^2 + 7x - 13$ 257

3) $2x^2 - 7x + 9$ 139

4) $2x^2 + 7x - 11$ 259

Multiple Choice Strategy with Equations

Store each potential answer (_____ STO → X)

Type in ~~equation~~ left hand side. Type in right hand side. See what matches up.

~~1 is correct, 0 is incorrect~~

*Check all potential answers

1. Which value of p is the solution of $5p-1=2p+20$?

1) $\frac{19}{7}$ $\frac{88}{7} \neq \frac{178}{7}$

2) $\frac{19}{3}$ $\frac{92}{3} \neq \frac{98}{3}$

3) 3 $14 \neq 26$

4) 7 $34=34$

2. What is the value of x in the equation $2(x-4)=4(2x+1)$?

1) -2 $-12=-12$

2) 2 $-4 \neq 20$

3) $-\frac{1}{2}$ $-9 \neq 0$

4) $\frac{1}{2}$ $-7 \neq 8$

3. Solve for x: $15x-3(3x+4)=6$

(1) 1 $-6 \neq 6$ (3) 3 $6=6$

(2) $-\frac{1}{2}$ $-15 \neq 6$ (4) $\frac{1}{3}$ $-10 \neq 6$

4. If $3(x-2)=2x+6$, the value of x is

$-6 \neq 6$ (1) 0 (3) 12 $30=30$

$9 \neq 16$ (2) 5 (4) 20 $54 \neq 46$

5. Which value of x is a solution of $\frac{5}{x}=\frac{x+13}{6}$?

1) -2 $-\frac{5}{2} \neq \frac{4}{6}$

2) -3 $-\frac{5}{3} \neq \frac{5}{3}$

3) -10 $-\frac{1}{2} \neq \frac{1}{2}$

4) -15 $-\frac{1}{3}=-\frac{1}{3}$

6. What is the solution of $\frac{k+4}{2}=\frac{k+9}{3}$?

1) 1 $\frac{5}{2} \neq \frac{10}{3}$

2) 5 $\frac{9}{2} \neq \frac{14}{3}$

3) 6 $5=5$

4) 14 $9 \neq \frac{23}{3}$

7. What is the value of x in the equation $\frac{2}{x} - 3 = \frac{26}{x}$?

1) -8 $\frac{-13}{4} = \frac{-13}{4}$

2) $-\frac{1}{8}$ $-19 \neq -208$

3) $\frac{1}{8}$ $13 \neq 208$

4) 8 $\frac{-11}{4} \neq \frac{13}{4}$

8. Which value of x is the solution of the equation $\frac{2x}{3} + \frac{x}{6} = 5$?

1) 6 $5 = 5$

2) 10 $\frac{25}{3} \neq 5$

3) 15 $\frac{25}{2} \neq 5$

4) 30 $25 \neq 5$

9. Solve for x: $\frac{3}{5}(x+2) = x - 4$

1) 8 $6 \neq 4$

2) 13 $9 = 9$

3) 15 $\frac{51}{5} \neq 11$

4) 23 $15 \neq 19$

10. Which value of x is the solution of $\frac{x}{3} + \frac{x+1}{2} = x$?

1) 1 $\frac{4}{3} \neq 1$

2) -1 $-\frac{1}{3} \neq -1$

3) 3 $3 = 3$

4) -3 $-2 \neq -3$

11. Which value of x is the solution of $\frac{2x-3}{x-4} = \frac{2}{3}$?

1) $-\frac{1}{4}$ $\frac{14}{17} \neq \frac{2}{3}$

2) $\frac{1}{4}$ $\frac{2}{3} = \frac{2}{3}$

3) -4 $\frac{11}{8} \neq \frac{2}{3}$

4) 4 ERROR

12. Which value of x satisfies the equation $\frac{7}{3}\left(x + \frac{9}{28}\right) = 20$?

1) 8.25 $20 = 20$

2) 8.89 $21.493 \neq 20$

3) 19.25 $45.\overline{6} \neq 20$

4) 44.92 $105.56\overline{3} \neq 20$

Equivalent Expressions

Use multiple choice strategy! See if the two expressions are equal.

$\rightarrow 6.597\ldots E12$

1. A computer application generates a sequence of musical notes using the function $f(n) = 6(16)^n$, where n is the number of the note in the sequence and $f(n)$ is the note frequency in hertz. Which function will generate the same note sequence as $f(n)$?

1) $g(n) = 12(2)^{4n}$ 1.31...E13
3) $p(n) = 12(4)^{2n}$ 1.31...E13
2) $h(n) = 6(2)^{4n}$ 6.597.. E12
4) $k(n) = 6(8)^{2n}$ 6.917.. E18

431

2. The function $f(x) = 3x^2 + 12x + 11$ can be written in vertex form as

1) $f(x) = (3x+6)^2 - 25$ 1271
3) $f(x) = 3(x+2)^2 - 1$ 431
2) $f(x) = 3(x+6)^2 - 25$ 743
4) $f(x) = 3(x+2)^2 + 7$ 439

3. Mario's \$15,000 car depreciates in value at a rate of 19% per year. The value, V, after t years can be modeled by the function $V = 15{,}000(0.81)^t$. Which function is equivalent to the original function? 1823...

1) $V = 15{,}000(0.9)^{9t}$ 1.14...
3) $V = 15{,}000(0.9)^{\frac{t}{9}}$ 13342..
2) $V = 15{,}000(0.9)^{2t}$ 1823...
4) $V = 15{,}000(0.9)^{\frac{t}{2}}$ 8857..

$\rightarrow$ 7857...

4. Nora inherited a savings account that was started by her grandmother 25 years ago. This scenario is modeled by the function $A(t) = 5000(1.013)^{t+25}$, where $A(t)$ represents the value of the account, in dollars, t years after the inheritance. Which function below is equivalent to $A(t)$?

1) $A(t) = 5000[(1.013^t)]^{25}$ 126279...
3) $A(t) = (5000)^t(1.013)^{25}$ 1.34.. E37
2) $A(t) = 5000[(1.013)^t + (1.013)^{25}]$ 12595...
4) $A(t) = 5000(1.013)^t(1.013)^{25}$ 7857...

3.2 E14

5. The number of bacteria grown in a lab can be modeled by $P(t) = 300 \bullet 2^{4t}$, where t is the number of hours. Which expression is equivalent to $P(t)$?

1) $300 \bullet 8^t$ 3.2... E11
3) $300^t \bullet 2^4$ 9.44..E25
2) $300 \bullet 16^t$ 3.2...E14
4) $300^{2t} \bullet 2^{2t}$ 3.6..E55

9543...

6. The growth of a certain organism can be modeled by $C(t) = 10(1.029)^{24t}$, where $C(t)$ is the total number of cells after t hours. Which function is approximately equivalent to $C(t)$?

1) $C(t) = 240(.083)^{24t}$ 0
3) $C(t) = 10(1.986)^t$ 9545...
2) $C(t) = 10(.083)^t$ 1.55..E-10
4) $C(t) = 240(1.986)^{\frac{t}{24}}$ 319...

Evaluating Expressions

Substitute the value in for x.

store

$f(8) = \frac{1}{2}(8)^2 - (\frac{1}{4}(8)+3) = 27$

1. If $f(x) = \frac{1}{2}x^2 - \left(\frac{1}{4}x + 3\right)$, what is the value of $f(8)$?

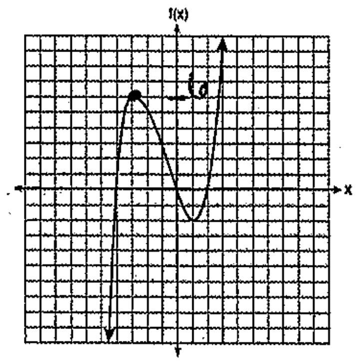
1) 11
2) 17
3) 27
4) 33

2. If $k(x) = 2x^2 - 3\sqrt{x}$, then $k(9)$ is $k(9) = 2(9)^2 - 3\sqrt{9} = 153$

1) 315
2) 307
3) 159
4) 153

3. The graph of $f(x)$ is shown below.
What is the value of $f(-3)$?

1) 6
2) 2
3) −2
4) −4



4. Lynn, Jude, and Anne were given the function $f(x) = -2x^2 + 32$, and they were asked to find $f(3)$. Lynn's answer was 14, Jude's answer was 4, and Anne's answer was ±4. Who is correct?

$f(3) = -2(3)^2 + 32 = 14$

1) Lynn, only
2) Jude, only
3) Anne, only
4) Both Lynn and Jude

5. Faith wants to use the formula $C(f) = \frac{5}{9}(f - 32)$ to convert degrees Fahrenheit, f, to degrees Celsius, $C(f)$. If Faith calculated $C(68)$, what would her result be?

$C(68) = \frac{5}{9}(68-32) = 20$

1) 20° Celsius
2) 20° Fahrenheit
3) 154° Celsius
4) 154° Fahrenheit

6. If $f(n) = (n-1)^2 + 3n$, which statement is true?

1) $f(3) = -2$ $\quad f(3) = (3-1)^2 + 3(3) = 13$
2) $f(-2) = 3$ $\quad f(-2) = (-2-1)^2 + 3(-2) = 3$
3) $f(-2) = -15$ $\quad f(-2) = (-2-1)^2 + 3(-2) = 3$
4) $f(-15) = -2$ $\quad f(-15) = (-15-1)^2 + 3(-15) = 211$

7. Which value of x results in equal outputs for $j(x) = 3x - 2$ and $b(x) = |x + 2|$?

1) -2 $j(-2)=3(-2)-2=-8$ $b(-2)=|-2+2|=0$

2) 2 $j(2)=3(2)-2=4$ $b(2)=|2+2|=4$

3) $\frac{2}{3}$ $j(\frac{2}{3})=3(\frac{2}{3})-2=0$ $b(\frac{2}{3})=|\frac{2}{3}+2|=\frac{8}{3}$

4) 4 $j(4)=3(4)-2=10$ $b(4)=|4+2|=6$

8. As x increases beyond 25, which function will have the largest value?

1) $f(x) = 1.5^x$ $1.5^{26}=37876$...
2) $g(x) = 1.5x + 3$ $1.5(26)+3=42$
3) $h(x) = 1.5x^2$ $1.5(26)^2=1014$
4) $k(x) = 1.5x^3 + 1.5x^2$ $1.5(26)^3+1.5(26)^2=27378$

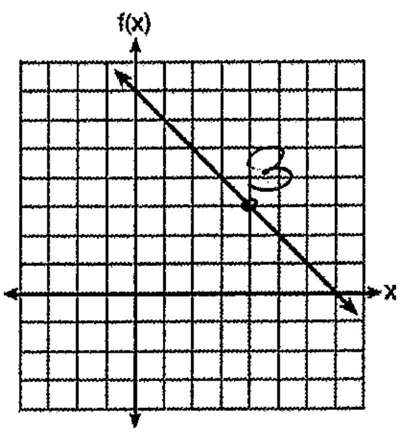
9. What is the largest integer, x, for which the value of $f(x) = 5x^4 + 30x^2 + 9$ will be greater than the value of $g(x) = 3^x$?

1) 7 $5(7)^4+30(7)^2+9=13484$ $3^7=2187$ ✓
2) 8 $5(8)^4+30(8)^2+9=22409$ $3^8=6561$ ✓
3) 9 $5(9)^4+30(9)^2+9=35244$ $3^9=19683$ ✓
4) 10 $5(10)^4+30(10)^2+9=53009$ $3^{10}=59049$ X

10. The function $g(x)$ is defined as $g(x) = -2x^2 + 3x$. The value of $g(-3)$ is

1) -27 $g(-3)=-2(-3)^2+3(-3)=-27$
2) -9
3) 27
4) 45

11. The functions $f(x)$, $q(x)$, and $p(x)$ are shown below.



$q(x) = (x - 1)^2 - 6$

~~$q(3)=(3-1)^2-6=-2$~~

$q(4)=(4-1)^2-6=3$

x	p(x)
2	5
3	4
4	3
5	4
6	5

When the input is 4, which functions have the same output value? 3

1) $f(x)$ and $q(x)$, only
2) $f(x)$ and $p(x)$, only
3) $q(x)$ and $p(x)$, only
4) $f(x)$, $q(x)$, and $p(x)$

Number Properties

$2(4+3)=2\bullet 4+2\bullet 3$	Distributive Property
$4\bullet 6=6\bullet 4$	Commutative Property of Multiplication
$2+7=7+2$	Commutative Property of Addition
$5+(2+3)=(5+2)+3$	Associative Property of Addition
$5\bullet(4\bullet 3)=(5\bullet 4)\bullet 3$	Associative Property of Multiplication
$4+0=4$	Additive Identity
$7\bullet 1=7$	Multiplicative Identity
$5+-5=0$	Additive Inverse
$4\bullet\frac{1}{4}=1$	Multiplicative Inverse
$3\bullet 0=0$	Multiplication Property of Zero
$4(3x^2+2)-9=8x^2+7\rightarrow 4(3x^2+2)=8x^2+16$	Addition Property of Equality
$2x+8=4x+4\rightarrow 8=2x+4$	Subtraction Property of Equality
$2x^2=8x+10\rightarrow x^2=4x+5$	Division Property of Equality
$\frac{2x+5}{3}=5\rightarrow 2x+5=15$	Multiplication Property of Equality

ASSociative property has two sets of parenthesis
Commutative property has numbers commute (move)
Identity is where you start and end with the same thing
Inverse is when you end up with the identity element (Addition 0, Multiplication 1)

1. Which property is illustrated by the equation $ax+ay=a(x+y)$?
1) associative
2) commutative
3) distributive
4) identity

2. The statement $2+0=2$ is an example of the use of which property of real numbers?
1) associative
2) additive identity
3) additive inverse
4) distributive

3. Which equation illustrates the associative property?
1) $x+y+z=x+y+z$
2) $x(y+z)=xy+xz$
3) $x+y+z=z+y+x$
4) $(x+y)+z=x+(y+z)$ ( )( )

4. If M and A represent integers, $M+A=A+M$ is an example of which property?
(1) commutative
(2) associative
(3) distributive
(4) closure

5. Which property is illustrated by the equation $\frac{3}{2}x+0=\frac{3}{2}x$?
1) commutative property of addition
2) additive inverse property
3) distributive property
4) additive identity property

6. Which equation illustrates the distributive property?

(1) $5(a+b)=5a+5b$ (3) $a+(b+c)=(a+b)+c$

(2) $a+b=b+a$ (4) $a+0=a$

7. Which equation illustrates the multiplicative inverse property?

1) $a\cdot 1=a$ 3) $a\left(\frac{1}{a}\right)=1$

2) $a\cdot 0=0$ 4) $(-a)(-a)=a^2$

8. A teacher asked the class to solve the equation $3(x+2)=21$. Robert wrote $3x+6=21$ as his first step. Which property did he use?

1) associative property
2) commutative property
3) distributive property
4) zero property of addition

9. Britney is solving a quadratic equation. Her first step is shown below.

Commutative

Problem: $3x^2-8-10x=3(2x+3)$ distributive

Step 1: $3x^2-10x-8=6x+9$

Which two properties did Britney use to get to step 1?

I. addition property of equality
II. commutative property of addition
III. multiplication property of equality
IV. distributive property of multiplication over addition

1) I and III
2) I and IV
3) II and III
4) II and IV

10. A part of Jennifer's work to solve the equation $2(6x^2-3)=11x^2-x$ is shown below.

Given: $2(6x^2-3)=11x^2-x$

Step 1: $12x^2-6=11x^2-x$

Which property justifies her first step?

1) identity property of multiplication
2) multiplication property of equality
3) commutative property of multiplication
4) distributive property of multiplication over subtraction

11. When solving the equation $12x^2-7x=6-2(x^2-1)$, Evan wrote $12x^2-7x=6-2x^2+2$ as his first step. Which property justifies this step?

1) subtraction property of equality
2) multiplication property of equality
3) associative property of multiplication
4) distributive property of multiplication over subtraction

Rational vs. Irrational

Rational	Irrational
Ends of continues with a pattern	Never ends with no pattern
Fraction	π
Perfect Square Radicals	Non Perfect Square Radicals

Addition/Subtraction: If at least one number is irrational, the result is irrational.
Multiplication/Division: If one number is irrational, the result is irrational. If both are irrational, the result can either be rational or irrational.
***If an irrational number is involved, the result is almost always irrational.**

1. Which statement is *not* always true?
1) The product of two irrational numbers is irrational. I·I = R or I X
2) The product of two rational numbers is rational. R·R=R ✓
3) The sum of two rational numbers is rational. R+R=R ✓
4) The sum of a rational number and an irrational number is irrational. R+I=I ✓

2. Which statement is *not* always true?
1) The sum of two rational numbers is rational. R+R=R ✓
2) The product of two irrational numbers is rational. I·I = R or I X
3) The sum of a rational number and an irrational number is irrational. R+I=I ✓
4) The product of a nonzero rational number and an irrational number is irrational. R·I=I ✓

3. Which expression results in a rational number? PS PS
1) PS $\sqrt{121} - \sqrt{21}$ NPS R−I=I
2) $\sqrt{25} \cdot \sqrt{50}$ PS NPS R·I=I
3) $\sqrt{36} \div \sqrt{225}$ R÷R=R
4) $3\sqrt{5} + 2\sqrt{5}$ NPS NPS I+I=I

R · I=I

4. The product of $\sqrt{576}$ and $\sqrt{684}$ is
1) irrational because both factors are irrational
2) rational because both factors are rational
3) irrational because one factor is irrational
4) rational because one factor is rational

5. Given the following expressions: I I I·I=R or I

I. $-\frac{5}{8}+\frac{3}{5}$ R+R=R III. $\left(\sqrt{5}\right)\cdot\left(\sqrt{5}\right)$ $\sqrt{5}\cdot\sqrt{5}=5$ R

II. $\frac{1}{2}+\sqrt{2}$ NPS R+I=I IV. $3\cdot\left(\sqrt{49}\right)$ PS R·R=R

Which expression(s) result in an irrational number?
1) II, only
2) III, only
3) I, III, IV
4) II, III, IV

6. Which expression results in a rational number? $L=\sqrt{2}$ I NPS

1) $L+M$ ~~3)~~ $N+P$ $M=3\sqrt{3}$ I NPS

2) $M+N$ 4) $P+L$ $N=\sqrt{16}$ R PS

$P=\sqrt{9}$ R PS

7. State whether $7-\sqrt{2}$ (NPS) is rational or irrational. Explain your answer.

R − I = I A rational minus an irrational is irrational.

8. Determine if the product of $3\sqrt{2}$ (I) and $8\sqrt{18}$ (I) is rational or irrational. Explain your answer. (I · I = R or I)

$3\sqrt{2}\cdot 8\sqrt{18} = 24(6) = R$

Rational, ~~b/c~~ the product of two irrational numbers could be rational or irrational. In this case, it was rational.

9. Jakob is working on his math homework. He decides that the sum of the expression $\frac{1}{3}+\frac{6\sqrt{5}}{7}$ must be rational because it is a fraction. Is Jakob correct? Explain.

R + I = I No, a rational plus an irrational is irrational

10. Is the sum of $3\sqrt{2}$ and $4\sqrt{2}$ rational or irrational? Explain your answer.

I + I = I

Irrational. Irrational plus irrational is irrational.

11. Ms. Fox asked her class "Is the sum of 4.2 (R) and $\sqrt{2}$ (I NPS) rational or irrational?" Patrick answered that the sum would be irrational. State whether Patrick is correct or incorrect. Justify your reasoning.

R + I = I

Yes, rational plus irrational is irrational.

12. A teacher wrote the following set of numbers on the board:
Explain why $a+b$ is irrational, but $b+c$ is rational.

$a=\sqrt{20}$ (NPS, I) $b=2.5$ (R) $c=\sqrt{225}$ (PS, R)

a+b: I+R = I. Irrational + rational is irrational.

b+C: R+R=R. Rational + rational is rational.

$\sqrt{225}$ is rational b/c it is a perfect square.

13. Is the product of $\sqrt{16}$ (PS, R) and $\frac{4}{7}$ (R) rational or irrational? Explain your reasoning.

R·R=R

rational times a rational is rational.

Polynomial Standard Form

A polynomial in standard form has the term with the highest exponent first and is followed by terms in decreasing exponential order.
The number in front of the term with the highest exponent is called the leading coefficient

highest exponent   # in front of highest exponent

1. An expression of the fifth degree is written with a leading coefficient of seven and a constant of six. Which expression is correctly written for these conditions?

1) ~~$6x^5 + x^4 + 7$~~
2) ~~$7x^6 - 6x^4 + 5$~~
3) ~~$6x^7 - x^5 + 5$~~
4) $7x^5 + 2x^2 + 6$

2. Mrs. Allard asked her students to identify which of the polynomials below are in standard form and explain why.

I. $15x^4 - 6x + 3x^2 - 1$ ✗
II. $12x^3 + 8x + 4$ ✓
III. $2x^5 + 8x^2 + 10x$ ✓

Which student's response is correct?

1) Tyler said I and II because the coefficients are decreasing.
2) Susan said only II because all the numbers are decreasing.
3) Fred said II and III because the exponents are decreasing.
4) Alyssa said II and III because they each have three terms.

3. When multiplying polynomials for a math assignment, Pat found the product to be $-4x + 8x^2 - 2x^3 + 5$. He then had to state the leading coefficient of this polynomial. Pat wrote down -4. Do you agree with Pat's answer? Explain your reasoning.

No. The leading coefficient is the coefficient of the term with the highest exponent.

4. Write the following polynomials in standard form, state their degree and leading coefficient.

a) $9x^3 - x^4 + 1 + 2x^6$

$2x^6 - x^4 + 9x^3 + 1$

Degree: 6
Leading coefficient: 2

b) $5y + 4 - 3y^5$

$-3y^5 + 5y + 4$

Degree: 5
Leading coefficient: −3

Operations with Polynomials

Adding: Combine like terms

Subtracting: Distribute the negative and then combine like terms (Keep Change Change)

***Subtracting from: from comes first!**

Multiplying: Box Method (Multiply to determine what's in the boxes, add to combine like terms)

Dividing: Divide every term in the numerator by the denominator

*Use Multiple Choice Strategy if Multiple Choice

1. If $A = 3x^2 + 5x - 6$ and $B = -2x^2 - 6x + 7$, then $A - B$ equals

1) $-5x^2 - 11x + 13$
2) $5x^2 + 11x - 13$
3) $-5x^2 - x + 1$
4) $5x^2 - x + 1$

$(3x^2+5x-6)-(-2x^2-6x+7)$

$$\begin{array}{r} 3x^2+5x-6 \\ +\ 2x^2+6x-7 \\ \hline 5x^2+11x-13 \end{array}$$

2. What is the result when $6x^2 - 13x + 12$ is subtracted from $-3x^2 + 6x + 7$?

1) $3x^2 - 7x + 19$
2) $9x^2 - 19x + 5$
3) $9x^2 - 7x + 19$
4) $-9x^2 + 19x - 5$

$(-3x^2+6x+7)-(6x^2-13x+12)$

$$\begin{array}{r} -3x^2+6x+7 \\ +\ -6x^2+13x-12 \\ \hline -9x^2+19x-5 \end{array}$$

3. What is the result when $4x^2 - 17x + 36$ is subtracted from $2x^2 - 5x + 25$?

1) $6x^2 - 22x + 61$
2) $2x^2 - 12x + 11$
3) $-2x^2 - 22x + 61$
4) $-2x^2 + 12x - 11$

$(2x^2-5x+25)-(4x^2-17x+36)$

$$\begin{array}{r} 2x^2-5x+25 \\ +\ -4x^2+17x-36 \\ \hline -2x^2+12x-11 \end{array}$$

4. Which expression is equivalent to $2(3g - 4) - 1(8g + 3)$?

1) $-2g - 1$
2) $-2g - 5$
3) $-2g - 7$
4) $-2g - 11$

$6g-8-8g-3$ $\quad -2g-11$

5. What is the product of $2x + 3$ and $4x^2 - 5x + 6$?

1) $8x^3 - 2x^2 + 3x + 18$
2) $8x^3 - 2x^2 - 3x + 18$
3) $8x^3 + 2x^2 - 3x + 18$
4) $8x^3 + 2x^2 + 3x + 18$

$(2x+3)(4x^2-5x+6)$

	$4x^2$	$-5x$	$+6$
$2x$	$8x^3$	$-10x^2$	$+12x$
$+3$	$+12x^2$	$-15x$	$+18$

$8x^3+2x^2-3x+18$

6. The expression $3(x^2 - 1) - 1(x^2 - 7x + 10)$ is equivalent to

1) $2x^2 - 7x + 7$
2) $2x^2 + 7x - 13$
3) $2x^2 - 7x + 9$
4) $2x^2 + 7x - 11$

$3x^2-3-x^2+7x-10$

$2x^2+7x-13$

7. Express in simplest form: $(3x^2+4x-8)-(-2x^2+4x+2)$

$$\begin{array}{r} 3x^2+4x-8 \\ +2x^2-4x-2 \\ \hline 5x^2-10 \end{array}$$

8. Express the product of $2x^2+7x-10$ and $x+5$ in standard form.

	$2x^2$	$+7x$	-10
x	$2x^3$	$+7x^2$	$-10x$
$+5$	$+10x^2$	$+35x$	-50

$2x^3+17x^2+25x-50$

9. Multiply $(2x^2+3x-2)(x-2)$

	$2x^2$	$+3x$	-2
x	$2x^3$	$+3x^2$	$-2x$
-2	$-4x^2$	$-6x$	$+4$

$2x^3-x^2-8x+4$

10. Write the expression $5x+4x^2(2x+7)-6x^2-9x$ as a polynomial in standard form.

$5x+8x^3+28x^2-6x^2-9x$

$8x^3+22x^2-4x$

11. Given that $f(x)=2x+1$, find $g(x)$ if $g(x)=2[f(x)]^2-1$.

$2(2x+1)^2-1$

$2(4x^2+4x+1)-1$

$8x^2+8x+2-1$

$8x^2+8x+1$

	$2x$	$+1$
$2x$	$4x^2$	$+2x$
$+1$	$+2x$	$+1$

$4x^2+4x+1$

Solving Linear Equations and Inequalities

1) Get rid of fractions (Multiply by the LCD)
2) Get rid of parenthesis (Distribute)
3) Combine like terms on each side
4) Bring all variables to one side
5) Isolate variable (add/subtract first, divide last)

*When dividing/multiplying by a negative in an inequality, switch the inequality!
*Be careful which direction the inequality sign is facing when you write your solution

1. What is the value of x in the equation $\left(\frac{x-2}{3}\right)+\left(\frac{1}{6}\right)=\left(\frac{5}{6}\right)$

1) 4
2) 6
3) 8
4) 11

$2(x-2)+1=5$

$2x-4+1=5$

$2x-3=5$

$+3 \quad +3$

$\frac{2x}{2}=\frac{8}{2}$

$x=4$

2. The solution to $-2(1-4x)=3x+8$ is

1) $\frac{6}{11}$
2) 2
3) $-\frac{10}{7}$
4) -2

$-2+8x=3x+8$

$-3x \quad -3x$

$-2+5x=8$

$+2 \quad +2$

$\frac{5x}{5}=\frac{10}{5}$

$x=2$

3. An equation is given below.

$4(x-7)=0.3(x+2)+2.11$

The solution to the equation is

1) 8.3
2) 8.7
3) 3
4) -3

$4x-28=.3x+.6+2.11$

$4x-28=.3x+2.71$

$-.3x \quad -.3x$

$3.7x-28=2.71$

$+28 \quad +28$

$\frac{3.7x}{3.7}=\frac{30.71}{3.7}$

$x=8.3$

4. Which value of x satisfies the equation $\frac{5}{6}\left(\frac{3}{8}-x\right)=16$?

1) -19.575
2) -18.825
3) -16.3125
4) -15.6875

$48\left(\frac{5}{16}\right)-\left(\frac{5}{6}x\right)=(16)48$

$15-40x=768$

$-15 \quad -15$

$\frac{-40x}{-40}=\frac{753}{-40}$

$x=-18.825$

5. What is the solution to the equation $\frac{3}{5}\left(x+\frac{4}{3}\right)=1.04$?

1) $3.0\overline{6}$
~~2)~~ 0.4
3) $-0.4\overline{8}$
4) $-0.709\overline{3}$

$5\left(\frac{3}{5}x\right)+5\left(\frac{4}{5}\right)=(1.04)5$

$3x+4=5.2$

$-4 \quad -4$

$\frac{3x}{3}=\frac{1.2}{3}$

$x=.4$

6. What is the solution to the inequality $2+\frac{4}{9}x \geq 4+x$?

~~1)~~ $x \leq -\frac{18}{5}$
2) $x \geq -\frac{18}{5}$
3) $x \leq \frac{54}{5}$
4) $x \geq \frac{54}{5}$

$9(2)+9\left(\frac{4}{9}x\right) \geq 9(4)+9(x)$

$18+4x \geq 36+9x$

$-4x \quad -4x$

$18 \geq 36+5x$

$-36 \quad -36$

$\frac{-18}{5} \geq \frac{5x}{5}$

$\frac{-18}{5} \geq x$

$x \leq -\frac{18}{5}$

7. The inequality $7-\frac{2}{3}x < x-8$ is equivalent to

~~1)~~ $x>9$
2) $x>-\frac{3}{5}$
3) $x<9$
4) $x<-\frac{3}{5}$

$3(7)-3\left(\frac{2}{3}x\right)<3(x)-3(8)$

$21-2x<3x-24$

$+2x \quad +2x$

$21<5x-24$

$+24 \quad +24$

$\frac{45}{5}<\frac{5x}{5}$

$9<x$

$x>9$

8. When $3x+2 \leq 5(x-4)$ is solved for x, the solution is

1) $x \leq 3$
2) $x \geq 3$
3) $x \leq -11$
~~4)~~ $x \geq 11$

$3x+2 \leq 5x-20$

$-3x \quad -3x$

$2 \leq 2x-20$

$+20 \quad +20$

$\frac{22}{2} \leq \frac{2x}{2}$

$11 \leq x$

$x \geq 11$

9. What is the solution to $2h+8 > 3h-6$

~~1)~~ $h<14$
2) $h<\frac{14}{5}$
3) $h>14$
4) $h>\frac{14}{5}$

$-2h \quad -2h$

$8>h-6$

$+6 \quad +6$

$14>h$

$h<14$

10. The solution to $4p + 2 < 2(p + 5)$ is

1) $p > -6$
2) $p < -6$
3) $p > 4$
4) $p < 4$

$4p+2 < 2p+10$
$-2p \quad -2p$
$2p+2 < 10$
$-2 \quad -2$
$2p < 8$
$\frac{2p}{2} < \frac{8}{2}$
$p < 4$

11. Which value would be a solution for x in the inequality $47 - 4x < 7$?

1) -13
2) -10
3) 10
4) 11

$-47 \quad -47$
$\frac{-4x}{-4} < \frac{-40}{-4}$
$x > 10$

switch inequality when dividing by a negative.

12. Given the set $\{x \mid -2 \le x \le 2, \text{where } x \text{ is an integer}\}$, what is the solution of $-2(x - 5) < 10$?

1) 0, 1, 2
2) 1, 2
3) −2, −1, 0
4) −2, −1

−2
−1
0
1
2

$-2x+10 < 10$
$-10 \quad -10$
$\frac{-2x}{-2} < \frac{0}{-2}$
$x > 0$

13. Solve the equation below algebraically for the exact value of x.

$6 - \frac{2}{3}(x + 5) = 4x$

$18 - 2(x+5) = 12x$
$18 - 2x - 10 = 12x$
$-2x + 8 = 12x$
$+2x \quad +2x$
$\frac{8}{14} = \frac{14x}{14}$
$x = \frac{8}{14}$
$x = \frac{4}{7}$

14. Solve the inequality below:

$1.8 - 0.4y \ge 2.2 - 2y$

$+2y \quad +2y$
$1.8 + 1.6y \ge 2.2$
$-1.8 \quad -1.8$
$\frac{1.6y}{1.6} \ge \frac{.4}{1.6}$
$y \ge .25$

15. Solve algebraically for x: $3600 + 1.02x < 2000 + 1.04x$

$-1.02x \qquad -1.02x$

$3600 < 2000 + .02x$

$-2000 \qquad -2000$

$\frac{1600}{.02} < \frac{.02x}{.02}$

$80{,}000 < x$

$x > 80{,}000$

16. Solve the inequality below to determine and state the smallest possible value for x in the solution set.
$3(x + 3) \le 5x - 3$

$3x + 9 \le 5x - 3$

$-3x \qquad -3x$

$9 \le 2x - 3$

$+3 \qquad +3$

$\frac{12}{2} \le \frac{2x}{2}$

$6 \le x$

$x \ge 6$

6 is the smallest possible value of x

17. Determine the smallest integer that makes $-3x + 7 - 5x < 15$ true.

$-8x + 7 < 15$

$-7 \qquad -7$

$\frac{-8x}{-8} < \frac{8}{-8}$

$x > -1$ switch the inequality

0 is the smallest integer value of x

18. Given $2x + ax - 7 > -12$, determine the largest integer value of a when $x = -1$.

$2(-1) + a(-1) - 7 > -12$

$-2 - a - 7 > -12$

$-9 - a > -12$

$+9 \qquad +9$

$\frac{-a}{-1} > \frac{-3}{-1}$ switch inequality

$a < 3$

2 is the largest integer value.

19. Solve for x algebraically: $7x - 3(4x - 8) \le 6x + 12 - 9x$
If x is a number in the interval $[4, 8]$, state all integers that satisfy the given inequality.
Explain how you determined these values.

$7x - 3(4x - 8) \le 6x + 12 - 9x$

$7x - 12x + 24 \le -3x + 12$

$-5x + 24 \le -3x + 12$

$+5x \qquad +5x$

$24 \le 2x + 12$

$-12 \qquad -12$

$\frac{12}{2} \le \frac{2x}{2}$

$6 \le x$

$x \ge 6$

4
5
6
7
8

6, 7, 8

They are greater than or equal to 6 and in between 4 and 8.

Literal Equations

Follow same steps as equation solving. Don't combine unlike terms.
When isolating, add or subtract first, divide last

1. If $abx - 5 = 0$, what is x in terms of a and b?

~~1)~~ $x = \frac{5}{ab}$

2) $x = -\frac{5}{ab}$

3) $x = 5 - ab$

4) $x = ab - 5$

$$abx - 5 = 0$$
$$+5 \quad +5$$
$$\frac{abx}{ab} = \frac{5}{ab}$$
$$x = \frac{5}{ab}$$

2. The formula for electrical power, P, is $P = I^2R$, where I is current and R is resistance. The formula for I in terms of P and R is

1) $I = \left(\frac{P}{R}\right)^2$

~~2)~~ $I = \sqrt{\frac{P}{R}}$

3) $I = (P - R)^2$

4) $I = \sqrt{P - R}$

$$\frac{P}{R} = \frac{I^2R}{R}$$
$$\sqrt{\frac{P}{R}} = \sqrt{I^2}$$
$$\sqrt{\frac{P}{R}} = I$$

3. Boyle's Law involves the pressure and volume of gas in a container. It can be represented by the formula $P_1V_1 = P_2V_2$. When the formula is solved for P_2, the result is

1) $P_1V_1V_2$

2) $\frac{V_2}{P_1V_1}$

~~3)~~ $\frac{P_1V_1}{V_2}$

4) $\frac{P_1V_2}{V_1}$

$$\frac{P_1V_1}{V_2} = \frac{P_2\cancel{V_2}}{\cancel{V_2}}$$
$$P_2 = \frac{P_1V_1}{V_2}$$

4. The formula for the sum of the degree measures of the interior angles of a polygon is $S = 180(n - 2)$. Solve for n, the number of sides of the polygon, in terms of S.

$$S = 180n - 360$$
$$+360 \quad\quad +360$$
$$\frac{S + 360}{180} = \frac{\cancel{180}n}{\cancel{180}}$$
$$n = \frac{S + 360}{180}$$

5. The formula for converting degrees Fahrenheit (F) to degrees Kelvin (K) is:

$$9(K) = \frac{5}{9}(F + 459.67)\,9$$

Solve for F, in terms of K.

$$9K = 5(F + 459.67)$$
$$9K = 5F + 2298.35$$
$$-2298.35 \quad\quad -2298.35$$
$$\frac{9K - 2298.35}{5} = \frac{\cancel{5}F}{\cancel{5}}$$
$$F = \frac{9K - 2298.35}{5}$$

6. The equation for the volume of a cylinder is $V = \pi r^2 h$. The positive value of r, in terms of h and V, is

~~1)~~ $r = \sqrt{\frac{V}{\pi h}}$

2) $r = \sqrt{V\pi h}$

3) $r = 2V\pi h$

4) $r = \frac{V}{2\pi}$

$$\frac{V}{\pi h} = \frac{\pi r^2 h}{\pi h}$$

$$\sqrt{\frac{V}{\pi h}} = \sqrt{r^2} \rightarrow r = \sqrt{\frac{V}{\pi h}}$$

7. The formula $F_g = \frac{GM_1M_2}{r^2}$ calculates the gravitational force between two objects where G is the gravitational constant, M_1 is the mass of one object, M_2 is the mass of the other object, and r is the distance between them. Solve for the positive value of r in terms of F_g, G, M_1, and M_2.

$$r^2(F_g) = \left(\frac{Gm_1m_2}{r^2}\right)r^2 \quad \frac{r^2 F_g}{F_g} = \frac{Gm_1m_2}{F_g}$$

$$\sqrt{r^2} = \sqrt{\frac{Gm_1m_2}{F_g}} \rightarrow r = \sqrt{\frac{Gm_1m_2}{F_g}}$$

8. If $ax + 3 = 7 - bx$, what is x expressed in terms of a and b?

1) $\frac{4}{ab}$ ~~3)~~ $\frac{4}{a+b}$

2) $-\frac{4}{ab}$ 4) $-\frac{4}{a+b}$

$$ax + 3 = 7 - bx$$
$$+bx \quad +bx$$
$$ax + bx + 3 = 7$$
$$-3 \quad -3$$

$$\rightarrow ax + bx = 4$$
$$\frac{x(a+b)}{a+b} = \frac{4}{a+b} \rightarrow x = \frac{4}{a+b}$$

9. Using the formula for the volume of a cone, express r in terms of V, h, and π.

From Reference Sheet

$$3(V) = \left(\frac{1}{3}\pi r^2 h\right)3$$

$$\frac{3V}{\pi h} = \frac{\pi r^2 h}{\pi h}$$

$$\rightarrow \sqrt{r^2} = \sqrt{\frac{3V}{\pi h}}$$

$$r = \sqrt{\frac{3V}{\pi h}}$$

10. The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. Express b_1 in terms of A, h, and b_2. The area of a trapezoid is 60 square feet, its height is 6 ft, and one base is 12 ft. Find the number of feet in the other base.

$$2(A) = \left(\frac{1}{2}h(b_1 + b_2)\right)2$$

$$2A = h(b_1 + b_2)$$

$$2A = hb_1 + hb_2$$
$$-hb_2 \quad -hb_2$$

$$\rightarrow \frac{2A - hb_2}{h} = \frac{hb_1}{h}$$

$$b_1 = \frac{2A - hb_2}{h}$$

$$b_1 = \frac{2(60) - 6(12)}{6}$$

$$b_1 = 8$$

Modeling Linear Functions
Per/Eachx + one time fee
Per or each goes in front of x, the one time fee or one time starting amount goes at the end. x represents what the amount you're paying per is for. For example, if something costs $5 per hour, x is hours.
The slope is per/each.
The y-intercept is your one time fee or one time starting amount.
Same: Set the two equations equal to each other

per → 1 time fee

1. The cost of airing a commercial on television is modeled by the function $C(n) = 110n + 900$, where n is the number of times the commercial is aired. Based on this model, which statement is true?

1) The commercial costs $0 to produce and $110 per airing up to $900.
2) The commercial costs $110 to produce and $900 each time it is aired.
3) The commercial costs $900 to produce and $110 each time it is aired.
4) The commercial costs $1010 to produce and can air an unlimited number of times.

2. A cell phone company charges $60.00 a month for up to 1 gigabyte of data. The cost of additional data is $0.05 per megabyte. If d represents the number of additional megabytes used and c represents the total charges at the end of the month, which linear equation can be used to determine a user's monthly bill?

1) $c = 60 - 0.05d$
2) $c = 60.05d$
3) $c = 60d - 0.05$
4) $c = 60 + 0.05d$

3. A plumber has a set fee for a house call and charges by the hour for repairs. The total cost of her services can be modeled by $c(t) = 125t + 95$. Which statements about this function are true?

1 time fee

I. A house call fee costs $95.
II. The plumber charges $125 per hour.
III. The number of hours the job takes is represented by t.

1) I and II, only
2) I and III, only
3) II and III, only
4) I, II, and III

4. A company that manufactures radios first pays a start-up cost, and then spends a certain amount of money to manufacture each radio. If the cost of manufacturing r radios is given by the function $c(r) = 5.25r + 125$, then the value 5.25 best represents

per/each

1) the start-up cost
2) the profit earned from the sale of one radio
3) the amount spent to manufacture each radio
4) the average number of radios manufactured

per one time fee

5. The cost of airing a commercial on television is modeled by the function $C(n) = 110n + 900$, where n is the number of times the commercial is aired. Based on this model, which statement is true?

1) The commercial costs $0 to produce and $110 per airing up to $900.
2) The commercial costs $110 to produce and $900 each time it is aired.
3) The commercial costs $900 to produce and $110 each time it is aired.
4) The commercial costs $1010 to produce and can air an unlimited number of times.

6. A satellite television company charges a one-time installation fee and a monthly service charge. The total cost is modeled by the function $y = 40 + 90x$. Which statement represents the meaning of each part of the function? Per

1) y is the total cost, x is the number of months of service, $90 is the installation fee, and $40 is the service charge per month.
2) y is the total cost, x is the number of months of service, $40 is the installation fee, and $90 is the service charge per month.
3) x is the total cost, y is the number of months of service, $40 is the installation fee, and $90 is the service charge per month.
4) x is the total cost, y is the number of months of service, $90 is the installation fee, and $40 is the service charge per month.

7. Each day, a local dog shelter spends an average of $2.40 on food per dog. The manager estimates the shelter's daily expenses, assuming there is at least one dog in the shelter, using the function $E(x) = 30 + 2.40x$. Which statements regarding the function $E(x)$ are correct? Per

I. x represents the number of dogs at the shelter per day. ✓
II. x represents the number of volunteers at the shelter per day. ✗
III. 30 represents the shelter's total expenses per day. ✗
IV. 30 represents the shelter's nonfood expenses per day. ✓

1) I and III
2) I and IV
3) II and III
4) II and IV

8. Last weekend, Emma sold lemonade at a yard sale. The function $P(c) = .50c - 9.96$ represented the profit, $P(c)$, Emma earned selling c cups of lemonade. Sales were strong, so she raised the price for this weekend by 25 cents per cup. Which function represents her profit for this weekend?

.50 Per
+.25
.75 Per

1) $P(c) = .25c - 9.96$
2) $P(c) = .50c - 9.71$
3) $P(c) = .50c - 10.21$
4) $P(c) = .75c - 9.96$

9. The amount Mike gets paid weekly can be represented by the expression $2.50a + 290$, where a is the number of cell phone accessories he sells that week. What is the constant term in this expression and what does it represent?

1) $2.50a$, the amount he is guaranteed to be paid each week
2) $2.50a$, the amount he earns when he sells a accessories
3) 290, the amount he is guaranteed to be paid each week
4) 290, the amount he earns when he sells a accessories

per

10. A car leaves Albany, NY, and travels west toward Buffalo, NY. The equation $D = 280 - 59t$ can be used to represent the distance, D, from Buffalo after t hours. In this equation, the 59 represents the

1) car's distance from Albany
2) speed of the car
3) distance between Buffalo and Albany
4) number of hours driving

miles per hour

11. A gardener is planting two types of trees:

Type A is 36 inches tall and grows at a rate of 15 inches per year.

Type B is 48 inches tall and grows at a rate of 10 inches per year.

Algebraically determine exactly how many years it will take for these trees to be the same height.

equal

$A(x) = 15x + 36$

$B(x) = 10x + 48$

$15x + 36 = 10x + 48$

$-10x \quad -10x$

$5x + 36 = 48$

$-36 \quad -36$

$\frac{5x}{5} = \frac{12}{5}$

$x = 2.4$

12. A local business was looking to hire a landscaper to work on their property. They narrowed their choices to two companies. Flourish Landscaping Company charges a flat rate of \$120 per hour. Green Thumb Landscapers charges \$70 per hour plus a \$1600 equipment fee. Write a system of equations representing how much each company charges. Determine and state the number of hours that must be worked for the cost of each company to be the same. If it is estimated to take at least 35 hours to complete the job, which company will be less expensive? Justify your answer.

$f(x) = 120x$

$g(x) = 70x + 1600$

Flourish is less expensive.

$120x = 70x + 1600$

$-70x \quad -70x$

$\frac{50x}{50} = \frac{1600}{50}$

$x = 32$

$f(35) = 120(35) = 4200$

$g(35) = 70(35) + 1600 = 4050$

13. Ian is borrowing \$1000 from his parents to buy a notebook computer. He plans to pay them back at the rate of \$60 per month. Ken is borrowing \$600 from his parents to purchase a snowboard. He plans to pay his parents back at the rate of \$20 per month. Write an equation that can be used to determine after how many months the boys will owe the same amount. Determine algebraically and state in how many months the two boys will owe the same amount. State the amount they will owe at this time.

Ian

$I(x) = 1000 - 60x$

Ken

$K(x) = 600 - 20x$

$1000 - 60x = 600 - 20x$

$+60x \quad +60x$

$1000 = 600 + 40x$

$-600 \quad -600$

$\frac{400}{40} = \frac{40x}{40}$

$10 = x$

$K(10) = 600 - 20(10)$

$K(x) = 400$

14. Next weekend Marnie wants to attend either carnival A or carnival B. Carnival A charges \$6 for admission and an additional \$1.50 per ride. Carnival B charges \$2.50 for admission and an additional \$2 per ride.

a) In function notation, write $A(x)$ to represent the total cost of attending carnival A and going on x rides. In function notation, write $B(x)$ to represent the total cost of attending carnival B and going on x rides.

b) Determine the number of rides Marnie can go on such that the total cost of attending each carnival is the same. equal

c) Marnie wants to go on five rides. Determine which carnival would have the lower total cost. Justify your answer.

$A(x) = 1.50x + 6$

$B(x) = 2x + 2.50$

$1.50x + 6 = 2x + 2.50$

$-1.50x \quad -1.50x$

$6 = .50x + 2.50$

$-2.50 \quad -2.50$

$\frac{3.50}{.50} = \frac{.50x}{.50}$

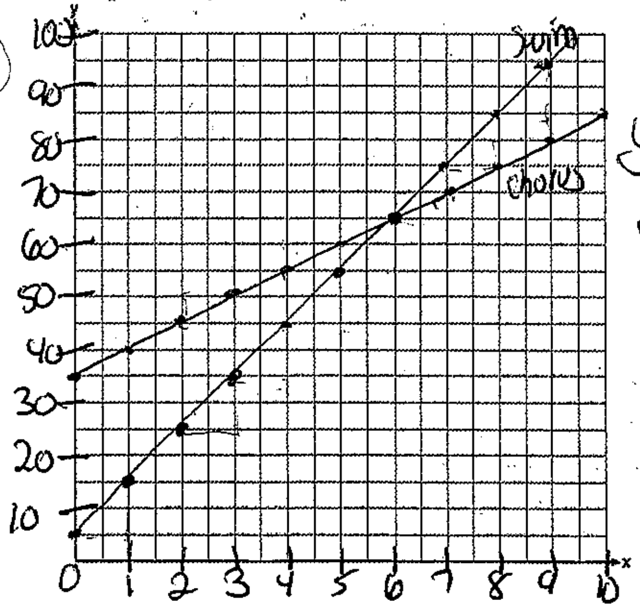
$7 = x$

$A(5) = 1.50(5) + 6 = 13.50$

$B(5) = 2(5) + 2.50 = 12.50$

Carnival B

15. Central High School had five members on their swim team in 2010. Over the next several years, the team increased by an average of 10 members per year. The same school had 35 members in their chorus in 2010. The chorus saw an increase of 5 members per year. Write a system of equations to model this situation, where x represents the number of years since 2010. Graph this system of equations on the set of axes below.



Swim

$y = 5 + 10x$

$y = 10x + 5$

x	y
0	5
1	15
2	25
3	35
4	45
5	55
6	65
7	75
8	85
9	95
10	105

Chorus

$y = 35 + 5x$

$y = 5x + 35$

x	y
0	35
1	40
2	45
3	50
4	55
5	60
6	65
7	70
8	75
9	80
10	85

Explain in detail what each coordinate of the point of intersection of these equations means in the context of this problem.

(6, 65)

6 years after 2010 (2016), the two clubs will have 65 members, the same amount.

Systems of Equations with Elimination

1) Choose a variable to cancel and multiply each equation by the other's coefficient
*multiply by negative if they are the same sign
2) Add equations together
3) Solve equation for one variable
4) Substitute answer in to either equation to find the second variable
*You can find an equivalent system by multiplying either equation by any constant
*For word problems, the first equation is just $x + y =$ for an amount. The second equation is usually a money equation.

***For money, the second equation is** $.01p$, $.05n$, $.10d$, *or* $.25q$

1. $9(c - 2d = 14)$
   $2(3c + 9d = 27)$

   $9c - 18d = 126$
   $+6c + 18d = 54$
   $15c = 180$
   $\frac{15c}{15} = \frac{180}{15}$
   $c = 12$

   $3c + 9d = 27$
   $3(12) + 9d = 27$
   $36 + 9d = 27$
   $-36 \quad -36$
   $\frac{9d}{9} = \frac{-9}{9}$
   $d = -1$

2. $3(3a - b = 3)$
   $1(a + 3b = 11)$

   $9a - 3b = 9$
   $+a + 3b = 11$
   $\frac{10a}{10} = \frac{20}{10}$
   $a = 2$

   $a + 3b = 11$
   $2 + 3b = 11$
   $-2 \quad -2$
   $\frac{3b}{3} = \frac{9}{3}$
   $b = 3$

3. $1(2x + y = 3)$
   $2(-x + 3y = -12)$

   $2x + y = 3$
   $+-2x + 6y = -24$
   $\frac{7y}{7} = \frac{-21}{7}$
   $y = -3$

   $2x + y = 3$
   $2x - 3 = 3$
   $+3 \quad +3$
   $\frac{2x}{2} = \frac{6}{2}$
   $x = 3$

4. $1(2x + 3y = 12)$
   $3(5x - y = 13)$

   $2x + 3y = 12$
   $+15x - 3y = 39$
   $\frac{17x}{17} = \frac{51}{17}$
   $x = 3$

   $2x + 3y = 12$
   $2(3) + 3y = 12$
   $6 + 3y = 12$
   $-6 \quad -6$
   $\frac{3y}{3} = \frac{6}{3}$
   $y = 2$

5. $2(-3x + 4y = 12)$
   $3(2x + y = -8)$

   $-6x + 8y = 24$
   $+6x + 3y = -24$
   $\frac{11y}{11} = \frac{0}{11}$
   $y = 0$

   $2x + y = -8$
   $2x + 0 = -8$
   $\frac{2x}{2} = \frac{-8}{2}$
   $x = -4$

6. $3(2x + 4y = -4)$
   $-2(3x + 5y = -3)$

   $6x + 12y = -12$
   $+-6x - 10y = 6$
   $\frac{2y}{2} = \frac{-6}{2}$
   $y = -3$

   $2x + 4y = -4$
   $2x + 4(-3) = -4$
   $2x - 12 = -4$
   $+12 \quad +12$
   $\frac{2x}{2} = \frac{8}{2}$
   $x = 4$

7. Which system of equations will yield the same solution as the system below?

$$2(x - y = 3)$$
$$2x - 3y = -1$$

1) $-2x - 2y = -6$
   $2x - 3y = -1$

2) $-2x + 2y = 3$
   $2x - 3y = -1$

3) $2x - 2y = 6$
   $2x - 3y = -1$

4) $3x + 3y = 9$
   $2x - 3y = -1$

8. Which system of equations does *not* have the same solution as the system below?

$$4x + 3y = 10$$

$$-6(-6x - 5y = -16) \quad 36x + 30y = 96$$

1) $-12x - 9y = -30$

$12x + 10y = 32$

2) $20x + 15y = 50$

$-18x - 15y = -48$

3) $24x + 18y = 60$

$-24x - 20y = -64$

4) $40x + 30y = 100$

$36x + 30y = -96$

9. A system of equations is given below.

$$x + 2y = 5$$

$$2(2x + y = 4) \quad 4x + 2y = 8$$

Which system of equations does *not* have the same solution?

1) $3x + 6y = 15$

$2x + y = 4$

2) $4x + 8y = 20$

$2x + y = 4$

3) $x + 2y = 5$

$6x + 3y = 12$

4) $x + 2y = 5$

$4x + 2y = 12$

10. Which system of equations has the same solution as the system below?

$$2x + 2y = 16$$

$$2(3x - y = 4) = 6x - 2y = 8$$

1) $2x + 2y = 16$

$6x - 2y = 4$

2) $2x + 2y = 16$

$6x - 2y = 8$

3) $x + y = 16$

$3x - y = 4$

4) $6x + 6y = 48$

$6x + 2y = 8$

11. Which system of equations would have the same solution as the system:

$$-3(x + y = 5) \quad -3x - 3y = -15$$

$$3x + 2y = 10$$

1) $3x + 2y = 5$

$x + y = 10$

2) $-3x - 3y = -15$

$3x + 2y = 10$

3) $-3x - 3y = 5$

$3x + 2y = 10$

4) $2x + 2y = 5$

$3x + 2y = 10$

12. Lizzy has 30 coins that total $4.80. All of her coins are dimes, *D*, and quarters, *Q*. Which system of equations models this situation?

~~1) $D+Q=4.80$~~
$.10D+.25Q=30$

2) $D+Q=30$
$.10D+.25Q=4.80$

3) $D+Q=30$
$.25D+.10Q=4.80$

~~4) $D+Q=4.80$~~
$.25D+.10Q=30$

13. Alicia purchased *H* half-gallons of ice cream for $3.50 each and *P* packages of ice cream cones for $2.50 each. She purchased 14 items and spent $43. Which system of equations could be used to determine how many of each item Alicia purchased?

1) $3.50H+2.50P=43$
$H+P=14$

2) $3.50P+2.50H=43$
$P+H=14$

3) $3.50H+2.50P=14$
~~$H+P=43$~~

4) $3.50P+2.50H=14$
~~$P+H=43$~~

14. The Celluloid Cinema sold 150 tickets to a movie. Some of these were child tickets and the rest were adult tickets. A child ticket cost $7.75 and an adult ticket cost $10.25. If the cinema sold $1470 worth of tickets, which system of equations could be used to determine how many adult tickets, *a*, and how many child tickets, *c*, were sold?

1) $a+c=150$
$10.25a+7.75c=1470$

~~2) $a+c=1470$~~
$10.25a+7.75c=150$

3) $a+c=150$
$7.75a+10.25c=1470$

4) $a+c=1470$
$7.75a+10.25c=150$

15. During its first week of business, a market sold a total of 108 apples and oranges. The second week, five times the number of apples and three times the number of oranges were sold. A total of 452 apples and oranges were sold during the second week. Determine how many apples and how many oranges were sold the first week.

$-5(a+o=108)$
$1(5a+3o=452)$

$-5a-5o=-540$
$+\ 5a+3o=452$

$\frac{-2o}{-2}=\frac{-88}{-2}$
$o=44$

$a+o=108$
$a+44=108$
$-44\quad -44$
$a=64$

16. Dylan has a bank that sorts coins as they are dropped into it. A panel on the front displays the total number of coins inside as well as the total value of these coins. The panel shows 90 coins with a value of $17.55 inside of the bank. If Dylan only collects dimes and quarters, write a system of equations in two variables or an equation in one variable that could be used to model this situation. Using your equation or system of equations, algebraically determine the number of quarters Dylan has in his bank.

$$-.10(d+q=90)$$
$$1(.10d+.25q=17.55)$$

$$-.10d-.10q=-9$$
$$+\ .10d+.25q=17.55$$
$$\frac{.15q}{.15}=\frac{8.55}{.15}$$
$$q=57$$

17. Mo's farm stand sold a total of 165 pounds of apples and peaches. She sold apples for $1.75 per pound and peaches for $2.50 per pound. If she made $337.50, how many pounds of peaches did she sell?

$$-1.75(a+p=165)$$
$$1(1.75a+2.50p=337.50)$$
$$-1.75a-1.75p=-288.75$$
$$+\ 1.75a+2.50p=337.50$$
$$\frac{.75p}{.75}=\frac{48.75}{.75}$$
$$p=65$$

$$a+p=165$$
$$a+65=165$$
$$-65\quad -65$$
$$a=100$$

18. Last week, a candle store received $355.60 for selling 20 candles. Small candles sell for $10.98 and large candles sell for $27.98. How many large candles did the store sell?

$$-10.98(s+l=20)$$
$$1(10.98s+27.98l=355.60)$$
$$-10.98s-10.98l=-219.6$$
$$+\ 10.98s+27.98l=355.60$$
$$\frac{17l}{17}=\frac{136}{17}$$
$$l=8$$

$$s+l=20$$
$$s+8=20$$
$$-8\quad -8$$
$$s=12$$

19. ~~The math department needs to buy new textbooks and laptops for the computer science classroom. The textbooks cost $116.00 each, and the laptops cost $439.00 each. If the math department has $6500 to spend and purchases 30 textbooks, how many laptops can they buy?~~

~~$t+l=30$~~

~~$116t+439l=6$~~

20. Byron has 72 coins in his piggy bank. The piggy bank contains only dimes and quarters. If he has $14.70 in his piggy bank, how many dimes does he have in his piggy bank?

$-.10(d+q=72)$

$1(.10d+.25q=14.70)$

$-.10d-.10q=-7.2$

$.10d+.25q=14.70$

$\frac{.15q}{.15}=\frac{7.5}{.15}$

$q=50$

$d+q=72$

$d+50=72$

$-50\quad -50$

$d=22$

21. An animal shelter spends $2.35 per day to care for each cat and $5.50 per day to care for each dog. Pat noticed that the shelter spent $89.50 caring for cats and dogs on Wednesday. Write an equation to represent the possible numbers of cats and dogs that could have been at the shelter on Wednesday. Pat said that there might have been 8 cats and 14 dogs at the shelter on Wednesday. Are Pat's numbers possible? Use your equation to justify your answer. Later, Pat found a record showing that there were a total of 22 cats and dogs at the shelter on Wednesday. How many cats were at the shelter on Wednesday?

$2.35c+5.50d=89.50$

$2.35(8)+5.50(14)=89.50$

$95.8 \neq 89.5$

No

$-2.35(c+d=22)$

$1(2.35c+5.50d=89.50)$

$-2.35c-2.35d=-51.7$

$+2.35c+5.50d=89.50$

$\frac{3.15d}{3.15}=\frac{37.8}{3.15}$

$d=12$

$c+d=22$

$c+12=22$

$-12\quad -12$

$c=10$

22. For a class picnic, two teachers went to the same store to purchase drinks. One teacher purchased 18 juice boxes and 32 bottles of water, and spent \$19.92. The other teacher purchased 14 juice boxes and 26 bottles of water, and spent \$15.76. Write a system of equations to represent the costs of a juice box, j, and a bottle of water, w. Kara said that the juice boxes might have cost 52 cents each and that the bottles of water might have cost 33 cents each. Use your system of equations to justify that Kara's prices are *not* possible. Solve your system of equations to determine the ~~actual cost, in dollars,~~ of each juice box and each bottle of water.

$-14(18j+32w=19.92)$
$18(14j+26w=15.76)$

$18(.52)+32(.33)=19.92$
$19.92=19.92$ ✓
$14(.52)+26(.33)=15.76$
$15.86 \neq 15.76$
No!

$-252j-448w=-278.88$
$252j+468w=283.68$
$\frac{20w}{20}=\frac{4.8}{20}$
$w=.24$
$18j+32(.24)=19.92$
$18j+7.68=19.92$
$-7.68 \quad -7.68$
$\frac{18j}{18}=\frac{12.24}{18}$
$j=.68$

5 sodas

23. Two friends went to a restaurant and ordered one plain pizza and two sodas. Their bill totaled \$15.95. Later that day, five friends went to the same restaurant. They ordered three plain pizzas and each person had one soda. Their bill totaled \$45.90. Write and solve a system of equations to determine the price of one plain pizza. [Only an algebraic solution can receive full credit.]

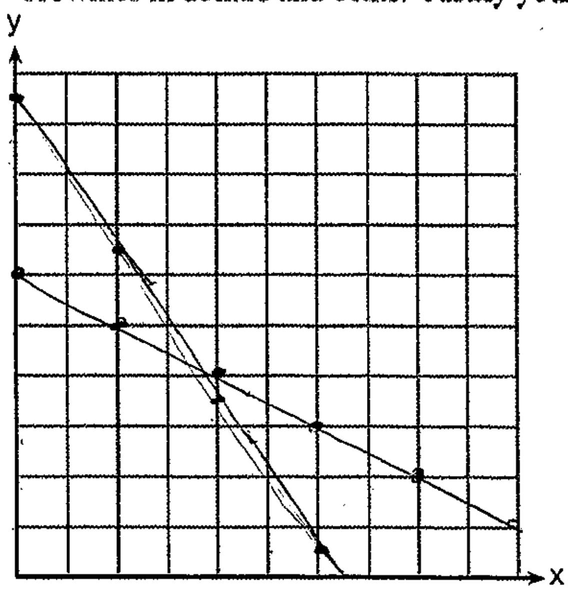
$-3(1p+2s=15.95)$
$1(3p+5s=45.90)$

$-3p-6s=-47.85$
$+3p+5s=45.90$
$\frac{-s}{-1}=\frac{-1.95}{-1}$
$s=1.95$

$p+2s=15.95$
$p+2(1.95)=15.95$
$p+3.9=15.95$
$-3.9 \quad -3.9$
$p=12.05$

24. Franco and Caryl went to a bakery to buy desserts. Franco bought 3 packages of cupcakes and 2 packages of brownies for \$19. Caryl bought 2 packages of cupcakes and 4 packages of brownies for \$24. Let x equal the price of one package of cupcakes and y equal the price of one package of brownies. Write a system of equations that describes the given situation. On the set of axes below, graph the system of equations. Determine the exact cost of one package of cupcakes and the exact cost of one package of brownies in dollars and cents. Justify your solution

Cupcakes: \$3.50
Brownies: \$4.25



$3x+2y=19$
$-3x \quad -3x$
$\frac{2y}{2}=\frac{-3x+19}{2}$
$y=-\frac{3}{2}x+9.5$

$2x+4y=24$
$-2x \quad -2x$
$\frac{4y}{4}=\frac{-2x+24}{4}$
$y=-\frac{1}{2}x+6$

$-2(3x+2y=19)$
$3(2x+4y=24)$
$-6x-4y=-38$
$+6x+12y=72$
$\frac{8y}{8}=\frac{34}{8}$
$y=4.25$

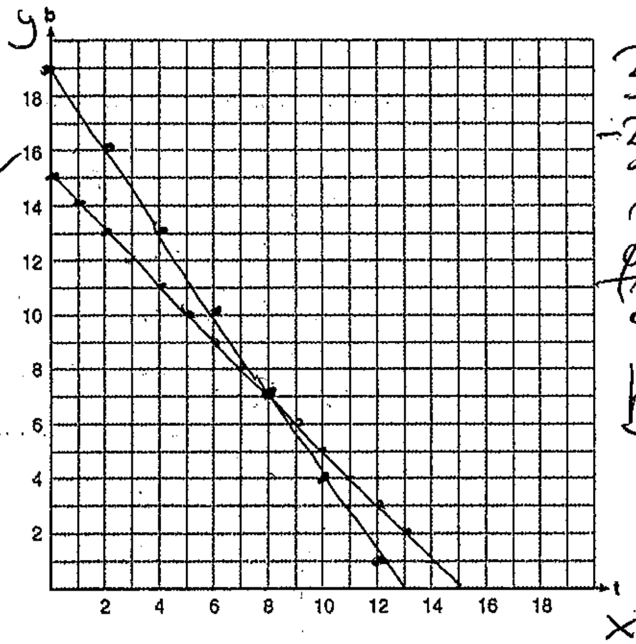
$3x+2y=19$
$3x+2(4.25)=19$
$3x+8.5=19$
$-8.5 \quad -8.5$
$\frac{3x}{3}=\frac{10.5}{3}$
$x=3.5$

25. A recreation center ordered a total of 15 tricycles and bicycles from a sporting goods store. The number of wheels for all the tricycles and bicycles totaled 38. Write a linear system of equations that models this scenario, where t represents the number of tricycles and b represents the number of bicycles ordered. On the set of axes below, graph this system of equations.

$t+b=15$

$-t \quad -t$

$b=-t+15$



$3t+2b=38$

$-3t \quad -3t$

$\frac{2b}{2}=\frac{-3t+38}{2}$

$b=-\frac{3}{2}t+19$

Based on your graph of this scenario, could the recreation center have ordered 10 tricycles? Explain your reasoning.

$3(t+b=15)$

$-1(3t+2b=38)$

$3t+3b=45$

$-3t-2b=-38$

$b=7$

$t+b=15$

$t+7=15$

$-7 \quad -7$

$t=8$

No, they ordered 8.

Systems of Inequalities

$<$: shade below dashed line $\quad\quad$ $\leq$: shade below solid line

$>$: shade above dashed line $\quad\quad$ $\geq$: shade above solid line

The solution set is the region that both graphs are shaded. Mark with an S.

*For word problems, the first inequality is just $x + y$ for an amount. The second inequality is usually a money inequality.

1. Graph the following systems of inequalities on the set of axes below:
Based upon your graph, explain why $(6, 1)$ is a solution to this system and why $(-6, 7)$ is *not* a solution to this system.

$2y \geq 3x - 16$

$y + 2x > -5$

$\frac{2y}{2} \geq \frac{3x-16}{2}$

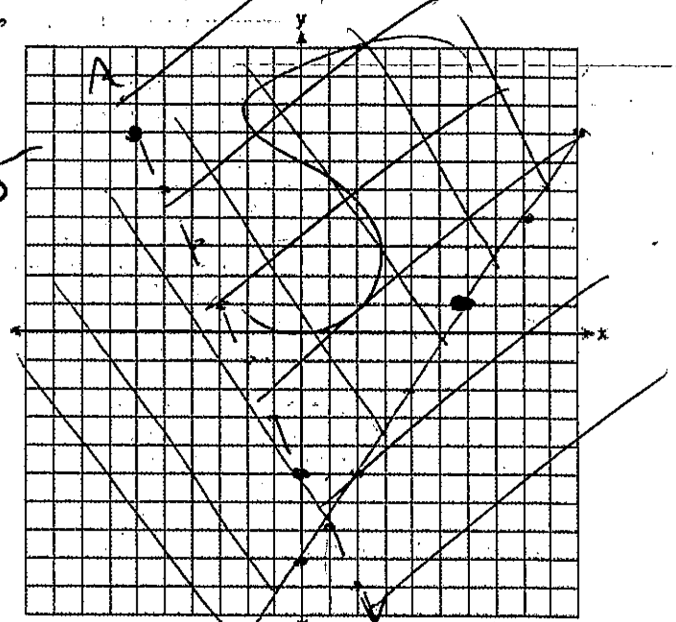
$y \geq \frac{3}{2}x - 8$

$y + 2x > -5$
$\quad -2x \quad -2x$

$y > -2x - 5$

(6,1) is in the solution set. On the line is included if it is a solid line.

(-6,7) is not in the solution set. On the line is not included if it's a dashed line.



2. On the set of axes below, graph the following system of inequalities:
Determine if the point $(1, 2)$ is in the solution set. Explain your answer.

$2y + 3x \leq 14$

$4x - y < 2$

$4x - y < 2$
$-4x \quad\quad -4x$

$\frac{-y}{-1} < \frac{-4x+2}{-1}$

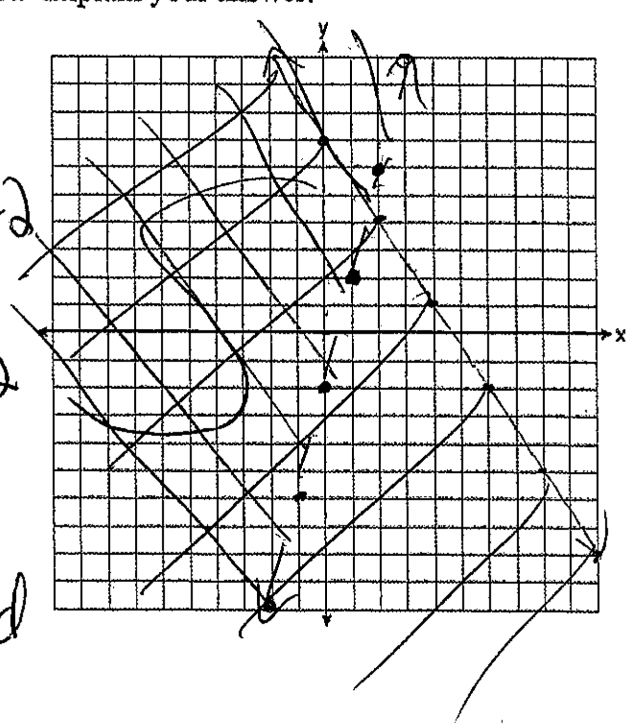
$y > 4x - 2$

$2y + 3x \leq 14$
$\quad -3x \quad -3x$

$\frac{2y}{2} \leq \frac{-3x+14}{2}$

$y \leq -\frac{3}{2}x + 7$

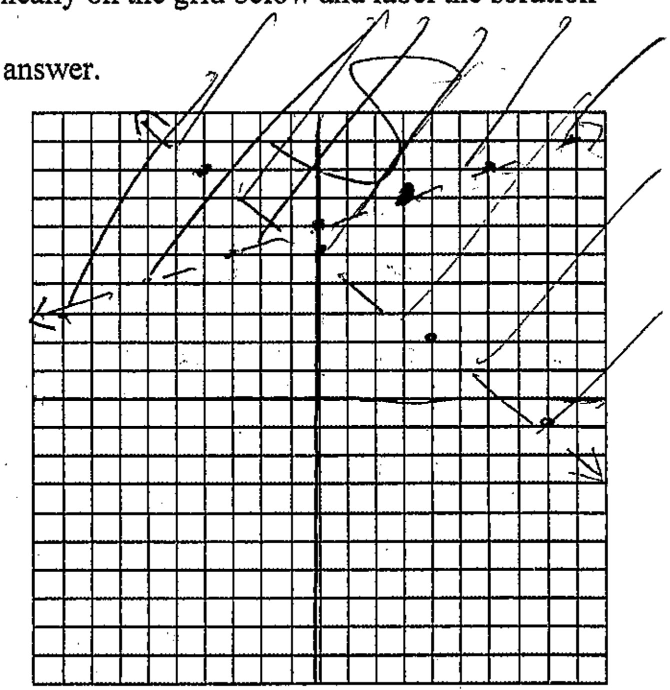
(1,2) is not in the solution set. the dashed line is not included.



3. Solve the following system of inequalities graphically on the grid below and label the solution S.
Is the point (3, 7) in the solution set? Explain your answer.

$3x + 4y > 20$

$x < 3y - 18$

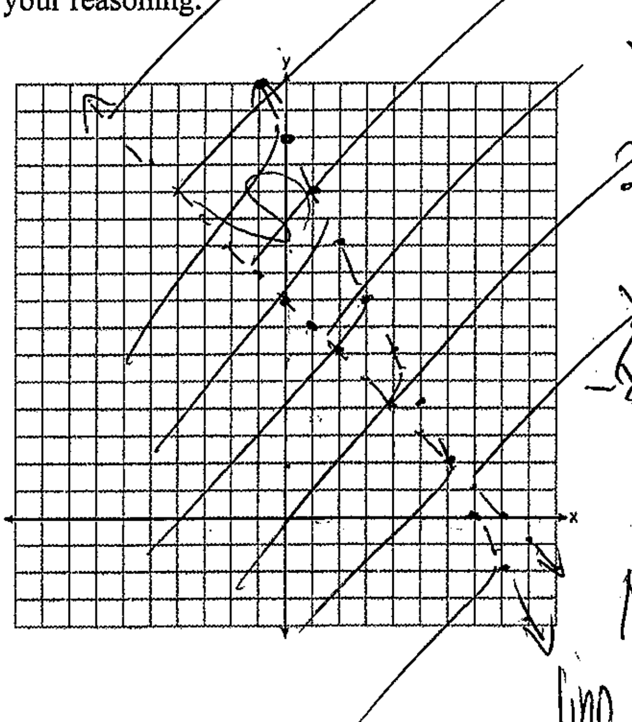


$3x+4y>20$
$-3x \quad -3x$
$\frac{4y}{4} > \frac{-3x+20}{4}$
$y > -\frac{3}{4}x+5$

$x < 3y-18$
$+18 \quad +18$
$\frac{x+18}{3} < \frac{3y}{3}$
$\frac{1}{3}x+6 < y$
$y > \frac{1}{3}x+6$

No, the dashed line is not included in the solution set

4. The sum of two numbers, x and y, is more than 8. When you double x and add it to y, the sum is less than 14. Graph the inequalities that represent this scenario on the set of axes below.
Kai says that the point (6, 2) is a solution to this system. Determine if he is correct and explain your reasoning.



$x+y>8$

$2x+y<14$

$x+y>8$
$-x \quad -x$
$y>-x+8$

$2x+y<14$
$-2x \quad -2x$
$y<-2x+14$

No, the dashed line is not included.

5. Jordan works for a landscape company during his summer vacation. He is paid $12 per hour for mowing lawns and $14 per hour for planting gardens. He can work a maximum of 40 hours per week, and would like to earn at least $250 this week. If m represents the number of hours mowing lawns and g represents the number of hours planting gardens, which system of inequalities could be used to represent the given conditions?

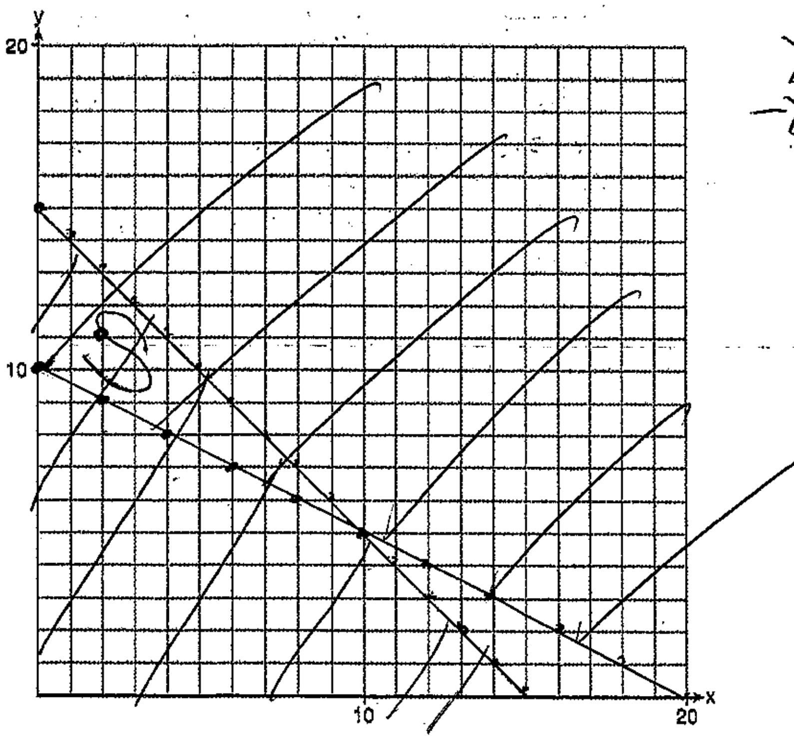
1) $m + g \leq 40$
   $12m + 14g \geq 250$

2) $m + g \geq 40$
   $12m + 14g \leq 250$

3) $m + g \leq 40$
   $12m + 14g \leq 250$

4) $m + g \geq 40$
   $12m + 14g \geq 250$

6. Edith babysits for x hours a week after school at a job that pays $4 an hour. She has accepted a job that pays $8 an hour as a library assistant working y hours a week. She will work both jobs. She is able to work no more than 15 hours a week, due to school commitments. Edith wants to earn at least $80 a week, working a combination of both jobs. Write a system of inequalities that can be used to represent the situation. Graph these inequalities on the set of axes below.



$x + y \leq 15$
$-x \quad -x$
$y \leq -x + 15$

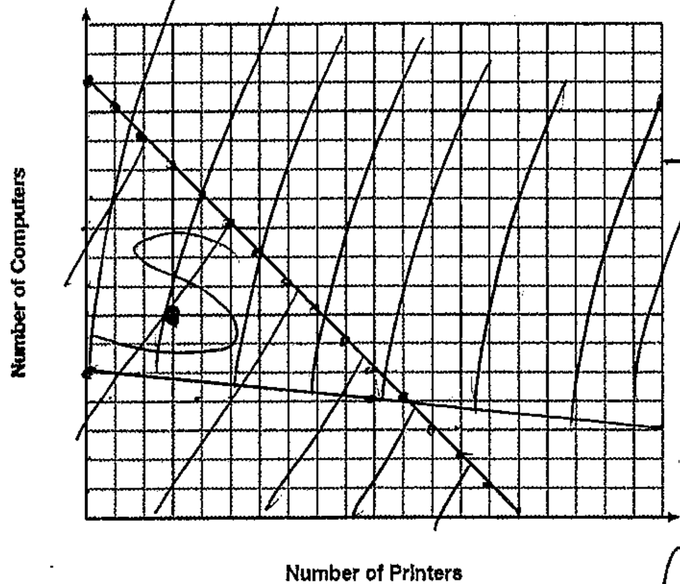
$4x + 8y \geq 80$
$-4x \quad -4x$
$\frac{8y}{8} \geq \frac{-4x+80}{8}$
$y \geq -\frac{1}{2}x + 10$

Determine and state one combination of hours that will allow Edith to earn *at least* $80 per week while working *no more than* 15 hours.

(2, 11)

2 hours babysitting and 11 hours as a library assistant.

7. An on-line electronics store must sell at least \$2500 worth of printers and computers per day. Each printer costs \$50 and each computer costs \$500. The store can ship a maximum of 15 items per day. On the set of axes below, graph a system of inequalities that models these constraints. Determine a combination of printers and computers that would allow the electronics store to meet all of the constraints. Explain how you obtained your answer.



$\cancel{x} + y \le 15$ $\quad$ $50x + 500y \ge 2500$

$\cancel{-x} \quad -x$ $\quad$ $-50x \quad -50x$

$y \le -x + 15$ $\quad$ $\frac{500y}{500} \ge \frac{-50x + 2500}{500}$

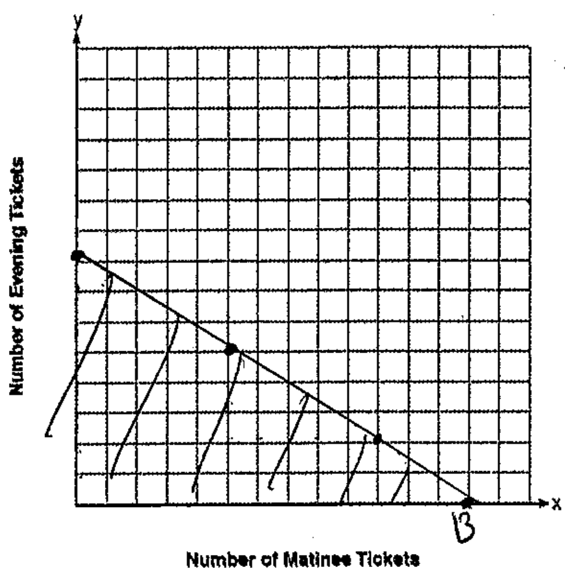
$y \ge -\frac{1}{10}x + 5$

(3,7)

3 printers and 7 computers

(3,7) is in the solution set.

8. Myranda received a movie gift card for \$100 to her local theater. Matinee tickets cost \$7.50 each and evening tickets cost \$12.50 each. If x represents the number of matinee tickets she could purchase, and y represents the number of evening tickets she could purchase, write an inequality that represents all the possible ways Myranda could spend her gift card on movies at the theater.. On the set of axes below, graph this inequality.
What is the maximum number of matinee tickets Myranda could purchase with her gift card? Explain your answer.



$7.50x + 12.50y \le 100$

$-7.50x \qquad -7.50x$

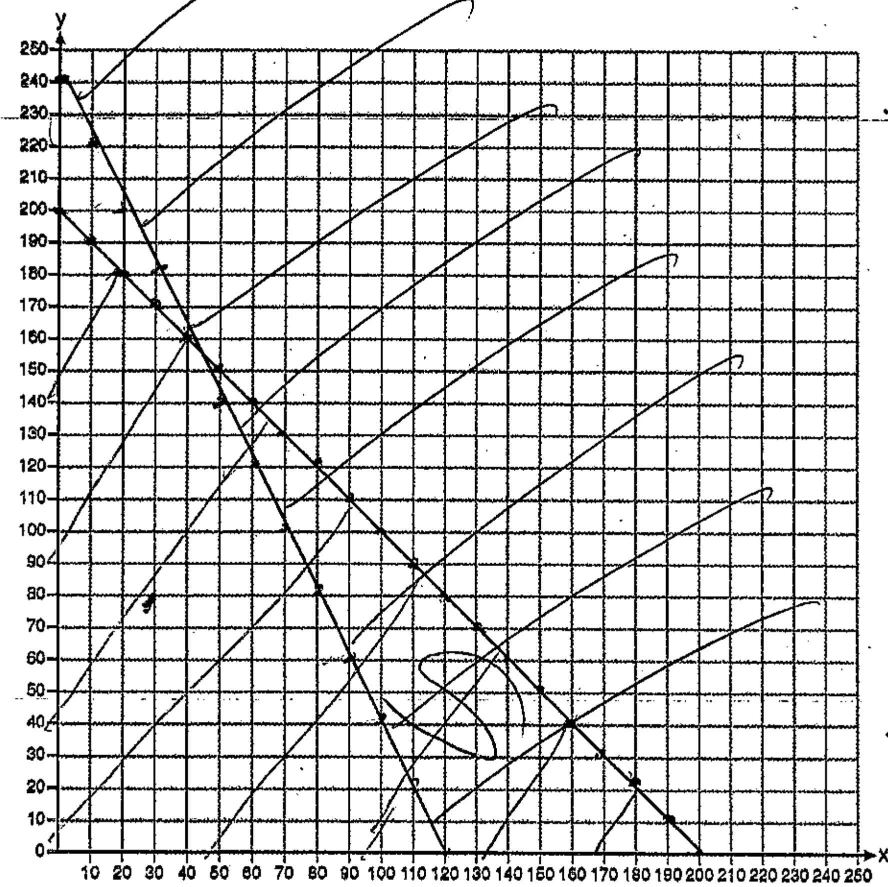
$\frac{12.50y}{12.50} \le \frac{-7.50x + 100}{12.50}$

$y \le -\frac{3}{5}x + 8$

13 matinee tickets.

13 is the largest x value in the solution set.

9. The Reel Good Cinema is conducting a mathematical study. In its theater, there are 200 seats. Adult tickets cost \$12.50 and child tickets cost \$6.25. The cinema's goal is to sell at least \$1500 worth of tickets for the theater. Write a system of linear inequalities that can be used to find the possible combinations of adult tickets, x, and child tickets, y, that would satisfy the cinema's goal. Graph the solution to this system of inequalities on the set of axes below. Label the solution with an S. Marta claims that selling 30 adult tickets and 80 child tickets will result in meeting the cinema's goal. Explain whether she is correct or incorrect, based on the graph drawn.



$x+y \le 200$

$-x \quad -x$

$y \le -x+200$

$12.50x+6.25y \ge$

$-12.50x \quad -12.$

$\frac{6.25y}{6.25} \ge \frac{-12.50x+15}{6.25}$

$y \ge -2x+240$

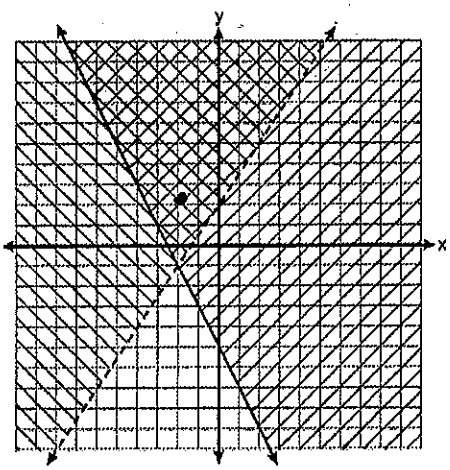
No, (30,80) is not in the solution set.

Systems of Inequalities

The solution to a system of inequalities is the region where they are both shaded. The solid line is included, the dashed line is not included.

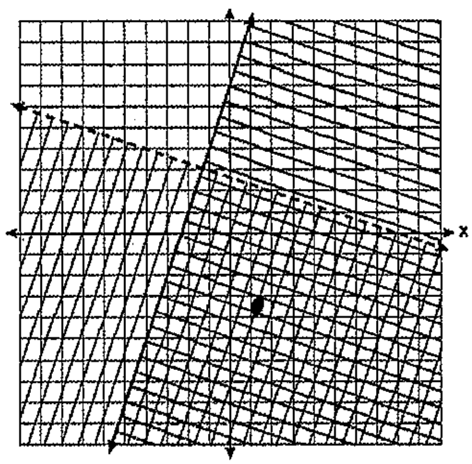
1. Which ordered pair is in the solution set of systems of inequalities graphed below?

1) (-2,-1)   3) (-2,2)
2) (-2,-4)   4) (2,-2)

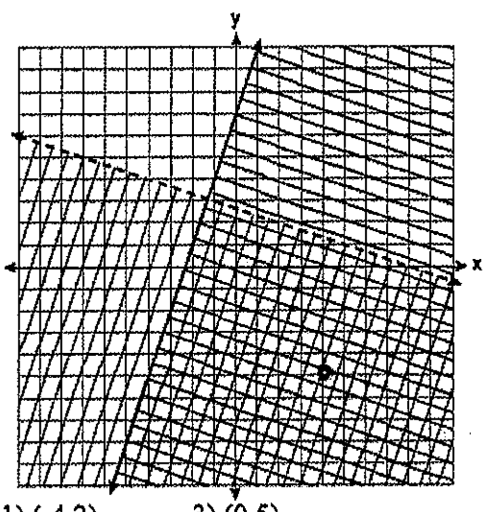


2. Which ordered pair is in the solution set of the linear the linear system of inequalities shown in the graph below?

1)	(1,−4)	3)	(5,3)
2)	(−5,7)	4)	(−7,−2)

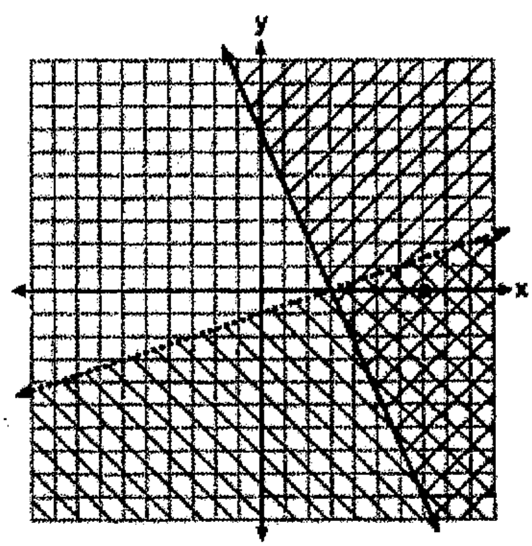


3. Which ordered pair is in the solution set of the system of linear inequalities graphed below?



1) (-4,2)   3) (0,5)
2) (-4,0)   4) (4,-5)

4. What is one point that lies in the solution set of the system of inequalities graphed below?



(1) (7,0)   (3) (0,7)
(2) (3,0)   (4) (−3,5)

Modeling Exponential Functions

Exponential/ Interest/Depreciation Problems: $A = P(1 \pm r)^t$, where A is the current amount, P is the initial amount (y-intercept), r is the rate as a decimal (divide by 100), and t is time.
Given an exponential function: What is in front of the parenthesis is the INITIAL amount, what is inside the parenthesis is 1 + the rate or 1 – the rate.

Example: $A = 500(1.2)^t$: 500 is initial amount, rate is .2 or 20% growth (1 + .2)

$A = 500(0.8)^t$: 500 is initial amount, rate is .2 or 20% decay (1 - .2)

1. Anne invested \$1000 in an account with a 1.3% annual interest rate. She made no deposits or withdrawals on the account for 2 years. If interest was compounded annually, which equation represents the balance in the account after the 2 years?

1) $A = 1000(1 - 0.013)^2$
2) $A = 1000(1 + 0.013)^2$
3) $A = 1000(1 - 1.3)^2$
4) $A = 1000(1 + 1.3)^2$

2. Dylan invested \$600 in a savings account at a 1.6% annual interest rate. He made no deposits or withdrawals on the account for 2 years. The interest was compounded annually. Find, to the *nearest cent*, the balance in the account after 2 years.

$A=A$
$P=600$
$r=.016$
$t=2$

$A=P(1+r)^t$ $A=600(1+.016)^2$
$A=619.35$

3. Cassandra bought an antique dresser for \$500. If the value of her dresser increases 6% annually, what will be the value of Cassandra's dresser at the end of 3 years to the *nearest dollar*?

$A=A$
$P=500$
$r=.06$
$t=3$

$A=P(1+r)^t$
$A=500(1+.06)^3$
$A=\$596$

4. Sheba opened a retirement account with \$36,500. Her account grew at a rate of 7% per year compounded annually. She made no deposits or withdrawals on the account. At the end of 20 years, what was the account worth, to the *nearest dollar*?

$A=A$
$P=36{,}500$
$r=.07$
$t=20$

$A=P(1+r)^t$
$A=36{,}500(1+.07)^{20}$
$A=141243$

5. Kirsten invested \$1000 in an account at an annual interest rate of 3%. She made no deposits or withdrawals on the account for 5 years. The interest was compounded annually. Write an equation to present the value, v, of Kirsten's account after 5 years.

$A=V$
$P=1000$
$r=.03$
$t=5$

$A=P(1+r)^t$
$V=1000(1+.03)^5$

6. Marilyn collects old dolls. She purchases a doll for \$450. Research shows this doll's value will increase by 2.5% each year. Write an equation that determines the value, V, of the doll t years after purchase. Assuming the doll's rate of appreciation remains the same, will the doll's value be doubled in 20 years? Justify your reasoning.

$A = V$
$P = 450$
$r = .025$
$t = t$

$A = P(1+r)^t$

$V = 450(1+.025)^t$

$V = 450(1+.025)^{20}$
$V = 737.38$

$450(2) = 900$
No!
$737 < 900$

7. The function $V(t) = 1350(1.017)^t$ represents the value $V(t)$, in dollars, of a comic book t years after its purchase. The yearly rate of appreciation of the comic book is

1) 17%
2) 1.7%
3) 1.017%
4) 0.017%

1.017
-1
$.017(100) = 1.7\%$

$1+r = 1.017$

8. Milton has his money invested in a stock portfolio. The value, $v(x)$, of his portfolio can be modeled with the function $v(x) = 30,000(0.78)^x$, where x is the number of years since he made his investment. Which statement describes the rate of change of the value of his portfolio?

1) It decreases 78% per year.
2) It decreases 22% per year.
3) It increases 78% per year.
4) It increases 22% per year.

$\cancel{1} - r = .78$
$-1 \quad -1$
$\frac{-r}{-1} = \frac{-.22}{-1}$

$r = .22(100)$
$r = 22\%$

9. The equation $A = 1300(1.02)^7$ is being used to calculate the amount of money in a savings account. What does 1.02 represent in this equation?

1) 0.02% decay
2) 0.02% growth
3) 2% decay
4) 2% growth

$1 + r = 1.02$
$-1 \quad -1$
$r = .02(100)$
$r = 2\%$

10. A car's depreciated value can be represented by the function $v(t) = 25500(.83)^t$. What was the initial value of the car and what is the depreciation rate?

Initial value: 25500

$\cancel{1} - r = .83$
$-1 \quad -1$
$-r = -.17$
$\frac{-r}{-1} = \frac{-.17}{-1}$
$r = .17(100)$
$r = 17\%$

11. The value, $v(t)$, of a car depreciates according to the function $v(t) = P(.85)^t$, where P is the purchase price of the car and t is the time, in years, since the car was purchased. State the percent that the value of the car *decreases* by each year. Justify your answer.

$\cancel{1}-r=.85$
$-1 \quad -1$
$\frac{-r}{-1}=\frac{-.15}{-1}$
$r=.15(100)$
$r=15\%$

12. Some banks charge a fee on savings accounts that are left inactive for an extended period of time. The equation $y = 5000(0.98)^x$ represents the value, y, of one account that was left inactive for a period of x years. What is the y-intercept of this equation and what does it represent?

1) 0.98, the percent of money in the account initially
2) 0.98, the percent of money in the account after x years
3) 5000, the amount of money in the account initially
4) 5000, the amount of money in the account after x years

initial value

13. The number of carbon atoms in a fossil is given by the function $y = 5100(0.95)^x$, where x represents the number of years since being discovered. What is the percent of change each year? Explain how you arrived at your answer.

1-r is what is inside the parenthesis

$\cancel{1}-r=.95$
$-1 \quad -1$
$\frac{-r}{-1}=\frac{-.05}{-1}$
$r=.05(100)$
$r=5\%$

14. A population of rabbits in a lab, $p(x)$, can be modeled by the function $p(x) = 20(1.014)^x$, where x represents the number of days since the population was first counted. Explain what 20 and 1.014 represent in the context of the problem.

20 is the initial number of rabbits
1.4% is the growth rate.

$1.014=1+r$
$-1 \quad -1$
$100(.014)=r$
$r=1.4\%$

15. The breakdown of a sample of a chemical compound is represented by the function $p(t) = 300(0.5)^t$, where $p(t)$ represents the number of milligrams of the substance and t represents the time, in years. In the function $p(t)$, explain what 0.5 and 300 represent.

300 is the initial number of milligrams of the substance.

50% is the rate of decrease

$\cancel{1}-r=.5$
$-1 \quad -1$
$\frac{-r}{-1}=\frac{-.5}{-1}$
$r=.5(100)$
$r=50\%$

Linear vs. Exponential

Linear	Exponential
Add/Subtract Constant Amount	**Multiply/Divide Constant Amount** Add/subtract increasing/decreasing amount
Per x + 1TF	AP1RT
	Percent/Rate

Linear increases/decreases by a constant amount.
Exponential increases/decreases by a constant **percent**!

1. One characteristic of all linear functions is that they change by
1) equal factors over equal intervals
2) unequal factors over equal intervals
3) equal differences over equal intervals
4) unequal differences over equal intervals

2. Which statement below is true about linear functions?
1) Linear functions grow by equal factors over equal intervals.
2) Linear functions grow by equal differences over equal intervals.
3) Linear functions grow by equal differences over unequal intervals.
4) Linear functions grow by unequal factors over equal intervals.

3. The table below shows the average yearly balance in a savings account where interest is compounded annually. No money is deposited or withdrawn after the initial amount is deposited.

Year	Balance, in Dollars
0	380.00 +≈180
10	562.49
20	832.63
30	1232.49
40	1824.39 +≈900
50	2700.54

Which type of function best models the given data?
1) linear function with a negative rate of change
2) linear function with a positive rate. of change
3) exponential decay function
4) exponential growth function

4. The number of people who attended a school's last six basketball games increased as the team neared the state sectional games. The table below shows the data. State the type of function that best fits the given data. Justify your choice of a function type.

Game	13	14	15	16	17	18
Attendance	348	435	522	609	696	783

+87 +87 +87 +87 +87

Linear, it is increasing by a constant amount.

5. The function, $t(x)$, is shown in the table below. Determine whether $t(x)$ is linear or exponential. Explain your answer.

Linear, it is decreasing by a constant amount.

x	t(x)	
−3	10	−2.5
−1	7.5	−2.5
1	5	−2.5
3	2.5	−2.5
5	0	−2.5

6. Caleb claims that the ordered pairs shown in the table below are from a nonlinear function. State if Caleb is correct. Explain your reasoning.

x	f(x)	
0	2	+2
1	4	+4
2	8	+8
3	16	

Yes, it is not increasing by a constant amount.

7. The tables below show the values of four different functions for given values of x. Which table represents a linear function?

~~1)~~ $f(x)$
2) $g(x)$
3) $h(x)$
4) $k(x)$

x	f(x)	
1	12	+7
2	19	+7
3	26	+7
4	33	

x	g(x)	
1	−1	+2
2	1	+4
3	5	+8
4	13	

x	h(x)	
1	9	+3
2	12	+5
3	17	+7
4	24	

x	k(x)	
1	−2	+6
2	4	+10
3	14	+14
4	28	

8. Which table of values represents a linear relationship?

1)

x	f(x)	
−1	−3	+1
0	−2	+3
1	1	+5
2	6	+7
3	13	

2)

x	f(x)	
−1	$\frac{1}{2}$	$+\frac{1}{2}$
0	1	+1
1	2	+2
2	4	+4
3	8	

~~3)~~

x	f(x)	
−1	−3	+2
0	−1	+2
1	1	+2
2	3	+2
3	5	

4)

x	f(x)	
−1	−1	+1
0	0	+1
1	1	+7
2	8	+19
3	27	

9. During physical education class, Andrew recorded the exercise times in minutes and heart rates in beats per minute (bpm) of four of his classmates. Which table best represents a linear model of exercise time and heart rate?

1) Student 1

Exercise Time (in minutes)	Heart Rate (bpm)
0	60
1	65
2	70
3	75
4	80

+5 +5 +5 +5

2) Student 2

Exercise Time (in minutes)	Heart Rate (bpm)
0	62
1	70
2	83
3	88
4	90

+8 +13 +5 +2

3) Student 3

Exercise Time (in minutes)	Heart Rate (bpm)
0	58
1	65
2	70
3	75
4	79

+7 +5 +5 +4

4) Student 4

Exercise Time (in minutes)	Heart Rate (bpm)
0	62
1	65
2	66
3	73
4	75

+3 +1 +7 +2

·3 ·3 ·3

10. Determine and state whether the sequence 1, 3, 9, 27, . . . displays exponential behavior. Explain how you arrived at your decision.

Yes, it is increasing by a constant factor.

+10

11. Ian is saving up to buy a new baseball glove. Every month he puts $10 into a jar. Which type of function best models the total amount of money in the jar after a given number of months?

1) linear
2) exponential
3) quadratic
4) square root

12. The highest possible grade for a book report is 100. The teacher deducts 10 points for each day the report is late. Which kind of function describes this situation?

1) linear −10
2) quadratic
3) exponential growth
4) exponential decay

13. Which situation is *not* a linear function?

1) A gym charges a membership fee of $10.00 down and $10.00 per month.
2) A cab company charges $2.50 initially and $3.00 per mile.
3) A restaurant employee earns $12.50 per hour.
4) A $12,000 car depreciates 15% per year.

exponential percents

14. Which scenario represents exponential growth? multiplication/percents → exponential

1) A water tank is filled at a rate of 2 gallons/minute.
2) A vine grows 6 inches every week.
3) A species of fly doubles its population every month during the summer.
4) A car increases its distance from a garage as it travels at a constant speed of 25 miles per hour.

15. Which situation could be modeled by using a linear function? exponential

1) a bank account balance that grows at a rate of 5% per year, compounded annually
2) a population of bacteria that doubles every 4.5 hours → exponential
3) the cost of cell phone service that charges a base amount plus 20 cents per minute
4) the concentration of medicine in a person's body that decays by a factor of one-third every hour → exponential

16. Which of the three situations given below is best modeled by an exponential function?

I. A bacteria culture doubles in size every day → exponential
II. A plant grows by 1 inch every 4 days. → +1, linear
III. The population of a town declines by 5% every 3 years → exponential

1) I, only
2) II, only
3) I and II
4) I and III

17. Which situation could be modeled with an exponential function? → percent exponential

1) the amount of money in a savings account where $150 is deducted every month.
2) the amount of money in Suzy's piggy bank which she adds $10 to each week.
3) the amount of money in a certificate of deposit that gets 4% interest each year
4) the amount of money in Jaclyn's wallet which increases and decreases by a different amount each week.

18. Which situation could be modeled with a linear function?

1) the height of a ball that is thrown in the air quadratic
2) the price of a car that depreciates 20% per year exponential
3) the amount of money Jonathan pays for a certain number of gallons of gas at $3.85 per gallon
4) a bacteria colony which doubles in number every 4 hours → exponential

19. Which situation could be modeled with an exponential function?

1) the amount of money in a savings account where $150 is deducted every month.
2) the amount of money in Suzy's piggy bank which she adds $10 to each week.
3) the amount of money in a certificate of deposit that gets 4% interest each year
4) the amount of money in Jaclyn's wallet which increases and decreases by a different amount each week.

exponential

Factoring

Greatest Common Factor: GCF(   )

Difference of Two Squares: $(\sqrt{\ }_1 + \sqrt{\ }_2)(\sqrt{\ }_1 - \sqrt{\ }_2)$

Trinomials: (x   )(x   )

1) First sign comes down
2) The two signs must multiply for the last sign
3) Find two numbers that multiply to the last number and add/subtract to the middle number.

Bridge Method: (Trinomial with a leading coefficient bigger than 1)

1) Build a bridge between the first and last numbers (Multiply)
2) Factor Trinomial Normally
3) Pay the toll (Divide by the leading coefficient)

*If possible, reduce the fraction

If they divide nicely, divide them

If not, put the denominator in front of the variable inside the parenthesis

***Factor further if necessary**

Factor each expression

1. $\frac{4x}{4} + \frac{8}{4}$

   $4(x+2)$

2. $\frac{12x}{6} + \frac{18}{6}$

   $6(2x+3)$

3. $\frac{x^2}{x} - \frac{7x}{x}$

   $x(x-7)$

4. $\frac{2x^2}{2x} - \frac{4xy}{2x}$

   $2x(x-2y)$

5. $\frac{5x^2y}{5x} - \frac{20x}{5x}$

   $5x(xy-4)$

6. $\sqrt{x^2} - \sqrt{64}$

   $(x+8)(x-8)$

7. $\sqrt{y^2} - \sqrt{36}$

   $(y+6)(y-6)$

8. $\sqrt{4t^2} - \sqrt{25}$

   $(2t+5)(2t-5)$

9. $9x^2 - 16y^4$

   $(3x+4y^2)(3x-4y^2)$

10. $36 - 25x^2$

    $(6+5x)(6-5x)$

11. $100y^4 - 49t^6$

$(10y^2+7t^3)(10y^2-7t^3)$

12. $1 - 9x^8y^4$

$(1+3x^4y^2)(1-3x^4y^2)$

13. $x^2 + 4x - 12$

$(x+6)(x-2)$

14. $y^2 + 3y + 2$

$(y+2)(y+1)$

15. $m^2 - 8m + 15$

$(m-5)(m-3)$

16. $x^2 - 8x - 20$

$(x-10)(x+2)$

17. $y^2 + 5y - 14$

$(y+7)(y-2)$

18. $x^2 + x - 12$

$(x+4)(x-3)$

19. $x^2 - 3x - 10$

$(x-5)(x+2)$

20. $x^2 - 7x + 12$

$(x-4)(x-3)$

21. $x^2 - 9x - 36$

$(x-12)(x+3)$

22. $y^2 - 21y + 110$

$(y-11)(y-10)$

23. $x^4 + 4x^2 - 12$

$(x^2+6)(x^2-2)$

24. $x^6 - 6x^3 + 9$

$(x^3-3)(x^3-3)$

25. $x^4 - 8x^2 - 9$

$(x^2-9)(x^2+1)$

$(x+3)(x-3)(x^2+1)$

26. $x^4 + x^2 - 2$

$(x^2+2)(x^2-1)$

$(x^2+2)(x+1)(x-1)$

27. $\frac{2x^2}{2} - \frac{50}{2}$

$2(x^2-25)$

$2(x+5)(x-5)$

28. $\frac{2x^2}{2} - \frac{8x}{2} - \frac{10}{2}$

$2(x^2-4x-5)$

$2(x-5)(x+1)$

29. $\frac{3x^2}{3} + \frac{9x}{3} - \frac{12}{3}$

$3(x^2+3x-4)$

$3(x+4)(x-1)$

30. $\frac{6x^2}{6} - \frac{54}{6}$

$6(x^2-9)$

$6(x+3)(x-3)$

31. $\frac{2x^2}{2} + \frac{14x}{2} + \frac{24}{2}$

$2(x^2+7x+12)$

$2(x+4)(x+3)$

32. $\frac{5x^2}{5} - \frac{500}{5}$

$5(x^2-100)$

$5(x+10)(x-10)$

33. $\frac{ax^2}{a} - \frac{2ax}{a} - \frac{8a}{a}$

$a(x^2-2x-8)$

$a(x-4)(x+2)$

34. $\frac{yx^2}{y} - \frac{64y}{y}$

$y(x^2-64)$

$y(x+8)(x-8)$

35. $\frac{12x^2}{3} - \frac{75}{3}$

$3(4x^2-25)$

$3(2x+5)(2x-5)$

36. $x^4 - 81$

$(x^2+9)(x^2-9)$

$(x^2+9)(x+3)(x-3)$

37. $2y^2 - 5y - 7$

$y^2-5y-14$ $\quad(2y-7)(y+1)$

$(y-\frac{7}{2})(y+\frac{2}{2})$

38. $2x^2 + 15x - 8$

$x^2+15x-16$

$(x+\frac{16}{2})(x-\frac{1}{2})$

$(x+8)(2x-1)$

39. $2x^2 + 7x - 4$

x^2+7x-8

$(x+\frac{8}{2})(x-\frac{1}{2})$

$(x+4)(2x-1)$

40. $6x^2 - 11x - 10$

$x^2-11x-60$

$(x-\frac{15}{6})(x+\frac{4}{6})$

$(x-\frac{5}{2})(x+\frac{2}{3})$

$(2x-5)(3x+2)$

41. $2x^2 - 9x - 18$

$x^2-9x-36$

$(x-\frac{12}{2})(x+\frac{3}{2})$

$(x-6)(2x+3)$

42. $3x^2 + 2x - 8$

$x^2+2x-24$

$(x+\frac{6}{3})(x-\frac{4}{3})$

$(x+2)(3x-4)$

43. $8x^2 + 7x - 1$

x^2+7x-8

$(x+\frac{8}{8})(x-\frac{1}{8})$

$(x+1)(8x-1)$

44. $6x^2 + x - 12$

x^2+x-72

$(x+\frac{9}{6})(x-\frac{8}{6})$

$(x+\frac{3}{2})(x-\frac{4}{3})$

$(2x+3)(3x-4)$

Solving Quadratic Equations by Factoring

Quadratic Equations

Mr. x^2 wants to party. Before he can party, all of his friends have to come over. Once all of his friends come over, they party! At the party, they want to blow bubbles(factor).

*Divide out an integer GCF if possible

1) Bring all terms to the side with x^2 (x^2 should be positive)

2) Factor (Follow the steps of factoring)

3) Set each factor equal to zero (T-chart) and solve

1. $y^2 - 5y - 6 = 0$

$(y-6)(y+1)=0$

$y-6=0 \quad y+1=0$

$y=6 \quad y=-1$

2. $x^2 + 4x = 0$

$x(x+4)=0$

$x=0 \quad x+4=0$

$x=-4$

3. $a^2 - 8a = 20$

$a^2-8a-20=0$

$(a-10)(a+2)=0$

$a-10=0 \quad a+2=0$

$a=10 \quad a=-2$

4. $3x^2 = 48$

$\frac{3x^2-48}{3}=\frac{0}{3}$

$x^2-16=0$

$(x+4)(x-4)=0$

$x+4=0 \quad x-4=0$

$x=-4 \quad x=4$

5. $x^2 + 8x = 20$

$x^2+8x-20=0$

$(x+10)(x-2)=0$

$x=-10 \quad x=2$

6. $x^2 + 3x = 8x - 4$

$x^2-5x+4=0$

$(x-4)(x-1)=0$

$x=4 \quad x=1$

7. $n^2 = 3n + 18$

$n^2-3n-18=0$

$(n-6)(n+3)=0$

$n=6 \quad n=-3$

8. $2x^2 + 3x = 5$

$2x^2+3x-5=0$

$x^2+3x-10=0$

$(x+5)(x-2)$

$(2x+5)(x-1)=0$

$2x+5=0 \quad x-1=0$

$\frac{2x}{2}=\frac{-5}{2}$

$x=-\frac{5}{2} \quad x=1$

9. $4x^2 = 64$

$\frac{4x^2-64}{4}=\frac{0}{4}$

$x^2-16=0$

$(x+4)(x-4)=0$

$x=-4 \quad x=4$

10. $4x^2 + 4x - 3 = 0$

$x^2+4x-12$

$(x+6)(x-2)$

$(x+\frac{3}{2})(x-\frac{1}{2})$

$0=(2x+3)(2x-1)$

$2x+3=0 \quad 2x-1=0$

$\frac{2x}{2}=\frac{-3}{2} \quad \frac{2x}{2}=\frac{1}{2}$

$x=-\frac{3}{2} \quad x=\frac{1}{2}$

Solving Quadratic Equations Using Completing the Square

1. Which equation has the same solution as $x^2 - 6x - \cancel{12} = 0$?

1) $(x+3)^2 = 21$
2) ~~2)~~ $(x-3)^2 = 21$
3) $(x+3)^2 = 3$
4) $(x-3)^2 = 3$

$\left(\frac{b}{2}\right)^2$ $\left(\frac{-6}{2}\right)^2$ 9

$+12 \quad +12$
$x^2-6x=12$
$x^2-6x+\boxed{9}=12+\boxed{9}$
$(x-3)(x-3)=21$
$(x-3)^2=21$

2. Which equation has the same solutions as $x^2 - 8x + \cancel{3} = 0$?

(1) $(x-8)^2 = 16$
(2) $(x-8)^2 = 13$
~~(3)~~ $(x-4)^2 = 13$
(4) $(x-4)^2 = 61$

$\left(\frac{b}{2}\right)^2$ $\left(\frac{-8}{2}\right)^2$ 16

$-3 \quad -3$
$x^2-8x=-3$
$x^2-8x+\boxed{16}=-3+\boxed{16}$
$(x-4)(x-4)=13$
$(x-4)^2=13$

3. When solving the equation $x^2 - 8x - \cancel{7} = 0$ by completing the square, which equation is a step in the process?

$\left(\frac{-8}{2}\right)^2$ 16

1) $(x-4)^2 = 9$
2) ~~2)~~ $(x-4)^2 = 23$
3) $(x-8)^2 = 9$
4) $(x-8)^2 = 23$

$+7 \quad +7$
$x^2-8x=7$
$x^2-8x+\boxed{16}=7+\boxed{16}$
$(x-4)(x-4)=23$
$(x-4)^2=23$

4. Which equation has the same solutions as $x^2 + 6x - \cancel{7} = 0$?

$\left(\frac{6}{2}\right)^2$ 9

1) $(x+3)^2 = 2$
2) $(x-3)^2 = 2$
3) $(x-3)^2 = 16$
4) ~~4)~~ $(x+3)^2 = 16$

$+7 \quad +7$
$x^2+6x=7$
$x^2+6x+\boxed{9}=7+\boxed{9}$
$(x+3)(x+3)=16$
$(x+3)^2=16$

5. Which equation has the same solution as $x^2 + 8x - 33 = 0$?

$\left(\frac{8}{2}\right)^2$ 16

1) ~~1)~~ $(x+4)^2 = 49$
2) $(x-4)^2 = 49$
3) $(x+4)^2 = 17$
4) $(x-4)^2 = 17$

$(x+4)^2=49$

$x^2+8x-\cancel{33}=0$
$+33 \quad +33$
$x^2+8x=33$
$x^2+6x+\boxed{16}=33+\boxed{16}$
$(x+4)(x+4)=49$

6. The method of completing the square was used to solve the equation $2x^2 - 12x + 6 = 0$. Which equation is a correct step when using this method?

$\frac{2x^2}{2} \quad \frac{12x}{2} \quad \frac{6}{2} \quad \frac{0}{2}$

$\left(\frac{-6}{2}\right)^2$ 9

1) ~~1)~~ $(x-3)^2 = 6$
2) $(x-3)^2 = -6$
3) $(x-3)^2 = 3$
4) $(x-3)^2 = -3$

$x^2-6x+\cancel{3}=0$
$-3 \quad -3$
$x^2-6x=-3$
$x^2-6x+\boxed{9}=-3+\boxed{9}$
$(x-3)(x-3)=6$
$(x-3)^2=6$

7. The quadratic equation $x^2 - 6x = 12$ is rewritten in the form $(x+p)^2 = q$, where q is a constant. What is the value of p?

1) -12
2) -9
3) -3
4) 9

$\left(\frac{-6}{2}\right)^2$

9

$x^2 - 6x + \boxed{9} = 12 + \boxed{9}$

$(x-3)(x-3) = 21$

$(x-3)^2 = 21$

$(x+p)^2 = q$

$\left(\frac{-8}{2}\right)^2 = 16$

8. What are the solutions to the equation $x^2 - 8x = 10$?

1) $4 \pm \sqrt{10}$
2) $4 \pm \sqrt{26}$
3) $-4 \pm \sqrt{10}$
4) $-4 \pm \sqrt{26}$

$x^2 - 8x + \boxed{16} = 10 + \boxed{16}$

$(x-4)(x-4) = 26$

$\sqrt{(x-4)^2} = \sqrt{26}$

$x - \cancel{4} = \pm\sqrt{26}$
$+4 \quad +4$

$x = 4 \pm \sqrt{26}$

$\left(\frac{4}{2}\right)^2 = 4$

9. What are the roots of the equation $x^2 + 4x - 16 = 0$?

1) $2 \pm 2\sqrt{5}$
2) $-2 \pm 2\sqrt{5}$
3) $2 \pm 4\sqrt{5}$
4) $-2 \pm 4\sqrt{5}$

$+16 \quad +16$

$x^2 + 4x = 16$

$x^2 + 4x + \boxed{4} = 16 + \boxed{4}$

$(x+2)(x+2) = 20$

$\sqrt{(x+2)^2} = \sqrt{20}$

$x + \cancel{2} = \pm\sqrt{20}$
$-2 \quad -2$

$x = -2 \pm \sqrt{20}$

$x = -2 \pm 2\sqrt{5}$

$\sqrt{20}$

$\sqrt{4}\ \sqrt{5}$

$2\sqrt{5}$

$\left(\frac{-6}{2}\right)^2 = 9$

10. If the quadratic formula is used to find the roots of the equation $x^2 - 6x - 19 = 0$, the correct roots are

1) $3 \pm 2\sqrt{7}$
2) $-3 \pm 2\sqrt{7}$
3) $3 \pm 4\sqrt{14}$
4) $-3 \pm 4\sqrt{14}$

$+19 \quad +19$

$x^2 - 6x = 19$

$x^2 - 6x + \boxed{9} = 19 + \boxed{9}$

$(x-3)(x-3) = 28$

$\sqrt{(x-3)^2} = \sqrt{28}$

$x - \cancel{3} = \pm\sqrt{28}$
$+3 \quad +3$

$x = 3 \pm \sqrt{28}$

$x = 3 \pm 2\sqrt{7}$

$\sqrt{28}$

$\sqrt{4}\ \sqrt{7}$

$2\sqrt{7}$

11. What are the solutions to the equation $3(x-4)^2 = 27$?

1) 1 and 7
2) -1 and -7
3) $4 \pm \sqrt{24}$
4) $-4 \pm \sqrt{24}$

$\frac{3}{3} \quad \frac{27}{3}$

$\sqrt{(x-4)^2} = \sqrt{9}$

$x - 4 = \pm 3$
$+4 \quad +4$

$x = 4 \pm 3$

$x = 4 + 3 = 7$

$x = 4 - 3 = 1$

12. Solve the equation $x^2 - 6x = 15$ by completing the square.

$\left(\frac{b}{2}\right)^2 \quad \left(\frac{-6}{2}\right)^2 = 9$

$x^2 - 6x + \boxed{9} = 15 + \boxed{9}$

$(x-3)(x-3) = 24$

$\sqrt{(x-3)^2} = \sqrt{24}$

$x - \cancel{3} = \pm\sqrt{24}$
$+3 \quad +3$

$x = 3 \pm \sqrt{24}$

$x = 3 \pm 2\sqrt{6}$

$\sqrt{24}$

$\sqrt{4}\ \sqrt{6}$

$2\sqrt{6}$

13. Solve the following equation by completing the square: $x^2 + 4x = 2$

$\left(\frac{b}{2}\right)^2$

$\left(\frac{4}{2}\right)^2 = 4$

$x^2 + 4x + \boxed{4} = 2 + \boxed{4}$

$(x+2)(x+2) = 6$

$\sqrt{(x+2)^2} = \sqrt{6}$

$x + \cancel{2} = \pm\sqrt{6}$
$-2 \quad -2$

$x = -2 \pm \sqrt{6}$

Modeling Quadratics

Picture frame questions: Add 2x to the original length AND width.

Area of a rectangle: $A = lw$

1. A school is building a rectangular soccer field that has an area of 6000 square yards. The soccer field must be 40 yards longer than its width. Determine algebraically the dimensions of the soccer field, in yards.

l: $x+40 = 60+40 = 100$
w: $x = 60$
A: 6000

$A = lw$
$6000 = x(x+40)$
$6000 = x^2+40x$
$-6000 \quad -6000$

$x^2+40x-6000=0$
$(x+100)(x-60)=0$
$x+100=0 \quad x-60=0$
$-100 \; -100 \quad +60 \; +60$
$\cancel{x=-100} \quad x=60$

100 by 60

2. A landscaper is creating a rectangular flower bed such that the width is half of the length. The area of the flower bed is 34 square feet. Write and solve an equation to determine the width of the flower bed, to the *nearest tenth of a foot.*

l: x
w: $\frac{1}{2}x$
A: 34

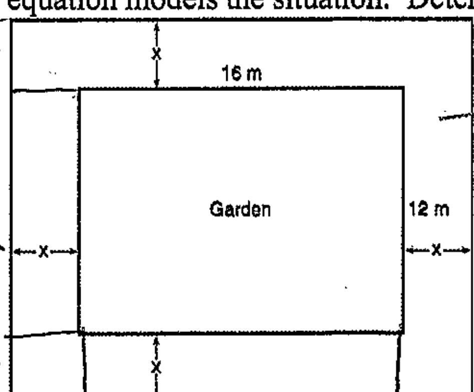
$A = lw$
$34 = x(\frac{1}{2}x)$
$2(34) = (\frac{1}{2}x^2)2$

$\sqrt{68} = \sqrt{x^2}$
$8.2 = x$

$\frac{1}{2}(8.2) = 4.1$

3. A rectangular garden measuring 12 meters by 16 meters is to have a walkway installed around it with a width of *x* meters, as shown in the diagram below. Together, the walkway and the garden have an area of 396 square meters.

Write an equation that can be used to find *x*, the width of the walkway. Describe how your equation models the situation. Determine and state the width of the walkway, in meters.



x
12
x
2x+12
x 16 x
2x+16

$A = lw$
$396 = (2x+12)(2x+16)$
$396 = 4x^2+56x+192$
$-396 \quad -396$
$\frac{0}{4} = \frac{4x^2}{4}+\frac{56x}{4}-\frac{204}{4}$
$0 = x^2+14x-51$
$0 = (x+17)(x-3)$
$x+17=0 \quad x-3=0$
$-17 \; -17 \quad +3 \; +3$
$\cancel{x=-17} \quad x=3$

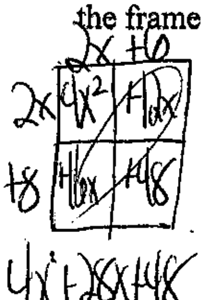
width = 3

	2x	+12
2x	$4x^2$	+24x
+16	+32x	+192

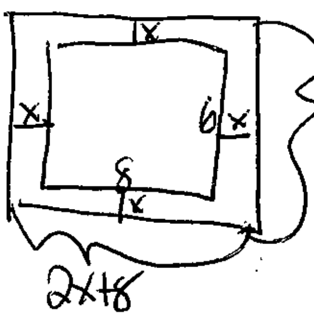
$4x^2+56x+192$

4. A rectangular picture measures 6 inches by 8 inches. Simon wants to build a wooden frame for the picture so that the framed picture takes up a maximum area of 100 square inches on his wall. The pieces of wood that he uses to build the frame all have the same width. Write an equation or inequality that could be used to determine the maximum width of the pieces of wood for the frame Simon could create. Explain how your equation or inequality models the situation. Solve the equation or inequality to determine the maximum width of the pieces of wood used for the frame to the *nearest tenth of an inch.*

$a=1$
$b=7$
$c=-13$



$4x^2+28x+48$



$A=lw$

$100=(2x+6)(2x+8)$

$100=4x^2+28x+48$
$-100 \qquad -100$

$\frac{0}{4}=\frac{4x^2}{4}+\frac{28x}{4}-\frac{52}{4}$

$0=x^2+7x-13$

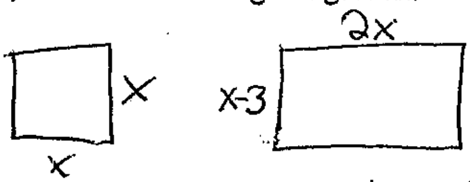
$x=\frac{-7\pm\sqrt{(7)^2-4(1)(-13)}}{2(1)}$

$x=\frac{-7\pm\sqrt{101}}{2}$

$x=1.5$

5. New Clarendon Park is undergoing renovations to its gardens. One garden that was originally a square is being adjusted so that one side is doubled in length, while the other side is decreased by three meters. The new rectangular garden will have an area that is 25% more than the original square garden. Write an equation that could be used to determine the length of a side of the original square garden. Explain how your equation models the situation. Determine the area, in square meters, of the new rectangular garden.

$x^2-8x=0$
$x(x-8)=0$
$x=0 \quad x=8$
$A=1.25(8)$
$A=80$



$A=x^2 \qquad A=1.25x^2$

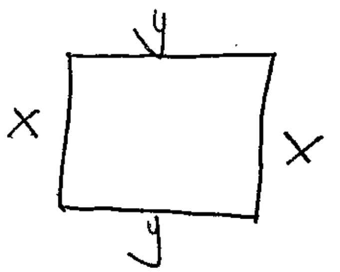
$A=lw$

$1.25x^2=2x(x-3)$

$1.25x^2=2x^2-6x$
$-1.25x^2 \quad -1.25x^2$

$\frac{0}{.75}=\frac{.75x^2}{.75}-\frac{6x}{.75}$

6. A contractor has 48 meters of fencing that he is going to use as the perimeter of a rectangular garden. The length of one side of the garden is represented by x, and the area of the garden is 108 square meters. Determine, algebraically, the dimensions of the garden in meters.



$2x+2y=48$

$\frac{xy}{x}=\frac{108}{x}$

$y=\frac{108}{x}$

$x\left(2x+2\left(\frac{108}{x}\right)\right)=(48)x$

$2x^2+2(108)=48x$

$2x^2+216=48x$
$-48x \quad -48x$

$\frac{2x^2}{2}-\frac{48x}{2}+\frac{216}{2}=\frac{0}{2}$

$x^2-24x+108$

$(x-18)(x-6)$

$x=18 \quad x=6$

18×6

Zeros, Vertex, Axis of Symmetry

The zeros (roots) hit the x axis. Graph the equation in the calculator and look at the graph.
The vertex (maximum/minimum) is the turning point.
The axis of symmetry (AOS) is the vertical line that cuts the graph in half. $x = \#$

1. The zeros of the function $f(x) = (x+2)^2 - 25$ are
1) -2 and 5
2) -3 and 7
3) -5 and 2
4) -7 and 3

2. The zeros of the function $f(x) = 2x^2 - 4x - 6$ are
1) 3 and -1
2) 3 and 1
3) -3 and 1
4) -3 and -1

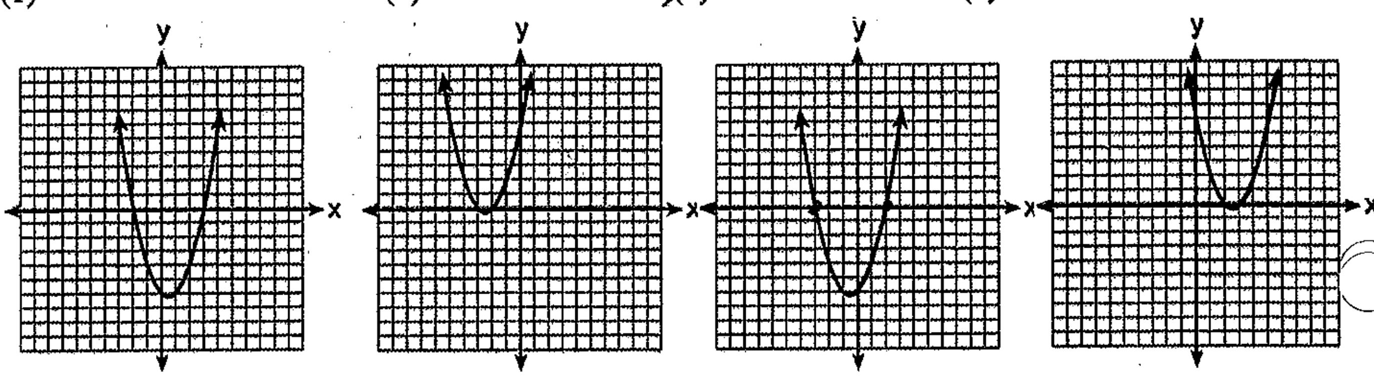
3. The zeros of the function $p(x) = x^2 - 2x - 24$ are
1) -8 and 3
2) -6 and 4
3) -4 and 6
4) -3 and 8

4. For which function defined by a polynomial are the zeros of the polynomial -4 and -6?
1) $y = x^2 - 10x - 24$
2) $y = x^2 + 10x + 24$
3) $y = x^2 + 10x - 24$
4) $y = x^2 - 10x + 24$

5. If $4x^2 - 100 = 0$, the roots of the equation are
1) -25 and 25
2) -25, only
3) -5 and 5
4) -5, only

6. The graphs below represent functions defined by polynomials. For which function are the zeros of the polynomials 2 and -3?

(1) (2) (3) (4)

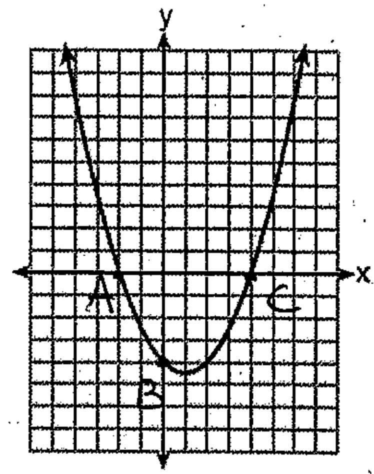


7. Which polynomial function has zeros at -3, 0, and 4?

1) $f(x) = (x+3)(x^2+4)$
2) $f(x) = (x^2-3)(x-4)$
3) $f(x) = x(x+3)(x-4)$
4) $f(x) = x(x-3)(x+4)$

8. The graph of $y = \frac{1}{2}x^2 - x - 4$ is shown below. The points $A(-2,0)$, $B(0,-4)$, and $C(4,0)$ lie on this graph.

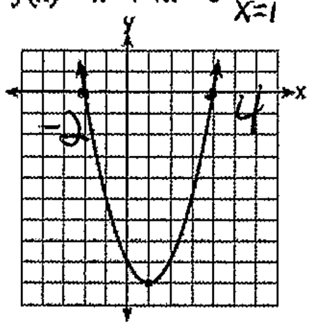
Which of these points can determine the zeros of the equation $y = \frac{1}{2}x^2 - x - 4$?

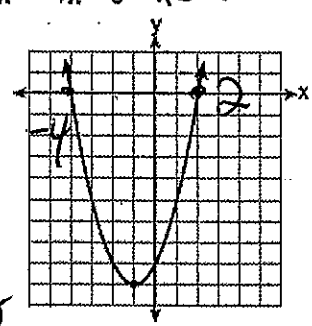


1) A, only
2) B, only
3) A and C, only
4) A, B, and C

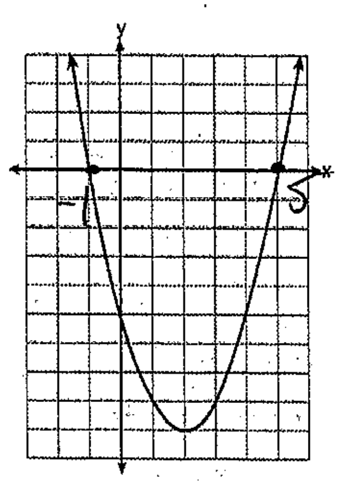
9. Which function has zeros of -4 and 2?

1) $f(x) = x^2 + 7x - 8$ $x=-8$ $x=1$
3) $g(x) = x^2 - 7x - 8$ $x=8$ $x=-1$

2)


4)


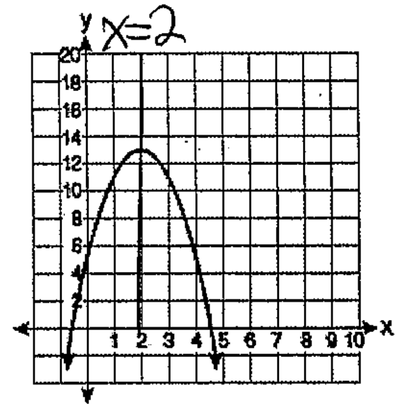
10. The graph of $f(x)$ is shown below.



Based on this graph, what are the roots of the equation $f(x) = 0$?

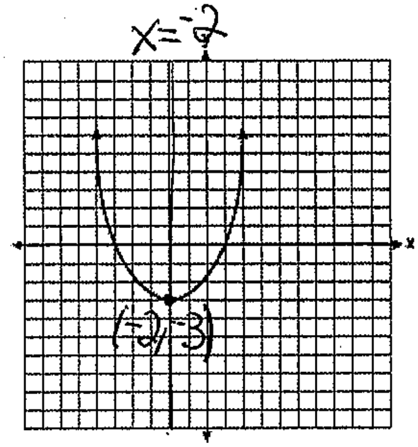
1) 1 and −5
2) −1 and 5
3) 2 and −9
4) −1 and −5 and 5

11. What is the equation of the axis of symmetry of the parabola shown in the diagram below?



1) $x = -0.5$
2) $x = 2$
3) $x = 4.5$
4) $x = 13$

12. What are the vertex and the axis of symmetry of the parabola shown in the diagram below?

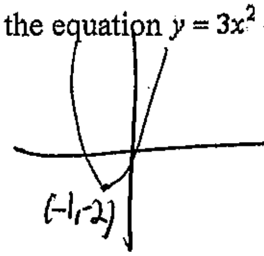


1) The vertex is $(-2,-3)$, and the axis of symmetry is $x = -2$.
2) The vertex is $(-2,-3)$, and the axis of symmetry is $y = -2$.
3) The vertex is $(-3,-2)$, and the axis of symmetry is $y = -2$.
4) The vertex is $(-3,-2)$, and the axis of symmetry is $x = -2$.

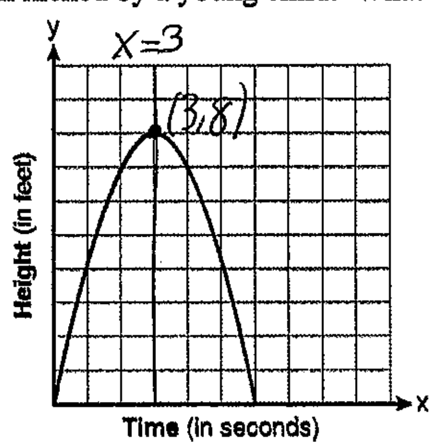
type in calc

13. What is the vertex of the graph of the equation $y = 3x^2 + 6x + 1$?

1) $(-1,-2)$
2) $(-1,10)$
3) $(1,-2)$
4) $(1,10)$



14. The graph below represents the parabolic path of a ball kicked by a young child. What are the vertex and the axis of symmetry for the parabola?



1) vertex: (3,8); axis of symmetry: $x = 3$
2) vertex: (3,8); axis of symmetry: $y = 3$
3) vertex: (8,3); axis of symmetry: $x = 3$
4) vertex: (8,3); axis of symmetry: $y = 3$

Vertex Form of a Parabola

Axis of Symmetry Method

1) Find the axis of symmetry graphically or using $x = -\frac{b}{2a}$
2) Find the y coordinate of the vertex by substituting axis of symmetry into equation
3) Substitute vertex is into $f(x) = a(x-v)^2 + t$ where (v,t) is the vertex

Completing the Square Method

1) $f(x) + c = a(x^2 + bx)$
2) Add the *distributed value* of $\left(\frac{b}{2}\right)^2$ to both sides
3) Factor the trinomial
4) Re-write as a binomial squared
5) Isolate $f(x)$

Rewrite the following equations in vertex form and state the vertex

1. $f(x) = x^2 + 6x + 2$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{-6}{2(1)}$

$x = -3$

$y = (-3)^2 + 6(-3) + 2$

$y = -7$ $(-3,-7)$

$y = (x+3)^2 - 7$

CTS Method

$f(x) - 2 = x^2 + 6x$

$9 + f(x) - 2 = x^2 + 6x + 9$

$f(x) + 7 = (x+3)^2$
$\quad -7 \qquad -7$

$f(x) = (x+3)^2 - 7$

$(-3,-7)$

2. $f(x) = x^2 - 8x + 3$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{8}{2(1)}$

$x = 4$

$y = (4)^2 - 8(4) + 3$

$y = -13$ $(4,-13)$

$y = (x-4)^2 - 13$

CTS Method

$f(x) - 3 = x^2 - 8x$

$16 + f(x) - 3 = x^2 - 8x + 16$

$f(x) + 13 = (x-4)^2$
$\quad -13 \qquad -13$

$f(x) = (x-4)^2 - 13$

$(4,-13)$

3. $f(x) = 2x^2 + 12x - 6$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{-12}{2(2)}$

$x = -3$

$f(-3) = 2(-3)^2 + 12(-3) - 6$

$f(-3) = -24$

$(-3,-24)$

$f(x) = 2(x+3)^2 - 24$

CTS Method

$f(x) + 6 = 2(x^2 + 6x)$ 18

$18 + f(x) + 6 = 2(x^2 + 6x + 9)$

$f(x) + 24 = 2(x+3)^2$
$\quad -24 \qquad -24$

$f(x) = 2(x+3)^2 - 24$

$(-3,-24)$

4. $y = 4x^2 + 8x - 6$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{-8}{2(4)}$

$x = -1$

$y = 4(-1)^2 + 8(-1) - 6$

$y = -10$

$(-1,-10)$

$y = 4(x+1)^2 - 10$

CTS Method

$y + 6 = 4(x^2 + 2x)$

$4 + y + 6 = 4(x^2 + 2x + 1)$

$y + 10 = 4(x+1)^2$
$\quad -10 \qquad -10$

$y = 4(x+1)^2 - 10$

$(-1,-10)$

5. $f(x) = -x^2 + 4x + 16$

CTS Method

$f(x) - 16 = -1(x^2 - 4x)$

$4 + f(x) - 16 = -1(x^2 - 4x + 4)$

$f(x) - 20 = -1(x-2)^2$

$f(x) = -1(x-2)^2 + 20$

$(2,20)$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{-4}{2(-1)}$

$x = 2$

$f(2) = -(2)^2 + 4(2) + 16$

$f(2) = 20$

$(2,20)$

$f(x) = -1(x-2)^2 + 20$

6. $f(x) = x^2 + 12x + 2$

CTS Method

$f(x) - 2 = x^2 + 12x$

$36 + f(x) - 2 = x^2 + 12x + 36$

$f(x) + 34 = (x+6)^2$

$f(x) = (x+6)^2 - 34$

$(-6,-34)$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{-12}{2(1)}$

$x = -6$

$f(-6) = (-6)^2 + 12(-6) + 2$

$f(-6) = -34 \quad (-6,-34)$

$f(x) = (x+6)^2 - 34$

7. $f(x) = -x^2 + 14x + 20$

CTS Method

$f(x) - 20 = -1(x^2 - 14x)$

$-49 + f(x) - 20 = -1(x^2 - 14x + 49)$

$f(x) - 69 = -1(x-7)^2$

$f(x) = -1(x-7)^2 + 69$

$(7,69)$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{-14}{2(-1)}$

$x = 7$

$f(7) = -(7)^2 + 14(7) + 20$

$f(7) = 69$

$(7,69)$

8. $f(x) = 4x^2 + 12x - 28$

CTS Method

$f(x) + 28 = 4(x^2 + 3x)$

$9 + f(x) + 28 = 4(x^2 + 3x + \frac{9}{4})$

$f(x) + 37 = 4(x + \frac{3}{2})^2$

$f(x) = 4(x + \frac{3}{2})^2 - 37$

$(-\frac{3}{2}, -37)$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{-12}{2(4)}$

$x = -\frac{3}{2}$

$f(-\frac{3}{2}) = 4(-\frac{3}{2})^2 + 12(-\frac{3}{2}) - 28$

$f(-\frac{3}{2}) = -37$

$(-\frac{3}{2}, -37)$

8. Identify the turning point of the function $f(x) = x^2 - 2x + 8$ by writing its equation in vertex form.

$1 + f(x) - 8 = x^2 - 2x + 1$

$f(x) - 7 = (x-1)^2$

$f(x) = (x-1)^2 + 7$

$(1,7)$

9. Given the function $f(x) = -x^2 + 8x + 9$, state whether the vertex represents a maximum or minimum point for the function. Explain your answer.
Rewrite $f(x)$ in vertex form by completing the square.

maximum because it opens down.

$f(x) - 9 = -1(x^2 - 8x)$

$-16 + f(x) - 9 = -1(x^2 - 8x + 16)$

$f(x) - 25 = -1(x-4)^2$

$f(x) = -1(x-4)^2 + 25$

Modeling Parabolas

The initial height is the y-intercept/constant term.
The domain is from [0,second zero].
The vertex is the turning point. 2^{nd} Trace (Calc): Maximum/Minimum
The object hits the ground at the second zero
To find the zeros algebraically, set the equation equal to zero, factor, set each factor equal to 0.

1. The expression $-4.9t^2 + 50t + \underline{2}$ represents the height, in meters, of a toy rocket t seconds after launch. The initial height of the rocket, in meters, is

y-intercept

1) 0
2) 2
3) 4.9
4) 50

2. The height of a ball Doreen tossed into the air can be modeled by the function $h(x) = -4.9x^2 + 6x + 5$, where x is the time elapsed in seconds, and $h(x)$ is the height in meters. The number 5 in the function represents

1) the initial height of the ball
   y-intercept
2) the time at which the ball reaches the ground
3) the time at which the ball was at its highest point
4) the maximum height the ball attained when thrown in the air

3. A toy rocket is launched from the ground straight upward. The height of the rocket above the ground, in feet, is given by the equation $h(t) = -16t^2 + 64t$, where t is the time in seconds. Determine the domain for this function in the given context. Explain your reasoning.

[0,4]
the ~~ball~~ rocket hits the ground at $t=4$.

4. The function $h(t) = -16t^2 + 144$ represents the height, $h(t)$, in feet, of an object from the ground at t seconds after it is dropped. A realistic domain for this function is

1) $-3 \le t \le 3$
2) $0 \le t \le 3$
3) $0 \le h(t) \le 144$
4) all real numbers

[0,3] the zero is 3

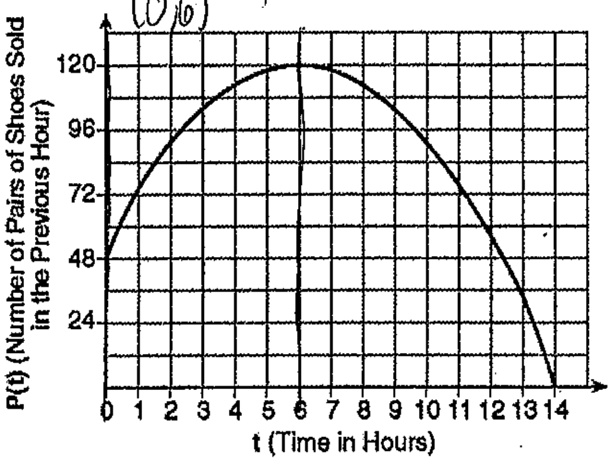
5. Morgan throws a ball up into the air. The height of the ball above the ground, in feet, is modeled by the function $h(t) = -16t^2 + 24t$, where t represents the time, in seconds, since the ball was thrown. What is the appropriate domain for this situation?

1) $0 \le t \le 1.5$ 1.5 is a zero
2) $0 \le t \le 9$
3) $0 \le h(t) \le 1.5$
4) $0 \le h(t) \le 9$

domain is x values

6. A manager wanted to analyze the online shoe sales for his business. He collected data for the number of pairs of shoes sold each hour over a 14-hour time period. He created a graph to model the data, as shown below.
The manager believes the set of integers would be the most appropriate domain for this model. Explain why he is *incorrect*. State the entire interval for which the number of pairs of shoes sold is increasing. Determine the average rate of change between the sixth and fourteenth hours, and explain what it means in the context of the problem.

increasing
(0,6)



Shoes can be sold at fractional hours. Also, they can't be sold at negative hours.

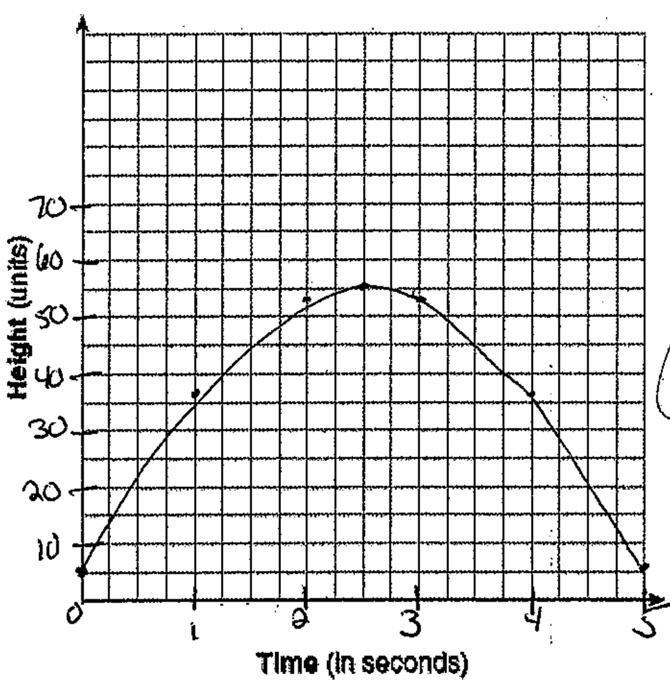
$\frac{f(b)-f(a)}{b-a}$

x	y
6	120
14	0

$\frac{0-120}{14-6} = -15$

On average, between 6 and 14 hours, the number of pairs of shoes sold decreased by 15 pairs per hour

7. Alex launched a ball into the air. The height of the ball can be represented by the equation $h = -8t^2 + 40t + 5$, where h is the height, in units, and t is the time, in seconds, after the ball was launched. Graph the equation from $t = 0$ to $t = 5$ seconds.
State the coordinates of the vertex and explain its meaning in the context of the problem.



x	y
0	5
1	37
2	53
3	53
4	37
5	5

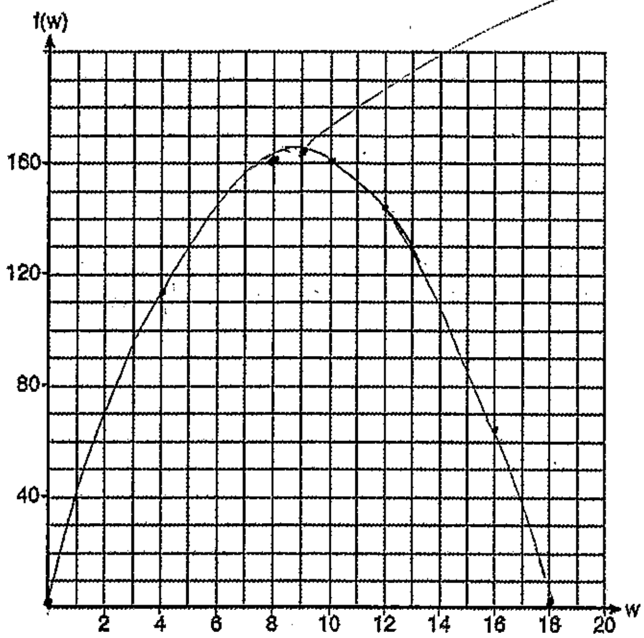
(2.5, 55)

2nd Trace, Maximum

At 2.5 seconds, the maximum height of the ball is 55 units

8. Paul plans to have a rectangular garden adjacent to his garage. He will use 36 feet of fence to enclose three sides of the garden. The area of the garden, in square feet, can be modeled by $f(w) = w(36 - 2w)$, where w is the width in feet. On the set of axes below, sketch the graph of $f(w)$. Explain the meaning of the vertex in the context of the problem.

2nd Trace, Maximum
(9, 162)



x	y
0	0
4	112
8	160
~~12~~	~~144~~
~~18~~	
9	162
10	160
12	144
16	64
18	0

At a width of 9 feet, the maximum area of the garden will be 162 square feet.

9. An Air Force pilot is flying at a cruising altitude of 9000 feet and is forced to eject from her aircraft. The function $h(t) = -16t^2 + 128t + 9000$ models the height, in feet, of the pilot above the ground, where t is the time, in seconds, after she is ejected from the aircraft. Determine and state the vertex of $h(t)$. Explain what the second coordinate of the vertex represents in the context of the problem. After the pilot was ejected, what is the maximum number of feet she was above the aircraft's cruising altitude? Justify your answer.

2nd Trace, maximum (4, 9256) The maximum height of the pilot was 9,256 feet.

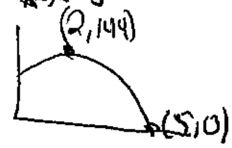
$$\begin{array}{r} 9256 \\ -9000 \\ \hline 256 \text{ feet} \end{array}$$

10. Let $h(t) = -16t^2 + 64t + 80$ represent the height of an object above the ground after t seconds. Determine the number of seconds it takes to achieve its maximum height. Justify your answer. State the time interval, in seconds, during which the height of the object *decreases*. Explain your reasoning.

2nd Trace, maximum *Adjust window 2nd Trace, zero

(2, 144) (5, 0)

2 seconds



decreases (2, 5)

It is decreasing from its maximum until it hits the ground.

11. When an apple is dropped from a tower 256 feet high, the function $h(t) = -16t^2 + 256$ models the height of the apple, in feet, after t seconds. Determine, algebraically, the number of seconds it takes the apple to hit the ground.

Factor

$f(x) = 0$

$$\frac{0}{-16} = \frac{-16x^2 + 256}{-16}$$

$$0 = x^2 - 16$$

$$0 = (x+4)(x-4)$$

$$\cancel{x = -4} \quad x = 4$$

Can't have negative time

4 seconds

12. The height, H, in feet, of an object dropped from the top of a building after t seconds is given by $H(t) = -16t^2 + 144$. How many feet did the object fall between one and two seconds after it was dropped? Determine, algebraically, how many seconds it will take for the object to reach the ground.

$f(x) = 0$

$$H(1) = -16(1)^2 + 144 = 128$$

$$H(2) = -16(2)^2 + 144 = 80$$

$$128 - 80 = 48 \text{ feet}$$

$$\frac{0}{-16} = \frac{-16t^2 + 144}{-16}$$

$$0 = t^2 - 9$$

$$0 = (t+3)(t-3)$$

$$\cancel{t = -3} \quad t = 3$$

3 Seconds

13. The height, H, in feet, of an object dropped from the top of a building after t seconds is given by $H(t) = -16t^2 + 144$. How many feet did the object fall between one and two seconds after it was dropped? Determine, algebraically, how many seconds it will take for the object to reach the ground.

Same as 12

Functions

A function is when each x value corresponds ("talks") to only one y value (x does not repeat).
A graph is a function if it passes the vertical line test (no vertical line can touch twice)

1. Which relation is *not* a function?
1) $\{(2,4),(1,2),(0,0),(-1,2),(-2,4)\}$
2) $\{(2,4),(1,1),(0,0),(-1,1),(-2,4)\}$
3) $\{(2,2),(1,1),(0,0),(-1,1),(-2,2)\}$
4) $\{(2,2),(1,1),(0,0),(1,-1),(2,-2)\}$ x values repeat

2. Which set is a function?
1) $\{(3,4),(3,5),(3,6),(3,7)\}$
2) $\{(1,2),(3,4),(4,3),(2,1)\}$
3) $\{(6,7),(7,8),(8,9),(6,5)\}$
4) $\{(0,2),(3,4),(0,8),(5,6)\}$

3. Which set of ordered pairs does *not* represent a function?
1) $\{(3,-2), (-2,3), (4,-1), (-1,4)\}$
2) $\{(3,-2), (3,-4), (4,-1), (4,-3)\}$ x values repeat
3) $\{(3,-2), (4,-3), (5,-4), (6,-5)\}$
4) $\{(3,-2), (5,-2), (4,-2), (-1,-2)\}$

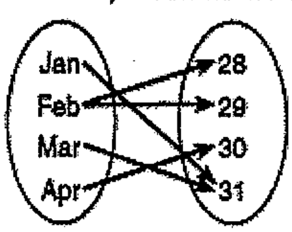
4. A function is defined as $\{(0,1),(2,3),(5,8),(7,2)\}$. Isaac is asked to create one more ordered pair for the function. Which ordered pair can he add to the set to keep it a function?
1) $(0,2)$ 0 would repeat
2) $(5,3)$ 5 would repeat
3) $(7,0)$ 7 would repeat
4) $(1,3)$ 1 would not repeat

5. A function is shown in the table below.
If included in the table, which ordered pair, $(-4,1)$ or $(1,-4)$, would result in a relation that is no longer a function? Explain your answer.

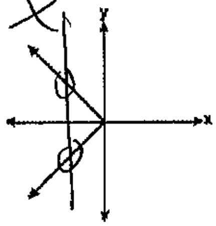
x	f(x)
–4	2
–1	–4
0	–2
3	16

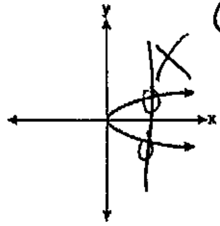
(-4,1) would make -4 repeat. There would be two outputs for one input.

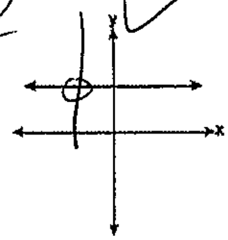
6. A mapping is shown in the diagram below.
This mapping is
1) a function, because Feb has two outputs, 28 and 29
2) a function, because two inputs, Jan and Mar, result in the output 31
3) not a function, because Feb has two outputs, 28 and 29
4) not a function, because two inputs, Jan and Mar, result in the output 31

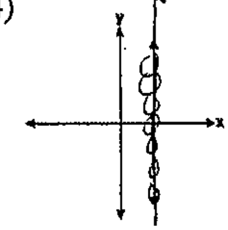


7. Which graph represents a function?

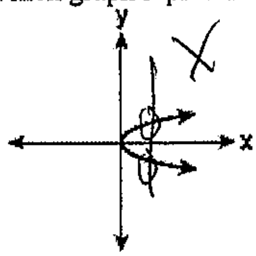
1)


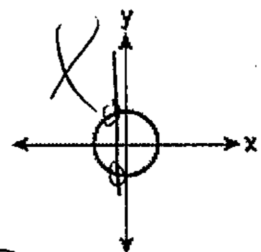
2)


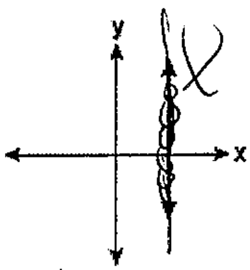
3)


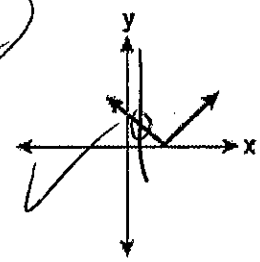
4)


8. Which graph represents a function?

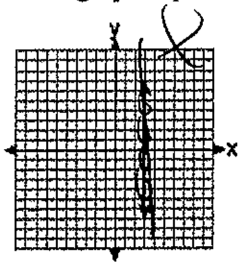
1)


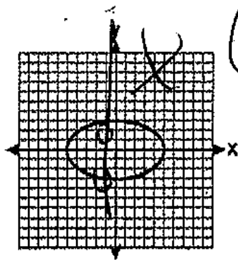
3)


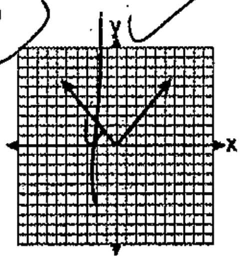
2)


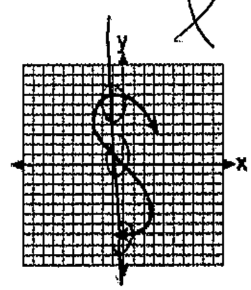
4)


9. Which graph represents a function?

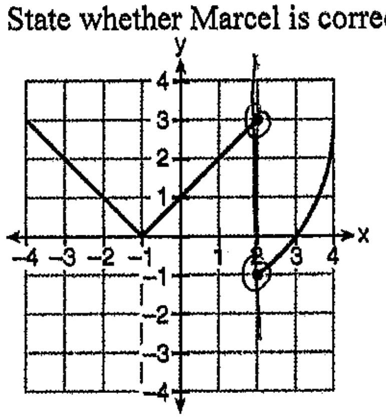
1)


2)


3)


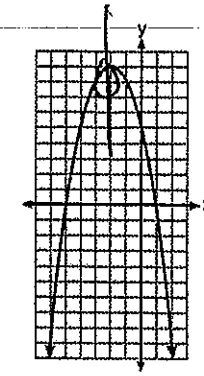
4)


10. Marcel claims that the graph below represents a function.
State whether Marcel is correct. Justify your answer.



No, it does not pass the vertical line test. 2 has 2 outputs.

11. A relation is graphed on the set of axes below.
Based on this graph, the relation is



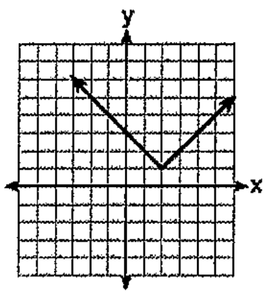
1) a function because it passes the horizontal line test
2) a function because it passes the vertical line test
3) not a function because it fails the horizontal line test
4) not a function because it fails the vertical line test

12. Which relation does *not* represent a function?

1)

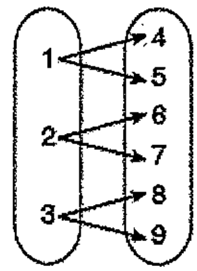
x	1	2	3	4	5	6
y	3.2	4	5.1	6	7.4	8.8

2)



3) $y = 3\sqrt{x+1} - 2$

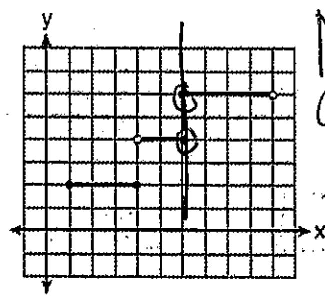
4)



Each input has multiple outputs

13. Four relations are shown below. State which relation(s) are functions. Explain why the other relation(s) are *not* functions.

III and IV are functions.



I

Not a function because it does not pass the vertical line test.

$\{(1,2), (2,5), (3,8), (2,-5), (1,-2)\}$

II

Not a function because 2 has multiple outputs

x	y
−4	1
0	3
4	5
6	6

III

$y = x^2$

IV

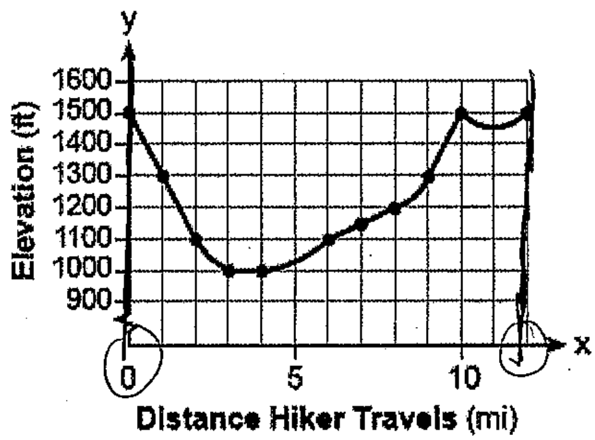
Domain and Range
Domain is all possible x values
Range is all possible y values
If given an equation, use your calculator
If given a graph, using your pencil, for domain travel vertically along the x axis and for range travel horizontally along the y axis to see where the values begin and end.
Modeling Domain: The answer is almost always:
Non-Negative Integers {0,1,2,3,4,...}: If 0 is possible
OR
Positive Integers {1,2,3,4,...}: If 0 is not possible (workers needed to complete a job)

1. What is the domain of the relation shown below?

$\{(4,2),(1,1),(0,0),(1,-1),(4,-2)\}$

1) $\{0,1,4\}$
2) $\{-2,-1,0,1,2\}$
3) $\{-2,-1,0,1,2,4\}$
4) $\{-2,-1,0,0,1,1,1,2,4,4\}$

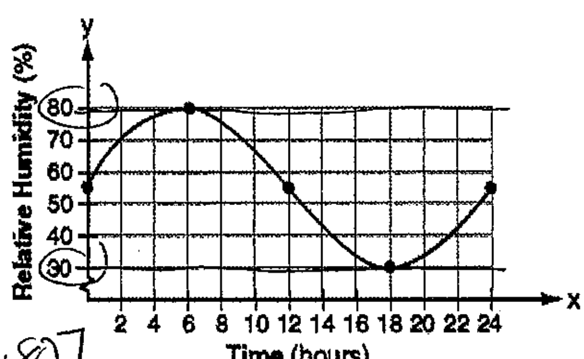
2. The accompanying graph shows the elevation of a certain region in New York State as a hiker travels along a trail.



What is the domain of this function?
(1) $1{,}000 \le x \le 1{,}500$
(2) $1{,}000 \le y \le 1{,}500$
(3) $0 \le x \le 12$
(4) $0 \le y \le 12$

[0,12]

3. A meteorologist drew the accompanying graph to show the changes in relative humidity during a 24-hour period in New York City.

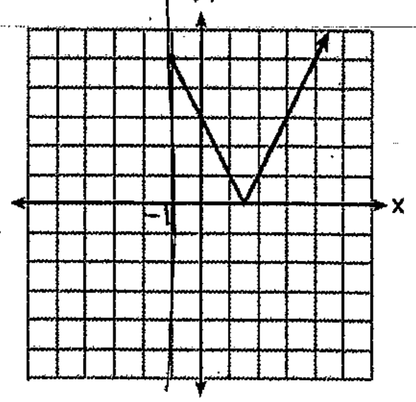


What is the range of this set of data?
(1) $0 \le y \le 24$
(2) $0 \le x \le 24$
(3) $30 \le y \le 80$
(4) $30 \le x \le 80$

[30,80]

$[-1, \infty)$

4. The function $f(x)$ is graphed below.



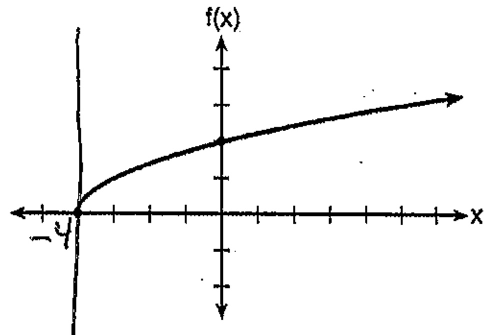
The domain of this function is

1) all positive real numbers
2) all positive integers
3) $x \geq 0$
4) $x \geq -1$

5. The graph of the function $f(x) = \sqrt{x+4}$ is shown below.
The domain of the function is

1) $\{x|x > 0\}$
2) $\{x|x \geq 0\}$
3) $\{x|x > -4\}$
4) $\{x|x \geq -4\}$

$[-4, \infty)$



6. If the domain of the function $f(x) = 2x^2 - 8$ is $\{-2, 3, 5\}$, then the range is

1) $\{-16, 4, 92\}$
2) $\{-16, 10, 42\}$
3) $\{0, 10, 42\}$
4) $\{0, 4, 92\}$

x	y
-2	0
3	10
5	42

7. The function $f(x) = 2x^2 + 6x - 12$ has a domain consisting of the integers from -2 to 1, inclusive. Which set represents the corresponding range values for $f(x)$?

1) $\{-32, -20, -12, -4\}$
2) $\{-16, -12, -4\}$
3) $\{-32, -4\}$
4) $\{-16, -4\}$

x	y
-2	-16
-1	-16
0	-12
1	-4

8. If the function $f(x) = x^2$ has the domain $\{0, 1, 4, 9\}$, what is its range?

1) $\{0, 1, 2, 3\}$
2) $\{0, 1, 16, 81\}$
3) $\{0, -1, 1, -2, 2, -3, 3\}$
4) $\{0, -1, 1, -16, 16, -81, 81\}$

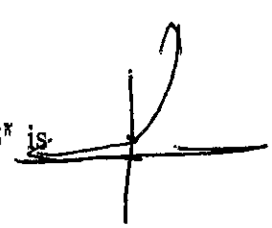
x	y
0	0
1	1
4	16
9	81

9. Let f be a function such that $f(x) = 2x - 4$ is defined on the domain $2 \leq x \leq 6$. The range of this function is

1) $0 \leq y \leq 8$
2) $0 \leq y < \infty$
3) $2 \leq y \leq 6$
4) $-\infty < y < \infty$

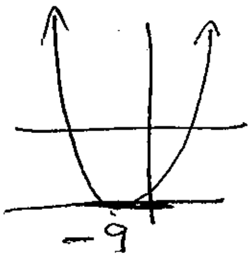
x	y
2	0
3	2
4	4
5	6
6	8

10. The range of the function defined as $y = 5^x$ is



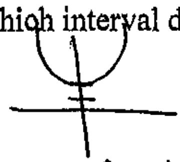
1) $y < 0$
2) $y > 0$
3) $y \leq 0$
4) $y \geq 0$

11. The range of the function $f(x) = x^2 + 2x - 8$ is all real numbers



1) less than or equal to -9
2) greater than or equal to -9
3) less than or equal to -1
4) greater than or equal to -1

12. If $f(x) = x^2 + 2$, which interval describes the range of this function?



1) $(-\infty, \infty)$
2) $[0, \infty)$
3) $[2, \infty)$
4) $(-\infty, 2]$

13. Officials in a town use a function, C, to analyze traffic patterns. $C(n)$ represents the rate of traffic through an intersection where n is the number of observed vehicles in a specified time interval. What would be the most appropriate domain for the function?

1) $\{\ldots -2, -1, 0, 1, 2, 3, \ldots\}$
2) $\{-2, -1, 0, 1, 2, 3\}$
3) $\{0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}\}$
4) $\{0, 1, 2, 3, \ldots\}$

14. Which domain would be the most appropriate set to use for a function that predicts the number of household online-devices in terms of the number of people in the household?

1) integers
2) whole numbers
3) irrational numbers
4) rational numbers

15. A store sells self-serve frozen yogurt sundaes. The function $C(w)$ represents the cost, in dollars, of a sundae weighing w ounces. An appropriate domain for the function is

1) integers
2) rational numbers
3) nonnegative integers
4) nonnegative rational numbers

16. If the function h(x) represents the number of full hours that it takes a person to assemble x sets of tires in a factory, which would be an appropriate domain for the function?

(1) the set of real numbers
(2) the set of negative integers
(3) the set of integers
(4) the set of non-negative integers

17. An online company lets you download songs for \$0.99 each after you have paid a \$5 membership fee. Which domain would be most appropriate to calculate the cost to download songs?

1) rational numbers greater than zero
2) whole numbers greater than or equal to one
3) integers less than or equal to zero
4) whole numbers less than or equal to one

18. At an ice cream shop, the profit, $P(c)$, is modeled by the function $P(c) = 0.87c$, where c represents the number of ice cream cones sold. An appropriate domain for this function is

1) an integer ≤ 0
2) an integer ≥ 0
3) a rational number ≤ 0
4) a rational number ≥ 0

19. The daily cost of production in a factory is calculated using $c(x) = 200 + 16x$, where x is the number of complete products manufactured. Which set of numbers best defines the domain of $c(x)$?

1) integers
2) positive real numbers
3) positive rational numbers
4) whole numbers

Transforming Functions

If adding to f(x), the graph moves up or down

If adding to x, the graph moves left or right (the opposite direction in which you would think)

$y = f(x) + a$ moves UP a units

$y = f(x) - a$ moves DOWN a units

$y = f(x + a)$ moves LEFT a units

$y = f(x - a)$ moves RIGHT a units

If the x is negated, the graph is reflected over the y axis

If the f(x) (aka y) is negated, the graph is reflected over the x axis

$y = f(-x)$ reflect over y axis

$y = -f(x)$ reflect over x axis

If a is positive, the vertex is a minimum and the graph opens upward

If a is negative, the vertex is a maximum and the graph opens downward

$y = af(x)$ Vertical Dilation

If $|a| > 1$, vertical stretch, narrower

If $|a| < 1$, vertical shrink, wider

$y = f(ax)$ Horizontal Dilation

If $|a| > 1$, Horizontal shrink

If $|a| < 1$, Horizontal stretch

right 2
up 3

1. Compared to the graph of $f(x) = x^2$, the graph of $g(x) = (x - 2)^2 + 3$ is the result of translating $f(x)$

1) 2 units up and 3 units right
2) 2 units down and 3 units up
3) 2 units right and 3 units up
4) 2 units left and 3 units right

right 3
down 2

2. Given the parent function $f(x) = x^3$, the function $g(x) = (x - 1)^3 - 2$ is the result of a shift of $f(x)$

1) 1 unit left and 2 units down
2) 1 unit left and 2 units up
3) 1 unit right and 2 units down
4) 1 unit right and 2 units up

(x+3)

3. If the original function $f(x) = 2x^2 - 1$ is shifted to the left 3 units to make the function $g(x)$, which expression would represent $g(x)$?

1) $2(x - 3)^2 - 1$
2) $2(x + 3)^2 - 1$
3) $2x^2 + 2$
4) $2x^2 - 4$

4. Joey's math class is studying the basic quadratic function, $f(x) = x^2$. Each student is supposed to make two new functions by adding or subtracting a constant to the function. Joey chooses the functions $g(x) = x^2 - 5$ and $h(x) = x^2 + 2$. What transformations would map $f(x)$ to $g(x)$ and $f(x)$ to $h(x)$?

down 5   up 2

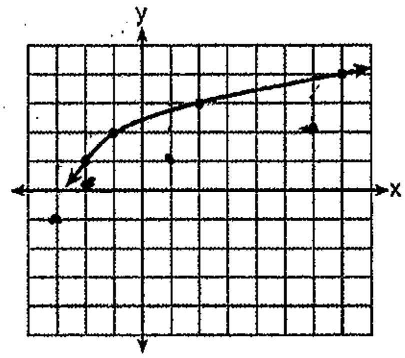
(1) shift left 5, shift right 2
(2) shift right 5, shift left 2
(3) shift up 5, shift down 2
(4) shift down 5, shift up 2

5. Describe the effect that each transformation below has on the function $f(x) = |x|$, where $a > 0$.

$g(x) = |x - a|$ right a units

$h(x) = |x| - a$ down a units

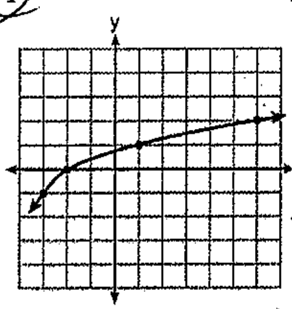
6. The graph of $y = f(x)$ is shown below.



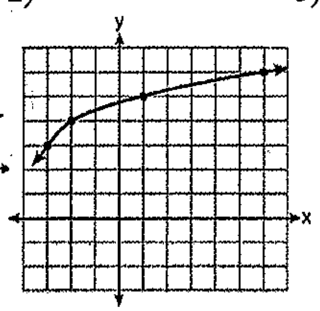
left 1 down 2

What is the graph of $y = f(x + 1) - 2$?

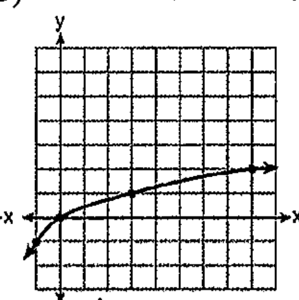
1)



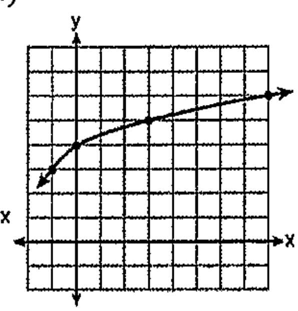
2)



3)



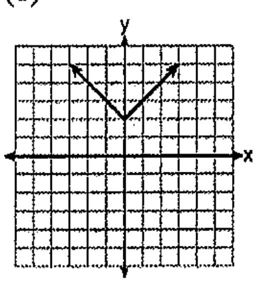
4)



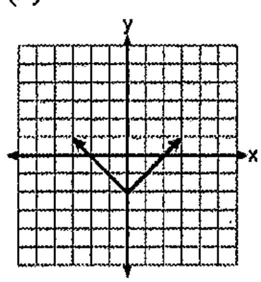
right 2

7. Which graph represents the equation $y = |x - 2|$?

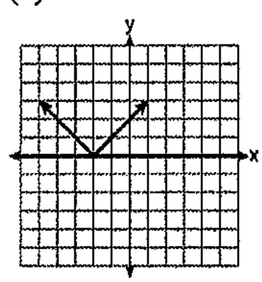
(1)



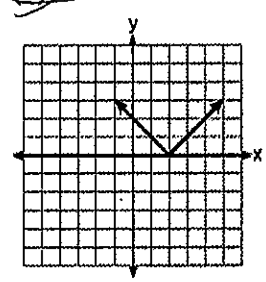
(2)



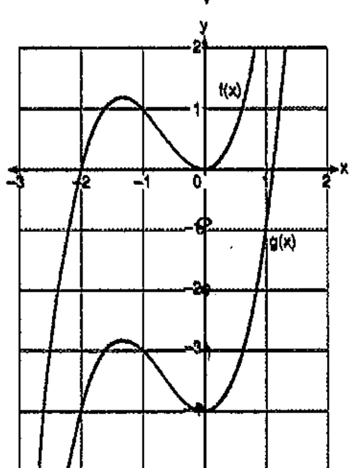
(3)



(4)



8. In the diagram below, $f(x) = x^3 + 2x^2$ is graphed. Also graphed is $g(x)$, the result of a translation of $f(x)$. Determine an equation of $g(x)$. Explain your reasoning.



$g(x) = x^3 + 2x^2 - 4$

The graph was translated down 4.

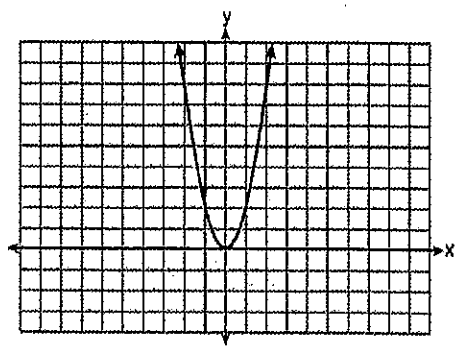
down 4

narrower right 2 up 1

9. How does the graph of $f(x) = 3(x-2)^2 + 1$ compare to the graph of $g(x) = x^2$?

1) The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.
2) The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.
3) The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.
4) The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.

10. The graph of the equation $y = ax^2$ is shown below.



opens downward wider

If a is multiplied by $-\frac{1}{2}$, the graph of the new equation is

1) wider and opens downward
2) wider and opens upward
3) narrower and opens downward
4) narrower and opens upward

11. When the function $f(x) = x^2$ is multiplied by the value a, where $a > 1$, the graph of the new function, $g(x) = ax^2$ upward narrower

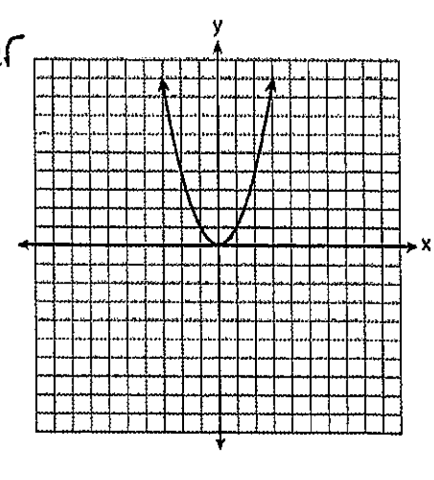
1) opens upward and is wider
2) opens upward and is narrower
3) opens downward and is wider
4) opens downward and is narrower

wider wider

12. In the functions $f(x) = kx^2$ and $g(x) = |kx|$, k is a positive integer. If k is replaced by $\frac{1}{2}$, which statement about these new functions is true?

1) The graphs of both $f(x)$ and $g(x)$ become wider.
2) The graph of $f(x)$ becomes narrower and the graph of $g(x)$ shifts left.
3) The graphs of both $f(x)$ and $g(x)$ shift vertically.
4) The graph of $f(x)$ shifts left and the graph of $g(x)$ becomes wider.

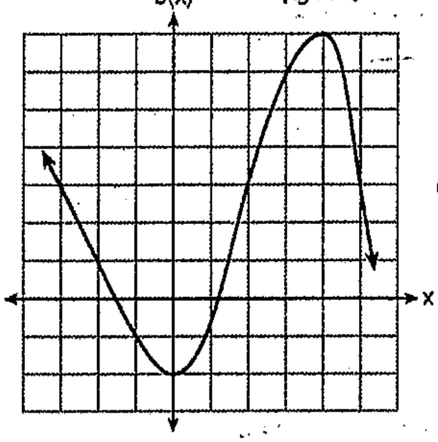
13. The graph of the equation $y = x^2$ is shown below. Which statement best describes the change in this graph when the coefficient of x^2 is multiplied by 4? narrower



1) The parabola becomes wider.
2) The parabola becomes narrower.
3) The parabola will shift up four units.
4) The parabola will shift right four units.

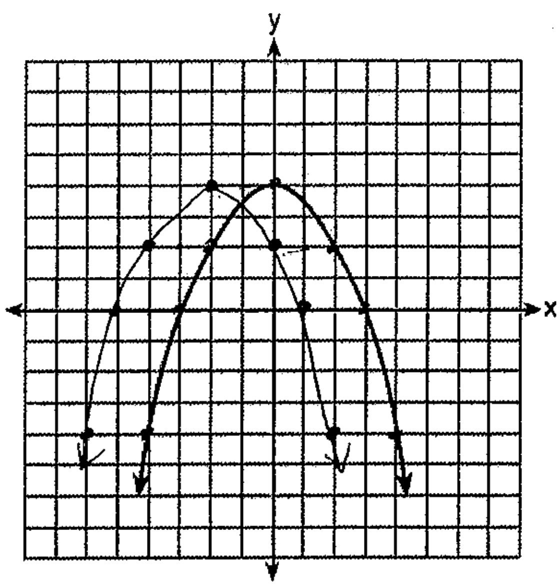
14. Richard is asked to transform the graph of $b(x)$ below. The graph of $b(x)$ is transformed using the equation $h(x) = b(x-2)-3$. Describe how the graph of $b(x)$ changed to form the graph of $h(x)$.

right 2 down 3



It is shifted right 2 and down 3.

15. The graph of the function $p(x)$ is represented below. On the same set of axes, sketch the function $p(x+2)$. left 2



Average rate of change: $\frac{f(b)-f(a)}{b-a}$

Use a table to organize your values. If given an equation, type it into $y=$. If given a graph, pull the values from the graph. $f(b)$ and $f(a)$ are y values. b and a are x values.
If asked which interval has the greatest rate of change, find the average rate of change for each interval.
"On average, between a and b, the function is increasing/decreasing x units per unit of time."

1. Joey enlarged a 3-inch by 5-inch photograph on a copy machine. He enlarged it four times. The table below shows the area of the photograph after each enlargement.

Enlargement	0	1	2	3	4
Area (square inches)	15	18.8	23.4	29.3	36.6

What is the average rate of change of the area from the original photograph to the fourth enlargement, to the *nearest tenth*?
1) 4.3
2) 4.5
3) 5.4
4) 6.0

$\frac{f(b)-f(a)}{b-a}$ $\frac{36.6-15}{4-0}=5.4$

2. A family is traveling from their home to a vacation resort hotel. The table below shows their distance from home as a function of time.
Determine the average rate of change between hour 2 and hour 7, including units.

Time (hrs)	0	2	5	7
Distance (mi)	0	140	375	480

$\frac{f(b)-f(a)}{b-a}$ $\frac{480-140}{7-2}=68$ miles per hour.

3. The table below shows the average diameter of a pupil in a person's eye as he or she grows older.
What is the average rate of change, in millimeters per year, of a person's pupil diameter from age 20 to age 80?
1) 2.4
2) 0.04
3) −2.4
4) −0.04

$\frac{f(b)-f(a)}{b-a}$ $\frac{2.3-4.7}{80-20}=-.04$

Age (years)	Average Pupil Diameter (mm)
20	4.7
30	4.3
40	3.9
50	3.5
60	3.1
70	2.7
80	2.3

4. The table below represents the height of a bird above the ground during flight, with $P(t)$ representing height in feet and t representing time in seconds. Calculate the average rate of change from 3 to 9 seconds, in feet per second.

t	P(t)
0	6.71
3	6.26
4	6
9	3.41

$\frac{f(b)-f(a)}{b-a}$ $\frac{3.41-6.26}{9-3} = -.475$

5. An astronaut drops a rock off the edge of a cliff on the Moon. The distance, $d(t)$, in meters, the rock travels after t seconds can be modeled by the function $d(t) = 0.8t^2$. What is the average speed, in meters per second, of the rock between 5 and 10 seconds after it was dropped?

type into y=

1) 12
2) 20
3) 60
4) 80

x	y
5	20
10	80

$\frac{f(b)-f(a)}{b-a}$ $\frac{80-20}{10-5} = 12$

type into y=

6. A population of rabbits in a lab, $p(x)$, can be modeled by the function $p(x) = 20(1.014)^x$, where x represents the number of days since the population was first counted. Determine, to the *nearest tenth*, the average rate of change from day 50 to day 100.

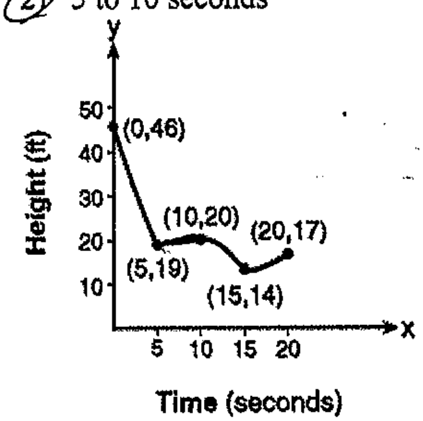
x	y
50	40.08
100	80.32

$\frac{f(b)-f(a)}{b-a}$ $\frac{80.32-40.08}{100-50} = .8$ rabbits per day

7. The graph below models the height of a remote-control helicopter over 20 seconds during flight.
Over which interval does the helicopter have the *slowest* average rate of change?

1) 0 to 5 seconds
2) 5 to 10 seconds
3) 10 to 15 seconds
4) 15 to 20 seconds

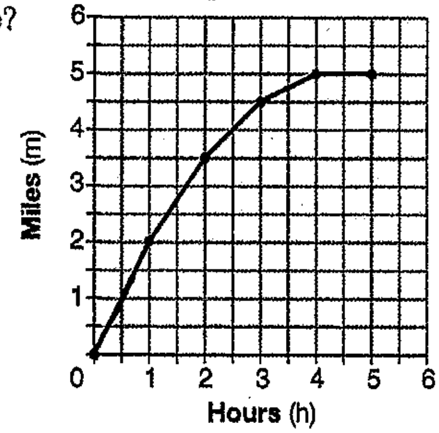
least slope



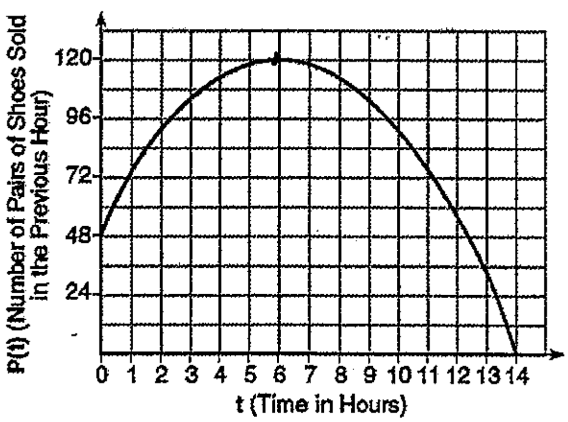
8. The graph below shows the distance in miles, *m*, hiked from a camp in *h* hours. Which hourly interval had the greatest rate of change?

highest slope

1) hour 0 to hour 1
2) hour 1 to hour 2
3) hour 2 to hour 3
4) hour 3 to hour 4



9. A manager wanted to analyze the online shoe sales for his business. He collected data for the number of pairs of shoes sold each hour over a 14-hour time period. He created a graph to model the data, as shown below. Determine the average rate of change between the sixth and fourteenth hours, and explain what it means in the context of the problem.



x	y
6	120
14	0

$$\frac{f(b)-f(a)}{b-a}$$

$$\frac{0-120}{14-6} = -15 \text{ pairs of shoes per hour.}$$

On average, from 6 to 14 hours, the pairs of shoes decreased by 15 pairs per hour.

10. The table below shows the year and the number of households in a building that had high-speed broadband internet access.
For which interval of time was the average rate of change the *smallest*?

1) 2002 - 2004 +12
2) 2003 - 2005 +17
3) 2004 - 2006 +19
4) 2005 - 2007 +14

Number of Households	11 +5	16 +7	23 +10	33 +9	42 +5	47
Year	2002	2003	2004	2005	2006	2007

11. The table below shows the cost of mailing a postcard in different years. During which time interval did the cost increase at the greatest average rate?

1) 1898-1971 $\frac{6-1}{1971-1898} = .068..$
2) 1971-1985 $\frac{14-6}{1985-1971} = .571\sim$
3) 1985-2006 $\frac{24-14}{2006-1985} = .476\sim$
4) 2006-2012 $\frac{35-24}{2012-2006} = 1.8\overline{3}$

Year	1898	1971	1985	2006	2012
Cost (¢)	1	6	14	24	35

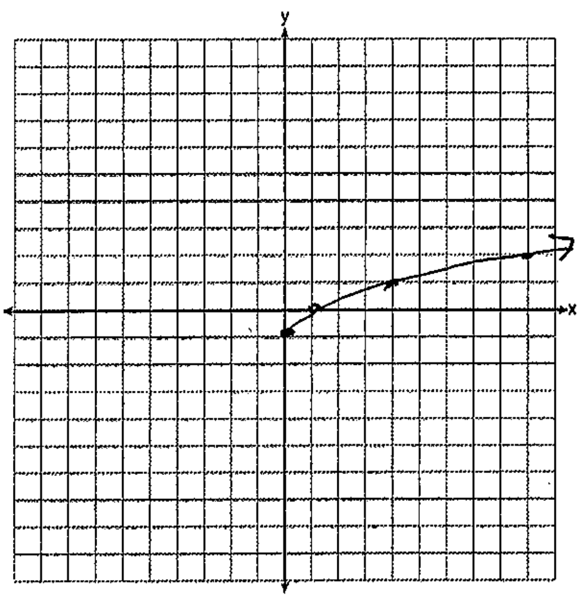
Graphing Non-Linear Functions

1) Get y by itself
2) Type into y =
3) If there is an interval/domain, plot only nice points between those values: no arrows
   If not, plot all "nice" points that fit on the graph (usually -10 to 10) and use arrows

* $a < x \le b$ means all numbers between a and b

*For quadratic and absolute value, find a mirror image

1. Draw the graph of $y = \sqrt{x} - 1$ on the set of axes below.

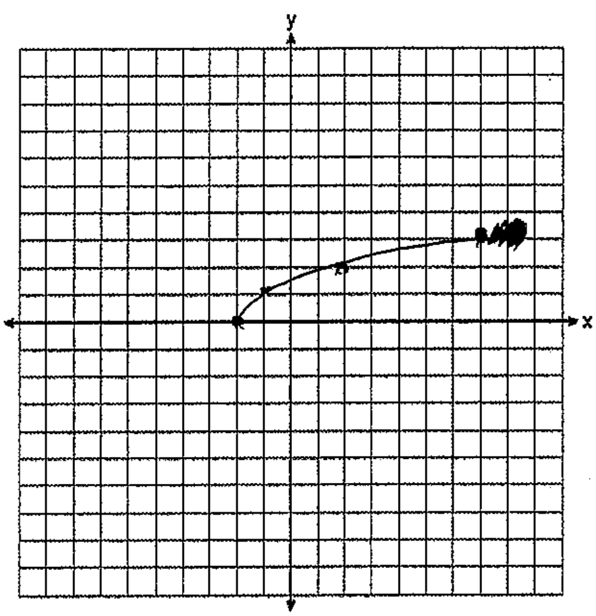


x	y
0	-1
1	0
4	1
9	2

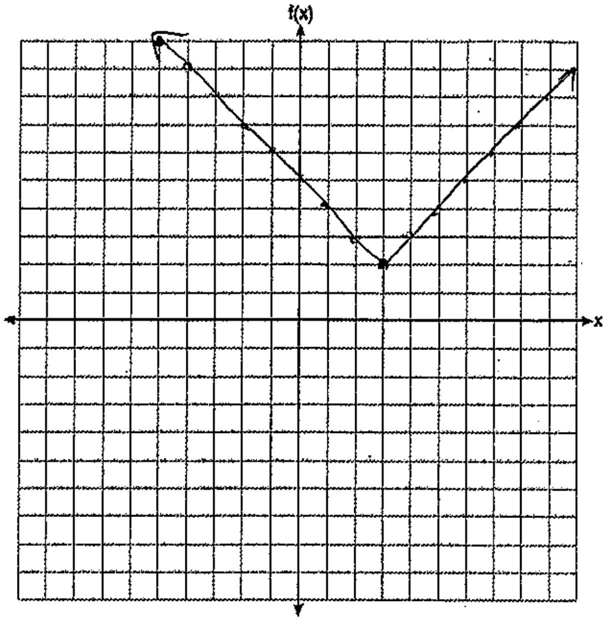
no arrows

2. Graph $f(x) = \sqrt{x+2}$ over the domain $-2 \le x \le 7$.

x	y
-2	0
-1	1
2	2
7	3

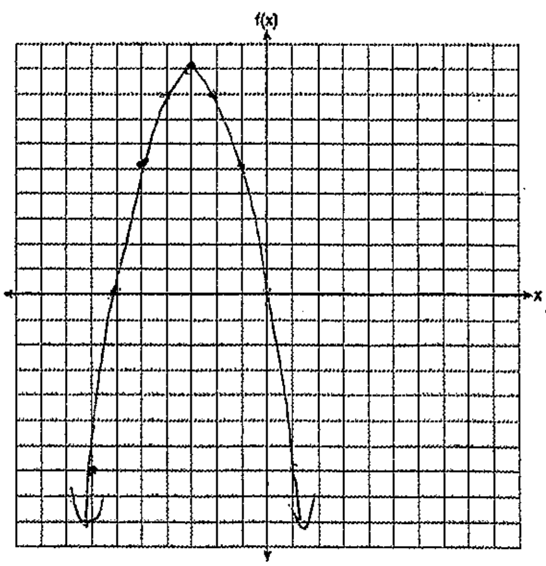


3. On the set of axes below, graph $f(x) = |x - 3| + 2$.



X	y
-5	10
-4	9
-3	8
-2	7
-1	6
0	5
1	4
2	3
3	2
4	3
5	4
6	5
7	6
8	7
9	8
10	9

4. Graph the function $f(x) = -x^2 - 6x$ on the set of axes below. State the coordinates of the vertex of the graph.

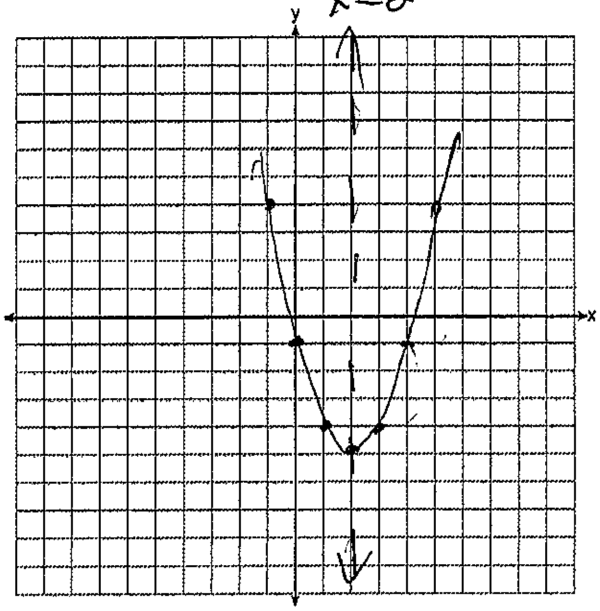


X	y
-7	-7
-6	0
-5	5
-4	8
-3	9
-2	8
-1	5
0	0
1	-7

vertex (-3,9)

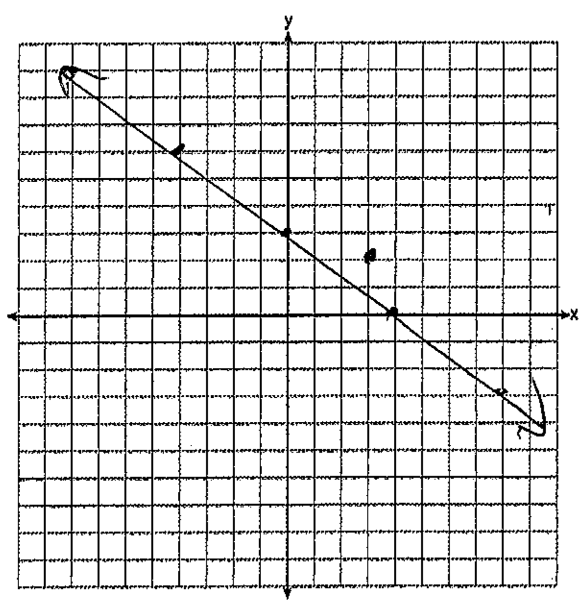
5. On the set of axes below, draw the graph of $y = x^2 - 4x - 1$. State the equation of the axis of symmetry.

axis of symmetry
$x=2$



X	y
-1	4
0	-1
1	-4
2	-5
3	-4
4	-1
5	4

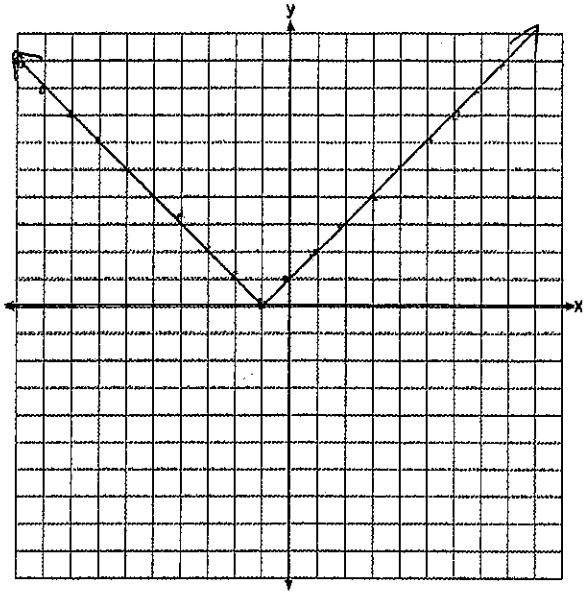
6. On the set of axes below, draw the graph of the equation $y = -\frac{3}{4}x + 3$. Is the point (3,2) a solution to the equation? Explain your answer based on the graph drawn.



X	y
-8	9
-4	6
0	3
4	0
8	-3

No, (3,2) is not on the line.

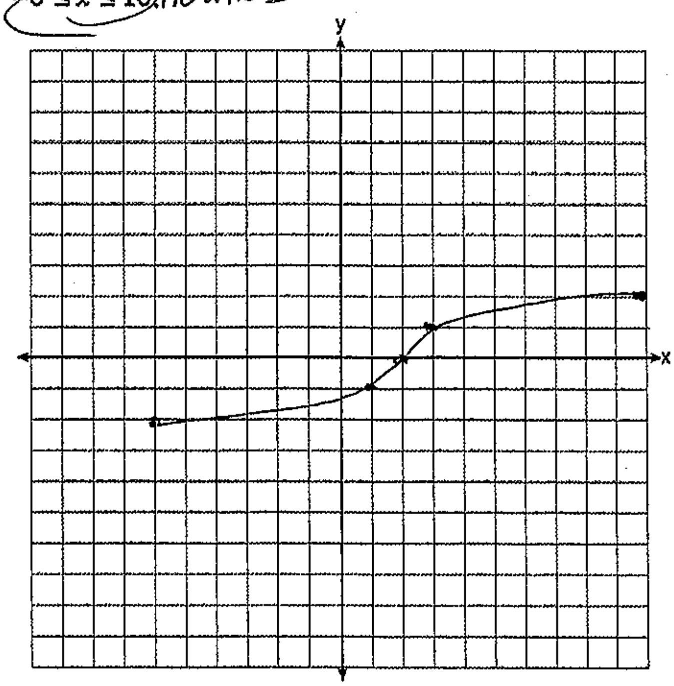
7. On the set of axes below, graph the function $y = |x+1|$.



x	y
-10	9
-9	8
-8	7
-7	6
-6	5
-5	4
-4	3
-3	2
-2	1
-1	0
0	1
1	2
2	3
3	4
4	5
5	6
6	7
7	8
8	9
9	10

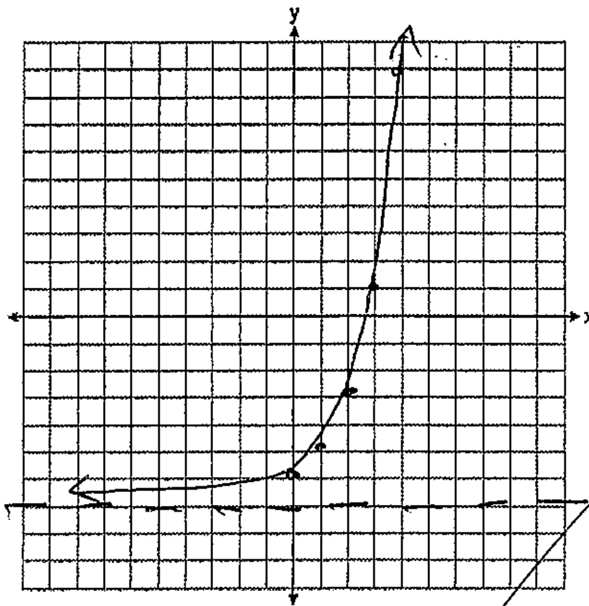
math:4

8. On the set of axes below, graph the function represented by $y = \sqrt[3]{x-2}$ for the domain $-6 \le x \le 10$. no arrows



x	y
-6	-2
1	-1
2	0
3	1
10	2

9. Graph the function $f(x) = 2^x - 7$ on the set of axes below.



x	y
0	-6
1	-5
2	-3
3	1
4	9

If $g(x) = 1.5x - 3$, determine if $f(x) > g(x)$ when $x = 4$. Justify your answer.

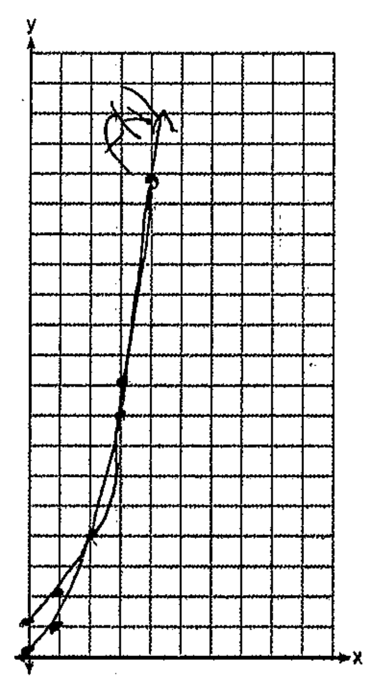
$g(4) = 1.5(4) - 3 = 3$ $9 > 3$

yes

10. Graph $f(x) = x^2$ and $g(x) = 2^x$ for $x \geq 0$ on the set of axes below.

f(x)

x	y
0	0
1	1
2	4
3	9
4	16



g(x)

x	y
0	1
1	2
2	4
3	8
4	16

State which function, $f(x)$ or $g(x)$, has a greater value when $x = 20$. Justify your reasoning.

$g(x)$ $2^{20} > 20^2$

Graphing Piecewise Functions

Create a table of values for each piece

Graph all points

*Make sure the border value without the equals gets an open circle.

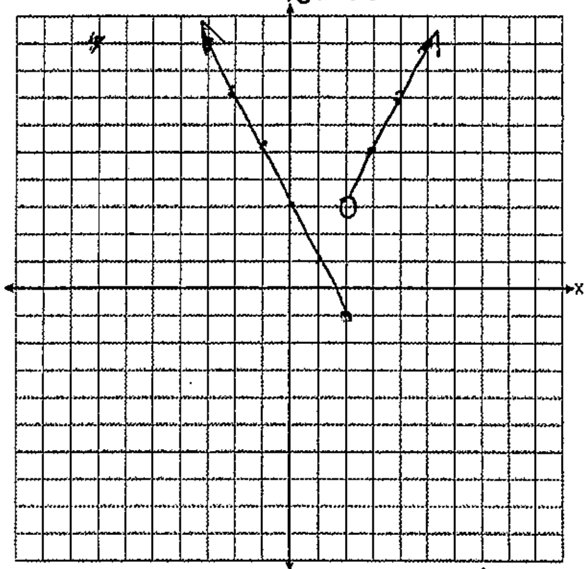
1. Graph the following on the set of axes below:

$$f(x)=\begin{cases}3-2x, & x\le 2\\ 2x-1, & x>2\end{cases}$$

closed circle

This value must be in both tables

open circle



$3-2x$

x	y
-3	9
-2	7
-1	5
0	3
1	1
2	-1

$2x-1$

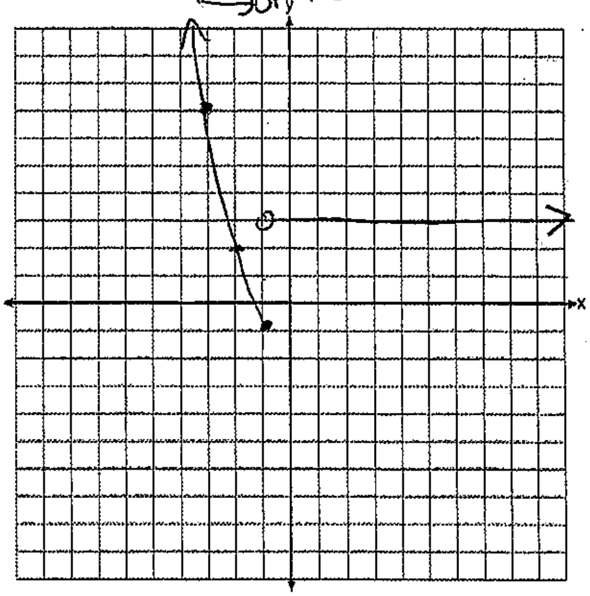
x	y
2	3
3	5
4	7
5	9

2. Graph the following on the set of axes below:

$$f(x)=\begin{cases}x^2-2, & x\le -1\\ 3, & x>1\end{cases}$$

closed circle

open circle



x^2-2

x	y
-3	7
-2	2
-1	-1

3

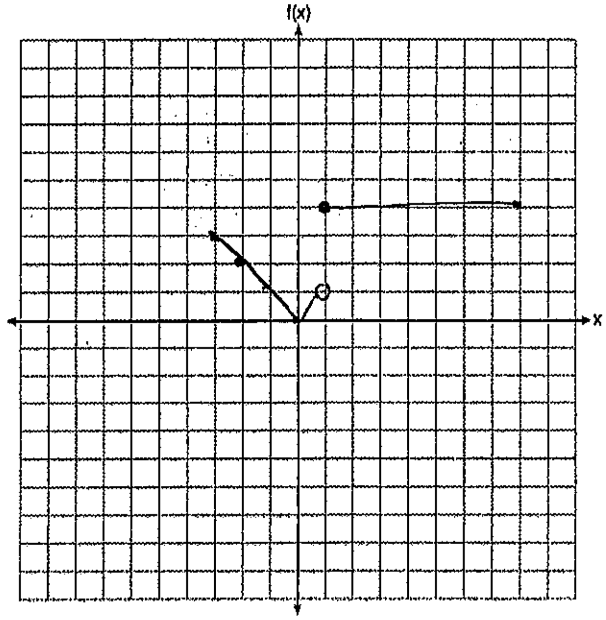
x	y
-1	3
0	3
1	3
2	3
↓	3

3. Graph the following function on the set of axes below.

$$f(x) = \begin{cases} |x|, & -3 \le x < 1 \\ 4, & 1 \le x \le 8 \end{cases}$$

open circle

closed circle



$|x|$

x	y
-3	3
-2	2
-1	1
0	0
1	1

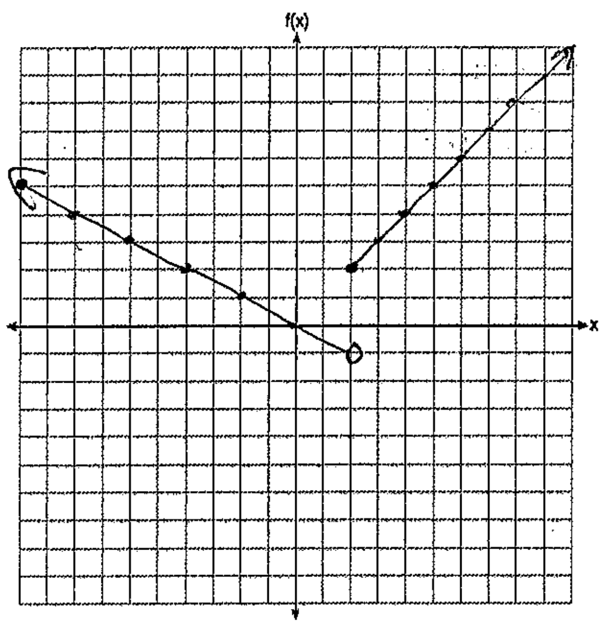
4

x	y
1	4
2	4
3	4
4	4
5	4
6	4
7	4
8	4

4. On the set of axes below, graph the piecewise function: $f(x) = \begin{cases} -\frac{1}{2}x, & x < 2 \\ x, & x \ge 2 \end{cases}$

open circle

closed circle



$-\frac{1}{2}x$

x	y
-10	5
-8	4
-6	3
-4	2
-2	1
0	0
2	-1

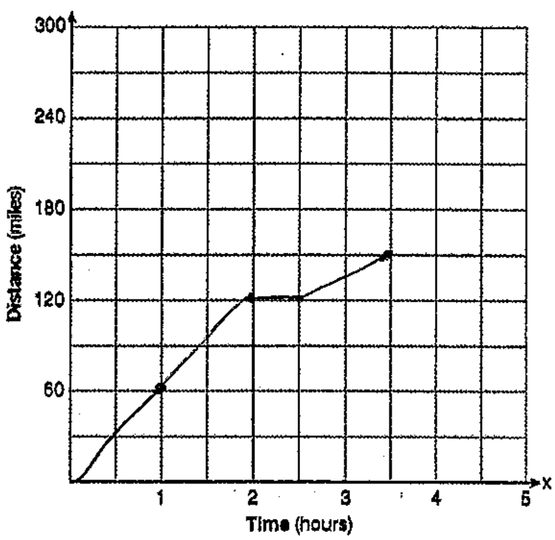
x

x	y
~~2~~	~~2~~
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10

Irregular Graphs

If the y axis is distance, 0 slope means there is no movement. The greater the slope, the faster the movement.
If the y axis is rate, 0 slope means the rate is staying the same. The greater the slope, the greater the rate is increasing.

1. A driver leaves home for a business trip and drives at a constant speed of 60 miles per hour for 2 hours. Her car gets a flat tire, and she spends 30 minutes changing the tire. She resumes driving and drives at 30 miles per hour for the remaining one hour until she reaches her destination. On the set of axes below, draw a graph that models the driver's distance from home.



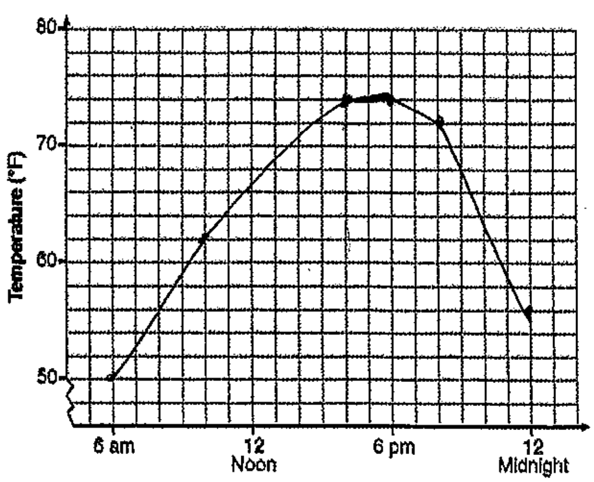
2. One spring day, Elroy noted the time of day and the temperature, in degrees Fahrenheit. His findings are stated below.

10 AM $50+4(3)=62$

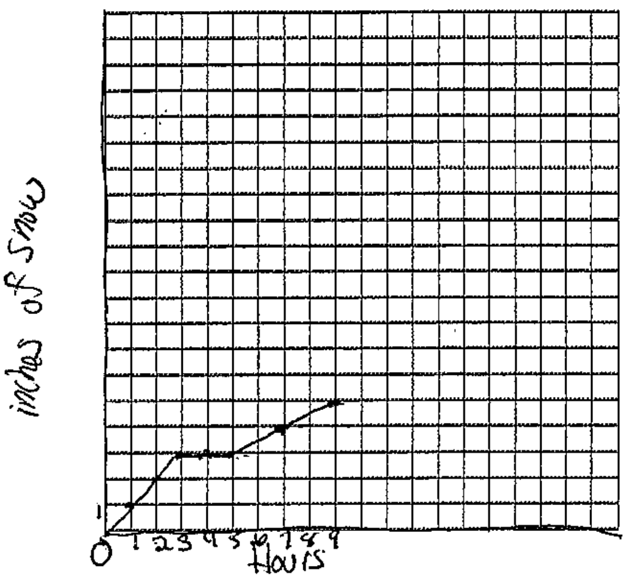
At 6 a.m., the temperature was 50°F. For the next 4 hours, the temperature rose 3° per hour. The next 6 hours, it rose 2° per hour. The temperature then stayed steady until 6 p.m. For the next 2 hours, the temperature dropped 1° per hour. The temperature then dropped steadily until the temperature was 56°F at midnight.
On the set of axes below, graph Elroy's data.

4pm $62+6(2)=74$

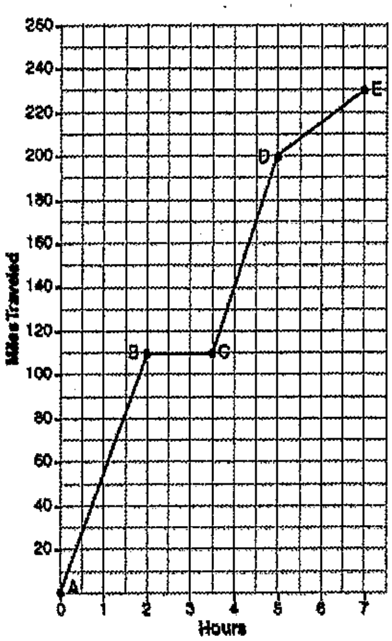
8AM $74-2(1)=72$



3. During a snowstorm, a meteorologist tracks the amount of accumulating snow. For the first three hours of the storm, the snow fell at a constant rate of one inch per hour. The storm then stopped for two hours and then started again at a constant rate of one-half inch per hour for the next four hours. On the grid below, draw and label a graph that models the accumulation of snow over time using the data the meteorologist collected.



4. The graph below models Craig's trip to visit his friend in another state. In the course of his travels, he encountered both highway and city driving. Based on the graph, during which interval did Craig most likely drive in the city? Explain your reasoning. Explain what might have happened in the interval between *B* and *C*. Determine Craig's average speed, to the *nearest tenth of a mile per hour*, for his entire trip.



He drove in the city from D to E because it is the slowest rate where the car was in motion.

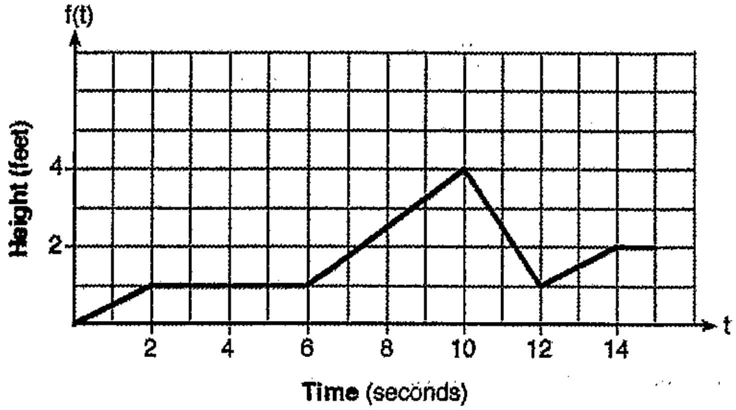
He may have stopped for lunch between B and C because he did not move although time was passing.

Average rate of change

x	y
0	0
7	230

$\frac{f(b)-f(a)}{b-a}$ $\quad \frac{230-0}{7-0} = 3.3$ mph

5. The graph of $f(t)$ models the height, in feet, that a bee is flying above the ground with respect to the time it traveled in t seconds. State all time intervals when the bee's rate of change is zero feet per second. Explain your reasoning.



(2,6)
(14,15)

The height did not change as time passed.

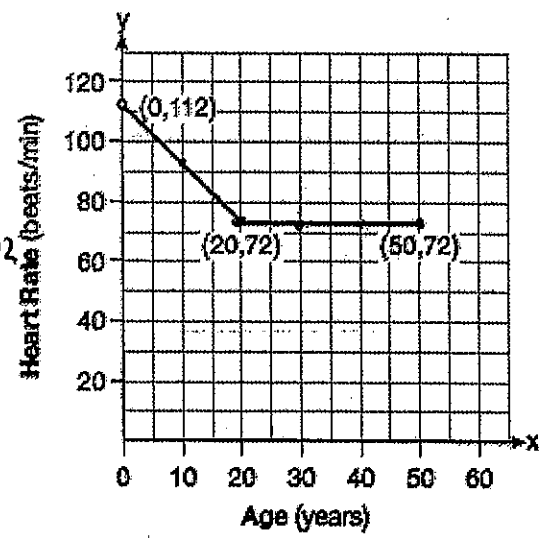
6. A graph of average resting heart rates is shown below. The average resting heart rate for adults is 72 beats per minute, but doctors consider resting rates from 60-100 beats per minute within normal range.

Which statement about average resting heart rates is *not* supported by the graph?

1) A 10-year-old has the same average resting heart rate as a 20-year-old. X
2) A 20-year-old has the same average resting heart rate as a 30-year-old.
3) A 40-year-old may have the same average resting heart rate for ten years. 40 and 50 are both 72
4) The average resting heart rate for teenagers steadily decreases.

Yes, decreases from 13 to 20.

Average Resting Heart Rate by Age

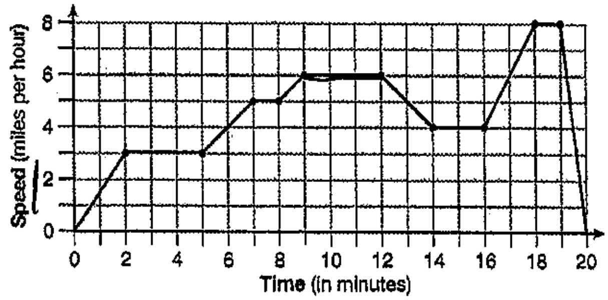


7. The graph below represents a jogger's speed during her 20-minute jog around her neighborhood.

Which statement best describes what the jogger was doing during the 9 – 12 minute interval of her jog?

1) She was standing still.
2) She was increasing her speed.
3) She was decreasing her speed
4) She was jogging at a constant rate.

The speed did not change



Identifying Functions

If asked which equation represents a table or graph, type the equation into the calculator and see if it matches the table or graph.

Look Carefully!

1. The table below represents the function F. The equation that represents this function is

x	3	4	6	7	8
$F(x)$	9	17	65	129	257

1) $F(x) = 3^x$
2) $F(x) = 3x$
3) $F(x) = 2^x + 1$
4) $F(x) = 2x + 3$

2. A laboratory technician studied the population growth of a colony of bacteria. He recorded the number of bacteria every other day, as shown in the partial table below.
Which function would accurately model the technician's data?

t (time, in days)	0	2	4
f(t) (bacteria)	25	15,625	9,765,625

1) $f(t) = 25^t$
2) $f(t) = 25^{t+1}$
3) $f(t) = 25t$
4) $f(t) = 25(t+1)$

3. Which function is shown in the table below?

x	f(x)
−2	$\frac{1}{9}$
−1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

1) $f(x) = 3x$
2) $f(x) = x + 3$
3) $f(x) = -x^3$
4) $f(x) = 3^x$

4. Marc bought a new laptop for $1250. He kept track of the value of the laptop over the next three years, as shown in the table below.

Years After Purchase	Value in Dollars
1	1000
2	800
3	640

Which function can be used to determine the value of the laptop for x years after the purchase?

1) $f(x) = 1000(1.2)^x$
2) $f(x) = 1000(0.8)^x$
3) $f(x) = 1250(1.2)^x$
4) $f(x) = 1250(0.8)^x$

5. Which chart could represent the function $f(x) = -2x + 6$?

1)

x	f(x)
0	6
2	10
4	14
6	18

2)

x	f(x)
0	4
2	6
4	8
6	10

3)

x	f(x)
0	8
2	10
4	12
6	14

4)

x	f(x)
0	6
2	2
4	–2
6	–6

6. The table below shows the temperature, $T(m)$, of a cup of hot chocolate that is allowed to chill over several minutes, m.

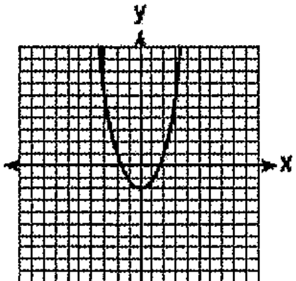
Which expression best fits the data for $T(m)$?

1) $150(0.85)^m$
2) $150(1.15)^m$
3) $150(0.85)^{m-1}$
4) $150(1.15)^{m-1}$

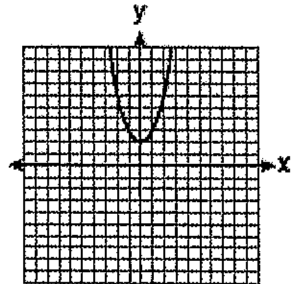
Time, m (minutes)	0	2	4	6	8
Temperature, T(m) (°F)	150	108	78	56	41

7. Which graph represents $y = \sqrt{x-2}$?

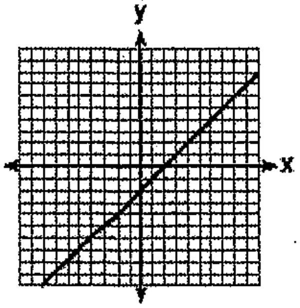
1)



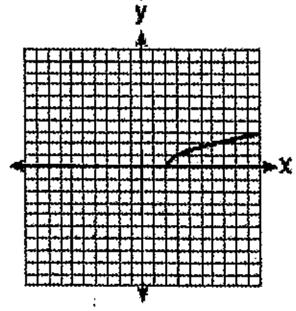
3)



2)

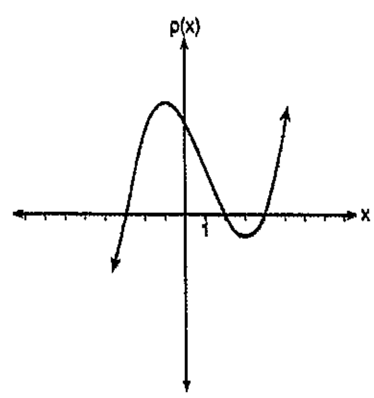


4)

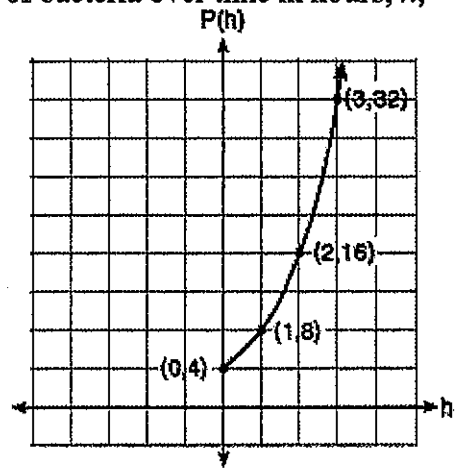


8. Based on the graph below, which expression is a possible factorization of $p(x)$?

1) $(x+3)(x-2)(x-4)$
2) $(x-3)(x+2)(x+4)$
3) $(x+3)(x-5)(x-2)(x-4)$
4) $(x-3)(x+5)(x+2)(x+4)$



9. Vinny collects population data, $P(h)$, about a specific strain of bacteria over time in hours, h, as shown in the graph below.



Which equation represents the graph of $P(h)$?

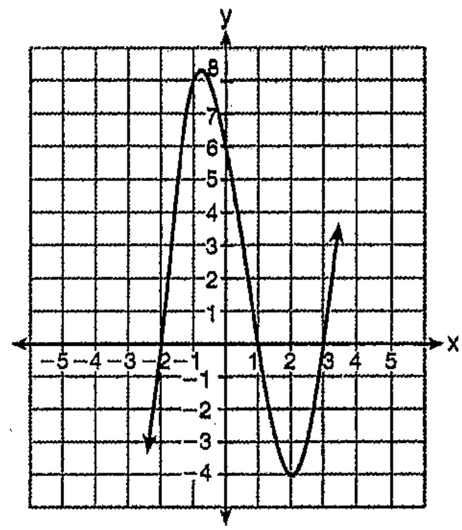
1) $P(h) = 4(2)^h$
2) $P(h) = \frac{46}{5}h + \frac{6}{5}$
3) $P(h) = 3h^2 + 0.2h + 4.2$
4) $P(h) = \frac{2}{3}h^3 - h^2 + 3h + 4$

10. Which equation(s) represent the graph below?

I $\quad y = (x+2)(x^2 - 4x - 12)$

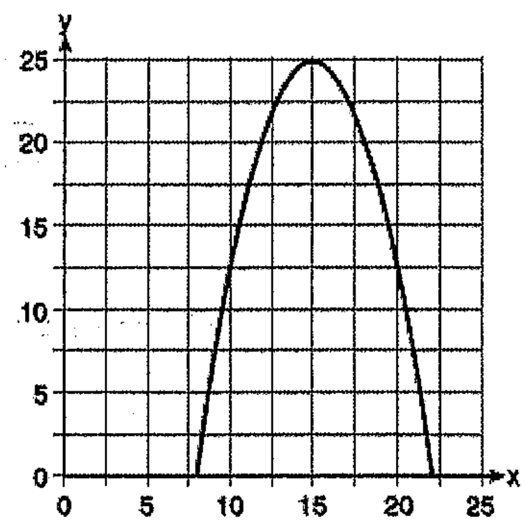
II $\quad y = (x-3)(x^2 + x - 2)$

III $\quad y = (x-1)(x^2 - 5x - 6)$



1) I, only
2) II, only
3) I and II
4) II and III

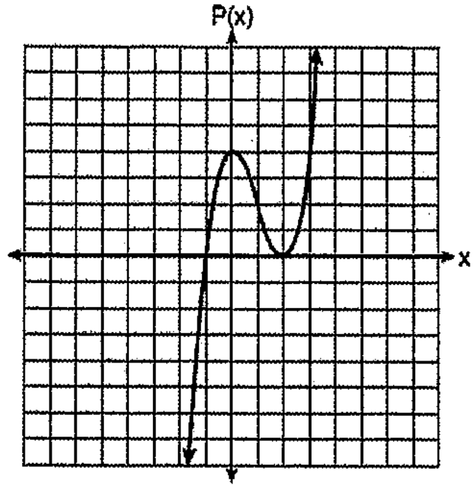
11. The graph of a quadratic function is shown below.



An equation that represents the function could be

1) $q(x) = \frac{1}{2}(x+15)^2 - 25$
2) $q(x) = -\frac{1}{2}(x+15)^2 - 25$
3) $q(x) = \frac{1}{2}(x-15)^2 + 25$
4) $q(x) = -\frac{1}{2}(x-15)^2 + 25$

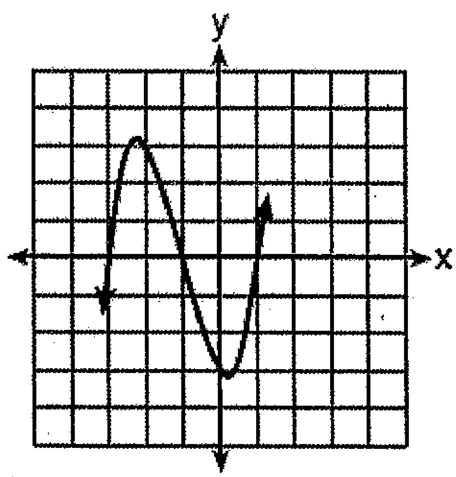
12. Wenona sketched the polynomial $P(x)$ as shown on the axes below.



Which equation could represent $P(x)$?

1) $P(x) = (x+1)(x-2)^2$
2) $P(x) = (x-1)(x+2)^2$
3) $P(x) = (x+1)(x-2)$
4) $P(x) = (x-1)(x+2)$

13. A cubic function is graphed on the set of axes below.



Which function could represent this graph?

1) $f(x) = (x-3)(x-1)(x+1)$
2) $g(x) = (x+3)(x+1)(x-1)$
3) $h(x) = (x-3)(x-1)(x+3)$
4) $k(x) = (x+3)(x+1)(x-3)$

Determining if a point is on the graph

Substitute x and y into the equation
If the two sides are equal, yes!
If the two sides are not equal, no!

or see if the point is in the table of values.

1. Which point is *not* on the graph represented by $y = x^2 + 3x - 6$?

1) (−6,12) $12 = (-6)^2 + 3(-6) - 6 = 12$ ✓
2) (−4,−2) $-2 = (-4)^2 + 3(-4) - 6 = -2$ ✓
3) (2,4) $4 = (2)^2 + 3(2) - 6 = 4$ ✓
4) (3,−6) $-6 = (3)^2 + 3(3) - 6 = 12$ X

2. Which ordered pair would *not* be a solution to $y = x^3 - x$?

1) (−4,−60) $-60 = (-4)^3 + 4 = -60$ ✓
2) (−3,−24) $-24 = (-3)^3 + 3 = -24$ ✓
3) (−2,−6) $-6 = (-2)^3 + 2 = -6$ ✓
4) (−1,−2) $-2 = (-1)^3 + 1 = 0$ X

3. Which ordered pair below is *not* a solution to $f(x) = x^2 - 3x + 4$?

1) (0,4) $4 = (0)^2 - 3(0) + 4 = 4$ ✓
2) (1.5, 1.75) $1.75 = (1.5)^2 - 3(1.5) + 4 = 1.75$ ✓
3) (5, 14) $14 = (5)^2 - 3(5) + 4 = 14$ ✓
4) (−1, 6) $6 = (-1)^2 - 3(-1) + 4 = 8$ X

4. Which point is *not* in the solution set of the equation $3y + 2 = x^2 - 5x + 17$?

1) (−2, 10) $3(10) + 2 = (-2)^2 - 5(-2) + 17$ $32 = 31$ X
2) (−1, 7) $3(7) + 2 = (-1)^2 - 5(-1) + 17$ $23 = 23$ ✓
3) (2, 3) $3(3) + 2 = (2)^2 - 5(2) + 17$ $11 = 11$ ✓
4) (5, 5) $3(5) + 2 = (5)^2 - 5(5) + 17$ $17 = 17$ ✓

5. How many of the equations listed below represent the line passing through the points (2, 3) and (4, −7)?

$x=2$ $y=3$ $x=4$ $y=-7$

$5(2)+3=13$ ✓ $5x + y = 13$ $5(4)+7=13$ X

$3+7=-5(2-4)$ ✓ $y + 7 = -5(x - 4)$ $-7+7=-5(4-4)$ ✓

✓ $3=-5(2)+13$ ✓ $y = -5x + 13$ $-7=-5(4)+13$ ✓

X $3-7=5(2-4)$ $y - 7 = 5(x - 4)$ $-7-7=5(4-4)$ X

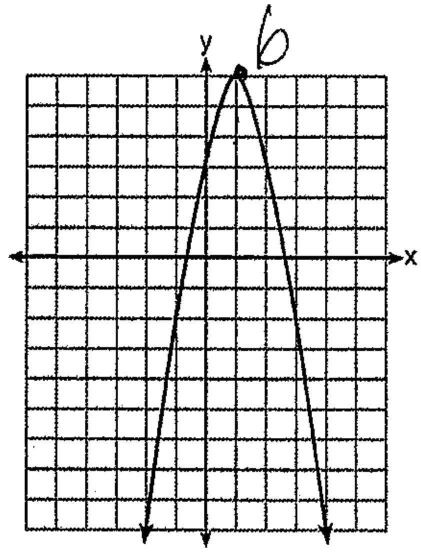
1) 1
2) 2
3) 3
4) 4

Key Points

To compare key points, find the key point for each function. Use the graph, the table (2nd graph), and the calculate menu (2nd Trace).

1. Let f be the function represented by the graph below.

6



Let g be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$. Determine which function has the larger maximum value. Justify your answer. ↘ type into calc

x	y
1	6.5
2	9
3	10.5
4	11
5	10.5
6	9
7	6.5

11

g(x)

11>6

2. Which quadratic function has the largest maximum?

1) $h(x) = (3 - x)(2 + x)$ 2nd Trace, max 6.25

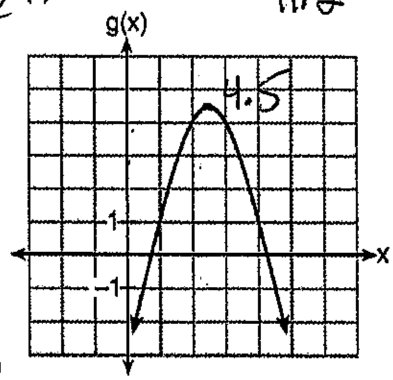
3) $k(x) = -5x^2 - 12x + 4$ 2nd Trace, max, adjust y max 11.2

2)

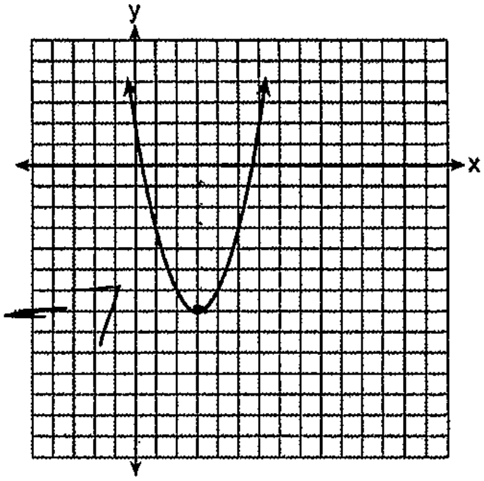
x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

≈9

4)



3. The graph representing a function is shown below.



Which function has a minimum that is *less* than the one shown in the graph?

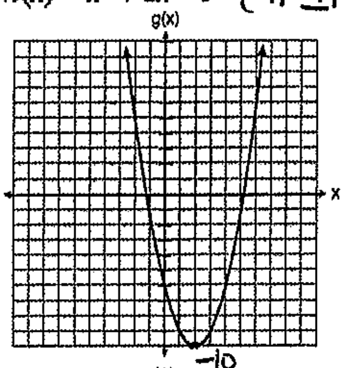
2nd Trace min

1) $y = x^2 - 6x + 7$ −2
2) $y = |x+3| - 6$ −6
3) $y = x^2 - 2x - 10$ −11 adjust y min
4) $y = |x-8| + 2$ 2

2nd Trace min

4. Which of the quadratic functions below has the *smallest* minimum value?

1) $h(x) = x^2 + 2x - 6$ (−1, −7)
2)



(2)

3) $k(x) = (x+5)(x+2)$ (−3.5, −2.25)
4)

x	f(x)
−1	−2
0	−5
1	−6
2	−5
3	−2

−6

5. Which statement is true about the quadratic functions $g(x)$, shown in the table below, and $f(x) = (x-3)^2 + 2$?

vertex: (3,2)
Zeros: None
AOS: X=3

X	Y
0	11
1	6
2	3
3	2
4	3
5	6
6	11

x	g(x)
0	4
1	−1
2	−4
3	−5
4	−4
5	−1
6	4

Vertex: (3, −5)
Zeros: ≈0.5, ≈5.5
AOS: x=3

1) They have the same vertex. X
2) They have the same zeros. X
3) They have the same axis of symmetry. ✓
4) They intersect at two points. X

6. Which quadratic function has the largest maximum over the set of real numbers?

1) $f(x) = -x^2 + 2x + 4$ (1, 5)

(2)

x	k(x)
−1	−1
0	3
1	5
2	5
3	3
4	−1

≈ 5.5

3) $g(x) = -(x-5)^2 + 5$ (5, 5) 2nd Trace, maximum

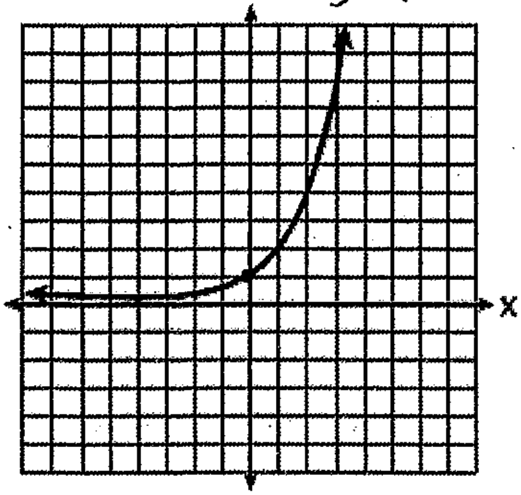
4)

x	h(x)
−2	−9
−1	−3
0	1
1	3
2	3
3	1

≈ 3.5

7. Three functions are shown below.

f(x) b=1



b=−1

x	h(x)
−5	30
−4	14
−3	6
−2	2
−1	0
0	−1
1	−1.5
2	−1.75

b=3

$g(x) = 3^x + 2$

Which statement is true?

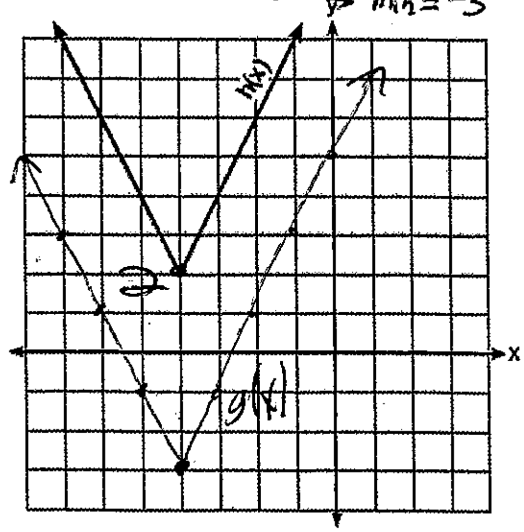
1) The y-intercept for $h(x)$ is greater than the y-intercept for $f(x)$. −1 > 1 ✗
2) The y-intercept for $f(x)$ is greater than the y-intercept for $g(x)$. 1 > 3 ✗
3) The y-intercept for $h(x)$ is greater than the y-intercept for both $g(x)$ and $f(x)$. −1 > 1 ✗ −1 > 3 ✗
(4) The y-intercept for $g(x)$ is greater than the y-intercept for both $f(x)$ and $h(x)$. 3 > 1 ✓ 3 > −1 ✓

8. The function $h(x)$, which is graphed below, and the function $g(x) = 2|x+4| - 3$ are given. y min = −3

Which statements about these functions are true?

I. $g(x)$ has a lower minimum value than $h(x)$. ✓
II. For all values of x, $h(x) < g(x)$. ✗
III. For any value of x, $g(x) \neq h(x)$. ✓

1) I and II, only
(2) I and III, only
3) II and III, only
4) I, II, and III



Sequences:
Arithmetic: add a constant difference, Geometric: multiply by a common ratio
Explicit Formulas (From Reference Sheet)

Arithmetic: $a_n = a_1 + (n-1)d$ Geometric: $a_n = a_1(r)^{n-1}$

Recursive Formulas

Arithmetic: $a_1 =$ $a_n = a_{n-1} + d$ Geometric: $a_1 =$ $a_n = ra_{n-1}$

1. Write an explicit AND recursive equation for the following sequence and find the tenth term.

19, 16, 13, 10 ... arithmetic

-3 -3 -3

$a_1=19$
$d=-3$

$a_n=a_1+(n-1)d$
$a_n=19+(n-1)(-3)$

$a_n=19-3n+3$
$a_n=-3n+22$
$a_{10}=-3(10)+22$
$a_{10}=-8$

$a_1=19$
$a_n=a_{n-1}-3$

2. Write an explicit AND recursive equation for the following sequence and find the ninth term.

2,8,32,128,... geometric

·4 ·4 ·4

$a_1=2$
$r=4$

$a_n=a_1(r)^{n-1}$
$a_n=2(4)^{n-1}$
$a_9=2(4)^{9-1}$
$a_9=131,072$

$a_1=2$
$a_n=4a_{n-1}$

3. Write an explicit AND recursive equation for the following sequence and find the eighth term.

2, 6, 18, 54, ... geometric

·3 ·3 ·3

$a_1=2$
$r=3$

$a_n=a_1(r)^{n-1}$
$a_n=2(3)^{n-1}$
$a_8=2(3)^{8-1}$
$a_8=4374$

$a_1=2$
$a_n=3a_{n-1}$

4. Write an explicit AND recursive equation for the following sequence and find the 20th term.

63, 57, 51, 45, ... arithmetic

-6 -6 -6

$a_1=63$
$d=-6$

$a_n=a_1+(n-1)d$
$a_n=63+(n-1)(-6)$
$a_n=63-6n+6$
$a_n=-6n+69$

$a_{20}=-6(20)+69$
$a_{20}=-51$

$a_1=63$
$a_n=a_{n-1}-6$

5. Write an explicit AND recursive equation for the following sequence and find the 7th term.

3, -12, 48, -192, ... geometric

·-4 ·-4 ·-4

$a_1=3$
$r=-4$

$a_n=a_1(r)^{n-1}$
$a_n=3(-4)^{n-1}$
$a_7=3(-4)^{7-1}$
$a_7=12,288$

$a_1=3$
$a_n=-4a_{n-1}$

Evaluating Recursive Sequences

a_{n-1} means the previous term!

1) Start with the term after the one they give you
2) Substitute the previous term in for a_{n-1}

n is the term that you are finding

$a_n \to a_{n-1}$ means the same thing as $a_{n+1} \to a_n$

1. Find the first 4 terms of the sequence $a_n = 2a_{n-1} + 4$ where $a_1 = 3$.

$a_1 = 3$ $\quad a_2 = 2a_1 + 4$ $\quad a_2 = 2(3) + 4$ $\quad a_2 = 6 + 4$ $\quad a_2 = 10$

$a_3 = 2a_2 + 4$ $\quad a_3 = 2(10) + 4$ $\quad a_3 = 24$

$a_4 = 2a_3 + 4$ $\quad a_4 = 2(24) + 4$ $\quad a_4 = 52$

3, 10, 24, 52

2. Find the first 4 terms of the recursive sequence $a_1 = -3$, $a_n = 4 - 3a_{n-1}$

$a_1 = -3$ $\quad a_2 = 4 - 3a_1$ $\quad a_2 = 4 - 3(-3)$ $\quad a_2 = 13$

$a_3 = 4 - 3a_2$ $\quad a_3 = 4 - 3(13)$ $\quad a_3 = -35$

$a_4 = 4 - 3a_3$ $\quad a_4 = 4 - 3(-35)$ $\quad a_4 = 109$

-3, 12, -35, 109

3. If $a_n = 3a_{n-1} - 4$ and $a_1 = 9$, find a_5

$a_1 = 9$ $\quad a_2 = 3a_1 - 4$ $\quad a_2 = 3(9) - 4$ $\quad a_2 = 23$

$a_3 = 3a_2 - 4$ $\quad a_3 = 3(23) - 4$ $\quad a_3 = 65$

$a_4 = 3a_3 - 4$ $\quad a_4 = 3(65) - 4$ $\quad a_4 = 191$

$a_5 = 3a_4 - 4$ $\quad a_5 = 3(191) - 4$ $\quad a_5 = 569$

4. If $f(1) = 3$ and $f(n) = -2f(n-1) + 1$, then $f(5) =$

1) -5
2) 11
3) 21
4) 43

$f(2) = -2(f(1)) + 1$ $\quad f(2) = -2(3) + 1$ $\quad f(2) = -5$

$f(3) = -2f(2) + 1$ $\quad f(3) = -2(-5) + 1$ $\quad f(3) = 11$

$f(4) = -2f(3) + 1$ $\quad f(4) = -2(11) + 1$ $\quad f(4) = -21$

$f(5) = -2f(4) + 1$ $\quad f(5) = -2(-21) + 1$ $\quad f(5) = 43$

5. Write the first five terms of the recursive sequence defined below.

$a_1 = 0$

$a_n = 2(a_{n-1})^2 - 1$, for $n > 1$

0, -1, 1, 1, 1

$a_1 = 0$

$a_2 = 2(a_1)^2 - 1$ $\quad a_2 = 2(0)^2 - 1$ $\quad a_2 = -1$

$a_3 = 2(a_2)^2 - 1$ $\quad a_3 = 2(-1)^2 - 1$ $\quad a_3 = 1$

$a_4 = 2(a_3)^2 - 1$ $\quad a_4 = 2(1)^2 - 1$ $\quad a_4 = 1$

$a_5 = 2(a_4)^2 - 1$ $\quad a_5 = 2(1)^2 - 1$ $\quad a_5 = 1$

6. Find the third term in the recursive sequence $a_{k+1} = 2a_k - 1$, where $a_1 = 3$.

$a_1 = 3$ $\quad a_2 = 2a_1 - 1$ $\quad a_3 = 2a_2 - 1$

$a_2 = 2(3) - 1$ $\quad a_3 = 2(5) - 1$

$a_2 = 5$ $\quad a_3 = 9$

7. Find the first four terms of the recursive sequence defined below.

$a_1 = -3$

$a_n = a_{(n-1)} - n$

$a_2 = a_1 - 2$ $\quad a_3 = a_2 - 3$ $\quad a_4 = a_3 - 4$

$a_2 = -3 - 2$ $\quad a_3 = -5 - 3$ $\quad a_4 = -8 - 4$

$a_2 = -5$ $\quad a_3 = -8$ $\quad a_4 = -12$

$-3, -5, -8, -12$

8. If $a_n = n(a_{n-1})$ and $a_1 = 1$, what is the value of a_5?

1) 5
2) 20
3) 120
4) 720

$a_1 = 1$ $\quad a_2 = 2(a_1)$ $\quad a_3 = 3a_2$ $\quad a_4 = 4a_3$ $\quad a_5 = 5a_4$

$a_2 = 2(1)$ $\quad a_3 = 3(2)$ $\quad a_4 = 4(6)$ $\quad a_5 = 5(24)$

$a_2 = 2$ $\quad a_3 = 6$ $\quad a_4 = 24$ $\quad a_5 = 120$

9. A sequence is defined recursively by $f(1) = 16$ and $f(n) = f(n-1) + 2n$. Find $f(4)$.

(1) 32 (2) 30 (3) 28 (4) 34

$f(1) = 16$ $\quad f(2) = f(1) + 2(2)$ $\quad f(3) = f(2) + 2(3)$ $\quad f(4) = f(3) + 2(4)$

$f(2) = 16 + 4$ $\quad f(3) = 20 + 6$ $\quad f(4) = 26 + 8$

$f(2) = 20$ $\quad f(3) = 26$ $\quad f(4) = 34$

10. Given the function $f(n)$ defined by the following:
Which set could represent the range of the function?

1) $\{2, 4, 6, 8, \dots\}$
2) $\{2, -8, 42, -208, \dots\}$
3) $\{-8, -42, -208, 1042, \dots\}$
4) $\{-10, 50, -250, 1250, \dots\}$

$f(1) = 2$

$f(n) = -5f(n-1) + 2$

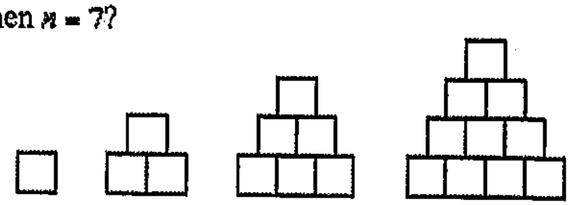
$f(2) = -5f(1) + 2$ $\quad f(3) = -5f(2) + 2$ $\quad f(4) = -5f(3) + 2$

$f(2) = -5(2) + 2$ $\quad f(3) = -5(-8) + 2$ $\quad f(4) = -5(42) + 2$

$f(2) = -8$ $\quad f(3) = 42$ $\quad f(4) = -208$

11. A sequence of blocks is shown in the diagram below.
This sequence can be defined by the recursive function $a_1 = 1$ and $a_n = a_{n-1} + n$. Assuming the pattern continues, how many blocks will there be when $n = 7$?

1) 13
2) 21
3) 28
4) 36



1 $\quad$ 3 $\quad$ 6 $\quad$ 10

$a_1 = 1$ $\quad a_2 = 3$ $\quad a_3 = 6$ $\quad a_4 = 10$

$a_5 = a_4 + 5$ $\quad a_6 = a_5 + 6$ $\quad a_7 = a_6 + 7$

$a_5 = 10 + 5$ $\quad a_6 = 15 + 6$ $\quad a_7 = 21 + 7$

$a_5 = 15$ $\quad a_6 = 21$ $\quad a_7 = 28$

Regression Equations
Turn Stat Diagnostics On (Mode, STATDIAGNOSTICS ON)
To write regression equations:

1) Stat, Edit
2) Stat, Calc, 4: LinReg or 0: ExpReg

r is the correlation coefficient. Negative slope has negative correlation coefficient, positive slope has positive correlation coefficient.
The closer the correlation coefficient is to 1 or -1, the stronger the correlation. The closer the correlation coefficient is to 0, the weaker the correlation is.
Read and round carefully! You may be asked to round to different values within different parts of the same question.

1. Which of the following correlation coefficients represents the strongest linear relationship?
(1) 0.79 (2) 0.36 (3) 0.12 (4) -0.87

2. Bella recorded data and used her graphing calculator to find the equation for the line of best fit. She then used the correlation coefficient to determine the strength of the linear fit. Which correlation coefficient represents the strongest linear relationship?
1) 0.9
2) 0.5
3) -0.3
4) -0.8

3. The results of a linear regression are shown below.
Which phrase best describes the relationship between x and y?
1) strong negative correlation
2) strong positive correlation
3) weak negative correlation
4) weak positive correlation

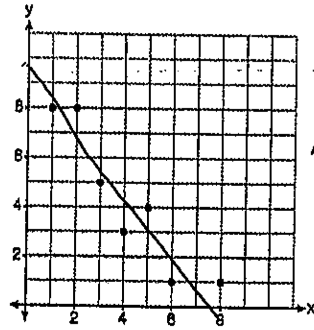
$y = ax + b$
$a = -1.15785$
$b = 139.3171772$
$r = -0.896557832$
$r^2 = 0.8038159461$

4. Which calculator output shows the strongest linear relationship between x and y?

(1)	(2)	(3)	(4)
Lin Reg	Lin Reg	Lin Reg	Lin Reg
$y = a + bx$	$y = a + bx$	$y = a + bx$	$y = a + bx$
$a = 59.026$	$a = .7$	$a = 2.45$	$a = -2.9$
$b = 6.767$	$b = 24.2$	$b = .95$	$b = 24.1$
$r = .8643$	$r = .8361$	$r = .6022$	$r = -.8924$

5. Analysis of data from a statistical study shows a linear relationship in the data with a correlation coefficient of -0.524. Which statement best summarizes this result?
1) There is a strong positive correlation between the variables.
2) There is a strong negative correlation between the variables.
3) There is a moderate positive correlation between the variables.
4) There is a moderate negative correlation between the variables.

6. What is the correlation coefficient of the linear fit of the data shown below, to the *nearest hundredth*?



Negative

1) 1.00
2) 0.93
3) −0.93
4) −1.00 → not a perfect fit

7. The percentage of students scoring 85 or better on a mathematics final exam and an English final exam during a recent school year for seven schools is shown in the table below. Write the linear regression equation for these data, rounding all values to the *nearest hundredth*. State the correlation coefficient of the linear regression equation, to the *nearest hundredth*. Explain the meaning of this value in the context of these data.

Percentage of Students Scoring 85 or Better	
Mathematics, x	English, y
27	46
12	28
13	45
10	34
30	56
45	67
20	42
L_1	L_2

$y = .96x + 23.95$

$r = .92$

There is a stong positive correlation between the percentage of students scoring 85 or better on Math and English.

8. Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below. Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*. The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Between the 2nd and 3rd hour.

Hours (x)	Average Number of Spores (y)
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380
L_1	L_2

Stat, Edit

Stat, Calc, 0: ExpReg

$y = 4.168(3.981)^x$

9. Omar has a piece of rope. He ties a knot in the rope and measures the new length of the rope. He then repeats this process several times. Some of the data collected are listed in the table below. State, to the *nearest tenth*, the linear regression equation that approximates the length, y, of the rope after tying x knots. Explain what the y-intercept means in the context of the problem. Explain what the slope means in the context of the problem. Stat, edit

Stat, Calc, 4: LinReg

$y = -8.5x + 99.2$

L1	Number of Knots	4	5	6	7	8
L2	Length of Rope (cm)	64	58	49	39	31

Slope: Each knot decreases the length of the rope by 8.5 cm.

y int: 99.2, the length of the rope before any knots is 99.2 cm

10. At Mountain Lakes High School, the mathematics and physics scores of nine students were compared as shown in the table below. State the correlation coefficient, to the *nearest hundredth*, for the line of best fit for these data. Explain what the correlation coefficient means with regard to the context of this situation.

make sure StatDiagnostics are on

L1	**Mathematics**	55	93	89	60	90	45	64	76	89
L2	**Physics**	66	89	94	52	84	56	66	73	92

Stat, Edit

Stat, Calc, 4: LinReg

$r = .92$

There is a strong, positive relationship between Math and Physics scores.

11. The data table below shows the median diameter of grains of sand and the slope of the beach for 9 naturally occurring ocean beaches. Write the linear regression equation for this set of data, rounding all values to the *nearest thousandth*. Using this equation, predict the slope of a beach, to the *nearest tenth of a degree*, on a beach with grains of sand having a median diameter of 0.65 mm.

Stat, edit

Stat, calc, 4: LinReg

L1	Median Diameter of Grains of Sand, in Millimeters (x)	0.17	0.19	0.22	0.235	0.235	0.3	0.35	0.42	0.85
L2	Slope of Beach, in Degrees (y)	0.63	0.7	0.82	0.88	1.15	1.5	4.4	7.3	11.3

$y = 17.159x - 2.476$

$y = 17.159(.65) - 2.476$

$y = 8.7°$

12. Erica, the manager at Stellarbeans, collected data on the daily high temperature and revenue from coffee sales. Data from nine days this past fall are shown in the table below. State the linear regression function, $f(t)$, that estimates the day's coffee sales with a high temperature of t. Round all values to the *nearest integer*. State the correlation coefficient, r, of the data to the *nearest hundredth*. Does r indicate a strong linear relationship between the variables? Explain your reasoning.

Stat, Edit

Stat, Calc, 4: LinReg

		Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9
L1	High Temperature, t	54	50	62	67	70	58	52	46	48
L2	Coffee Sales, f(t)	\$2900	\$3080	\$2500	\$2380	\$2200	\$2700	\$3000	\$3620	\$3720

$y = -58x + 6182$

$r = -.94$

Yes, because r is close to -1, there is a strong, negative relationship between temperature and coffee sales.

13. The data given in the table below show some of the results of a study comparing the height of a certain breed of dog, based upon its mass. Write the linear regression equation for these data, where x is the mass and y is the height. Round all values to the *nearest tenth*. State the value of the correlation coefficient to the *nearest tenth*, and explain what it indicates.

r

L_1	**Mass (kg)**	4.5	5	4	3.5	5.5	5	5	4	4	6	3.5	5.5
L_2	**Height (cm)**	41	40	35	38	43	44	37	39	42	44	31	30

Stat, Edit
Stat, Calc, 4: LinReg
$y = 1.9x + 29.8$

$r = .3$
There is a weak, positive relationship between mass and height of this breed of dog.

14. An application developer released a new app to be downloaded. The table below gives the number of downloads for the first four weeks after the launch of the app.
Write an exponential equation that models these data. Use this model to predict how many downloads the developer would expect in the 26th week if this trend continues. Round your answer to the nearest download.

L_1	Number of Weeks	1	2	3	4
L_2	Number of Downloads	120	180	270	405

Stat, Edit
Stat, Calc, 0: ExpReg
$y = 80(1.5)^x$
$y = 80(1.5)^{26} = 3030140$

15. Emma recently purchased a new car. She decided to keep track of how many gallons of gas she used on five of her business trips. The results are shown in the table below.
Write the linear regression equation for these data where miles driven is the independent variable. (Round all values to the *nearest hundredth*.)

Miles Driven	Number of Gallons Used
150	7
200	10
400	19
600	29
1000	51
L_1	L_2

Stat, Edit
Stat, Calc, 4: LinReg
$y = .05x - .92$

16. A nutritionist collected information about different brands of beef hot dogs. She made a table showing the number of Calories and the amount of sodium in each hot dog.
a) Write the correlation coefficient for the line of best fit. Round your answer to the *nearest hundredth*.
r
b) Explain what the correlation coefficient suggests in the context of this problem.

Calories per Beef Hot Dog	Milligrams of Sodium per Beef Hot Dog
186	495
181	477
176	425
149	322
184	482
190	587
158	370
139	322
L_1	L_2

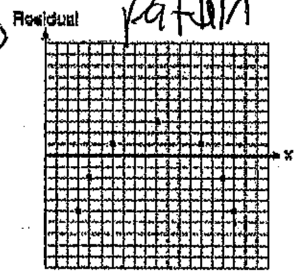
Stat, Edit
Stat, Calc, 4: LinReg
$r = .94$
There is a strong, positive collelation between calories and mg of sodium in beef hot dogs.

Residual Plots
Residual is actual – predicted, but that is generally given to you.
Plot the x column and the residual column
Good fit: Points are *randomly* scattered above and below the x-axis.
Bad Fit: There is a pattern (likely a parabola).

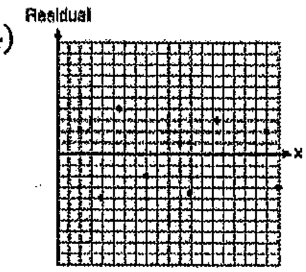
1. Which statistic would indicate that a linear function would *not* be a good fit to model a data set?

(1) $r = -0.93$ (2) $r = 1$ (3)

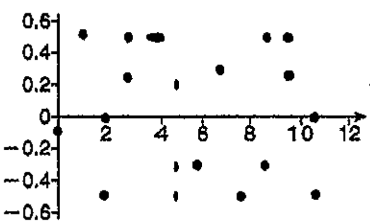
Pattern



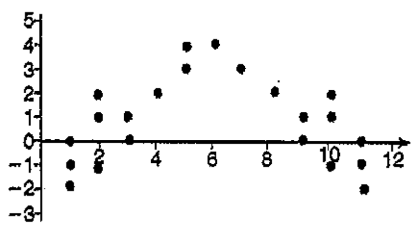
(4)



2. The residual plots from two different sets of bivariate data are graphed below. Explain, using evidence from graph A and graph B, which graph indicates that the model for the data is a good fit.



Graph A

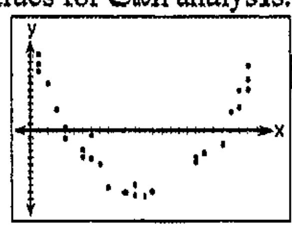


Graph B

Graph A is a good fit because the points are randomly scattered above and below the x-axis

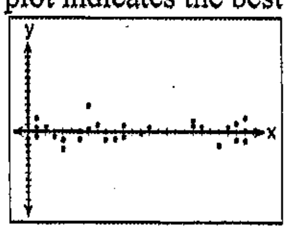
3. After performing analyses on a set of data, Jackie examined the scatter plot of the residual values for each analysis. Which scatter plot indicates the best linear fit for the data?

1)

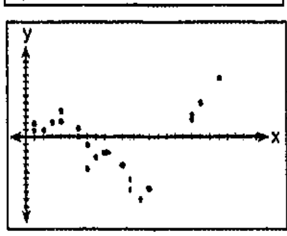


pattern

3)

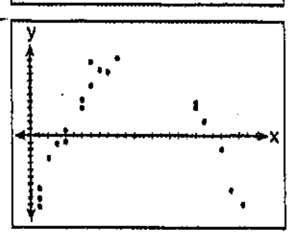


2)



pattern

4)



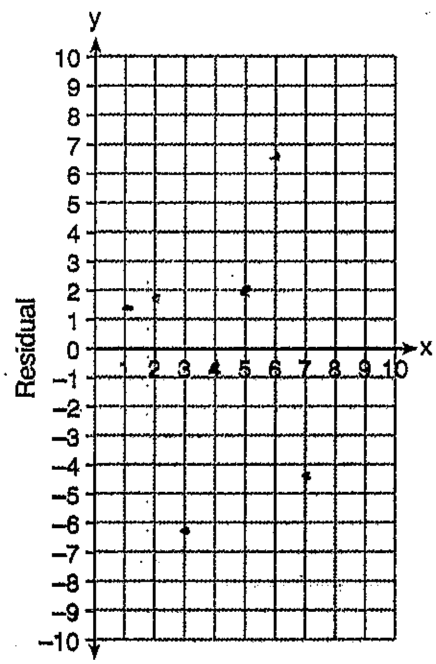
pattern

4. Use the data below to write the regression equation ($y = ax + b$) for the raw test score based on the hours tutored. Round all values to the *nearest hundredth.*

Tutor Hours, x	Raw Test Score	Residual (Actual – Predicted)
1	30	1.3
2	37	1.9
3	35	–6.4
4	47	–0.7
5	56	2.0
6	67	6.6
7	62	–4.7

Equation: $y = 6.32x + 22.43$

Create a residual plot on the axes below, using the residual scores in the table above.



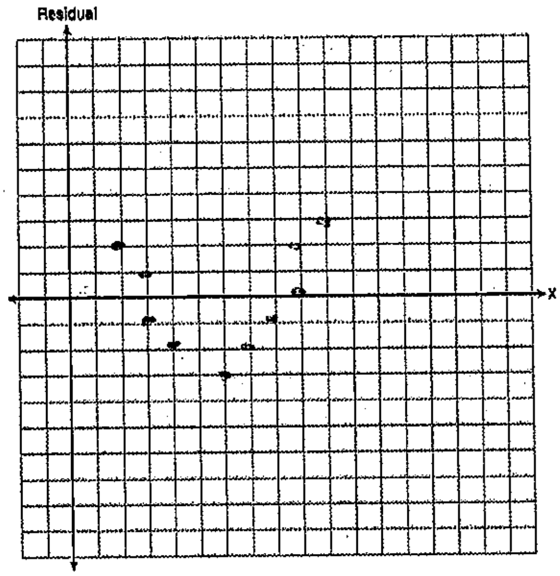
Based on the residual plot, state whether the equation is a good fit for the data. Justify your answer.

Good fit because the points are randomly scattered above and below the x-axis.

5. The table below represents the residuals for a line of best fit. Plot these residuals on the set of axes below. Using the plot, assess the fit of the line for these residuals and justify your answer.

x	2	3	3	4	6	7	8	9	9	10
Residual	2	1	−1	−2	−3	−2	−1	2	0	3

It is a bad fit because there is a pattern.

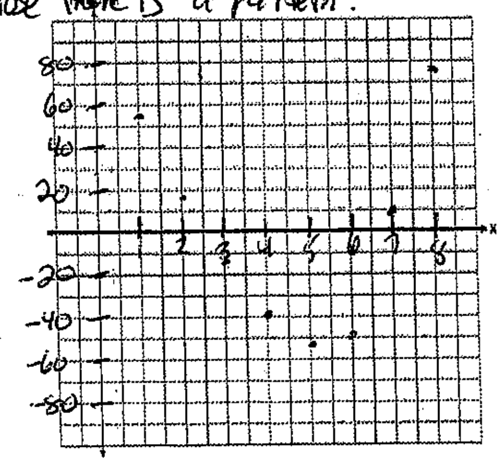


6. Justin recently opened a new on-line business selling custom photo frames. During the first eight weeks of business he recorded the number of frames he sold weekly in the first two columns of the accompanying table. He also calculated the residual scores, rounded to the nearest integer, and recorded them in the third column of the table.

a) Write the linear regression equation, $y = ax + b$, for the weekly number of photo frames sold, rounding all values to the nearest hundredth. Stat, Edit Stat, Calc, 4 LinReg $y = 43.49x - 87.57$

b) Justin wants to determine if he chose the best type of regression to model the data. Create a residual plot on the accompanying axes, and determine if the equation is a good fit for the data. Justify your answer. No, it's a bad fit because there is a pattern.

Week	Number of Photo Frames Sold	Residual
(x)	(y)	
1	11	55
2	17	18
3	28	-15
4	46	-40
5	75	-54
6	123	-50
7	227	10
8	338	78



Scale ≥ $\frac{\text{max}}{\#\text{of boxes}}$

scale ≥ $\frac{78}{10}$

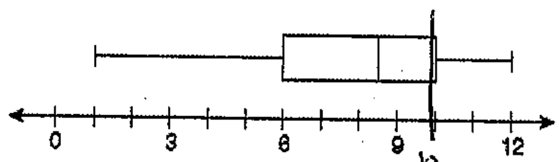
scale ≥ 7.8

scale = 10

Box Plot

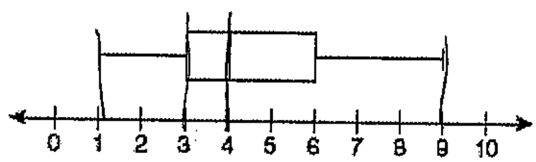
The first dash is the minimum
The second dash in the first (lower) quartile
The third dash is the median (second quartile)
The fourth dash is the third (upper) quartile
The fifth dash is the maximum
Range = maximum – minimum
Interquartile Range = Q3 – Q1
Each section is 25% of the data

1. What is the value of the third quartile shown on the box-and-whisker plot below?



1) 6   (3) 10
2) 8.5   4) 12

2. A movie theater recorded the number of tickets sold daily for a popular movie during the month of June. The box-and-whisker plot shown below represents the data for the number of tickets sold, in hundreds.

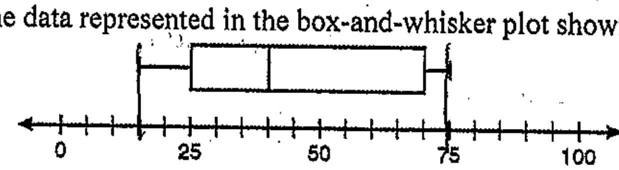


Which conclusion can be made using this plot?

1) The second quartile is 600. 400 X
2) The mean of the attendance is 400. X
Mean is not part of box plot.
3) The range of the attendance is 300 to 600. 100 to 900 X
4) Twenty-five percent of the attendance is between 300 and 400. ✓ 25% between each line

3. What is the range of the data represented in the box-and-whisker plot shown below?

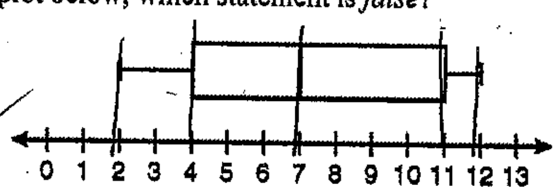
1) 40
2) 45
(3) 60
4) 100



Range=max−min
Range=75−15
Range=60

4. Based on the box-and-whisker plot below, which statement is *false*?

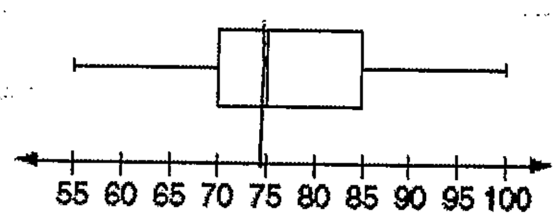
1) The median is 7. ✓
2) The range is 12. 12−2=10 X
3) The first quartile is 4. ✓
4) The third quartile is 11. ✓



5. The accompanying box-and-whisker plot represents the scores earned on a science test.

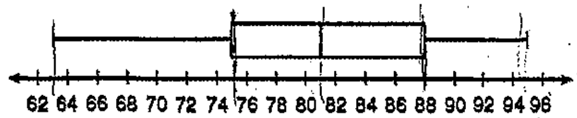
What is the median score?

(1) 70   (3) 77
(2) 75   (4) 85



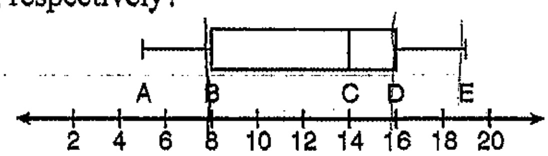
6. The box-and-whisker plot below represents a set of grades in a college statistics class. Which interval contains exactly 50% of the grades?

1) 63-88 75%, 3 lines
2) 63-95 100%, 4 lines
3) 75-81 25%, 1 line
4) 75-88 50%, 2 lines

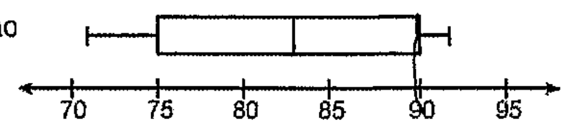


7. The box-and-whisker plot shown below represents the number of magazine subscriptions sold by members of a club.
Which statistical measures do points *B*, *D*, and *E* represent, respectively?

1) minimum, median, maximum
2) first quartile, median, third quartile
3) first quartile, third quartile, maximum
4) median, third quartile, maximum



8. The box plot below summarizes the data for the average mo Fahrenheit for Orlando, Florida.
The third quartile is



1) 92
2) 90
3) 83
4) 71

9. Which statistic can *not* be determined from a box plot representing the scores on a math test in Mrs. DeRidder's algebra class?

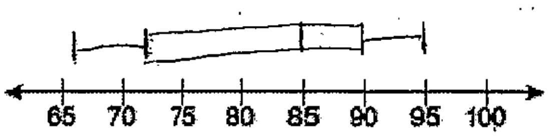
1) the lowest score
2) the median score
3) the highest score
4) the score that occurs most frequently

10. The test scores from Mrs. Gray's math class are shown below. Construct a box-and-whisker plot to display these data.

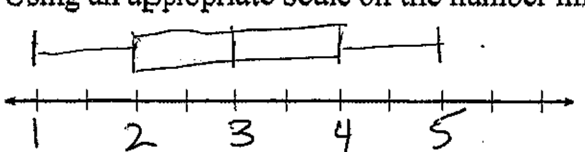
Stat, Edit
Stat, Calc, 1-Var stats

72, 73, 66, 71, 82, 85, 95, 85, 86, 89, 91, 92

min 66
Q1 72.5
Med 85
Q3 90
max 95



11. Robin collected data on the number of hours she watched television on Sunday through Thursday nights for a period of 3 weeks. The data are shown in the table below.
Using an appropriate scale on the number line below, construct a box plot for the 15 values.



	Sun	Mon	Tues	Wed	Thurs
Week 1	4	3	3.5	2	2
Week 2	4.5	5	2.5	3	1.5
Week 3	4	3	1	1.5	2.5

min 1
Q1 2
Med 3
Q3 4
max 5

Stat, Edit
Stat, Calc, 1-Var stats

Unit Conversions

Unit Conversions

To cancel out units, multiply by the conversion. The unit to cancel should have one on top and one on bottom.

1. The following conversion was done correctly: $\frac{3\cancel{\text{miles}}}{1\cancel{\text{hour}}} \cdot \frac{1\cancel{\text{hour}}}{60\text{ minutes}} \cdot \frac{5280\cancel{\text{feet}}}{1\cancel{\text{mile}}} \cdot \frac{12\text{ inches}}{1\cancel{\text{foot}}}$

What were the final units for this conversion?

1) minutes per foot
2) minutes per inch
3) feet per minute
4) inches per minute

2. Olivia entered a baking contest. As part of the contest, she needs to demonstrate how to measure a gallon of milk if she only has a teaspoon measure. She converts the measurement using the ratios below:

Which ratio is *incorrectly* written in Olivia's conversion?

$$\frac{4\cancel{\text{ quarts}}}{1\text{ gallon}} \cdot \frac{2\cancel{\text{ pints}}}{1\cancel{\text{ quart}}} \cdot \frac{2\text{ cups}}{1\cancel{\text{ pint}}} \cdot \frac{\frac{1}{4}\text{ cup}}{4\cancel{\text{ tablespoons}}} \cdot \frac{3\text{ teaspoons}}{1\text{ tablespoon}}$$

cup

1) $\frac{4\text{ quarts}}{1\text{ gallon}}$
2) $\frac{2\text{ pints}}{1\text{ quart}}$
3) $\frac{\frac{1}{4}\text{ cup}}{4\text{ tablespoons}}$
4) $\frac{3\text{ teaspoons}}{1\text{ tablespoon}}$

3. A construction worker needs to move 120 ft^3 of dirt by using a wheelbarrow. One wheelbarrow load holds 8 ft^3 of dirt and each load takes him 10 minutes to complete. One correct way to figure out the number of hours he would need to complete this job is

1) $\frac{120\cancel{ft^3}}{1} \cdot \frac{10\text{ min}}{1\cancel{\text{ load}}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{1\cancel{\text{ load}}}{8\cancel{ft^3}}$ X
2) $\frac{120\text{ ft}^3}{1} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{8\text{ ft}^3}{10\text{ min}} \cdot \frac{1}{1\text{ load}}$ X
3) $\frac{120\text{ ft}^3}{1} \cdot \frac{1\text{ load}}{10\text{ min}} \cdot \frac{8\text{ ft}^3}{1\text{ load}} \cdot \frac{1\text{ hr}}{60\text{ min}}$ X
4) $\frac{120\cancel{ft^3}}{1} \cdot \frac{1\cancel{\text{ load}}}{8\cancel{ft^3}} \cdot \frac{10\cancel{\text{ min}}}{1\cancel{\text{ load}}} \cdot \frac{1\text{ hr}}{60\cancel{\text{ min}}}$ ✓

4. The Utica Boilermaker is a 15-kilometer road race. Sara is signed up to run this race and has done the following training runs:

I. 10 miles
II. 44,880 feet
III. 15,560 yards

Which run(s) are at least 15 kilometers?

1) I, only
2) II, only
3) I and III
4) II and III

$10\text{ mi} \cdot \frac{1.609\text{ km}}{1\text{ mi}} = 16.09\text{ km}$

$44{,}880\text{ ft} \cdot \frac{1\text{ mi}}{5280\text{ ft}} \cdot \frac{1.609\text{ km}}{1\text{ mi}} = 13.67\text{ km}$

$15{,}560\text{ yd} \cdot \frac{1\text{ mi}}{1760\text{ yd}} \cdot \frac{1.609\text{ km}}{1\text{ mi}}$ 14.2

5. A typical marathon is 26.2 miles. Allan averages 12 kilometers per hour when running in marathons. Determine how long it would take Allan to complete a marathon, to the *nearest tenth of an hour*. Justify your answer.

$26.2\cancel{\text{mi}} \cdot \frac{1.609\text{ km}}{1\cancel{\text{ mi}}} \cdot \frac{1\text{ hr}}{12\text{ km}}$

$\frac{26.2(1.609)}{12} = 3.5$