# *Common Core Algebra I Key Understandings*

## Can I put this into the calculator?!

## Pre-Algebra
**Rounding:**
Nearest unit(foot, inch, meter, degree, etc.): 0 decimal places
Nearest tenth: 1 decimal place
Nearest hundredth: 2 decimal places
Nearest thousandth: 3 decimal places
Nearest ten-thousandth: 4 decimal places
*Draw a line after the place you are rounding to, the next digit determines whether you round up or down
**Adding/Subtracting:**
-If same sign, add the numbers and keep the sign
-If different sign, subtract the numbers and take the sign of the bigger number
**Multiplying/Dividing:**
-If same sign, the answer is positive
-If different signs, the answer is negative
**Absolute value is the distance from 0, distance is always positive.**

| Rational | Irrational |
|---|---|
| Ends of continues with a pattern | Never ends with no pattern |
| Fraction | $\pi$ |
| Perfect Square Radicals | Non Perfect Square Radicals |

Addition/Subtraction: If at least one number is irrational, the result is irrational.
Multiplication/Division: If one number is irrational, the result is irrational. If both are irrational, the result can either be rational or irrational.
***If an irrational number is involved, the result is almost always irrational.**
**Order of Operations: PEMDAS**

## Geometric Measurements
**Area:**
Rectangle: A=l$\bullet$w
Triangle: $A = \dfrac{1}{2}l\bullet w$

**Perimeter:**
Add all sides

*To find missing side of right triangle, use Pythagorean Theorem: $a^2 + b^2 = c^2$

# Number Properties:

| | |
|---|---|
| $2(4 + 3) = 2 \bullet 4 + 2 \bullet 3$ | Distributive Property |
| $4 \bullet 6 = 6 \bullet 4$ | Commutative Property of Multiplication |
| $2 + 7 = 7 + 2$ | Commutative Property of Addition |
| $5 + (2 + 3) = (5 + 2) + 3$ | Associative Property of Addition |
| $5 \bullet (4 \bullet 3) = (5 \bullet 4) \bullet 3$ | Associative Property of Multiplication |
| $4 + 0 = 4$ | Additive Identity |
| $7 \bullet 1 = 7$ | Multiplicative Identity |
| $5 + -5 = 0$ | Additive Inverse |
| $4 \bullet \dfrac{1}{4} = 1$ | Multiplicative Inverse |
| $3 \bullet 0 = 0$ | Multiplication Property of Zero |
| $4(3x^2 + 2) - 9 = 8x^2 + 7 \rightarrow 4(3x^2 + 2) = 8x^2 + 16$ | Addition Property of Equality |
| $2x + 8 = 4x + 4 \rightarrow 8 = 2x + 4$ | Subtraction Property of Equality |
| $2x^2 = 8x + 10 \rightarrow x^2 = 4x + 5$ | Division Property of Equality |
| $\dfrac{2x + 5}{3} = 5 \rightarrow 2x + 5 = 15$ | Multiplication Property of Equality |

**\*ASSociative property has two sets of parenthesis**
**\*Commutative property has numbers commute (move)**
**\*Identity is where you start and end with the same thing**
**\*Inverse is when you end up with the identity element (Addition 0, Multiplication 1)**
**Additive Inverse: Negate every term**
**Multiplicative Inverse: Find the reciprocal (flip)**

## Equation Solving
1) Get rid of fractions (Multiply by the LCD)
2) Get rid of parenthesis (Distribute)
3) Combine like terms on each side
4) Bring all variables to one side
5) Isolate variable (add/subtract first, divide last)

= sign: equation
$<, \leq, >, \geq$ sign: inequality
Neither sign: expression

**Word Problems:**
**Exponential**/ Interest/Depreciation Problems: $A = P(1 \pm r)^t$, where A is the current amount, P is the initial amount, r is the rate as a decimal (divide by 100), and t is time.
Given an exponential function: What is in front of the parenthesis is the INITIAL amount, what is inside the parenthesis is 1 + the rate or 1 – the rate.
**Example:** $A = 500(1.2)^t$: 500 is initial amount, rate is .2 or 20% growth (1 + .2)

$A = 500(0.8)^t$: 500 is initial amount, rate is .2 or 20% decay (1 - .2)

**Linear**: per x plus the one time fee
To determine when two things are the same, set them equal to each other.
To determine which of two things to choose for a given amount, substitute given amount into each function.
Profit = amount made – amount spent
**Writing Equations/Inequalities**
**-less than and subtracted from: switch the order in which you see the terms**
-*is* less than and *is* more than are inequalities
-quantity means parenthesis
**In terms of x:**
**-When given something is less than, more than, twice, etc. of something else, call something x and express everything else in terms of x.**
-Consecutive integers are one apart (x, x + 1, x + 2)
-Consecutive even and odd integers are two apart (x, x + 2, x + 4)
**Solving Inequalities:**

| < less than | open circle | shade left |
|---|---|---|
| ≤ less than or equal to | closed circle | shade left |
| > greater than | open circle | shade right |
| ≥ greater than or equal to | closed circle | shade right |

**Multiple Choice:** The answer must be *in* the solution set
**\*When multiplying or dividing by a negative in an inequality, switch the inequality!**

**-To solve a system of equations algebraically:**
**Addition Method (Variables are lined up):**
1) Choose a variable to cancel and multiply each equation by the other's coefficient
*multiply by negative if they are the same sign
2) Add equations together
3) Solve equation for one variable
4) Substitute answer in to either equation to find the second variable.
**\*For word problems, the first equation is just** $x + y =$

**Substitution Method (One of the variables is isolated):**
1) Substitute one equation into the other
2) Solve equation for one variable
3) Substitute answer in to either equation to find the second variable

# Sequences:
**Arithmetic: has a constant difference (addition)**
**Geometric: has a common ratio (multiply)**
**Formulas are on the reference sheet**
**Arithmetic:** $a_n = a_1 + (n-1)d$ , **Geometric:** $a_n = a_1(r)^{n-1}$

$a_n = nth\ term$, $a_1 = first\ term$, $n = the\ term\ you're\ finding\ (leave\ as\ n\ to\ find\ equation)$,
$d = common\ difference\ (what\ you're\ adding), r = common\ ratio(what\ you're\ multiplying)$
*Keep $n-1$ as $n-1$ when writing the equation
**Recursive Sequence: Find the term after the one they give you by substituting the previous term.** $a_{n-1}$ means the previous term. $a_{n-1}$ or $a_{n+1}$ is recursive and essentially the same thing.

# Polynomials
$ax^b$, a is the coefficient, x is the base, b is the exponent
The degree of the polynomial is the highest exponent
Terms are separated by plus or minus signs


Adding: Combine like terms
Subtracting: Distribute the negative and then combine like terms (Keep Change Change)
**\*Subtracting from: from comes first!**
Multiplying: Box Method (Multiply to determine what's in the boxes, add to combine like terms)
Dividing: Divide every term in the numerator by the denominator


# Exponents
**Multiplying: Add exponents** $\qquad\qquad\qquad\qquad\quad$ $x^2 \bullet x^3 = x^{2+3} = x^5$

**Dividing: Subtract exponents:** $\qquad\qquad\qquad\quad$ $\dfrac{x^8}{x^5} = x^{8-5} = x^3$

**When raising a power to a power, multiply exponents:** $(x^2)^3 = x^{2\bullet 3} = x^6$

Anything to the zero power is equal to 1 $\qquad\qquad$ $x^0 = 1$

**Negative exponents are fractions!** $\qquad\qquad\quad$ $x^{-2} = \dfrac{1}{x^2}$

**Drop what is being raised to the negative power**


**If exponent is outside parenthesis, everything gets it** $\qquad \left(\dfrac{xy}{z}\right)^3 = \dfrac{x^3 y^3}{z^3}$


# Functions
-A function is when each x value "talks" to only one y value (x does not repeat)
-A graph is a function if it passes the vertical line test (no vertical line can touch twice)
-To evaluate functions, substitute the value in for the variables
*\* $f(x)$ is the same thing as $m(n)$ is the same thing as $y$*
**Domain is all possible x values**
Fraction: set denominator equal to zero, answer is all real numbers except your answer
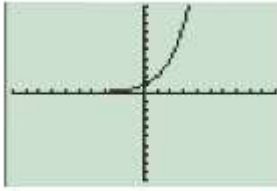Radicals: set $radicand \geq 0$
If no restrictions: all real numbers
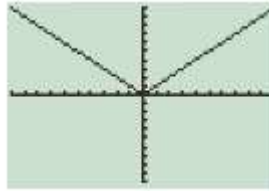
**Range is all possible y values**
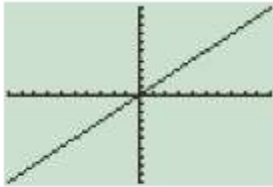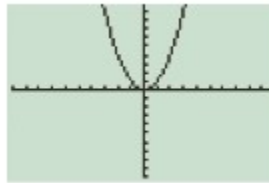**\*Real life domain problems are almost always non-negative integers or whole numbers**
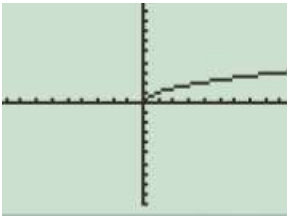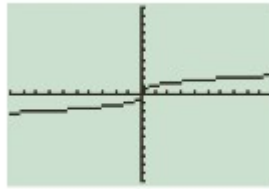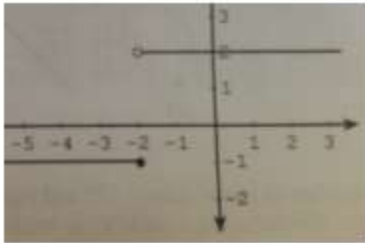
**Types of Functions**

Exponential

Absolute Value

Linear

Quadratic

Square Root

Cube Root
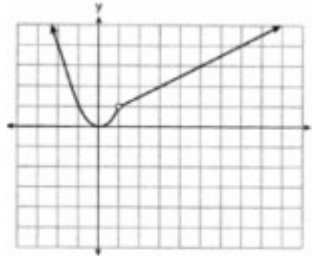
Step

Piecewise

**To determine whether a function is linear or exponential:**

| Linear | Exponential |
|---|---|
| **Add/Subtract Constant Amount** | **Multiply/Divide Constant Amount** |
| | Add/subtract increasing/decreasing amount |
| Per x + 1TF | AP1RT |
| | **Percent/Rate** |
| **Look at Scatter Plot (Turn on Stat Plot in y=, zoom 9)** ||

# Graphing:

-Standard form of a line is y = mx + b, m = slope, b = y intercept
-To graph a line, find slope and y intercept by ISOLATING y OR type into y =
-To graph a straight lines, Yanci Has X-Ray Vision (YHXV), y = # is horizontal line, x = # is vertical line

-To graph anything else:
*Get y by itself
*Type into y =
*If there is an interval/domain, plot only nice points between those values: no arrows
$a < x \le b$ (All numbers between a and b)
*If not, plot all "nice" points from -10 to 10 and use arrows
*For quadratic and absolute value, find a mirror image
-Slope = $\dfrac{rise}{run} = \dfrac{y_2 - y_1}{x_2 - x_1}$, positive slope is uphill, negative slope is downhill

Average rate of change: $\dfrac{f(b) - f(a)}{b - a}$, you may have to find $f(a)$ and $f(b)$ by substituting

-To solve a system of equations graphically, find the point(s) of intersection.
-To solve a system of inequalities:
$<$ : shade below dashed line             $\le$ : shade below solid line
$>$: shade above dashed line             $\ge$ : shade above solid line
The solution set is the region that both graphs are shaded. Mark with an S.

**To determine if a point is on a graph, substitute x and y into the equation**
A linear function has a constant difference being added
An exponential function has a constant factor (or percent or rate)
Each half of an absolute value function has a constant difference being added
Each half of a quadratic function **does not** have a constant different being added
A function is increasing if the y value increases as its x value increases (average rate of change is positive)
A function is decreasing if the y value decreases as its x value increases (average rate of change is negative)
A function is neither increasing nor decreasing at the starting point, turning point(s), and endpoints
A function is positive when the y values are above 0 (above the x axis)
A function is negative when the y values are below 0 (below the x axis)

**Parabolas:** (Graphically)
-The roots/zeroes hit the x axis
-The vertex is the turning point
-The axis of symmetry (AOS) cuts it in half, x = #

**Transforming Functions:**
If adding to f(x), the graph moves up or down
If adding to x, the graph moves left or right (the opposite direction in which you would think)
$y = f(x) + a$ moves UP a units
$y = f(x) - a$ moves DOWN a units
$y = f(x + a)$ moves LEFT a units
$y = f(x - a)$ moves RIGHT a units

If the x is negated, the graph is reflected over the y axis
If the f(x) (aka y) is negated, the graph is reflected over the x axis
$y = f(-x)$ reflect over y axis
$y = -f(x)$ reflect over x axis
If a is positive, the vertex is a minimum and the graph opens upward
If a is negative, the vertex is a maximum and the graph opens downward

$y = af(x)$ Vertical Dilation
If $|a| > 1$, vertical stretch, narrower
If $|a| < 1$, vertical shrink, wider
$y = f(ax)$ Horizontal Dilation
If $|a| > 1$, Horizontal shrink
If $|a| < 1$, Horizontal stretch

# Factoring/Quadratics:
**Greatest Common Factor**
GCF(    )
Divide each term by the GCF, put the GCF outside the parenthesis
**Difference of Two Squares**
$(\sqrt{1} + \sqrt{2})(\sqrt{1} - \sqrt{2})$
Recognize subtraction of two perfect squares
Two sets of parenthesis, different signs, square root of each
**Trinomials**
(x    )(x    )
1) First sign comes down
2) The two signs must multiply for the last sign
3) Find two numbers that multiply to the last number and add/subtract to the middle number
*Write out all of your pairs of factors starting with 1 and itself
**Bridge Trinomials**
1) Multiply the first and last
2) Factor the trinomial normally
3) Pay the toll (Divide by the leading coefficient, what you multiplied by in step 1)
*If they divide nicely, divide them. If the fractions reduce, reduce them. If not, put the denominator in front of the variable inside the parenthesis.
**Can You Factor Any Further?**
Go back through the list and try to factor further

**Quadratic Equations**

Mr. $x^2$ wants to party. Before he can party, all of his friends have to come over. Once all of his friends come over, they party! At the party, they want to blow bubbles. If the bubble machine is broken, they sing the quadratic formula song.

*Divide out an integer GCF if possible

1) Bring all terms to the side with $x^2$ ($x^2$ should be positive)
2) Factor (Follow the steps of factoring)
3) Set each factor equal to zero (T-chart) and solve

**If you can't factor:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) List a, b, and c values
2) Substitute values into formula
3) Solve what is underneath the radical
4) Reduce the radical (If necessary)
        a) Make perfect square and non perfect square radicals
        b) Find the largest perfect square that divides into the radicand
        c) Take the square root of the perfect square, keep the non perfect square in the radical
5) Reduce the fraction by dividing out a GCF of EVERY term (If necessary)

**Completing the Square**

*Divide out a GCF if possible

1) Bring both variable terms to one side and the constant term to the other side

2) Add $\left(\dfrac{b}{2}\right)^2$ to both sides

3) Factor the trinomial (Both factors must be the same)
4) Rewrite the factors as a binomial squared
5) Take the square root of both sides (The right hand side should have a $\pm$)
6) Add or subtract to isolate x

**To find the roots algebraically:**

Replace y or f(x) with 0, solve the quadratic equation (Mr. $x^2$)

**To put quadratic function into vertex form:**

Variation of completing the square

1) Factor out integer GCF of the two variable terms (If necessary)

2) Add $\left(\dfrac{b}{2}\right)^2$ and subtract the distributed value

*Add if the GCF is negative

3) Factor the trinomial (Both factors must be the same)
4) Rewrite the factors as a binomial squared

$f(x) = (x - a)^2 + b$, where (a,b) is the vertex

## Statistics:

**Scatter Plots:** Positive slope is a positive correlation, negative slope is negative correlation, and if there is no line of best fit, there is no correlation.

**r = correlation coefficient.** The closer r is to -1 or 1, the stronger the correlation/relationship. The closer r is to 0, the weaker the correlation/relationship.

*You Must Turn On Diagnostics to view r ( $2^{nd}$ 0, (*Cata* log))

**Regression Equations:**
Type into Stat: Edit, Stat: Calc: Choose the appropriate regression
**Make sure you include the actual equation with y = and x and read the question(s) carefully**
To determine whether linear or exponential, see table in FUNCTIONS.

**Residual Plots:**
Residual is actual – predicted, but is generally given to you.
**Plot the x column and the residual column**
If the points are scattered randomly above and below the x-axis, the regression equation is a good choice. If there is a pattern (likely a parabola), it is a bad choice.

**Measures of Central Tendency:**
To find measures of central tendency (mean, median, quartiles, standard deviation): Type into Stat: Edit, Stat: Calc:1 – Var Stats. Use $L_2$ if there is a frequency list.

Range = max – min
Interquartile Range = Q3 – Q1
Mode is the number with the highest frequency (repeats the most)
$\sigma x$ = **population standard deviation,** Sx = Sample standard deviation
The smaller the standard deviation, the more consistent the data is (closer to the mean)

**Box (and Whisker) Plot:** Need min, Q1, Med, Q3, and max, box connects the quartiles
**Dot Plot:** Stack dots or x's vertically at each value on a number line
**Histograms:** Cumulative frequency means add up the frequencies, make each bar the same width, use a lightning bolt if the first interval is not 0, **title** and label!

Skew left: Most of the data is to the left of the median or peaks
Skew right: Most of the data is to the right of the median or peaks
Symmetric: The data is evenly spread left and right in relation to the median or peaks
Uniform (Histograms): All the bars are the same height

Causal relation is when one thing causes the other to happen
Univariate data has one measurement involved while bivariate data has two measurements involved
Bias is bad. An unbiased sample is completely random and should have no influence on the topic
Qualitative has to do with quality(opinions/colors), **quantitative has to do with quantity(measurable numbers).**

## Miscellaneous:
**Two Way Probability (Contingency Tables)**
**Joint** is two things happening. The denominator is total total.
**Marginal** is one thing happening. The denominator is total total.
**Conditional** is one thing happens GIVEN THAT or IF another happens. The condition is what you are given, what comes *after* GIVEN THAT or IF. The denominator is the condition.

**Multiple Choice Strategies**
10 STO $\rightarrow$ X, 15 STO $\rightarrow$ Y (Alpha 1)
**Variables in the problem, numbers in the answers:**
Substitute each answer in for the variable. Use $2^{nd}$ Math (Test) for equals or inequalities.
**Variables in the problem, variables in the answers:**
Type in original problem, $2^{nd}$ Math (Test), =, Type in each answer
**0 is not equivalent, 1 is equivalent**

**Conversions:** To cancel out units, have unit on top and on bottom
Example: $\dfrac{75 \text{ km}}{1 \text{ hr}} \times \dfrac{1,000 \text{ m}}{1 \text{ km}} \times \dfrac{1 \text{ hr}}{60 \text{ min}}$