Name:

Common Core Geometry Common Regents Questions!

Mr. Schlansky

Identifying Transformations

Check for orientation!!! (The direction of the letters) The only transformation that changes orientation is a line reflection (an even amount of reflections will preserve orientation). Translation = slide Rotation = turn Reflection = flip Dilation = change size (enlarge or shrink)

1. In the diagram below, which single transformation was used to map triangle *A* onto triangle *B*?

- 1) line reflection
- 2) rotation
- 3) dilation
- 4) translation



2. In the diagram below, line m is parallel to line n. Figure 2 is the image of Figure 1 after a reflection over line m. Figure 3 is the image of Figure 2 after a reflection over line n. Which single transformation would carry Figure 1 onto Figure 3?

- 1) a dilation
- 2) a rotation
- 3) a reflection
- 4) a translation

3. In the diagram below, congruent figures 1, 2, and 3 are drawn.

Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?

- 1) a reflection followed by a translation
- 2) a rotation followed by a translation
- 3) a translation followed by a reflection
- 4) a translation followed by a rotation



4. A sequence of transformations maps rectangle ABCD onto rectangle A"B"C"D", as shown in the diagram below.

Which sequence of transformations maps ABCD onto A'B'C'D' and then maps A'B'C'D' onto A''B''C''D''?

- 1) a reflection followed by a rotation
- 2) a reflection followed by a translation
- 3) a translation followed by a rotation
- 4) a translation followed by a reflection

5. Which sequence of transformations will map $\triangle ABC$ onto $\triangle A'B'C'$?

- 1) reflection and translation
- 2) rotation and reflection
- 3) translation and dilation
- 4) dilation and rotation



В

C

C

B

D

D

A

6. Identify which sequence of transformations could map pentagon ABCDE onto pentagon A"B"C"D"E", as shown below.



- 1) dilation followed by a rotation
- 2) translation followed by a rotation
- 3) line reflection followed by a translation
- 4) line reflection followed by a line reflection

7. On the set of axes below, rectangle *ABCD* can be proven congruent to rectangle *KLMN* using which transformation?

- 1) rotation
- 2) translation
- 3) reflection over the *x*-axis
- 4) reflection over the *y*-axis



8. Triangle *ABC* and triangle *DEF* are graphed on the set of axes below. Which sequence of transformations maps triangle *ABC* onto triangle *DEF*?

- 1) a reflection over the *x*-axis followed by a reflection over the *y*-axis
- 2) a 180° rotation about the origin followed by a reflection over the line y = x
- a 90° clockwise rotation about the origin followed by a reflection over the *y*-axis
- a translation 8 units to the right and 1 unit up followed by a 90° counterclockwise rotation about the origin



9. Given: $\triangle AEC$, $\triangle DEF$, and $\overline{FE} \perp \overline{CE}$

What is a correct sequence of similarity transformations that shows $\triangle AEC \sim \triangle DEF$?

- 1) a rotation of 180 degrees about point E followed by a horizontal translation
- 2) a counterclockwise rotation of 90 degrees about point *E* followed by a horizontal translation
- 3) a rotation of 180 degrees about point *E* followed by a dilation with a scale factor of 2 centered at point *E*
- 4) a counterclockwise rotation of 90 degrees about point *E* followed by a dilation with a scale factor of 2 centered at point *E*



10. In the diagram below, $\triangle ABC \cong \triangle DEF$.



Which sequence of transformations maps $\triangle ABC$ onto $\triangle DEF$?

- 1) a reflection over the x-axis followed by 3) a rotation of 180° about the origin a translation
- 2) a reflection over the y-axis followed by 4) a counterclockwise rotation of 90° a translation

followed by a translation about the origin followed by a translation

11. On the set of axes below, $\triangle ABC$ has vertices at A(-2, 0), B(2, -4), C(4, 2), and $\triangle DEF$ has vertices at D(4, 0), E(-4, 8), F(-8, -4).



Which sequence of transformations will map $\triangle ABC$ onto $\triangle DEF$?

1)	a dilation of $\triangle ABC$ by a scale factor of 3)	a dilation of $\triangle ABC$ by a scale factor of
	2 centered at point A	2 centered at the origin, followed by a
		rotation of 180° about the origin
2)	a dilation of $\triangle ABC$ by a scale factor of 4)	a dilation of $\triangle ABC$ by a scale factor of
	$\frac{1}{2}$ centered at point A	$\frac{1}{2}$ centered at the origin, followed by a
		rotation of 180° about the origin

Rigid Motion Properties

A rigid motion preserves size and angle measure producing a congruent figure They all produce a congruent figure except dilation.

1. Which transformation would *not* always produce an image that would be congruent to the original figure? 3) rotation 4) reflection 2) dilation

1) translation

2. The vertices of ΔJKL have coordinates J(5,1), K(-2,-3), and L(-4,1). Under which transformation is the image $\Delta J'K'L'$ not congruent to ΔJKL ?

1) a translation of two units to the right and two units down 3) a reflection over the x-axis

2) a counterclockwise rotation of 180 degrees around the origin 4) a dilation with a scale factor

of 2 and centered at the origin

3. If $\triangle A'B'C'$ is the image of $\triangle ABC$, under which transformation will the triangles *not* be congruent?

1) reflection over the x-axis 3) dilation centered at the origin with scale factor 2

2) translation to the left 5 and down 4 4) rotation of 270° counterclockwise about the origin

4. Under which transformation would $\triangle A'B'C'$, the image of $\triangle ABC$, not be congruent to $\triangle ABC?$

- 1) reflection over the *y*-axis
- 2) rotation of 90° clockwise about the origin
- 3) translation of 3 units right and 2 units down
- 4) dilation with a scale factor of 2 centered at the origin

5. The image of $\triangle DEF$ is $\triangle D'E'F$. Under which transformation will be triangles *not* be congruent?

1)	a reflection through the origin	3)	a dilation with a scale factor of 1 centered at $(2, 3)$
2)	a reflection over the line $y = x$	4)	a dilation with a scale factor of $\frac{3}{2}$ centered
			at the origin

6. The vertices of $\triangle PQR$ have coordinates P(2,3), Q(3,8), and R(7,3). Under which transformation of $\triangle PQR$ are distance and angle measure preserved? 2) $(x,y) \rightarrow (x+2,3y)$ 3) $(x,y) \rightarrow (2x,y+3)$ 4) $(x,y) \rightarrow (x+2,y+3)$ 1) $(x, y) \rightarrow (2x, 3y)$

7. Which transformation would result in the perimeter of a triangle being different from the perimeter of its image?

- 1) $(x, y) \rightarrow (y, x)$
- 2) $(x,y) \rightarrow (x,-y)$
- 3) $(x, y) \rightarrow (4x, 4y)$
- 4) $(x,y) \rightarrow (x+2,y-5)$

Rigid Motion Proofs

To prove triangles are congruent/similar using rigid motions/transformations				
1) Identify the transformations (Check for orientat	ion to determine if reflection)			
On the grid: reflect/rotate/dilate first				
Off the grid: translate first				
Translate to				
Reflect Δ over				
Rotate Δ about pointuntil it maps onto	Δ			
Dilation Δ centered at point by a scale factor of $\frac{image}{original}$				
Congruence Similarity				
2) A and are rigid motions.	2) A dilation and preserve angle measure			
3) A rigid motion preserves size and angle	producing a similar figure.			
measure producing a congruent figure.				

1. Triangle A'B'C' is the image of triangle *ABC* after a translation of 2 units to the right and 3 units up. Is triangle *ABC* congruent to triangle *A'B'C'*? Explain why.

2. After a reflection over a line, $\Delta A'B'C'$ is the image of ΔABC . Explain why triangle *ABC* is congruent to triangle $\Delta A'B'C'$.

3. The graph below shows $\triangle ABC$ and its image, $\triangle A^{"}B^{"}C^{"}$. Describe a sequence of rigid motions which would map $\triangle ABC$ onto $\triangle A^{"}B^{"}C^{"}$.



4. As graphed on the set of axes below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a sequence of transformations.

Is $\triangle A'B'C'$ congruent to $\triangle ABC$? Use the properties of rigid motion to explain your answer.



5. Triangle *ABC* and triangle *ADE* are graphed on the set of axes below. Describe a transformation that maps triangle *ABC* onto triangle *ADE*. Explain why this transformation makes triangle *ADE* similar to triangle *ABC*.



6. Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.



7. Prove that $\triangle ABC \cong \triangle A'B'C'$ using rigid motions.



8. Prove that $\triangle ABC \cong \triangle A'B'C'$ using rigid motions.



9. Describe a sequence of transformation that would map $\triangle ADE$ onto $\triangle ABC$. What is the relationship between $\triangle ADE$ and $\triangle ABC$? Explain your answer.



10. Prove that $\triangle ABC \cong \triangle XYZ$ using rigid motions.



Regular Polygon Rotations

To determine the minimum number of degrees a regular polygon must be rotated to be mapped onto itself:

1) The minimum rotation is $\frac{360}{n}$.

2) Any multiple of that will also map the regular polygon onto itself!

1. What is the minimum number of degrees a regular decagon must be rotated to be mapped onto itself?

2. What is the minimum number of degrees a regular hexagon must be rotated to be carried onto itself?

3. A regular pentagon is shown in the diagram below.

If the pentagon is rotated clockwise around its center, the minimum number of degrees it must be rotated to carry the pentagon onto itself is

- 1) 54°
- 2) 72°
- 3) 108°
- 4) 360°

4. Which regular polygon has a minimum rotation of 45° to carry the polygon onto itself? (1) octagon (3) hexagon

(2) decagon (4) pentagon



5. The regular polygon below is rotated about its center. Which angle of rotation will carry the figure onto itself?

- 1) 60°
- 2) 108°
- 3) 216°
- 4) 540°



6.	Which	of the	following rotations	would not map	an equilateral	triangle ont	o itself?
(1)) 120°		(3) 180°				
(2)) 240°		(4) 480°				

7. In which regular polygon would a rotation of rotation of 144° carry the shape onto itself?

(1) octagon(3) hexagon(2) square(4) pentagon

8. Which of the following rotations would not map a regular pentagon onto itself?(1) 144 (3) 216

(2) 120 (4) 720

9. Which of the following regular polygons has rotational symmetry of 480°?

- (1) pentagon (3) octagon
- (2) hexagon (4) decagon

10. Which rotation about its center will carry a regular decagon onto itself?

- 1) 54°
- 2) 162°
- 3) 198°
- 4) 252°

To map a shape onto itself:

Translation/Dilation: Never.

Reflection: The line of reflection must be a line of symmetry (cuts shape in half). Rotation: Center of rotation must be the center of the shape. Use common sense for degree measure.

1. In the diagram below, a square is graphed in the coordinate plane.

A reflection over which line does *not* carry the square onto itself?

- 1) x = 5
- 2) *y* = 2
- 3) y = x
- 4) x + y = 4



2. On the set of axes below, Geoff drew rectangle ABCD.

What of the following transformations would map the rectangle onto itself?

- 3. Circle *K* is shown in the graph below.

Which of the following transformations map circle K onto itself?

- 1) $r_{y=x}$ 3) r_{x-axis} 2) r 4) r
- 2) $r_{x=-5}$ 4) $r_{y=2}$





4. Which transformation does not map the circle below onto itself?



5. As shown in the graph below, the quadrilateral is a rectangle.



Which transformation would *not* map the rectangle onto itself?

- 1) a reflection over the *x*-axis
- 3) a rotation of 180° about the origin
- 2) a reflection over the line x = 4
- 4) a rotation of 180° about the point (4,0)
- 6. Which figure always has exactly four lines of reflection that map the figure onto itself?
- 1) square

3) regular octagon

2) rectangle

- 4) equilateral triangle
- 7. Which transformation would not carry a square onto itself?
- 1) a reflection over one of its diagonals
- 2) a 90° rotation clockwise about its center
- 3) a 180° rotation about one of its vertices
- 4) a reflection over the perpendicular bisector of one side

Candy Corn Problems

If the bases are not involved: $\frac{top}{top} = \frac{bottom}{bottom} = \frac{side}{side}$ If bases are involved: separate your triangles!

1. In the diagram of $\triangle ADC$ below, $\overline{EB} \parallel \overline{DC}$, AE = 9, ED = 5, and AB = 9.2.

What is the length of \overline{AC} , to the *nearest tenth*?

- 1) 5.1
- 2) 5.2
- 3) 14.3
- 4) 14.4



2. In the diagram of $\triangle ABC$, points *D* and *E* are on \overline{AB} and \overline{CB} , respectively, such that $\overline{AC} \parallel \overline{DE}$.



If AD = 24, DB = 12, and DE = 4, what is the length of \overline{AC} ?

- 1) 8
- 2) 12
- 3) 16
- 4) 72

3. Given $\triangle MRO$ shown below, with trapezoid *PTRO*, MR = 9, MP = 2, and PO = 4.



 What is the length of \overline{TR} ?

 1) 4.5
 3) 3

 2) 5
 4) 6

4. To find the distance across a pond from point B to point C, a surveyor drew the diagram below. The measurements he made are indicated on his diagram. Use the surveyor's information to determine and state the distance from point B to point C, to the *nearest yard*.



5. In the diagram below, triangle ACD has points B and E on sides \overline{AC} and \overline{AD} , respectively, such that $\overline{BE} \parallel \overline{CD}$, AB = 1, BC = 3.5, and AD = 18.



What is the length of \overline{AE} , to the *nearest tenth*?

6. In the diagram of $\triangle ABC$ shown below, $\overline{DE} \parallel \overline{BC}$. If $\overline{AE} = 6$, $\overline{DE} = 10$, and $\overline{AC} = 9$, find \overline{BC}



7. In the diagram of $\triangle ABC$ below, \overline{DE} is parallel to \overline{AB} , CD = 15, AD = 9, and AB = 40. Find the length of \overline{DE} .



8. In the diagram below of $\triangle PQR$, \overline{ST} is drawn parallel to \overline{PR} , PS = 2, SQ = 5, and TR = 5What is the length of \overline{QR} ?



When an altitude is drawn to a right triangle HLLS and SAAS

 $\frac{H}{L} = \frac{L}{S} \quad \frac{S}{A} = \frac{A}{S}$ If L is involved, use HLLS If A is involved, use SAAS **Know how to reduce radicals:** LEG ALTITUDE SEG SEG HYPOTENUSE

C

В

D

- 1) Separate into perfect square and non perfect square
 - 2) Take the square root of the perfect square

1. In the diagram below of right triangle ACB, altitude \overline{CD} is drawn to hypotenuse \overline{AB} .



2. In the diagram below of right triangle ABC, altitude \overline{CD} is drawn to hypotenuse \overline{AB} .

If AD = 3 and DB = 12, what is the length of altitude \overline{CD} ? 1) 6 2) $6\sqrt{5}$ 3) 3 4) $3\sqrt{5}$ 3. If $\overline{AD} = 3$ and $\overline{DB} = 27$, find \overline{CD}

4. In the diagram below of right triangle *ACB*, altitude \overline{CD} intersects \overline{AB} at *D*. If AD = 3 and DB = 4, find the length of \overline{CD} in simplest radical form.



5. Triangle ABC shown below is a right triangle with altitude \overline{AD} drawn to the hypotenuse \overline{BC} .

- If BD = 2 and DC = 10, what is the length of \overline{AB} ? 1) $2\sqrt{2}$ 2) $2\sqrt{5}$ 3) $2\sqrt{6}$
- 4) $2\sqrt{30}$

B 2 D 10 C

6. In right triangle *ABC* shown in the diagram below, altitude \overline{BD} is drawn to hypotenuse \overline{AC} , CD = 12, and AD = 3.



4) 9

2 D 10 C

7. In the diagram below of right triangle *ABC*, altitude \overline{BD} is drawn to hypotenuse \overline{AC} , AC = 16, and CD = 7.

What is the length of \overline{BD} ? 1) $3\sqrt{7}$ 2) $4\sqrt{7}$ 3) $7\sqrt{3}$

4) 12

8. In the diagram below of $\triangle ABC$, $\angle ABC$ is a right angle, AC = 12, AD = 8, and altitude \overline{BD} is drawn.

What is the length of \overline{BC} ?

- 1) $4\sqrt{2}$
- 2) $4\sqrt{3}$
- 3) $4\sqrt{5}$
- 4) $4\sqrt{6}$





9. In $\triangle RST$ shown below, altitude \overline{SU} is drawn to \overline{RT} at U. If SU = h, UT = 12, and RT = 42, which value of h will make $\triangle RST$ a right triangle with $\angle RST$ as a right angle?

- 1) $6\sqrt{3}$
- 2) $6\sqrt{10}$
- 3) ₆√14
- 4) $6\sqrt{35}$



11. Kirstie is testing values that would make triangle *KLM* a right triangle when \overline{LN} is an altitude, and KM = 16, as shown below.



12. In the diagram below, \overline{CD} is the altitude drawn to the hypotenuse \overline{AB} of right triangle ABC.

Which lengths would *not* produce an altitude that measures $6\sqrt{2}$?

- 1) AD = 2 and DB = 36
- 2) AD = 3 and AB = 24
- 3) AD = 6 and DB = 12
- 4) AD = 8 and AB = 17



h

U

R

Т

Corresponding Parts of Congruent Triangles are Congruent

Redraw the shapes so it is more clear to see what parts correspond to each other

1. After a counterclockwise rotation about point *X*, scalene triangle *ABC* maps onto $\triangle RST$, as shown in the diagram below.



2. In the diagram below, a sequence of rigid motions maps ABCD onto JKLM.

Which of the following statements must be true?



3. In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$,

 $\angle C$ onto $\angle Z$, and \overline{AC} onto \overline{XZ} .



Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

Determine and state whether $\angle A \cong \angle Y$. Explain why.

4. The image of $\triangle ABC$ after a rotation of 90° clockwise about the origin is $\triangle DEF$, as shown below.

Which statement is true?

- 1) $BC \cong DE$
- 2) $\overline{AB} \cong \overline{DF}$
- 3) $\angle C \cong \angle E$
- 4) $\angle A \cong \angle D$



5. Triangle MNP is the image of triangle JKL after a 120° counterclockwise rotation about point Q. If the measure of angle L is 47° and the measure of angle N is 57°, determine the measure of angle M. Explain how you arrived at your answer.



6. In the diagram below, a sequence of rigid motions m *ABCD* onto *JKLM*.

If $m \angle A = 82^\circ$, $m \angle B = 104^\circ$, and $m \angle L = 121^\circ$, the measur of $\angle M$ is

- 1) 53°
- 2) 82°
- 3) 104°
- 4) 121°



7. In the diagram below, $\triangle ABC$ with sides 13, 15, and 16, is mapped onto $\triangle DEF$ after a clockwise rotation of 90° about point *P*. If DE = 2x - 1, what is the value of *x*?



To determine if a proportion is correct

Look at the letters vertically and horizontally One direction, the letters should correspond Second direction, the letters should be in the same triangle *It does not matter which direction does which

1. As shown in the diagram below, \overline{AB} and \overline{CD} intersect at *E*, and $\overline{AC} \parallel \overline{BD}$.

Given $\triangle AEC \sim \triangle BED$, which equation is true?

- $\begin{array}{c} 1) \quad \frac{CE}{DE} = \frac{EB}{EA} \\ 2) \quad \frac{AE}{BE} = \frac{AC}{BD} \\ 3) \quad \frac{EC}{AE} = \frac{BE}{ED} \end{array}$
- $\frac{4}{EC} = \frac{AC}{BD}$

2. In the diagram below of right triangle AED, $\overline{BC} \parallel \overline{DE}$. Which statement is always true?

1)	$\frac{AC}{BC} = \frac{DE}{AE}$
2)	$\frac{AB}{AD} = \frac{BC}{DE}$
3)	$\frac{AC}{CE} = \frac{BC}{DE}$
4)	$\frac{DE}{BC} = \frac{DB}{AB}$



Е

B

D

3. In the diagram below, $\Delta QRX \sim \Delta TUV$. Which of the following statements is *not* true?



4. Given that $\Delta DEF \sim \Delta HIJ$, which is the correct statement about their corresponding sides?



5. In the diagram below, $\triangle ABC \sim \triangle RST$.



6. In the diagram below, ΔMOY is the image of ΔNOE after a dilation followed by a reflection. Which of the following statements is true?



7. In the diagram below, $\triangle DEF$ is the image of $\triangle ABC$ after a clockwise rotation of 180° and a dilation where AB = 3, BC = 5.5, AC = 4.5, DE = 6, FD = 9, and EF = 11.

Which relationship must always be true?

- $\frac{1}{m \angle D} = \frac{1}{2}$
- $\frac{1}{m \angle C} = \frac{2}{1}$
- 3) $\frac{m \angle A}{m \angle C} = \frac{m \angle F}{m \angle D}$
- $\frac{4}{m\angle B} = \frac{m\angle C}{m\angle F}$



8. Scalene triangle ABC is similar to triangle DEF. Which statement is *false*?

- 1) AB:BC=DE:EF
- 2) AC:DF=BC:EF
- 3) $\angle ACB \cong \angle DFE$
- 4) $\angle ABC \cong \angle EDF$

To show triangles are similar:

The ANGLES of similar triangles are congruent

The SIDES of similar triangles are in proportion

1) AA (2 pairs of corresponding angles are congruent)

2) SAS (2 pairs of corresponding sides are in proportion and the corresponding angles between them are congruent)

3) SSS (3 pairs of corresponding sides are in proportion)

*Congruent triangles must be similar. Similar triangles are not necessarily congruent.

1. Triangles *RST* and *XYZ* are drawn below. If RS = 6, ST = 14, XY = 9, YZ = 21, and $\angle S \cong \angle Y$, is $\triangle RST$ similar to $\triangle XYZ$? Justify your answer.



2. Using the information given below, which set of triangles can not be proven similar?





3. In the diagram below, $\angle GRS \cong \angle ART$, GR = 36, SR = 45, AR = 15, and RT = 18. Which triangle similarity statement is correct?

1) $\triangle GRS \sim \triangle ART$ by \overrightarrow{AA} .



- 3) $\triangle GRS \sim \triangle ART$ by SSS.
- 4) $\triangle GRS$ is not similar to $\triangle ART$.



- 2) DE = 8, DF = 10, and $\angle A \cong \angle D$
- 3) DE = 36, DF = 64, and $\angle C \cong \angle F$
- 4) $DE = 15, DF = 20, \text{ and } \angle C \cong \angle F$



E

5. Triangles *ABC* and *DEF* are drawn below.

If AB = 9, BC = 15, DE = 6, EF = 10, and $\angle B \cong \angle E$, which statement is true?

- 1) $\angle CAB \cong \angle DEF$ 2) $\underline{AB} = \underline{FE}$
- $\overline{CB} = \overline{DE}$
- 3) $\triangle ABC \sim \triangle DEF$
- $\frac{AB}{DE} = \frac{FE}{CB}$

6. In the diagram below, $\triangle ABC \sim \triangle ADE$.

Which measurements are justified by this similarity?

- 1) AD = 3, AB = 6, AE = 4, and AC = 12
- 2) AD = 5, AB = 8, AE = 7, and AC = 10
- 3) AD = 3, AB = 9, AE = 5, and AC = 10
- 4) AD = 2, AB = 6, AE = 5, and AC = 15



7. Skye says that the two triangles below are congruent. Margaret says that the two triangles are similar. Are Skye and Margaret both correct? Explain why.



8. If $\triangle ABC$ is mapped onto $\triangle DEF$ after a line reflection and $\triangle DEF$ is mapped onto $\triangle XYZ$ after a translation, the relationship between $\triangle ABC$ and $\triangle XYZ$ is that they are always

- 1) congruent and similar
- 2) congruent but not similar
- 3) similar but not congruent
- 4) neither similar nor congruent

Right Triangles If only sides are involved, use Pythagorean theorem! $(a^2 + b^2 = c^2)$ **If an angle is involved, use SOHCAHTOA 1) Label each side with O, A, and H 2) Determine whether to use sine, cosine, or tangent (Which two are involved?) 3) Substitute into appropriate formula** *If finding a side, cross multiply and solve *If finding an angle, use \sin^{-1} , \cos^{-1} , or \tan^{-1}

1. In $\triangle ABC$ below, the measure of $\angle A = 90^\circ$, AB = 6, AC = 8, and BC = 10.



2. In triangle *MCT*, the measure of $\angle T = 90^\circ$, *MC* = 85 cm, *CT* = 84 cm, and *TM* = 13 cm. Which ratio represents the sine of $\angle C$?

1)	13	3	3)	13
	85			84
2)	84	4	ł)	84
	85			13

3. As shown in the diagram below, a ladder 12 feet long leans against a wall and makes an angle of 72° with the ground.

Find, to the *nearest tenth of a foot*, the distance from the wall to the base of the ladder.

12 ft

4. The diagram below shows the path a bird flies from the top of a 9.5-foot-tall sunflower to a point on the ground 5 feet from the base of the sunflower.

To the *nearest tenth of a degree*, what is the measure of angle *x*?

- 1) 27.8
- 2) 31.8
- 3) 58.2
- 4) 62.2



5. From the top of an apartment building, the angle of depression to a car parked on the street below is 38 degrees, as shown in the diagram below. The car is parked 80 feet from the base of the building. Find the height of the building, to the *nearest tenth of a foot*.



6. As shown in the diagram below, a building casts a 72-foot shadow on the ground when the angle of elevation of the Sun is 40°.

How tall is the building, to the *nearest foot*?

- 1) 46
- 2) 60
- 3) 86
- 4) 94



7. A carpenter leans an extension ladder against a house to reach the bottom of a window 30 feet above the ground. As shown in the diagram below, the ladder makes a 70° angle with the ground. To the *nearest foot*, determine and state the length of the ladder.



8. In the diagram of right triangle *ABC* shown below, AB = 14 and AC = 9. What is the measure of $\angle A$, to the *nearest degree*?



9. A 28-foot ladder is leaning against a house. The bottom of the ladder is 6 feet from the base of the house. Find the measure of the angle formed by the ladder and the ground, to the *nearest degree*.

10. Bob places an 18-foot ladder 6 feet from the base of his house and leans it up against the side of his house. Find, to the *nearest degree*, the measure of the angle the bottom of the ladder makes with the ground.

11. As shown in the diagram below, an island (1) is due north of a marina (M). A boat house (H) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.

Determine and state, to the *nearest tenth of a mile*, the distance from the boat house (H) to the island (I). Determine and state, to the *nearest tenth of a mile*, the distance from the island (I) to the marina (M).



Acute Angles in a Right Triangle

 $\sin A = \cos B$: In a right triangle, the sine of one acute angle is equal to the cosine of the other acute angle

A + B = 90: The two acute angles in a right triangle are complementary

1. In scalene triangle *ABC* shown in the diagram below, $m \angle C = 90^\circ$.

Which equation is always true?

1) $\sin A = \sin B$ 2) $\cos A = \cos B$

- $\begin{array}{l} 2) \quad \cos A = \cos B \\ 3) \quad \cos A = \sin C \end{array}$
- $\frac{3}{2} \cos A = \sin C$
- 4) $\sin A = \cos B$



- 2. In $\triangle ABC$, the complement of $\angle B$ is $\angle A$. Which statement is always true?
- 1) $\tan \angle A = \tan \angle B$
- 2) $\sin \angle A = \sin \angle B$
- 3) $\cos \angle A = \tan \angle B$
- 4) $\sin \angle A = \cos \angle B$
- 3. Given: Right triangle ABC with right angle at C. If $\sin A$ increases, does $\cos B$ increase or decrease? Explain why.

4. In right triangle ABC, $m \angle C = 90^{\circ}$. If $\cos B = \frac{5}{13}$, which function also equals $\frac{5}{13}$? 1) $\tan A$ 2) $\tan B$ 3) $\sin A$ 4) $\sin B$

5. In right triangle ABC, $m \angle C = 90^\circ$ and $AC \neq BC$. Which trigonometric ratio is equivalent to $\sin B$?

1)	cos A	3)	tan A
2)	cos <i>B</i>	4)	tan B

6. In right triangle *ABC* with the right angle at *C*, $\sin A = 2x + 0.1$ and $\cos B = 4x - 0.7$. Determine and state the value of *x*. Explain your answer.

7. If $sin(3x + 2)^\circ = cos(4x - 10)^\circ$, what is the value of x to the *nearest tenth*? (1) 7.6 (2) 12.0 (3) 14.0 (4) 26.9

8. If sin(2x + 7)° = cos(4x - 7)°, what is the value of x?
1) 7
2) 15
3) 21

4) 30

9. I	n a right triangle, sin(40 – x)° -	$= \cos(3x)^{\circ}$. What is the value of x	?
1)	10	3) 20	
2)	15	4) 25	

10. In a right triangle, the acute angles have the relationship sin(2x + 4) = cos(46). What is the value of x?

- 1) 20
- 2) 21
- 3) 24
- 4) 25

11. Find the value of *R* that will make the equation $\sin 73^\circ = \cos R$ true when $0^\circ < R < 90^\circ$. Explain your answer.

12. Which expression is always equivalent to $\sin x$ when $0^\circ < x < 90^\circ$?

- 1) $\cos(90^\circ x)$
- 2) $\cos(45^\circ x)$
- 3) $\cos(2x)$
- 4) $\cos x$

Parallelogram Properties



A rectangle and rhombus have all of the properties of the parallelogram.

A square has all of the properties of the parallelogram, rectangle, and rhombus. A trapezoid has one pair of opposite sides parallel and one pair of opposite sides not parallel.

An isosceles trapezoid is a trapezoid that has congruent legs and congruent diagonals. **For properties questions, draw the shape!**

- 1. Which of the following is not true of all rectangles?
- 1) Consecutive sides are perpendicular
- 2) Opposite sides are parallel
- 3) Diagonals are perpendicular to each other
- 4) Diagonals bisect each other
- 2. Which of the following is true about rhombuses?
- 1) Consecutive sides are perpendicular
- 2) Opposite sides are congruent
- 3) Consecutive angles are congruent
- 4) Diagonals are congruent
- 3. Which of the following is *not* true about all parallelograms?
- 1) Diagonals bisect each other
- 2) Diagonals are perpendicular to each other
- 3) Opposite angles are congruent
- 4) Consecutive angles are supplementary
- 4. A quadrilateral whose diagonals bisect each other and are perpendicular is a
- 1) rhombus 3) trapezoid
- 2) rectangle 4) parallelogram

5. If the diagonals of a quadrilateral do *not* bisect each other, then the quadrilateral could be a

- 1) rectangle
- 2) rhombus
- 3) square
- 4) trapezoid

6. Which statement is true about every parallelogram?

- 1) All four sides are congruent.
- 2) The interior angles are all congruent.
- 3) Two pairs of opposite sides are congruent.
- 4) The diagonals are perpendicular to each other.

7. Which quadrilateral has diagonals that always bisect its angles and also bisect each other?

- 1) rhombus
- 2) rectangle
- 3) parallelogram
- 4) isosceles trapezoid

8. The diagonals of a quadrilateral are congruent but do not bisect each other. This quadrilateral is

- 1) an isosceles trapezoid
- 2) a parallelogram
- 3) a rectangle
- 4) a rhombus

9. Given three distinct quadrilaterals, a square, a rectangle, and a rhombus, which quadrilaterals must have perpendicular diagonals?

- 1) the rhombus, only
- 2) the rectangle and the square
- 3) the rhombus and the square
- 4) the rectangle, the rhombus, and the square
- 10. A parallelogram must be a rhombus when its
- 1) Diagonals are congruent.
- 2) Opposite sides are parallel.
- 3) Diagonals are perpendicular.
- 4) Opposite angles are congruent.

- 11. A parallelogram must be a rectangle when its
- 1) diagonals are perpendicular
- 2) diagonals are congruent
- 3) opposite sides are parallel
- 4) opposite sides are congruent
- 12. A rectangle must be a square when its
- 1) consecutive sides are perpendicular
- 2) diagonals are congruent
- 3) diagonals are perpendicular to each other
- 4) opposite sides are parallel
- 13. A rhombus must be a square when its
- 1) consecutive sides are congruent
- 2) diagonals are congruent
- 3) opposite angles are congruent
- 4) diagonals are perpendicular to each other
- 14. A parallelogram must be a rectangle when its
- 1) consecutive sides are congruent
- 2) opposite angles are congruent
- 3) consecutive sides are perpendicular
- 4) opposite sides are parallel
- 15. Which of the following properties does not make a parallelogram a rhombus?
- 1) diagonals bisect the angles
- 2) diagonals are perpendicular to each other
- 3) opposite angles are congruent
- 4) consecutive sides are congruent
- 16. Which of the following properties does not make a rhombus a square?
- 1) Diagonals are congruent
- 2) Diagonals are perpendicular to each other
- 3) Consecutive sides are perpendicular
- 4) Consecutive angles are congruent
- 17. Which property is true of all rhombuses but not of all rectangles?
- 1) opposite sides are parallel
- 2) diagonals are perpendicular to each other
- 3) diagonals bisect each other
- 4) opposite angles are congruent

18. Which set of statements would describe a parallelogram that can always be classified as a rhombus?

- I. Diagonals are perpendicular bisectors of each other.
- II. Diagonals bisect the angles from which they are drawn.

III. Diagonals form four congruent isosceles right triangles.

- 1) I and II3) II and III
- 2) I and III 4) I, II, and III
- 19. If ABCD is a parallelogram, which statement would prove that ABCD is a rhombus?

1)	$\angle ABC \cong \angle CDA$	3)	$\overrightarrow{AC} \perp \overrightarrow{BD}$
2)	$\overline{AC} \cong \overline{BD}$	4)	$\overline{AB} \perp \overline{CD}$

20. In parallelogram *ABCD*, diagonals \overline{AC} and \overline{BD} intersect at *E*. Which statement does *not* prove parallelogram *ABCD* is a rhombus?

- 1) $\overline{AC} \cong \overline{DB}$
- 2) $\overline{AB} \cong \overline{BC}$
- 3) $\overline{AC} \perp \overline{DB}$
- 4) \overline{AC} bisects $\angle DCB$

21. In the diagram below, parallelogram *ABCD* has diagonals \overline{AC} and \overline{BD} that intersect at point *E*.

Which expression is not always true?

- 1) $\angle DAE \cong \angle BCE$
- 2) $\angle DEC \cong \angle BEA$
- 3) $\overline{AC} \cong \overline{DB}$
- 4) $\overline{DE} \cong \overline{EB}$



22. In the diagram below, isosceles trapezoid ABCD has diagonals \overline{AC} and \overline{BD} that intersect at point *E*.

Which expression is not always true?

- 1) $AC \cong DB$
- 2) $\overline{DC} \parallel \overline{AB}$
- 3) $\overline{DE} \cong \overline{AE}$
- 4) $\overline{AD} \cong \overline{CB}$



Triangles/Parallel Lines Cut By a Transversal/Angles of Parallelograms

- The three angles of a triangle add to equal 180°. Look for triangles.
 *The four angles of a quadrilateral add to 360°.
- 2) Linear pairs add to 180°. Look for linear pairs.
- 3) Vertical angles are congruent. Look for an X (intersecting lines).
- 4) Given congruent sides: Isosceles triangle has congruent angles opposite congruent sides.
- 5) Given equilateral triangle: Equilateral triangle has angles 60, 60, 60.
- 6) Given angle bisector: An angle bisector cuts an angle into two congruent halves.
- 7) **Given parallel**: Extend parallel lines and transversal. Follow the transversal and fill in all 8 angles. If angles are the same (both acute or both obtuse), the angles are congruent. If the angles are different (one acute and one obtuse), the angles are supplementary (add to 180).
- 8) **Given parallelogram**: Opposite angles are congruent and consecutive angles are supplementary (add to 180)

1. In the diagram below, \overrightarrow{RCBT} and $\triangle ABC$ are shown with $\mathbf{m} \angle A = 60$ and $\mathbf{m} \angle ABT = 125$.

What is $m \angle ACR$?

- 1) 125
- 2) 115
- 3) 65
- 4) 55



2. In the diagram below of isosceles triangle ABC, $\overline{AB} \cong \overline{CB}$ and angle bisectors \overline{AD} , \overline{BF} , and \overline{CE} are drawn and intersect at X. If $m\angle BAC = 50^{\circ}$, find $m\angle AXC$.



3. In the diagram of $\triangle JEA$ below, $m \angle JEA = 90$ and $m \angle EAJ = 48$. Line segment MS connects points M and S on the triangle, such that $m \angle EMS = 59$.

What is $m \angle JSM$?

- 1) 163
- 2) 121
- 3) 42
- 4) 17



4. In the diagram below, $\triangle LMO$ is isosceles with LO = MO.



5. In the diagram below of $\triangle ACD$, *B* is a point on \overline{AC} such that $\triangle ADB$ is an equilateral triangle, and $\triangle DBC$ is an isosceles triangle with $\overline{DB} \cong \overline{BC}$. Find m $\angle C$.



6. Given $\triangle ABC$ with $m \angle B = 62^{\circ}$ and side \overline{AC} extended to *D*, as shown below.



7. In the diagram below, $\overline{AEFB} \| \overline{CGD}$, and \overline{GE} and \overline{GF} are drawn.

If $m \angle EFG = 32^\circ$ and $m \angle AEG = 137^\circ$, what is $m \angle EGF$?

- 1) 11°
- 2) 43°
- 3) 75°
- 4) 105°



8. Steve drew line segments *ABCD*, *EFG*, *BF*, and *CF* as shown in the diagram below. Scalene ΔBFC is formed.

Which statement will allow Steve to prove ABCD || EFG?

- 1) $\angle CFG \cong \angle FCB$
- 2) $\angle ABF \cong \angle BFC$
- 3) $\angle EFB \cong \angle CFB$
- 4) $\angle CBF \cong \angle GFC$



- 9. As shown in the diagram below, $\overrightarrow{ABC} \parallel \overrightarrow{EFG}$ and $\overrightarrow{BF} \cong \overrightarrow{EF}$. If $m \angle CBF = 42.5^\circ$, then $m \angle EBF$ is
- 1) 42.5°
- 2) 68.75°
- 3) 95°
- 4) 137.5°





Which statement is true?

- 1) m $\angle D = 28^{\circ}$
- 2) m∠A = 43°
- 3) $m \angle ACD = 71^{\circ}$
- 4) m $\angle BCE = 109^{\circ}$



11. In the diagram below, \overline{DE} divides \overline{AB} and \overline{AC} proportionally, $m \angle C = 26^{\circ}$, $m \angle A = 82^{\circ}$, and \overline{DF} bisects $\angle BDE$. The measure of angle DFB is 1) 36° 3) 72° 2) 54° 4) 82° 0 E

12. In the diagram below of parallelogram *ROCK*, $m \angle C$ is 70° and $m \angle ROS$ is 65°. What is $m \angle KSO$?

- 1) 45°
- 2) 110°



F

B

13. In the diagram below, *ABCD* is a parallelogram, \overline{AB} is extended through *B* to *E*, and \overline{CE} is drawn.

If $\overline{CE} \cong \overline{BE}$ and $m \angle D = 112^{\circ}$, what is $m \angle E$? 1) 44° 2) 56° 3) 68°

4) 112°

W

V

R

Е

D

14. In the diagram of parallelogram *FRED* shown below. \overline{ED} is extended to *A*. and \overline{AF} is drawn such that $\overline{AF} \cong \overline{DF}$.



- 1) 124°
- 2) 112°
- 3) 68°
- 4) 56°



Q

A

16. In parallelogram JNIE shown below, diagonal \overline{JI} is drawn, $\overline{UP} \perp \overline{JN}$, \overline{JI} intersects \overline{UP} at R. If $m \angle JNI = 120$ and $m \angle JIN = 20$, find $m \angle PRI$ and $m \angle RJU$.



17. In the diagram below of parallelogram *ROCK*, $m \angle C$ is 70° and $m \angle ROS$ is 65°.



18. In parallelogram *ABCD* shown below, the bisectors of $\angle ABC$ and $\angle DCB$ meet at *E*, a point on \overline{AD} .

If $m \angle A = 68^\circ$, determine and state $m \angle BEC$.



19. The diagram below shows parallelogram *LMNO* with diagonal \overline{LN} , $m \angle M = 118^\circ$, and $m \angle LNO = 22^\circ$.

Explain why m∠NLO is 40 degrees.



Volume = (<u>Area of the base</u>)(height), if it comes to a point, multiply by $\frac{1}{3}$. Area of the base is USUALLY A = lw (rectangle/square) or $A = \pi r^2$ (circle)

Most volume formulas are on the reference sheet. Be careful. B = area of the base General Prism: $V = (area \ base)(height)$ Rectangular prism: V = lwhCylinder: $V = \pi r^2 h$ Pyramid: $V = \frac{1}{3} lwh$ Cone: $V = \frac{1}{3} \pi r^2 h$ Sphere: $V = \frac{4}{3} \pi r^3$

1. A cylinder has a diameter of 10 inches and a height of 2.3 inches. What is the volume of this cylinder, to the *nearest tenth of a cubic inch*?

2. What is the volume of a rectangular prism whose length is 4 cm, width is 6 cm, and height is 5 cm?

3. What is the volume of a cube if each side of the cube measures 8 in?

4. What is the volume of a cylinder whose height is 12 inches and whose diameter is 20 inches in terms of π ?

5. Find the volume of a sphere that has a diameter of 12 in in terms of π .

6. A regular pyramid has a square base with an edge length of 14 and an altitude of 24. Find its volume.



7. Find the volume of a cone with a height of 12 in and a diameter of 8 in rounded to the nearest hundredth.

8. Find the volume of the object below if the diameter is 18.2 meters. Round your answer to the *nearest cubic meter*.



Volume with Algebra

Substitute into appropriate volume formula Solve the equation *To get rid of a fraction, multiply by the denominator *To get rid of cubed, take the cubed root (final step)

1. A brick in the shape of a rectangular prism has a base that measures 3 inches by 5 inches. If the volume of the brick is 90 cubic inches, what is the height of the brick?

2. A right circular cylinder has a volume of 1,000 cubic inches and a height of 8 inches. What is the radius of the cylinder to the *nearest tenth of an inch*?

3. The base of a pyramid is a rectangle with a width of 6 cm and a length of 8 cm. Find, in centimeters, the height of the pyramid if the volume is 288 cm^3 .

 Find the radius of a sphere with a volume of 576π cubic units. Find the answer to the nearest tenth of a unit.



5. The volume of a cylinder is $12,566.4 \text{ cm}^3$. The height of the cylinder is 8 cm. Find the radius of the cylinder to the *nearest tenth of a centimeter*.

6. The Parkside Packing Company needs a rectangular shipping box. The box must have a length of 11 inches and a width of 8 inches. Find, to the *nearest tenth of an inch*, the minimum height of the box such that the volume is *at least* 800 cubic inches.

7. If the volume of a sphere is 36π , what is the radius of the sphere? (1) 3 (2) 6 (3) 12 (4) 24

8. Find the length of the radius of a cylinder to the *nearest tenth* if it has a volume of $60 \text{ } cm^3$ and a height of 10 cm.

3 dimensional rotations ALMOST ALWAYS form a cylinder or cone

Reflect the shape in 2 dimensions and connect the images with curves

1. Which object is formed when right triangle RST shown below is rotated around leg RS?

- 1) a pyramid with a square base
- 2) an isosceles triangle
- 3) a right triangle
- 4) a cone

s

w

2. If the rectangle below is continuously rotated about side *w*, which solid figure is formed?

- 1) pyramid
- 2) rectangular prism
- 3) cone
- 4) cylinder

3. If you rotated the shaded figure below about line *m*, which solid would result from the revolution?

	m <		
(1) cylinder	(2) cone	(3) cube	(4) sphere

4.

If you rotated the triangular region of the figure below about line *m*, what solid would result from the revolution?

(1) cylinder (2) cone (3) cube (4) sphere

5. What shape will be formed if the circle in the graph is rotated continuously about the line ℓ?
(1) a sphere
(2) a donut
(3) a cylinder
(4) a cone



7. Circle O is centered at the origin. In the diagram below, a quarter of circle O is graphed.

Which three-dimensional figure is generated when the quarter circle is continuously rotated about the *y*-axis?

- 1) cone
- 2) sphere
- 3) cylinder
- 4) hemisphere



8. If an equilateral triangle is continuously rotated around one of its medians, which 3dimensional object is generated?

- 1) cone
- 2) pyramid
- 3) prism
- 4) sphere

9. A student has a rectangular postcard that he folds in half lengthwise. Next, he rotates it continuously about the folded edge. Which three-dimensional object below is generated by this rotation?





10. In the diagram below, right triangle *ABC* has legs whose lengths are 4 and 6. What is the volume of the three-dimensional object formed by continuously rotating the right triangle around \overline{AB} ?



11. In the rectangle below, $\overline{UN} = 8in$ and $\overline{KN} = 3in$. Find the volume of the three dimensional object created by rotating rectangle FUNK continuously about side \overline{FK}



12. In the diagram of right triangle *ABC* shown below, AB = 14 and AC = 9. What is the volume of the three dimensional object formed when the triangle is continuously rotated about side \overline{BC}



13. A rectangle whose length and width are 10 and 6, respectively, is shown below. The rectangle is continuously rotated around a straight line to form an object whose volume is 150π .

Wh	nich line could	the rectangle be rotated around?
1)	a long side	3) the vertical line of symmetry
2)	a short side	4) the horizontal line of symmetry

6		
	10	

Finding Center and Radius of a Circle Using Completing the Square

 $(x-a)^2 + (y-b)^2 = r^2$ where (a,b) is the center and r is the radius To put into center-radius form: COMPLETE THE SQUARE TWICE To find center: Negate what is in the parenthesis. If there are no parentheses, the coordinate is 0.

Radius is the square root of the right hand side

Completing the Square

- 1) Write the x's together, y's together, and move constant to the other side $x^2 + bx + y^2 + by = c$
- 2) Add $\left(\frac{b}{2}\right)^2$ to both sides for each variable

3) Factor each trinomial (Both factors must be the same)

4) Rewrite the factors as a binomial squared

1.
$$x^2 + y^2 + 16x + 6y + 9 = 0$$

2. $x^2 + y^2 - 12x - 14y = 15$

3.
$$x^{2} + 8y + 10 + y^{2} - 4x = 6$$

4. $x^{2} + 4x + 12 + y^{2} - 2y - 1 = 22$

5. What are the coordinates of the center of a circle whose equation is

- $x^2 + y^2 16x + 6y + 53 = 0?$
- 1) (-8,-3)
- 2) (-8,3)
- 3) (8,-3)
- 4) (8,3)

6. The equation of a circle is $x^2 + y^2 + 6y = 7$. What are the coordinates of the center and the length of the radius of the circle?

- 1) center (0,3) and radius 4
- 2) center (0, -3) and radius 4
- 3) center (0,3) and radius 16
- 4) center (0, -3) and radius 16

7. What are the coordinates of the center and length of the radius of the circle whose equation is $x^2 + 6x + y^2 - 4y = 23$?

- 1) (3,-2) and 36
- 2) (3,-2) and 6
- 3) (-3, 2) and 36
- 4) (-3, 2) and 6

8. What is an equation of a circle whose center is (1,4) and diameter is 10?

1) $x^{2} - 2x + y^{2} - 8y = 8$ 2) $x^{2} + 2x + y^{2} + 8y = 8$ 3) $x^{2} - 2x + y^{2} - 8y = 83$ 4) $x^{2} + 2x + y^{2} + 8y = 83$

9. What is an equation of circle O shown in the graph below?

1)
$$x^2 + 10x + y^2 + 4y = -13$$

- 2) $x^2 10x + y^2 4y = -13$
- 3) $x^2 + 10x + y^2 + 4y = -25$
- 4) $x^2 10x + y^2 4y = -25$



Line Dilations THE IMAGE IS ALWAYS PARALLEL! SLOPE IS ALWAYS THE SAME! Conceptual:

Determine if the point is on the line by substituting the x and y coordinates into the equation of the line.

If the point is on the line: Same y intercept (Exact same equation). If the point is on the line: Different y intercept.

Writing the equation:

If center is origin: Multiply scale factor and original b to find new b **If center is on the line:** The image is the same equation as the original.

1. A line segment is dilated by a scale factor of 2 centered at a point not on the line segment. Which statement regarding the relationship between the given line segment and its image is true?

- The line segments are perpendicular, and the image is one-half of the length of the given line segment.
- The line segments are perpendicular, and the image is twice the length of the given line segment.
- The line segments are parallel, and the image is twice the length of the given line segment.
 - 4) The line segments are parallel, and the image is one-half of the length of the given line segment.

2. The image of $\triangle ABC$ after a dilation of scale factor k centered at point A is $\triangle ADE$, as shown in the diagram below.

Which statement is always true?

- 1) 2AB = AD
- 2) $\overline{AD} \perp \overline{DE}$
- 3) AC = CE
- 4) $\overline{BC} \parallel \overline{DE}$



3. The line whose equation is 3x - 5y = 4 is dilated by a scale factor of $\frac{5}{3}$ centered at the

origin. Which statement is correct?

- 1) The image of the line has the same slope as the pre-image but a different *y*-intercept.
- 2) The image of the line has the same *y*-intercept as the pre-image but a different slope.
- 3) The image of the line has the same slope and the same *y*-intercept as the preimage.
- 4) The image of the line has a different slope and a different *y*-intercept from the pre-image.

4. A line that passes through the points whose coordinates are (1, 1) and (5, 7) is dilated by a scale factor of 3 and centered at the origin. The image of the line

- 1) is perpendicular to the original line
- 2) is parallel to the original line
- 3) passes through the origin
- 4) is the original line

5. The line y = -5x-1 is dilated by a scale factor of 2 and centered at the origin. Write an equation that represents the image of the line after the dilation.

6. The line y = -2x + 4 is dilated by a scale factor of $\frac{5}{2}$ and centered at the origin. Write an equation that represents the image of the line after the dilation.

7. The line y = 2x - 4 is dilated by a scale factor of $\frac{3}{2}$ and centered at the origin. Which equation represents the image of the line after the dilation?

- 1) y = 2x 42) y = 2x - 6
- 2) y = 2x = 03) y = 3x - 4
- $(j) = 5 \times 4$
- $4) \quad y = 3x 6$

8. What is an equation of the image of the line $y = \frac{3}{2}x - 4$ after a dilation of a scale factor of $\frac{3}{4}$ centered at the origin?

1)
$$y = \frac{9}{8}x - 4$$

2) $y = \frac{9}{8}x - 3$
3) $y = \frac{3}{2}x - 4$
4) $y = \frac{3}{2}x - 3$

9. The equation of line h is 2x + y = 1. Line m is the image of line h after a dilation of scale factor 4 with respect to the origin. What is the equation of the line m?

1) y = -2x + 1

- 2) y = -2x + 43) y = 2x + 4
- 4) y = 2x + 1

10. Line y = 3x - 1 is transformed by a dilation with a scale factor of 2 and centered at (3, 8). The line's image is 1) y = 3x - 82) y = 3x - 43) y = 3x - 24) y = 3x - 111. Line MN is dilated by a scale factor of 2 centered at the point (0, 6). If MN is represented by y = -3x + 6, which equation can represent $\overrightarrow{M'N'}$, the image of $\overrightarrow{MN?}$ 1) y = -3x + 12

2) y = -3x + 6

- 3) y = -6x + 12
- 4) y = -6x + 6

12. Line *n* is represented by the equation 3x + 4y = 20. Determine and state the equation of line p, the image of line n, after a dilation of scale factor $\frac{1}{3}$ centered at the point (4, 2). Explain your answer.

13. Aliyah says that when the line 4x + 3y = 24 is dilated by a scale factor of 2 centered at the point (3,4), the equation of the dilated line is $y = -\frac{4}{3}x + 16$. Is Aliyah correct? Explain why.

Dilating Segments with Perimeter and Area

Multiply the original segment and scale factor to find the image. Multiply the original perimeter and scale factor to find the image perimeter. Multiply the original area and the (*scale factor*)² to find the image area. *You may have to use distance formula to find original segment. *The center of dilation does not effect the size of the image

1. A line segment with a length of 5 is dilated by a scale factor of 4. What is the length of its image?

2. A line segment has a length of 12 and is dilated by $\frac{1}{2}$. What is the length of its image?

3. A three-inch line segment is dilated by a scale factor of 6 and centered at its midpoint. What is the length of its image?

- 1) 9 inches
- 2) 2 inches
- 3) 15 inches
- 4) 18 inches

4. The coordinates of the endpoints of \overline{AB} are A(2, 3) and B(5, -1). Determine the length of $\overline{A'B'}$, the image of \overline{AB} , after a dilation of $\frac{1}{2}$ centered at the origin.



5. Line segment A'B', whose endpoints are (4, -2) and (16, 14), is the image of \overline{AB} after a dilation of $\frac{1}{2}$ centered at the origin. What is the length of \overline{AB} ?



6. Given square *RSTV*, where RS = 9 cm. If square *RSTV* is dilated by a scale factor of 3 about a given center, what is the perimeter, in centimeters, of the image of *RSTV* after the dilation?

- 1) 12
- 2) 27
- 3) 36
- 4) 108

7. Triangle *RJM* has an area of 6 and a perimeter of 12. If the triangle is dilated by a scale factor of 3 centered at the origin, what are the area and perimeter of its image, triangle R'J'M'?

- 1) area of 9 and perimeter of 15
- 2) area of 18 and perimeter of 36
- 3) area of 54 and perimeter of 36
- 4) area of 54 and perimeter of 108

8. Rectangle *A'B'C'D'* is the image of rectangle *ABCD* after a dilation centered at point *A* by a scale factor of $\frac{2}{3}$. Which statement is correct?

1) Rectangle A'B'C'D' has a perimeter that is $\frac{2}{3}$ the perimeter of rectangle ABCD.

²⁾ Rectangle *A'B'C'D'* has a perimeter that is $\frac{3}{2}$ the perimeter of rectangle *ABCD*.

³⁾ Rectangle *A'B'C'D'* has an area that is $\frac{2}{3}$ the area of rectangle *ABCD*.

⁴⁾ Rectangle *A'B'C'D'* has an area that is $\frac{3}{2}$ the area of rectangle *ABCD*.

Equation of a line through a point

- 1) Find m using parallel (same slope) or perpendicular (negative reciprocal slopes).
- 2) Substitute into $y y_1 = m(x x_1)$. Don't forget to negate x_1 and y_1 .
- 3) If it's multiple choice, you may have to distribute and isolate y.

1. What is the equation of a line that passes through the point (-3, -11) and is parallel to the line whose equation is 2x - y = 4?

1)
$$y = 2x + 5$$

2) $y = 2x - 5$
3) $y = \frac{1}{2}x + \frac{25}{2}$
4) $y = -\frac{1}{2}x - \frac{25}{2}$

2. What is an equation of the line that passes through the point (-2, 5) and is perpendicular to the line whose equation is $y = \frac{1}{2}x + 5$?

1)
$$y-5 = \frac{1}{2}(x+2)$$

2) $y-5 = -2(x+2)$
3) $y+5 = \frac{1}{2}(x-2)$
4) $y+5 = -2(x-2)$

3. What is an equation of the line that contains the point (3, -1) and is perpendicular to the line whose equation is y = -3x + 2?

1)
$$y = -3x + 8$$

2) $y = -3x$
3) $y = \frac{1}{3}x$
4) $y = \frac{1}{3}x - 2$

4. An equation of the line that passes through (2, -1) and is parallel to the line 2y + 3x = 8 is

1)
$$y+1 = -\frac{3}{2}(x-2)$$

2) $y+1 = \frac{2}{3}(x-2)$
3) $y-1 = -\frac{3}{2}(x+2)$
4) $y-1 = \frac{2}{3}(x+2)$

5. What is an equation of the line that is perpendicular to the line whose equation is $y = \frac{3}{5}x - 2$ and that passes through the point (3, -6)? ¹⁾ $y = \frac{5}{3}x - 11$ ²⁾ $y = -\frac{5}{3}x + 11$ ³⁾ $y = -\frac{5}{3}x - 1$ ⁴⁾ $y = \frac{5}{3}x + 1$

6. The equation of a line is $y = \frac{2}{3}x + 5$. What is an equation of the line that is perpendicular to the given line and that passes through the point (4,2)?

1) $y = \frac{2}{3}x - \frac{2}{3}$ 2) $y = \frac{3}{2}x - 4$ 3) $y = -\frac{3}{2}x + 7$ 4) $y = -\frac{3}{2}x + 8$

7. What is an equation of the line that passes through the point (6, 8) and is perpendicular to a line with equation $y = \frac{3}{2}x + 5$?

1)
$$y-8 = \frac{3}{2}(x-6)$$

2) $y-8 = -\frac{2}{3}(x-6)$
3) $y+8 = \frac{3}{2}(x+6)$
4) $y+8 = -\frac{2}{3}(x+6)$

8. What is an equation of a line which passes through (6, 9) and is perpendicular to the line whose equation is 4x - 6y = 15?

1) $y-9 = -\frac{3}{2}(x-6)$ 2) $y-9 = \frac{2}{3}(x-6)$ 3) $y+9 = -\frac{3}{2}(x+6)$ 4) $y+9 = \frac{2}{3}(x+6)$

Partitions

- 1) Find $\frac{\Delta x}{p}$ and $\frac{\Delta y}{p}$ where p is the number of partitions.
- 2) Count those values out on the graph between the two endpoints
- 3) Circle and state the point that matches the given ratio.

BE CAREFUL WHICH POINT YOU START FROM! *Expect to have to use your scrap graph paper

1. The coordinates of the endpoints of \overline{AB} are A(-6, -5) and B(4,0). Point *P* is on \overline{AB} . Determine and state the coordinates of point *P*, such that AP:PB is 2:3.

2. What are the coordinates of the point on the directed line segment from G(-4, -7) to O(4,5) that partitions the segment into a ratio of 3 to 1?

3. Directed line segment IQ has endpoints whose coordinates are I(-7,8) and Q(-1,-4). Determine the coordinates of point *J* that divides the segment in the ratio 1 to 5.



4. Directed line segment *SB* has endpoints whose coordinates are S(-6,3) and B(9,-2). Determine the coordinates of point *J* that divides the segment in the ratio 2 to 3.

5. What are the coordinates of the point on the directed line segment from P(-1,6) to S(5,3) that partitions the segment into a ratio of 1 to 2?

6. Directed line segment *JK* has endpoints whose coordinates are J(8,6) and Q(-10,-3). Determine the coordinates of point *O* that divides the segment in the ratio 5 to 4.





1. As shown in the diagram below, secants \overrightarrow{PWR} and \overrightarrow{PTS} are drawn to circle O from external point P.

If $m \angle RPS = 35^{\circ}$ and $\widehat{mRS} = 121^{\circ}$, determine and state \widehat{mWT} .



2. In the diagram of circle *O*, \overline{PQ} is tangent to *O* at *Q* and \overline{PRT} is a secant. If $m \angle P = 56$ and mQT = 192, find mQR



3. AC = 150°, AH = 70°, find m \angle APH



4. In the diagram below of circle *O*, chords \overline{AB} and \overline{CD} intersect at *E*. If $\widehat{mAC} = 72^{\circ}$ and $\underline{m}\angle AEC = 58^{\circ}$, how many degrees are in \underline{mDB} ?



5. In the diagram below of circle O, chords \overline{AE} and \overline{DC} intersect at point B, such that $\widehat{mAC} = 36$ and $\widehat{mDE} = 20$. What is $\underline{m\angle ABC}$?



6. In the diagram below of circle *O*, chords \overline{AB} and \overline{CD} intersect at *E*. If $m \angle AEC = 34$ and $\widehat{mAC} = 50$, what is \widehat{mDB} ?



7. In the diagram below, secants \overline{RST} and \overline{RQP} , drawn from point *R*, intersect circle *O* at *S*, *T*, *Q*, and *P*.

If RS = 6, ST = 4, and RP = 15, what is the length of \overline{RQ} ?



8. In the diagram below of circle O, \overline{PA} is tangent to circle O at A, and \overline{PBC} is a secant with points B and C on the circle.

If PA = 8 and PB = 4, what is the length of \overline{BC} ?



9. In the accompanying diagram, cabins B and G are located on the shore of a circular lake, and cabin L is located near the lake. Point D is a dock on the lake shore and is collinear with cabins B and L. The road between cabins G and L is 8 miles long and is tangent to the lake. The path between cabin L and dock D is 4 miles long.

What is the length, in miles, of \overline{BD} ?



(Not drawn to scale)

10. In the diagram below of circle *O*, chords \overline{AB} and \overline{CD} intersect at *E*. If CE = 10, ED = 6, and AE = 4, what is the length of \overline{EB} ?



11. If $\overline{BR} = 10$, $\overline{BE} = 4$, $\overline{AE} = 8$, find \overline{ES}



12. In the diagram of circle *O* below, chord \overline{AB} intersects chord \overline{CD} at *E*, DE = 2x + 8, EC = 3, AE = 4x - 3, and EB = 4. What is the value of *x*?



Area of a Sector = $\frac{\theta \pi r^2}{360}$ If given area of a sector, use algebra to solve for missing variable

1. In circle O, $m \angle AOC = 70$ and $\overline{AO} = 2$ in. Find the area of sector COA to the nearest square inch.



2. In circle O, if $\angle BOY = 80^{\circ}$ and $\overline{BO} = 8 \ cm$, find the area of sector BOY in terms of π .



3. In circle O, $m \angle AOC = 65$ and $\overline{DO} = 6$ in. Find the area of sector COB in terms of π

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4. Determine and state, in terms of π , the area of a sector that intercepts a 40° arc of a circle with a radius of 4.5.

5. A circle with a diameter of 10 cm and a central angle of 30° is drawn below. What is the area, to the *nearest tenth of a square centimeter*, of the sector formed by the 30° angle?

- 1) 5.2
- 2) 6.5
- 3) 13.1
- 4) 26.2



6. In the diagram below of circle O, GO = 8 and $m \angle GOJ = 60^{\circ}$. What is the area, in terms of π , of the shaded region?

- 1) $\frac{4\pi}{2}$
- 3 2) <u>20 л</u>
- 3 3) <u>32 л</u>
- 4) $\frac{3}{160\pi}$
- 7. In the diagram below of circle O, the area of the shaded sector AOC is $12\pi \text{ in}^2$ and the length of \overrightarrow{OA} is 6 inches. Determine and state $m \angle AOC$.



8. The area of a sector of a circle with a radius measuring 15 cm is 75π cm². What is the measure of the central angle that forms the sector?

- 1) 72°
 3) 144°
- 2) 120° 4) 180°

9. In the diagram below of circle *O*, the area of sector *STO* is $48\pi in^2$ and the length of \overline{OP} is 12 inches. Determine and state $m \angle SOT$



10. In circle O, diameters \overline{BOD} and \overline{COA} intersect at the center of the circle O. If the area of sector OCD = 240π square inches and $m\angle AOD = 80$, find the measure of \overline{OB} to the nearest tenth of an inch.



11. In the diagram below, the circle has a radius of 25 inches. The area of the *unshaded* sector is 500π in².

Determine and state the degree measure of angle Q, the central angle of the shaded sector.



Arc Length: $s = \theta r$, where s = arc length, θ = central angle (in radians), r = radius

1. In circle O, the measure of central angle AOB is 3 radians and the length of \overline{OB} is 2 cm. What is the measure of arc AB?



2. What is the measure of the central angle below?



3. What is the measure of the radius of a sector whose arc length is 12 inches and has a central angle of 4 radians?

4. A wheel has a radius of 18 inches. Which distance, to the *nearest inch*, does the wheel travel when it rotates through an angle of $\frac{2\pi}{5}$ radians?

5. What is the measure of a central angle in degrees whose arc length is 6 meters and whose radius measures 8 meters?

6. In the diagram below, the circle shown has radius 10. Angle B intercepts an arc with a length of 2π .

What is the measure of angle *B*, in radians?

- 1) $10 + 2\pi$
- 2) 20*π*
- 3) $\frac{\pi}{5}$
- 4)
- $\frac{5}{\pi}$



7. In circle O, the measure of central angle AOB is $\frac{\pi}{2}$ radians and the length of arc AB is 10 cm. What is the measure of radius OB to the nearest tenth of a cm?

8. In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle A intercepts an arc of length π , and angle *B* intercepts an arc of length $\frac{13\pi}{8}$.

Dominic thinks that angles A and B have the same radian measure. State whether Dominic is correct or not. Explain why.

