Name: Schlansky

Common Core Geometry Common Regents Questions!

Mr. Schlansky

Identifying Transformations

Check for orientation!!! (The direction of the letters)

The only transformation that changes orientation is a line reflection (an even amount of reflections will preserve orientation).

Translation = slide

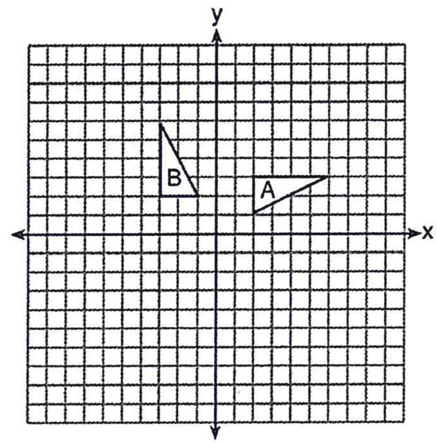
Rotation = turn

Reflection = flip

Dilation = change size (enlarge or shrink)

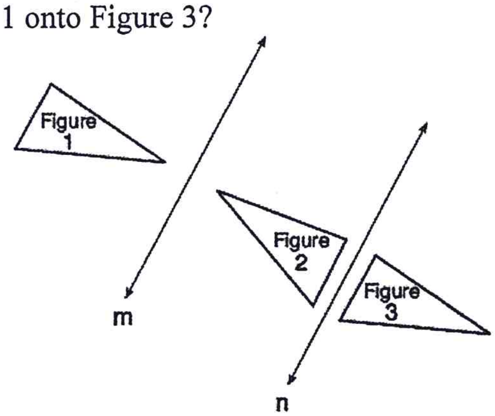
1. In the diagram below, which single transformation was used to map triangle *A* onto triangle *B*?

1) line reflection
2) rotation
3) dilation
4) translation



2. In the diagram below, line *m* is parallel to line *n*. Figure 2 is the image of Figure 1 after a reflection over line *m*. Figure 3 is the image of Figure 2 after a reflection over line *n*. Which single transformation would carry Figure 1 onto Figure 3?

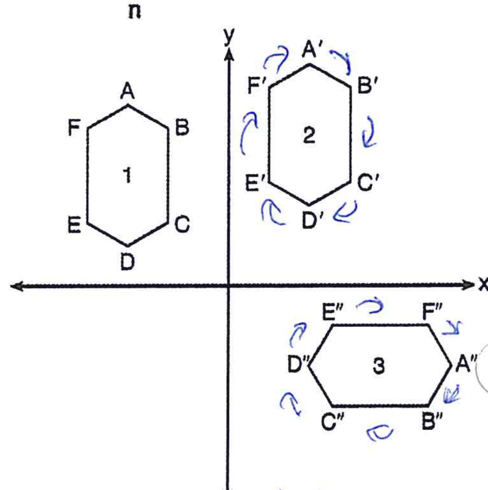
1) a dilation
2) a rotation
3) a reflection
4) a translation



3. In the diagram below, congruent figures 1, 2, and 3 are drawn.

Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?

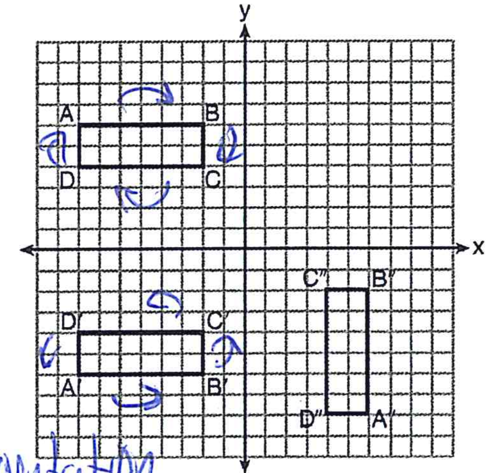
1) a reflection followed by a translation
2) a rotation followed by a translation
3) a translation followed by a reflection
4) a translation followed by a rotation



same orientation

4. A sequence of transformations maps rectangle $ABCD$ onto rectangle $A''B''C''D''$, as shown in the diagram below.
Which sequence of transformations maps $ABCD$ onto $A'B'C'D'$ and then maps $A'B'C'D'$ onto $A''B''C''D''$?

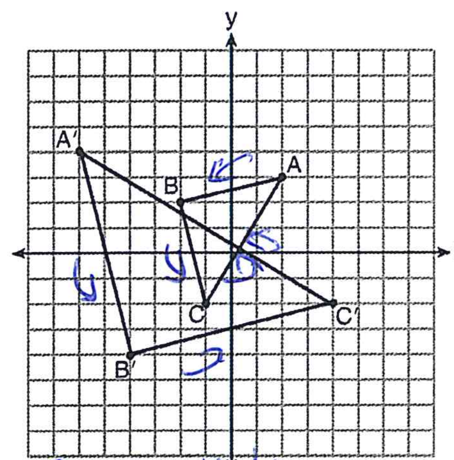
1) a reflection followed by a rotation
2) a reflection followed by a translation
3) a translation followed by a rotation
4) a translation followed by a reflection



opposite orientation

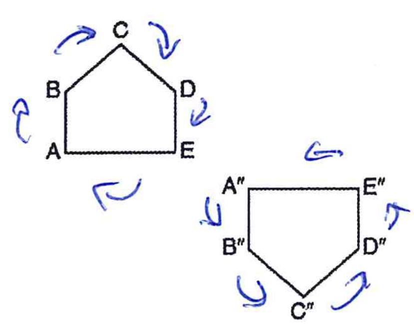
5. Which sequence of transformations will map $\triangle ABC$ onto $\triangle A'B'C'$?

1) reflection and translation
2) rotation and reflection
3) translation and dilation
4) dilation and rotation



same orientation

6. Identify which sequence of transformations could map pentagon $ABCDE$ onto pentagon $A''B''C''D''E''$, as shown below.

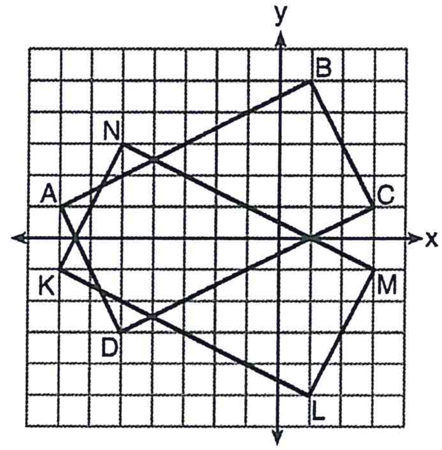


opposite orientation
reflection

1) dilation followed by a rotation
2) translation followed by a rotation
3) line reflection followed by a translation
4) line reflection followed by a line reflection

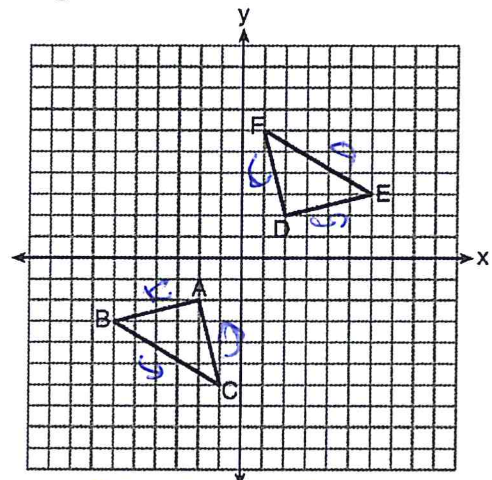
7. On the set of axes below, rectangle *ABCD* can be proven congruent to rectangle *KLMN* using which transformation?

1) rotation
2) translation
3) reflection over the x-axis
4) reflection over the y-axis



8. Triangle *ABC* and triangle *DEF* are graphed on the set of axes below. Which sequence of transformations maps triangle *ABC* onto triangle *DEF*?

1) a reflection over the x-axis followed by a reflection over the y-axis
2) a 180° rotation about the origin followed by a reflection over the line $y = x$
3) a 90° clockwise rotation about the origin followed by a reflection over the y-axis
4) a translation 8 units to the right and 1 unit up followed by a 90° counterclockwise rotation about the origin

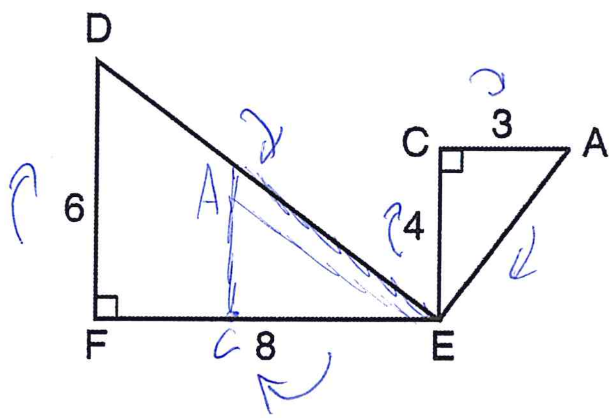


Same orientation

9. Given: $\triangle AEC$, $\triangle DEF$, and $\overline{FE} \perp \overline{CE}$

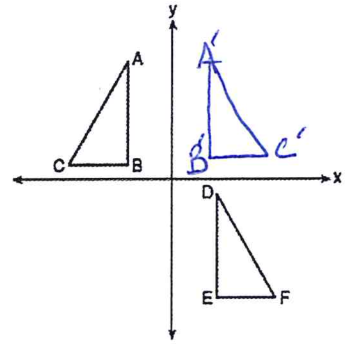
What is a correct sequence of similarity transformations that shows $\triangle AEC \sim \triangle DEF$?

1) a rotation of 180 degrees about point *E* followed by a horizontal translation
2) a counterclockwise rotation of 90 degrees about point *E* followed by a horizontal translation
3) a rotation of 180 degrees about point *E* followed by a dilation with a scale factor of 2 centered at point *E*
4) a counterclockwise rotation of 90 degrees about point *E* followed by a dilation with a scale factor of 2 centered at point *E*



Same orientation

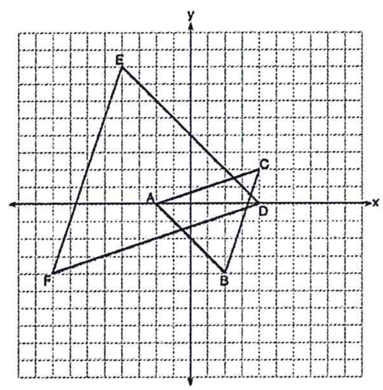
10. In the diagram below, $\triangle ABC \cong \triangle DEF$.



Which sequence of transformations maps $\triangle ABC$ onto $\triangle DEF$?

1) a reflection over the x-axis followed by a translation
2) a reflection over the y-axis followed by a translation
3) a rotation of 180° about the origin followed by a translation
4) a counterclockwise rotation of 90° about the origin followed by a translation

11. On the set of axes below, $\triangle ABC$ has vertices at $A(-2,0)$, $B(2,-4)$, $C(4,2)$, and $\triangle DEF$ has vertices at $D(4,0)$, $E(-4,8)$, $F(-8,-4)$.



Which sequence of transformations will map $\triangle ABC$ onto $\triangle DEF$?

1) a dilation of $\triangle ABC$ by a scale factor of 2 centered at point A
2) a dilation of $\triangle ABC$ by a scale factor of $\frac{1}{2}$ centered at point A
3) a dilation of $\triangle ABC$ by a scale factor of 2 centered at the origin, followed by a rotation of 180° about the origin
4) a dilation of $\triangle ABC$ by a scale factor of $\frac{1}{2}$ centered at the origin, followed by a rotation of 180° about the origin

Rigid Motion Properties
A rigid motion preserves size and angle measure producing a congruent figure
They all produce a congruent figure except dilation.

1. Which transformation would *not* always produce an image that would be congruent to the original figure?
1) translation   2) dilation   3) rotation   4) reflection

2. The vertices of $\triangle JKL$ have coordinates $J(5,1)$, $K(-2,-3)$, and $L(-4,1)$. Under which transformation is the image $\triangle J'K'L'$ *not* congruent to $\triangle JKL$?
1) a translation of two units to the right and two units down
2) a counterclockwise rotation of 180 degrees around the origin
3) a reflection over the x-axis
4) a dilation with a scale factor of 2 and centered at the origin

3. If $\triangle A'B'C'$ is the image of $\triangle ABC$, under which transformation will the triangles *not* be congruent?
1) reflection over the x-axis
2) translation to the left 5 and down 4
3) dilation centered at the origin with scale factor 2
4) rotation of 270° counterclockwise about the origin

4. Under which transformation would $\triangle A'B'C'$, the image of $\triangle ABC$, *not* be congruent to $\triangle ABC$?
1) reflection over the y-axis
2) rotation of 90° clockwise about the origin
3) translation of 3 units right and 2 units down
4) dilation with a scale factor of 2 centered at the origin

5. The image of $\triangle DEF$ is $\triangle D'E'F'$. Under which transformation will he triangles *not* be congruent?
1) a reflection through the origin
2) a reflection over the line $y = x$
3) a dilation with a scale factor of 1 centered at (2, 3)
4) a dilation with a scale factor of $\frac{3}{2}$ centered at the origin

6. The vertices of $\triangle PQR$ have coordinates $P(2,3)$, $Q(3,8)$, and $R(7,3)$. Under which transformation of $\triangle PQR$ are distance and angle measure preserved?
1) $(x,y) \to (2x, 3y)$ 2) $(x,y) \to (x+2, 3y)$ 3) $(x,y) \to (2x, y+3)$ 4) $(x,y) \to (x+2, y+3)$

dilations   dilation   dilation

7. Which transformation would result in the perimeter of a triangle being different from the perimeter of its image?
1) $(x,y) \to (y,x)$
2) $(x,y) \to (x,-y)$
3) $(x,y) \to (4x,4y)$
4) $(x,y) \to (x+2, y-5)$

dilations

Rigid Motion Proofs

To prove triangles are congruent/similar using rigid motions/transformations

1) Identify the transformations (**Check for orientation to determine if reflection**)
On the grid: reflect/rotate/dilate first
Off the grid: translate first
Translate ________ to ________

Reflect Δ________ over

Rotate Δ________ about point __until it maps onto Δ________

Dilation Δ________ centered at point ____ by a scale factor of $\frac{image}{original}$

Congruence	Similarity
2) A _______ and _______ are rigid motions. 3) **A rigid motion preserves size and angle measure producing a congruent figure.**	2) A dilation and ________ preserve angle measure producing a similar figure.

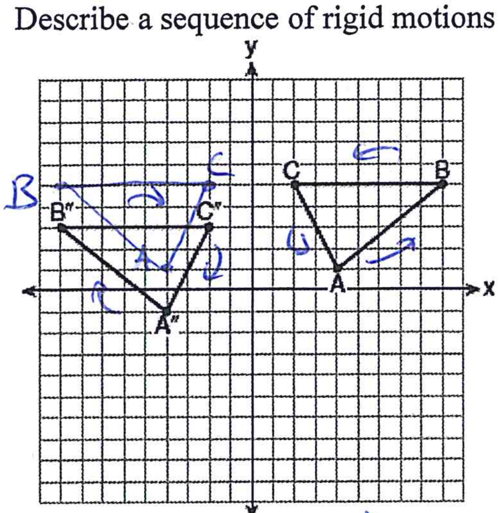
1. Triangle $A'B'C'$ is the image of triangle ABC after a translation of 2 units to the right and 3 units up. Is triangle ABC congruent to triangle $A'B'C'$? Explain why.

2) Yes, a translation is a rigid motion.
3) A rigid motion preserves size and angle measure producing a congruent figure.

2. After a reflection over a line, $\Delta A'B'C'$ is the image of ΔABC. Explain why triangle ABC is congruent to triangle $\Delta A'B'C'$.

2) A reflection is a rigid motion.
3) A rigid motion preserves size and angle measure producing a congruent figure.

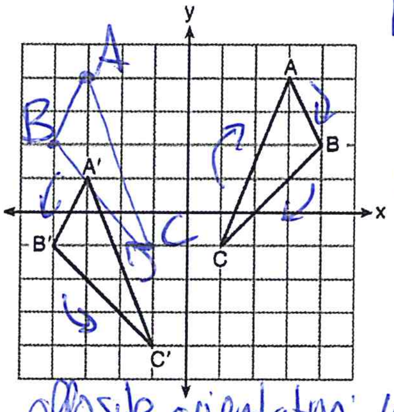
3. The graph below shows ΔABC and its image, $\Delta A''B''C''$.
Describe a sequence of rigid motions which would map ΔABC onto $\Delta A''B''C''$.



1) Reflect Δ ABC over the y axis followed by a translation 2 units down.

orientation opposite
line reflection.

4. As graphed on the set of axes below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a sequence of transformations.
Is $\triangle A'B'C'$ congruent to $\triangle ABC$? Use the properties of rigid motion to explain your answer.

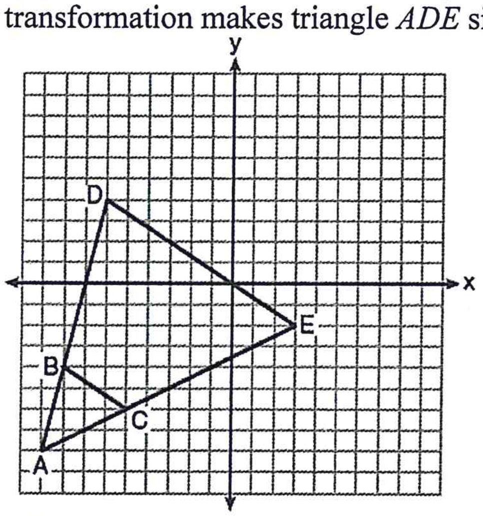


1) Reflect $\triangle ABC$ over the y axis followed by a translation 3 units down.
2) Yes, a reflection and translation are rigid motions.
3) A rigid motion preserves size and angle measure producing a congruent figure

opposite orientation: reflection

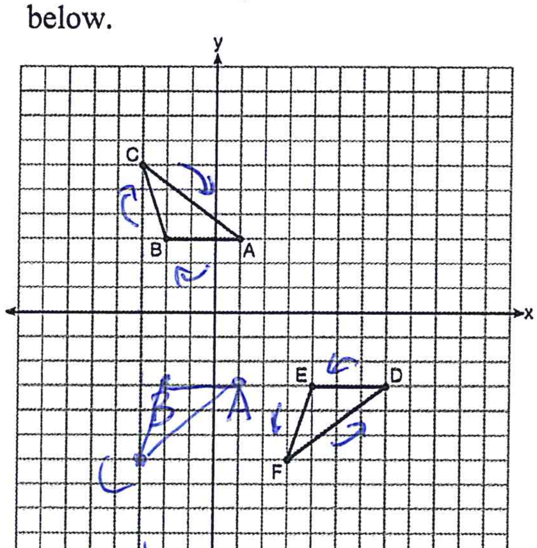
5. Triangle *ABC* and triangle *ADE* are graphed on the set of axes below.
Describe a transformation that maps triangle *ABC* onto triangle *ADE*. Explain why this transformation makes triangle *ADE* similar to triangle *ABC*.

scale factor $= \frac{\text{image}}{\text{original}}$



1) Dilate $\triangle ABC$ by a scale factor of $\frac{DE}{BC}$ centered at A.
2) A dilation preserves angle measure producing a similar figure.

6. Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.

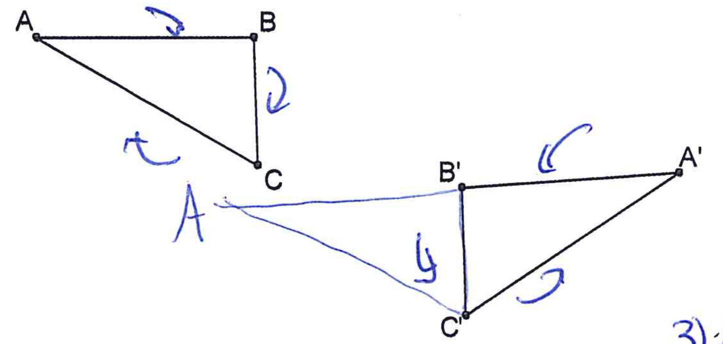


1) Reflect $\triangle ABC$ over the x-axis followed by a translation 6 units to the right.
2) A reflection and translation are rigid motions.
3) A rigid motion preserves size and angle measure producing a congruent figure

opposite orientation reflection

opposite orientation
reflection

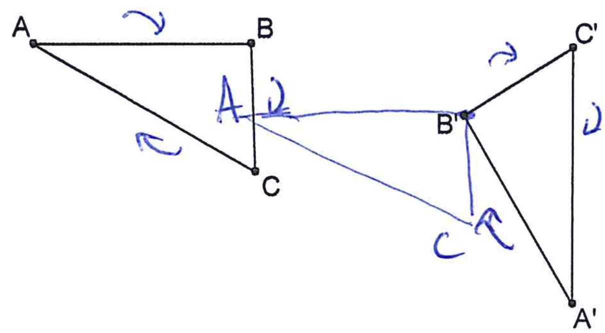
7. Prove that $\Delta ABC \cong \Delta A'B'C'$ using rigid motions.



1) Translate $\overline{BC}$ to $\overline{B'C'}$ followed by reflecting ΔABC over $\overline{BC}$.
2) A translation and reflection are rigid motions
3) A rigid motion preserves size and angle measure producing a congruent figure.

8. Prove that $\Delta ABC \cong \Delta A'B'C'$ using rigid motions.

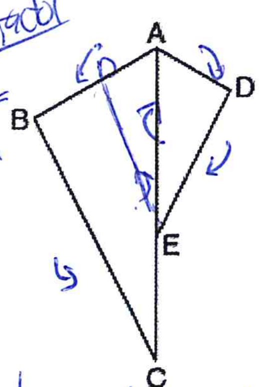
Same orientation



1) Translate B to B' followed by rotating ΔABC about B until it maps onto $\Delta A'B'C'$.
2) A translation and rotation are rigid motions.
3) A rigid motion preserves size and angle measure producing a congruent figure.

9. Describe a sequence of transformation that would map ΔADE onto ΔABC. What is the relationship between ΔADE and ΔABC? Explain your answer.

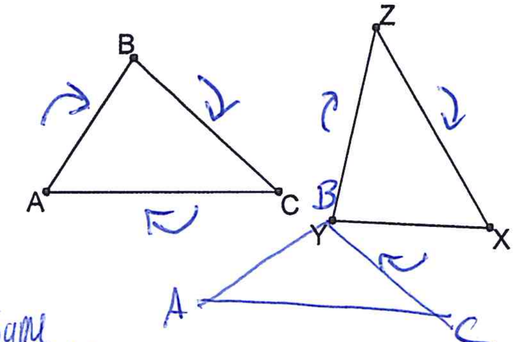
scale factor $\frac{\text{image}}{\text{original}}$



1) Reflect ΔADE over $\overline{AE}$ followed by translating ΔADE by a scale factor of $\frac{\overline{AB}}{\overline{AD}}$ centered at A.
2) They are similar. A dilation preserves angle measure producing a similar figure.

opposite orientation: reflection

10. Prove that $\Delta ABC \cong \Delta XYZ$ using rigid motions.



Same orientation

1) translate B to Y followed by rotating ΔABC centered at B until it maps onto ΔXYZ.
2) A translation and rotation are rigid motions.
3) A rigid motion preserves size and angle measure producing a congruent figure.

Regular Polygon Rotations

To determine the minimum number of degrees a regular polygon must be rotated to be mapped onto itself:

1) The minimum rotation is $\frac{360}{n}$.

2) Any multiple of that will also map the regular polygon onto itself!

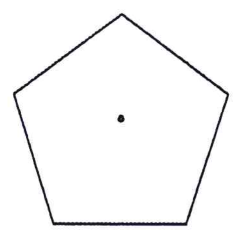
1. What is the minimum number of degrees a regular decagon must be rotated to be mapped onto itself?

$\frac{360}{n}$ $\frac{360}{10} = 36°$

2. What is the minimum number of degrees a regular hexagon must be rotated to be carried onto itself?

$\frac{360}{n}$ $\frac{360}{6} = 60°$

3. A regular pentagon is shown in the diagram below.



If the pentagon is rotated clockwise around its center, the minimum number of degrees it must be rotated to carry the pentagon onto itself is

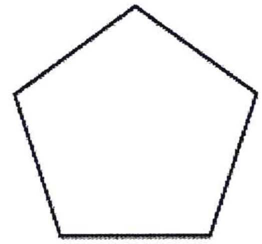
1) 54°
2) ~~72°~~
3) 108°
4) 360°

$\frac{360}{n}$ $\frac{360}{5} = 72°$

4. Which regular polygon has a minimum rotation of 45° to carry the polygon onto itself?

~~(1)~~ octagon $\frac{360}{8} = 45$ (3) hexagon $\frac{360}{6} = 60$
(2) decagon $\frac{360}{10} = 36$ (4) pentagon $\frac{360}{5} = 72$

5. The regular polygon below is rotated about its center. Which angle of rotation will carry the figure onto itself?



1) 60°
2) 108°
3) 216° $72(3)$
4) 540°

$\frac{360}{n}$ $\frac{360}{5} = 72°$

6. Which of the following rotations would not map an equilateral triangle onto itself?

(1) 120° ✓
(3) 180°
(2) 240° ✓ $120(2)$
$120(4)$ (4) 480° ✓

$\frac{360}{n}$ $\frac{360}{3} = 120°$

7. In which regular polygon would a rotation of rotation of 144° carry the shape onto itself?

(1) octagon $\frac{360}{8} = 45$
(3) hexagon $\frac{360}{6} = 60$
(2) square $\frac{360}{4} = 90$
(4) pentagon $\frac{360}{5} = 72$

$72(2) = 144$

8. Which of the following rotations would not map a regular pentagon onto itself?

(1) 144 $72(2)$
(3) 216 $72(3)$
(2) 120
(4) 720 $72(10)$

$\frac{360}{n}$ $\frac{360}{5} = 72$

9. Which of the following regular polygons has rotational symmetry of 480°?

(1) pentagon $\frac{360}{5} = 72$
(3) octagon $\frac{360}{8} = 45$
(2) hexagon $\frac{360}{6} = 60$
(4) decagon $\frac{360}{10} = 36$

$60(8) = 480$

10. Which rotation about its center will carry a regular decagon onto itself?

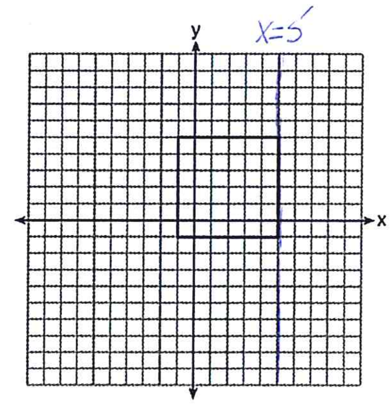
1) 54°
2) 162°
3) 198°
4) 252°

$\frac{360}{n}$ $\frac{360}{10} = 36$

$36(7) = 252$

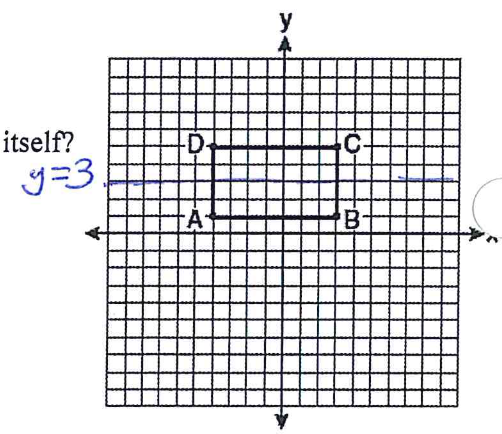
To map a shape onto itself:
Translation/Dilation: Never.
Reflection: **The line of reflection must be a line of symmetry** (cuts shape in half).
Rotation: **Center of rotation must be the center of the shape.** Use common sense for degree measure.

1. In the diagram below, a square is graphed in the coordinate plane.



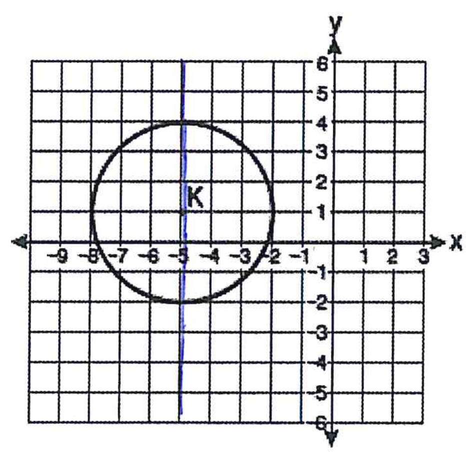
A reflection over which line does *not* carry the square onto itself?
1) $x = 5$
2) $y = 2$
3) $y = x$
4) $x + y = 4$

2. On the set of axes below, Geoff drew rectangle *ABCD*.



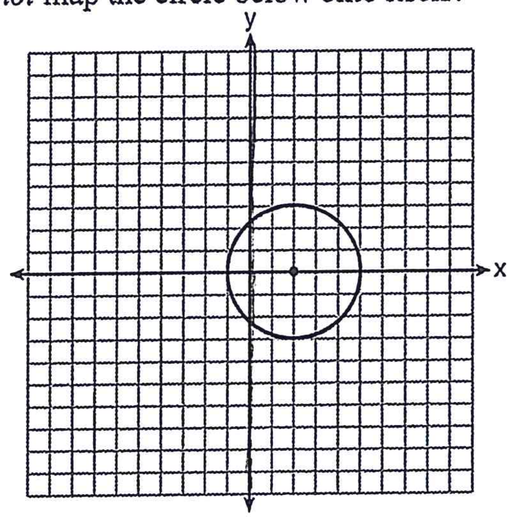
What of the following transformations would map the rectangle onto itself?
1) r_{y-axis}
2) $r_{y=3}$
3) R_{90}
4) $T_{1,0}$

3. Circle *K* is shown in the graph below.



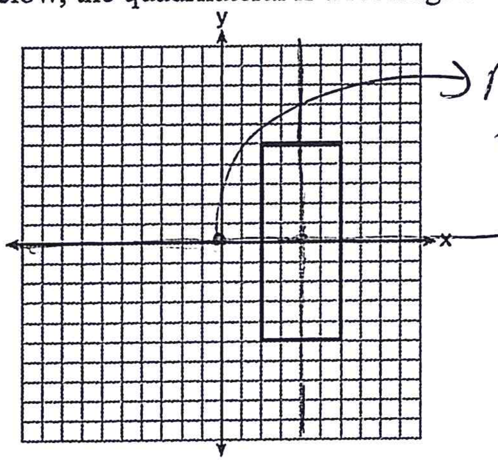
Which of the following transformations map circle K onto itself?
1) $r_{y=x}$
2) $r_{x=-5}$
3) r_{x-axis}
4) $r_{y=2}$

4. Which transformation does *not* map the circle below onto itself?



1) $r_{x\text{-}axis}$
2) $r_{y\text{-}axis}$
3) $r_{x=2}$
4) $T_{0,0}$

5. As shown in the graph below, the quadrilateral is a rectangle.



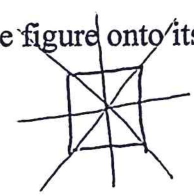
not center of the shape

Which transformation would *not* map the rectangle onto itself?

1) a reflection over the x-axis
2) a reflection over the line $x = 4$
3) a rotation of 180° about the origin
4) a rotation of 180° about the point $(4, 0)$

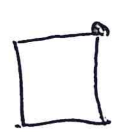
6. Which figure always has exactly four lines of reflection that map the figure onto itself?

1) square
2) rectangle
3) regular octagon
4) equilateral triangle



7. Which transformation would *not* carry a square onto itself?

1) a reflection over one of its diagonals
2) a 90° rotation clockwise about its center
3) a 180° rotation about one of its vertices
4) a reflection over the perpendicular bisector of one side



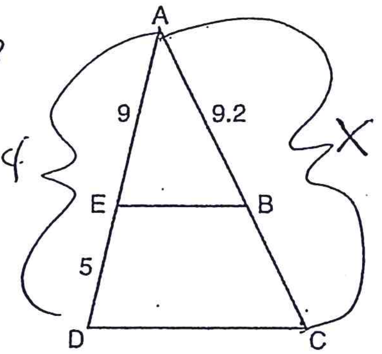
not the center of the shape

Candy Corn Problems

If the bases are not involved: $\dfrac{top}{top} = \dfrac{bottom}{bottom} = \dfrac{side}{side}$

If bases are involved: separate your triangles!

1. In the diagram of $\triangle ADC$ below, $\overline{EB} \parallel \overline{DC}$, $AE = 9$, $ED = 5$, and $AB = 9.2$.



What is the length of $\overline{AC}$, to the *nearest tenth*?

1) 5.1
2) 5.2
3) 14.3
4) 14.4

bases not involved

$\dfrac{top}{top} = \dfrac{side}{side}$

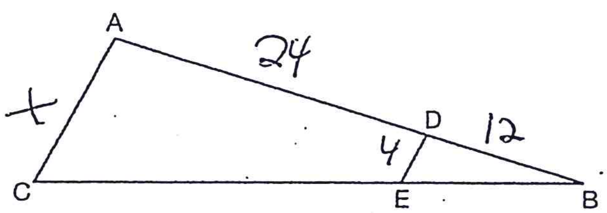
$\dfrac{9}{9.2} = \dfrac{14}{x}$

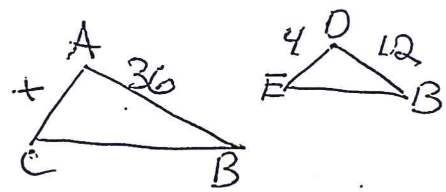
$\dfrac{9x}{9} = \dfrac{128.8}{9}$

$x = 14.3$

2. In the diagram of $\triangle ABC$, points D and E are on $\overline{AB}$ and $\overline{CB}$, respectively, such that $\overline{AC} \parallel \overline{DE}$.

bases involved
separate





If $AD = 24$, $DB = 12$, and $DE = 4$, what is the length of $\overline{AC}$?

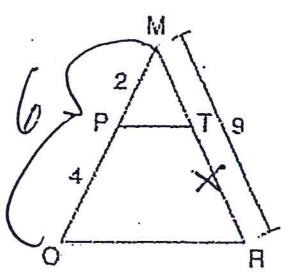
1) 8
2) 12
3) 16
4) 72

$\dfrac{x}{4} = \dfrac{36}{12}$

$\dfrac{12x}{12} = \dfrac{144}{12}$

$x = 12$

3. Given $\triangle MRO$ shown below, with trapezoid $PTRO$, $MR = 9$, $MP = 2$, and $PO = 4$.



bases not involved

$\dfrac{bottom}{bottom} = \dfrac{side}{side}$

$\dfrac{4}{x} = \dfrac{6}{9}$

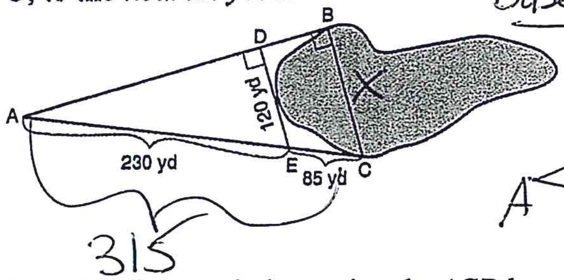
What is the length of $\overline{TR}$?

$\dfrac{6x}{6} = \dfrac{36}{6}$

$x = 6$

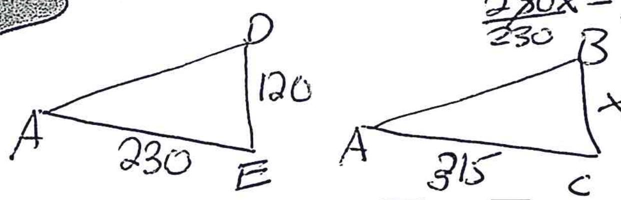
1) 4.5
2) 5
3) 3
4) 6

4. To find the distance across a pond from point B to point C, a surveyor drew the diagram below. The measurements he made are indicated on his diagram. Use the surveyor's information to determine and state the distance from point B to point C, to the *nearest yard.*



bases involved

separate

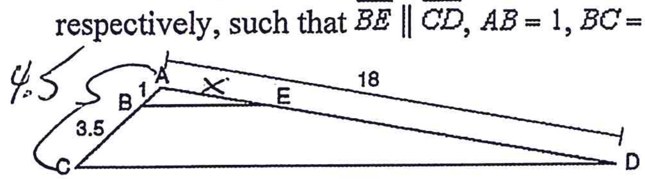


$\frac{230}{315} = \frac{120}{x}$

$\frac{230x}{230} = \frac{37800}{230}$

$x = 164$

5. In the diagram below, triangle ACD has points B and E on sides $\overline{AC}$ and $\overline{AD}$, respectively, such that $\overline{BE} \parallel \overline{CD}$, $AB = 1$, $BC = 3.5$, and $AD = 18$.



What is the length of $\overline{AE}$, to the *nearest tenth*?

bases not involved

$\frac{\text{top}}{\text{top}} = \frac{\text{side}}{\text{side}}$

$\frac{1}{x} = \frac{4.5}{18}$

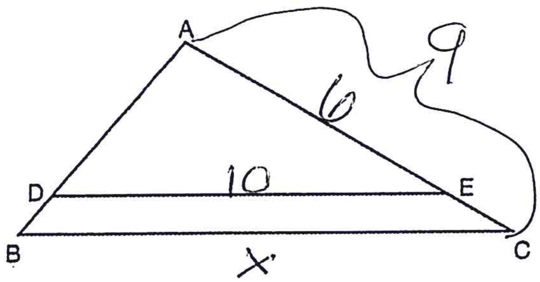
$\frac{4.5x}{4.5} = \frac{18}{4.5}$

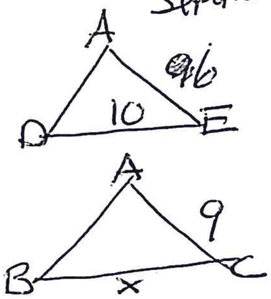
$x = 4.0$

6. In the diagram of $\triangle ABC$ shown below, $\overline{DE} \parallel \overline{BC}$. bases involved

If $\overline{AE} = 6$, $\overline{DE} = 10$, and $\overline{AC} = 9$, find $\overline{BC}$

separate





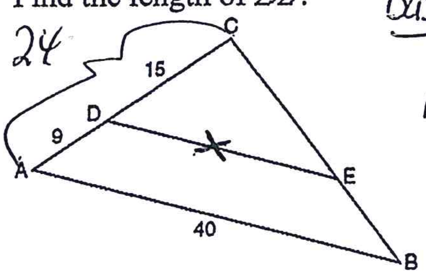
$\frac{10}{x} = \frac{6}{9}$

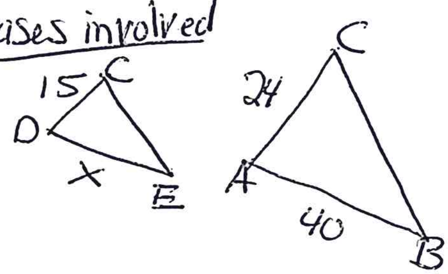
$\frac{6x}{6} = \frac{90}{6}$

$x = 15$

7. In the diagram of $\triangle ABC$ below, $\overline{DE}$ is parallel to $\overline{AB}$, $CD = 15$, $AD = 9$, and $AB = 40$. Find the length of $\overline{DE}$.

bases involved





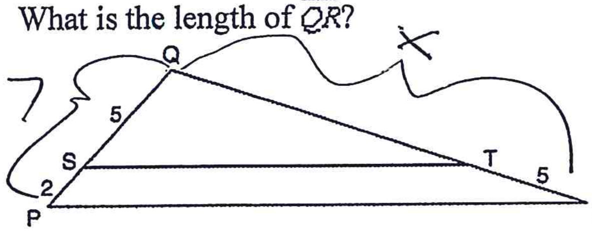
$\frac{15}{24} = \frac{x}{40}$

$\frac{24x}{24} = \frac{600}{24}$

$x = 25$

8. In the diagram below of $\triangle PQR$, $\overline{ST}$ is drawn parallel to $\overline{PR}$, $PS = 2$, $SQ = 5$, and $TR = 5$

What is the length of $\overline{QR}$?



bases not involved

$\frac{\text{side}}{\text{side}} = \frac{\text{bottom}}{\text{bottom}}$

$\frac{7}{x} = \frac{2}{5}$

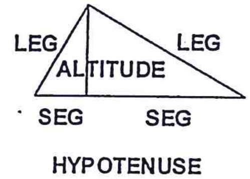
$\frac{2x}{2} = \frac{35}{2}$

$x = 17.5$

When an altitude is drawn to a right triangle
HLLS and SAAS

$$\frac{H}{L}=\frac{L}{S} \qquad \frac{S}{A}=\frac{A}{S}$$

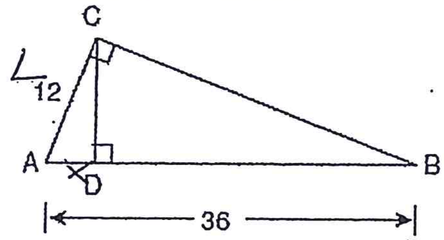
If L is involved, use HLLS
If A is involved, use SAAS



Know how to reduce radicals:

1) Separate into perfect square and non perfect square
2) Take the square root of the perfect square

1. In the diagram below of right triangle ACB, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

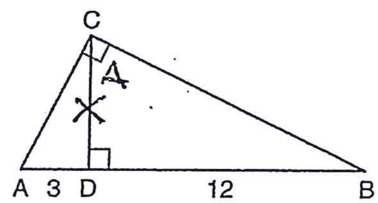


If $AB = 36$ and $AC = 12$, what is the length of $\overline{AD}$?

1) 32
2) 6
3) 3
4) 4

$\frac{H}{L}=\frac{L}{S}$ $\quad \cancel{\frac{36}{12}=\frac{12}{x}}$ $\quad \frac{36x}{36}=\frac{144}{36}$ $\quad x=4$

2. In the diagram below of right triangle ABC, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

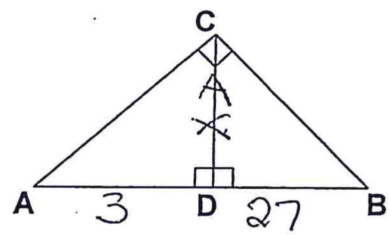


If $AD = 3$ and $DB = 12$, what is the length of altitude $\overline{CD}$?

1) 6
2) $6\sqrt{5}$
3) 3
4) $3\sqrt{5}$

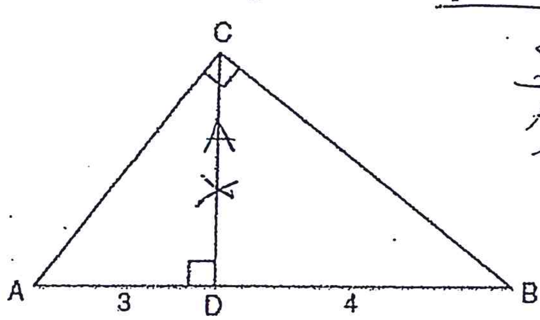
$\frac{S}{A}=\frac{A}{S}$ $\quad \frac{3}{x}=\frac{x}{12}$ $\quad \sqrt{x^2}=\sqrt{36}$ $\quad x=6$

3. If $\overline{AD} = 3$ and $\overline{DB} = 27$, find $\overline{CD}$



$\frac{S}{A}=\frac{A}{S}$ $\quad \frac{3}{x}=\frac{x}{27}$ $\quad \sqrt{x^2}=\sqrt{81}$ $\quad x=9$

4. In the diagram below of right triangle ACB, altitude $\overline{CD}$ intersects $\overline{AB}$ at D. If $AD = 3$ and $DB = 4$, find the length of $\overline{CD}$ in simplest radical form.



$\frac{S}{A}=\frac{A}{S}$

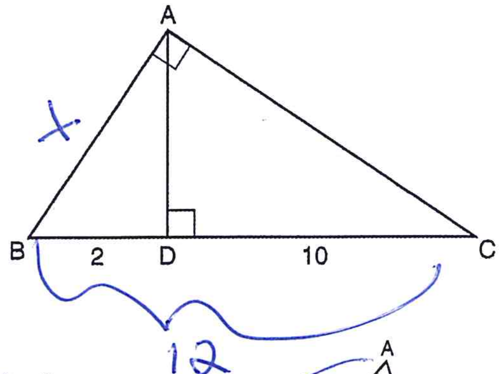
$\frac{3}{x}=\frac{x}{4}$

$\sqrt{x^2}=\sqrt{12}$

$\sqrt{4}\ \sqrt{3}$

$x=2\sqrt{3}$

5. Triangle ABC shown below is a right triangle with altitude $\overline{AD}$ drawn to the hypotenuse $\overline{BC}$.



If $BD = 2$ and $DC = 10$, what is the length of $\overline{AB}$?

1) $2\sqrt{2}$
2) $2\sqrt{5}$
3) $2\sqrt{6}$
4) $2\sqrt{30}$

$\frac{H}{L} = \frac{L}{S}$

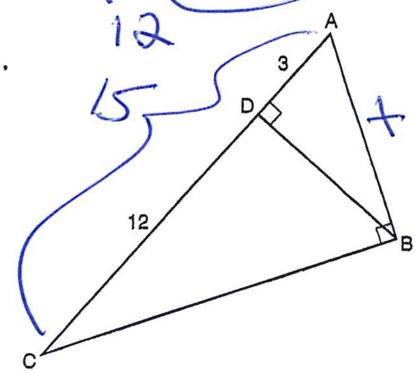
$\cancel{\frac{12}{x} = \frac{x}{2}}$

$\sqrt{x^2} = \sqrt{24}$

$\sqrt{4}\sqrt{6}$

$x = 2\sqrt{6}$

6. In right triangle ABC shown in the diagram below, altitude $\overline{BD}$ is drawn to hypotenuse $\overline{AC}$, $CD = 12$, and $AD = 3$.



What is the length of $\overline{AB}$?

1) $5\sqrt{3}$
2) 6
3) $3\sqrt{5}$
4) 9

$\frac{H}{L} = \frac{L}{S}$

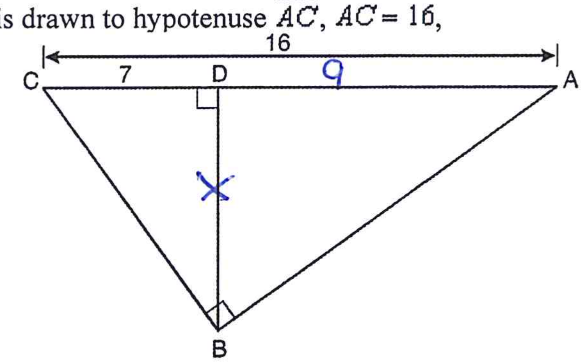
$\frac{15}{x} = \frac{x}{3}$

$\sqrt{x^2} = \sqrt{45}$

$\sqrt{9}\sqrt{5}$

$x = 3\sqrt{5}$

7. In the diagram below of right triangle ABC, altitude $\overline{BD}$ is drawn to hypotenuse $\overline{AC}$, $AC = 16$, and $CD = 7$.



$\frac{S}{A} = \frac{A}{S}$

What is the length of $\overline{BD}$?

1) $3\sqrt{7}$
2) $4\sqrt{7}$
3) $7\sqrt{3}$
4) 12

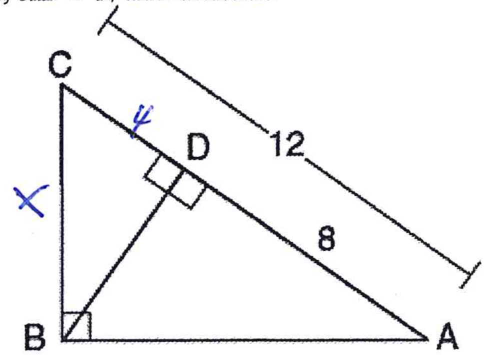
$\frac{7}{x} = \frac{x}{9}$

$\sqrt{x^2} = \sqrt{63}$

$\sqrt{9}\sqrt{7}$

$x = 3\sqrt{7}$

8. In the diagram below of $\triangle ABC$, $\angle ABC$ is a right angle, $AC = 12$, $AD = 8$, and altitude $\overline{BD}$ is drawn.



$\frac{H}{L} = \frac{L}{S}$

What is the length of $\overline{BC}$?

1) $4\sqrt{2}$
2) $4\sqrt{3}$
3) $4\sqrt{5}$
4) $4\sqrt{6}$

$\frac{12}{x} = \frac{x}{4}$

$\sqrt{x^2} = \sqrt{48}$

$\sqrt{16}\sqrt{3}$

$4\sqrt{3}$

9. In $\triangle RST$ shown below, altitude $\overline{SU}$ is drawn to $\overline{RT}$ at U.
If $SU = h$, $UT = 12$, and $RT = 42$, which value of h will make $\triangle RST$ a right triangle with $\angle RST$ as a right angle?

1) $6\sqrt{3}$
2) $6\sqrt{10}$
3) $6\sqrt{14}$
4) $6\sqrt{35}$

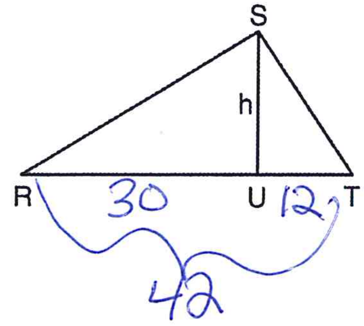
$\frac{S}{A} = \frac{A}{S}$

$\frac{30}{h} = \frac{h}{12}$

$\sqrt{h^2} = \sqrt{360}$

$\sqrt{36}\sqrt{10}$

$6\sqrt{10}$



10. In the diagram of right triangle ABC, $\overline{CD}$ intersects hypotenuse $\overline{AB}$ at D.
If $AD = 4$ and $DB = 6$, which length of $\overline{AC}$ makes $\overline{CD} \perp \overline{AB}$?

1) $2\sqrt{6}$
2) $2\sqrt{10}$
3) $2\sqrt{15}$
4) $4\sqrt{2}$

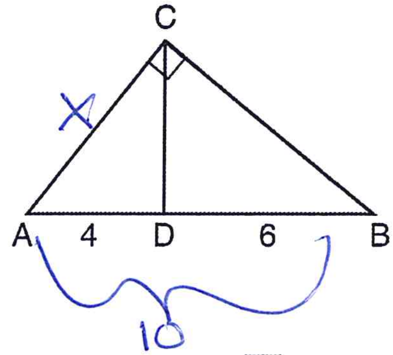
$\frac{H}{L} = \frac{L}{S}$

$\frac{10}{x} = \frac{x}{4}$

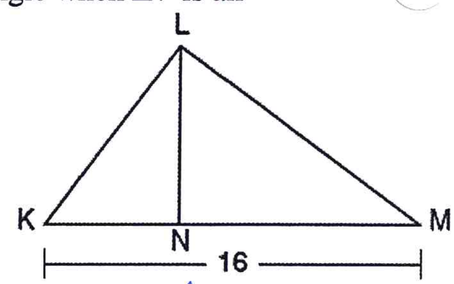
$\sqrt{x^2} = \sqrt{40}$

$\sqrt{4}\sqrt{10}$

$2\sqrt{10}$



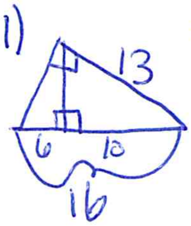
11. Kirstie is testing values that would make triangle KLM a right triangle when $\overline{LN}$ is an altitude, and $KM = 16$, as shown below.



Which lengths would make triangle KLM a right triangle?

1) $LM = 13$ and $KN = 6$
2) $LM = 12$ and $NM = 9$
3) $KL = 11$ and $KN = 7$
4) $LN = 8$ and $NM = 10$

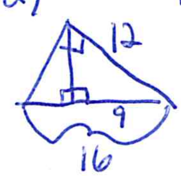
1)



$\frac{16}{13} = \frac{13}{10}$

$160 \neq 169$

2)



$\frac{16}{12} = \frac{12}{9}$

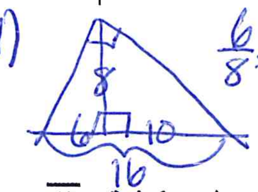
$144 = 144$ ✓

3)

$\frac{16}{11} = \frac{11}{7}$

$121 \neq 112$

4)



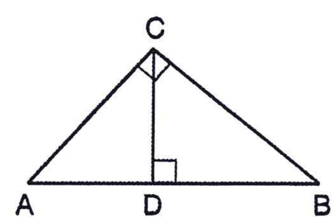
$\frac{6}{8} = \frac{8}{10}$

$64 \neq 60$

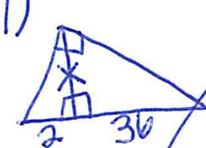
12. In the diagram below, $\overline{CD}$ is the altitude drawn to the hypotenuse $\overline{AB}$ of right triangle ABC.

Which lengths would *not* produce an altitude that measures $6\sqrt{2}$?

1) $AD = 2$ and $DB = 36$
2) $AD = 3$ and $AB = 24$
3) $AD = 6$ and $DB = 12$
4) $AD = 8$ and $AB = 17$



1)



$\frac{2}{x} = \frac{x}{36}$

$\sqrt{x^2} = \sqrt{72}$

$\sqrt{36}\sqrt{2}$

$x = 6\sqrt{2}$

2)

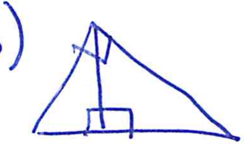
$\frac{3}{x} = \frac{x}{21}$

$\sqrt{x^2} = \sqrt{63}$

$\sqrt{9}\sqrt{7}$

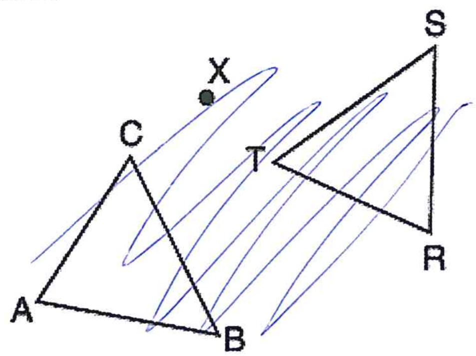
$x = 3\sqrt{7}$

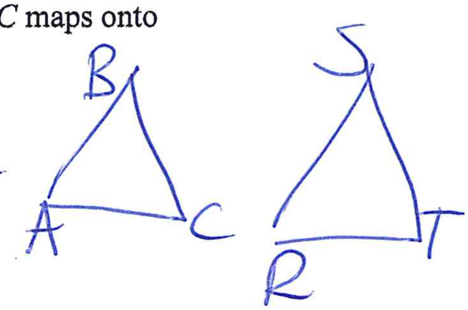
3)



Corresponding Parts of Congruent Triangles are Congruent
Redraw the shapes so it is more clear to see what parts correspond to each other

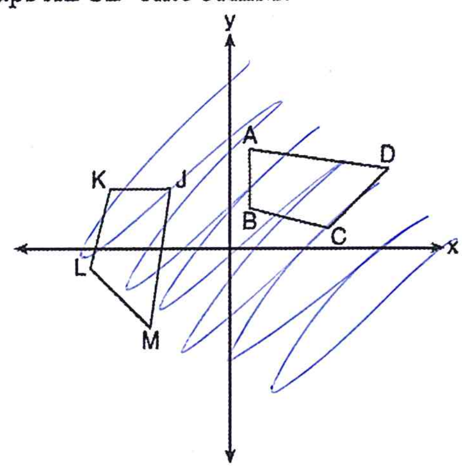
1. After a counterclockwise rotation about point *X*, scalene triangle *ABC* maps onto $\triangle RST$, as shown in the diagram below.



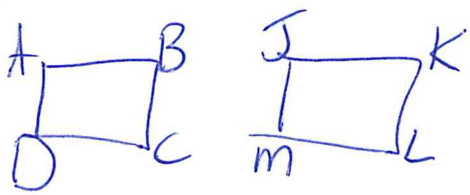


Which statement must be true?
~~1)~~ $\angle A \cong \angle R$ ✓
2) $\angle A \cong \angle S$ X
3) $\overline{CB} \cong \overline{TR}$ X
4) $\overline{CA} \cong \overline{TS}$ X

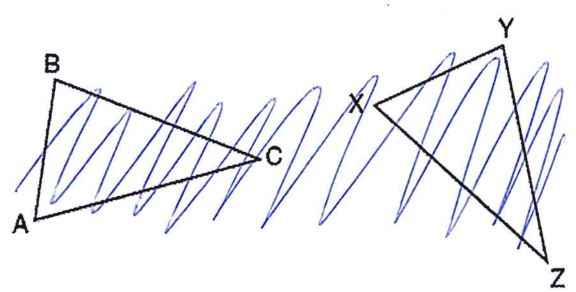
2. In the diagram below, a sequence of rigid motions maps *ABCD* onto *JKLM*.

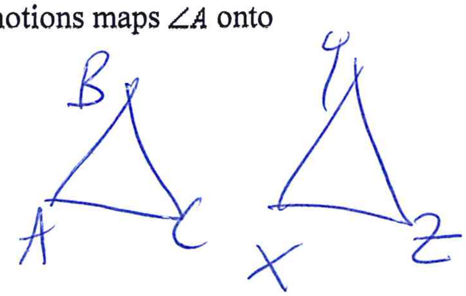


Which of the following statements must be true?
1) $\angle L \cong \angle B$ X
~~2)~~ $\angle A \cong \angle J$ ✓
3) $\overline{JK} \cong \overline{AC}$ X
4) $\overline{JM} \cong \overline{AB}$ X



3. In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$,
$\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$.

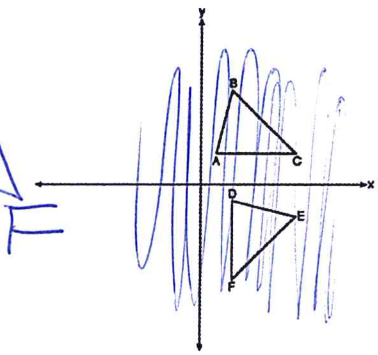




Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why. Yes, corresponding sides of congruent triangles are congruent.

Determine and state whether $\angle A \cong \angle Y$. Explain why. No, those angles don't correspond to each other.

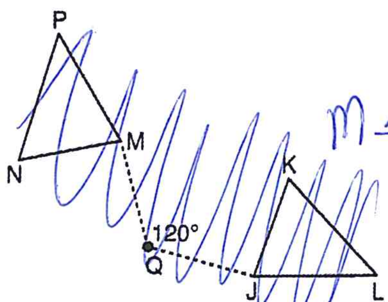
4. The image of $\triangle ABC$ after a rotation of 90° clockwise about the origin is $\triangle DEF$, as shown below.



Which statement is true?

1) $\overline{BC} \cong \overline{DE}$ X
2) $\overline{AB} \cong \overline{DF}$ X
3) $\angle C \cong \angle E$ X
4) $\angle A \cong \angle D$ ✓

5. Triangle MNP is the image of triangle JKL after a 120° counterclockwise rotation about point Q. If the measure of angle L is 47° and the measure of angle N is 57°, determine the measure of angle M. Explain how you arrived at your answer.



$47+57+x=180$

$104+x=180$

$-104 \quad -104$

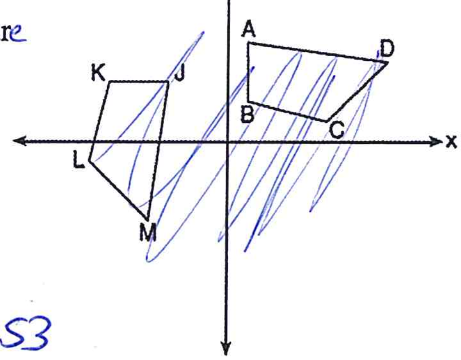
$x=76$

Corresponding angles of congruent triangles are congruent.
A rotation is a rigid motion.
A rigid motion preserves size and angle measure producing a congruent figure

6. In the diagram below, a sequence of rigid motions maps $ABCD$ onto $JKLM$.

If $m\angle A = 82°$, $m\angle B = 104°$, and $m\angle L = 121°$, the measure of $\angle M$ is

1) 53° ✓
2) 82°
3) 104°
4) 121°



angles of a quad add to 360.

$104+82+121+x=360$

$307+x=360$

$-307 \quad -307$

$x=53$

7. In the diagram below, $\triangle ABC$ with sides 13, 15, and 16, is mapped onto $\triangle DEF$ after a clockwise rotation of 90° about point P.

If $DE = 2x - 1$, what is the value of x?

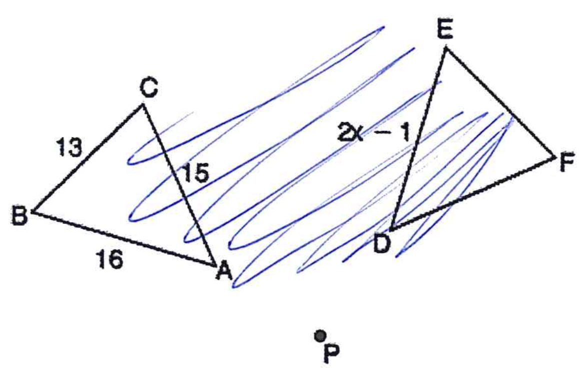
$2x-1=16$

$+1 \quad +1$

$2x=17$

$\frac{2x}{2}=\frac{17}{2}$

$x=8.5$



To determine if a proportion is correct
Look at the letters vertically and horizontally
One direction, the letters should correspond
Second direction, the letters should be in the same triangle
*It does not matter which direction does which

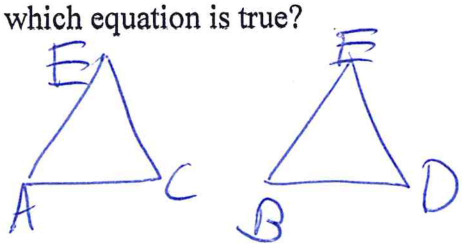
1. As shown in the diagram below, $\overline{AB}$ and $\overline{CD}$ intersect at E, and $\overline{AC} \parallel \overline{BD}$. Given $\triangle AEC \sim \triangle BED$, which equation is true?

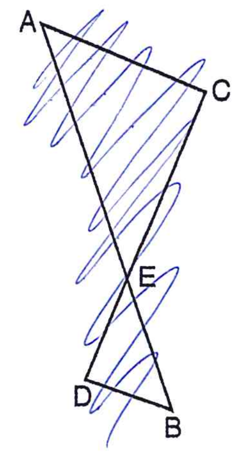
1) $\dfrac{CE}{DE} = \dfrac{EB}{EA}$

2) $\dfrac{AE}{BE} = \dfrac{AC}{BD}$

3) $\dfrac{EC}{AE} = \dfrac{BE}{ED}$

4) $\dfrac{ED}{EC} = \dfrac{AC}{BD}$





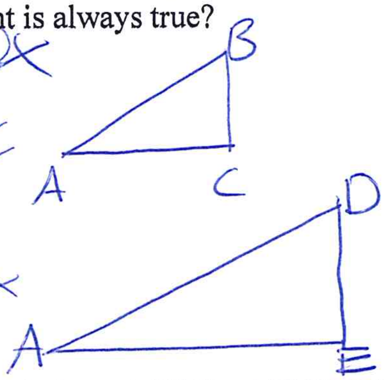
2. In the diagram below of right triangle AED, $\overline{BC} \parallel \overline{DE}$. Which statement is always true?

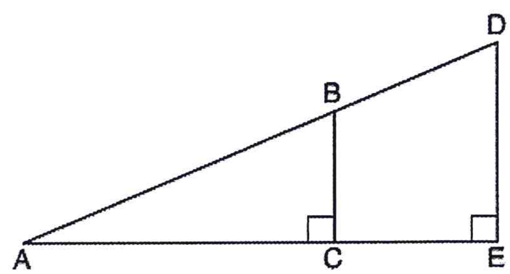
1) $\dfrac{AC}{BC} = \dfrac{DE}{AE}$

2) $\dfrac{AB}{AD} = \dfrac{BC}{DE}$

3) $\dfrac{AC}{CE} = \dfrac{BC}{DE}$

4) $\dfrac{DE}{BC} = \dfrac{DB}{AB}$





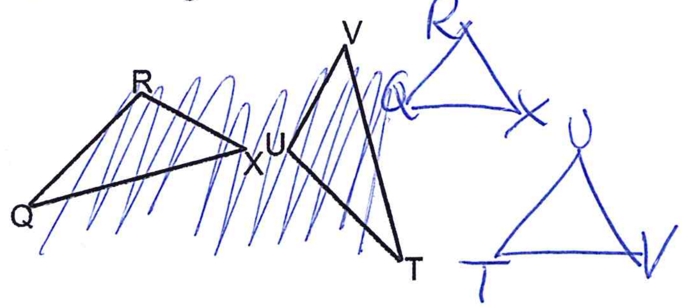
3. In the diagram below, $\Delta QRX \sim \Delta TUV$. Which of the following statements is *not* true?

1) $\dfrac{\overline{QR}}{\overline{TU}} = \dfrac{\overline{QX}}{\overline{TV}}$

2) $\dfrac{\angle X}{\angle V} = \dfrac{\angle Q}{\angle T}$

3) $\dfrac{\overline{RX}}{\overline{UV}} = \dfrac{\overline{VT}}{\overline{XQ}}$

4) $\dfrac{\overline{QX}}{\overline{QR}} = \dfrac{\overline{TV}}{\overline{TU}}$



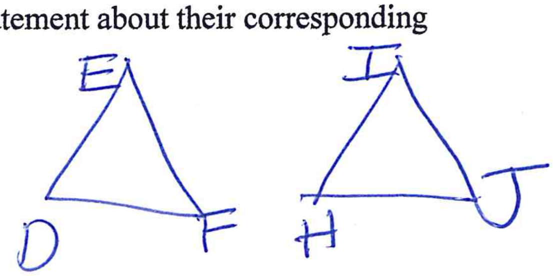
4. Given that $\Delta DEF \sim \Delta HIJ$, which is the correct statement about their corresponding sides?

1) $\dfrac{\overline{EF}}{\overline{IJ}} = \dfrac{\overline{DE}}{\overline{HI}} = \dfrac{\overline{DF}}{\overline{HJ}}$

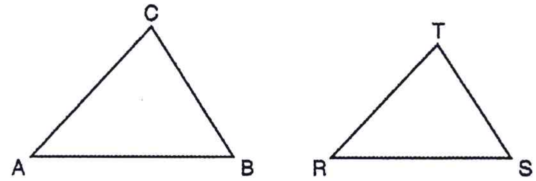
2) $\dfrac{\overline{EF}}{\overline{HI}} = \dfrac{\overline{IJ}}{\overline{DE}} = \dfrac{\overline{DF}}{\overline{HJ}}$

3) $\dfrac{\overline{DE}}{\overline{HI}} = \dfrac{\overline{EF}}{\overline{HJ}} = \dfrac{\overline{DF}}{\overline{IJ}}$

4) $\dfrac{\overline{DE}}{\overline{JI}} = \dfrac{\overline{EF}}{\overline{HJ}} = \dfrac{\overline{FD}}{\overline{HI}}$



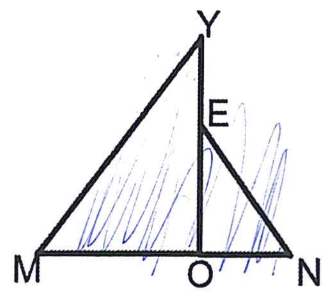
5. In the diagram below, $\triangle ABC \sim \triangle RST$.



Which statement is *not* true?

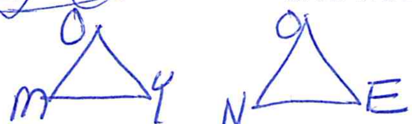
1) $\angle A \cong \angle R$
2) $\frac{AB}{RS} = \frac{BC}{ST}$
3) $\frac{AB}{BC} = \frac{ST}{RS}$
4) $\frac{AB + BC + AC}{RS + ST + RT} = \frac{AB}{RS}$

6. In the diagram below, ΔMOY is the image of ΔNOE after a dilation followed by a reflection. Which of the following statements is true?

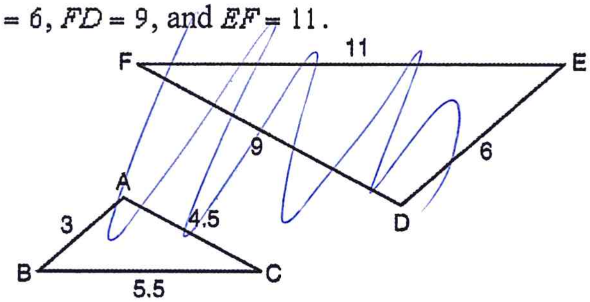


1) $\frac{\overline{MO}}{\overline{ON}} = \frac{\overline{NE}}{\overline{MY}}$ 2) $\frac{perimeter\ MOY}{perimeter\ NOE} = \frac{\overline{EN}}{\overline{YM}}$

3) $\frac{\angle M}{\angle N} = \frac{\angle Y}{\angle E}$ 4) $\frac{area\ MOY}{area\ NOE} = \frac{\overline{YM}}{\overline{EN}}$



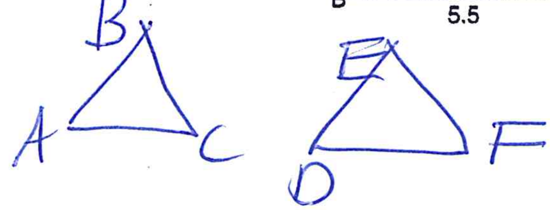
7. In the diagram below, $\triangle DEF$ is the image of $\triangle ABC$ after a clockwise rotation of 180° and a dilation where $AB = 3$, $BC = 5.5$, $AC = 4.5$, $DE = 6$, $FD = 9$, and $EF = 11$.



Which relationship must always be true?

1) $\frac{m\angle A}{m\angle D} = \frac{1}{2}$
2) $\frac{m\angle C}{m\angle F} = \frac{2}{1}$
3) $\frac{m\angle A}{m\angle C} = \frac{m\angle F}{m\angle D}$
4) $\frac{m\angle B}{m\angle E} = \frac{m\angle C}{m\angle F}$

ratio of angles is always 1:1

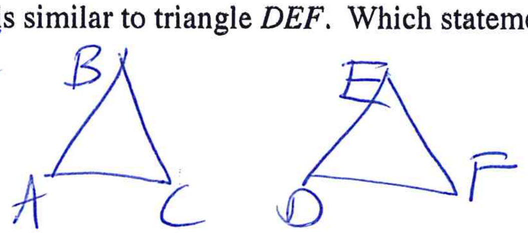


1) $\frac{AB}{BC} = \frac{DE}{EF}$

2) $\frac{AC}{DF} = \frac{BC}{EF}$

8. Scalene triangle ABC is similar to triangle DEF. Which statement is *false*?

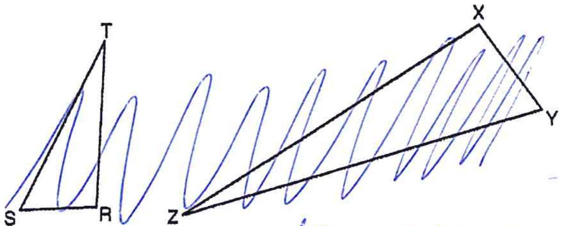
1) $AB:BC = DE:EF$
2) $AC:DF = BC:EF$
3) $\angle ACB \cong \angle DFE$
4) $\angle ABC \cong \angle EDF$

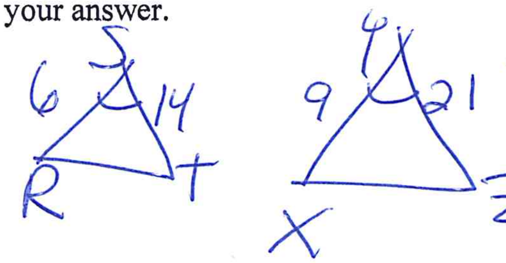


To show triangles are similar:
The ANGLES of similar triangles are congruent
The SIDES of similar triangles are in proportion
1) AA (2 pairs of corresponding angles are congruent)
2) SAS (2 pairs of corresponding sides are in proportion and the corresponding angles between them are congruent)
3) SSS (3 pairs of corresponding sides are in proportion)
*Congruent triangles must be similar. Similar triangles are not necessarily congruent.

1. Triangles RST and XYZ are drawn below. If $RS = 6$, $ST = 14$, $XY = 9$, $YZ = 21$, and $\angle S \cong \angle Y$, is $\triangle RST$ similar to $\triangle XYZ$? Justify your answer.

Determine if sides are in proportion.



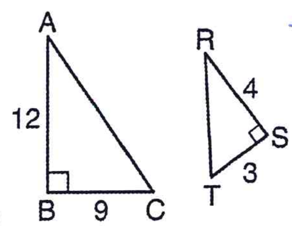


$\frac{6}{9} = \frac{14}{21}$

$126 = 126$ ✓

Yes, SAS. Two pairs of corresponding sides are in proportion and the angle between them is congruent.

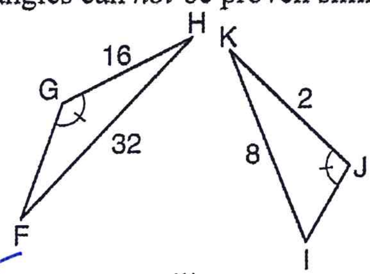
2. Using the information given below, which set of triangles can *not* be proven similar?

1)


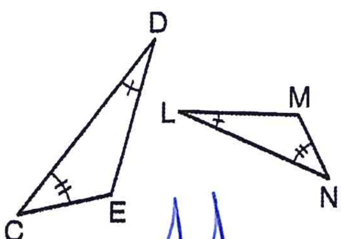
$\frac{3}{9} = \frac{4}{12}$

$36 = 36$ ✓

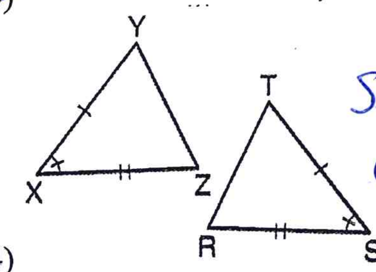
SAS

~~3)~~


not SAS, SSS, or AA

2)


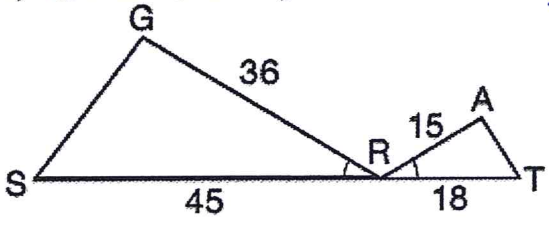
AA

4)


SAS congruence. Congruent triangles are similar

3. In the diagram below, $\angle GRS \cong \angle ART$, $GR = 36$, $SR = 45$, $AR = 15$, and $RT = 18$. Which triangle similarity statement is correct?

1) $\triangle GRS \sim \triangle ART$ by AA.
2) $\triangle GRS \sim \triangle ART$ by SAS.
3) $\triangle GRS \sim \triangle ART$ by SSS.
~~4)~~ $\triangle GRS$ is not similar to $\triangle ART$.



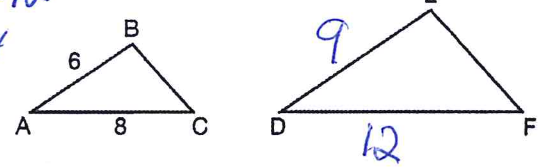
$\frac{15}{36} = \frac{18}{45}$

$675 \neq 648$

the sides are not in proportion

4. In the diagram below, $\triangle ABC \sim \triangle DEF$.
If $AB = 6$ and $AC = 8$, which statement will justify similarity by SAS?

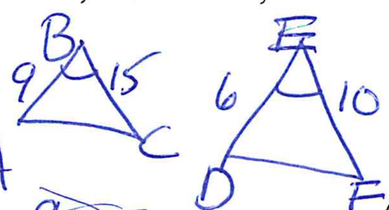
1) $DE = 9$, $DF = 12$, and $\angle A \cong \angle D$ $\quad \frac{6}{9} = \frac{8}{12} \quad 72 = 72$
2) $DE = 8$, $DF = 10$, and $\angle A \cong \angle D$ $\quad \frac{6}{8} = \frac{8}{10} \quad 60 \neq 64$
3) $DE = 36$, $DF = 64$, and $\angle C \cong \angle F$
4) $DE = 15$, $DF = 20$, and $\angle C \cong \angle F$ Not SAS



5. Triangles ABC and DEF are drawn below.

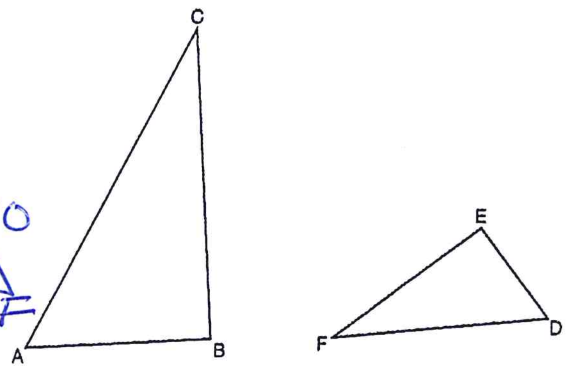
If $AB = 9$, $BC = 15$, $DE = 6$, $EF = 10$, and $\angle B \cong \angle E$, which statement is true?

1) $\angle CAB \cong \angle DEF$ ✗
2) $\frac{AB}{CB} = \frac{FE}{DE}$ ✗
3) $\triangle ABC \sim \triangle DEF$ ✓ SAS
4) $\frac{AB}{DE} = \frac{FE}{CB}$ ✗



$\frac{9}{6} = \frac{15}{10}$

$90 = 90$ ✓



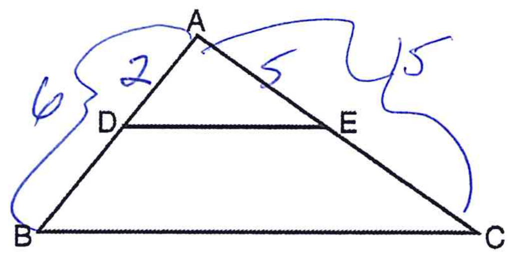
6. In the diagram below, $\triangle ABC \sim \triangle ADE$.

The sides must be in proportion

Which measurements are justified by this similarity?

1) $AD = 3$, $AB = 6$, $AE = 4$, and $AC = 12$
2) $AD = 5$, $AB = 8$, $AE = 7$, and $AC = 10$
3) $AD = 3$, $AB = 9$, $AE = 5$, and $AC = 10$
4) $AD = 2$, $AB = 6$, $AE = 5$, and $AC = 15$ $\quad \frac{2}{5} = \frac{6}{15}$

$30 = 30$ ✓



7. Skye says that the two triangles below are congruent. Margaret says that the two triangles are similar. Are Skye and Margaret both correct? Explain why.

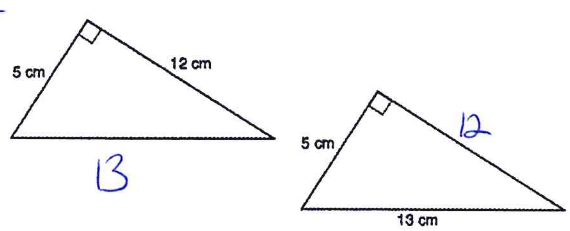
$a^2 + b^2 = c^2$

$5^2 + 12^2 = x^2$

$25 + 144 = x^2$

$\sqrt{169} = \sqrt{x^2}$

$13 = x$



They are congruent by SSS. Congruent figures are similar.

8. If $\triangle ABC$ is mapped onto $\triangle DEF$ after a line reflection and $\triangle DEF$ is mapped onto $\triangle XYZ$ after a translation, the relationship between $\triangle ABC$ and $\triangle XYZ$ is that they are always

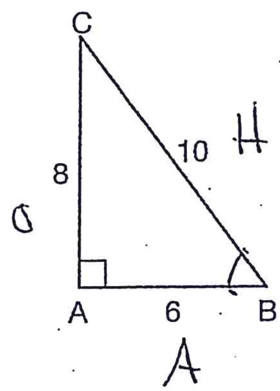
1) congruent and similar
2) congruent but not similar
3) similar but not congruent
4) neither similar nor congruent

A rigid motion preserves size and angle measure producing a congruent figure.

congruent figures are similar

Right Triangles
If only sides are involved, use Pythagorean theorem! ($a^2 + b^2 = c^2$)
If an angle is involved, use SOHCAHTOA
1) Label each side with O, A, and H
2) Determine whether to use sine, cosine, or tangent (Which two are involved?)
3) Substitute into appropriate formula
***If finding a side, cross multiply and solve**
***If finding an angle, use $\sin^{-1}$, $\cos^{-1}$, *or* $\tan^{-1}$**

1. In ΔABC below, the measure of $\angle A = 90°$, $AB = 6$, $AC = 8$, and $BC = 10$.



Which ratio represents the cosine of $\angle B$?

1) $\frac{10}{8}$

2) $\frac{8}{6}$

~~3)~~ $\frac{6}{10}$

4) $\frac{8}{10}$

$\cos\theta = \frac{A}{H}$

$\cos B = \frac{6}{10}$

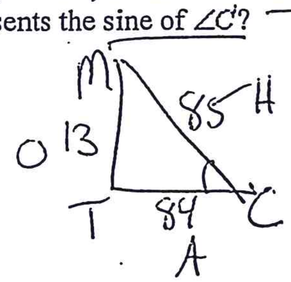
2. In triangle MCT, the measure of $\angle T = 90°$, $MC = 85$ cm, $CT = 84$ cm, and $TM = 13$cm. Which ratio represents the sine of $\angle C$?

~~1)~~ $\frac{13}{85}$

2) $\frac{84}{85}$

3) $\frac{13}{84}$

4) $\frac{84}{13}$



$\sin\theta = \frac{O}{H}$

$\sin C = \frac{13}{85}$

3. As shown in the diagram below, a ladder 12 feet long leans against a wall and makes an angle of 72° with the ground.

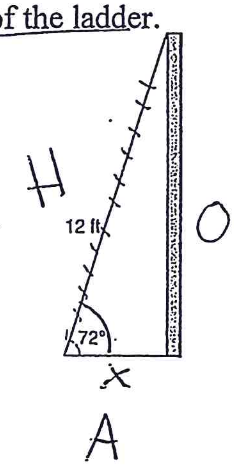
Find, to the *nearest tenth of a foot,* the distance from the wall to the base of the ladder.

$\cos\theta = \frac{A}{H}$

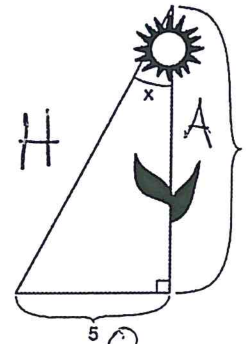
$\cos 72 = \frac{x}{12}$

$\cancel{\frac{.3090}{1} = \frac{x}{12}}$

$x = 3.7$



4. The diagram below shows the path a bird flies from the top of a 9.5-foot-tall sunflower to a point on the ground 5 feet from the base of the sunflower.



To the *nearest tenth of a degree*, what is the measure of angle x?

1) 27.8
2) 31.8
3) 58.2
4) 62.2

$\tan\theta = \frac{O}{A}$

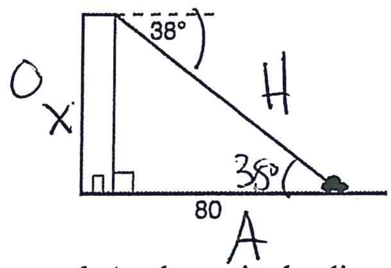
$\tan^{-1} \tan x = \tan^{-1}\frac{5}{9.5}$

$x = \tan^{-1}\left(\frac{5}{9.5}\right)$

$x = 27.8$

5. From the top of an apartment building, the angle of depression to a car parked on the street below is 38 degrees, as shown in the diagram below. The car is parked 80 feet from the base of the building. Find the height of the building, to the *nearest tenth of a foot.*

*angle of depression = angle of elevation



$\tan\theta = \frac{O}{A}$

$\tan 38 = \frac{x}{80}$

$\frac{.7813}{1} = \frac{x}{80}$

$x = 62.5$

6. As shown in the diagram below, a building casts a 72-foot shadow on the ground when the angle of elevation of the Sun is 40°.
How tall is the building, to the *nearest foot*?

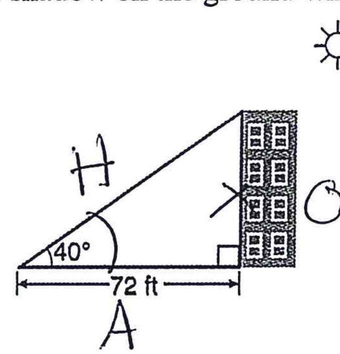
1) 46
2) 60
3) 86
4) 94

$\tan\theta = \frac{O}{A}$

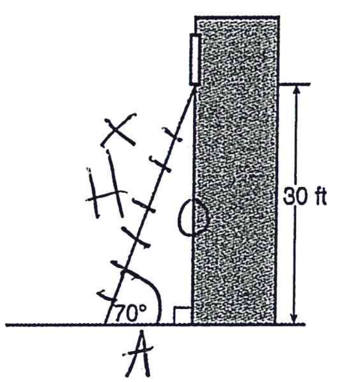
$\tan 40 = \frac{x}{72}$

$\frac{.8391}{1} = \frac{x}{72}$

$x = 60$



7. A carpenter leans an extension ladder against a house to reach the bottom of a window 30 feet above the ground. As shown in the diagram below, the ladder makes a 70° angle with the ground. To the *nearest foot*, determine and state the length of the ladder.



$\sin\theta = \frac{O}{H}$

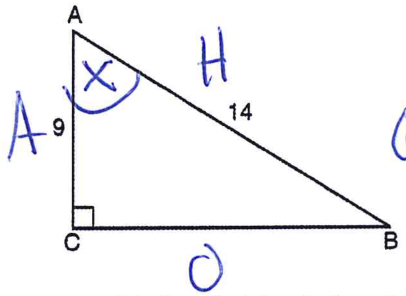
$\sin 70 = \frac{30}{x}$

$\frac{.9397}{1} = \frac{30}{x}$

$\frac{.9397x}{.9397} = \frac{30}{.9397}$

$x = 32$

8. In the diagram of right triangle ABC shown below, $AB = 14$ and $AC = 9$. What is the measure of $\angle A$, to the *nearest degree*?



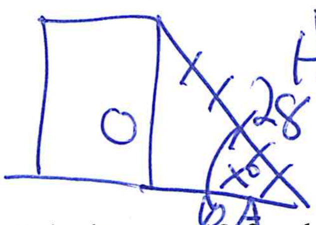
$\cos\theta = \frac{A}{H}$

$\cos^{-1}\cos x = \cos^{-1}\frac{9}{14}$

$x = \cos^{-1}\left(\frac{9}{14}\right)$

$\to x =$

9. A 28-foot ladder is leaning against a house. The bottom of the ladder is 6 feet from the base of the house. Find the measure of the angle formed by the ladder and the ground, to the *nearest degree*.



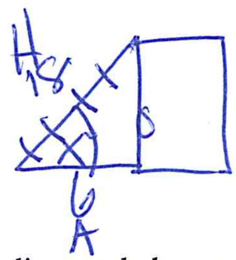
$\cos\theta = \frac{A}{H}$

$\cos^{-1}\cos x = \cos^{-1}\frac{6}{28}$

$\to x = \cos^{-1}\left(\frac{6}{28}\right)$

$x = 78°$

10. Bob places an 18-foot ladder 6 feet from the base of his house and leans it up against the side of his house. Find, to the *nearest degree*, the measure of the angle the bottom of the ladder makes with the ground.



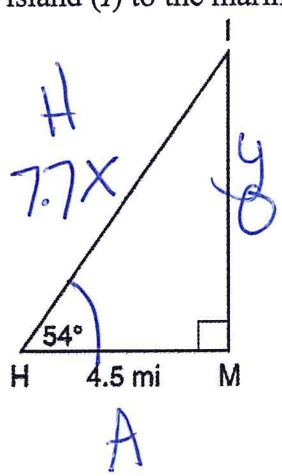
$\cos\theta = \frac{A}{H}$

$\cos^{-1}\cos x = \cos^{-1}\frac{6}{18}$

$x = \cos^{-1}\left(\frac{6}{18}\right)$

$\to x = 71°$

11. As shown in the diagram below, an island (I) is due north of a marina (M). A boat house (H) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.
Determine and state, to the *nearest tenth of a mile*, the distance from the boat house (H) to the island (I). Determine and state, to the *nearest tenth of a mile*, the distance from the island (I) to the marina (M).



$\cos\theta = \frac{A}{H}$

$\cos 54 = \frac{4.5}{x}$

$\cancel{\frac{.5878}{1} = \frac{4.5}{x}}$

$\cancel{.5878x = 4.5}$

$\frac{.5878x}{.5878} = \frac{4.5}{.5878}$

$\to x = 7.7$

$\tan\theta = \frac{O}{A}$

$\tan 54 = \frac{x}{4.5}$

$\cancel{\frac{1.3764}{1} = \frac{x}{4.5}}$

$x = 6.2$

$a^2 + b^2 = c^2$

$4.5^2 + x^2 = 7.7^2$

$20.25 + x^2 = 59.2$

$-20.25 \quad -20.2$

$\sqrt{x^2} = \sqrt{39.04}$

$x = 6.2$

Acute Angles in a Right Triangle

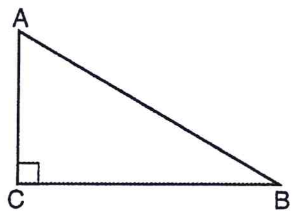
$\sin A = \cos B$**: In a right triangle, the sine of one acute angle is equal to the cosine of the other acute angle**

$A + B = 90$**: The two acute angles in a right triangle are complementary**

1. In scalene triangle ABC shown in the diagram below, $m\angle C = 90°$.

Which equation is always true?

1) $\sin A = \sin B$
2) $\cos A = \cos B$
3) $\cos A = \sin C$
4) $\sin A = \cos B$



2. In $\triangle ABC$, the complement of $\angle B$ is $\angle A$. Which statement is always true?

1) $\tan\angle A = \tan\angle B$
2) $\sin\angle A = \sin\angle B$
3) $\cos\angle A = \tan\angle B$
4) $\sin\angle A = \cos\angle B$

3. Given: Right triangle ABC with right angle at C. If $\sin A$ increases, does $\cos B$ increase or decrease? Explain why.

increases because $\sin A = \cos B$

In a right $\triangle$, the sine of one acute angle is equal to the cosine of the other acute angle

4. In right triangle ABC, $m\angle C = 90°$. If $\cos B = \frac{5}{13}$, which function also equals $\frac{5}{13}$?

1) $\tan A$
2) $\tan B$
3) $\sin A$
4) $\sin B$

$\sin A = \cos B$

5. In right triangle ABC, $m\angle C = 90°$ and $AC \neq BC$. Which trigonometric ratio is equivalent to $\sin B$?

1) $\cos A$
2) $\cos B$
3) $\tan A$
4) $\tan B$

$\sin A = \cos B$

the letters don't matter as long as they are the acute angles.

6. In right triangle ABC with the right angle at C, $\sin A = 2x + 0.1$ and $\cos B = 4x - 0.7$. Determine and state the value of x. Explain your answer.

In a right triangle, the sine of one acute angle is equal to the cosine of the other acute angle.

$\sin A = \cos B$

$2x + 0.1 = 4x - 0.7$

$-2x \quad -2x$

$0.1 = 2x - 0.7$

$+0.7 \quad +0.7$

$\frac{0.8}{2} = \frac{2x}{2}$

$0.4 = x$

sin A = cos B   A+B=90

7. If $\sin(3x+2)^\circ = \cos(4x-10)^\circ$, what is the value of x to the *nearest tenth*?

(1) 7.6 (2) 12.0 ~~(3)~~ 14.0 (4) 26.9

$3x+2+4x-10=90$
$7x-8=90$
$+8\quad +8$
$\frac{7x}{7}=\frac{98}{7}$
$x=14$

sin A = cos B

8. If $\sin(2x+7)^\circ = \cos(4x-7)^\circ$, what is the value of x?

1) 7
~~2)~~ 15
3) 21
4) 30

$A+B=90$
$2x+7+4x-7=90$
$\frac{6x}{6}=\frac{90}{6}$
$x=15$

9. In a right triangle, $\sin(40-x)^\circ = \cos(3x)^\circ$. What is the value of x?

sin A = cos B

1) 10
2) 15
3) 20
~~4)~~ 25

$A+B=90$

$40-x+3x=90$
$2x+40=90$
$-40\quad -40$
$\frac{2x}{2}=\frac{50}{2}$
$x=25$

sin A = cos B

10. In a right triangle, the acute angles have the relationship $\sin(2x+4) = \cos(46)$. What is the value of x?

~~1)~~ 20
2) 21
3) 24
4) 25

$A+B=90$
$2x+4+46=90$
$2x+50=90$
$-50\quad -50$
$\frac{2x}{2}=\frac{40}{2}$
$x=20$

11. Find the value of R that will make the equation $\sin 73^\circ = \cos R$ true when $0^\circ < R < 90^\circ$. Explain your answer.

sin A = cos B

$A+B=90$
$73+R=90$
$-73\quad -73$
$R=17$

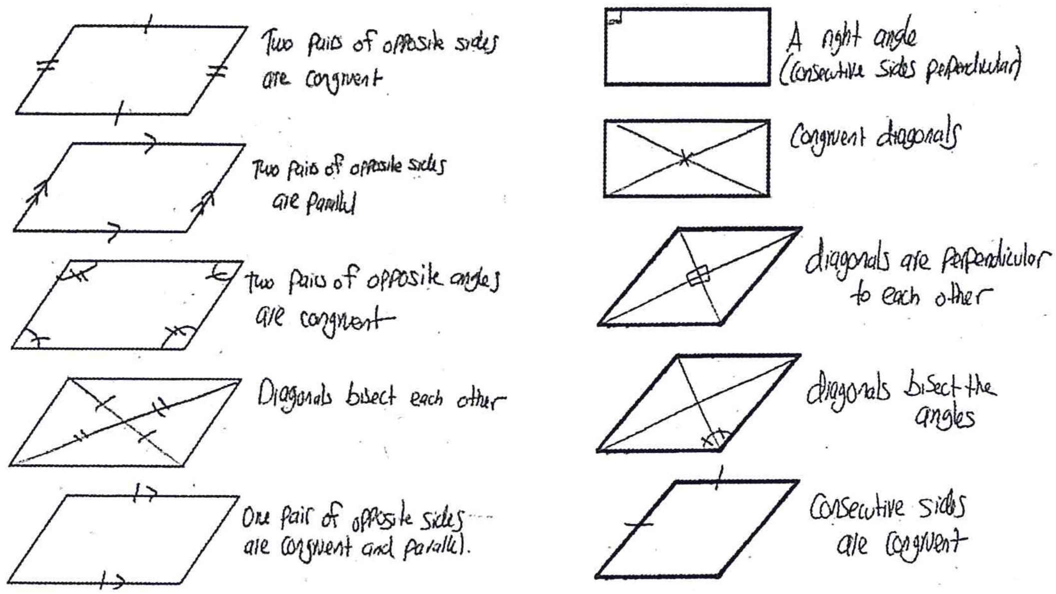
the acute angles of a right triangle are complementary.

12. Which expression is always equivalent to $\sin x$ when $0^\circ < x < 90^\circ$?

~~1)~~ $\cos(90^\circ - x)$
2) $\cos(45^\circ - x)$
3) $\cos(2x)$
4) $\cos x$

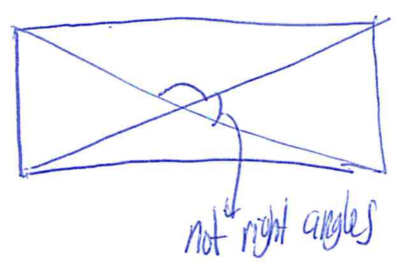
$\sin x=\cos y$
$x+y=90$
$-x\quad -x$
$y=90-x$

Parallelogram Properties

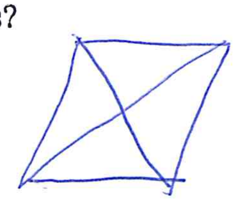


A rectangle and rhombus have all of the properties of the parallelogram.
A square has all of the properties of the parallelogram, rectangle, and rhombus.
A trapezoid has one pair of opposite sides parallel and one pair of opposite sides not parallel.
An isosceles trapezoid is a trapezoid that has congruent legs and congruent diagonals.
For properties questions, draw the shape!

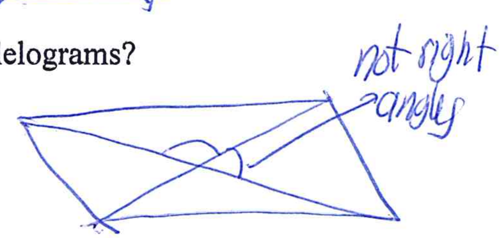
1. Which of the following is not true of all rectangles?
1) Consecutive sides are perpendicular
2) Opposite sides are parallel
3) Diagonals are perpendicular to each other
4) Diagonals bisect each other



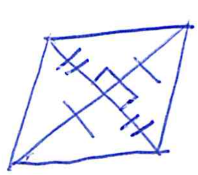
2. Which of the following is true about rhombuses?
1) Consecutive sides are perpendicular
2) Opposite sides are congruent
3) Consecutive angles are congruent
4) Diagonals are congruent



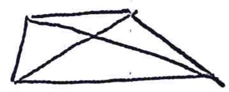
3. Which of the following is *not* true about all parallelograms?
1) Diagonals bisect each other
2) Diagonals are perpendicular to each other
3) Opposite angles are congruent
4) Consecutive angles are supplementary



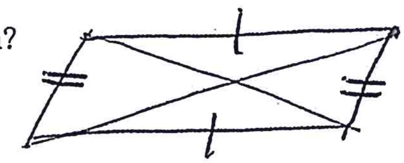
4. A quadrilateral whose diagonals bisect each other and are perpendicular is a
1) rhombus
2) rectangle
3) trapezoid
4) parallelogram



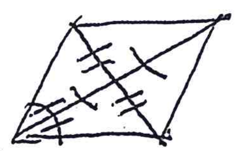
5. If the diagonals of a quadrilateral do *not* bisect each other, then the quadrilateral could be a
1) rectangle
2) rhombus
3) square
4) trapezoid



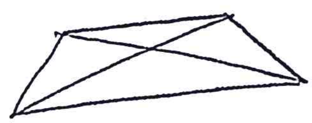
6. Which statement is true about every parallelogram?
1) All four sides are congruent.
2) The interior angles are all congruent.
3) Two pairs of opposite sides are congruent.
4) The diagonals are perpendicular to each other.



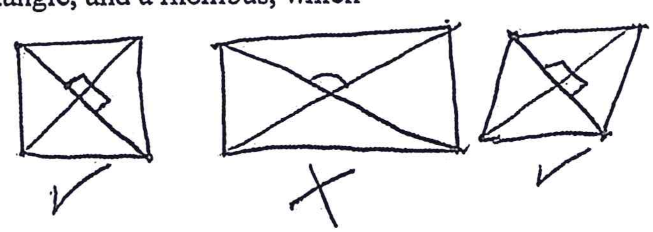
7. Which quadrilateral has diagonals that always bisect its angles and also bisect each other?
1) rhombus
2) rectangle
3) parallelogram
4) isosceles trapezoid



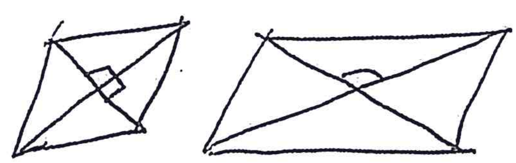
8. The diagonals of a quadrilateral are congruent but do not bisect each other. This quadrilateral is
1) an isosceles trapezoid
2) a parallelogram
3) a rectangle
4) a rhombus



9. Given three distinct quadrilaterals, a square, a rectangle, and a rhombus, which quadrilaterals must have perpendicular diagonals?
1) the rhombus, only
2) the rectangle and the square
3) the rhombus and the square
4) the rectangle, the rhombus, and the square



10. A parallelogram must be a rhombus when its which property proves a rhombus?
1) Diagonals are congruent.
2) Opposite sides are parallel.
3) Diagonals are perpendicular.
4) Opposite angles are congruent.

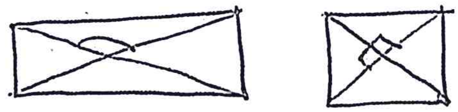


which property proves a rectangle

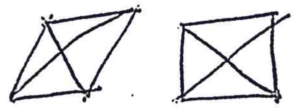
11. A parallelogram must be a rectangle when its
1) diagonals are perpendicular
2) diagonals are congruent
3) opposite sides are parallel
4) opposite sides are congruent

which property proves a square

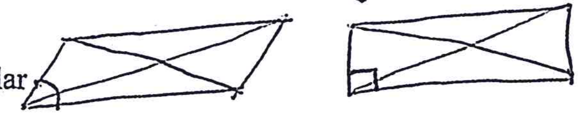
12. A rectangle must be a square when its
1) consecutive sides are perpendicular
2) diagonals are congruent
3) diagonals are perpendicular to each other
4) opposite sides are parallel



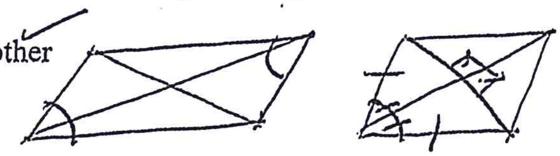
13. A rhombus must be a square when its which property proves a square
1) consecutive sides are congruent
2) diagonals are congruent
3) opposite angles are congruent
4) diagonals are perpendicular to each other



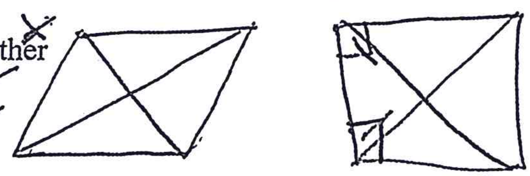
14. A parallelogram must be a rectangle when its which property proves a rectangle
1) consecutive sides are congruent
2) opposite angles are congruent
3) consecutive sides are perpendicular
4) opposite sides are parallel



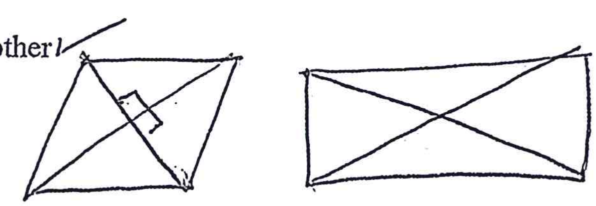
15. Which of the following properties does not make a parallelogram a rhombus?
1) diagonals bisect the angles
2) diagonals are perpendicular to each other
3) opposite angles are congruent
4) consecutive sides are congruent



16. Which of the following properties does not make a rhombus a square?
1) Diagonals are congruent
2) Diagonals are perpendicular to each other
3) Consecutive sides are perpendicular
4) Consecutive angles are congruent



17. Which property is true of all rhombuses but not of all rectangles?
1) opposite sides are parallel
2) diagonals are perpendicular to each other
3) diagonals bisect each other
4) opposite angles are congruent



18. Which set of statements would describe a parallelogram that can always be classified as a rhombus?

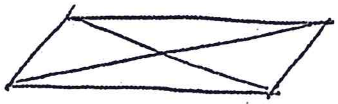
I. Diagonals are perpendicular bisectors of each other.

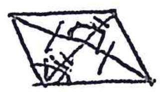
II. Diagonals bisect the angles from which they are drawn.

III. Diagonals form four congruent isosceles right triangles.

 must be a square all ~~...~~ squares are rhombuses

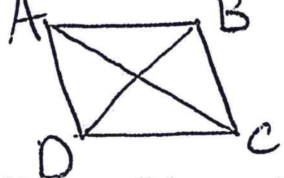
1) I and II
2) I and III
3) II and III
4) I, II, and III





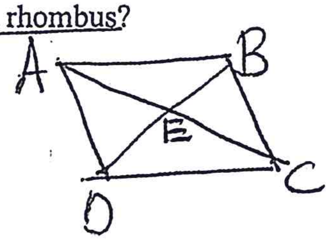
19. If *ABCD* is a parallelogram, which statement would prove that <u>*ABCD* is a rhombus</u>?

1) $\angle ABC \cong \angle CDA$ opposite angles $\cong$
2) $\overline{AC} \cong \overline{BD}$ diagonals $\cong$
3) $\overline{AC} \perp \overline{BD}$ diagonals perpendicular to each other
4) $\overline{AB} \perp \overline{CD}$ opposite sides $\perp$

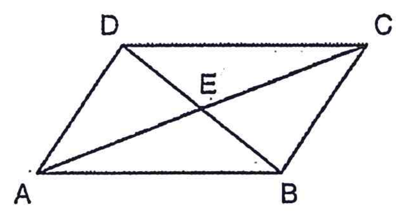


20. In parallelogram *ABCD*, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at *E*. Which statement does *not* prove parallelogram *ABCD* is a rhombus?

1) $\overline{AC} \cong \overline{DB}$ $\cong$ diagonals
2) $\overline{AB} \cong \overline{BC}$ consecutive sides $\cong$
3) $\overline{AC} \perp \overline{DB}$ $\perp$ diagonals
4) $\overline{AC}$ bisects $\angle DCB$ diagonals bisect the angles



21. In the diagram below, parallelogram *ABCD* has diagonals $\overline{AC}$ and $\overline{BD}$ that intersect at point *E*.



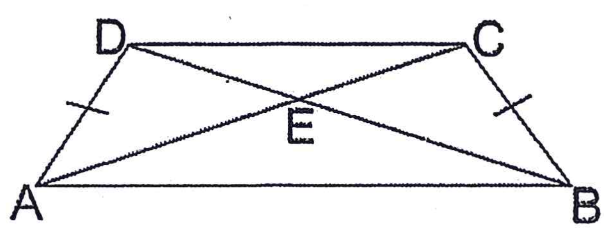
Which expression is *not* always true?

1) $\angle DAE \cong \angle BCE$ $\cong$ alternate interior angles
2) $\angle DEC \cong \angle BEA$ $\cong$ vertical angles
3) $\overline{AC} \cong \overline{DB}$ $\cong$ diagonals
4) $\overline{DE} \cong \overline{EB}$ diagonals bisect each other

22. In the diagram below, isosceles trapezoid ABCD has diagonals $\overline{AC}$ and $\overline{BD}$ that intersect at point *E*.

Which expression is *not* always true?

1) $\overline{AC} \cong \overline{DB}$ $\cong$ diagonals
2) $\overline{DC} \parallel \overline{AB}$ opposite sides $\parallel$
3) $\overline{DE} \cong \overline{AE}$ diagonals bisect each other and $\cong$
4) $\overline{AD} \cong \overline{CB}$ opposite $\cong$ legs



Triangles/Parallel Lines Cut By a Transversal/Angles of Parallelograms

1) The three angles of a triangle add to equal 180°. **Look for triangles.**
   *The four angles of a quadrilateral add to 360°.
2) Linear pairs add to 180°. **Look for linear pairs.**
3) Vertical angles are congruent. Look for an X (intersecting lines).
4) **Given congruent sides:** Isosceles triangle has congruent angles opposite congruent sides.
5) **Given equilateral triangle:** Equilateral triangle has angles 60, 60, 60.
6) **Given angle bisector:** An angle bisector cuts an angle into two congruent halves.
7) **Given parallel:** Extend parallel lines and transversal. Follow the transversal and fill in all 8 angles. If angles are the same (both acute or both obtuse), the angles are congruent. If the angles are different (one acute and one obtuse), the angles are supplementary (add to 180).
8) **Given parallelogram:** Opposite angles are congruent and consecutive angles are supplementary (add to 180)

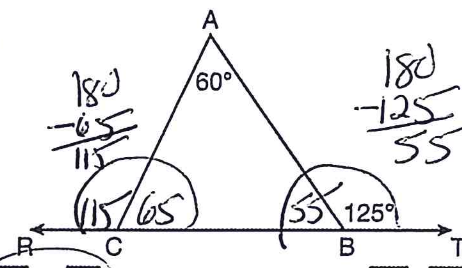
1. In the diagram below, $\overleftrightarrow{RCBT}$ and $\triangle ABC$ are shown with $m\angle A = 60$ and $m\angle ABT = 125$.

What is $m\angle ACR$?
1) 125
~~2) 115~~
3) 65
4) 55

$\triangle CAB$
$55+60+x=180$
$115+x=180$
$-115 \quad -115$
$x=65$

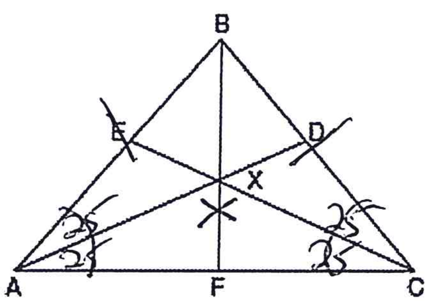
180
-65
115

180
-125
55



2. In the diagram below of isosceles triangle ABC, $\overline{AB} \cong \overline{CB}$ and angle bisectors $\overline{AD}$, $\overline{BF}$, and $\overline{CE}$ are drawn and intersect at X.
If $m\angle BAC = 50°$, find $m\angle AXC$.

cuts an angle into 2 ≅ halves



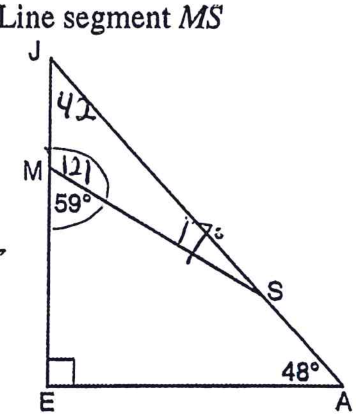
$25+25+x=180$
$50+x=180$
$-50 \quad -50$
$x=130$

3. In the diagram of $\triangle JEA$ below, $m\angle JEA = 90$ and $m\angle EAJ = 48$. Line segment MS connects points M and S on the triangle, such that $m\angle EMS = 59$.

$\triangle JEA$
$90+48+x=180$
$138+x=180$
$-138 \quad -138$
$x=42$

What is $m\angle JSM$?
1) 163
2) 121
3) 42
~~4) 17~~

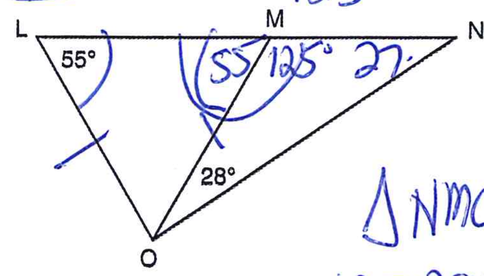
180
-59
121



$\triangle JMS$
$42+121+x=180$
$163+x=180$
$-163 \quad -163$
$x=17$

$\frac{180}{-55}$ 125

4. In the diagram below, $\triangle LMO$ is isosceles with $LO = MO$.



If $m\angle L = 55$ and $m\angle NOM = 28$, what is $m\angle N$?
1) 27
2) 28
3) 42
4) 70

$\triangle NMO$

$125+28+x=180$

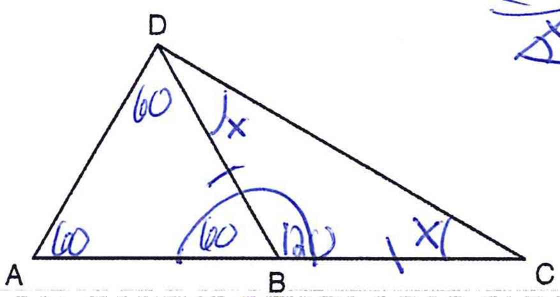
$153+x=180$

$-153 \quad -153$

$x=27$

5. In the diagram below of $\triangle ACD$, B is a point on $\overline{AC}$ such that $\triangle ADB$ is an equilateral triangle, and $\triangle DBC$ is an isosceles triangle with $\overline{DB} \cong \overline{BC}$. Find $m\angle C$.

60, 60, 60



$\frac{180}{-60}$ 120

$\triangle DBC$

$x+x+120=180$

$2x+120=180$

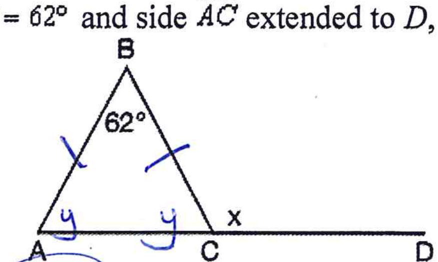
$-120 \quad -120$

$\frac{2x}{2}=\frac{60}{2}$

$x=30$

$m\angle C=30°$

6. Given $\triangle ABC$ with $m\angle B = 62°$ and side $\overline{AC}$ extended to D, as shown below.



$\triangle ABC$

$y+y+62=180$

$2y+62=180$

$-62 \quad -62$

$\frac{2y}{2}=\frac{118}{2}$

$y=59$

Which value of x makes $\overline{AB} \cong \overline{CB}$?

1) 59°
2) 62°
3) 118°
4) 121°

7. In the diagram below, $\overline{AEFB} \parallel \overline{CGD}$, and $\overline{GE}$ and $\overline{GF}$ are drawn.
If $m\angle EFG = 32°$ and $m\angle AEG = 137°$, what is $m\angle EGF$?

1) 11°
2) 43°
3) 75°
4) ~~4)~~ 105°

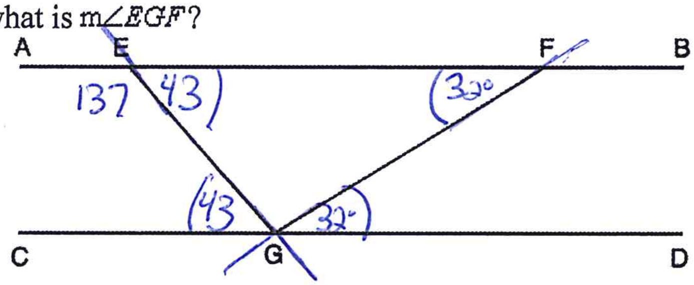
ΔEGF

$43+32+x=180$

$75+x=180$

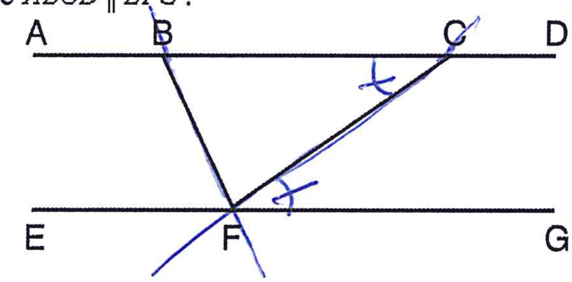
$-75 \quad -75$

$x=105$



8. Steve drew line segments *ABCD*, *EFG*, *BF*, and *CF* as shown in the diagram below.
Scalene ΔBFC is formed.
Which statement will allow Steve to prove $\overline{ABCD} \parallel \overline{EFG}$?

1) ~~1)~~ $\angle CFG \cong \angle FCB$ alternate interior angles
2) $\angle ABF \cong \angle BFC$ X
3) $\angle EFB \cong \angle CFB$ X
4) $\angle CBF \cong \angle GFC$ X



9. As shown in the diagram below, $\overleftrightarrow{ABC} \parallel \overleftrightarrow{EFG}$ and $\overline{BF} \cong \overline{EF}$.
If $m\angle CBF = 42.5°$, then $m\angle EBF$ is

1) 42.5°
2) ~~2)~~ 68.75°
3) 95°
4) 137.5°

ΔEBF

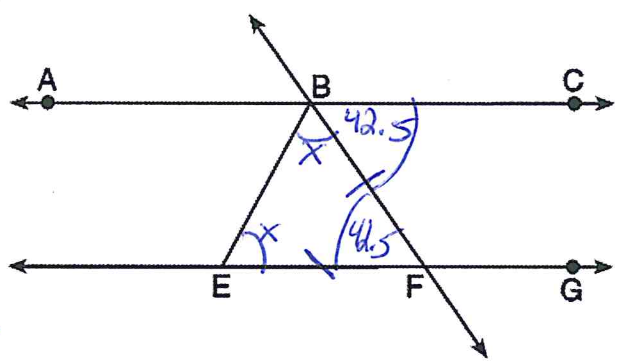
$x+x+42.5=180$

$2x+42.5=180$

$-42.5 \quad -42.5$

$\frac{2x}{2}=\frac{137.5}{2}$

$x=68.75$



10. In the diagram below, $\overline{AB} \parallel \overrightarrow{DEF}$, $\overline{AE}$ and $\overline{BD}$ intersect at *C*, $m\angle B = 43°$, and $m\angle CEF = 152°$.
Which statement is true?

1) $m\angle D = 28°$ X
2) $m\angle A = 43°$ X
3) ~~3)~~ $m\angle ACD = 71°$ ✓
4) $m\angle BCE = 109°$ X

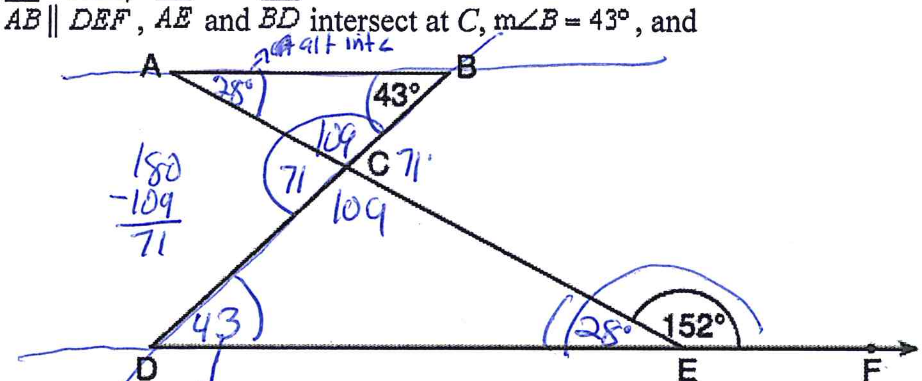
ΔDCE

$43+28+x=180$

$71+x=180$

$-71 \quad -71$

$x=109$



$\triangle DBF$

$54+72+x=180$

$126+x=180$

$-126 \quad -126$

$x=54$

parallel

11. In the diagram below, $\overline{DE}$ divides $\overline{AB}$ and $\overline{AC}$ proportionally, $m\angle C = 26°$, $m\angle A = 82°$, and $\overline{DF}$ bisects $\angle BDE$. $\rightarrow$ cuts in half

The measure of angle DFB is

1) 36°
2) ~~54°~~
3) 72°
4) 82°

$\triangle ADE$

$82+26+x=180$

$108+x=180$

$-108 \quad -108$

$x=72$

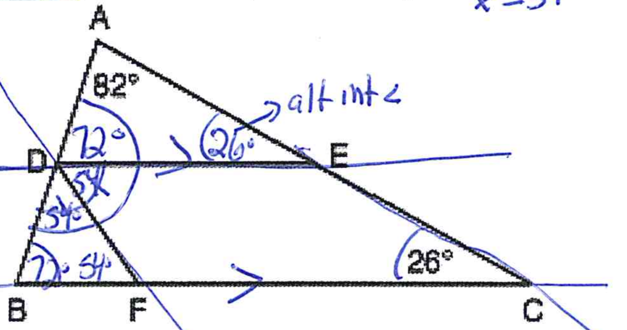
$72+x+x=180$

$72+2x=180$

$-72 \quad -72$

$2x=108$

$x=54$



12. In the diagram below of parallelogram $ROCK$, $m\angle C$ is 70° and $m\angle ROS$ is 65°. What is $m\angle KSO$?

1) 45°
2) 110°
3) 115°
4) ~~135°~~

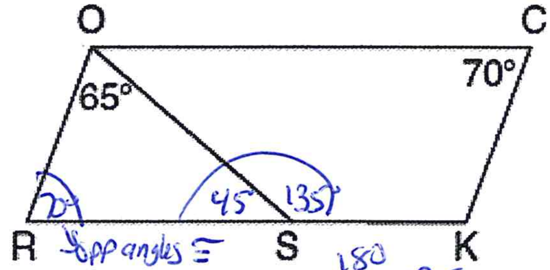
$\triangle ROS$

$70+65+x=180$

$135+x=180$

$-135 \quad -135$

$x=45$



13. In the diagram below, $ABCD$ is a parallelogram, $\overline{AB}$ is extended through B to E, and $\overline{CE}$ is drawn.

If $\overline{CE} \cong \overline{BE}$ and $m\angle D = 112°$, what is $m\angle E$?

1) ~~44°~~
2) 56°
3) 68°
4) 112°

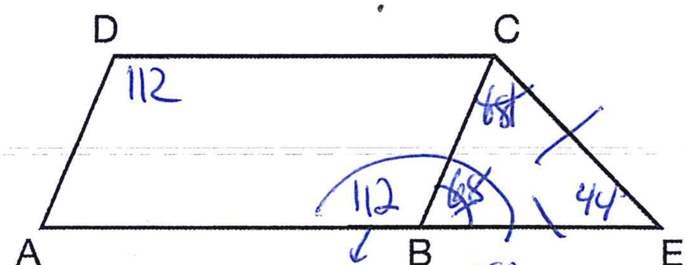
$\triangle CBE$

$68+68+x=180$

$136+x=180$

$-136 \quad -136$

$x=44$



14. In the diagram of parallelogram $FRED$ shown below. $\overline{ED}$ is extended to A. and $\overline{AF}$ is drawn such that $\overline{AF} \cong \overline{DF}$.

If $m\angle R = 124°$, what is $m\angle AFD$?

1) 124°
2) 112°
3) ~~68°~~
4) 56°

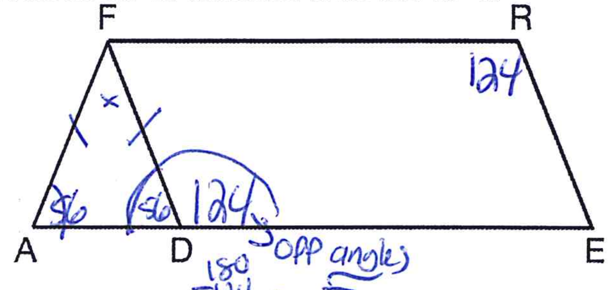
$\triangle AFD$

$56+56+x=180$

$112+x=180$

$-112 \quad -112$

$x=68$



15. In parallelogram $QRST$ shown below, diagonal $\overline{TR}$ is drawn, U and V are points on $\overline{TS}$ and $\overline{QR}$, respectively, and $\overline{UV}$ intersects $\overline{TR}$ at W.

If $m\angle S = 60°$, $m\angle SRT = 83°$, and $m\angle TWU = 35°$, what is $m\angle WVQ$?

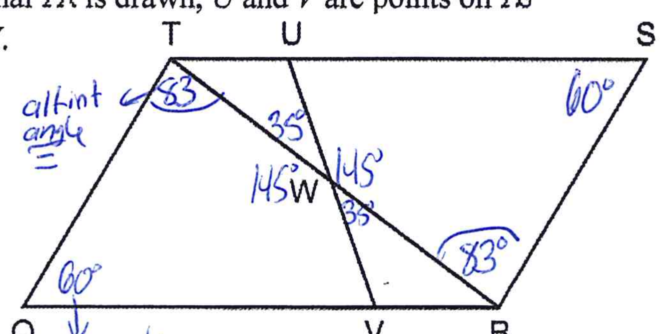
Quad QTWV

$60+83+145+x=360$

$x+288=360$

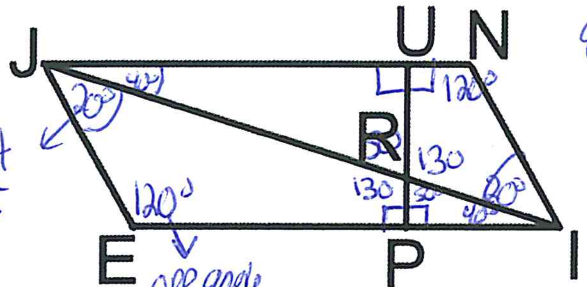
$-288 \quad -288$

$x=72$



right angles

16. In parallelogram JNIE shown below, diagonal $\overline{JI}$ is drawn, $\overline{UP} \perp \overline{JN}$, $\overline{JI}$ intersects $\overline{UP}$ at R. If $m\angle JNI = 120$ and $m\angle JIN = 20$, find $m\angle PRI$ and $m\angle RJU$.



alt int angle ≅

opp angle ≅

quad UNIR

$90+120+20+x=360$

$230+x=360$

$-230 \quad -230$

$x=130$

$\angle PRI = 50°$

$\angle RJU = 40°$

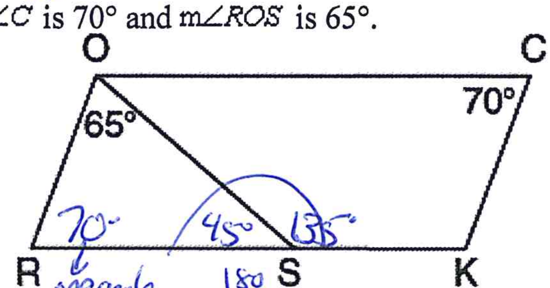
ΔPRI

$90+50+x=180$

$140+x=180$

$-140 \quad -140 \quad x=40$

17. In the diagram below of parallelogram *ROCK*, m$\angle C$ is 70° and m$\angle ROS$ is 65°.



opp angle ≅

$180 - 45 = 135$

What is m$\angle KSO$?

1) 45°
2) 110°
3) 115°
4) 135°

ΔRSO

$70+65+x=180$

$135+x=180$

$-135 \quad -135$

$x=45$

Consecutive angle are supplementary

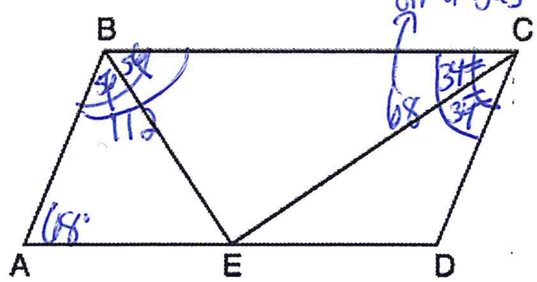
18. In parallelogram *ABCD* shown below, the bisectors of $\angle ABC$ and $\angle DCB$ meet at *E*, a point on $\overline{AD}$.

cut in half

If m$\angle A = 68°$, determine and state m$\angle BEC$.

$180 - 68 = 112$

opp angles ≅



~~ΔABE~~

~~56+68~~

ΔBEC

$56+34+x=180$

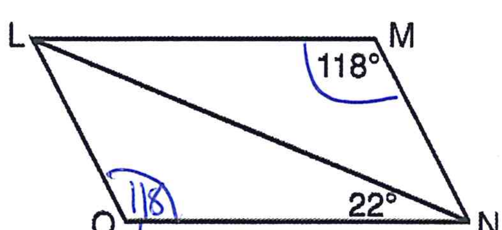
$90+x=180$

$-90 \quad -90$

$x=90$

19. The diagram below shows parallelogram *LMNO* with diagonal $\overline{LN}$, m$\angle M$ = 118°, and m$\angle LNO$ = 22°.

Explain why m$\angle NLO$ is 40 degrees.



1) opposite angles of a parallelogram are congruent

2) The angles of a triangle add to equal 180°.

ΔLON

$118+22+x=180$

$140+x=180$

$-140 \quad -140$

$x=40$

Volume

Volume = (Area of the base)(height), if it comes to a point, multiply by $\frac{1}{3}$.

Area of the base is USUALLY $A = lw$ (rectangle/square) or $A = \pi r^2$ (circle)

Most volume formulas are on the reference sheet. Be careful. B = area of the base
General Prism: $V = (area\ base)(height)$
Rectangular prism: $V = lwh$
Cylinder: $V = \pi r^2 h$
Pyramid: $V = \frac{1}{3} lwh$
Cone: $V = \frac{1}{3}\pi r^2 h$
Sphere: $V = \frac{4}{3}\pi r^3$

1. A cylinder has a diameter of 10 inches and a height of 2.3 inches. What is the volume of this cylinder, to the *nearest tenth of a cubic inch*?

$V = \pi r^2 h$
$V = \pi(5)^2(2.3)$
$V = 180.6\ in^3$

2. What is the volume of a rectangular prism whose length is 4 cm, width is 6 cm, and height is 5 cm?

$V = lwh$
$V = 4(6)(5)$
$V = 120\ cm^3$

3. What is the volume of a cube if each side of the cube measures 8 in?

$V = lwh$
$V = 8(8)(8)$
$V = 512\ in^3$

4. What is the volume of a cylinder whose height is 12 inches and whose diameter is 20 inches in terms of π?

$V=\pi r^2 h$

$V=\pi(10)^2(12)$

$V=1200\pi$

5. Find the volume of a sphere that has a diameter of 12 in in terms of π.

$V=\frac{4}{3}\pi r^3$

$V=\frac{4}{3}\pi(6)^3$

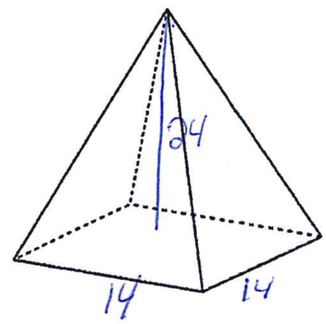
$V=288\pi$

6. A regular pyramid has a square base with an edge length of 14 and an altitude of 24. Find its volume.

$V=\frac{1}{3}lwh$

$V=\frac{1}{3}(14)(14)(24)$

$V=1568$ units^3



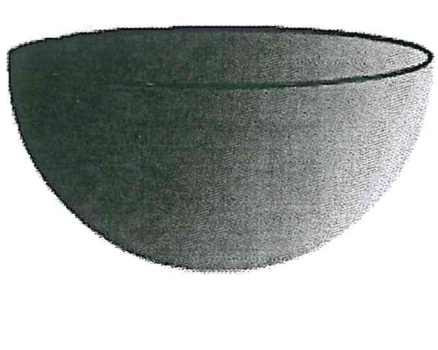
7. Find the volume of a cone with a height of 12 in and a diameter of 8 in rounded to the nearest hundredth.

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi(4)^2(12)$

$V=201.06$ in^3

8. Find the volume of the object below if the diameter is 18.2 meters. Round your answer to the *nearest cubic meter.*



hemisphere

$V=\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V=\frac{1}{2}\left(\frac{4}{3}\pi(9.1)^3\right)$

$V=1578$ m^3

Volume with Algebra
Substitute into appropriate volume formula
Solve the equation
***To get rid of a fraction, multiply by the denominator**
***To get rid of cubed, take the cubed root (final step)**

1. A brick in the shape of a rectangular prism has a base that measures 3 inches by 5 inches. If the volume of the brick is 90 cubic inches, what is the height of the brick?

$V = lwh$

$90 = 3(5)(x)$

$\frac{90}{15} = \frac{\cancel{15}x}{\cancel{15}}$

$6 = x$

2. A right circular cylinder has a volume of 1,000 cubic inches and a height of 8 inches. What is the radius of the cylinder to the *nearest tenth of an inch*?

$V = \pi r^2 h$

$\frac{1000}{8\pi} = \frac{\cancel{\pi} r^2 (8)}{\cancel{8\pi}}$

$\sqrt{39.7..} = \sqrt{r^2}$

$6.3 = r$

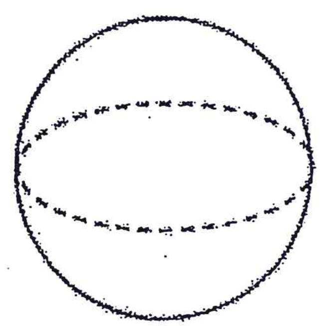
3. The base of a pyramid is a rectangle with a width of 6 cm and a length of 8 cm. Find, in centimeters, the height of the pyramid if the volume is 288 cm^3.

$V = \frac{1}{3} lwh$ $\qquad x = 18$

$288 = \frac{1}{3}(6)(8)(x)$

$\frac{288}{16} = \frac{16x}{16}$

4. Find the radius of a sphere with a volume of 576π cubic units. Find the answer to the *nearest tenth of a unit.* ____________



$V = \frac{4}{3}\pi r^3$ $\qquad r = 7.6$

$3\left(576\pi = \frac{4}{\cancel{3}}\pi r^3\right)\cancel{3}$

$\frac{1728\cancel{\pi}}{4\pi} = \frac{4\cancel{\pi} r^3}{4\pi}$

$\sqrt[3]{432} = \sqrt[3]{r^3}$

5. The volume of a cylinder is 12,566.4 cm^3. The height of the cylinder is 8 cm. Find the radius of the cylinder to the *nearest tenth of a centimeter.*

$V = \pi r^2 h$

$\frac{12566.4}{8\pi} = \frac{\pi r^2(8)}{8\pi}$

$\sqrt{500..} = \sqrt{r^2}$

$22.4 = r$

6. The Parkside Packing Company needs a rectangular shipping box. The box must have a length of 11 inches and a width of 8 inches. Find, to the *nearest tenth of an inch,* the minimum height of the box such that the volume is *at least* 800 cubic inches.

$V = lwh$

$800 = 11(8)(x)$

$\frac{800}{88} = \frac{88x}{88}$

$9.1 = x$

7. If the volume of a sphere is 36π, what is the radius of the sphere?

(1) 3 (2) 6 (3) 12 (4) 24

$V = \frac{4}{3}\pi r^3$

$3(36\pi) = (\frac{4}{3}\pi r^3)3$

$\frac{108\pi}{4\pi} = \frac{4\pi r^3}{4\pi}$

$\sqrt[3]{27} = \sqrt[3]{r^3}$

$3 = r$

8. Find the length of the radius of a cylinder to the *nearest tenth* if it has a volume of 60 cm^3 and a height of 10 cm.

$V = \pi r^2 h$

$\frac{60}{10\pi} = \frac{\pi r^2(10)}{10\pi}$

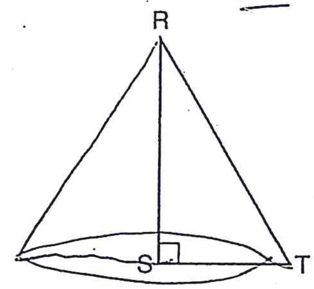
$\sqrt{1.9..} = \sqrt{r^2}$

$1.4 = r$

3 dimensional rotations ALMOST ALWAYS form a cylinder or cone

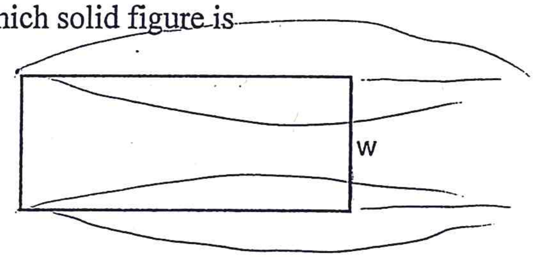
Reflect the shape in 2 dimensions and connect the images with curves

1. Which object is formed when right triangle *RST* shown below is rotated around leg $\overline{RS}$?



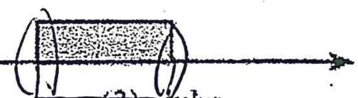
1) a pyramid with a square base
2) an isosceles triangle
3) a right triangle
4) a cone

2. If the rectangle below is continuously rotated about side *w*, which solid figure is formed?



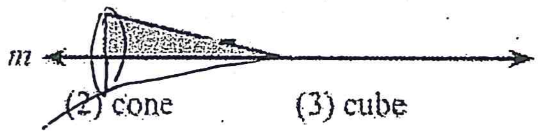
1) pyramid
2) rectangular prism
3) cone
4) cylinder

3. If you rotated the shaded figure below about line *m*, which solid would result from the revolution?



(1) cylinder (2) cone (3) cube (4) sphere

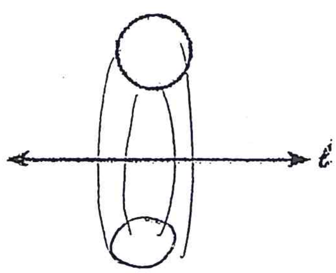
4. If you rotated the triangular region of the figure below about line *m*, what solid would result from the revolution?



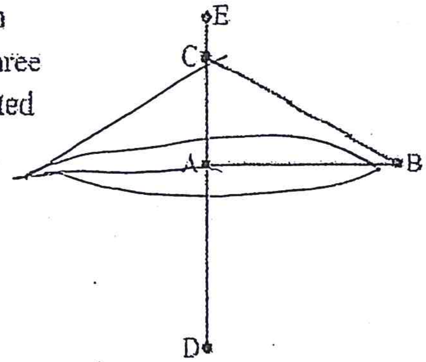
(1) cylinder (2) cone (3) cube (4) sphere

5. What shape will be formed if the circle in the graph is rotated continuously about the line ℓ?

(1) a sphere
(2) a donut
(3) a cylinder
(4) a cone



6. ΔABC is shown in the diagram to the right. If $m\angle A = 90$, what three dimensional object will be generated if ΔABC is rotated about $\overline{ED}$?

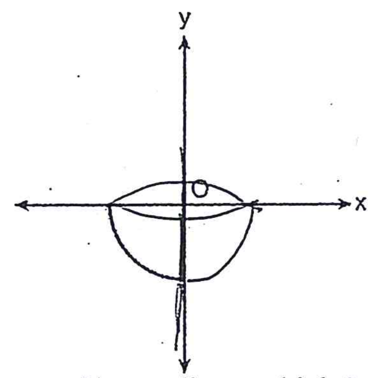


(1) another right triangle
(2) a cylinder
(3) a cone
(4) a pyramid

7. Circle O is centered at the origin. In the diagram below, a quarter of circle O is graphed.

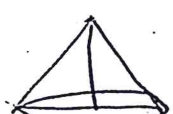
Which three-dimensional figure is generated when the quarter circle is continuously rotated about the y-axis?

1) cone
2) sphere
3) cylinder
4) hemisphere



8. If an equilateral triangle is continuously rotated around one of its medians, which 3-dimensional object is generated?

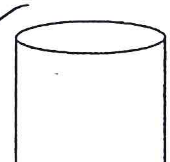
1) cone
2) pyramid
3) prism
4) sphere

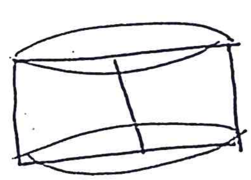


9. A student has a rectangular postcard that he folds in half lengthwise. Next, he rotates it continuously about the folded edge. Which three-dimensional object below is generated by this rotation?

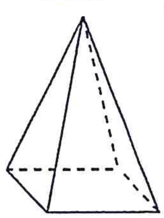
1)

3)

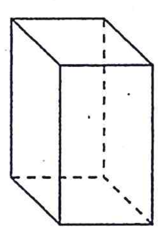




2)

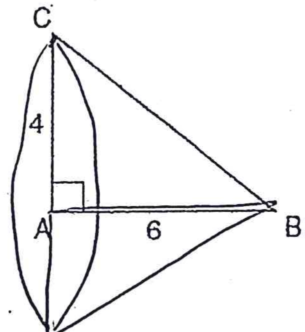


4)



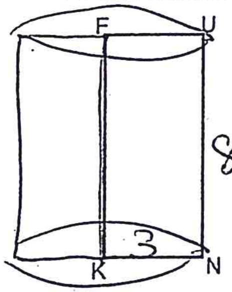
10. In the diagram below, right triangle ABC has legs whose lengths are 4 and 6. What is the volume of the three-dimensional object formed by continuously rotating the right triangle around $\overline{AB}$?

~~1)~~ 32π 3) 96π
2) 48π 4) 144π



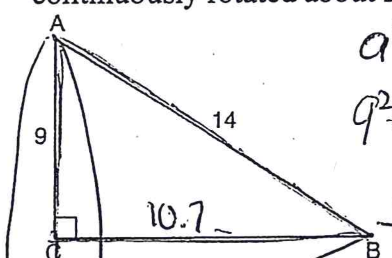
$V = \frac{1}{3}\pi r^2 h$
$V = \frac{1}{3}\pi(4)^2(6)$
$V = 32\pi$

11. In the rectangle below, $\overline{UN} = 8\,in$ and $\overline{KN} = 3\,in$. Find the volume of the three dimensional object created by rotating rectangle FUNK continuously about side $\overline{FK}$



$V = \pi r^2 h$
$V = \pi(3)^2(8)$
$V = 72\pi$

12. In the diagram of right triangle ABC shown below, $AB = 14$ and $AC = 9$. What is the volume of the three dimensional object formed when the triangle is continuously rotated about side $\overline{BC}$



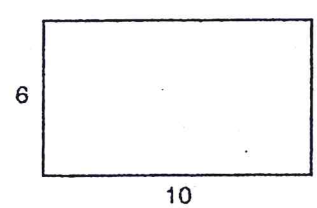
$a^2+b^2=c^2$
$9^2+x^2=14^2$
$81+x^2=196$
$-81 \quad -81$
$\sqrt{x^2}=\sqrt{115}$
$x = 10.7..$

$V = \frac{1}{3}\pi r^2 h$
$V = \frac{1}{3}\pi(9)^2(10.7..)$
$V \approx 910$

13. A rectangle whose length and width are 10 and 6, respectively, is shown below. The rectangle is continuously rotated around a straight line to form an object whose volume is 150π.

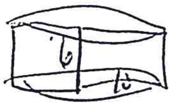
Which line could the rectangle be rotated around?

1) a long side ~~3)~~ the vertical line of symmetry
2) a short side 4) the horizontal line of symmetry



1)


$V = \pi(6)^2(10)$
$V = 360\pi$

2)


$V = \pi(10)^2(6)$
$V = 600\pi$

3)


$V = \pi(5)^2(6)$
$V = 150\pi$

4)


$V = \pi(3)^2(10)$
$V = 90\pi$

Finding Center and Radius of a Circle Using Completing the Square

$(x-a)^2+(y-b)^2=r^2$ where (a,b) is the center and r is the radius

To put into center-radius form: COMPLETE THE SQUARE TWICE

To find center: Negate what is in the parenthesis. If there are no parentheses, the coordinate is 0.

Radius is the square root of the right hand side

Completing the Square

1) Write the x's together, y's together, and move constant to the other side

$$x^2+bx+y^2+by=c$$

2) Add $\left(\frac{b}{2}\right)^2$ to both sides for each variable

3) Factor each trinomial (Both factors must be the same)

4) Rewrite the factors as a binomial squared

$\left(\frac{16}{2}\right)^2=64$

1. $x^2+y^2+16x+6y+\cancel{9}=0$ $\left(\frac{6}{2}\right)^2=9$

$-9 \quad -9$

$x^2+16x+y^2+6y=-9$

$x^2+16x+\boxed{64}+y^2+6y+\boxed{9}=-9+\boxed{64}+\boxed{9}$

$(x+8)(x+8)+(y+3)(y+3)=64$

$(x+8)^2+(y+3)^2=64$

center: $(-8,-3)$

$r=8$

$\left(\frac{-12}{2}\right)^2=36$ $\left(\frac{-14}{2}\right)^2=49$

2. $x^2+y^2-12x-14y=15$

$x^2-12x+y^2-14y=15$

$x^2-12x+\boxed{36}+y^2-14y+\boxed{49}=15+\boxed{36}+\boxed{49}$

$(x-6)(x-6)+(y-7)(y-7)=100$

$(x-6)^2+(y-7)^2=100$

Center: $(6,7)$

$r=10$

$\left(\frac{-4}{2}\right)^2=4$ $\left(\frac{8}{2}\right)^2=16$

3. $x^2+8y+\cancel{10}+y^2-4x=6$

$-10 \quad -10$

$x^2-4x+y^2+8y=-4$

$x^2-4x+\boxed{4}+y^2+8y+\boxed{16}=-4+\boxed{4}+\boxed{16}$

$(x-2)(x-2)+(y+4)(y+4)=16$

$(x-2)^2+(y+4)^2=16$

center: $(2,-4)$

$r=4$

$\left(\frac{4}{2}\right)^2=4$ $\left(\frac{-2}{2}\right)^2=1$

4. $x^2+4x+12+y^2-2y-1=22$

$x^2+4x+y^2-2y+\cancel{11}=22$

$-11 \quad -11$

$x^2+4x+y^2-2y=11$

$x^2+4x+\boxed{4}+y^2-2y+\boxed{1}=11+\boxed{4}+\boxed{1}$

$(x+2)(x+2)+(y-1)(y-1)=16$

$(x+2)^2+(y-1)^2=16$

Center: $(-2,1)$

$r=4$

$(-\frac{16}{2})^2=64$ $(\frac{6}{2})^2=9$

5. What are the coordinates of the center of a circle whose equation is $x^2+y^2-16x+6y+\cancel{53}=0$?

1) $(-8,-3)$
2) $(-8,3)$
3) $(8,-3)$ ✓
4) $(8,3)$

$\cancel{-53}$ -53

$x^2-16x+y^2+6y=-53$

$x^2-16x+\boxed{64}+y^2+6y+\boxed{9}=-53+\boxed{64}+\boxed{9}$

$(x-8)(x-8)+(y+3)(y+3)=20$

$(x-8)^2+(y+3)^2=20$

$(8,-3)$ $r=\sqrt{20}$

6. The equation of a circle is $x^2+y^2+6y=7$. What are the coordinates of the center and the length of the radius of the circle?

1) center $(0,3)$ and radius 4
2) center $(0,-3)$ and radius 4 ✓
3) center $(0,3)$ and radius 16
4) center $(0,-3)$ and radius 16

$(\frac{6}{2})^2=9$

$x^2+y^2+6y+\boxed{9}=7+\boxed{9}$

$x^2+(y+3)(y+3)=16$

$x^2+(y+3)^2=16$

$(0,-3)$ $r=4$

7. What are the coordinates of the center and length of the radius of the circle whose equation is $x^2+6x+y^2-4y=23$?

1) $(3,-2)$ and 36
2) $(3,-2)$ and 6
3) $(-3,2)$ and 36
4) $(-3,2)$ and 6 ✓

$(\frac{6}{2})^2=9$

$(\frac{-4}{2})^2=4$

$x^2+6x+\boxed{9}+y^2-4y+\boxed{4}=23+\boxed{9}+\boxed{4}$

$(x+3)(x+3)+(y-2)(y-2)=36$

$(x+3)^2+(y-2)^2=36$

$(-3,2)$ $r=6$

$r=5$

Since the coordinates are both positive, the x and y coefficients must both be negative.

8. What is an equation of a circle whose center is $(1,4)$ and diameter is 10?

1) $x^2-2x+y^2-8y=8$ ✓
2) ~~$x^2+2x+y^2+8y=8$~~
3) $x^2-2x+y^2-8y=83$
4) ~~$x^2+2x+y^2+8y=83$~~

$(\frac{-2}{2})^2=1$

$(\frac{-8}{2})^2=16$

1) $x^2-2x+\boxed{1}+y^2-8y+\boxed{16}=8+\boxed{1}+\boxed{16}$

$(x-1)^2+(y-4)^2=25$

$(1,4)$ $r=5$ ✓

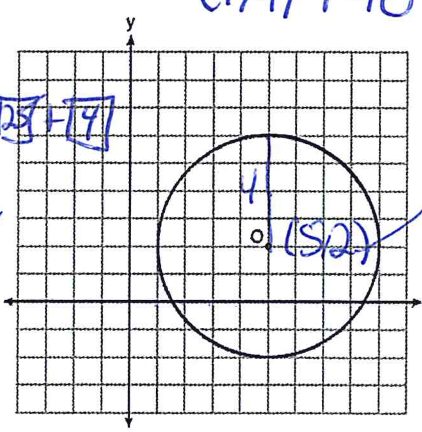
3) $x^2-2x+\boxed{1}+y^2-8y+\boxed{16}=83+\boxed{1}+\boxed{16}$

$(x-1)^2+(y-4)^2=100$

$(1,4)$ $r=10$ ✗

9. What is an equation of circle O shown in the graph below?

1) ~~$x^2+10x+y^2+4y=-13$~~
2) $x^2-10x+y^2-4y=-13$ ✓
3) ~~$x^2+10x+y^2+4y=-25$~~
4) $x^2-10x+y^2-4y=-25$



$(\frac{-10}{2})^2=25$

$(\frac{-4}{2})^2=4$

2) $x^2-10x+\boxed{25}+y^2-4y+\boxed{4}=-13+\boxed{25}+\boxed{4}$

$(x-5)^2+(y-2)^2=16$

$(5,2)$ $r=4$ ✓

4) $x^2-10x+\boxed{25}+y^2-4y+\boxed{4}=-25+\boxed{25}+\boxed{4}$

$(x-5)^2+(y-2)^2=4$

$(5,2)$ $r=2$ ✗

Line Dilations

THE IMAGE IS ALWAYS PARALLEL! SLOPE IS ALWAYS THE SAME!

Conceptual:

Determine if the point is on the line by substituting the x and y coordinates into the equation of the line.

If the point is on the line: Same y intercept (Exact same equation).

If the point is on the line: Different y intercept.

Writing the equation:

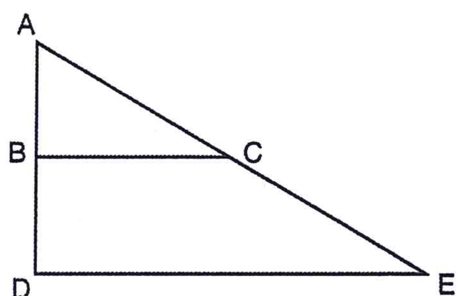
If center is origin: Multiply scale factor and original b to find new b

If center is on the line: The image is the same equation as the original.

1. A line segment is dilated by a scale factor of 2 centered at a point not on the line segment. Which statement regarding the relationship between the given line segment and its image is true?

1) The line segments are perpendicular, and the image is one-half of the length of the given line segment.
2) The line segments are perpendicular, and the image is twice the length of the given line segment.
3) The line segments are parallel, and the image is twice the length of the given line segment.
4) The line segments are parallel, and the image is one-half of the length of the given line segment.

2. The image of $\triangle ABC$ after a dilation of scale factor k centered at point A is $\triangle ADE$, as shown in the diagram below.



Which statement is always true?

1) $2AB = AD$
2) $\overline{AD} \perp \overline{DE}$
3) $AC = CE$
4) $\overline{BC} \parallel \overline{DE}$

$3(0) - 5(0) = 4$

$0 = 4$ x dif y-intercept

3. The line whose equation is $3x - 5y = 4$ is dilated by a scale factor of $\frac{5}{3}$ centered at the

(0,0) origin. Which statement is correct?

1) The image of the line has the same slope as the pre-image but a different y-intercept.
2) The image of the line has the same y-intercept as the pre-image but a different slope.
3) The image of the line has the same slope and the same y-intercept as the pre-image.
4) The image of the line has a different slope and a different y-intercept from the pre-image.

4. A line that passes through the points whose coordinates are (1, 1) and (5, 7) is dilated by a scale factor of 3 and centered at the origin. The image of the line

1) is perpendicular to the original line
2) is parallel to the original line
3) passes through the origin
4) is the original line

The origin is not on that line.

$\to$ same slope

5. The line $y = -5x - 1$ is dilated by a scale factor of 2 and centered at the origin. Write an equation that represents the image of the line after the dilation.

multiply scale factor and b

$b = 2(-1) = -2$
$m = -5$
$y = -5x - 2$

$\to$ same slope

6. The line $y = -2x + 4$ is dilated by a scale factor of $\frac{5}{2}$ and centered at the origin. Write an equation that represents the image of the line after the dilation.

multiply scale factor and b

$b = \frac{5}{2}(4) = 10$
$m = -2$
$y = -2x + 10$

$\to$ same slope

7. The line $y = 2x - 4$ is dilated by a scale factor of $\frac{3}{2}$ and centered at the origin. Which equation represents the image of the line after the dilation?

multiply scale factor and b

1) $y = 2x - 4$
2) $y = 2x - 6$
3) $y = 3x - 4$
4) $y = 3x - 6$

$b = \frac{3}{2}(-4) = -6$
$m = 2$
$y = 2x - 6$

$\to$ same slope

8. What is an equation of the image of the line $y = \frac{3}{2}x - 4$ after a dilation of a scale factor of $\frac{3}{4}$ centered at the origin?

1) $y = \frac{9}{8}x - 4$
2) $y = \frac{9}{8}x - 3$
3) $y = \frac{3}{2}x - 4$
4) $y = \frac{3}{2}x - 3$

multiply scale factor and b

$b = \frac{3}{4}(-4) = -3$
$m = \frac{3}{2}$
$y = \frac{3}{2}x - 3$

$2x + y = 1$
$-2x \quad -2x$
$y = -2x + 1$

Same slope

9. The equation of line h is $2x + y = 1$. Line m is the image of line h after a dilation of scale factor 4 with respect to the origin. What is the equation of the line m?

1) $y = -2x + 1$
2) ~~2)~~ $y = -2x + 4$
3) $y = 2x + 4$
4) $y = 2x + 1$

multiply scale factor and b

$$\boxed{\begin{array}{c} b = 4(1) = 4 \\ m = -2 \end{array}}$$

$y = -2x + 4$

on the line?

10. Line $y = 3x - 1$ is transformed by a dilation with a scale factor of 2 and centered at $(3, 8)$. The line's image is

1) $y = 3x - 8$
2) $y = 3x - 4$
3) $y = 3x - 2$
4) ~~4)~~ $y = 3x - 1$

$8 = 3(3) - 1$
$8 = 8$ ✓
Same slope
Same y int

on the line?

11. Line MN is dilated by a scale factor of 2 centered at the point $(0, 6)$. If $\overleftrightarrow{MN}$ is represented by $y = -3x + 6$, which equation can represent $\overleftrightarrow{M'N'}$, the image of $\overleftrightarrow{MN}$?

1) $y = -3x + 12$
2) ~~2)~~ $y = -3x + 6$
3) $y = -6x + 12$
4) $y = -6x + 6$

$6 = -3(0) + 6$
$6 = 6$ ✓

same slope
same y int

12. Line n is represented by the equation $3x + 4y = 20$. Determine and state the equation of line p, the image of line n, after a dilation of scale factor $\frac{1}{3}$ centered at the point $(4, 2)$. Explain your answer.

on the line?

If the center of dilation is on the line, the image is the same line.

$3(4) + 4(2) = 20$
$12 + 8 = 20$
$20 = 20$ ✓
Same slope
Same y int

line p
$3x + 4y = 20$

on the line?

13. Aliyah says that when the line $4x + 3y = 24$ is dilated by a scale factor of 2 centered at the point $(3, 4)$, the equation of the dilated line is $y = -\frac{4}{3}x + 16$. Is Aliyah correct? Explain why.

$4x + 3y = 24$
$-4x \quad -4x$

$\frac{3y}{3} = \frac{-4x + 24}{3}$

$y = -\frac{4}{3}x + 8$

$4 = -\frac{4}{3}(3) + 8$
$4 = 4$ ✓

Same slope
Same y-int.

No, the center of dilation is on the line so the image is the same line, which is not $y = -\frac{4}{3}x + 16$.

Dilating Segments with Perimeter and Area

Multiply the original segment and scale factor to find the image.
Multiply the original perimeter and scale factor to find the image perimeter.
Multiply the original area and the $(scale\ \ factor)^2$ to find the image area.
*You may have to use distance formula to find original segment.
*The center of dilation does not effect the size of the image

1. A line segment with a length of 5 is dilated by a scale factor of 4. What is the length of its image?

multiply

$5(4)=20$

2. A line segment has a length of 12 and is dilated by $\frac{1}{2}$. What is the length of its image?

multiply $\quad 12(\frac{1}{2})=6$

3. A three-inch line segment is dilated by a scale factor of 6 and centered at its midpoint. What is the length of its image? multiply $\quad 3(6)=18 \quad$ illelevant for Length

1) 9 inches
2) 2 inches
3) 15 inches
4) 18 inches

4. The coordinates of the endpoints of $\overline{AB}$ are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of $\overline{AB}$, after a dilation of $\frac{1}{2}$ centered at the origin.

Irrelevant for Length.

multiply

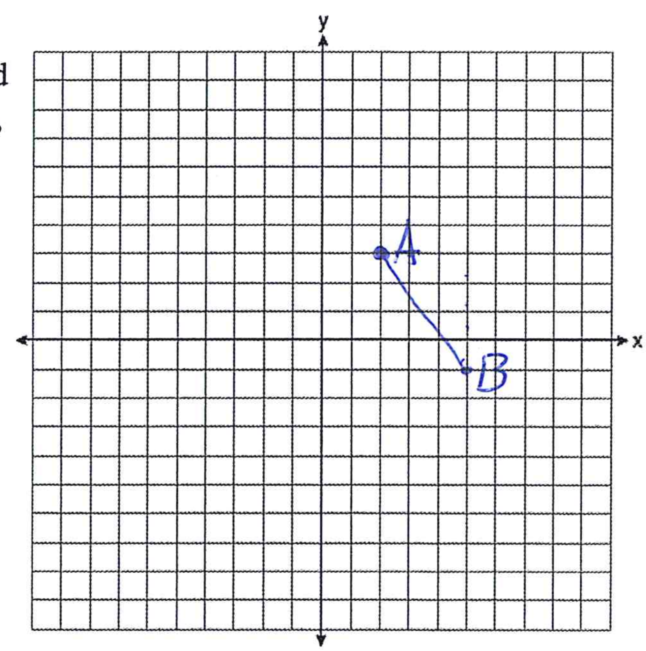
$d\overline{AB}=\sqrt{3^2+4^2}$

$d\overline{AB}=\sqrt{9+16}$

$d\overline{AB}=\sqrt{25}$

$d\overline{AB}=5$

$5(\frac{1}{2})=\frac{5}{2}$



5. Line segment $A'B'$, whose endpoints are $(4,-2)$ and $(16,14)$, is the image of $\overline{AB}$ after a dilation of $\frac{1}{2}$ centered at the origin. What is the length of $\overline{AB}$?

irrelevant

$d=\sqrt{\Delta x^2+\Delta y^2}$

$d=\sqrt{12^2+16^2}$

$d=\sqrt{144+256}$

$d=\sqrt{400}$

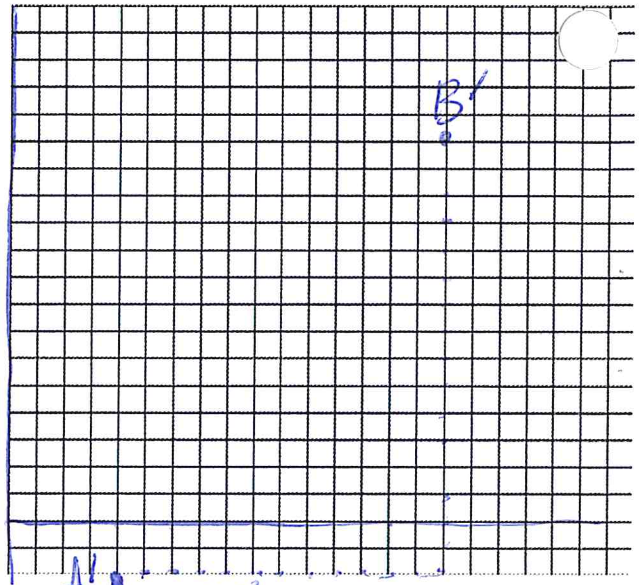
$d=20$

$\overline{AB}=40$

$\frac{1}{2}AB=A'B'$

$2(\frac{1}{2}AB)=(20)2$

$AB=40$



6. Given square $RSTV$, where $RS = 9$ cm. If square $RSTV$ is dilated by a scale factor of 3 about a given center, what is the perimeter, in centimeters, of the image of $RSTV$ after the dilation?

multiply

irrelevant

1) 12
2) 27
3) 36
4) 108

$9(3)=27$

27 27 27 27

$27(4)=108$

7. Triangle RJM has an area of 6 and a perimeter of 12. If the triangle is dilated by a scale factor of 3 centered at the origin, what are the area and perimeter of its image, triangle $R'J'M'$?

multiply

(scale factor)^2

1) area of 9 and perimeter of 15
2) area of 18 and perimeter of 36
3) area of 54 and perimeter of 36
4) area of 54 and perimeter of 108

$p=$ scale factor(perimeter) $=3(12)=36$

$a=$ (scale factor)^2(area) $=9(6)=54$

8. Rectangle $A'B'C'D'$ is the image of rectangle $ABCD$ after a dilation centered at point A by a scale factor of $\frac{2}{3}$. Which statement is correct?

1) Rectangle $A'B'C'D'$ has a perimeter that is $\frac{2}{3}$ the perimeter of rectangle $ABCD$.

2) Rectangle $A'B'C'D'$ has a perimeter that is $\frac{3}{2}$ the perimeter of rectangle $ABCD$.

3) Rectangle $A'B'C'D'$ has an area that is $\left(\frac{2}{3}\right)^2$ the area of rectangle $ABCD$.

4) Rectangle $A'B'C'D'$ has an area that is $\frac{3}{2}$ the area of rectangle $ABCD$.

Equation of a line through a point flip and negate

1) Find m using parallel (same slope) or perpendicular (negative reciprocal slopes).
2) Substitute into $y - y_1 = m(x - x_1)$. Don't forget to negate x_1 and y_1.
3) If it's multiple choice, you may have to distribute and isolate y.

1. What is the equation of a line that passes through the point $(-3,-11)$ (x_1, y_1) and is parallel to the line whose equation is $2x - y = 4$? same slope

1) $y = 2x + 5$
2) $y = 2x - 5$
3) $y = \frac{1}{2}x + \frac{25}{2}$
4) $y = -\frac{1}{2}x - \frac{25}{2}$

$-2x \quad -2x$
$\frac{-y}{-1} = \frac{-2x+4}{-1}$
$y = 2x - 4$
$m = 2$

$m\parallel = 2$
$x_1 = -3$
$y_1 = -11$

$y - y_1 = m(x - x_1)$
$y + 11 = 2(x + 3)$
$y + 11 = 2x + 6$
$\quad -11 \quad -11$
$y = 2x - 5$

2. What is an equation of the line that passes through the point $(-2, 5)$ (x_1, y_1) and is perpendicular to the line whose equation is $y = \frac{1}{2}x + 5$? negative reciprocal slopes

$m = \frac{1}{2}$

1) $y - 5 = \frac{1}{2}(x + 2)$
2) $y - 5 = -2(x + 2)$
3) $y + 5 = \frac{1}{2}(x - 2)$
4) $y + 5 = -2(x - 2)$

$y - y_1 = m(x - x_1)$
$y - 5 = -2(x + 2)$

$m\perp = -2$
$x_1 = -2$
$y_1 = 5$

3. What is an equation of the line that contains the point $(3, -1)$ (x_1, y_1) and is perpendicular to the line whose equation is $y = -3x + 2$? negative reciprocal slopes

1) $y = -3x + 8$
2) $y = -3x$
3) $y = \frac{1}{3}x$
4) $y = \frac{1}{3}x - 2$

$m = -3$

$m\perp = \frac{1}{3}$
$x_1 = 3$
$y_1 = -1$

$y - y_1 = m(x - x_1)$
$y + 1 = \frac{1}{3}(x - 3)$
$y + 1 = \frac{1}{3}x - 1$
$\quad -1 \quad -1$
$y = \frac{1}{3}x - 2$

4. An equation of the line that passes through $(2, -1)$ (x_1, y_1) and is parallel to the line $2y + 3x = 8$ is same slope

1) $y + 1 = -\frac{3}{2}(x - 2)$
2) $y + 1 = \frac{2}{3}(x - 2)$
3) $y - 1 = -\frac{3}{2}(x + 2)$
4) $y - 1 = \frac{2}{3}(x + 2)$

$-3x \quad -3x$
$\frac{2y}{2} = \frac{-3x+8}{2}$
$y = -\frac{3}{2}x + 4$
$m = -\frac{3}{2}$

$m\parallel = -\frac{3}{2}$
$x_1 = 2$
$y_1 = -1$

$y - y_1 = m(x - x_1)$
$y + 1 = -\frac{3}{2}(x - 2)$

$m = 3/5$

negative reciprocal slopes

5. What is an equation of the line that is perpendicular to the line whose equation is $y = \frac{3}{5}x - 2$ and that passes through the point $(3, -6)$?

$x_1 \; y_1$

$m\perp = -\frac{5}{3}$

$x_1 = 3$

$y_1 = -6$

1) $y = \frac{5}{3}x - 11$

2) $y = -\frac{5}{3}x + 11$

~~3)~~ $y = -\frac{5}{3}x - 1$

4) $y = \frac{5}{3}x + 1$

$y - y_1 = m(x - x_1)$

$y + 6 = -\frac{5}{3}(x - 3)$

$y + \cancel{6} = -\frac{5}{3}x + 5$
$\quad -6 \qquad\qquad -6$

$y = -\frac{5}{3}x - 1$

6. The equation of a line is $y = \frac{2}{3}x + 5$. What is an equation of the line that is perpendicular to the given line and that passes through the point $(4, 2)$?

Negative reciprocal slopes

$x_1 \; y_1$

1) $y = \frac{2}{3}x - \frac{2}{3}$

2) $y = \frac{3}{2}x - 4$

3) $y = -\frac{3}{2}x + 7$

~~4)~~ $y = -\frac{3}{2}x + 8$

$\boxed{m = \frac{2}{3}}$

$y - y_1 = m(x - x_1)$ $\quad m\perp = -\frac{3}{2}$

$y - 2 = -\frac{3}{2}(x - 4)$ $\quad x_1 = 4$, $y_1 = 2$

$y - \cancel{2} = -\frac{3}{2}x + 6$
$\quad +2 \qquad\qquad +2$

$y = -\frac{3}{2}x + 8$

7. What is an equation of the line that passes through the point $(6, 8)$ and is perpendicular to a line with equation $y = \frac{3}{2}x + 5$?

$x_1 \; y_1$

$m = \frac{3}{2}$

negative reciprocal slopes

1) $y - 8 = \frac{3}{2}(x - 6)$

~~2)~~ $y - 8 = -\frac{2}{3}(x - 6)$

3) $y + 8 = \frac{3}{2}(x + 6)$

4) $y + 8 = -\frac{2}{3}(x + 6)$

$y - y_1 = m(x - x_1)$

$y - 8 = -\frac{2}{3}(x - 6)$

$m\perp = -\frac{2}{3}$

$x_1 = 6$

$y_1 = 8$

8. What is an equation of a line which passes through $(6, 9)$ and is perpendicular to the line whose equation is $4x - 6y = 15$?

$x_1 \; y_1$

negative reciprocal slopes

$-4x \qquad -4x$

~~1)~~ $y - 9 = -\frac{3}{2}(x - 6)$

2) $y - 9 = \frac{2}{3}(x - 6)$

3) $y + 9 = -\frac{3}{2}(x + 6)$

4) $y + 9 = \frac{2}{3}(x + 6)$

$\frac{-6y}{-6} = \frac{-4x + 15}{-6}$

$y = \frac{2}{3}x - \frac{5}{2}$

$m\perp = -\frac{3}{2}$

$x_1 = 6$

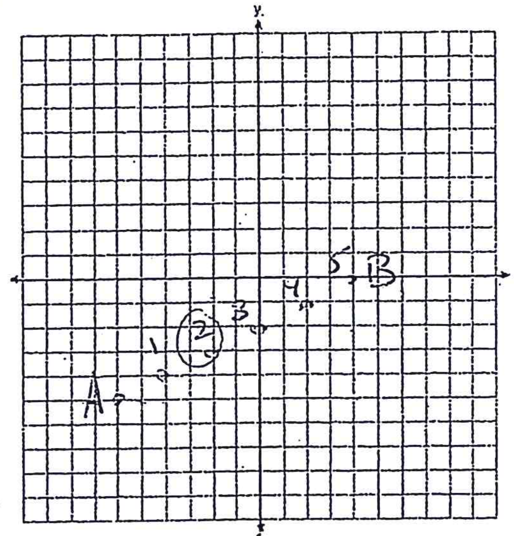
$y_1 = 9$

$y - y_1 = m(x - x_1)$

$y - 9 = -\frac{3}{2}(x - 6)$

Partitions

1) **Find** $\frac{\Delta x}{p}$ *and* $\frac{\Delta y}{p}$ where p is the number of partitions.
2) **Count those values out on the graph between the two endpoints**
3) **Circle and state the point that matches the given ratio.**
BE CAREFUL WHICH POINT YOU START FROM!



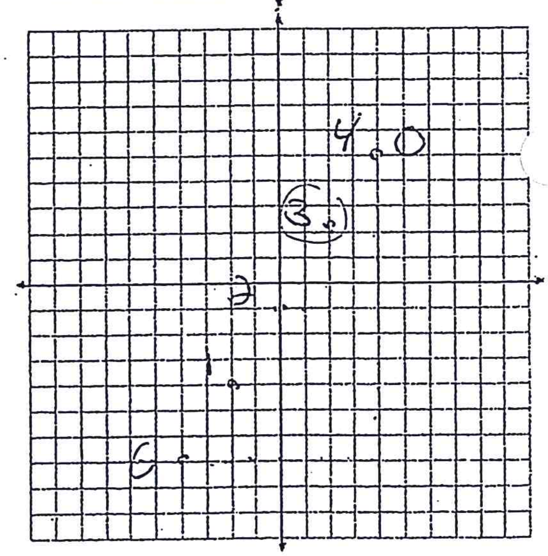
1. The coordinates of the endpoints of $\overline{AB}$ are $A(-6,-5)$ and $B(4,0)$. Point P is on $\overline{AB}$. Determine and state the coordinates of point P, such that $AP:PB$ is $2:3$. $p=5$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$

$\frac{10}{5}$ $\frac{5}{5}$

2 1

$(-2,-3)$



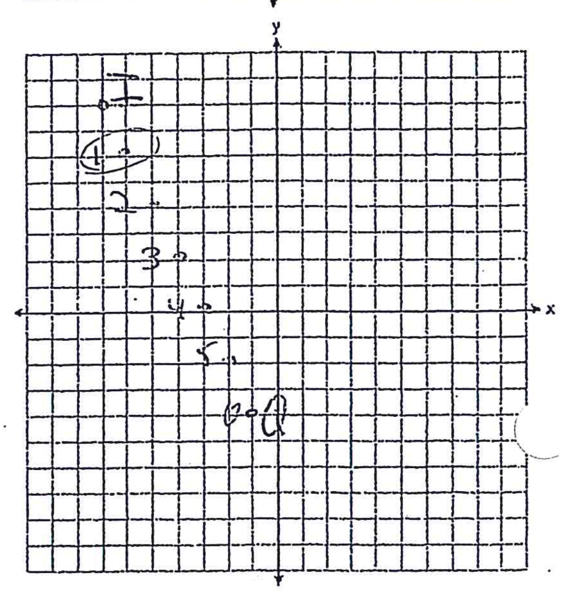
2. What are the coordinates of the point on the directed line segment from $G(-4,-7)$ to $O(4,5)$ that partitions the segment into a ratio of 3 to 1? $p=4$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$

$\frac{8}{4}$ $\frac{12}{4}$

2 3

$(2,2)$



3. Directed line segment IQ has endpoints whose coordinates are $I(-7,8)$ and $Q(-1,-4)$. Determine the coordinates of point J that divides the segment in the ratio 1 to 5. $p=6$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$

$\frac{6}{6}$ $\frac{12}{6}$

1 2

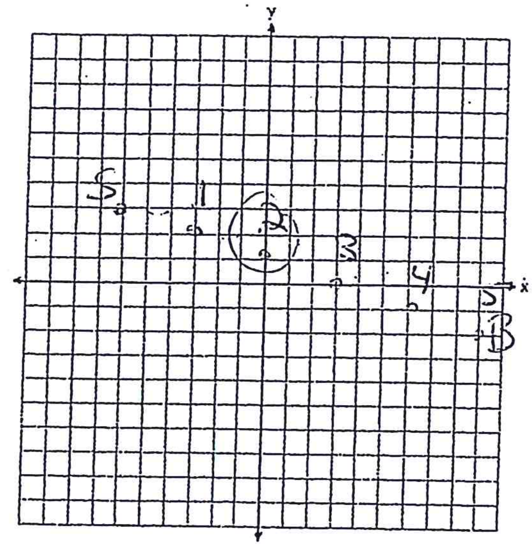
$(-6,6)$

4. Directed line segment SB has endpoints whose coordinates are $S(-6,3)$ and $B(9,-2)$. Determine the coordinates of point J that divides the segment in the ratio 2 to 3. $p=5$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$ $(0,1)$

$\frac{15}{5}$ $\frac{5}{5}$

3 1

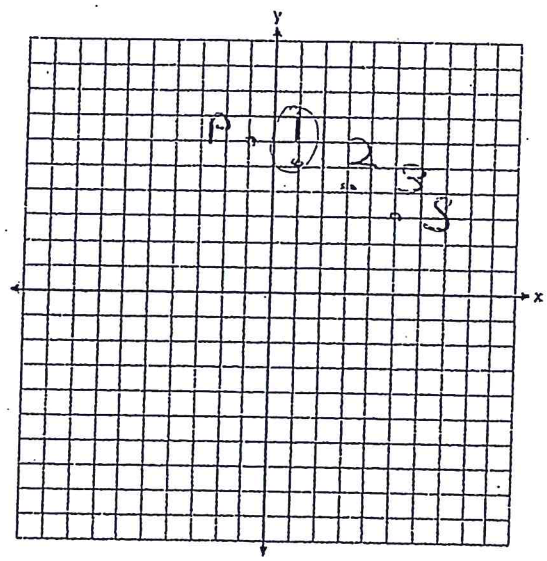


5. What are the coordinates of the point on the directed line segment from $P(-1,6)$ to $S(5,3)$ that partitions the segment into a ratio of 1 to 2? $P=3$

$\frac{\Delta x}{P}$ $\frac{\Delta y}{P}$ $(1,5)$

$\frac{6}{3}$ $\frac{3}{3}$

2 1

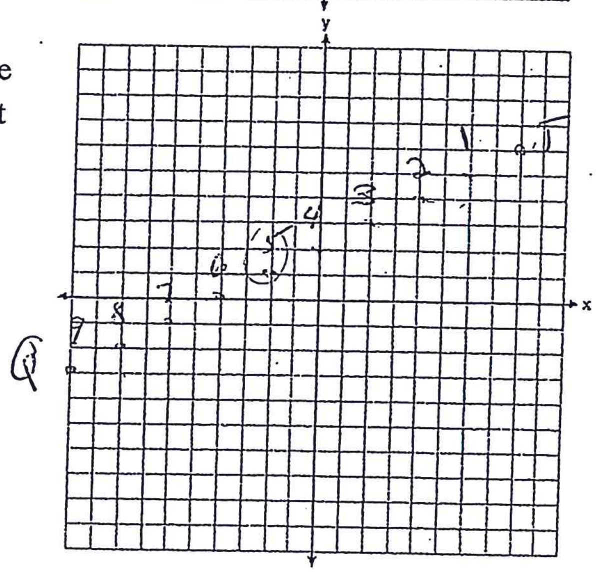


6. Directed line segment JK has endpoints whose coordinates are $J(8,6)$ and $Q(-10,-3)$. Determine the coordinates of point O that divides the segment in the ratio 5 to 4. $p=9$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$ $(-2,1)$

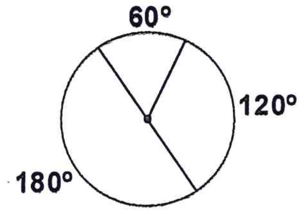
$\frac{18}{9}$ $\frac{9}{9}$

2 1

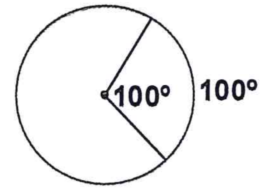


Circle Angle and Segment Rules:

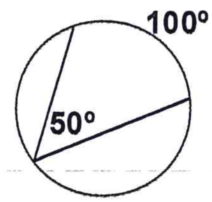
The arcs of a circle add to 360°
A diameter cuts a circle into 2 halves of 180° each



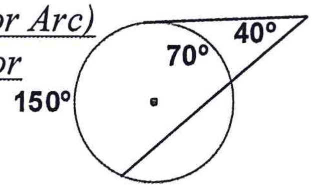
Central Angle: Has its vertex at the center of the circle
Central angle is equal to the measure of the intercepted arc

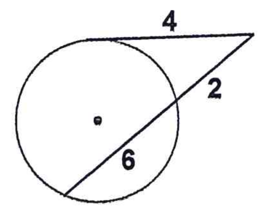


Inscribed Angle: Has its vertex on the circle
Inscribed angle is half of the measure of the intercepted arc

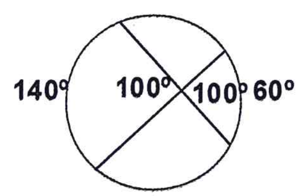


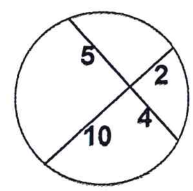
Exterior Angle:
Angles: *2(Exterior Angle) = (Major Arc – Minor Arc)*
Segments: *Whole • Exterior = Whole • Exterior*



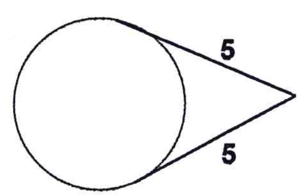


Intersecting Chords:
Angles: *2(Vertical Angle) – Arc + Arc*
Segments: *Part • Part = Part • Part*

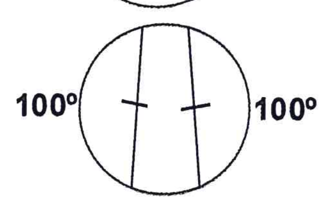




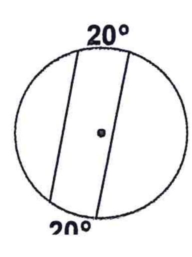
Two tangents drawn from the same point are congruent



Congruent chords intercept congruent arcs



Parallel chords intercept congruent arcs



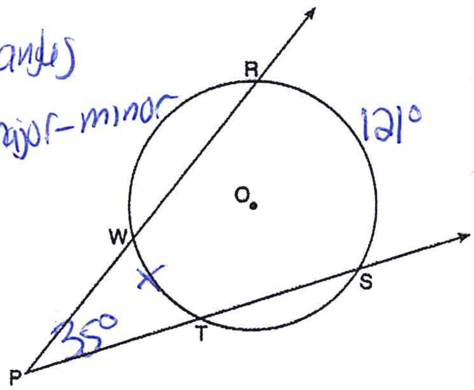
2(exterior angle) = major − minor

1. As shown in the diagram below, secants $\overrightarrow{PWR}$ and $\overrightarrow{PTS}$ are drawn to circle O from external point P.

If $m\angle RPS = 35°$ and $m\overset{\frown}{RS} = 121°$, determine and state $m\overset{\frown}{WT}$.

- exterior
- arcs and angles

2(EA) = major − minor



$2(EA) = major - minor$

$2(35) = 121 - x$

$70 = 121 - x$

$-121 \quad -121$

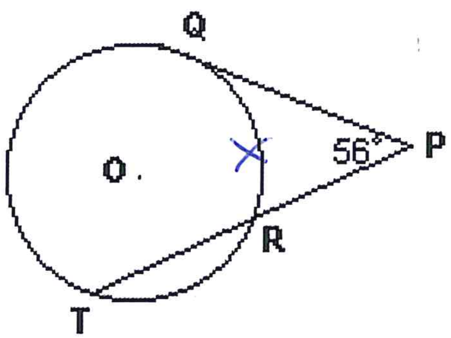
$\frac{-51}{-1} = \frac{-x}{-1}$

$51 = x$

2. In the diagram of circle O, $\overline{PQ}$ is tangent to O at Q and $\overline{PRT}$ is a secant. If $m\angle P = 56$ and $m\overset{\frown}{QT} = 192$, find $m\overset{\frown}{QR}$

- exterior
- arcs and angles

2(EA) = major − minor



$2(EA) = major - minor$

$2(56) = 192 - x$

$112 = 192 - x$

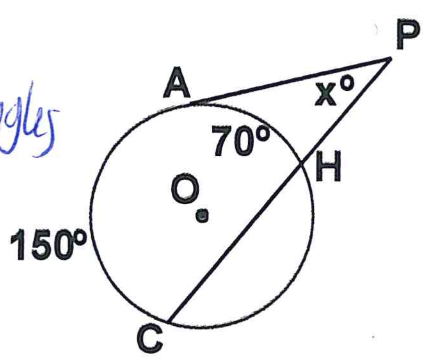
$-192 \quad -192$

$\frac{-80}{-1} = \frac{-x}{-1}$

$80 = x$

3. $\overset{\frown}{AC} = 150°$, $\overset{\frown}{AH} = 70°$, find $m\angle APH$

- exterior
- arcs and angles



$2(EA) = major - minor$

$2x = 150 - 70$

$\frac{2x}{2} = \frac{80}{2}$

$x = 40$

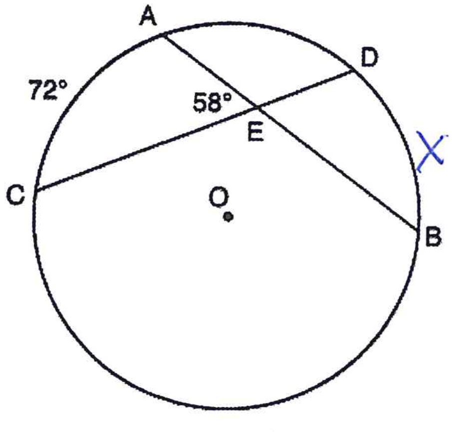
2(EA) = major − minor

2(vertical angle) = arc + arc

4. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $m\overset{\frown}{AC} = 72°$ and $m\angle AEC = 58°$, how many degrees are in $m\overset{\frown}{DB}$?

- interior
- arcs and angles

2(VA) = arc + arc



2(VA) = arc + arc

$2(58) = x + 72$

$116 = x + 72$

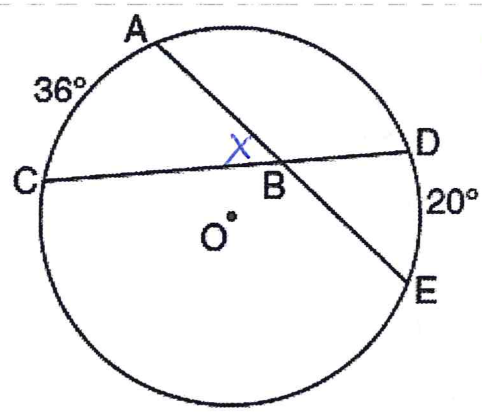
$-72 \quad -72$

$44 = x$

5. In the diagram below of circle O, chords $\overline{AE}$ and $\overline{DC}$ intersect at point B, such that $m\overset{\frown}{AC} = 36$ and $m\overset{\frown}{DE} = 20$. What is $m\angle ABC$?

- interior
- arcs and angles

2(VA) = arc + arc



2(VA) = arc + arc

$2x = 36 + 20$

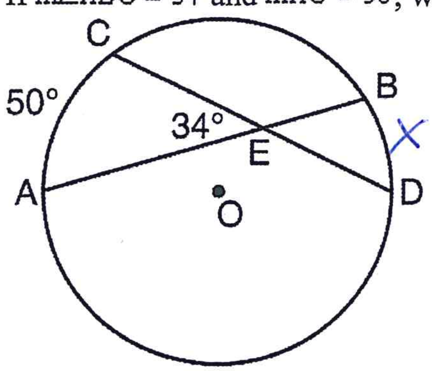
$\frac{2x}{2} = \frac{56}{2}$

$x = 28$

6. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $m\angle AEC = 34$ and $m\overset{\frown}{AC} = 50$, what is $m\overset{\frown}{DB}$?

- interior
- arcs and angles

2(VA) = arc + arc



2(VA) = arc + arc

$2(34) = x + 50$

$68 = x + 50$

$-50 \quad -50$

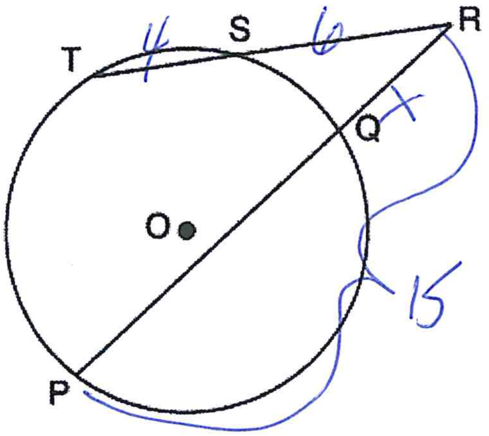
$18 = x$

whole · exterior = whole · exterior

- Segments
- exterior

w · e = w · e

7. In the diagram below, secants $\overline{RST}$ and $\overline{RQP}$, drawn from point R, intersect circle O at S, T, Q, and P.
If $RS = 6$, $ST = 4$, and $RP = 15$, what is the length of $\overline{RQ}$?



$w \cdot e = w \cdot e$

$15 \cdot x = 10 \cdot 6$

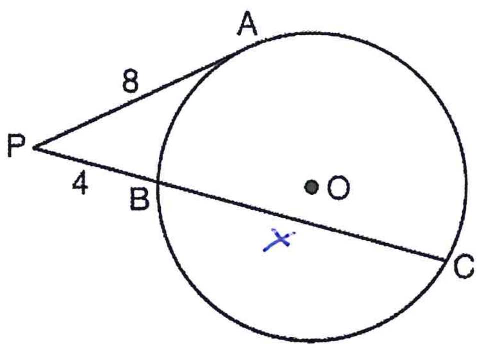
$\frac{\cancel{15}x}{15} = \frac{60}{15}$

$x = 4$

- Segments
- exterior

w · e = w · e

8. In the diagram below of circle O, $\overline{PA}$ is tangent to circle O at A, and $\overline{PBC}$ is a secant with points B and C on the circle.
If $PA = 8$ and $PB = 4$, what is the length of $\overline{BC}$?



$w \cdot e = w \cdot e$

$8 \cdot 8 = (x+4)4$

$64 = 4x + 16$

$-16 \quad -16$

$\frac{48}{4} = \frac{4x}{4}$

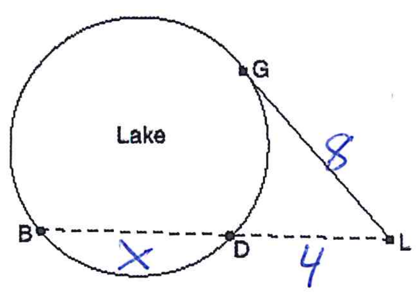
$x = 12$

9. In the accompanying diagram, cabins B and G are located on the shore of a circular lake, and cabin L is located near the lake. Point D is a dock on the lake shore and is collinear with cabins B and L. The road between cabins G and L is 8 miles long and is tangent to the lake. The path between cabin L and dock D is 4 miles long.

- segments
- exterior

w · e = w · e

What is the length, in miles, of $\overline{BD}$?



(Not drawn to scale)

$w \cdot e = w \cdot e$

$8 \cdot 8 = (x+4)4$

$64 = 4x + 16$

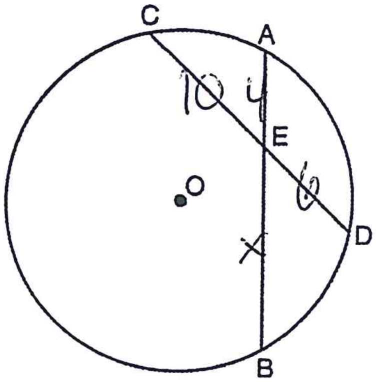
$-16 \quad -16$

$\frac{48}{4} = \frac{4x}{4}$

$12 = x$

Part · Part = Part · Part

10. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $CE = 10$, $ED = 6$, and $AE = 4$, what is the length of $\overline{EB}$?



$P \cdot P = P \cdot P$

$10 \cdot 6 = 4 \cdot x$

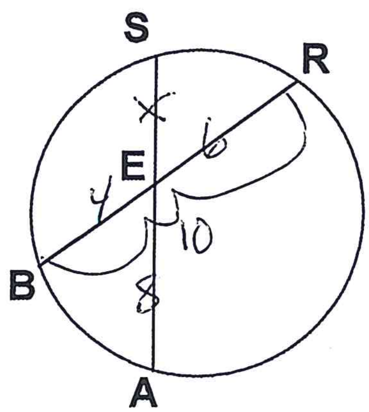
$60 = 4x$

$\frac{60}{4} = \frac{4x}{4}$

$15 = x$

– Segments
– interior
$P \cdot P = P \cdot P$

11. If $\overline{BR} = 10$, $\overline{BE} = 4$, $\overline{AE} = 8$, find $\overline{ES}$



$P \cdot P = P \cdot P$

$6 \cdot 4 = 8 \cdot x$

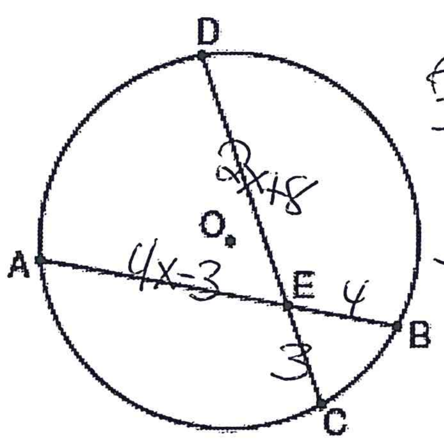
$24 = 8x$

$\frac{24}{8} = \frac{8x}{8}$

$3 = x$

– Segments
– interior
$P \cdot P = P \cdot P$
*10 is not a part

12. In the diagram of circle O below, chord $\overline{AB}$ intersects chord $\overline{CD}$ at E, $DE = 2x + 8$, $EC = 3$, $AE = 4x - 3$, and $EB = 4$.
What is the value of x?



$P \cdot P = P \cdot P$

$3(2x+8) = 4(4x-3)$

$6x + 24 = 16x - 12$

$-6x \quad -6x$

$24 = 10x - 12$

$+12 \quad +12$

$36 = 10x$

$\frac{36}{10} = \frac{10x}{10}$

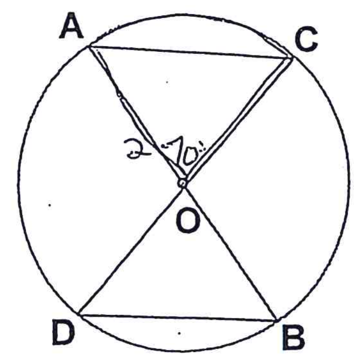
$3.6 = x$

– Segments
– interior
$P \cdot P = P \cdot P$

Area of a Sector $= \dfrac{\theta \pi r^2}{360}$

If given area of a sector, use algebra to solve for missing variable

1. In circle O, $m\angle AOC = 70$ and $\overline{AO} = 2\ in$. Find the area of sector COA to the nearest square inch.

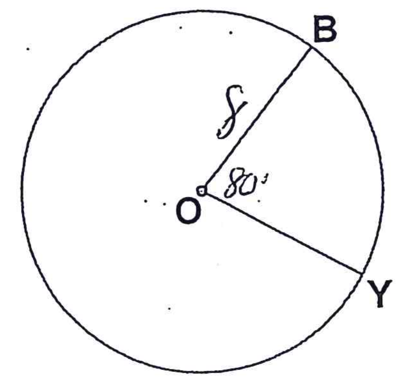


$A = \dfrac{\theta \pi r^2}{360}$

$A = \dfrac{70\pi(2)^2}{360}$

$A = 2\ in^2$

2. In circle O, if $\angle BOY = 80°$ and $\overline{BO} = 8\ cm$, find the area of sector BOY in terms of π.



$A = \dfrac{\theta \pi r^2}{360}$

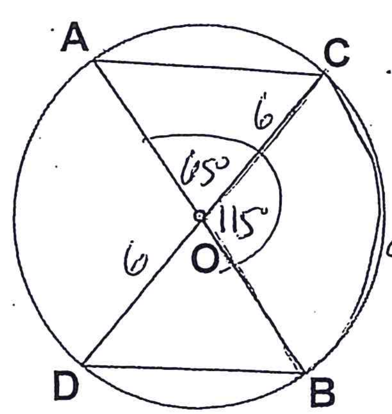
$A = \dfrac{80\pi(8)^2}{360}$

$A = \dfrac{128}{9}\pi$

3. In circle O, $m\angle AOC = 65$ and $\overline{DO} = 6\ in$. Find the area of sector COB in terms of π

Linear pair

$180 - 65 = 115$



All radii are congruent

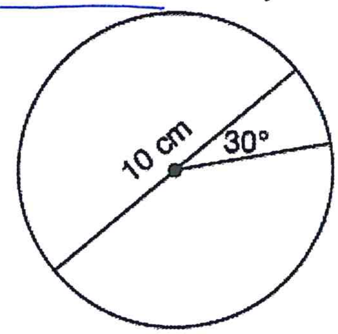
$A = \dfrac{\theta \pi r^2}{360}$

$A = \dfrac{115\pi(6)^2}{360}$

$A = \dfrac{23}{2}\pi$

5. A circle with a diameter of 10 cm and a central angle of 30° is drawn below. What is the area, to the *nearest tenth of a square centimeter*, of the sector formed by the 30° angle?

1) 5.2
2) 6.5 ✓
3) 13.1
4) 26.2



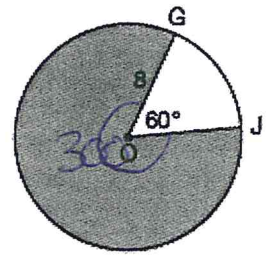
$A = \frac{\theta \pi r^2}{360}$

$A = \frac{30\pi(5)^2}{360}$

$A = 6.5$

6. In the diagram below of circle O, $GO = 8$ and $m\angle GOJ = 60°$. What is the area, in terms of π, of the shaded region?

1) $\frac{4\pi}{3}$
2) $\frac{20\pi}{3}$
3) $\frac{32\pi}{3}$
4) $\frac{160\pi}{3}$ ✓



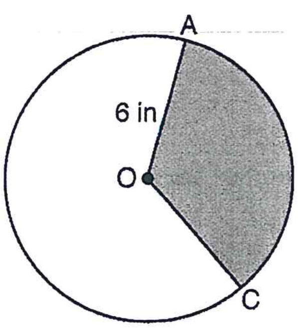
$\begin{array}{r} 360 \\ -60 \\ \hline 300 \end{array}$

$A = \frac{\theta \pi r^2}{360}$

$A = \frac{300\pi(8)^2}{360}$

$A = \frac{160}{3}\pi$

7. In the diagram below of circle O, the area of the shaded sector AOC is 12π in^2 and the length of $\overline{OA}$ is 6 inches. Determine and state $m\angle AOC$.



$A = \frac{\theta \pi r^2}{360}$

$\frac{12\cancel{\pi}}{1} = \frac{x\cancel{\pi}(6)^2}{360}$

$\frac{36x}{36} = \frac{4320}{36}$

$x = 120°$

8. The area of a sector of a circle with a radius measuring 15 cm is 75π cm^2. What is the measure of the central angle that forms the sector?

1) 72°
2) 120° ✓
3) 144°
4) 180°

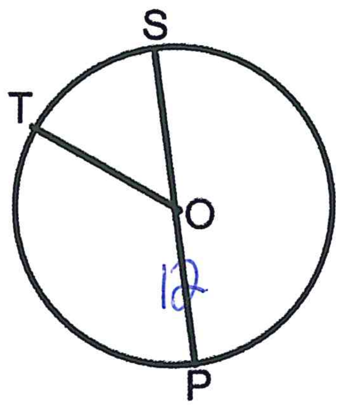
$A = \frac{\theta \pi r^2}{360}$

$\frac{75\cancel{\pi}}{1} = \frac{x\cancel{\pi}(15)^2}{360}$

$\frac{225x}{225} = \frac{27000}{225}$

$x = 120°$

9. In the diagram below of circle O, the area of sector STO is $48\pi\ in^2$ and the length of $\overline{OP}$ is 12 inches. Determine and state $m\angle SOT$



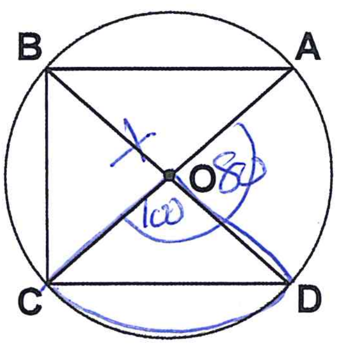
$$A=\frac{\theta\pi r^2}{360}$$

$$\frac{48\pi}{1}=\frac{x\pi(12)^2}{360}$$

$$\frac{17280}{144}=\frac{144x}{144}$$

$x=120°$

10. In circle O, diameters $\overline{BOD}$ and $\overline{COA}$ intersect at the center of the circle O. If the area of sector OCD = 240π square inches and $m\angle AOD = 80$, find the measure of $\overline{OB}$ to the nearest tenth of an inch.



$$\begin{array}{r} 180 \\ -80 \\ \hline 100 \end{array}$$

$$A=\frac{\theta\pi r^2}{360}$$

$$\frac{240\pi}{1}=\frac{100\pi(x)^2}{360}$$

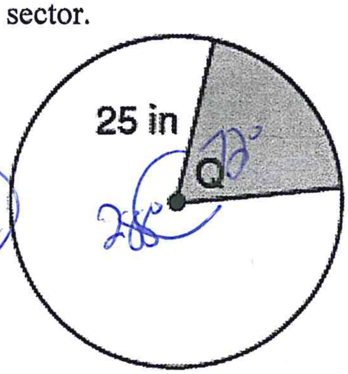
$$\frac{86400}{100}=\frac{100x^2}{100}$$

$$\sqrt{x^2}=\sqrt{864}$$

$x=29.4$

11. In the diagram below, the circle has a radius of 25 inches. The area of the *unshaded* sector is 500π in^2.
Determine and state the degree measure of angle Q, the central angle of the shaded sector.

$$\begin{array}{r} 360 \\ -288 \\ \hline 72° \end{array}$$



$$A=\frac{\theta\pi r^2}{360}$$

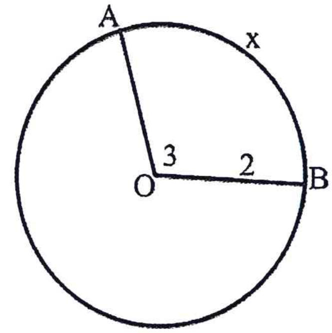
$$\frac{500\pi}{1}=\frac{x\pi(25)^2}{360}$$

$$\frac{625x}{625}=\frac{180000}{625}$$

$x=288°$

Arc Length: $s = \theta r$, where s = arc length, θ = central angle (in radians), r = radius

1. In circle O, the measure of central angle AOB is 3 radians and the length of $\overline{OB}$ is 2 cm. What is the measure of arc AB?

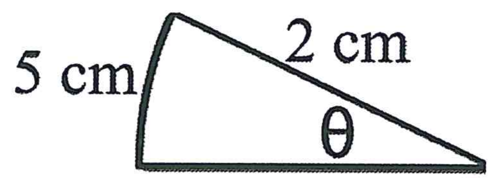


$S = \theta r$

$x = 3(2)$

$x = 6$

2. What is the measure of the central angle below?



$S = \theta r$

$\frac{S}{2} = \frac{x(2)}{2}$

$\frac{5}{2} = x$

3. What is the measure of the radius of a sector whose arc length is 12 inches and has a central angle of 4 radians?

$S = \theta r$

$\frac{12}{4} = \frac{4r}{4}$

$3 = r$

4. A wheel has a radius of 18 inches. Which distance, to the *nearest inch*, does the wheel travel when it rotates through an angle of $\frac{2\pi}{5}$ radians?

$S = \theta r$

$x = \frac{2\pi}{5}(18)$

$x = 22.6$

5. What is the measure of a central angle in degrees whose arc length is 6 meters and whose radius measures 8 meters?

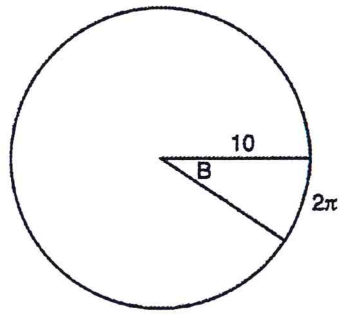
$S = \theta r$

$\frac{6}{8} = \frac{x8}{8}$

$x = \frac{3}{4}$

6. In the diagram below, the circle shown has radius 10. Angle B intercepts an arc with a length of 2π.
What is the measure of angle B, in radians?

1) $10 + 2\pi$
2) 20π
3) $\frac{\pi}{5}$
4) $\frac{5}{\pi}$

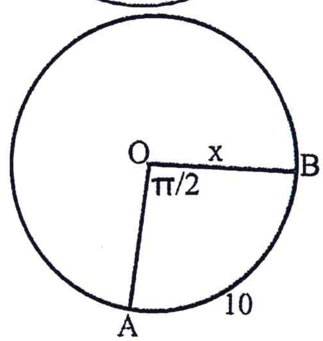


$S = \theta r$

$\frac{2\pi}{10} = \frac{x(10)}{10}$

$\frac{\pi}{5} = x$

7. In circle O, the measure of central angle AOB is $\frac{\pi}{2}$ radians and the length of arc AB is 10 cm. What is the measure of radius $\overline{OB}$ to the *nearest tenth of a cm*?



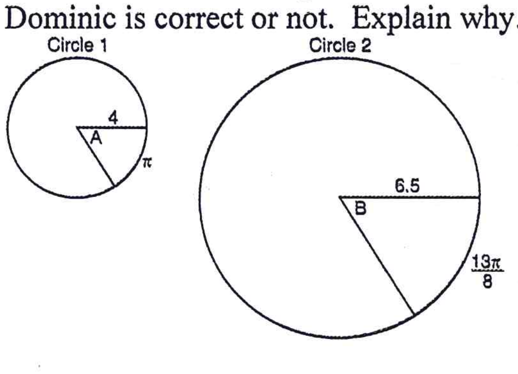
$S = \theta r$

$\cancel{10 = \frac{\pi}{2}(x)}$

$\frac{20}{\pi} = \frac{\pi x}{\pi}$

$6.4 = x$

8. In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle A intercepts an arc of length π, and angle B intercepts an arc of length $\frac{13\pi}{8}$.
Dominic thinks that angles A and B have the same radian measure. State whether Dominic is correct or not. Explain why.



$S = \theta r$

$\frac{\pi}{4} = \frac{x(4)}{4}$

$\frac{\pi}{4} = x$

Yes

$S = \theta r$

$\cancel{\frac{13\pi}{8} = \frac{x(6.5)}{1}}$

$\frac{13\pi}{52} = \frac{52x}{52}$

$\frac{\pi}{4} = x$