Name: Schlansky

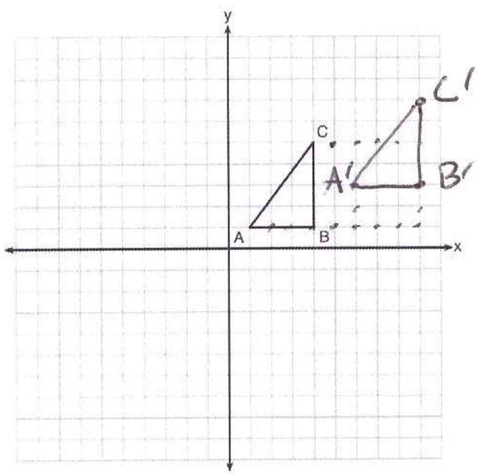
Common Core Geometry Regents Review Packet!

Mr. Schlansky

2024

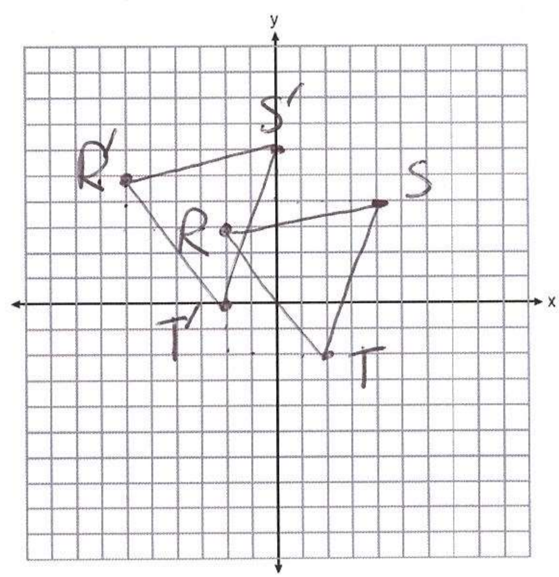
Performing Transformations
Reflections
Flip (Count to what you are reflecting over)
*Switch the coordinates for reflection over $y = x$
y = # is horizontal line, x = # is vertical line. You must graph these lines before you can reflect over them.
Rotations
$R_{90} = (-y, x)$
$R_{180} = (-x, -y)$
$R_{270} = (y, -x)$
Translation
Slide. Count out the translation on the grid
Dilations
If centered at the origin: multiply the coordinates by the scale factor
If centered at a point: Count from the center to each point the number of times of the scale factor.

1. In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. Graph and the image of $\triangle ABC$ after the translation five units to the right and two units up.



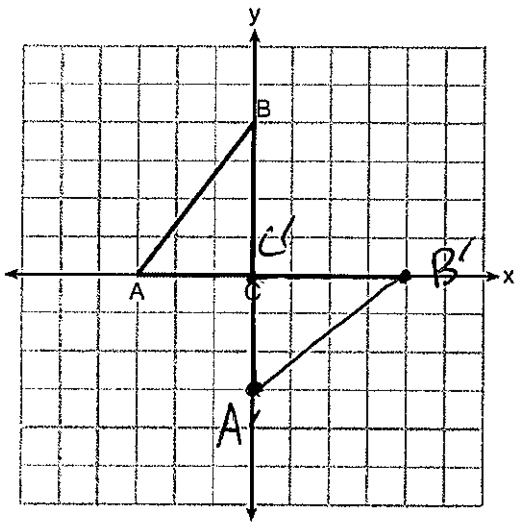
2. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$.

Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a translation 4 units to the left and 2 units up.



3. Triangle ABC is graphed on the set of axes below. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection over the line $y=x$.

Switch x and y

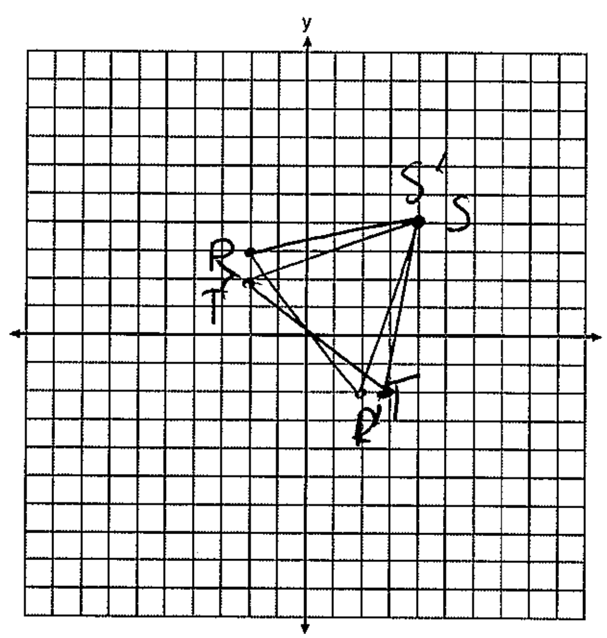


$A(-3,0) \rightarrow (0,-3)\ A'$
$B(0,4) \rightarrow (4,0)\ B'$
$C(0,0) \rightarrow (0,0)\ C'$

4. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a reflection in the line $y=x$.

Switch x and y

$R(-2,3) \rightarrow (3,-2)\ R'$
$S(4,4) \rightarrow (4,4)\ S'$
$T(2,-2) \rightarrow (-2,2)\ T'$

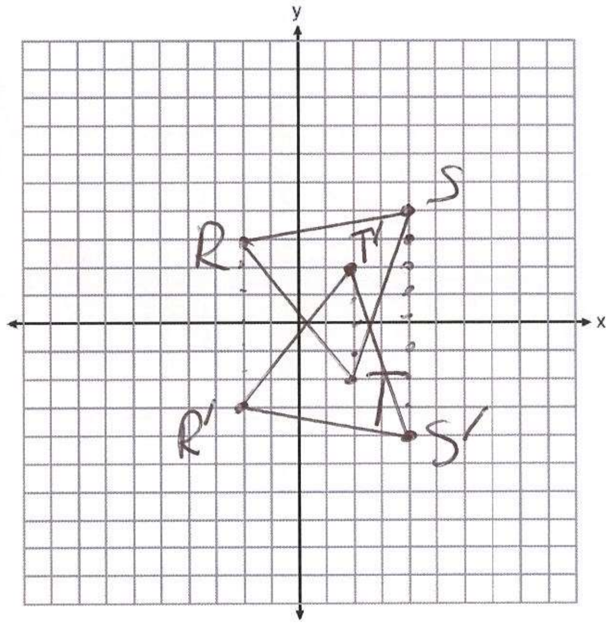


5. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a reflection in x-axis.

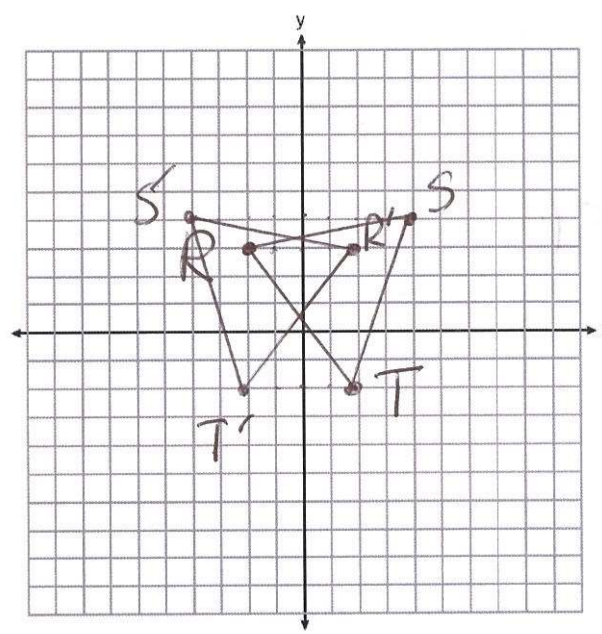
$R(-2,3) \rightarrow (-2,-3)$

$S(4,4) \rightarrow (4,-4)$

$T(2,-2) \Rightarrow (2,2)$

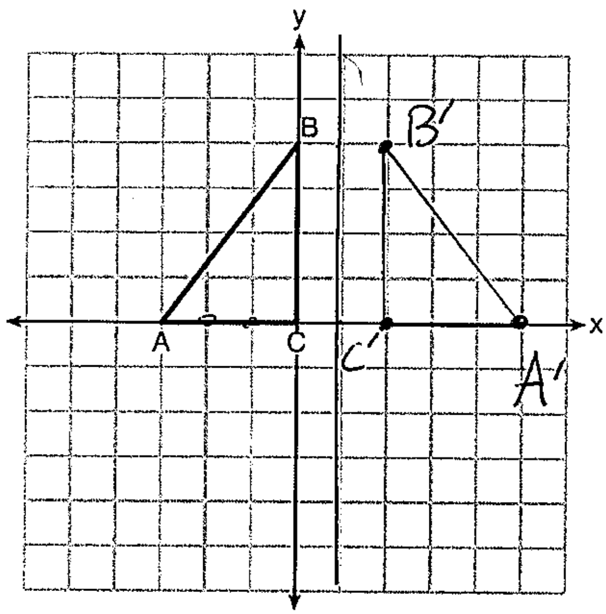


6. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a reflection in y-axis.



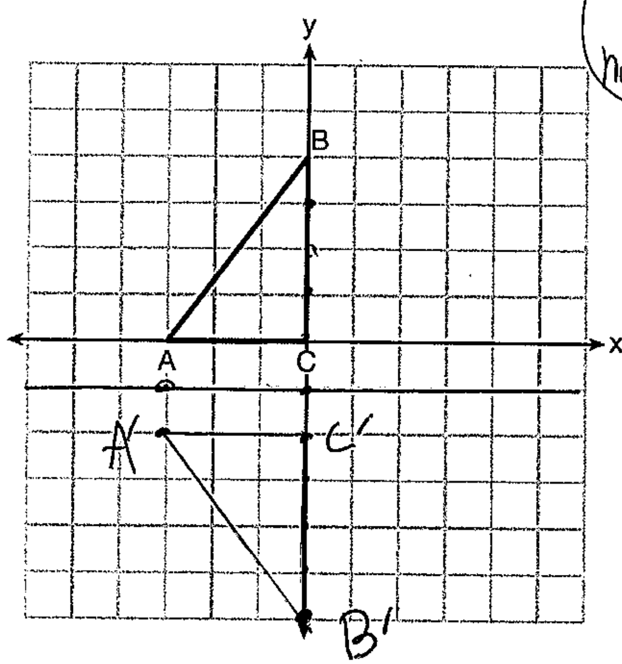
7. Triangle *ABC* is graphed on the set of axes below. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection over the line $x = 1$.

4H XV vertical



8. Triangle *ABC* is graphed on the set of axes below. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection over the line $y = -1$.

4H XV horizontal



$R^{CW}_{-270} = R^{CCW}_{90}(x,y) = (-y,x)$

$R_{-180} = R_{180}(x,y) = (-x,-y)$

$R_{-90} = R_{270}(x,y) = (y,-x)$

Switch and negate!

$(x,y) \to (-y,x)$ 90

$(-x,-y)$ 180

$(+y,-x)$ 270

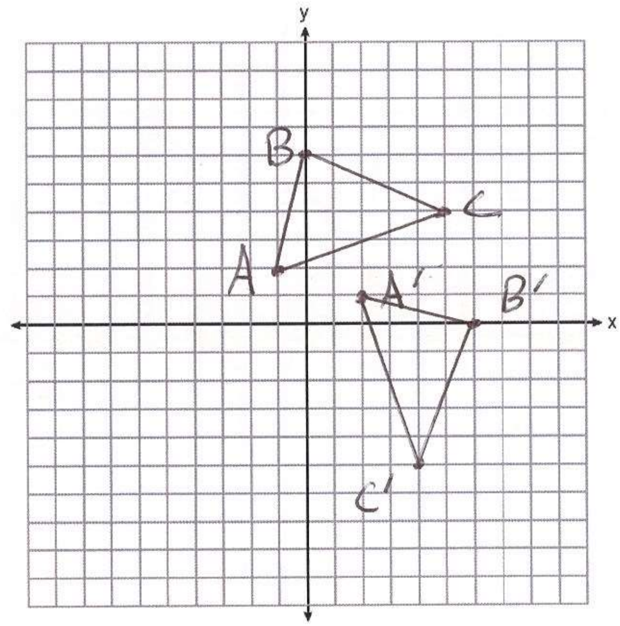
9. On the accompanying set of axes, graph $\triangle ABC$ with coordinates $A(-1,2)$, $B(0,6)$, and $C(5,4)$. Then graph $\triangle A'B'C'$, the image of $\triangle ABC$ after a counter-clockwise rotation of 270 centered at the origin.

$R_{270}(x,y) = (y,-x)$

$A(-1,2) \xrightarrow{y,-x} (2,1)A'$

$B(0,6) \to (6,0)B'$

$C(5,4) \to (4,-5)C'$



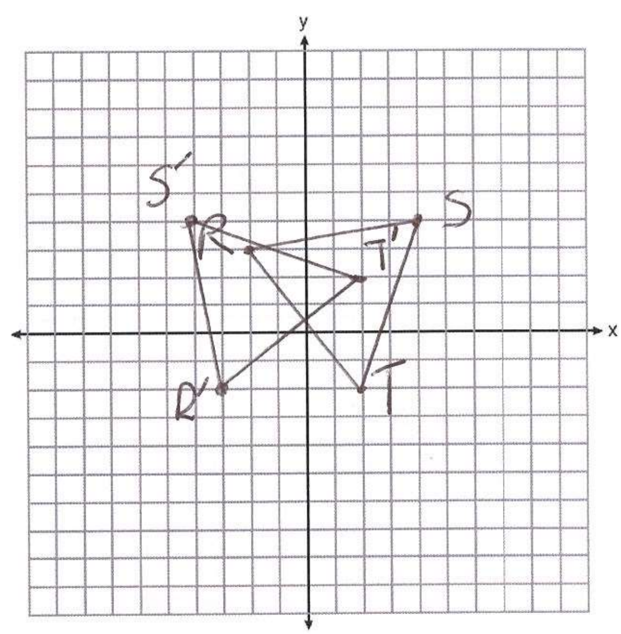
10. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a counter-clockwise rotation of 90 centered at the origin.

$R_{90}(x,y) = (-y,x)$

$R(-2,3) \xrightarrow{-y,x} (-3,-2)R'$

$S(4,4) \to (-4,4)S'$

$T(2,-2) \to (2,2)T'$

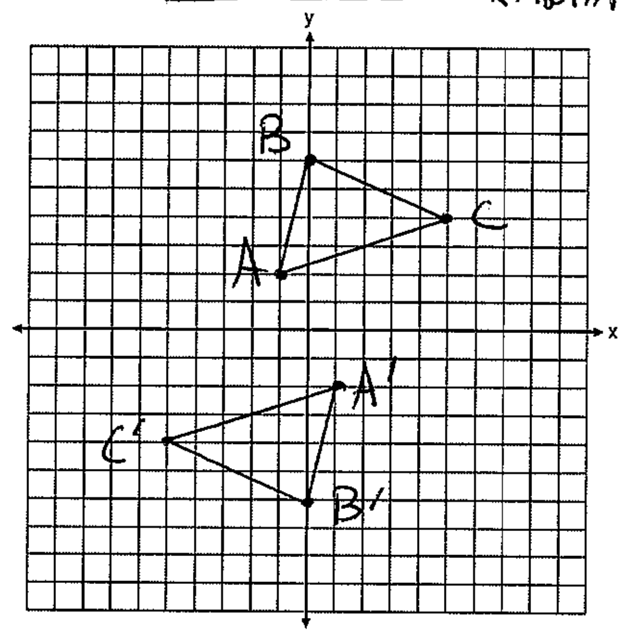


11. On the accompanying set of axes, graph $\triangle ABC$ with coordinates $A(-1,2)$, $B(0,6)$, and $C(5,4)$. Then graph $\triangle A'B'C'$, the image of $\triangle ABC$ after a clockwise rotation of 180 centered at the origin. $R_{-180}(x,y)=(-x,-y)$

$A(-1,2) \xrightarrow{-x,-y} (1,-2)\ A'$

$B(0,6) \rightarrow (0,-6)\ B'$

$C(5,4) \rightarrow (-5,-4)\ C'$

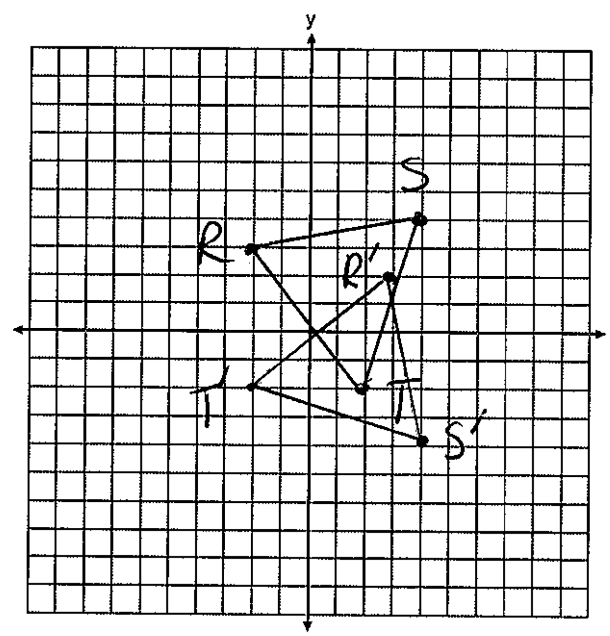


12. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a clockwise rotation of 90 centered at the origin. R_{-90} $R_{270}(x,y)=(y,-x)$

$R(-2,3) \xrightarrow{y,-x} (3,2)\ R'$

$S(4,4) \rightarrow (4,-4)\ S'$

$T(2,-2) \rightarrow (-2,-2)\ T'$



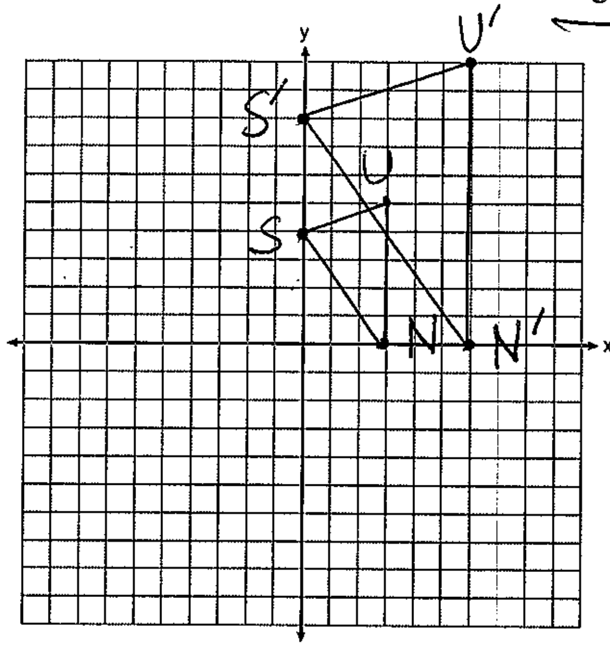
13. Triangle *SUN* has coordinates *S*(0,4), *U*(3,5), and *N*(3,0). On the accompanying grid, draw and label $\triangle SUN$. Then, graph and state the coordinates of $\triangle S'U'N'$, the image of $\triangle SUN$ after a dilation of 2 centered at the origin. multiply

$\cdot 2$

S(0,4) → (0,8) S'

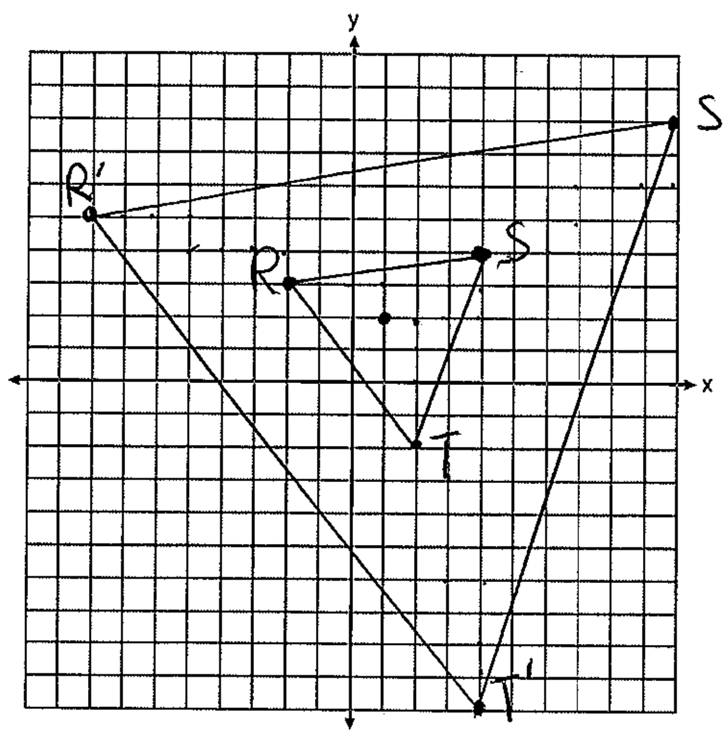
U(3,5) → (6,10) U'

N(3,0) → (6,0) N'



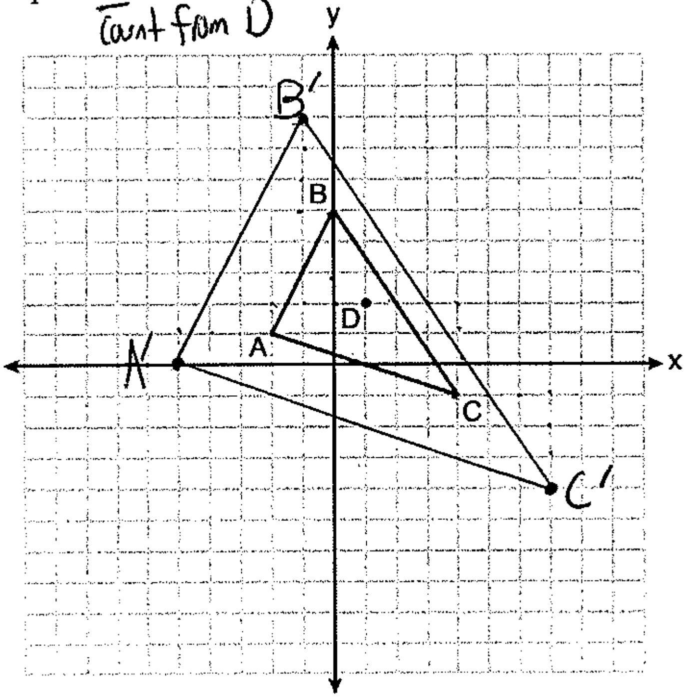
14. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$ and $\triangle R'S'T'$, the image of $\triangle RST$ after a dilation of 3 centered at (1,2).

Count from the center

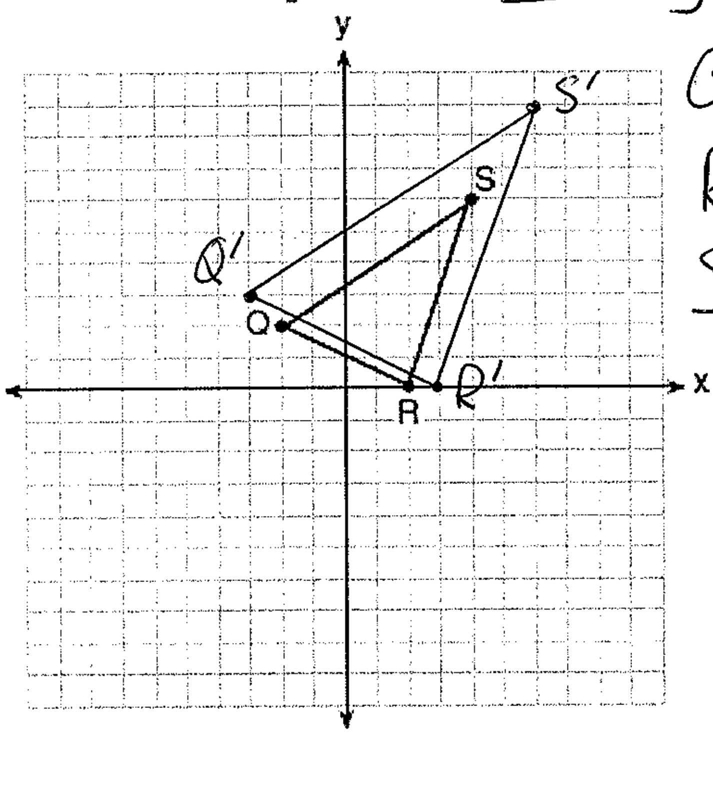


15 ~~9~~. Triangle ABC and point $D(1,2)$ are graphed on the set of axes below.
Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point D.

Count from D



16 ~~10~~. Triangle QRS is graphed on the set of axes below.
On the same set of axes, graph and label $\triangle Q'R'S'$, the image of $\triangle QRS$ after a dilation with a scale factor of $\frac{3}{2}$ centered at the origin. multiply



$\cdot \frac{3}{2}$

$Q(-2,2) \rightarrow (-3,3)$

$R(2,0) \rightarrow (3,0)$

$S(4,6) \rightarrow (6,9)$

Identifying Transformations

Check for orientation!!! (The direction of the letters)

The only transformation that changes orientation is a line reflection (an even amount of reflections will preserve orientation).

Translation = slide

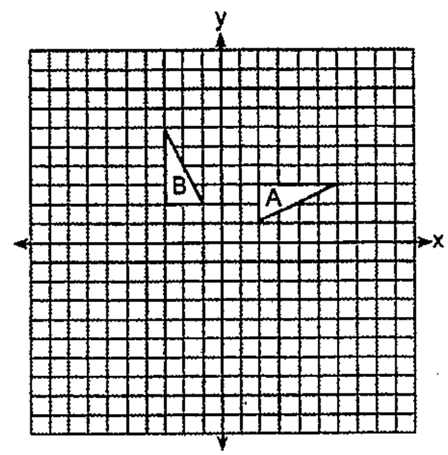
Rotation = turn

Reflection = flip

Dilation = change size (enlarge or shrink)

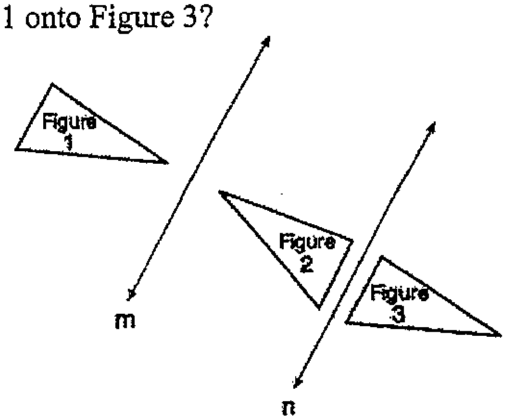
1. In the diagram below, which single transformation was used to map triangle A onto triangle B?

1) line reflection
2) rotation
3) dilation
4) translation



2. In the diagram below, line m is parallel to line n. Figure 2 is the image of Figure 1 after a reflection over line m. Figure 3 is the image of Figure 2 after a reflection over line n. Which single transformation would carry Figure 1 onto Figure 3?

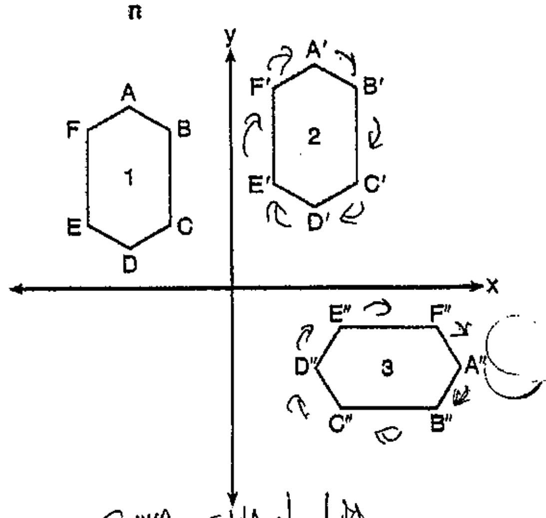
1) a dilation
2) a rotation
3) a reflection
4) a translation



3. In the diagram below, congruent figures 1, 2, and 3 are drawn.

Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?

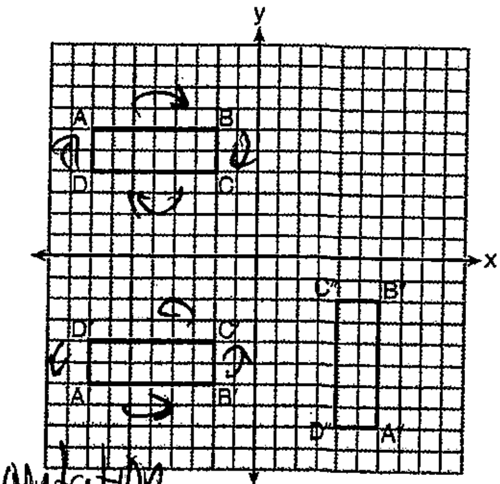
1) a reflection followed by a translation
2) a rotation followed by a translation
3) a translation followed by a reflection
4) a translation followed by a rotation



same orientation

4. A sequence of transformations maps rectangle $ABCD$ onto rectangle $A''B''C''D''$, as shown in the diagram below.
Which sequence of transformations maps $ABCD$ onto $A'B'C'D'$ and then maps $A'B'C'D'$ onto $A''B''C''D''$?

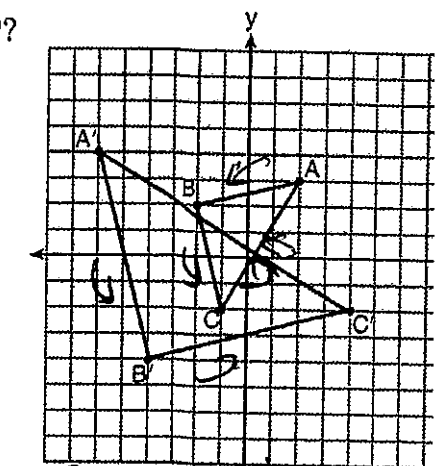
1) a reflection followed by a rotation
2) a reflection followed by a translation
3) a translation followed by a rotation
4) a translation followed by a reflection



opposite orientation

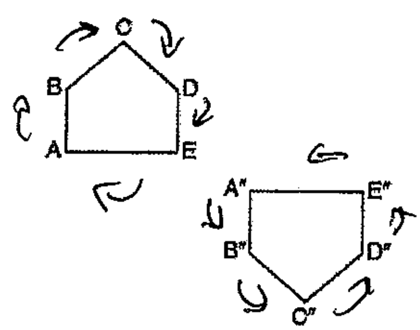
5. Which sequence of transformations will map $\triangle ABC$ onto $\triangle A'B'C'$?

1) reflection and translation
2) rotation and reflection
3) translation and dilation
4) dilation and rotation



same orientation

6. Identify which sequence of transformations could map pentagon $ABCDE$ onto pentagon $A''B''C''D''E''$, as shown below.

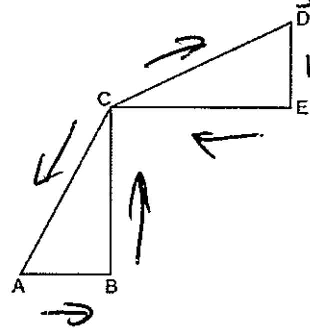


opposite orientation
reflection

1) dilation followed by a rotation
2) translation followed by a rotation
3) line reflection followed by a translation
4) line reflection followed by a line reflection

7. In the diagram below, $\triangle ABC \cong \triangle DEC$.

opposite orientation
single reflection

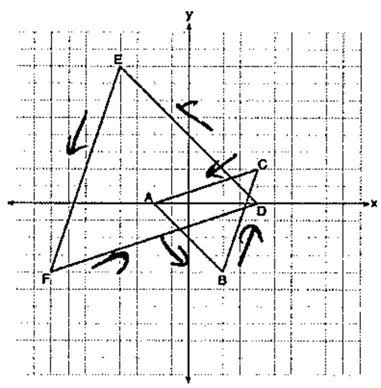


Which transformation will map $\triangle ABC$ onto $\triangle DEC$?

1) a rotation
2) a line reflection
3) a translation followed by a dilation
4) a line reflection followed by a second line reflection double reflection

8. On the set of axes below, $\triangle ABC$ has vertices at $A(-2,0)$, $B(2,-4)$, $C(4,2)$, and $\triangle DEF$ has vertices at $D(4,0)$, $E(-4,8)$, $F(-8,-4)$.

Same orientation
not single reflection



it got bigger so scale factor > 1

Which sequence of transformations will map $\triangle ABC$ onto $\triangle DEF$?

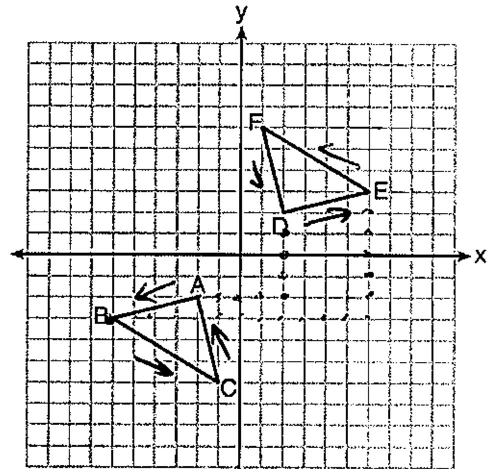
1) a dilation of $\triangle ABC$ by a scale factor of 2 centered at point A A would be invariant
2) a dilation of $\triangle ABC$ by a scale factor of $\frac{1}{2}$ centered at point A
3) a dilation of $\triangle ABC$ by a scale factor of 2 centered at the origin, followed by a rotation of 180° about the origin
4) a dilation of $\triangle ABC$ by a scale factor of $\frac{1}{2}$ centered at the origin, followed by a rotation of 180° about the origin

9. Triangle *ABC* and triangle *DEF* are graphed on the set of axes below. Which sequence of transformations maps triangle *ABC* onto triangle *DEF*?

1) a reflection over the x-axis followed by a reflection over the y-axis
2) a 180° rotation about the origin followed by a reflection over the line $y = x$
3) a 90° clockwise rotation about the origin followed by a reflection over the y-axis
4) a translation 8 units to the right and 1 unit up followed by a 90° counterclockwise rotation about the origin

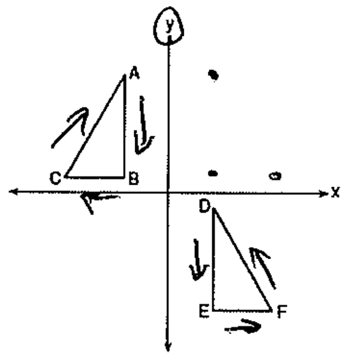
try each choice

try to avoid the rotation if possible



Same orientation
not single reflection

10. In the diagram below, $\triangle ABC \cong \triangle DEF$.



opposite orientation
single reflection

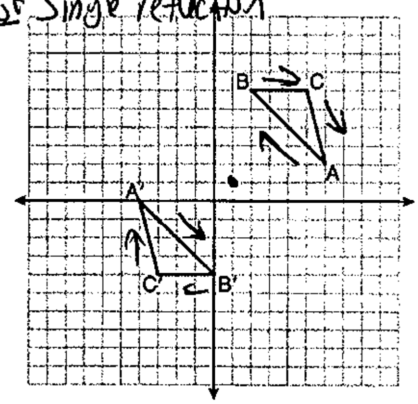
Which sequence of transformations maps $\triangle ABC$ onto $\triangle DEF$?

1) a reflection over the x-axis followed by a translation
2) a reflection over the y-axis followed by a translation
3) a rotation of 180° about the origin followed by a translation
4) a counterclockwise rotation of 90° about the origin followed by a translation

11. On the set of axes below, $\triangle ABC \cong \triangle A'B'C'$. Triangle *ABC* maps onto $\triangle A'B'C'$ after a

1) reflection over the line $y = -x$
2) reflection over the line $y = -x + 2$
3) rotation of 180° centered at (1,1)
4) rotation of 180° centered at the origin

Same orientation
not single reflection



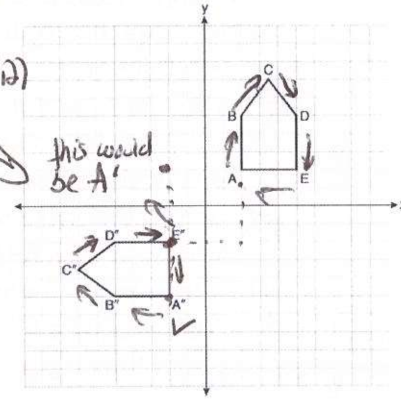
Same orientation
not single reflection

12. On the set of axes below, pentagon *ABCDE* is congruent to *A"B"C"D"E"*. Which describes a sequence of rigid motions that maps *ABCDE* onto *A"B"C"D"E"*?

$-y, x$

$A(2,2) \rightarrow (-2,2)$

1) a rotation of 90° counterclockwise about the origin followed by a reflection over the *x*-axis
2) a rotation of 90° counterclockwise about the origin followed by a translation down 7 units
3) a reflection over the *y*-axis followed by a reflection over the *x*-axis
4) a reflection over the *x*-axis followed by a rotation of 90° counterclockwise about the origin

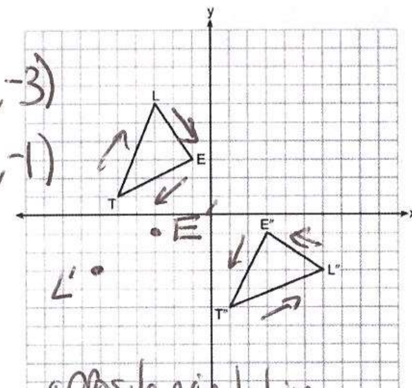


13. On the set of axes below, $\triangle LET$ and $\triangle L"E"T"$ are graphed in the coordinate plane where $\triangle LET \cong \triangle L"E"T"$.
Which sequence of rigid motions maps $\triangle LET$ onto $\triangle L"E"T"$?

1) a reflection over the *y*-axis followed by a reflection over the *x*-axis
2) a rotation of 180° about the origin
3) a rotation of 90° counterclockwise about the origin followed by a reflection over the *y*-axis
4) a reflection over the *x*-axis followed by a rotation of 90° clockwise about the origin

$L(-3,6) \rightarrow (-6,-3)$

$E(-1,3) \rightarrow (-3,-1)$

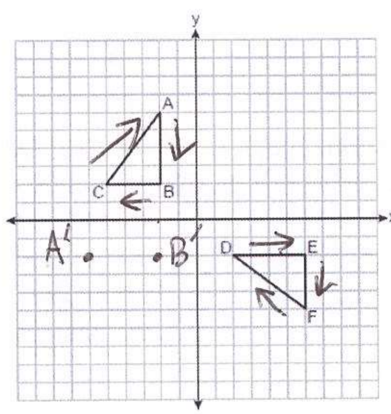


opposite orientation
single reflection

14. On the set of axes below, congruent triangles *ABC* and *DEF* are drawn.

$A(-2,6) \xrightarrow{-y,x} (-6,-2)$

$B(-2,2) \rightarrow (-2,-2)$



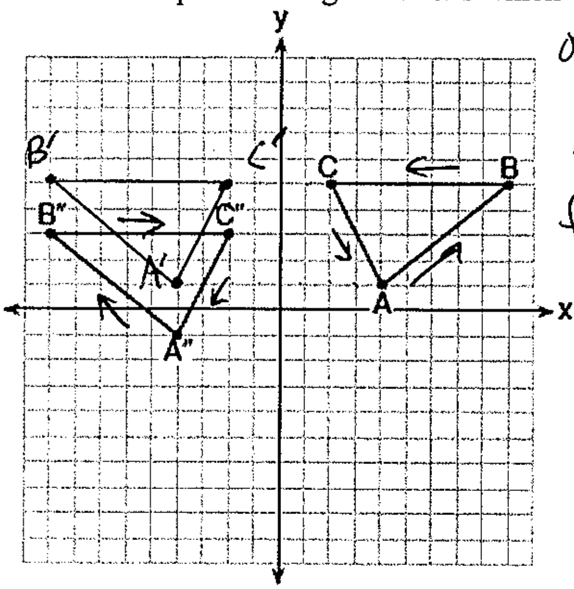
Same orientation
not single reflection

Which sequence of transformations maps $\triangle ABC$ onto $\triangle DEF$?

1) A counterclockwise rotation of 90 degrees about the origin, followed by a translation 8 units to the right.
2) A counterclockwise rotation of 90 degrees about the origin, followed by a reflection over the *y*-axis.
3) A counterclockwise rotation of 90 degrees about the origin, followed by a translation 4 units down.
4) A clockwise rotation of 90 degrees about the origin, followed by a reflection over the *x*-axis.

Identifying Transformations (Open Response)
CHECK FOR ORIENTATION!!!!
Same orientation (rotation first, then translation)
-Rotate any point the appropriate degree measure and direction.
-Translate the rest of the way by counting from that point to its image.
Opposite orientation (reflection first, then translation)
-Reflect over the appropriate axis (use y=x if it needs to be reflected diagonally)
-Translate the rest of the way by counting from any new point to its image.

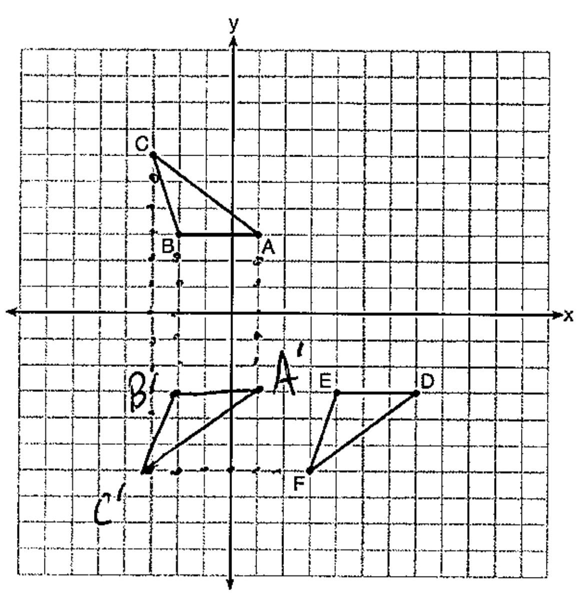
1. The graph below shows $\triangle ABC$ and its image, $\triangle A''B''C''$.
Describe a sequence of rigid motions which would map $\triangle ABC$ onto $\triangle A''B''C''$.



opposite orientation
reflection

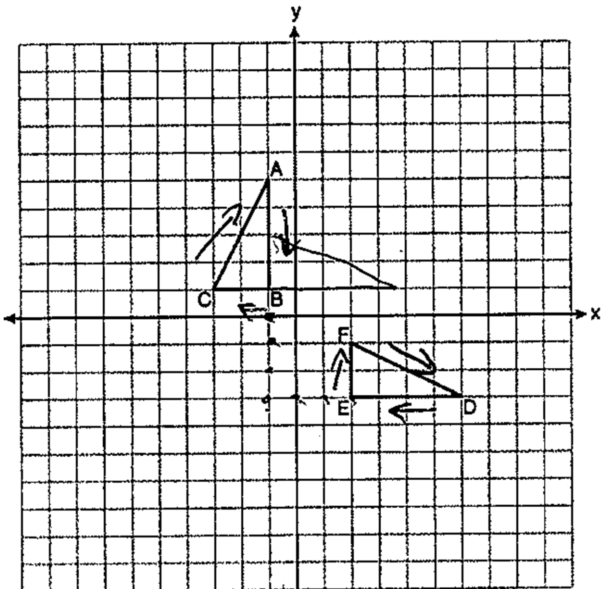
1) Reflect $\triangle ABC$ over the y-axis followed by a translation 2 units down

2. Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.



1) reflect $\triangle ABC$ over the x-axis followed by a translation 6 units to the right.

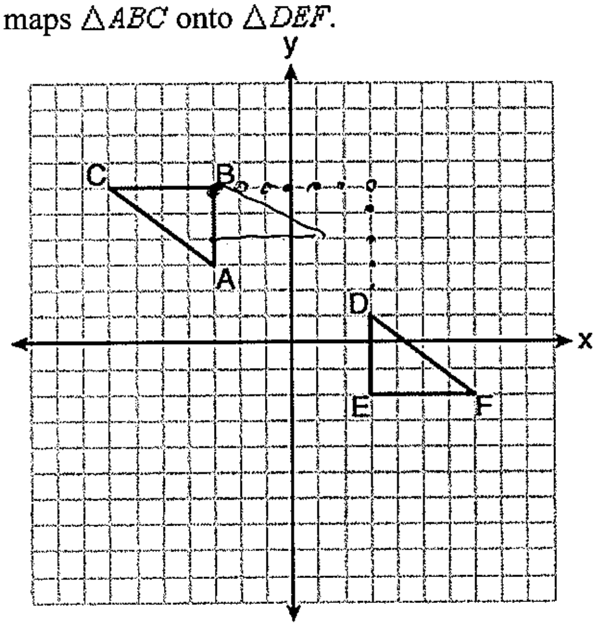
3. On the set of axes below, $\triangle ABC$ and $\triangle DEF$ are graphed. Describe a sequence of rigid motions that would map $\triangle ABC$ onto $\triangle DEF$.



rotate $\triangle ABC$ 90° clockwise centered at B followed by a translation 4 units down and 3 units right.

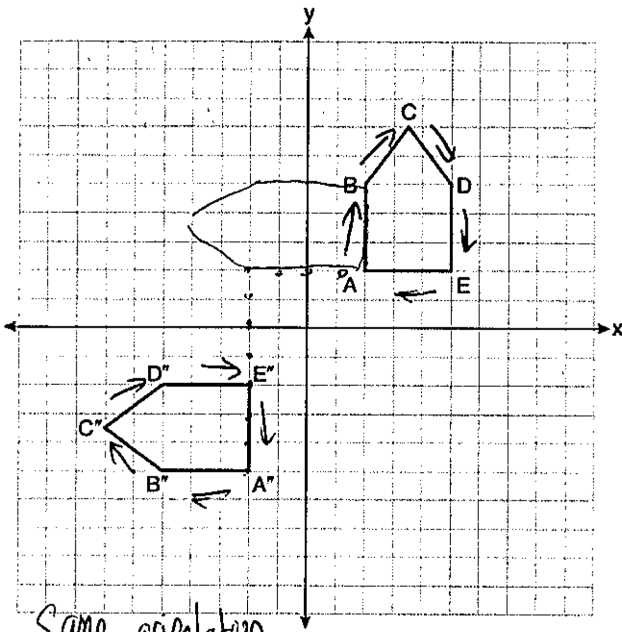
same orientation
rotation

4. On the set of axes below, $\triangle ABC \cong \triangle DEF$. Describe a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle DEF$.



rotate $\triangle ABC$ 180° counter-clockwise centered at B followed by a translation 6 units right and 5 units down

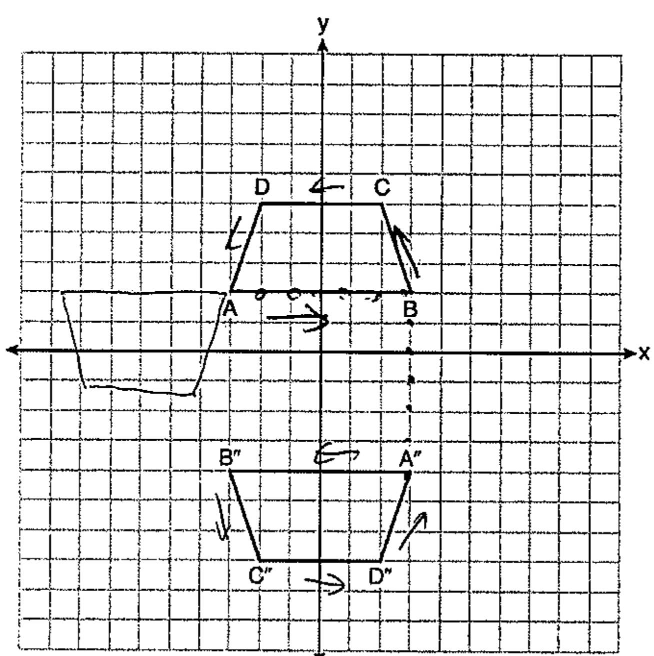
5. On the set of axes below, pentagon *ABCDE* is congruent to *A"B"C"D"E"*. Describe a sequence of rigid motions that maps pentagon *ABCDE* onto *A"B"C"D"E"*.



rotate ABCDE 90° counter-clockwise centered at A followed by a translation left 4 and down 7.

Same orientation
rotation

6. Trapezoids *ABCD* and *A"B"C"D"* are graphed on the set of axes below. Describe a sequence of transformations that maps trapezoid *ABCD* onto trapezoid *A"B"C"D"*.

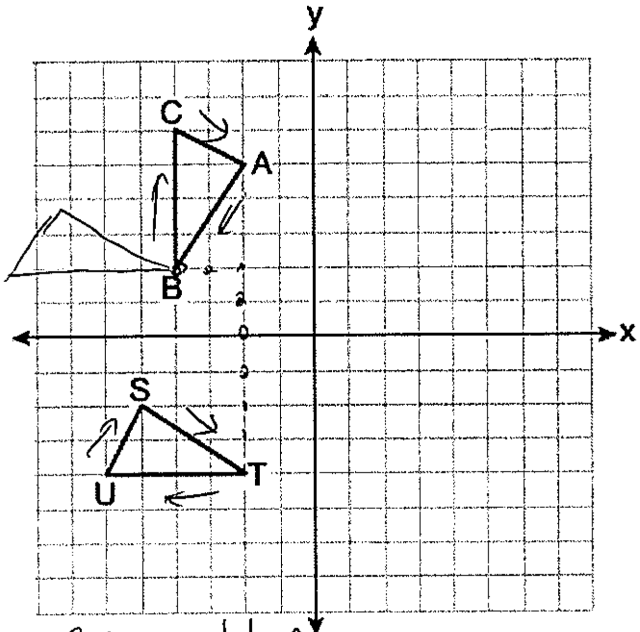


rotate ABCD 180° counter-clockwise centered at A followed by a translation 6 units right and 6 units down.

Same orientation
rotation

7. On the set of axes below, $\triangle ABC \cong \triangle STU$. Describe a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle STU$.

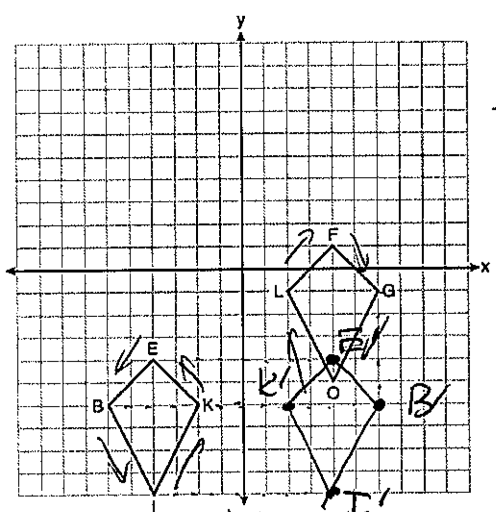
Rotate $\triangle ABC$ 90° counter-clockwise centered at B followed by a translation 2 right and 6 down.



Same orientation
rotation

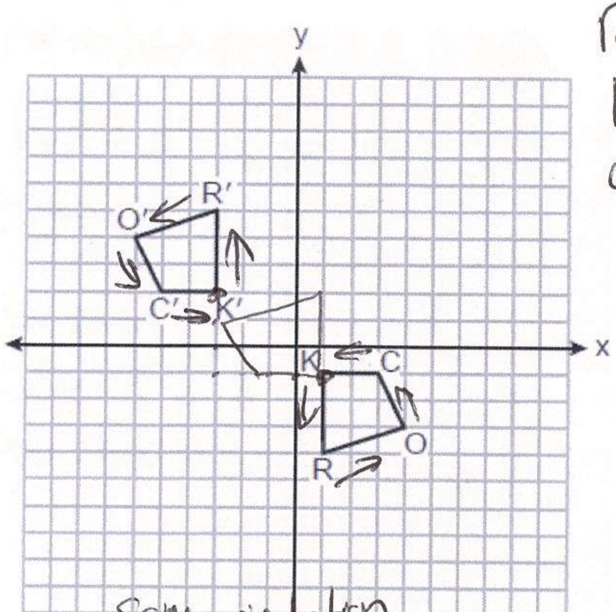
8. Quadrilaterals *BIKE* and *GOLF* are graphed on the set of axes below. Describe a sequence of transformations that maps quadrilateral *BIKE* onto quadrilateral *GOLF*.

Reflect BIKE over the y-axis followed by a translation 5 units up.



Opposite orientation
reflection

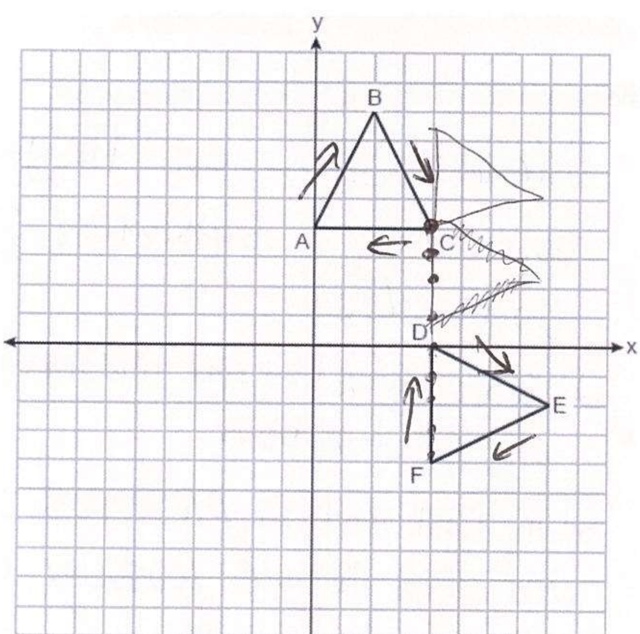
9. On the set of axes below, congruent quadrilaterals *ROCK* and *R'O'C'K'* are graphed. Describe a sequence of transformations that would map quadrilateral *ROCK* onto quadrilateral *R'O'C'K'*.



rotate ROCK 180° centered at K followed by a translation 4 left and 3 up

Same orientation
rotation

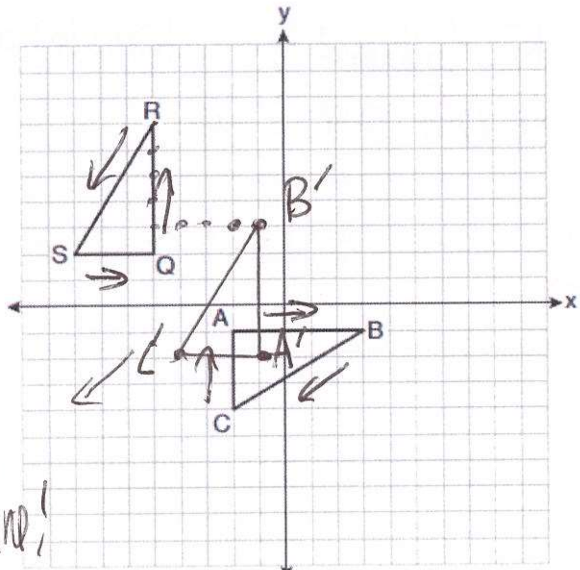
10. Triangles *ABC* and *DEF* are graphed on the set of axes below.



rotate $\triangle ABC$ 90° clockwise centered at C followed by a translation 8 units down

Same orientation
rotation

11. On the set of axes below, $\triangle ABC$ is graphed with coordinates $A(-2,-1)$, $B(3,-1)$, and $C(-2,-4)$. Triangle QRS, the image of $\triangle ABC$, is graphed with coordinates $Q(-5,2)$, $R(-5,7)$, and $S(-8,2)$. Describe a sequence of transformations that would map $\triangle ABC$ onto $\triangle QRS$.



~~Q(~~ $A(-2,-1) \xrightarrow{y,x} (-1,-2)$

~~R(~~ $B(3,-1) \rightarrow (-1,3)$

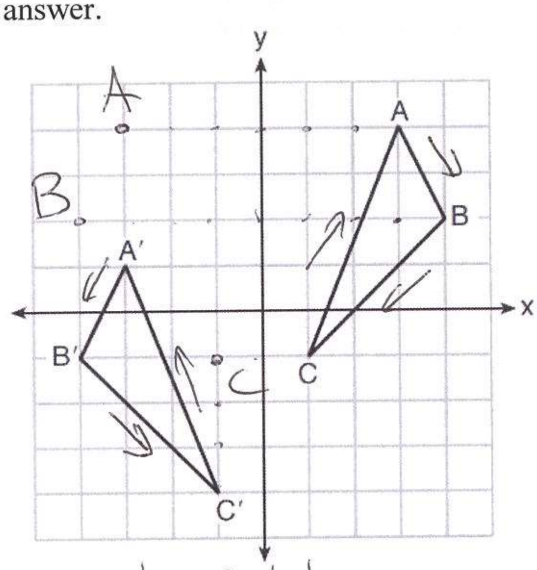
~~S(~~ $C(-2,-4) \rightarrow (-4,-2)$

Reflect $\triangle ABC$ over the line $y=x$ followed by a translation 4 units left and 4 units up

need a diagonal line! $y=x$!

opposite orientation reflection

12. As graphed on the set of axes below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a sequence of transformations.
Is $\triangle A'B'C'$ congruent to $\triangle ABC$? Use the properties of rigid motion to explain your answer.



1) Reflect $\triangle ABC$ over the y-axis followed by a translation 3 units down.

2) Yes, a reflection and translation are rigid motions.

3) A rigid motion preserves size and angle measure producing a congruent figure.

opposite orientation reflection

Rigid Motion Properties
A rigid motion preserves size and angle measure producing a congruent figure
They all produce a congruent figure except dilation.

1. Which transformation would *not* always produce an image that would be congruent to the original figure?
1) translation 2) dilation 3) rotation 4) reflection

2. The vertices of ΔJKL have coordinates $J(5,1)$, $K(-2,-3)$, and $L(-4,1)$. Under which transformation is the image $\Delta J'K'L'$ *not* congruent to ΔJKL?
1) a translation of two units to the right and two units down
2) a counterclockwise rotation of 180 degrees around the origin
3) a reflection over the x-axis
4) a dilation with a scale factor of 2 and centered at the origin

3. If $\triangle A'B'C'$ is the image of $\triangle ABC$, under which transformation will the triangles *not* be congruent?
1) reflection over the x-axis
2) translation to the left 5 and down 4
3) dilation centered at the origin with scale factor 2
4) rotation of 270° counterclockwise about the origin

4. Under which transformation would $\triangle A'B'C'$, the image of $\triangle ABC$, *not* be congruent to $\triangle ABC$?
1) reflection over the y-axis
2) rotation of 90° clockwise about the origin
3) translation of 3 units right and 2 units down
4) dilation with a scale factor of 2 centered at the origin

5. The image of $\triangle DEF$ is $\triangle D'E'F'$. Under which transformation will he triangles *not* be congruent?
1) a reflection through the origin
2) a reflection over the line $y = x$
3) a dilation with a scale factor of 1 centered at $(2, 3)$
4) a dilation with a scale factor of $\frac{3}{2}$ centered at the origin

6. The vertices of $\triangle PQR$ have coordinates $P(2,3)$, $Q(3,8)$, and $R(7,3)$. Under which transformation of $\triangle PQR$ are distance and angle measure preserved?
1) $(x,y) \rightarrow (2x,3y)$ 2) $(x,y) \rightarrow (x+2,3y)$ 3) $(x,y) \rightarrow (2x,y+3)$ 4) $(x,y) \rightarrow (x+2,y+3)$

dilations dilation dilation

7. Which transformation would result in the perimeter of a triangle being different from the perimeter of its image?
1) $(x,y) \rightarrow (y,x)$
2) $(x,y) \rightarrow (x,-y)$
3) $(x,y) \rightarrow (4x,4y)$
4) $(x,y) \rightarrow (x+2,y-5)$

dilations

Rigid Motion Proofs

To prove triangles are congruent using rigid motions/transformations

1) A _______ and _______ are rigid motions.

2) **A rigid motion preserves size and angle measure producing a congruent figure.**

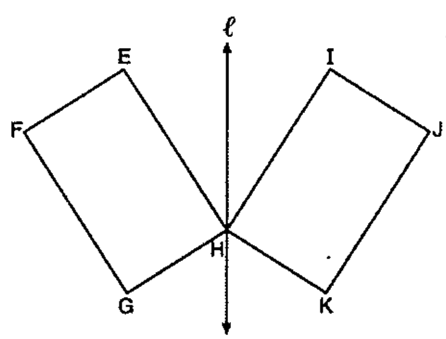
1. Triangle $A'B'C'$ is the image of triangle ABC after a translation of 2 units to the right and 3 units up. Is triangle ABC congruent to triangle $A'B'C'$? Explain why.

Yes, a translation is a rigid motion. A rigid motion preserves size and angle measure producing a congruent figure.

2. After a reflection over a line, $\Delta A'B'C'$ is the image of ΔABC. Explain why triangle ABC is congruent to triangle $\Delta A'B'C'$.

A reflection is a rigid motion. A rigid motion preserves size and angle measure producing a congruent figure.

3. In the diagram below, parallelogram *EFGH* is mapped onto parallelogram *IJKH* after a reflection over line ℓ. Use the properties of rigid motions to explain why parallelogram *EFGH* is congruent to parallelogram *IJKH*.



A reflection is a rigid motion. A rigid motion preserves size and angle measure producing a congruent figure.

4. The image of triangle ABC after a rotation of 200 degrees clockwise centered at the point (3,-1) is triangle DEF. Are the triangles congruent? Use the properties of rigid motions to explain your answer.

Yes, a rotation is a rigid motion. A rigid motion preserves size and angle measure producing a congruent figure.

Regular Polygon Rotations

To determine the minimum number of degrees a regular polygon must be rotated to be mapped onto itself:

1) The minimum rotation is $\frac{360}{n}$.
2) **Any multiple of that will also map the regular polygon onto itself!**

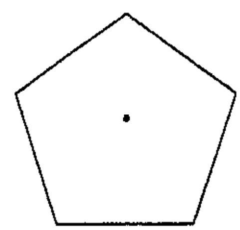
1. What is the minimum number of degrees a regular decagon must be rotated to be mapped onto itself?

$\frac{360}{n}$ $\frac{360}{10} = 36°$

2. What is the minimum number of degrees a regular hexagon must be rotated to be carried onto itself?

$\frac{360}{n}$ $\frac{360}{6} = 60°$

3. A regular pentagon is shown in the diagram below.



If the pentagon is rotated clockwise around its center, the minimum number of degrees it must be rotated to carry the pentagon onto itself is

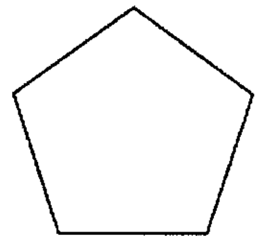
1) 54°
2) 72°
3) 108°
4) 360°

$\frac{360}{n}$ $\frac{360}{5} = 72°$

4. Which regular polygon has a minimum rotation of 45° to carry the polygon onto itself?

(1) octagon $\frac{360}{8} = 45$ (3) hexagon $\frac{360}{6} = 60$
(2) decagon $\frac{360}{10} = 36$ (4) pentagon $\frac{360}{5} = 72$

5. The regular polygon below is rotated about its center. Which angle of rotation will carry the figure onto itself?



1) 60°
2) 108°
3) 216° $\frac{216}{72}=3$ $\frac{360}{5}=72$
4) 540°

6. Which rotation would map a regular hexagon onto itself?

1) 45°
2) 150°
3) 240° $\frac{240}{60}=4$
4) 315°

$\frac{360}{6}=60$

7. Which rotation about its center will carry a regular decagon onto itself?

1) 54°
2) 162°
3) 198°
4) 252°

$\frac{360}{10}=36$ $\frac{252}{36}=7$

8. Which rotation about its center will carry a regular octagon onto itself?

1) 80°
2) 315° $\frac{315}{45}=7$ $\frac{360}{8}=45$
3) 280°
4) 120°

9. Which of the following rotations would <u>not</u> map a regular pentagon onto itself?

1) 144
2) 120
3) 216
4) 720

$\frac{360}{5}=72$

10. Which of the following rotations would not map an equilateral triangle onto itself?

1) 120°
2) 240°
3) 180°
4) 480°

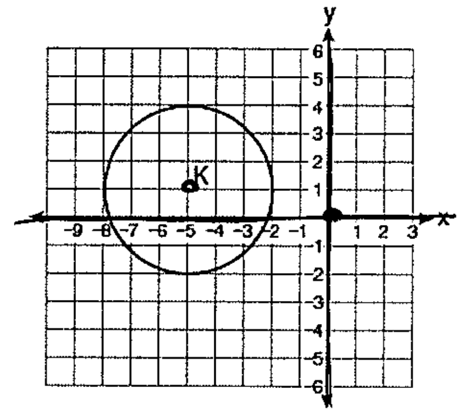
$\frac{360}{3}=120$

11. Which figure will not carry onto itself after a 120-degree rotation about its center?

1) equilateral triangle $\frac{360}{3}=120$ ✓
2) regular hexagon $\frac{360}{6}=60(2)=120$
3) regular octagon $\frac{360}{8}=45$ ✗
4) regular nonagon $\frac{360}{9}=40(3)=120$ ✓

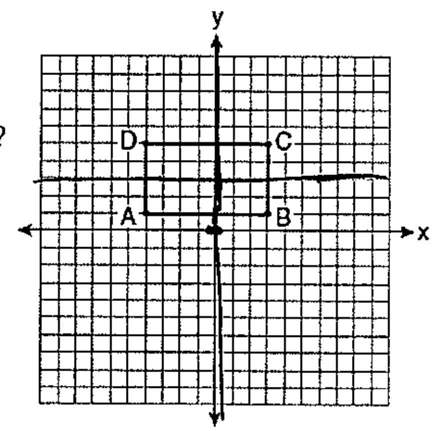
To map a shape onto itself:
Translation/Dilation: Never.
Reflection: **The line of reflection must be a line of symmetry** (cuts shape in half).
Rotation: **Center of rotation must be the center of the shape.** Use common sense for degree measure.

1. Circle K is shown in the graph below.
Which of the following transformations map circle K onto itself?



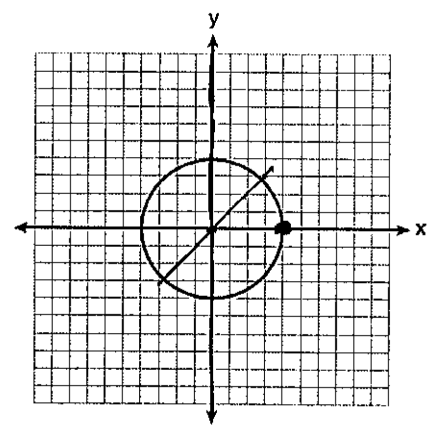
1) Reflection over the line $x-axis$
2) Reflection over the $y-axis$
3) Rotation of 90 centered at the origin
4) Rotation of 90 centered at K

2. On the set of axes below, Geoff drew rectangle *ABCD.*



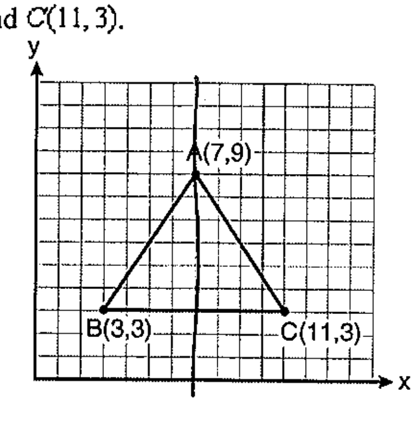
What of the following transformations would map the rectangle onto itself?
1) Reflection over the y axis
2) Reflection over the line $y=3$
3) Rotation of 180 centered at the origin
4) Translation one unit to the right

3. In the diagram below, which transformation does *not* map the circle onto itself?



1) Rotation of 80 centered at the origin
2) Reflection over the line $y=x$
3) Rotation of 180 centered at (4,0)
4) Reflection over the line $x=0$

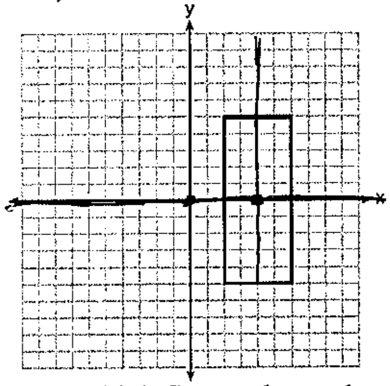
4. The vertices of the triangle in the diagram below are $A(7,9)$, $B(3,3)$, and $C(11,3)$.
Which transformation will map ΔABC onto itself?



1) Rotation of 60 centered at (3,3)
2) Reflection over the line $y=5$
3) Reflection over the line $x=7$
4) Translation 3 units up

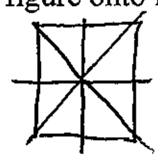
5. As shown in the graph below, the quadrilateral is a rectangle.
Which transformation would *not* map the rectangle onto itself?

1) a reflection over the x-axis
2) a reflection over the line $x = 4$
3) a rotation of 180° about the origin
4) a rotation of 180° about the point $(4, 0)$



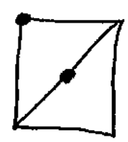
6. Which figure always has exactly four lines of reflection that map the figure onto itself?

1) square
2) rectangle
3) regular octagon
4) equilateral triangle



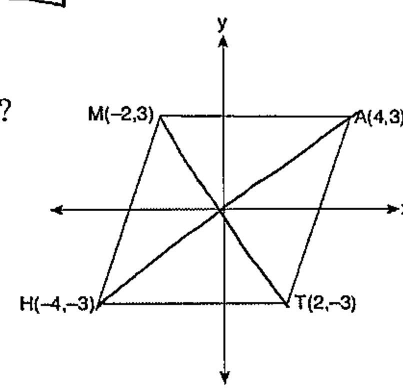
7. Which transformation would *not* carry a square onto itself?

1) a reflection over one of its diagonals
2) a 90° rotation clockwise about its center
3) a 180° rotation about one of its vertices
4) a reflection over the perpendicular bisector of one side



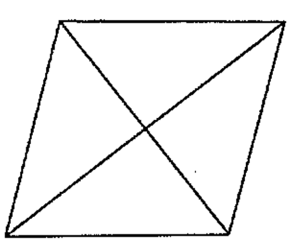
8. Which transformation carries the parallelogram below onto itself?

1) a reflection over $y = x$
2) a reflection over $y = -x$
3) a rotation of 90° counterclockwise about the origin
4) a rotation of 180° counterclockwise about the origin



9. The figure below shows a rhombus with noncongruent diagonals.
Which transformation would *not* carry this rhombus onto itself?

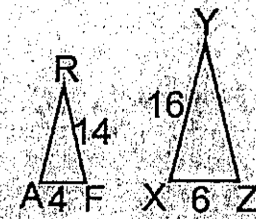
1) a reflection over the shorter diagonal
2) a reflection over the longer diagonal
3) a clockwise rotation of 90° about the intersection of the diagonals
4) a counterclockwise rotation of 180° about the intersection of the diagonals



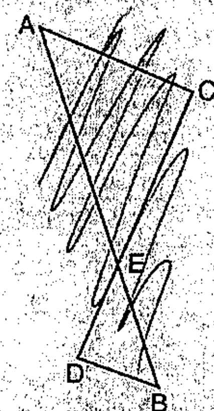
Scale factor $= \dfrac{image}{original}$

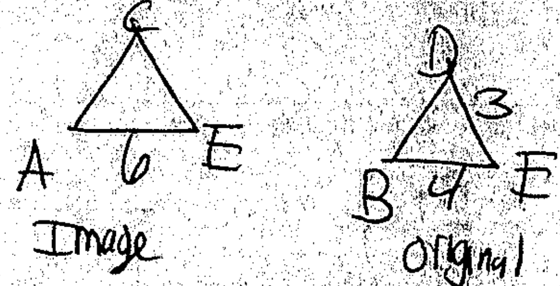
1. In the diagram below, $\triangle XYZ$ is the image of $\triangle ARF$ after a dilation.

What is the scale factor of the dilation? $\frac{6}{4}=\frac{3}{2}$



2. In the diagram below, $\triangle ACE$ is the image of $\triangle BDE$ after a sequence of transformations. If $\overline{AE}=6$, $\overline{DE}=3$, and $\overline{EB}=4$, what is the scale factor?

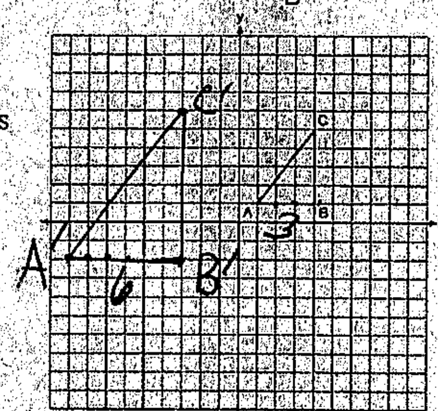




$\frac{6}{4}=3$

3. In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. The coordinates of its image after a sequence of transformations is $A'(-9,-2)$, $B'(-3,-2)$, *and* $C'(-3,6)$. What is the scale factor?

$\frac{6}{3}=2$



4. After a dilation with center $(0,0)$, the image of $\overline{DB}$ is $\overline{D'B'}$. If $DB=4.5$ and $D'B'=18$, the scale factor of this dilation is

1) $\frac{1}{5}$

2) 5

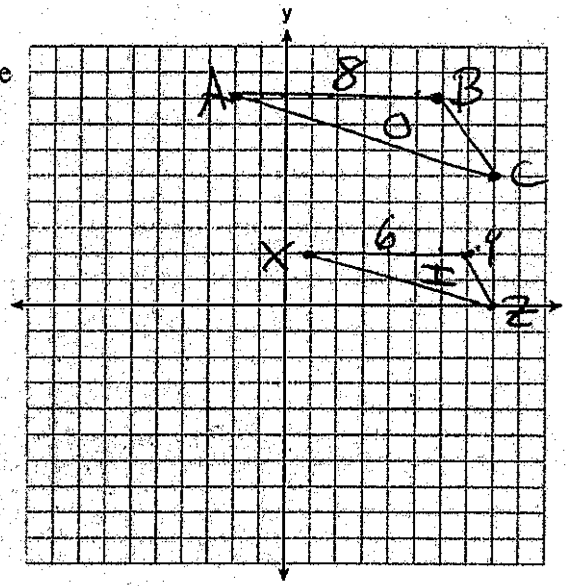
3) $\frac{1}{4}$

4) 4

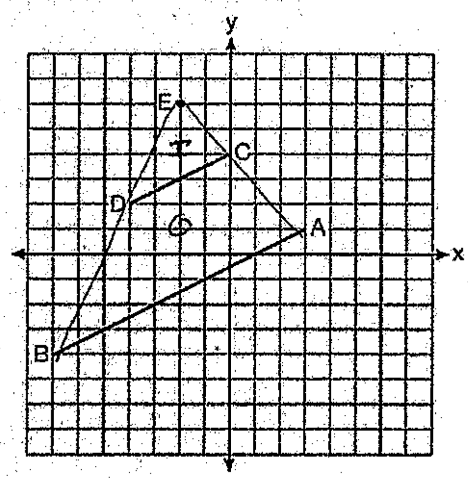
$\frac{18}{4.5}=4$

5. $\triangle ABC$ has coordinates $A(-2,8), B(6,8)$, *and* $C(8,5)$. The coordinates of $\triangle XYZ$, the image of $\triangle ABC$ after a sequence of transformations is $X(1,2), Y(7,2)$, *and* $Z(8,0)$. What is the scale factor?

$\frac{6}{8} = \frac{3}{4}$



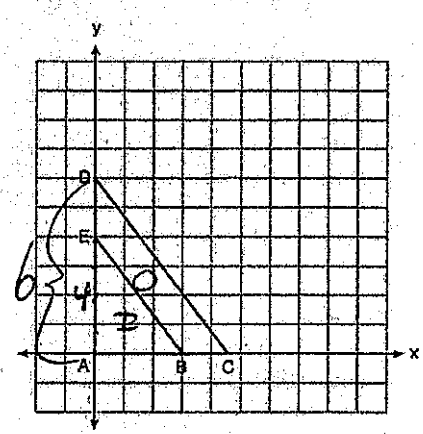
6. In the diagram below, $\overline{CD}$ is the image of $\overline{AB}$ after a dilation of scale factor k with center E.



Which ratio is equal to the scale factor k of the dilation?

1) $\frac{EC}{EA}$ $\frac{I}{O}$
2) $\frac{BA}{EA}$ $\frac{O}{O}$
3) $\frac{EA}{BA}$ $\frac{O}{O}$
4) $\frac{EA}{EC}$ $\frac{O}{I}$

7. In the diagram below, $\triangle ABE$ is the image of $\triangle ACD$ after a dilation centered at the origin. The coordinates of the vertices are $A(0,0)$, $B(3,0)$, $C(4.5,0)$, $D(0,6)$, and $E(0,4)$.



The scale factor of dilation is

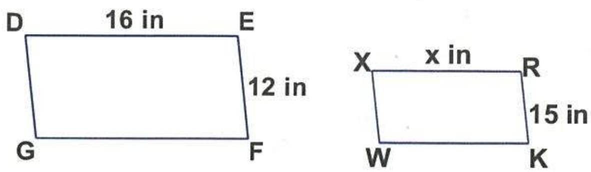
1) $\frac{2}{3}$
2) $\frac{3}{2}$
3) $\frac{3}{4}$
4) $\frac{4}{3}$

$\frac{4}{6} = \frac{2}{3}$

Similar Triangles with Parallel Lines

If the lines are parallel, the triangles are similar and the sides are in proportion.

1. Parallelogram DEFG is similar to parallelogram XRKW. Find x.

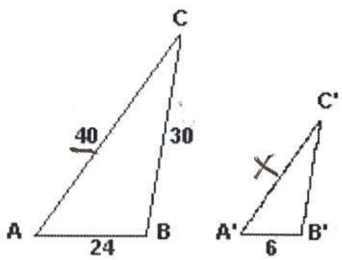


$\frac{16}{x}=\frac{12}{15}$

$\frac{12x}{12}=\frac{240}{12}$

$x=20$

2. In the diagram, ΔABC is similar to $\Delta A'B'C'$, $AB = 24$, $BC = 30$, and $CA = 40$. If the shortest side of $\Delta A'B'C'$ is 6, find the length of the longest side of $\Delta A'B'C'$.

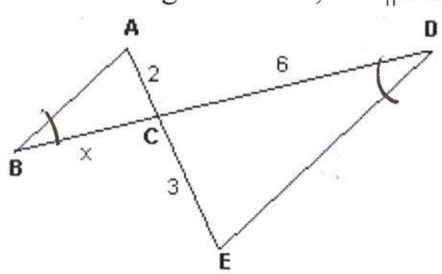


$\frac{40}{x}=\frac{24}{6}$

$\frac{24x}{24}=\frac{240}{24}$

$x=10$

3. In the diagram below, $\overline{AB} \parallel \overline{DE}$. If $AC = 2$, $CD = 6$, and $CE = 3$, what is BC ?



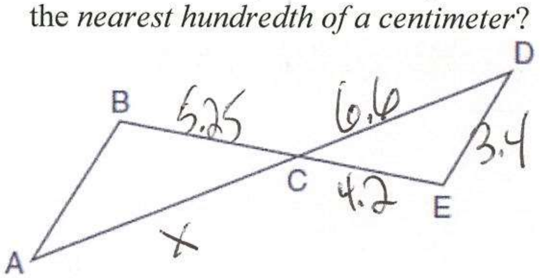
$\frac{x}{6}=\frac{2}{3}$

$\frac{3x}{3}=\frac{12}{3}$

$x=4$

4. In the diagram below, $\overline{AD}$ intersects $\overline{BE}$ at C, and $\overline{AB} \parallel \overline{DE}$.

If $CD = 6.6$ cm, $DE = 3.4$ cm, $CE = 4.2$ cm, and $BC = 5.25$ cm, what is the length of $\overline{AC}$, to the *nearest hundredth of a centimeter*?

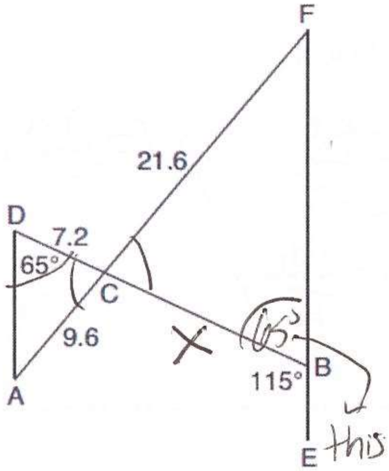


$\frac{x}{6.6}=\frac{5.25}{4.2}$

$\frac{4.2x}{4.2}=\frac{34.65}{4.2}$

$x=8.25$

5. In the diagram below, $\overline{AF}$, and $\overline{DB}$ intersect at C, and $\overline{AD}$ and $\overline{FBE}$ are drawn such that $m\angle D = 65°$, $m\angle CBE = 115°$, $DC = 7.2$, $AC = 9.6$, and $FC = 21.6$. What is the length of $\overline{CB}$?



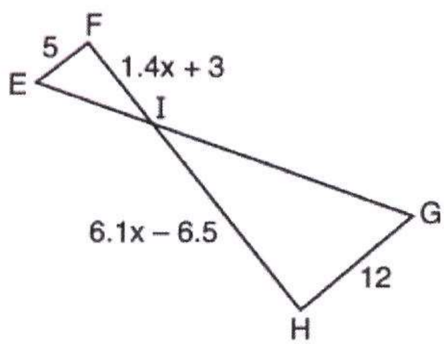
this makes the triangles similar

$$\frac{7.2}{x} = \frac{9.6}{21.6}$$

$$\frac{9.6x}{9.6} = \frac{155.52}{9.6}$$

$$x = 16.2$$

6. In the diagram below, $\overline{EF} \parallel \overline{HG}$, $EF = 5$, $HG = 12$, $FI = 1.4x + 3$, and $HI = 6.1x - 6.5$. What is the length of $\overline{HI}$?



*use equation solver if needed

$$\frac{5}{12} = \frac{1.4x+3}{6.1x-6.5}$$

$$12(1.4x+3) = 5(6.1x-6.5)$$

$$16.8x + 36 = 30.5x - 32.5$$

$$-16.8x \qquad -16.8x$$

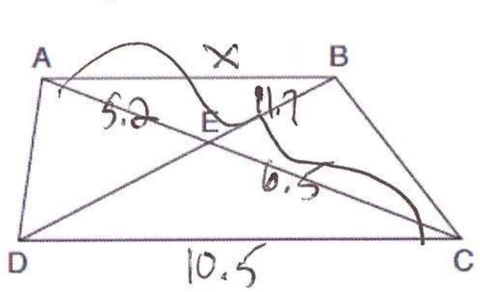
$$36 = 13.7x - 32.5$$

$$+32.5 \qquad +32.5$$

$$\frac{68.5}{13.7} = \frac{13.7x}{13.7}$$

$$5 = x$$

7. In trapezoid $ABCD$ below, $\overline{AB} \parallel \overline{CD}$.



$$\begin{array}{r} 11.7 \\ -5.2 \\ \hline 6.5 \end{array}$$

$$\frac{5.2}{6.5} = \frac{x}{10.5}$$

$$\frac{6.5x}{6.5} = \frac{54.6}{6.5}$$

$$x = 8.4$$

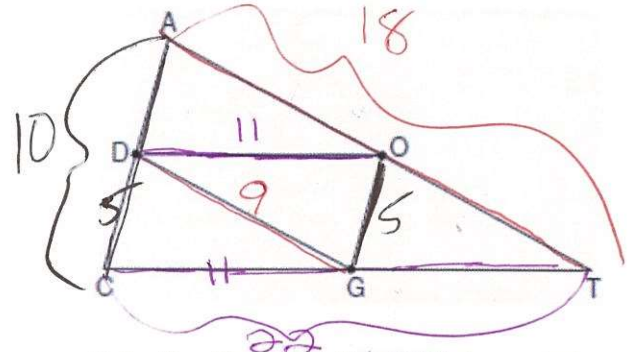
If $AE = 5.2$, $AC = 11.7$, and $CD = 10.5$, what is the length of $\overline{AB}$, to the *nearest tenth*?
1) 4.7 2) 6.5 3) 8.4 4) 13.1

Joining the Midpoints of a Triangle

The midsegments are half of the opposite parallel sides

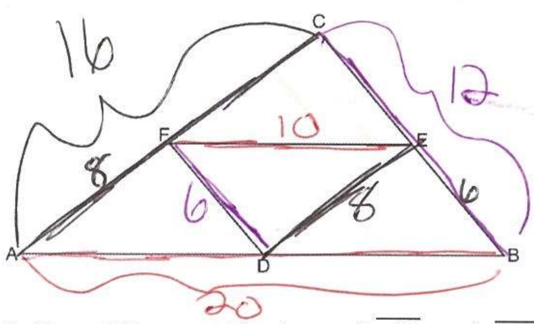
$2(midsegment) = opposite\ side$

1. In the diagram below of $\triangle ACT$, D is the midpoint of $\overline{AC}$, O is the midpoint of $\overline{AT}$, and G is the midpoint of $\overline{CT}$. If $AC = 10$, $AT = 18$, and $CT = 22$, what is the perimeter of parallelogram $CDOG$?



5+11+5+11= 32

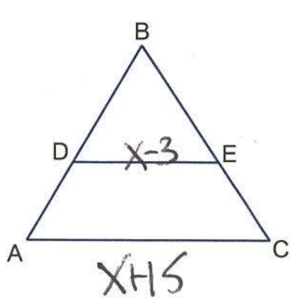
2. In the diagram of $\triangle ABC$ shown below, D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$, and F is the midpoint of $\overline{AC}$. If $AB = 20$, $BC = 12$, and $AC = 16$, what is the perimeter of trapezoid $ABEF$?



20+8+6+10

44

3. D and E are midpoints of $\overline{AB}$ and $\overline{BC}$ respectively. If $\overline{AC} = x+15$ and $\overline{DE} = x-3$, find the measure of $\overline{DE}$.



2(midsegment) = opposite side

$2(x-3) = x+15$

$2x-6 = x+15$

$-x \quad -x$

$x-6 = 15$

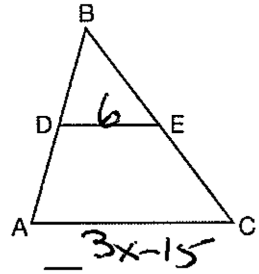
$+6 \quad +6$

$x=21$

$\overline{DE} = 21-3$

$\overline{DE} = 18$

4. In $\triangle ABC$, D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{BC}$. If $AC = 3x - 15$ and $DE = 6$, what is the value of x?

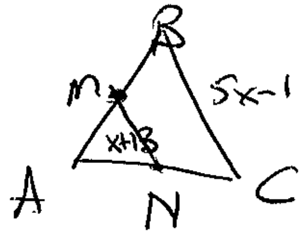


1) 6
2) 7
3) 9
4) 12

2(midsegment) = opposite side
$2(6) = 3x - 15$
$12 = 3x - 15$
$+15 \quad +15$
$\frac{27}{3} = \frac{3x}{3}$
$9 = x$

5. In $\triangle ABC$, M is the midpoint of $\overline{AB}$ and N is the midpoint of $\overline{AC}$. If $MN = x + 13$ and $BC = 5x - 1$, what is the length of $\overline{MN}$?

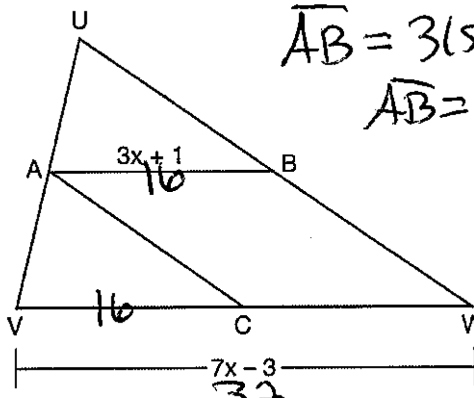
1) 3.5
2) 9
3) 16.5
4) 22



2(midsegment) = opposite side
$2(x+13) = 5x - 1$
$2x + 26 = 5x - 1$
$-2x \quad -2x$
$26 = 3x - 1$
$+1 \quad +1$
$\frac{27}{3} = \frac{3x}{3}$
$9 = x$

$MN = 9 + 13$
$MN = 22$

6. In the diagram of $\triangle UVW$ below, A is the midpoint of $\overline{UV}$, B is the midpoint of $\overline{UW}$, C is the midpoint of $\overline{VW}$, and $\overline{AB}$ and $\overline{AC}$ are drawn.



$\overline{AB} = 3(5) + 1$
$\overline{AB} = 16$

If $VW = 7x - 3$ and $AB = 3x + 1$, what is the length of $\overline{VC}$?

1) 5
2) 13
3) 16
4) 32

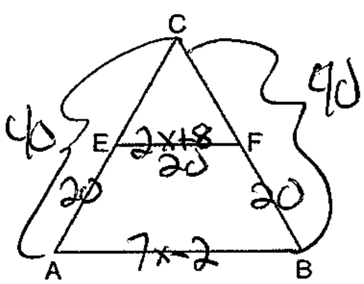
2(midsegment) = opposite side
$2(3x+1) = 7x - 3$
$6x + 2 = 7x - 3$
$-6x \quad -6x$
$2 = x - 3$
$+3 \quad +3$
$5 = x$

7. In the diagram of equilateral triangle ABC shown below, E and F are the midpoints of $\overline{AC}$ and $\overline{BC}$, respectively.

If $EF = 2x + 8$ and $AB = 7x - 2$, what is the perimeter of trapezoid $ABFE$?

1) 36
2) 60
3) 100
4) 120

$7(6) - 2 = 40$
$2(6) + 8 = 20$



2(midsegment) = opposite side
$2(2x+8) = 7x - 2$
$4x + 16 = 7x - 2$
$-4x \quad -4x$
$16 = 3x - 2$
$+2 \quad +2$
$\frac{18}{3} = \frac{3x}{3}$
$6 = x$

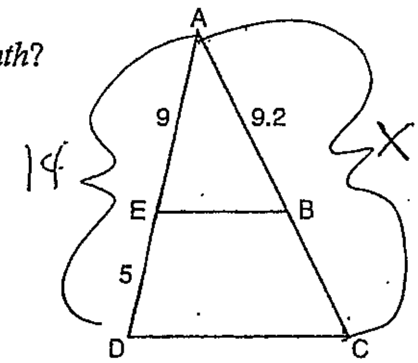
$20 + 20 + 20 + 40$
$\boxed{100}$

Candy Corn Problems

If the bases are not involved: $\frac{top}{top} = \frac{bottom}{bottom} = \frac{side}{side}$

If bases are involved: separate your triangles!

1. In the diagram of $\triangle ADC$ below, $\overline{EB} \parallel \overline{DC}$, $AE = 9$, $ED = 5$, and $AB = 9.2$.



What is the length of $\overline{AC}$, to the *nearest tenth*?

1) 5.1
2) 5.2
3) 14.3
4) 14.4

bases not involved

$\frac{top}{top} = \frac{side}{side}$

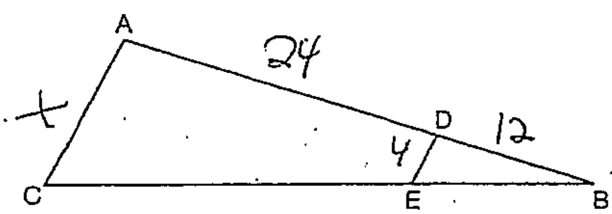
$\frac{9}{9.2} = \frac{14}{x}$

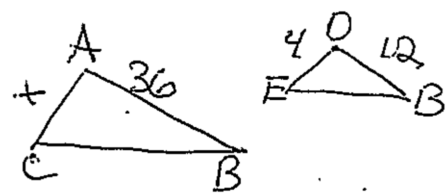
$\frac{9x}{9} = \frac{128.8}{9}$

$x = 14.3$

2. In the diagram of $\triangle ABC$, points D and E are on $\overline{AB}$ and $\overline{CB}$, respectively, such that $\overline{AC} \parallel \overline{DE}$.

bases involved
separate





If $AD = 24$, $DB = 12$, and $DE = 4$, what is the length of $\overline{AC}$?

1) 8
2) 12
3) 16
4) 72

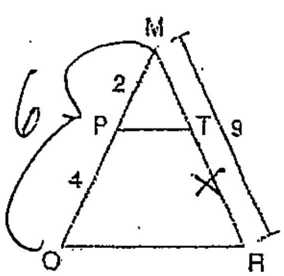
$\frac{x}{4} = \frac{36}{12}$

$\frac{12x}{12} = \frac{144}{12}$

$x = 12$

3. Given $\triangle MRO$ shown below, with trapezoid $PTRO$, $MR = 9$, $MP = 2$, and $PO = 4$.

bases not involved



$\frac{bottom}{bottom} = \frac{side}{side}$

$\frac{9}{x} = \frac{6}{9}$

What is the length of $\overline{TR}$?

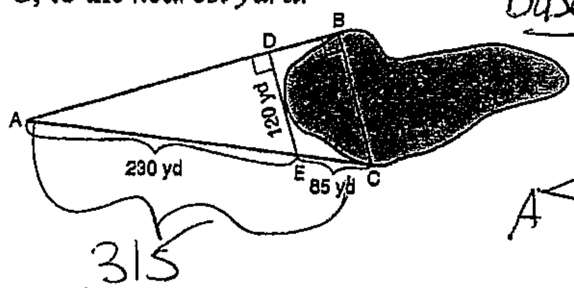
1) 4.5
2) 5
3) 3
4) 6

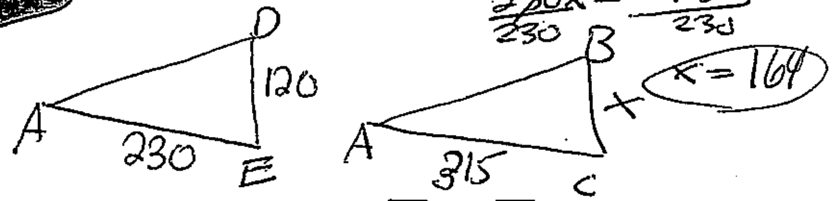
$\frac{6x}{6} = \frac{36}{6}$

$x = 6$

4. To find the distance across a pond from point B to point C, a surveyor drew the diagram below. The measurements he made are indicated on his diagram.
Use the surveyor's information to determine and state the distance from point B to point C, to the *nearest yard.*

bases involved
separate



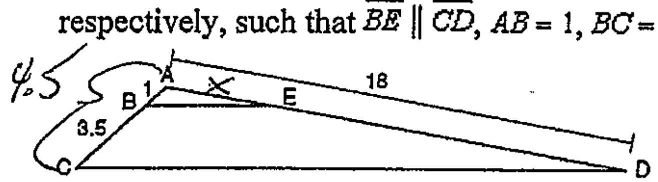


$\frac{230}{315} = \frac{120}{x}$

$\frac{230x}{230} = \frac{37800}{230}$

$x = 164$

5. In the diagram below, triangle ACD has points B and E on sides $\overline{AC}$ and $\overline{AD}$, respectively, such that $\overline{BE} \parallel \overline{CD}$, $AB = 1$, $BC = 3.5$, and $AD = 18$.



bases not involved

$\frac{\text{top}}{\text{top}} = \frac{\text{side}}{\text{side}}$

$\frac{1}{x} = \frac{4.5}{18}$

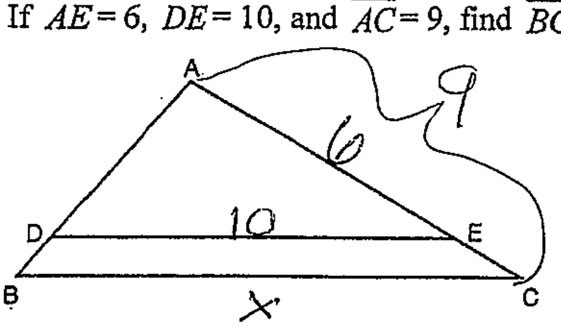
$\frac{4.5x}{4.5} = \frac{18}{4.5}$

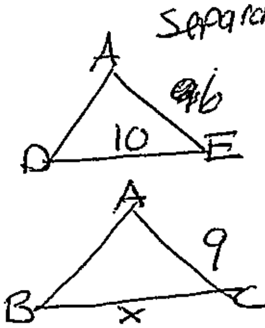
$x = 4.0$

What is the length of $\overline{AE}$, to the *nearest tenth*?

6. In the diagram of $\triangle ABC$ shown below, $\overline{DE} \parallel \overline{BC}$. bases involved
If $\overline{AE} = 6$, $\overline{DE} = 10$, and $\overline{AC} = 9$, find $\overline{BC}$

separate





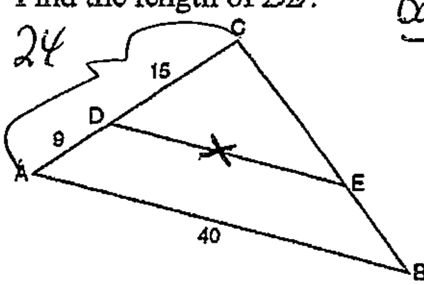
$\frac{10}{x} = \frac{6}{9}$

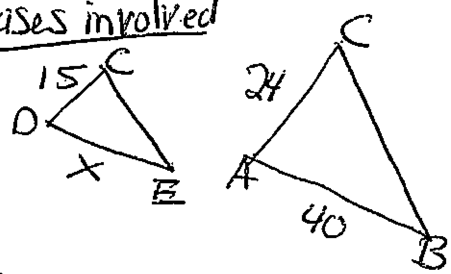
$\frac{6x}{6} = \frac{90}{6}$

$x = 15$

7. In the diagram of $\triangle ABC$ below, $\overline{DE}$ is parallel to $\overline{AB}$, $CD = 15$, $AD = 9$, and $AB = 40$.
Find the length of $\overline{DE}$.

bases involved



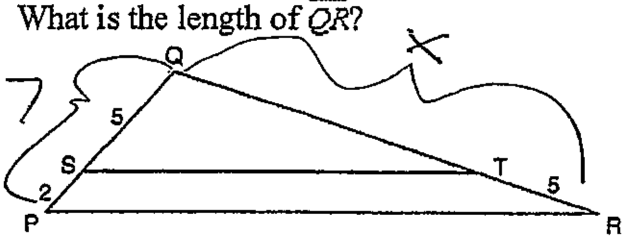


$\frac{15}{24} = \frac{x}{40}$

$\frac{24x}{24} = \frac{600}{24}$

$x = 25$

8. In the diagram below of $\triangle PQR$, $\overline{ST}$ is drawn parallel to $\overline{PR}$, $PS = 2$, $SQ = 5$, and $TR = 5$
What is the length of $\overline{QR}$?



bases not involved

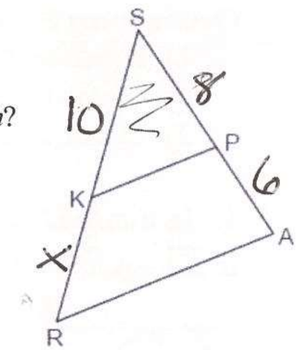
$\frac{\text{side}}{\text{side}} = \frac{\text{bottom}}{\text{bottom}}$

$\frac{7}{x} = \frac{2}{5}$

$\frac{2x}{2} = \frac{35}{2}$

$x = 17.5$

9. In the diagram of $\triangle SRA$ below, $\overline{KP}$ is drawn such that $\angle SKP \cong \angle SRA$.



If $SK = 10$, $SP = 8$, and $PA = 6$, what is the length of $\overline{KR}$, to the *nearest tenth*?

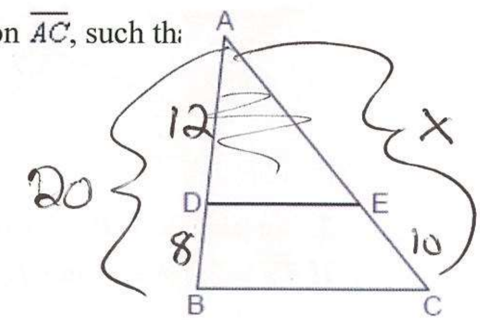
1) 4.8
2) 7.5
3) 8.0
4) 13.3

$\frac{\text{top}}{\text{top}} = \frac{\text{bottom}}{\text{bottom}}$

$\frac{10}{8} = \frac{x}{6}$ $\quad \frac{8x}{8} = \frac{60}{8}$

$x = 7.5$

10. In triangle ABC below, D is a point on $\overline{AB}$ and E is a point on $\overline{AC}$, such th



If $AD = 12$, $DB = 8$, and $EC = 10$, what is the length of $\overline{AC}$?

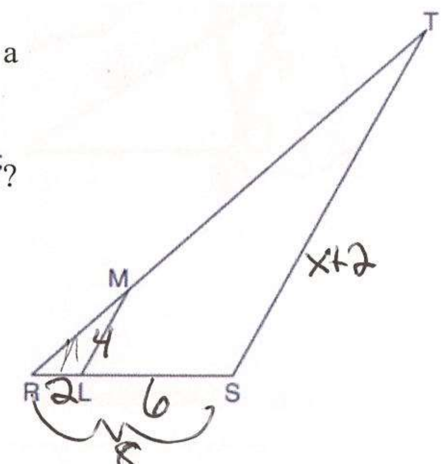
1) 15
2) 22
3) 24
4) 25

$\frac{\text{side}}{\text{side}} = \frac{\text{bottom}}{\text{bottom}}$

$\frac{20}{x} = \frac{8}{10}$ $\quad \frac{8x}{8} = \frac{200}{8}$

$x = 25$

11. In the diagram below of $\triangle RST$, L is a point on $\overline{RS}$, and M is a point on $\overline{RT}$, such that $LM \parallel ST$.



If $RL = 2$, $LS = 6$, $LM = 4$, and $ST = x + 2$, what is the length of $\overline{ST}$?

1) 10 2) 12 3) 14 4) 16

bases involved!

$\frac{4}{x+2} = \frac{2}{8}$

$2(x+2) = 32$

$2x + 4 = 32$
$-4 \quad -4$

$\frac{2x}{2} = \frac{28}{2}$

$x = 14$

12. In the diagram below of $\triangle CER$, $\overline{LA} \parallel \overline{CR}$.

If $CL = 3.5$, $LE = 7.5$, and $EA = 9.5$, what is the length of $\overline{AR}$, to the *nearest tenth*?

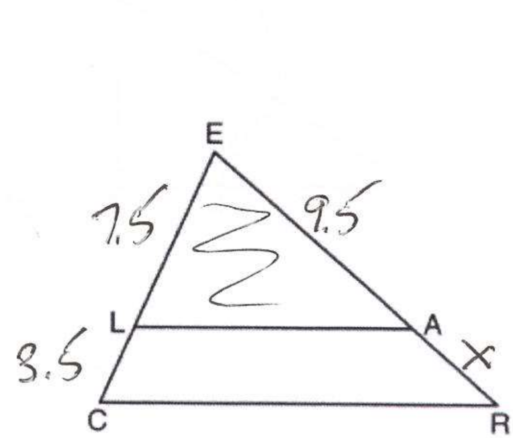
1) 5.5
2) 4.4
3) 3.0
4) 2.8

$\frac{\text{top}}{\text{top}} = \frac{\text{bottom}}{\text{bottom}}$

$\frac{7.5}{9.5} = \frac{3.5}{x}$

$\frac{7.5x}{7.5} = \frac{33.25}{7.5}$

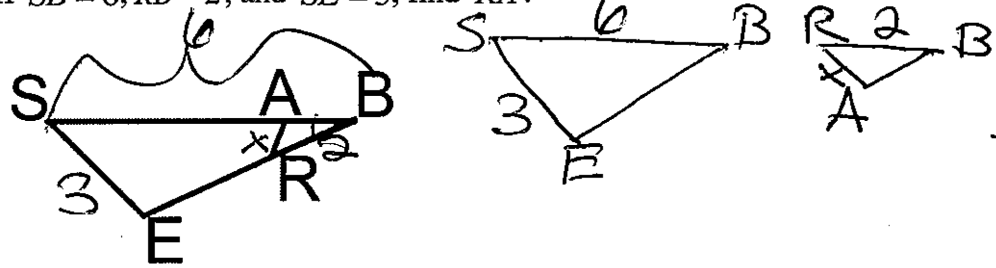
$x = 4.4$



Overlapping Similar Triangles

1) Separate the triangles and draw them with the same orientation
2) Match up the corresponding letters (use reflexive property)
3) Create a proportion and solve

1. In triangle *SEB*, *A* is on $\overline{SB}$, and *E* is on $\overline{EB}$ so that $\angle E \cong \angle BAR$.
If $\overline{SB} = 6, \overline{RB} = 2$, and $\overline{SE} = 3$, find $\overline{RA}$.

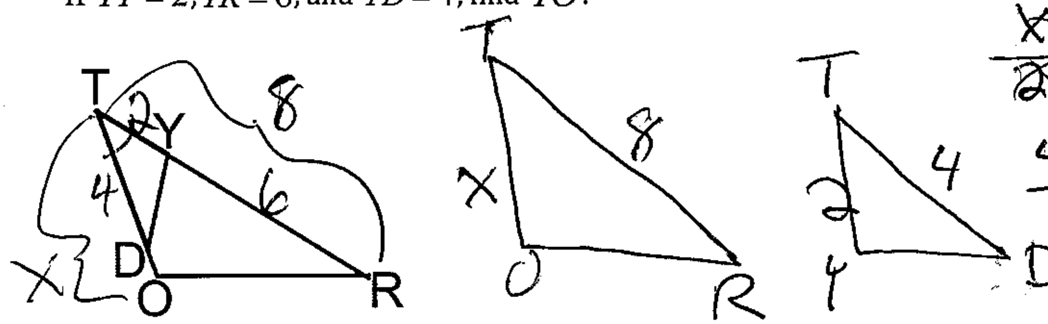


$\frac{6}{2} = \frac{3}{x}$

$\frac{6x}{6} = \frac{6}{6}$

$x = 1$

2. In triangle *TOR*, *Y* is on $\overline{TR}$, and *D* is on $\overline{TO}$ so that $\angle TYD \cong \angle ROT$.
If $\overline{TY} = 2, \overline{YR} = 6$, and $\overline{TD} = 4$, find $\overline{TO}$.

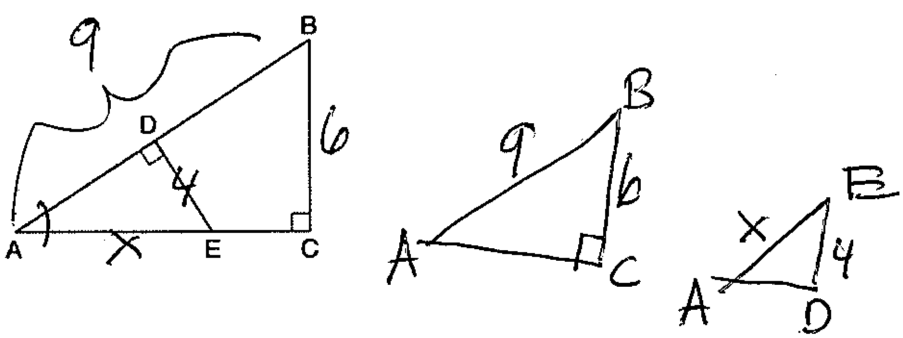


$\frac{x}{2} = \frac{8}{4}$

$\frac{4x}{4} = \frac{16}{4}$

$x = 4$

3. In $\triangle ABC$ shown below, $\angle ACB$ is a right angle, *E* is a point on $\overline{AC}$, and $\overline{ED}$ is drawn perpendicular to hypotenuse $\overline{AB}$. If $AB = 9$, $BC = 6$, and $DE = 4$, what is the length of $\overline{AE}$?

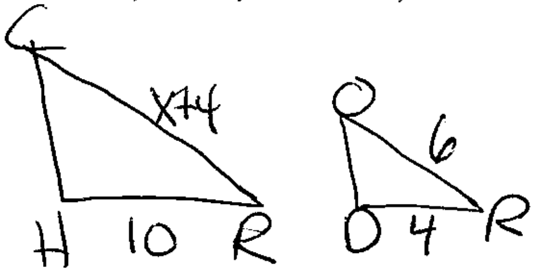


$\frac{9}{x} = \frac{6}{4}$

$\frac{6x}{6} = \frac{36}{6}$

$x = 6$

4. In triangle *CHR*, *O* is on $\overline{HR}$, and *D* is on $\overline{CR}$ so that $\angle H \cong RDO$. If $RD = 4$, $RO = 6$, and $OH = 4$, what is the length of $\overline{CD}$?



$\frac{x+4}{6} = \frac{10}{4}$

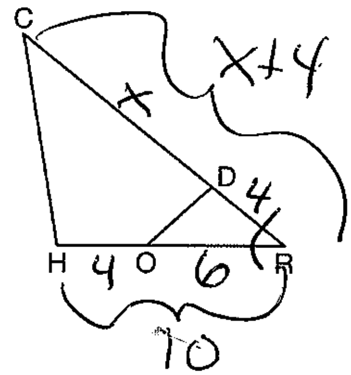
$4(x+4) = 60$

$4x + 16 = 60$

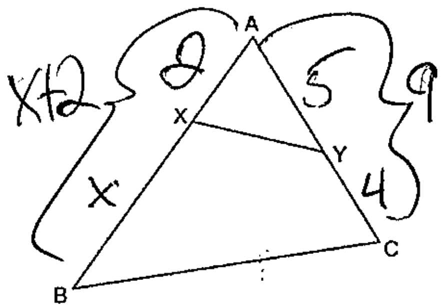
$-16 \quad -16$

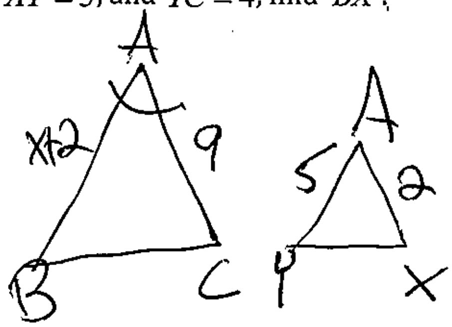
$\frac{4x}{4} = \frac{44}{4}$

$x = 11$



5. In the diagram below of $\triangle ABC$, *X* and *Y* are points on $\overline{AB}$ and $\overline{AC}$, respectively, such that $m\angle AYX = m\angle B$. If $\overline{AX} = 2$, $\overline{AY} = 5$, and $\overline{YC} = 4$, find $\overline{BX}$.





$\frac{x+2}{5} = \frac{9}{2}$

$2(x+2) = 45$

$2x + 4 = 45$

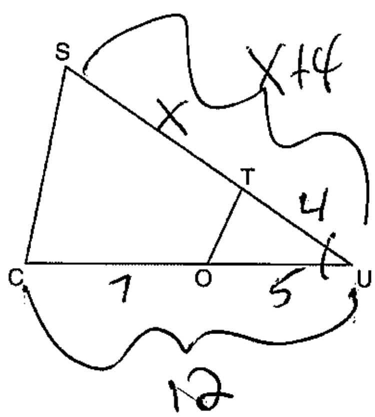
$-4 \quad -4$

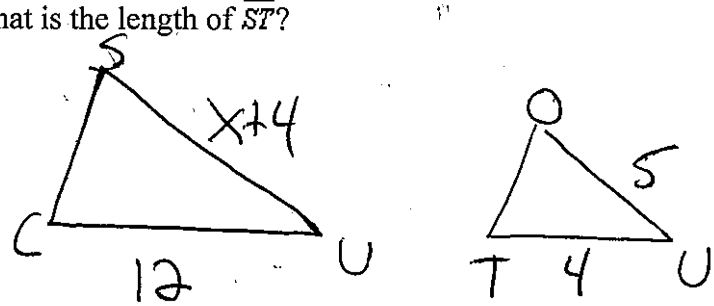
$\frac{2x}{2} = \frac{41}{2}$

$x = 20.5$

6. In $\triangle SCU$ shown below, points *T* and *O* are on $\overline{SU}$ and $\overline{CU}$, respectively. Segment *OT* is drawn so that $\angle C \cong \angle OTU$.

If $TU = 4$, $OU = 5$, and $OC = 7$, what is the length of $\overline{ST}$?





$\frac{x+4}{5} = \frac{12}{4}$

$4(x+4) = 60$

$4x + 16 = 60$

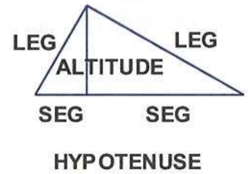
$-16 \quad -16$

$\frac{4x}{4} = \frac{44}{4}$

$x = 11$

When an altitude is drawn to a right triangle
HLLS and SAAS

$\frac{H}{L} = \frac{L}{S} \quad \frac{S}{A} = \frac{A}{S}$

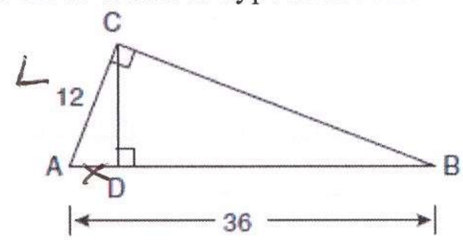


If L is involved, use HLLS
If A is involved, use SAAS

Know how to reduce radicals:

1) Separate into perfect square and non perfect square
2) Take the square root of the perfect square

1. In the diagram below of right triangle ACB, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.



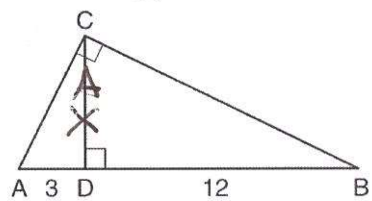
If $AB = 36$ and $AC = 12$, what is the length of $\overline{AD}$?

1) 32
2) 6
3) 3
4) 4

$\frac{H}{L} = \frac{L}{S}$ $\quad \frac{36}{12} = \frac{12}{x}$

$\frac{36x}{36} = \frac{144}{36}$ $\quad x = 4$

2. In the diagram below of right triangle ABC, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.



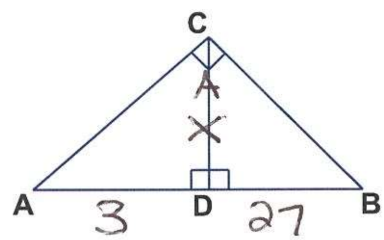
If $AD = 3$ and $DB = 12$, what is the length of altitude $\overline{CD}$?

1) 6
2) $6\sqrt{5}$
3) 3
4) $3\sqrt{5}$

$\frac{S}{A} = \frac{A}{S}$ $\quad \frac{3}{x} = \frac{x}{12}$ $\quad \sqrt{x^2} = \sqrt{36}$

$x = 6$

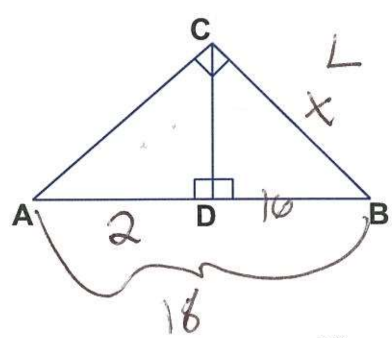
3. If $\overline{AD} = 3$ and $\overline{DB} = 27$, find $\overline{CD}$



$\frac{S}{A} = \frac{A}{S}$ $\quad \frac{3}{x} = \frac{x}{27}$ $\quad \sqrt{x^2} = \sqrt{81}$

$x = 9$

4. If $\overline{AD} = 2$ and $\overline{AB} = 18$, find $\overline{BC}$ to the nearest tenth

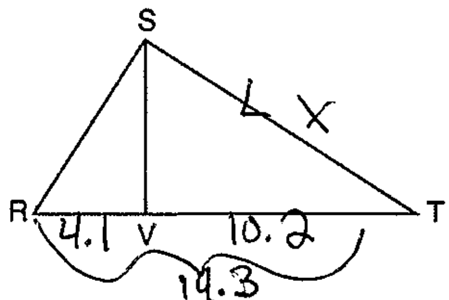


$\frac{H}{L} = \frac{L}{S}$ $\quad \frac{18}{x} = \frac{x}{16}$

$\sqrt{x^2} = \sqrt{288}$

$x = 17.0$

5. In right triangle RST below, altitude $\overline{SV}$ is drawn to hypotenuse $\overline{RT}$. If $RV = 4.1$ and $TV = 10.2$, what is the length of $\overline{ST}$, to the *nearest tenth*?



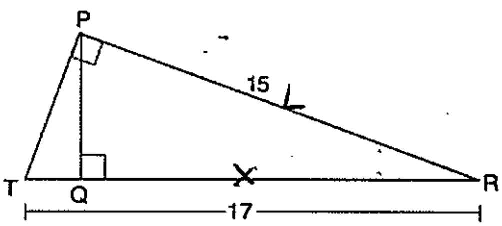
$\frac{H}{L}=\frac{L}{S}$

$\frac{14.3}{x}=\frac{x}{10.2}$

$\sqrt{x^2}=\sqrt{145.86}$

$x=12.1$

6. In right triangle PRT, $m\angle P = 90°$, altitude $\overline{PQ}$ is drawn to hypotenuse $\overline{RT}$, $RT = 17$, and $PR = 15$. Determine and state, to the *nearest tenth*, the length of $\overline{RQ}$.



$\frac{H}{L}=\frac{L}{S}$

$\frac{17}{15}=\frac{15}{x}$

$\frac{17x}{17}=\frac{225}{17}$

$x=13.2$

PS
1
4
9
16
25
36
49
64
81
100

7. Reduce the following radicals:

$\sqrt{45}$ — $\sqrt{9}\sqrt{5}$ — $3\sqrt{5}$

$\sqrt{50}$ — $\sqrt{25}\sqrt{2}$ — $5\sqrt{2}$

$\sqrt{162}$ — $\sqrt{81}\sqrt{2}$ — $9\sqrt{2}$

8. Triangle ABC shown below is a right triangle with altitude $\overline{AD}$ drawn to the hypotenuse $\overline{BC}$.

If $BD = 2$ and $DC = 10$, what is the length of $\overline{AB}$?

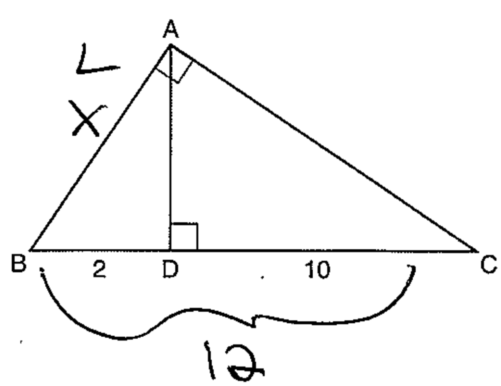
1) $2\sqrt{2}$
2) $2\sqrt{5}$
3) $2\sqrt{6}$
4) $2\sqrt{30}$

$\frac{12}{x}=\frac{x}{2}$

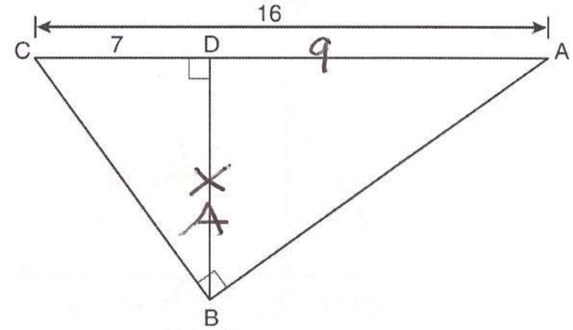
$\sqrt{x^2}=\sqrt{24}$

$\sqrt{4}\sqrt{6}$

$2\sqrt{6}$



9. In the diagram below of right triangle *ABC*, altitude $\overline{BD}$ is drawn to hypotenuse $\overline{AC}$, $AC = 16$, and $CD = 7$.



What is the length of $\overline{BD}$?

1) $3\sqrt{7}$
2) $4\sqrt{7}$
3) $7\sqrt{3}$
4) 12

$\frac{S}{A} = \frac{A}{S}$

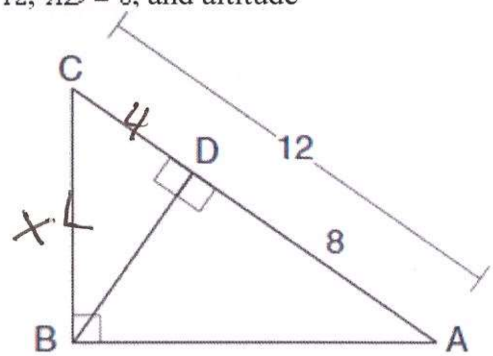
$\frac{7}{x} = \frac{x}{9}$

$\sqrt{x^2} = \sqrt{63}$

$\sqrt{9}\sqrt{7}$

$3\sqrt{7}$

10. In the diagram below of $\triangle ABC$, $\angle ABC$ is a right angle, $AC = 12$, $AD = 8$, and altitude $\overline{BD}$ is drawn.



What is the length of $\overline{BC}$?

1) $4\sqrt{2}$
2) $4\sqrt{3}$
3) $4\sqrt{5}$
4) $4\sqrt{6}$

$\frac{H}{L} = \frac{L}{S}$

$\frac{12}{x} = \frac{x}{4}$

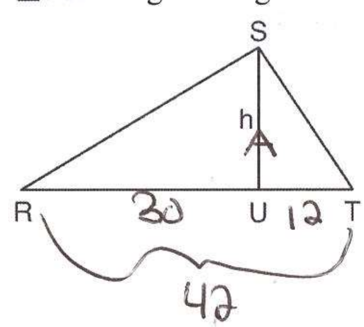
$\sqrt{x^2} = \sqrt{48}$

$\sqrt{16}\ \sqrt{3}$

$4\sqrt{3}$

11. In $\triangle RST$ shown below, altitude $\overline{SU}$ is drawn to $\overline{RT}$ at *U*.
If $SU = h$, $UT = 12$, and $RT = 42$, which value of *h* will make $\triangle RST$ a right triangle with $\angle RST$ as a right angle?

1) $6\sqrt{3}$
2) $6\sqrt{10}$
3) $6\sqrt{14}$
4) $6\sqrt{35}$



$\frac{S}{A} = \frac{A}{S}$

$\frac{30}{h} = \frac{h}{12}$

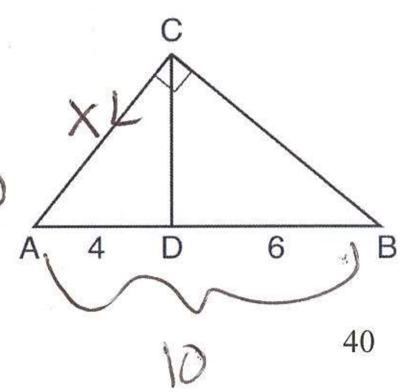
$\sqrt{h^2} = \sqrt{360}$

$\sqrt{36}\sqrt{10}$

$6\sqrt{10}$

12. In the diagram of right triangle *ABC*, $\overline{CD}$ intersects hypotenuse $\overline{AB}$ at *D*.
If $AD = 4$ and $DB = 6$, which length of $\overline{AC}$ makes $\overline{CD} \perp \overline{AB}$?

1) $2\sqrt{6}$
2) $2\sqrt{10}$
3) $2\sqrt{15}$
4) $4\sqrt{2}$



$\frac{H}{L} = \frac{L}{S}$

$\frac{10}{x} = \frac{x}{4}$

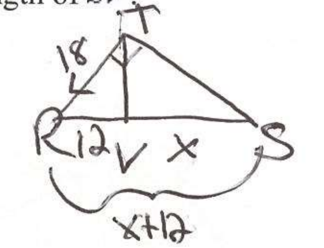
$\sqrt{x^2} = \sqrt{40}$

$\sqrt{4}\ \sqrt{10}$

$2\sqrt{10}$

13. In right triangle RST, altitude $\overline{TV}$ is drawn to hypotenuse $\overline{RS}$. If $RV = 12$ and $RT = 18$, what is the length of $\overline{SV}$?

1) $6\sqrt{5}$
2) 15
3) $6\sqrt{6}$
4) 27



$\frac{H}{L} = \frac{L}{S}$

$\frac{x+12}{18} = \frac{18}{12}$

$12(x+12) = 324$

$12x + 144 = 324$

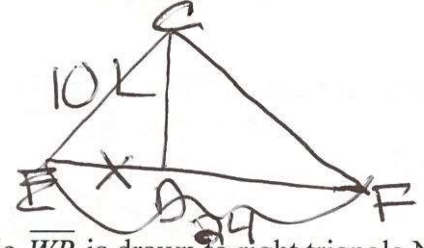
$-144 \quad -144$

$\frac{12x}{12} = \frac{180}{12}$

$x = 15$

14. Line segment $\overline{CD}$ is the altitude drawn to hypotenuse $\overline{EF}$ in right triangle ECF. If $EC = 10$ and $EF = 24$, then, to the *nearest tenth*, ED is

1) 4.2
2) 5.4
3) 15.5
4) 21.8



$\frac{H}{L} = \frac{L}{S}$

$\frac{24}{10} = \frac{10}{x}$

$\frac{24x}{24} = \frac{100}{24}$

$x = 4.2$

15. Altitude $\overline{WR}$ is drawn to right triangle NWQ. If $\overline{QW} = 8$ and $\overline{NQ} = 16$, find $\overline{WR}$ to the nearest tenth.

Both involved use both!

$\frac{S}{A} = \frac{A}{S}$

$\frac{12}{x} = \frac{x}{4}$

$\sqrt{x^2} = \sqrt{48}$

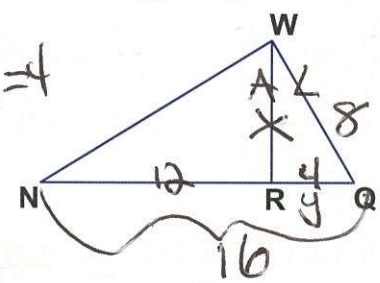
$x = 6.9$

$\frac{H}{L} = \frac{L}{S}$

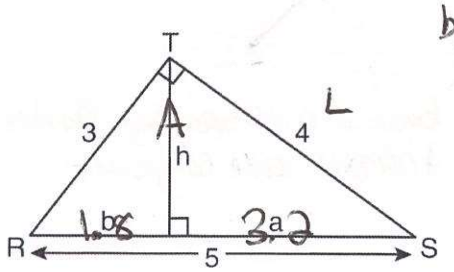
$\frac{16}{8} = \frac{8}{y}$

$\frac{16y}{16} = \frac{64}{16}$

$y = 4$



16. In the diagram below, $\triangle RST$ is a $3-4-5$ right triangle. The altitude, h, to the hypotenuse has been drawn. Determine the length of h.



both involved

$\frac{H}{L} = \frac{L}{S}$

$\frac{5}{4} = \frac{4}{a}$

$\frac{16}{5} = \frac{5a}{5}$

$3.2 = a$

$\frac{S}{A} = \frac{A}{S}$

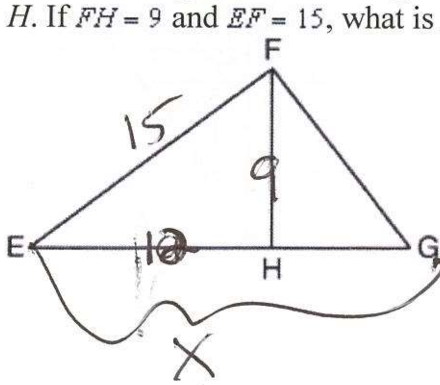
$\frac{1.8}{h} = \frac{h}{3.2}$

$\sqrt{h^2} = \sqrt{5.76}$

$h = 2.4$

17. In the diagram below of right triangle EFG, altitude $\overline{FH}$ intersects hypotenuse $\overline{EG}$ at H. If $FH = 9$ and $EF = 15$, what is EG?

Not enough information to do either!



$a^2 + b^2 = c^2$

$a^2 + 9^2 = 15^2$

$a^2 + 81 = 225$

$-81 \quad -81$

$\sqrt{a^2} = \sqrt{144}$

$a = 12$

$\frac{H}{L} = \frac{L}{S}$

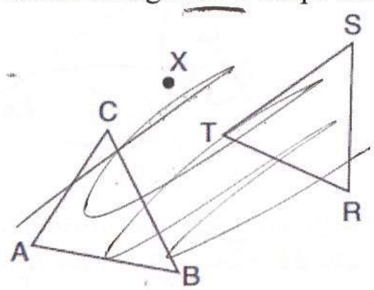
$\frac{x}{15} = \frac{15}{12}$

$\frac{12x}{12} = \frac{225}{12}$

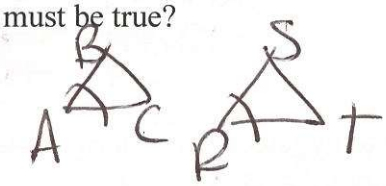
$x = 18.75$

Corresponding Parts of Congruent Triangles are Congruent
Redraw the shapes so it is more clear to see what parts correspond to each other

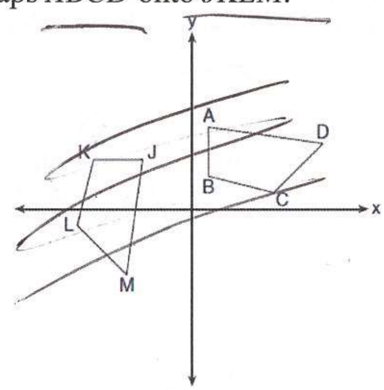
1. After a counterclockwise rotation about point *X*, scalene triangle *ABC* maps onto $\triangle RST$, as shown in the diagram below.



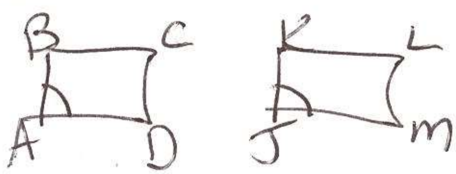
Which statement must be true?
1) $\angle A \cong \angle R$
2) $\angle A \cong \angle S$
3) $\overline{CB} \cong \overline{TR}$
4) $\overline{CA} \cong \overline{TS}$



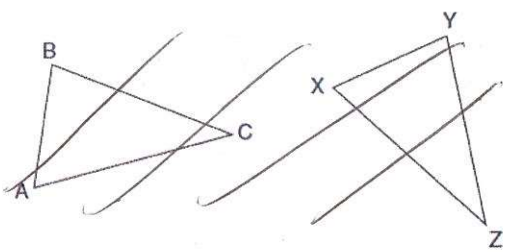
2. In the diagram below, a sequence of rigid motions maps *ABCD* onto *JKLM.*

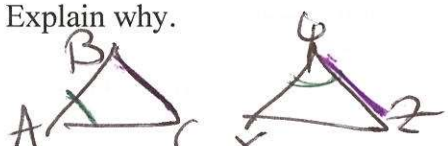


Which of the following statements must be true?
1) $\angle L \cong \angle B$
2) $\angle A \cong \angle J$
3) $\overline{JK} \cong \overline{AC}$
4) $\overline{JM} \cong \overline{AB}$



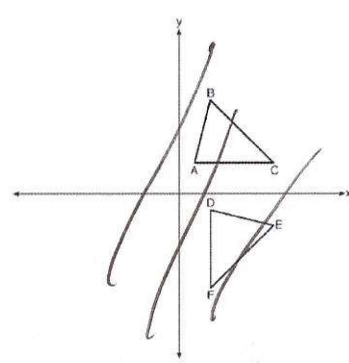
3. In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$. Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why. Determine and state whether $\angle A \cong \angle Y$. Explain why.



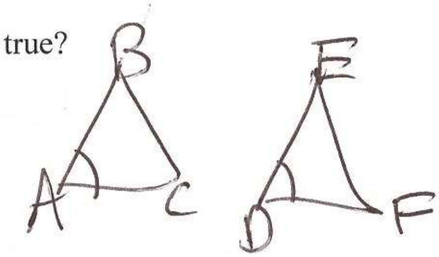


$\overline{BC} \cong \overline{YZ}$ because corresponding parts of congruent triangles are congruent
$\angle A \ncong \angle Y$ because they don't correspond

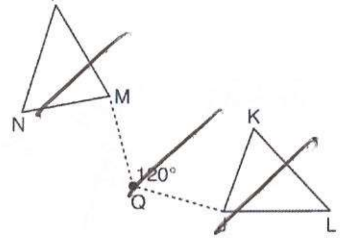
4. The image of $\triangle ABC$ after a rotation of 90° clockwise about the origin is $\triangle DEF$, as shown below.

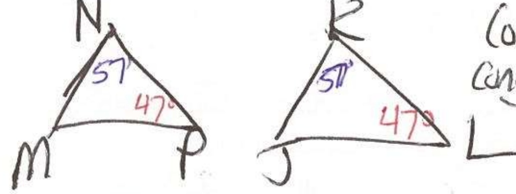


Which statement is true?
1) $\overline{BC} \cong \overline{DE}$
2) $\overline{AB} \cong \overline{DF}$
3) $\angle C \cong \angle E$
4) $\angle A \cong \angle D$



5. Triangle *MNP* is the image of triangle *JKL* after a 120° counterclockwise rotation about point *Q*. If the measure of angle *L* is 47° and the measure of angle *N* is 57°, determine the measure of angle *M*. Explain how you arrived at your answer.

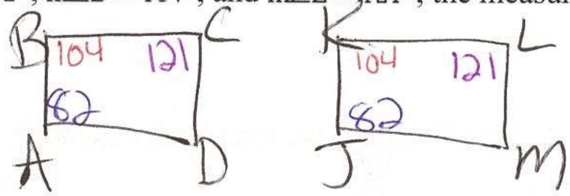


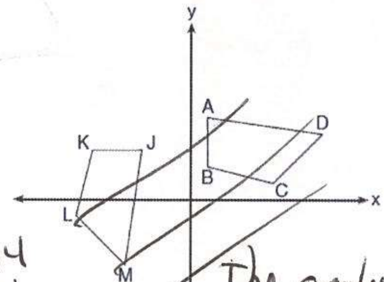


Corresponding angles of congruent triangles are congruent

$$\begin{array}{r} 57 \\ +47 \\ \hline 104 \end{array} \qquad \begin{array}{r} 180 \\ -104 \\ \hline 76° \end{array}$$

6. In the diagram below, a sequence of rigid motions maps *ABCD* onto *JKLM*.
If $m\angle A = 82°$, $m\angle B = 104°$, and $m\angle L = 121°$, the measure of $\angle M$ is
1) 53°
2) 82°
3) 104°
4) 121°

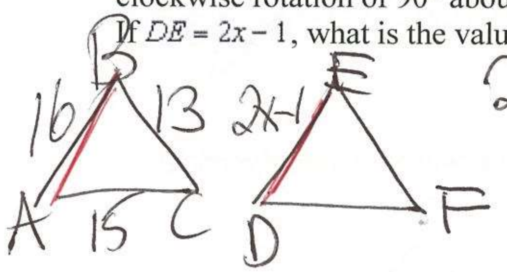




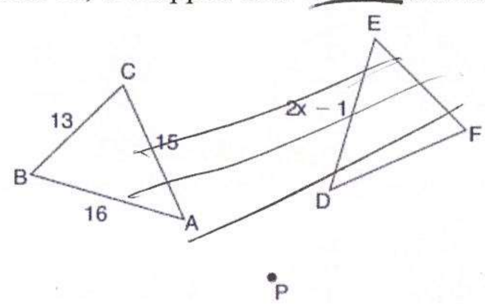
$$\begin{array}{r} 104 \\ +121 \\ +82 \\ \hline 307 \end{array} \qquad \begin{array}{r} 360 \\ -307 \\ \hline 53 \end{array}$$

The angles of a quadrilateral add to 360.

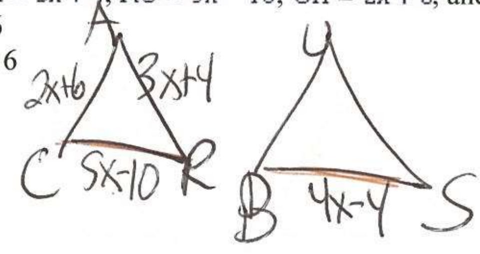
7. In the diagram below, $\triangle ABC$ with sides 13, 15, and 16, is mapped onto $\triangle DEF$ after a clockwise rotation of 90° about point *P*.
If $DE = 2x - 1$, what is the value of *x*?

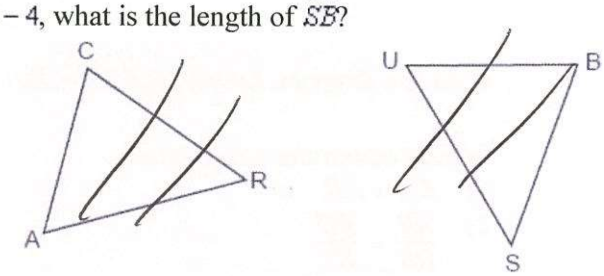


$2x - \cancel{1} = 16$
$+1 \quad +1$
$2x = 17$
$\frac{2x}{2} = \frac{17}{2}$
$x = 8.5$



8. In the diagram below, $\triangle CAR$ is mapped onto $\triangle BUS$ after a sequence of rigid motions.
If $AR = 3x + 4$, $RC = 5x - 10$, $CA = 2x + 6$, and $SB = 4x - 4$, what is the length of $\overline{SB}$?
1) 6
2) 16
3) 20
4) 28



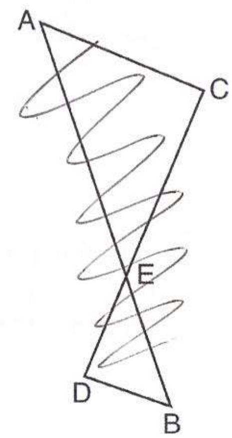


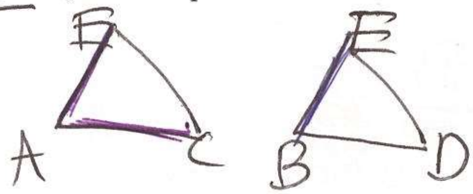
$5x - 10 = 4x - 4$
$-4x \quad -4x$
$x - 10 = -4$
$+10 \quad +10$
$x = 6$

$\overline{SB} = 4(6) - 4$
$\overline{SB} = 20$

To determine if a proportion is correct
Look at the letters vertically and horizontally
One direction, the letters should correspond
Second direction, the letters should be in the same triangle
*It does not matter which direction does which

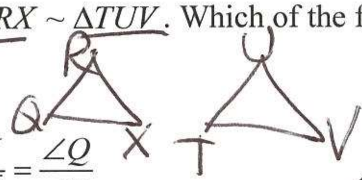
1. As shown in the diagram below, $\overline{AB}$ and $\overline{CD}$ intersect at E, and $\overline{AC} \parallel \overline{BD}$. Given $\triangle AEC \sim \triangle BED$, which equation is true?

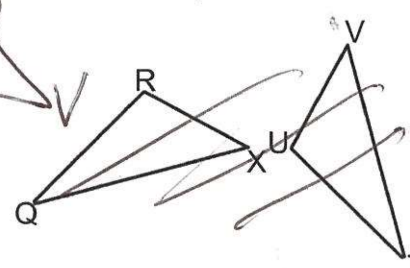




1) $\frac{CE}{DE} = \frac{EB}{EA}$
2) $\frac{AE}{BE} = \frac{AC}{BD}$
3) $\frac{EC}{AE} = \frac{BE}{ED}$
4) $\frac{ED}{EC} = \frac{AC}{BD}$

2. In the diagram below, $\Delta QRX \sim \Delta TUV$. Which of the following statements is *not* true?

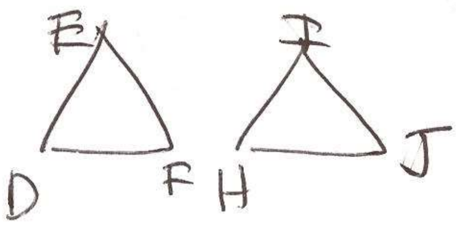




1) $\frac{\overline{QR}}{\overline{TU}} = \frac{\overline{QX}}{\overline{TV}}$
2) $\frac{\angle X}{\angle V} = \frac{\angle Q}{\angle T}$
3) $\frac{\overline{RX}}{\overline{UV}} = \frac{\overline{VT}}{\overline{XQ}}$
4) $\frac{\overline{QX}}{\overline{QR}} = \frac{\overline{TV}}{\overline{TU}}$

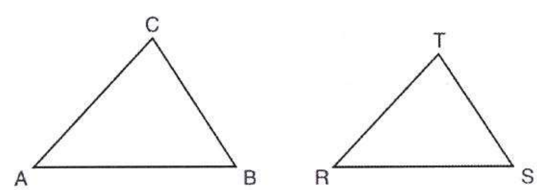
doesn't correspond

3. Given that $\Delta DEF \sim \Delta HIJ$, which is the correct statement about their corresponding sides?



1) $\frac{\overline{EF}}{\overline{IJ}} = \frac{\overline{DE}}{\overline{HI}}$
2) $\frac{\overline{EF}}{\overline{HI}} = \frac{\overline{IJ}}{\overline{DE}}$
3) $\frac{\overline{DE}}{\overline{HJ}} = \frac{\overline{EF}}{\overline{HI}}$
4) $\frac{\overline{DE}}{\overline{JI}} = \frac{\overline{EF}}{\overline{HJ}}$

4. In the diagram below, $\triangle ABC \sim \triangle RST$.



Which statement is *not* true?
1) $\angle A \cong \angle R$
2) $\frac{AB}{RS} = \frac{BC}{ST}$
3) $\frac{AB}{BC} = \frac{ST}{RS}$
4) $\angle B \cong \angle S$

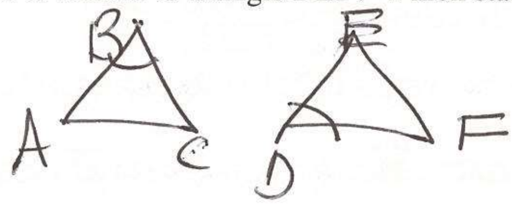
5. Scalene triangle *ABC* is similar to triangle *DEF*. Which statement is *false*?

1) $\dfrac{\overline{AB}}{\overline{BC}} = \dfrac{\overline{DE}}{\overline{EF}}$

2) $\dfrac{\overline{AC}}{\overline{DF}} = \dfrac{\overline{BC}}{\overline{EF}}$

3) $\angle ACB \cong \angle DFE$

4) $\angle ABC \cong \angle EDF$



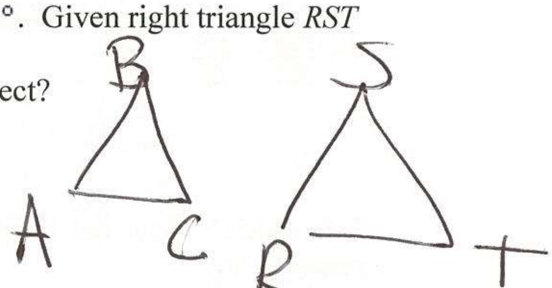
6. Given right triangle *ABC* with a right angle at *C*, $m\angle B = 61°$. Given right triangle *RST* with a right angle at *T*, $m\angle R = 29°$.
Which proportion in relation to $\triangle ABC$ and $\triangle RST$ is *not* correct?

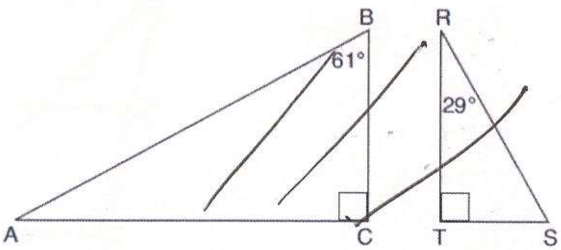
1) $\dfrac{AB}{RS} = \dfrac{RT}{AC}$

2) $\dfrac{BC}{ST} = \dfrac{AB}{RS}$

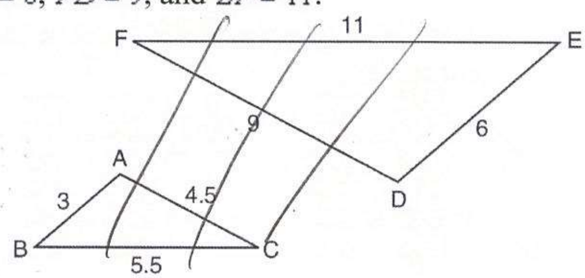
3) $\dfrac{BC}{ST} = \dfrac{AC}{RT}$

4) $\dfrac{AB}{AC} = \dfrac{RS}{RT}$





7. In the diagram below, $\triangle DEF$ is the image of $\triangle ABC$ after a clockwise rotation of 180° and a dilation where $AB = 3$, $BC = 5.5$, $AC = 4.5$, $DE = 6$, $FD = 9$, and $EF = 11$.



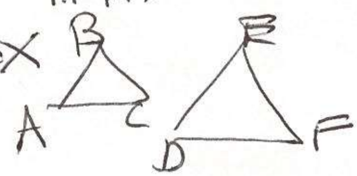
Which relationship must always be true?

1) $\dfrac{m\angle A}{m\angle D} = \dfrac{1}{2}$ X angles are always in 1:1 ratio

2) $\dfrac{m\angle C}{m\angle F} = \dfrac{2}{1}$ X

3) $\dfrac{m\angle A}{m\angle C} = \dfrac{m\angle F}{m\angle D}$ X

4) $\dfrac{m\angle B}{m\angle E} = \dfrac{m\angle C}{m\angle F}$ ✓



8. In the diagram below of isosceles triangle *AHE* with the vertex angle at *H*, $\overline{CB} \perp \overline{AE}$ and $\overline{FD} \perp \overline{AE}$.

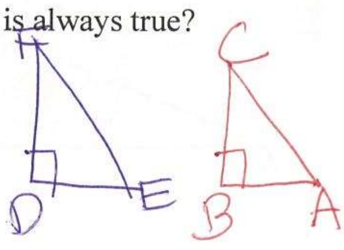
Which statement is always true?

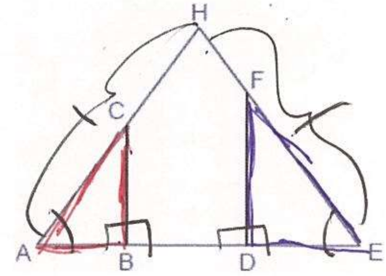
1) $\dfrac{AH}{AC} = \dfrac{EH}{EF}$

2) $\dfrac{AC}{EF} = \dfrac{AB}{ED}$

3) $\dfrac{AB}{ED} = \dfrac{CB}{FE}$

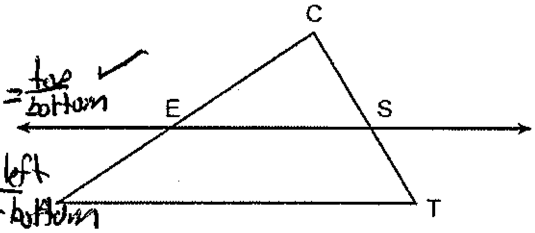
4) $\dfrac{AD}{AB} = \dfrac{BE}{DE}$





Candy Corn Problems: Is the Proportion True?
Have a picture of the original problem and the triangles separated.
If bases are not involved, see if it satisfies $\frac{top}{top} = \frac{bottom}{bottom} = \frac{side}{side}$
If bases are involved, separate the triangles and follow the same procedure from previous lesson.

1. In the diagram below of $\triangle ACT$, $\overleftrightarrow{ES}$ is drawn parallel to $\overline{AT}$ such that E is on $\overline{CA}$ and S is on $\overline{CT}$.
Which statement is always true?

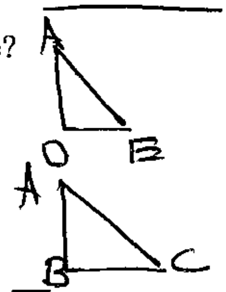


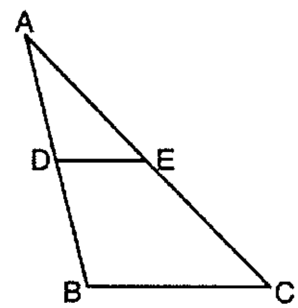
1) $\frac{CE}{CA} = \frac{CS}{ST}$ ~~top bottom~~ $\frac{\text{top}}{\text{side}} = \frac{\text{top}}{\text{bottom}}$
2) $\frac{CE}{ES} = \frac{EA}{AT}$
3) $\frac{CE}{EA} = \frac{CS}{ST}$ $\frac{\text{top}}{\text{bottom}} = \frac{\text{top}}{\text{bottom}}$ ✓
4) $\frac{CE}{ST} = \frac{EA}{CS}$ $\frac{\text{top left}}{\text{right bottom}}$ ✗

2. In $\triangle ABC$ below, $\overline{DE}$ is drawn such that D and E are on $\overline{AB}$ and $\overline{AC}$, respectively.

bases involved

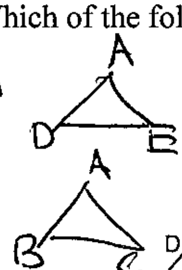
If $\overline{DE} \parallel \overline{BC}$, which equation will always be true?

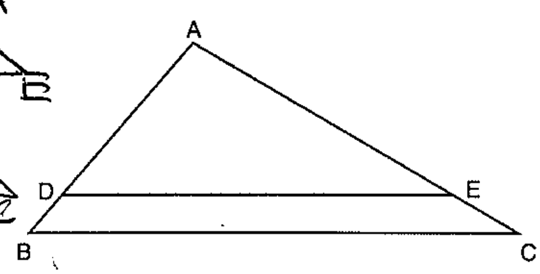




1) $\frac{AD}{DE} = \frac{DB}{BC}$
2) $\frac{AD}{DE} = \frac{AB}{BC}$
3) $\frac{AD}{BC} = \frac{DE}{DB}$
4) $\frac{AD}{BC} = \frac{DE}{AB}$

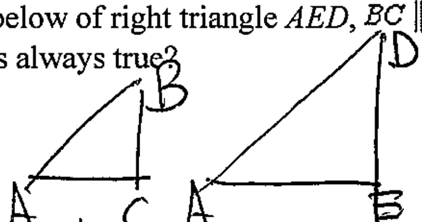
3. In the diagram of $\triangle ABC$ shown below, $\overline{DE} \parallel \overline{BC}$. Which of the following statements is *not* true?



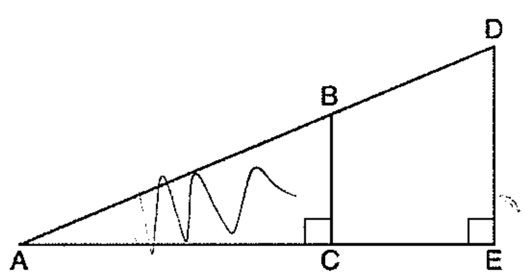


✓ 1) $\frac{\overline{AD}}{\overline{DE}} = \frac{\overline{AB}}{\overline{BC}}$
✓ 2) $\frac{\overline{BC}}{\overline{DE}} = \frac{\overline{CA}}{\overline{EA}}$
✗ 3) $\frac{\overline{AD}}{\overline{AE}} = \frac{\overline{DB}}{\overline{AC}}$ $\frac{\text{top}}{\text{top}} = \frac{\text{bottom}}{\text{side}}$
✓ 4) $\frac{\overline{DB}}{\overline{EC}} = \frac{\overline{AB}}{\overline{AC}}$ $\frac{\text{bottom}}{\text{bottom}} = \frac{\text{side}}{\text{side}}$

4. In the diagram below of right triangle AED, $\overline{BC} \parallel \overline{DE}$.
Which statement is always true?



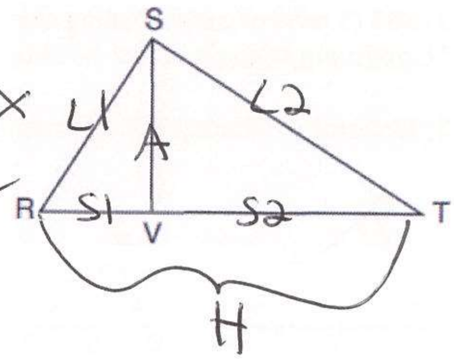
✗ 1) $\frac{AC}{BC} = \frac{DE}{AE}$
✓ 2) $\frac{AB}{AD} = \frac{BC}{DE}$
✗ 3) $\frac{AC}{CE} = \frac{BC}{DE}$ $\frac{\text{top}}{\text{bottom}} = \frac{\text{base}}{\text{base}}$
✗ 4) $\frac{DE}{BC} = \frac{DB}{AB}$ $\frac{\text{base}}{\text{bottom}} = \frac{\text{bottom}}{\text{top}}$



HLLS SAAS Problems: Is the Proportion True?

See if each proportion satisfies $\frac{H}{L} = \frac{L}{S}$ or $\frac{S}{A} = \frac{A}{S}$.

1. In right triangle RST below, altitude $\overline{SV}$ is drawn to hypotenuse $\overline{RT}$. Which of the following proportions is true?



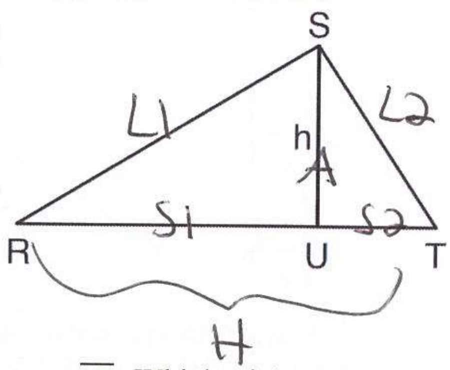
1) $\frac{\overline{RV}}{\overline{VS}} = \frac{\overline{VT}}{\overline{VS}}$ $\frac{S1}{A} = \frac{S2}{A}$ ✗

2) $\frac{\overline{RT}}{\overline{RS}} = \frac{\overline{RS}}{\overline{VT}}$ $\frac{H}{L_1} = \frac{L_1}{S_2}$ ✗

3) $\frac{\overline{RT}}{\overline{SV}} = \frac{\overline{SV}}{\overline{VT}}$ $\frac{H}{A} = \frac{A}{S_2}$ ✗

4) $\frac{\overline{RT}}{\overline{ST}} = \frac{\overline{ST}}{\overline{VT}}$ $\frac{H}{L_2} = \frac{L_2}{S_2}$ ✓

2. In right triangle RST below, altitude $\overline{SU}$ is drawn to hypotenuse $\overline{RT}$. Which of the following proportions is *not* true?



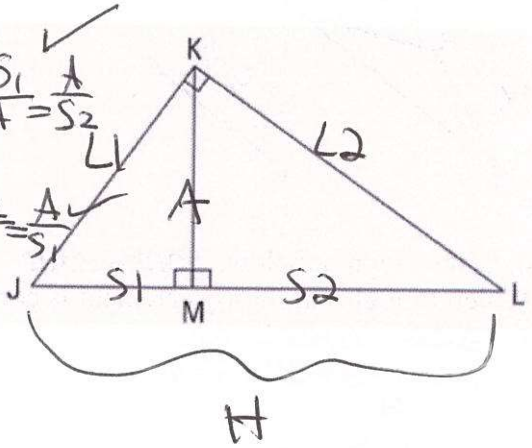
1) $\frac{\overline{RU}}{\overline{SU}} = \frac{\overline{SU}}{\overline{UT}}$ $\frac{S_1}{A} = \frac{A}{S_2}$ ✓

2) $\frac{\overline{SU}}{\overline{RU}} = \frac{\overline{RU}}{\overline{UT}}$ $\frac{A}{S_1} = \frac{S_1}{S_2}$ ✗

3) $\frac{\overline{RT}}{\overline{RS}} = \frac{\overline{RS}}{\overline{RU}}$ $\frac{H}{L_1} = \frac{L_1}{S_1}$ ✓

4) $\frac{\overline{TR}}{\overline{ST}} = \frac{\overline{ST}}{\overline{UT}}$ $\frac{H}{L_2} = \frac{L_2}{S_2}$ ✓

3. In right triangle JKL below, altitude $\overline{KM}$ is drawn to hypotenuse $\overline{JL}$. Which of the following proportions is *not* true?



1) $\frac{\overline{JL}}{\overline{JK}} = \frac{\overline{JK}}{\overline{JM}}$ $\frac{H}{L1} = \frac{L1}{S1}$ ✓

2) $\frac{\overline{JM}}{\overline{KM}} = \frac{\overline{KM}}{\overline{ML}}$ $\frac{S_1}{A} = \frac{A}{S_2}$ ✓

3) $\frac{\overline{JL}}{\overline{KL}} = \frac{\overline{KL}}{\overline{JM}}$ $\frac{H}{L_2} = \frac{L_2}{S_1}$ ✗

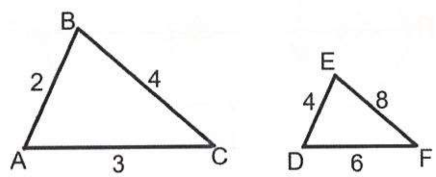
4) $\frac{\overline{ML}}{\overline{MK}} = \frac{\overline{MK}}{\overline{MJ}}$ $\frac{S_2}{A} = \frac{A}{S_1}$ ✓

To show triangles are similar:
The ANGLES of similar triangles are congruent
The SIDES of similar triangles are in proportion

1) AA (2 pairs of corresponding angles are congruent)
2) SAS (2 pairs of corresponding sides are in proportion and the corresponding angles between them are congruent)
3) SSS (3 pairs of corresponding sides are in proportion)

*Congruent triangles must be similar. Similar triangles are not necessarily congruent.

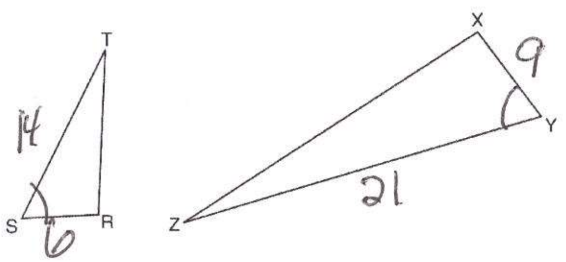
1. Determine whether the following triangles are similar. Explain your answer.



$\frac{2}{4} = \frac{4}{8} = \frac{3}{6}$ Yes, SSS

$\frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

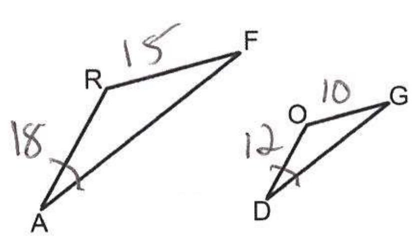
2. Triangles RST and XYZ are drawn below. If $RS = 6$, $ST = 14$, $XY = 9$, $YZ = 21$, and $\angle S \cong \angle Y$, is $\triangle RST$ similar to $\triangle XYZ$? Justify your answer.



$\frac{6}{9} = \frac{14}{21}$ Yes, SAS

$\frac{2}{3} = \frac{2}{3}$

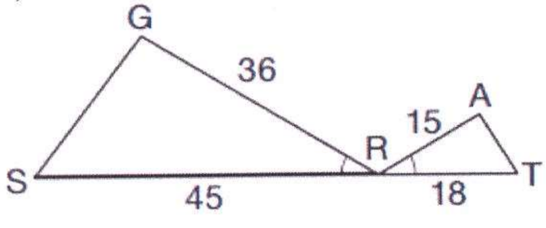
3. In the diagram below, $\overline{AR} = 18$, $\overline{RF} = 15$, $\overline{DO} = 12$, $\overline{OG} = 10$, and $\angle RAF \cong \angle ODG$. Must $\triangle ARF \sim \triangle DOG$? Explain your answer.



No, it's not SAS because the angle is not the angle in between the two sides.

4. In the diagram below, $\angle GRS \cong \angle ART$, $GR = 36$, $SR = 45$, $AR = 15$, and $RT = 18$. Which triangle similarity statement is correct?

1) $\triangle GRS \sim \triangle ART$ by AA.
2) $\triangle GRS \sim \triangle ART$ by SAS.
3) $\triangle GRS \sim \triangle ART$ by SSS.
4) $\triangle GRS$ is not similar to $\triangle ART$.

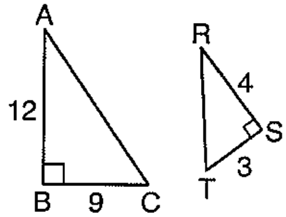


$\frac{36}{15} \neq \frac{45}{18}$

$\frac{12}{5} \neq \frac{5}{2}$

Not in proportion

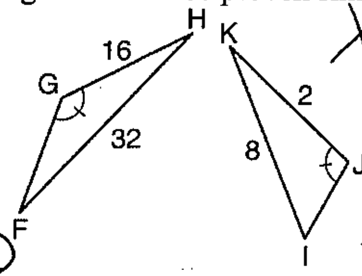
5. Using the information given below, which set of triangles can *not* be proven similar?

1) 

$\frac{12}{4} = \frac{9}{3}$

$3 = 3$

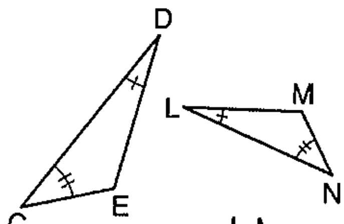
SAS ✓

3) 

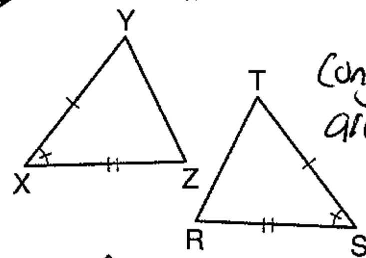
X angle is not in between the 2 sides and the sides are not in proportion

$\frac{16}{2} \neq \frac{32}{8}$

$8 \neq 4$

2) 

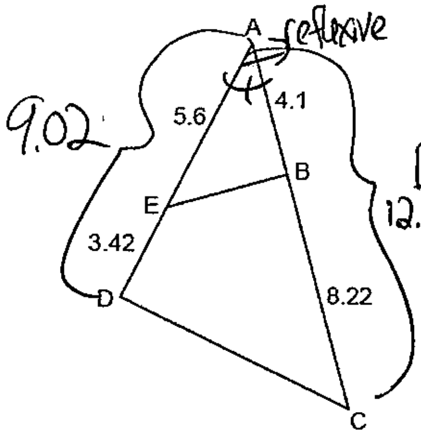
✓ AA

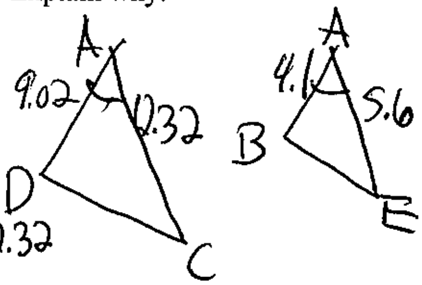
4) 

Congruent sides are in proportion

✓ SAS

6. In $\triangle ADC$ below, $\overline{EB}$ is drawn such that $AB = 4.1$, $AE = 5.6$, $BC = 8.22$, and $ED = 3.42$.. Is $\triangle ABE$ similar to $\triangle ADC$? Explain why.





$\frac{9.02}{4.1} = \frac{12.32}{5.6}$

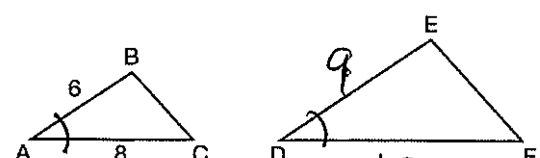
$2.2 = 2.2$ ✓

Yes, SAS

7. In the diagram below, $\triangle ABC \sim \triangle DEF$.
If $AB = 6$ and $AC = 8$, which statement will justify similarity by SAS?

1) $DE = 9$, $DF = 12$, and $\angle A \cong \angle D$
2) $DE = 8$, $DF = 10$, and $\angle A \cong \angle D$
3) $DE = 36$, $DF = 64$, and $\angle C \cong \angle F$
4) $DE = 15$, $DF = 20$, and $\angle C \cong \angle F$

not inbetween the 2 sides



1) $\frac{6}{9} = \frac{8}{12}$

$\frac{2}{3} = \frac{2}{3}$ ✓

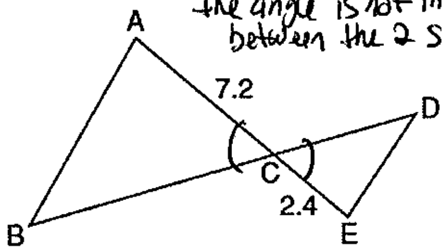
2) $\frac{6}{8} \neq \frac{8}{10}$

$\frac{3}{4} \neq \frac{4}{5}$

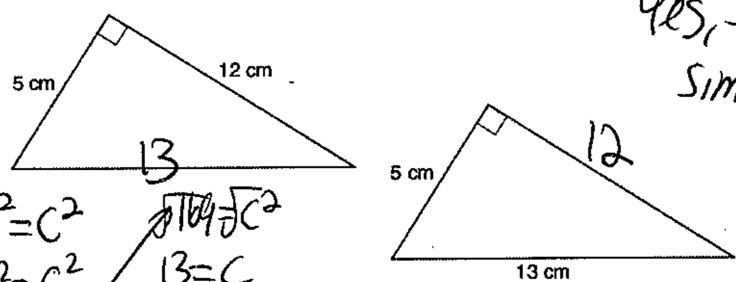
8. In the diagram below, $AC = 7.2$ and $CE = 2.4$.
Which statement is *not* sufficient to prove $\triangle ABC \sim \triangle EDC$?

1) $\overline{AB} \parallel \overline{ED}$ AA
2) $DE = 2.7$ and $AB = 8.1$ X
the angle is not in between the 2 sides
3) $CD = 3.6$ and $BC = 10.8$ SAS
4) $DE = 3.0$, $AB = 9.0$, $CD = 2.9$, and $BC = 8.7$ SSS

you should check that all of the sides are in proportion



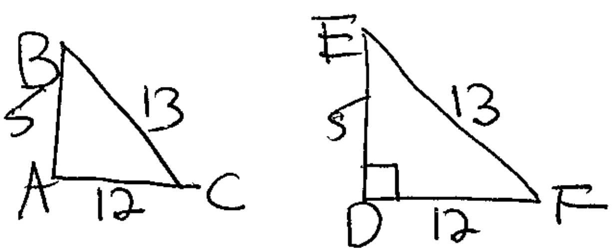
9. Skye says that the two triangles below are congruent. Margaret says that the two triangles are similar. Are Skye and Margaret both correct? Explain why.



$a^2+b^2=c^2$
$5^2+12^2=c^2$
$25+144=c^2$
$\sqrt{169}=\sqrt{c^2}$
$13=c$

Yes, they are congruent and similar by both SAS and SSS. Congruent sides are in proportion.

10. In $\triangle ABC$, $AB = 5$, $AC = 12$, and $m\angle A = 90°$. In $\triangle DEF$, $m\angle D = 90°$, $DF = 12$, and $EF = 13$. Brett claims $\triangle ABC \cong \triangle DEF$ and $\triangle ABC \sim \triangle DEF$. Is Brett correct? Explain why.



$a^2+b^2=c^2$
$5^2+12^2=c^2$
$25+144=c^2$
$\sqrt{169}=\sqrt{c^2}$
$13=c$

Yes, they are congruent and similar by both SAS and SSS.

11. If $\triangle ABC$ is mapped onto $\triangle DEF$ after a line reflection and $\triangle DEF$ is mapped onto $\triangle XYZ$ after a translation, the relationship between $\triangle ABC$ and $\triangle XYZ$ is that they are always

1) congruent and similar
2) congruent but not similar
3) similar but not congruent
4) neither similar nor congruent

Trigonometric Ratios (SOHCAHTOA)

1) Label each side with H, A, and O

2) Use SOHCAHTOA ($\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$, $\tan\theta = \frac{O}{A}$)

1. Find the following trig ratios for the given triangle.

$\sin A$ $\frac{O}{H} = \frac{8}{17}$

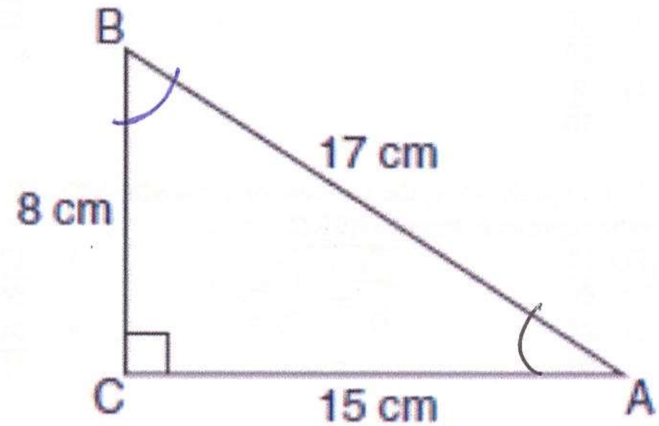
$\cos A$ $\frac{A}{H} = \frac{15}{17}$

$\tan A$ $\frac{O}{A} = \frac{8}{15}$

$\sin B$ $\frac{O}{H} = \frac{15}{17}$

$\cos B$ $\frac{A}{H} = \frac{8}{17}$

$\tan B$ $\frac{O}{A} = \frac{15}{8}$



2. Find the following trig ratios for the given triangle.

$\sin J$ $\frac{O}{H} = \frac{7}{25}$

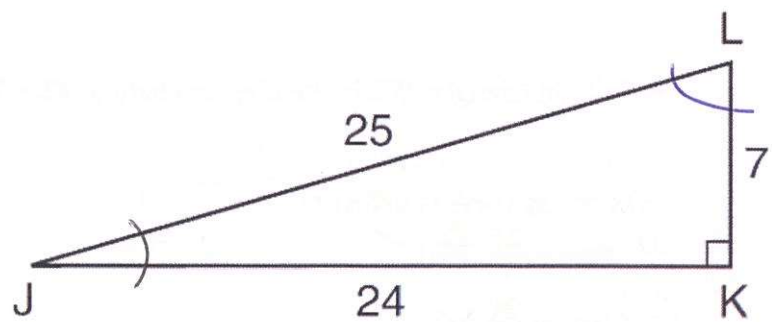
$\cos J$ $\frac{A}{H} = \frac{24}{25}$

$\tan J$ $\frac{O}{A} = \frac{7}{24}$

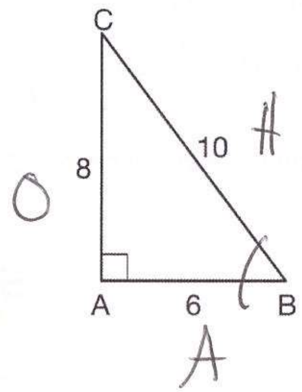
$\sin L = \frac{O}{H} = \frac{24}{25}$

$\cos L = \frac{A}{H} = \frac{7}{25}$

$\tan L = \frac{O}{A} = \frac{24}{7}$



3. In ΔABC below, the measure of $\angle A = 90°$, $AB = 6$, $AC = 8$, and $BC = 10$.



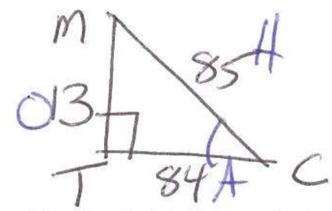
Which ratio represents the cosine of $\angle B$?

$\frac{A}{H} = \frac{6}{10}$

1) $\frac{10}{8}$
2) $\frac{8}{6}$
3) $\frac{6}{10}$
4) $\frac{8}{10}$

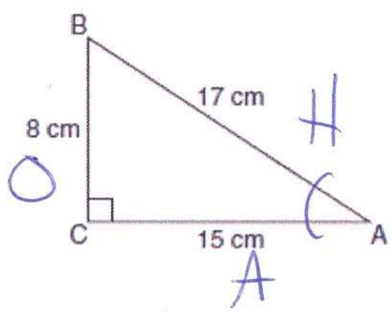
4. In triangle MCT, the measure of $\angle T = 90°$, $MC = 85$ cm, $CT = 84$ cm, and $TM = 13$cm. Which ratio represents the sine of $\angle C$?

$\frac{O}{H} = \frac{13}{85}$



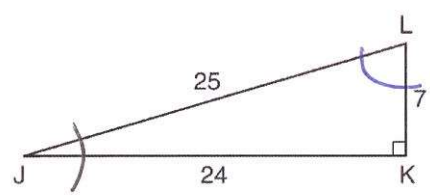
1) $\frac{13}{85}$
2) $\frac{84}{85}$
3) $\frac{13}{84}$
4) $\frac{84}{13}$

5. Which equation shows a correct trigonometric ratio for angle A in the right triangle below?



1) $\sin A = \frac{15}{17}$ $\frac{O}{H}$
2) $\tan A = \frac{8}{17}$ $\frac{O}{A}$
3) $\cos A = \frac{15}{17} = \frac{A}{H}$
4) $\tan A = \frac{5}{8} = \frac{O}{A}$

6. In right triangle JKL in the diagram below, $KL = 7$, $JK = 24$, $JL = 25$, and $\angle K = 90°$.



Which statement is *not* true?

1) $\tan L = \frac{24}{7}$ $\frac{O}{A}$
2) $\cos L = \frac{24}{25}$
3) $\tan J = \frac{7}{24}$
4) $\sin J = \frac{7}{25}$

Finding Sides and Angles with Trig

S $\frac{O}{H}$ C $\frac{A}{H}$ T $\frac{O}{A}$

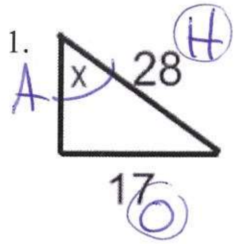
Finding Sides and Angles with Trig

1) Label each side with H, A, and O
2) Determine whether to use sine, cosine, or tangent (Which two are involved?)
3) Substitute into appropriate formula

*If finding a side, cross multiply and solve

*If finding an angle, use $\sin^{-1}$, $\cos^{-1}$, *or* $\tan^{-1}$

1.



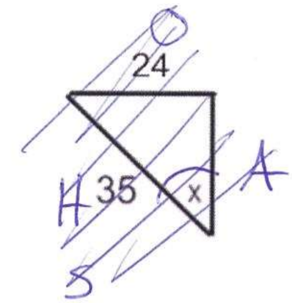
$\sin\theta = \frac{O}{H}$

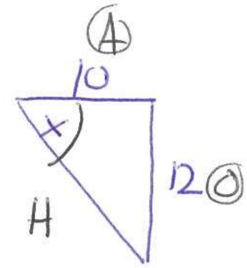
$\sin^{-1} \sin x = \sin^{-1} \frac{17}{28}$

$x = \sin^{-1}\left(\frac{17}{28}\right)$

$x = 37°$

2.





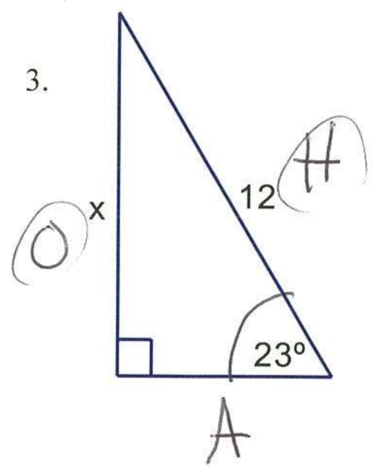
$\tan\theta = \frac{O}{A}$

$\tan^{-1} \tan x = \tan^{-1} \frac{12}{10}$

$x = \tan^{-1}\left(\frac{12}{10}\right)$

$x = 50°$

3.



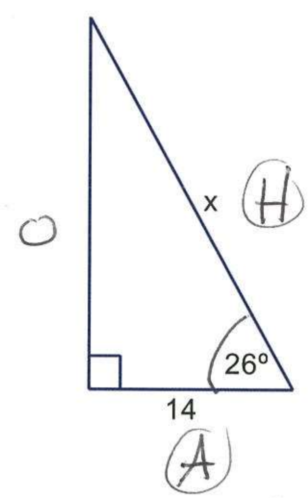
$\sin\theta = \frac{O}{H}$

$\frac{\sin 23}{1} = \frac{x}{12}$

$x = 12 \sin 23$

$x = 5$

4.



$\cos\theta = \frac{A}{H}$

$\frac{\cos 26}{1} = \frac{14}{x}$

$\frac{14}{\cos 26} = \frac{x \cos 26}{\cos 26}$

$16 = x$

$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$

5. As shown in the diagram below, a ladder 12 feet long leans against a wall and makes an angle of 72° with the ground.

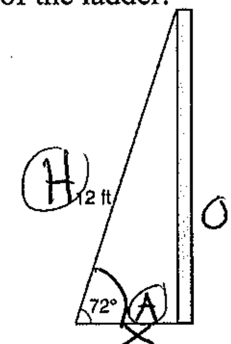
Find, to the *nearest tenth of a foot,* the distance from the wall to the base of the ladder.

$\cos\theta = \frac{A}{H}$

$\cos 72 = \frac{x}{12}$

$x = 12\cos 72$

$x = 3.7$



6. The diagram below shows the path a bird flies from the top of a 9.5-foot-tall sunflower to a point on the ground 5 feet from the base of the sunflower.

To the *nearest tenth of a degree*, what is the measure of angle x?

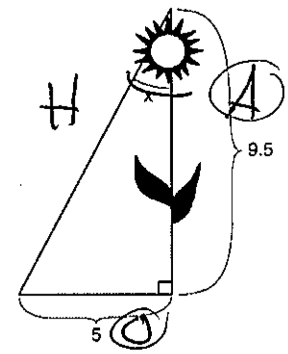
1) 27.8
2) 31.8
3) 58.2
4) 62.2

$\tan\theta = \frac{O}{A}$

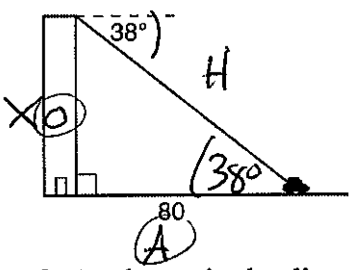
$\tan^{-1} \tan x = \tan^{-1}\frac{5}{9.5}$

$x = \tan^{-1}\left(\frac{5}{9.5}\right)$

$x = 27.8$



7. From the top of an apartment building, the angle of depression to a car parked on the street below is 38 degrees, as shown in the diagram below. The car is parked 80 feet from the base of the building. Find the height of the building, to the *nearest tenth of a foot.*



$\tan\theta = \frac{O}{A}$

$\tan 38 = \frac{x}{80}$

$x = 80\tan 38$

$x = 62.5$

8. As shown in the diagram below, a building casts a 72-foot shadow on the ground when the angle of elevation of the Sun is 40°.
How tall is the building, to the *nearest foot*?

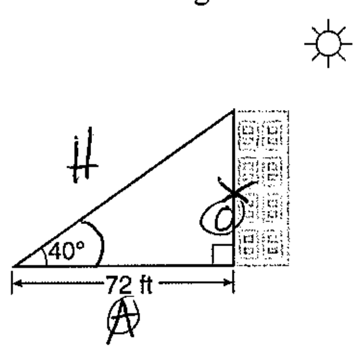
1) 46
2) 60
3) 86
4) 94

$\tan\theta = \frac{O}{A}$

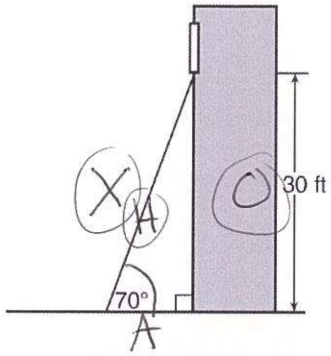
$\tan 40 = \frac{x}{72}$

$x = 72\tan 40$

$x = 60$



9. A carpenter leans an extension ladder against a house to reach the bottom of a window 30 feet above the ground. As shown in the diagram below, the ladder makes a 70° angle with the ground. To the *nearest foot*, determine and state the length of the ladder.



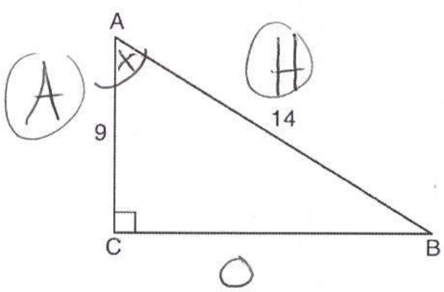
$\sin\theta = \frac{O}{H}$

$\frac{\sin 70}{1} = \frac{30}{x}$

$\frac{x \sin 70}{\sin 70} = \frac{30}{\sin 70}$

$x = 32$

10. In the diagram of right triangle ABC shown below, $AB = 14$ and $AC = 9$. What is the measure of $\angle A$, to the *nearest degree*?



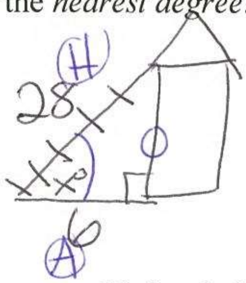
$\cos\theta = \frac{A}{H}$

$\cos^{-1}\cos x = \cos^{-1}\frac{9}{14}$

$x = \cos^{-1}\left(\frac{9}{14}\right)$

$x = 50°$

11. A 28-foot ladder is leaning against a house. The bottom of the ladder is 6 feet from the base of the house. Find the measure of the angle formed by the ladder and the ground, to the *nearest degree*.



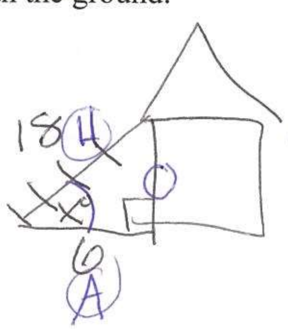
$\cos\theta = \frac{A}{H}$

$\cos^{-1}\cos x = \cos^{-1}\frac{6}{28}$

$x = \cos^{-1}\left(\frac{6}{28}\right)$

$x = 78°$

12. Bob places an 18-foot ladder 6 feet from the base of his house and leans it up against the side of his house. Find, to the *nearest degree*, the measure of the angle the bottom of the ladder makes with the ground.



$\cos\theta = \frac{A}{H}$

$\cos^{-1}\cos x = \cos^{-1}\frac{6}{18}$

$x = \cos^{-1}\left(\frac{6}{18}\right)$

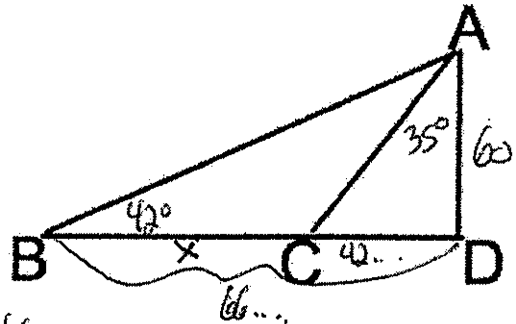
$x = 71°$

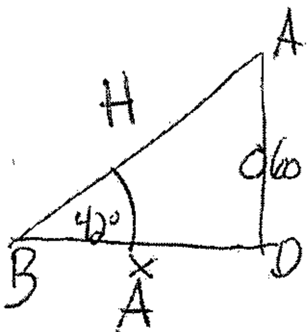
Name Schlansky
Mr. Schlansky

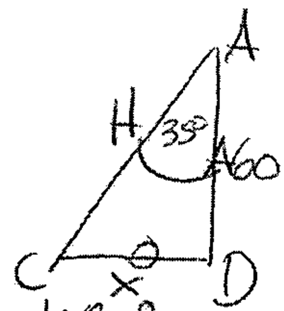
Date ____________
Geometry

Compound Right Triangles (Subtraction)

1. In the diagram below, $m\angle CAD = 35$, $m\angle ABD = 42$, and $m\overline{AD} = 60$. Find to the nearest tenth, $m\overline{BC}$.







$66... - 42.. = \boxed{24.6}$

$\frac{x\tan 42}{\tan 42} = \frac{60}{\tan 42}$

$x = 66...$

$\tan\theta = \frac{O}{A}$

$\frac{\tan 42}{1} = \frac{60}{x}$

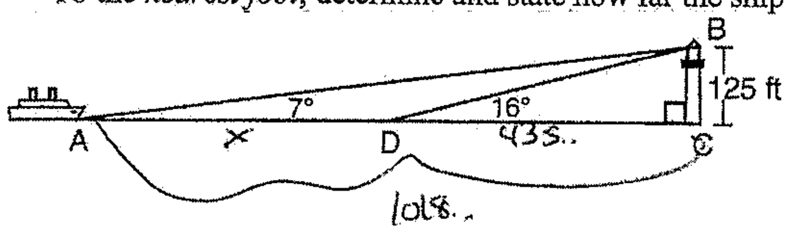
$\tan\theta = \frac{O}{A}$

$\frac{\tan 35}{1} = \frac{x}{60}$

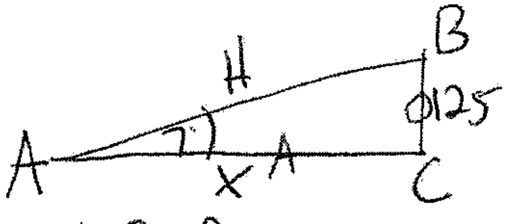
$x = 60\tan 35$

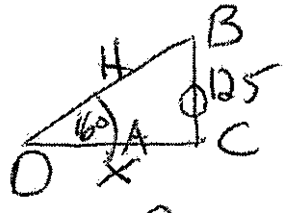
$x = 42...$

2. As shown in the diagram below, a ship is heading directly toward a lighthouse whose beacon is 125 feet above sea level. At the first sighting, point *A*, the angle of elevation from the ship to the light was 7°. A short time later, at point *D*, the angle of elevation was 16°.
To the *nearest foot*, determine and state how far the ship traveled from point *A* to point *D*.



$1018.. - 435.. = \boxed{582}$





$\tan\theta = \frac{O}{A}$

$\frac{\tan 7}{1} = \frac{125}{x}$

$\frac{x\tan 7}{\tan 7} = \frac{125}{\tan 7}$

$x = 1018..$

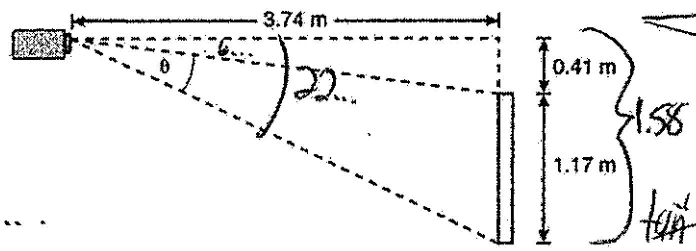
$\tan\theta = \frac{O}{A}$

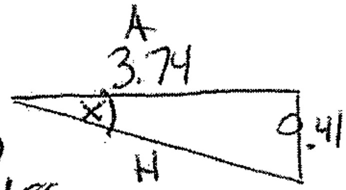
$\frac{\tan 16}{1} = \frac{125}{x}$

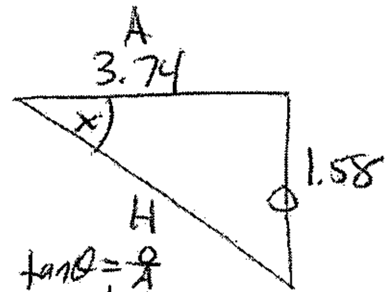
$\frac{x\tan 16}{\tan 16} = \frac{125}{\tan 16}$

$x = 435...$

3. As modeled below, a projector mounted on a ceiling is 3.74 m from a wall, where a whiteboard is displayed. The vertical distance from the ceiling to the top of the whiteboard is 0.41 m, and the height of the whiteboard is 1.17 m. Determine and state the projection angle, θ, to the *nearest tenth of a degree.*







$$\begin{array}{r} 22... \\ -\ 6... \\ \hline 16.6^\circ \end{array}$$

$\tan\theta = \frac{O}{A}$

$\tan^{-1}\tan x = \tan^{-1}\frac{.41}{3.74}$

$x = \tan^{-1}\left(\frac{.41}{3.74}\right)$

$x = 6...$

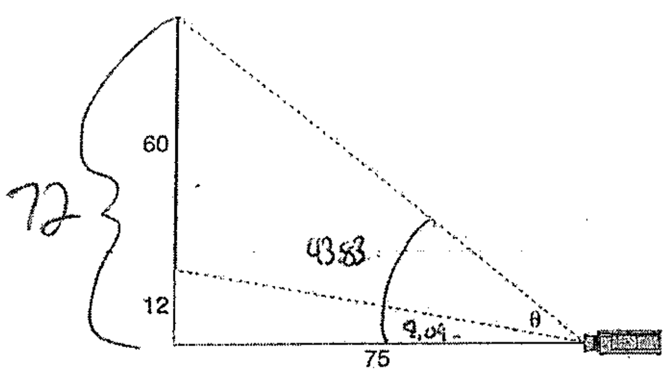
$\tan\theta = \frac{O}{A}$

$\tan^{-1}\tan x = \tan^{-1}\frac{1.58}{3.74}$

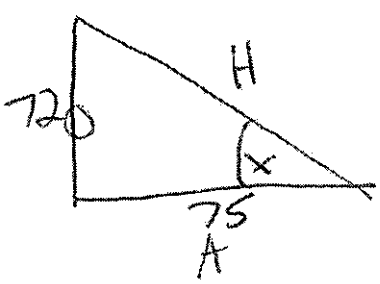
$x = \tan^{-1}\left(\frac{1.58}{3.74}\right)$

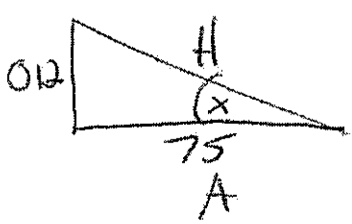
$x = 22...$

4. As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 feet off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the screen.
Determine and state, to the *nearest tenth of a degree*, the measure of θ, the projection angle.



$$\begin{array}{r} 43.83.. \\ -\ 9.09.. \\ \hline 34.7^\circ \end{array}$$





$\tan\theta = \frac{O}{A}$

$\tan^{-1}\tan x = \tan^{-1}\frac{72}{75}$

$x = \tan^{-1}\left(\frac{72}{75}\right)$

$x = 43.83..$

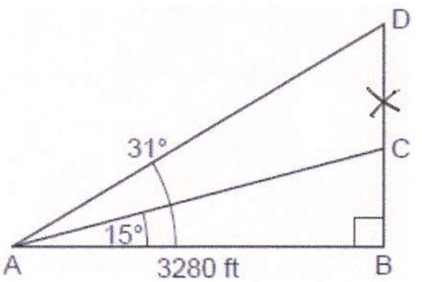
$\tan\theta = \frac{O}{A}$

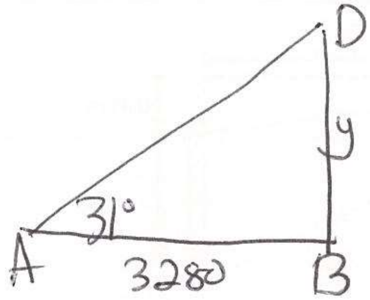
$\tan^{-1}\tan x = \tan^{-1}\frac{12}{75}$

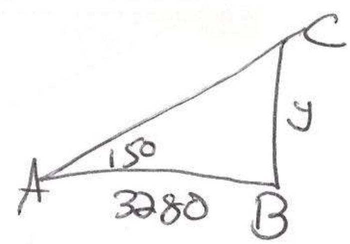
$x = \tan^{-1}\left(\frac{12}{75}\right)$

$x = 9.09..$

5. Cape Canaveral, Florida is where NASA launches rockets into space. As modeled in the diagram below, a person views the launch of a rocket from observation area *A*, 3280 feet away from launch pad *B*. After launch, the rocket was sighted at *C* with an angle of elevation of 15°. The rocket was later sighted at *D* with an angle of elevation of 31°. Determine and state, to the *nearest foot*, the distance the rocket traveled between the two sightings, *C* and *D*.







$\frac{\tan 31}{1} = \frac{y}{3280}$

$y = 3280 \tan 31$

$y = 1970...$

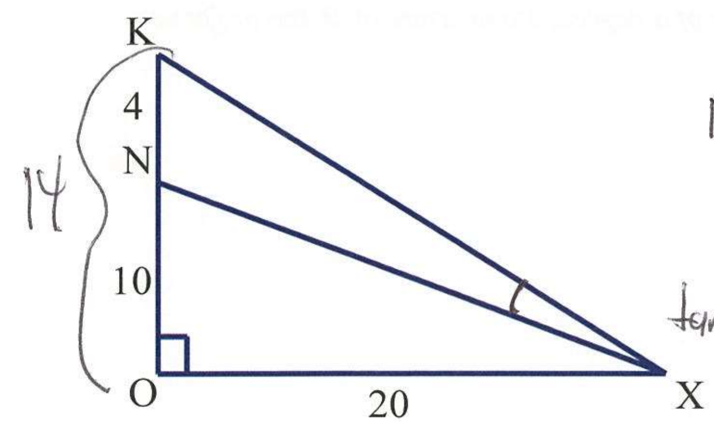
$\frac{\tan 15}{1} = \frac{y}{3280}$

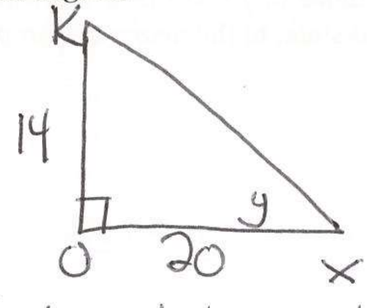
$y = 3280 \tan 15$

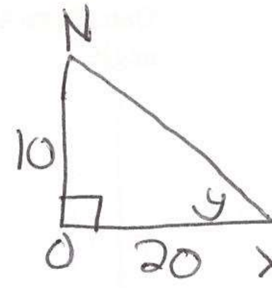
$y = 878..$

$1970.. - 878.. = 1092$

6. Find the measure of $\angle KXN$ below the *nearest degree*.







$\tan y = \frac{14}{20}$

$y = \tan^{-1} \frac{14}{20}$

$y = 34..$

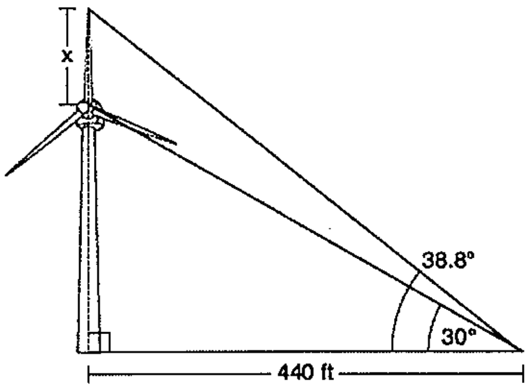
$\tan y = \frac{10}{20}$

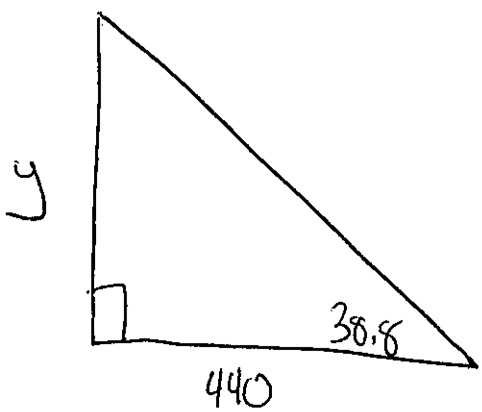
$y = \tan^{-1} \frac{10}{20}$

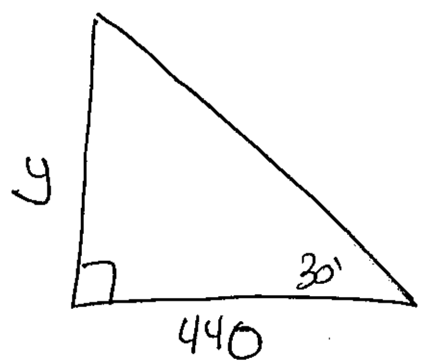
$y = 26..$

$34.. - 26.. = 8°$

7. Nick wanted to determine the length of one blade of the windmill pictured below. He stood at a point on the ground 440 feet from the windmill's base. Using surveyor's tools, Nick measured the angle between the ground and the highest point reached by the top blade and found it was 38.8°. He also measured the angle between the ground and the lowest point of the top blade, and found it was 30°. Determine and state a blade's length, x, to the *nearest foot*.







$\frac{\tan 38.8}{1} = \frac{y}{440}$

$y = 440 \tan 38.8$

~~y=353~~

$y = 353\ldots$

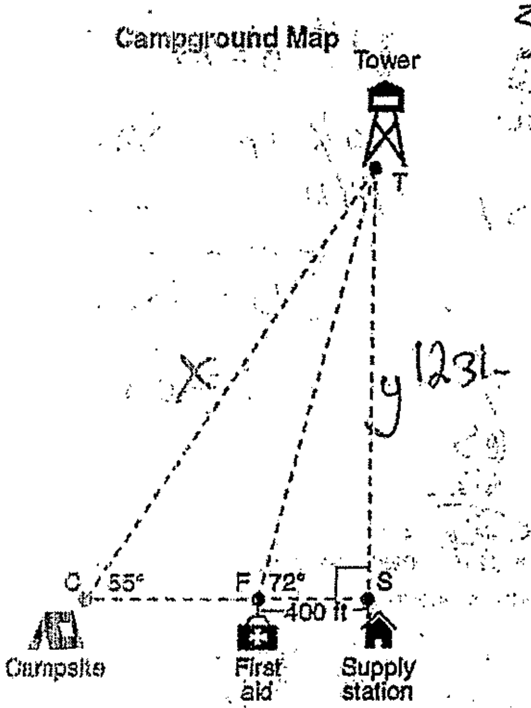
$\frac{\tan 30}{1} = \frac{y}{440}$

$y = 440 \tan 30$

$y = 254\ldots$

$$\begin{array}{r} 353.. \\ -254. \\ \hline \boxed{100} \end{array}$$

1. The map of a campground is shown below. Campsite C, first aid station F, and supply station S lie along a straight path. The path from the supply station to the tower, T, is perpendicular to the path from the supply station to the campsite. The length of path $\overline{FS}$ is 400 feet. The angle formed by path $\overline{TF}$ and path $\overline{FS}$ is 72°. The angle formed by path $\overline{TC}$ and path $\overline{CS}$ is 55°. Determine and state, to the *nearest foot*, the distance from the campsite to the tower.



ST is a side in both triangles

$\tan 72 = \frac{y}{400}$

$\frac{3.0777}{1} = \frac{y}{400}$

$y = 1231..$

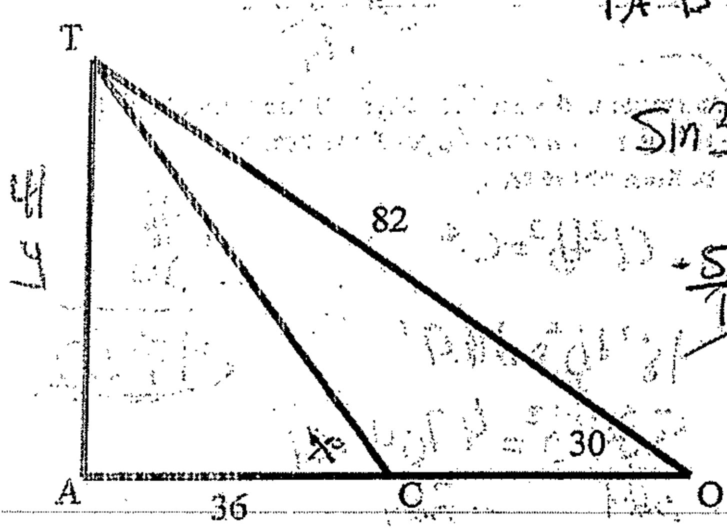
$\sin 55 = \frac{1231..}{x}$

$\frac{.8192}{1} = \frac{1231..}{x}$

$\frac{.8192x}{.8192} = \frac{1231..}{.8192}$

$x = 1503$

2. Find the measure of $\angle TCA$ in the diagram of right triangle TAO below to the nearest tenth of a degree.



TA is a side in both triangles

$\sin 30 = \frac{y}{82}$

$\frac{.5}{1} = \frac{y}{82}$

$y = 41$

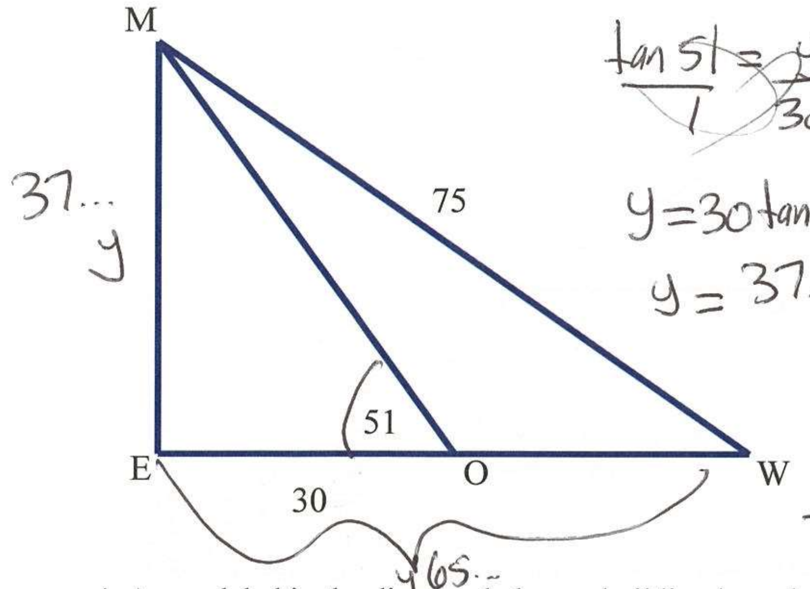
$\tan x = \frac{41}{36}$

$x = \tan^{-1} \frac{41}{36}$

$x = 48.7°$

3. Find the measure of $\overline{OW}$ in the diagram of right triangle MEW below to the nearest unit.

$m\overline{E}$ is in both triangles



$\tan 51 = \frac{y}{30}$

$y = 30 \tan 51$

$y = 37...$

$a^2 + b^2 = c^2$

$37..^2 + y^2 = 75^2$

$1372.. + y^2 = 5625$

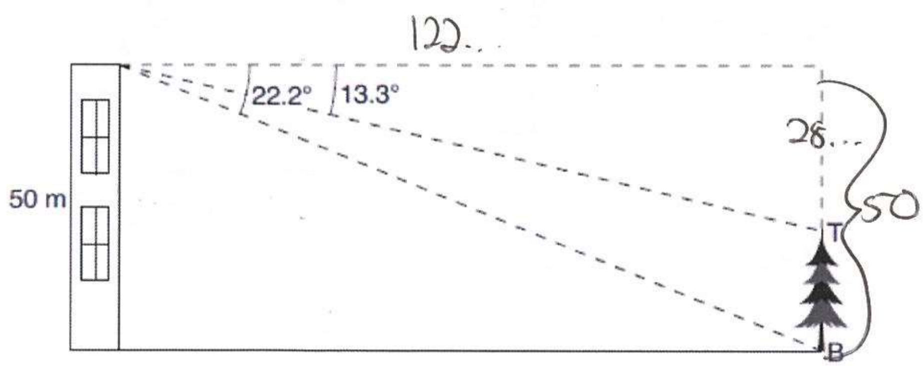
$-1372.. \quad -1372..$

$\sqrt{y^2} = \sqrt{4252..}$

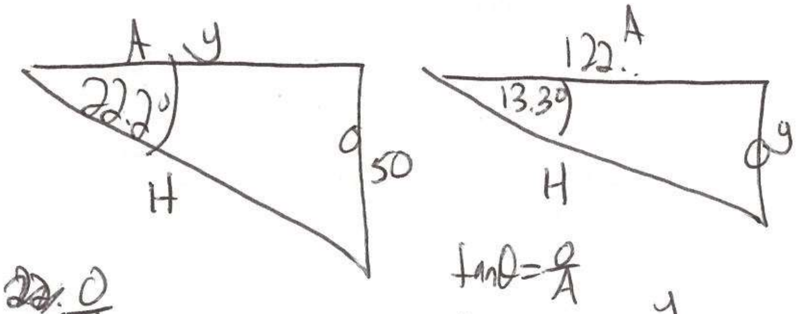
$y = 65..$

$65.. - 30 = 35$

4. As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, *T*, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, *B*, is 22.2°. Determine and state, to the *nearest meter*, the height of the tree.



$50 - 28.. = 21$



$\tan\theta = \frac{O}{A}$

$\tan 22.2 = \frac{50}{y}$

$y \tan 22.2 = 50$

$\frac{y \tan 22.2}{\tan 22.2} = \frac{50}{\tan 22.2}$

$y = 122...$

$\tan\theta = \frac{O}{A}$

$\tan 13.3 = \frac{y}{122..}$

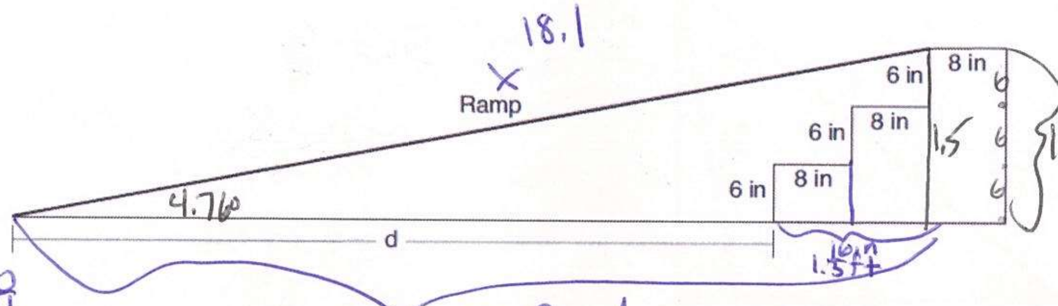
$y = 122.. \tan 13.3$

$y = 28...$

Compound Right Triangle Problems: Other

Problem Solve using SOHCAHTOA and/or Pythagorean Theorem

1. As modeled in the diagram below, an access ramp starts on flat ground and ends at the beginning of the top step. Each step is 6 inches tall and 8 inches deep. If the angle of elevation of the ramp is 4.76°, determine and state the length of the ramp, to the *nearest tenth of a foot*. Determine and state, to the *nearest tenth of a foot*, the horizontal distance, *d*, from the bottom of the stairs to the bottom of the ramp.



$\sin\theta = \frac{O}{H}$

$\sin 4.76 = \frac{1.5}{x}$

$x \sin 4.76 = 1.5$

$\frac{x \sin 4.76}{\sin 4.76} = \frac{1.5}{\sin 4.76}$

$x = 18.1\text{ft}$

18....

you could have also used Pythagorean Theorem instead

$\cos\theta = \frac{A}{H}$

$\cos 4.76 = \frac{y}{18.1}$

$y = 18.1 \cos 4.76$

$y = 18...$

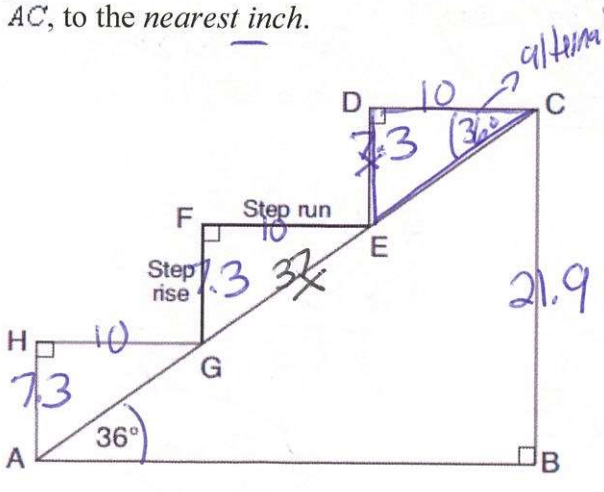
18...
− 1.3
16.7ft

$18\text{ in} \cdot \frac{1\text{ ft}}{12\text{ in}} = 1.5\text{ft}$

$16\text{ in} \cdot \frac{1\text{ ft}}{12\text{ in}} = 1.\bar{3}\text{ft}$

2. A homeowner is building three steps leading to a deck, as modeled by the diagram below. All three step rises, $\overline{HA}$, $\overline{FG}$, and $\overline{DE}$, are congruent, and all three step runs, $\overline{HG}$, $\overline{FE}$, and $\overline{DC}$, are congruent. Each step rise is perpendicular to the step run it joins. The measure of $\angle CAB = 36°$ and $\angle CBA = 90°$.

If each step run is parallel to $\overline{AB}$ and has a length of 10 inches, determine and state the length of each step rise, to the *nearest tenth of an inch*. Determine and state the length of $\overline{AC}$, to the *nearest inch*.



$\tan\theta = \frac{O}{A}$

$\tan 36 = \frac{x}{10}$

$x = 10 \tan 36$

$x = 7.3$

$7.3(3) = 21.9$

$\sin\theta = \frac{O}{H}$

$\sin 36 = \frac{21.9}{x}$

$x \sin 36 = 21.9$

$\frac{x \sin 36}{\sin 36} = \frac{21.9}{\sin 36}$

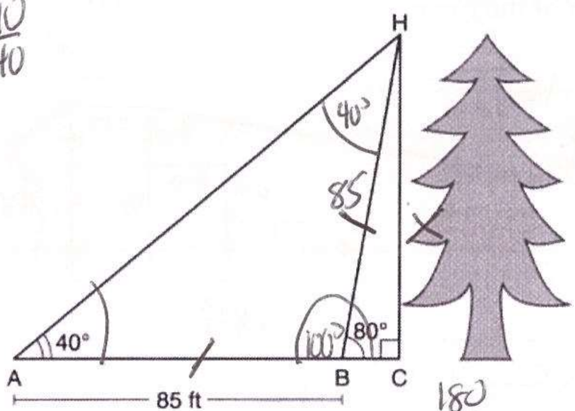
$x = 37$

you also could have used Pythagorean Theorem instead.

3. Barry wants to find the height of a tree that is modeled in the diagram below, where $\angle C$ is a right angle. The angle of elevation from point *A* on the ground to the top of the tree, *H*, is 40°. The angle of elevation from point *B* on the ground to the top of the tree, *H*, is 80°. The distance between points *A* and *B* is 85 feet. Barry claims that $\triangle ABH$ is isosceles. Explain why Barry is correct. Determine and state, to the *nearest foot*, the height of the tree.

40
H00
140

180
-140
40



180
-80
100

$\triangle ABH$ is isosceles because it has 2 ≅ angles.

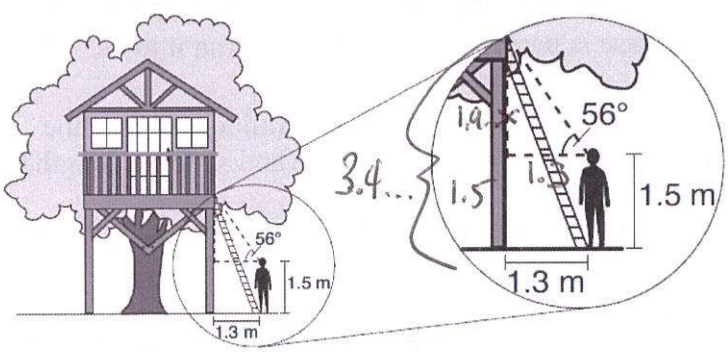
$\sin\theta = \frac{O}{H}$

$\frac{\sin 80}{1} = \frac{x}{85}$

$x = 85 \sin 80$

$x = 84$

4. David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground. Determine and state the minimum length of a ladder, to the *nearest tenth of a meter*, that David will need to buy for his treehouse.

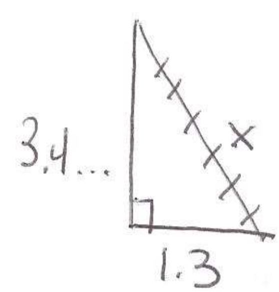


$\tan\theta = \frac{O}{A}$

$\frac{\tan 56}{1} = \frac{x}{1.3}$

$x = 1.3 \tan 56$

$x = 1.9..$



$a^2 + b^2 = c^2$

$1.3^2 + 3.4..^2 = x^2$

$\sqrt{13.4..} = \sqrt{x^2}$

$3.7 = x$

Acute Angles in a Right Triangle

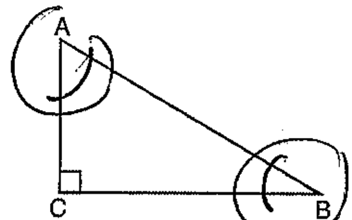
$\sin A = \cos B$: **In a right triangle, the sine of one acute angle is equal to the cosine of the other acute angle**

$A + B = 90$: **The two acute angles in a right triangle are complementary**

1. In scalene triangle ABC shown in the diagram below, $m\angle C = 90°$.

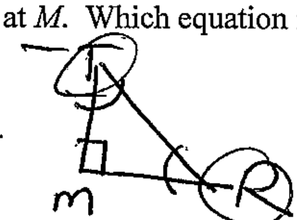
Which equation is always true?

1) $\sin A = \sin B$
2) $\cos A = \cos B$
3) $\cos A = \sin C$
4) $\sin A = \cos B$



2. Right triangle TMR is a scalene triangle with the right angle at M. Which equation is true?

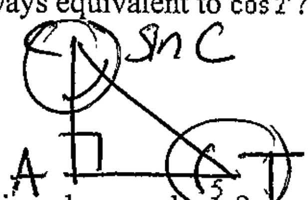
1) ~~$\sin M = \cos T$~~
2) ~~$\sin R = \cos R$~~
3) $\sin T = \cos R$
4) ~~$\sin T = \cos M$~~



3. Right triangle ACT has $m\angle A = 90°$. Which expression is always equivalent to $\cos T$?

1) $\cos C$
2) $\sin C$
3) $\tan T$
4) $\sin T$

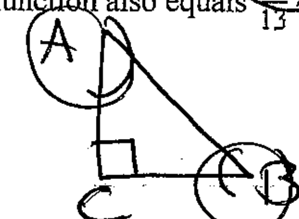
sin C



sinA=cosB

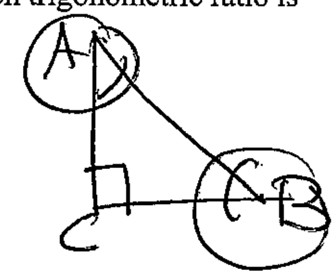
4. In right triangle ABC, $m\angle C = 90°$. If $\cos B = \frac{5}{13}$, which function also equals $\frac{5}{13}$?

1) $\tan A$
2) $\tan B$
3) $\sin A$
4) $\sin B$

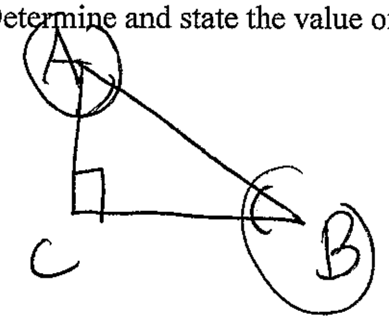


5. In right triangle ABC, $m\angle C = 90°$ and $AC \neq BC$. Which trigonometric ratio is equivalent to $\sin B$?

1) $\cos A$ cosA
2) $\cos B$
3) $\tan A$
4) $\tan B$



6. In right triangle ABC with the right angle at C, $\sin A = 2x + 0.1$ and $\cos B = 4x - 0.7$. Determine and state the value of x. Explain your answer.



$\sin A = \cos B$

$2x + 0.1 = 4x - 0.7$

$-2x \qquad -2x$

$0.1 = 2x - 0.7$

$+0.7 \qquad +0.7$

$\frac{0.8}{2} = \frac{2x}{2}$

$0.4 = x$

sin A = cos B

7. If $\sin(3x+2)^\circ = \cos(4x-10)^\circ$, what is the value of x to the *nearest tenth*?

(1) 7.6 (2) 12.0 (3) 14.0 (4) 26.9

$A+B=90$ $3x+2+4x-10=90$

$7x-8=90$

$+8$ $+8$

$\frac{7x}{7}=\frac{98}{7}$

$x=14$

sin A = cos B

8. If $\sin(2x+7)^\circ = \cos(4x-7)^\circ$, what is the value of x?

1) 7
2) 15
3) 21
4) 30

$A+B=90$

$2x+7+4x-7=90$

$\frac{6x}{6}=\frac{90}{6}$

$x=15$

sin A = cos B

9. In a right triangle, $\sin(40-x)^\circ = \cos(3x)^\circ$. What is the value of x?

1) 10
2) 15
3) 20
4) 25

$A+B=90$

$40-x+3x=90$

$2x+40=90$

-40 -40

$\frac{2x}{2}=\frac{50}{2}$

$x=25$

sin A = cos B

10. In a right triangle, the acute angles have the relationship $\sin(2x+4) = \cos(46)$. What is the value of x?

1) 20
2) 21
3) 24
4) 25

$A+B=90$

$2x+4+46=90$

$2x+50=90$

-50 -50

$\frac{2x}{2}=\frac{40}{2}$

$x=20$

11. Which expression is always equivalent to $\sin x$ when $0^\circ < x < 90^\circ$?

1) $\cos(90^\circ - x)$
2) $\cos(45^\circ - x)$
3) $\cos(2x)$
4) $\cos x$

sin A = cos B

$\sin x = \cos(90-x)$

$x+(90-x)=90$

sin A = cos B A+B=90

$\sin 40 = \cos 50$ $40+B=90$

-40 -40

$B=50$

12. Which of the following is equivalent to $\sin 40$?

1) $\sin 50$ 2) $\cos 50$ 3) $\cos 40$ 4) $\tan 50$

these must add to 90

sin 33

$90-57=33$

13. Which of the following is equivalent to $\cos 57$?

1) $\sin 57$ 2) $\sin 33$ 3) $\cos 33$ 4) $\cos 123$

$90-30=60$

14. Which expression is equal to $\sin 30^\circ$?

1) $\tan 30^\circ$
2) $\sin 60^\circ$
3) $\cos 60^\circ$
4) $\cos 30^\circ$

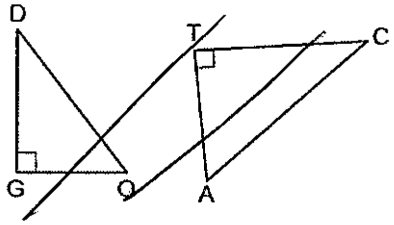
cos 60

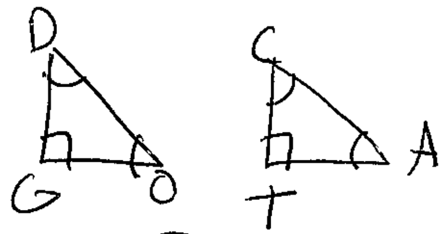
Trigonometry with Similar Triangles

Draw your own triangles separately!

Match up the corresponding angles and apply trigonometry rules from there.

1. In the diagram below, $\triangle DOG \sim \triangle CAT$, where $\angle G$ and $\angle T$ are right angles.





cos O = cos A

Which expression is always equivalent to $\sin D$?

1) $\cos A$ 3) $\tan A$

2) $\sin A$ sin A = cos B   4) $\cos C$

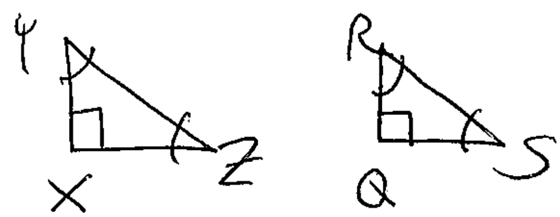
2. If scalene triangle XYZ is similar to triangle QRS and $m\angle X = 90°$, which equation is always true?

sin A = cos B

1) ~~$\sin Y = \sin S$~~   3) ~~$\cos Y = \sin Q$~~

2) ~~$\cos R = \cos Z$~~   4) $\sin R = \cos Z$

sin Y =

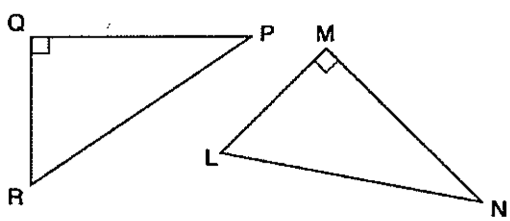


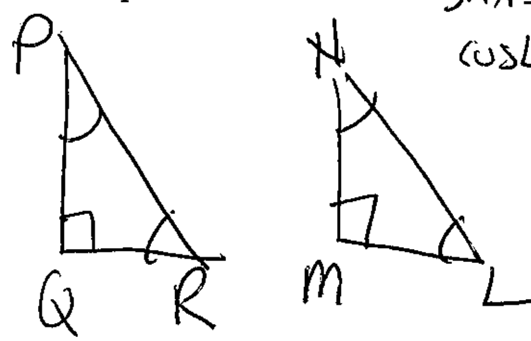
3. In the diagram below, right triangle PQR is transformed by a sequence of rigid motions that maps it onto right triangle NML. What ratio is equal to cos L?

sin A = cos B

cos L = sin N

sin P





1) $\sin R$

2) $\cos R$

3) $\sin P$

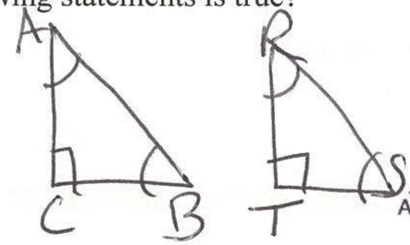
4) $\cos P$

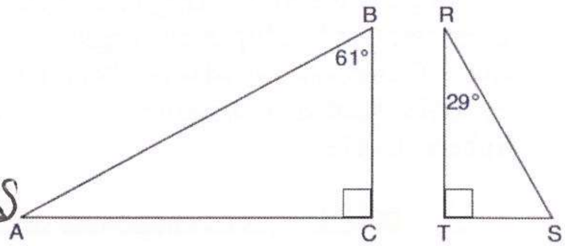
$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$

4. Given right triangle *ABC* with a right angle at *C*, m∠*B* = 61°. Given right triangle *RST* with a right angle at *T*, m∠*R* = 29°.
Which of the following statements is true?

1) $\sin A = \cancel{\cos C}$
2) $\sin B = \cos R$
3) $\sin S = \cos B$
4) $\sin C = \cancel{\cos T}$

$\sin B = \cos A$
$\downarrow$
$\cos R$





5. In the diagram below of △*HAR* and △*NTY*, angles *H* and *N* are right angles, and △*HAR* ~ △*NTY*. If *AR* = 13 and *HR* = 12, what is the measure of angle *Y*, to the *nearest degree*?

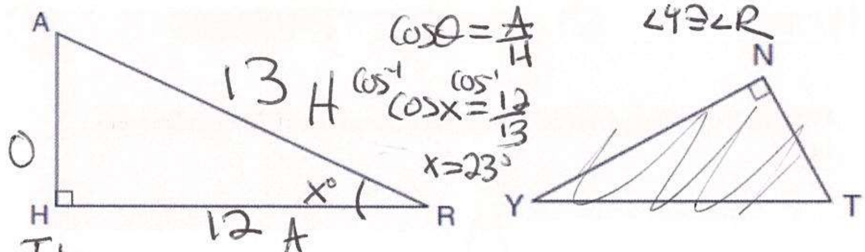
1) 23°
2) 25°
3) 65°
4) 67°

$\cos\theta = \frac{A}{H}$

$\angle Y \cong \angle R$

$\cos^{-1} \cos x = \frac{12}{13} \cos^{-1}$

$x = 23°$



6. Kayla was cutting right triangles from wood to use for an art project. Two of the right triangles she cut are shown below.

If △*ABC* ~ △*DEF*, with right angles *B* and *E*, *BC* = 15 cm, and *AC* = 17 cm, what is the measure of ∠*F*, to the *nearest degree*?

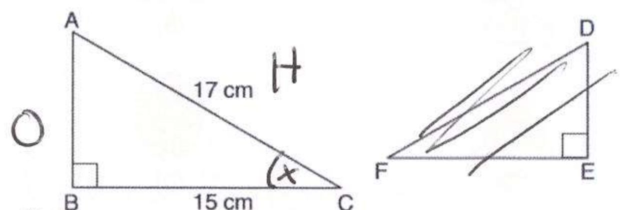
1) 28°
2) 41°
3) 62°
4) 88°

$\cos\theta = \frac{A}{H}$

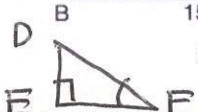
$\cos^{-1} \cancel{\cos} x = \frac{15}{17} \cos^{-1}$

$x = \cos^{-1}\left(\frac{15}{17}\right)$

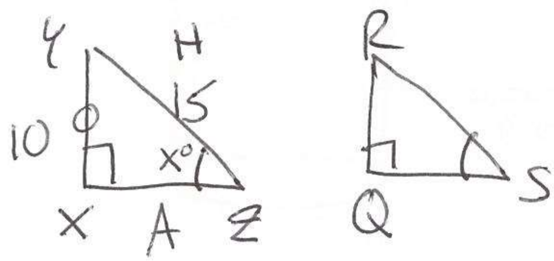
$x = 28°$



$\angle F \cong \angle C$



1. Scalene triangle *XYZ* is similar to triangle *QRS* and m∠*X* = 90°. If $\overline{XY} = 10$ and $\overline{ZY} = 15$, find the measure of ∠*S* to the *nearest tenth of a degree*.



$\sin\theta = \frac{O}{H}$

$\sin^{-1} \cancel{\sin} x = \frac{10}{15} \sin^{-1}$

$x = \sin^{-1}\left(\frac{10}{15}\right)$

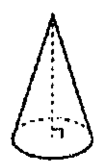
$x = 41.8°$

Cross Sections (2 dimensional slice of a 3 dimensional object):
The base of the shape is always one of its cross sections
Rectangular Prism: Rectangle, triangle
Cylinder: Circle, ellipse, rectangle
Cone: Circle, ellipse, triangle, "curved" rectangle
Pyramid: Rectangle, triangle
Sphere: Circle

1. Which type of shape can represent a two-dimensional cross-section of a sphere?
(1) circular (2) triangular (3) square (4) rectangular

2. Which is *not* a possible two-dimensional cross section of a three-dimensional cylinder?
(1) circle (2) rectangle (3) ellipes (4) triangle

3.
William is drawing pictures of cross sections of the right circular cone below.



Which drawing can *not* be a cross section of a cone?

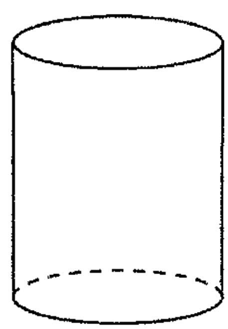

(1)


(3)


(2)


(4)

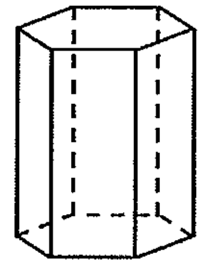
6. A plane intersects a cylinder perpendicular to its bases.
Vertical



This cross section can be described as a
1) rectangle
2) parabola
3) triangle
4) circle

7. A right hexagonal prism is shown below. A two-dimensional cross section that is perpendicular to the base is taken from the prism.

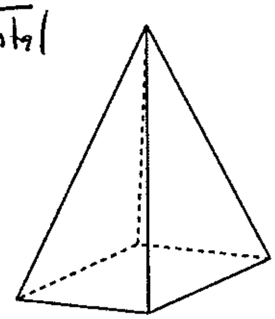
vertical



Which figure describes the two-dimensional cross section?
1) triangle
2) rectangle
3) pentagon
4) hexagon

8. In the diagram below, a plane intersects a square pyramid parallel to its base.

horizontal

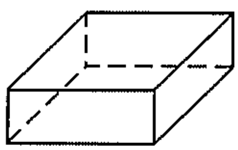


Which two-dimensional shape describes this cross section?

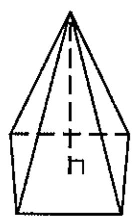
1) circle
2) square
3) triangle
4) pentagon

7. Which figure can have the same cross section as a sphere?

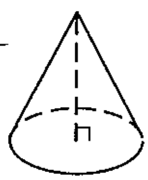
1)



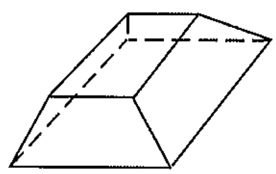
3)



2)



4)



vertical

8. A plane intersects a hexagonal prism. The plane is perpendicular to the base of the prism. Which two-dimensional figure is the cross section of the plane intersecting the prism?

1) triangle
2) trapezoid
3) hexagon
4) rectangle

9. The cross section of a regular pyramid contains the altitude of the pyramid. The shape of this cross section is a

Vertical

1) circle
2) square
3) triangle
4) rectangle

Volume

Volume = (<u>Area of the base</u>)(height), if it comes to a point, multiply by $\frac{1}{3}$.

Area of the base is USUALLY $A = lw$ (rectangle/square) or $A = \pi r^2$ (circle)

Most volume formulas are on the reference sheet. Be careful. B = area of the base

General Prism: $V = (area\ base)(height)$

Rectangular prism: $V = lwh$

Triangular prism: $V = \frac{1}{2}lwh$

Cylinder: $V = \pi r^2 h$

Pyramid: $V = \frac{1}{3}lwh$

Cone: $V = \frac{1}{3}\pi r^2 h$

Sphere: $V = \frac{4}{3}\pi r^3$

1. What is the volume of a rectangular prism whose length is 4 cm, width is 6 cm, and height is 5 cm?

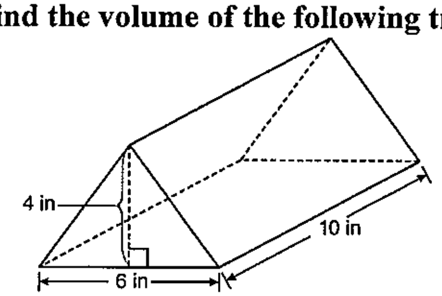
$V = lwh$
$V = 4(6)(5)$
$V = 120\ cm^3$

2. What is the volume of a cube if each side of the cube measures 8 in?

$V = lwh$
$V = 8(8)(8)$
$V = 512\ in^3$

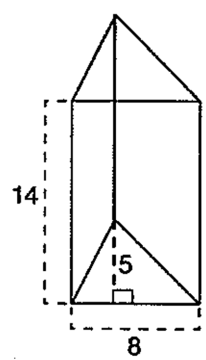
Find the volume of the following triangular prisms

3.



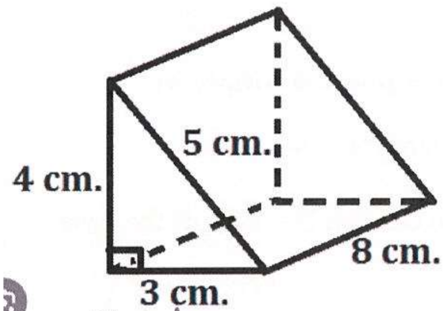
$V = \frac{1}{2}lwh$
$V = \frac{1}{2}(6)(10)(4)$
$V = 120\ in^3$

4.



$V = \frac{1}{2}lwh$
$V = \frac{1}{2}(8)(14)(5)$
$V = 280\ units^3$

5.

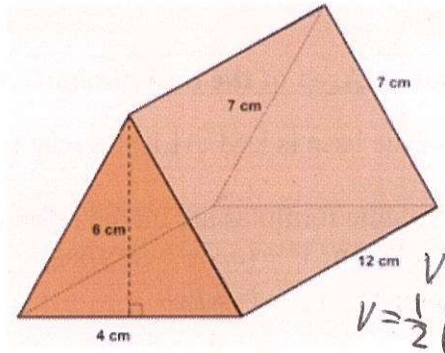


$V=\frac{1}{2}lwh$

$V=\frac{1}{2}(3)(8)(4)$

$V=48\ cm^3$

6.

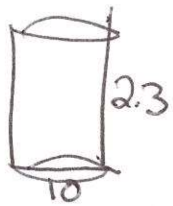


$V=\frac{1}{2}lwh$

$V=\frac{1}{2}(4)(12)(6)$

$V=144\ cm^3$

7. A cylinder has a diameter of 10 inches and a height of 2.3 inches. What is the volume of this cylinder, to the *nearest tenth of a cubic inch*?

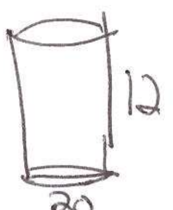


$V=\pi r^2h$

$V=\pi(5)^2(2.3)$

$V=180.6\ in^3$

8. What is the volume of a cylinder whose height is 12 inches and whose diameter is 20 inches in terms of π?



$V=\pi r^2h$

$V=\pi(10)^2(12)$

$V=1200\pi\ in^3$

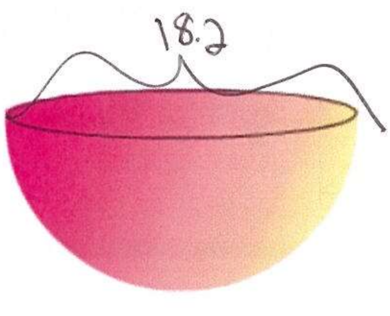
9. Find the volume of a sphere that has a diameter of $\overset{r=6}{\text{12 inches}}$ in terms of π.

$V=\frac{4}{3}\pi r^3$

$V=\frac{4}{3}\pi(6)^3$

$V=288\pi$

10. Find the volume of the object below if the diameter is 18.2 meters. Round your answer to the *nearest cubic meter*.



$V=\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V=\frac{1}{2}\left(\frac{4}{3}\pi(9.1)^3\right)$

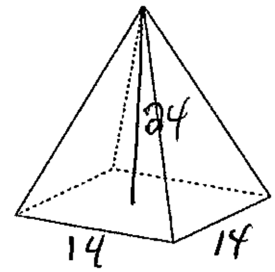
$V=1578\ m^3$

11. A regular pyramid has a square base with an edge length of 14 cm and an altitude of 24 cm. Find its volume.

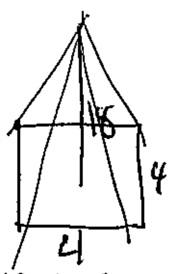
$V=\frac{1}{3}lwh$

$V=\frac{1}{3}(14)(14)(24)$

$V=1568 cm^3$



12. Find the volume of a square pyramid with a base with edge length 4 inches and a height of 18 inches.

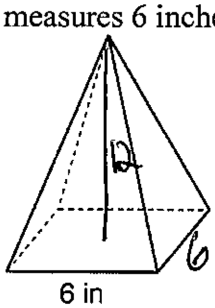


$V=\frac{1}{3}lwh$

$V=\frac{1}{3}(4)(4)(18)$

$V=96 in^3$

13. As shown in the diagram below, a regular pyramid has a square base whose side measures 6 inches. If the altitude of the pyramid measures 12 inches, find its volume.

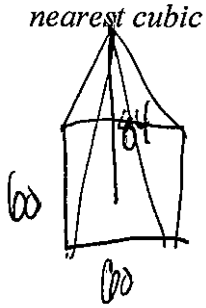


$V=\frac{1}{3}lwh$

$V=\frac{1}{3}(6)(6)(12)$

$V=144 in^3$

14. A child's tent can be modeled as a pyramid with a square base whose sides measure 60 inches and whose height measures 84 inches. What is the volume of the tent, to the *nearest cubic inch*?

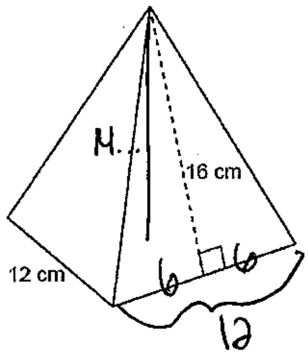


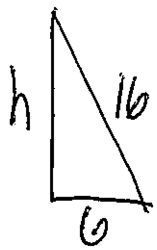
$V=\frac{1}{3}lwh$

$V=\frac{1}{3}(60)(60)(84)$

$V=100{,}800 in^3$

15. A candle in the shape of a right pyramid is modeled below. Each side of the square base measures 12 centimeters. The slant height of the pyramid measures 16 centimeters. Determine and state the volume of the candle, to the *nearest cubic centimeter*.





$a^2+b^2=c^2$

$6^2+h^2=16^2$

$36+h^2=256$

$-36 \quad -36$

$\sqrt{h^2}=\sqrt{220}$

$h=14..$

$V=\frac{1}{3}lwh$

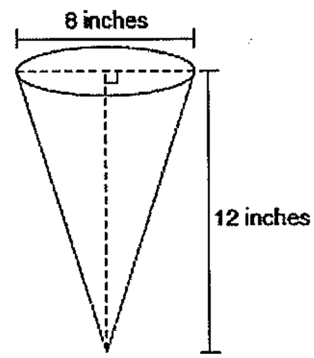
$V=\frac{1}{3}(12)(12)(14..)$

$V=712\text{ cm}^3$

16. In the diagram below, a right circular cone has a diameter of 8 inches and a height of 12 inches.

What is the volume of the cone to the *nearest cubic inch*?

1) 201 3) 603
2) 481 4) 804

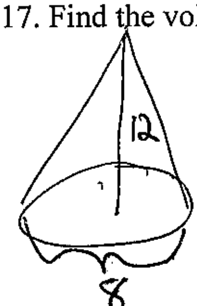


$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi(4)^2(12)$

$V=201..$

17. Find the volume of a cone with a height of 12 in and a diameter of 8 in in terms of π.

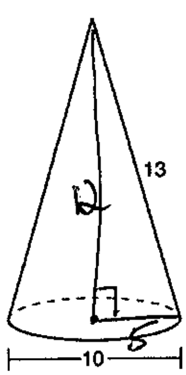


$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi(4)^2(12)$

$V=64\pi$

18. Determine and state the volume of the cone, in terms of π.



$a^2+b^2=c^2$

$5^2+b^2=13^2$

$25+b^2=169$

$-25 \quad -25$

$\sqrt{b^2}=\sqrt{144}$

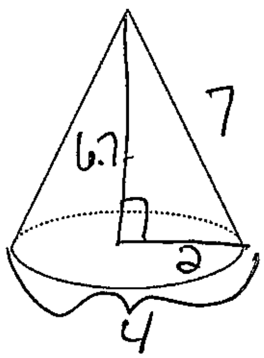
$b=12$

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi(5)^2(12)$

$V=100\pi$

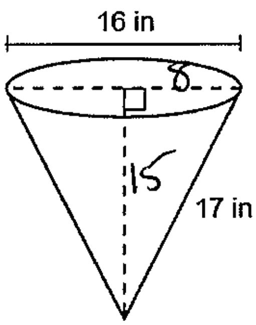
19. A cone has a base with a diameter of 4 and a slant height of 7.
Find its volume rounded to the *nearest* tenth.



$a^2+b^2=c^2$
$2^2+b^2=7^2$
$4+b^2=49$
$-4 \quad -4$
$\sqrt{b^2}=\sqrt{45}$
$b=6.7\ldots$

$V=\frac{1}{3}\pi r^2h$
$V=\frac{1}{3}\pi(2)^2(6.7\ldots)$
$V=28.1u^3$

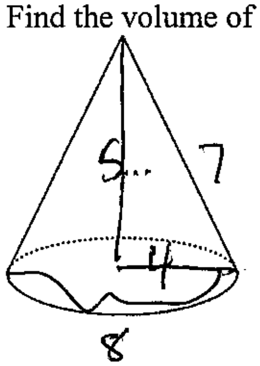
20. In the diagram below, a cone has a diameter of 16 inches and a slant height of 17 inches. What is the volume of the cone, in terms of π, in cubic inches?



$a^2+b^2=c^2$
$8^2+b^2=17^2$
$64+b^2=289$
$-64 \quad -64$
$\sqrt{b^2}=\sqrt{225}$
$b=15$

$V=\frac{1}{3}\pi r^2h$
$V=\frac{1}{3}\pi(8)^2(15)$
$V=320\pi \text{ in}^3$

21. In the diagram below, a right circular cone has a diameter of 8 and a slant height of 7. Find the volume of the cone rounded to the *nearest tenth.*



$a^2+b^2=c^2$
$4^2+b^2=7^2$
$16+b^2=49$
$-16 \quad -16$
$\sqrt{b^2}=\sqrt{33}$
$b=5\ldots$

$V=\frac{1}{3}\pi r^2h$
$V=\frac{1}{3}\pi(4)^2(5\ldots)$
$V=96.3u^3$

$\text{density} = \frac{\text{mass}}{\text{volume}}$ $\text{Population density} = \frac{\text{Population}}{\text{area}}$

Name Schlansky Date ____________
Mr. Schlansky Geometry

Density

1. Farmer John has a farm with a chicken pen in it. The chicken pen is rectangular measuring 5 yards by 7 yards. If there are 48 chickens in the pen, what is the population density to the nearest tenth of a chicken?

$Pd = \frac{P}{a}$

$Pd = \frac{48 \text{ chickens}}{35 \text{ yd}^2}$

$A = lw$
$A = 5(7)$
$A = 35 \text{ yd}^2$

2. Jennifer is having her Sweet 16 party on a giant circular patio that has a radius of 7.2 meters. If there are 83 people at the party, to the nearest tenth, what is the population density?

$Pd = \frac{P}{a}$

$Pd = \frac{83 \text{ ppl}}{162\ldots \text{yd}^2}$

$Pd = .5 \text{ ppl/yd}^2$

$A = \pi r^2$
$A = \pi(7.2)^2$
$A = 162\ldots \text{ m}^2$

3. For a music festival, a stage was built in the shape of a right triangle whose sides measure 6 yards, 8 yards, and 10 yards. At the end of the concert, all of the performers came out an performed together. There were a total of 62 performers on the stage. To the nearest tenth of a person, what was the population density on the stage?

$Pd = \frac{P}{a}$

$Pd = \frac{62 \text{ ppl}}{24 \text{ yd}^2}$

$Pd = 2.6 \text{ ppl/yd}^2$

$A = \frac{1}{2}bh$
$A = \frac{1}{2}(6)(8)$
$A = 24 \text{ yd}^2$

4. Town A has an area of 12 square miles. Town B has an area of 10 square miles. If town A has a population of 8,198 people and town B has a population of 7,384 people, which town has a greater population density? Justify your answer.

Town A

$Pd = \frac{P}{a}$

$Pd = \frac{8198 \text{ ppl}}{12 \text{ mi}^2}$

$Pd = 683\ldots \text{ ppl/mi}^2$

Town B

$Pd = \frac{P}{a}$

$Pd = \frac{7384 \text{ ppl}}{10 \text{ mi}^2}$

$Pd = 738.4 \text{ ppl/mi}^2$

Town B has a greater population density

5. A brick that weighs 1824 grams has dimensions that measure 4 cm by 3 cm by 8 cm. To the nearest tenth, what is the density of the brick?

$d = \frac{m}{V}$

$d = \frac{1824g}{96\ cm^3}$

$d = 19\ g/cm^3$

$V = lwh$

$V = 4(3)(8)$

$V = 96\ cm^3$

6. A cylindrical candleholder has a diameter of 4.5 cm and a height of 20 cm. If the candleholder has a mass of 2900 g, rounded to the nearest whole number, what is its density?

$d = \frac{m}{V}$

$d = \frac{2900g}{318...cm^3}$

$d = 9\ g/cm^3$

TYRE π

$V = \pi r^2 h$

$V = \pi(2.25)^2(20)$

$V = 318...\ cm^3$

7. What is the density of a solid sphere of clay that has a diameter of 3.2 inches and has a mass of 552 grams? Round your answer to the nearest tenth.

$d = \frac{m}{V}$

$d = \frac{552 grams}{17..\ in^3}$

$d = 32.2\ g/in^3$

TYRE π in

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(1.6)^3$

$V = 17...\ in^3$

8. A wooden cube has an edge length of 6 centimeters and a mass of 137.8 grams. Determine the density of the cube, to the *nearest thousandth*. State which type of wood the cube is made of, using the density table below.

lwh all equal

Type of Wood	Density (g/cm³)
Pine	0.373
Hemlock	0.431
Elm	0.554
Birch	0.601
Ash	0.638
Maple	0.676
Oak	0.711

$d = \frac{m}{V}$

$d = \frac{137.8g}{216\ cm^3}$

$d = .638\ g/cm^3$

Ash

$V = lwh$

$V = 6(6)(6)$

$V = 216\ cm^3$

Compound and Displaced Volume

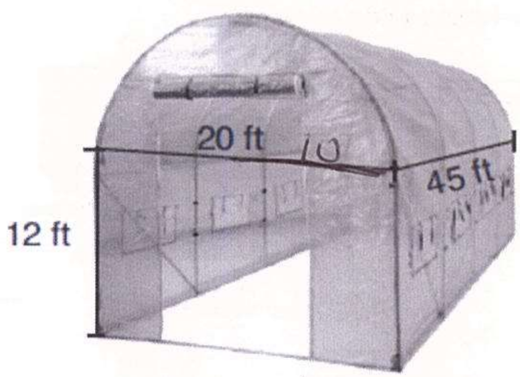
Compound Volume: If a shape is made up of multiple shapes on top of each other, find the volume of each and add them together.

Displaced Volume (Hollow): If a shape is being taken out of a bigger shape, find the volume of each and subtract them.

*If given thickness, draw a cross section and subtract double the thickness from each dimension.

Find the volume of each of the following shapes and round to the *nearest tenth* if necessary.

1.



rectangular prism

$V = lwh$

$V = 12(20)(45)$

$V = 10800$

half cylinder

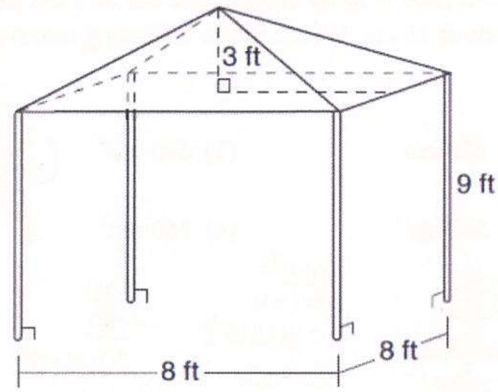
$V = \frac{1}{2}\pi r^2 h$

$V = \frac{1}{2}\pi(10)^2(45)$

$V = 7068...$

$10800 + 7068... = 17868.6$

2.



rectangular prism

$V = lwh$

$V = 8(8)(9)$

$V = 576$

pyramid

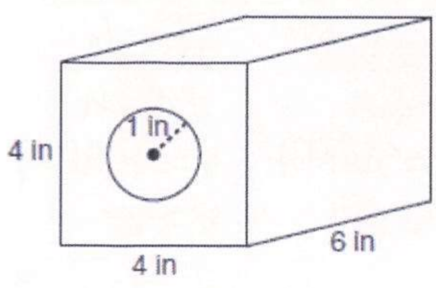
$V = \frac{1}{3}lwh$

$V = \frac{1}{3}(8)(8)(9)$

$V = 192$

$576 + 192 = 768$

3. A solid metal prism has a rectangular base with sides of 4 inches and 6 inches, and a height of 4 inches. A hole in the shape of a cylinder, with a radius of 1 inch, is drilled through the entire length of the rectangular prism. What is the approximate volume of the remaining solid, in cubic inches?



rectangular prism

$V = lwh$

$V = 4(4)(6)$

$V = 96$

cylinder

$V = \pi r^2 h$

$V = \pi(1)^2(6)$

$V = 18...$

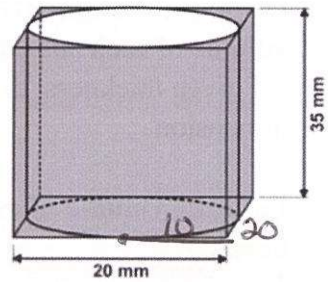
$96 - 18... = 77 \text{ in}^3$

4. A piece of hardware is constructed by drilling a cylindrical hole through a right prism with a square base that measures 20 mm on each side. The hole is 35 mm long, as shown in the diagram. Determine the volume of the remaining material once the hole has been drilled. Round your answer to the nearest cubic millimeter.

Rectangular Prism
$V=lwh$
$V=20(20)(35)$
$V=14000$

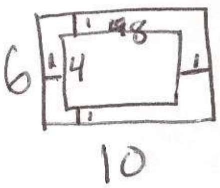
Cylinder
$V=\pi r^2 h$
$V=\pi(10)^2(35)$
$V=10995...$

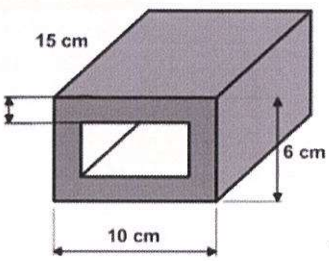
$14000-10995..=3004\,mm^3$



5. A box tube is to be constructed out of 1 cm thick metal that has a width of 10 cm, a height of 6 cm, and a depth of 15 cm. Which of the following represents the volume of the metal used?

(1) 420 cm^3
(2) 540 cm^3
(3) 640 cm^3
(4) 760 cm^3





Outside
$V=lwh$
$V=10(6)(15)$
$V=900$

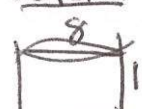
inside
$V=lwh$
$V=8(4)(15)$
$V=480$

$900-480=420\,cm^3$

6. The paper towel roll shown below has a diameter of 8 inches and the paper has a thickness of 3 inches. If the height of the paper towel roll is 12 inches, what is the volume of the paper towels? Round your answer to the nearest tenth of a cubic inch.

12

Outside



$V=\pi r^2 h$
$V=\pi(4)^2(12)$
$V=603..$

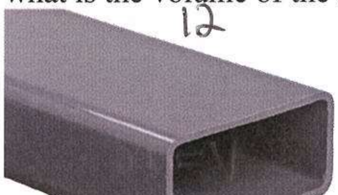
inside

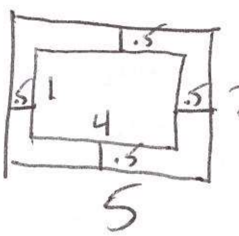
$V=\pi r^2 h$
$V=\pi(1)^2(12)$
$V=37...$

$603..-37..=565.5\,in^3$

7. A hollow metal pipe is in the shape of a rectangular prism that has a height of 12 cm. The length is 5 cm and the width is 2 cm. If the thickness is 0.5 cm all the way around, what is the volume of the metal?

12





Outside
$V=lwh$
$V=5(2)(12)$
$V=120$

inside
$V=lwh$
$V=4(1)(12)$
$V=48$

$120-48=72\,cm^3$

Volume with Algebra
Substitute into appropriate volume formula
Solve the equation
***To get rid of a fraction, multiply by the denominator**
***To get rid of cubed, take the cubed root (final step)**

1. A brick in the shape of a rectangular prism has a base that measures 3 inches by 5 inches. If the volume of the brick is 90 cubic inches, what is the height of the brick?

$V = lwh$

$90 = 3(5)(x)$

$\frac{90}{15} = \frac{15x}{15}$

$6 = x$

2. A right circular cylinder has a volume of 1,000 cubic inches and a height of 8 inches. What is the radius of the cylinder to the *nearest tenth of an inch*?

$V = \pi r^2 h$

$\frac{1000}{8\pi} = \frac{\pi r^2 (8)}{8\pi}$

$\sqrt{39.7..} = \sqrt{r^2}$

$6.3 = r$

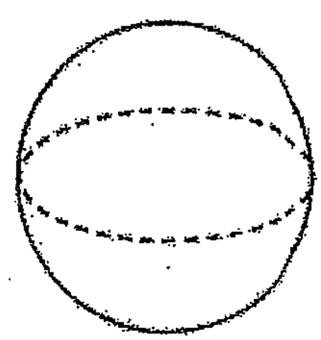
3. The base of a pyramid is a rectangle with a width of 6 cm and a length of 8 cm. Find, in centimeters, the height of the pyramid if the volume is 288 cm^3.

$V = \frac{1}{3} lwh$ $\qquad x = 18$

$288 = \frac{1}{3}(6)(8)(x)$

$\frac{288}{16} = \frac{16x}{16}$

4. Find the radius of a sphere with a volume of 576π cubic units. Find the answer to the *nearest tenth of a unit.*



$V = \frac{4}{3}\pi r^3$ $\qquad r = 7.6$

$3(576\pi) = (\frac{4}{3}\pi r^3)3$

$\frac{1728\pi}{4\pi} = \frac{4\pi r^3}{4\pi}$

$\sqrt[3]{432} = \sqrt[3]{r^3}$

Use equation solver as needed

5. The volume of a cylinder is 12,566.4 cm^3. The height of the cylinder is 8 cm. Find the radius of the cylinder to the *nearest tenth of a centimeter.*

$V = \pi r^2 h$

$\frac{12566.4}{8\pi} = \frac{\cancel{\pi} r^2 (\cancel{8})}{\cancel{8\pi}}$

$\sqrt{500..} = \sqrt{r^2}$

$22.4 = r$

6. A rectangular shipping box must have a length of 11 inches and a width of 8 inches. Find, to the *nearest tenth of an inch*, the height of the box such that the volume is 800 cubic inches.

$V = lwh$

$800 = 11(8)(x)$

$\frac{800}{88} = \frac{\cancel{88}x}{\cancel{88}}$

$9.09.. = x$

$9.1 = x$

7. If the volume of a sphere is 36π, what is the radius of the sphere?

(1) 3 (2) 6 (3) 12 (4) 24

$V = \frac{4}{3}\pi r^3$

$3(36\cancel{\pi}) = (\frac{4}{\cancel{3}}\cancel{\pi} r^3)\cancel{3}$

$\frac{108}{4} = \frac{\cancel{4}r^3}{\cancel{4}}$

$\sqrt[3]{27} = \sqrt[3]{r^3}$

$3 = r$

8. Find the length of the radius of a cylinder to the *nearest tenth* if it has a volume of 60 cm^3 and a height of 10 cm.

$V = \pi r^2 h$

$\frac{60}{10\pi} = \frac{\cancel{\pi} r^2 (\cancel{10})}{\cancel{10\pi}}$

$\sqrt{1.9..} = \sqrt{r^2}$

$1.4 = r$

9. The volume of a triangular prism is 70 in^3. The base of the prism is a right triangle with one leg whose measure is 5 inches. If the height of the prism is 4 inches, determine and state the length, in inches, of the other leg of the triangle.

$V = \frac{1}{2}lwh$

$70 = \frac{1}{2}(5)(x)(4)$

$\frac{70}{10} = \frac{\cancel{10}x}{\cancel{10}}$

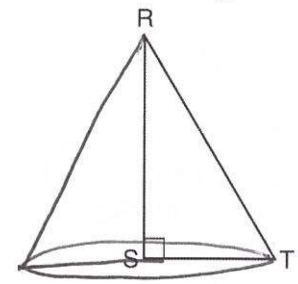
$7 = x$

78 80

3 dimensional rotations ALMOST ALWAYS form a cylinder or cone

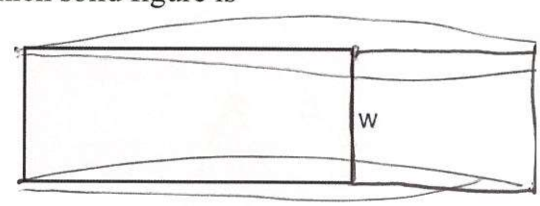
Reflect the shape in 2 dimensions and connect the images with curves

1. Which object is formed when right triangle *RST* shown below is rotated around leg $\overline{RS}$?



1) a pyramid with a square base
2) an isosceles triangle
3) a right triangle
4) a cone

2. If the rectangle below is continuously rotated about side *w*, which solid figure is formed?

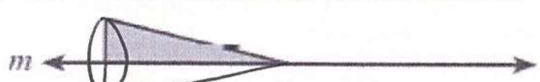


1) pyramid
2) rectangular prism
3) cone
4) cylinder

3. If you rotated the shaded figure below about line *m*, which solid would result from the revolution?

(1) cylinder (2) cone ~~(3) cube~~ (4) sphere

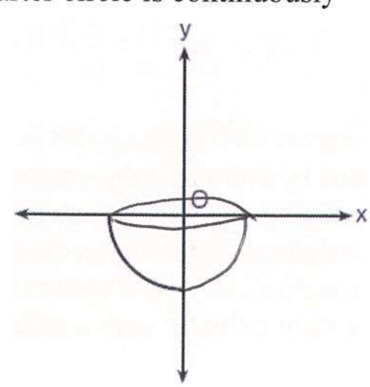
4. If you rotated the triangular region of the figure below about line *m*, what solid would result from the revolution?



(1) cylinder (2) cone (3) cube (4) sphere

5. Circle *O* is centered at the origin. In the diagram below, a quarter of circle *O* is graphed.

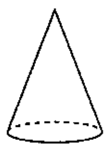
Which three-dimensional figure is generated when the quarter circle is continuously rotated about the *y*-axis?

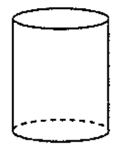


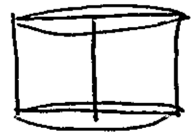
1) cone
2) sphere
3) cylinder
4) hemisphere

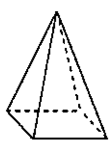
6. If an equilateral triangle is continuously rotated around one of its medians, which 3-dimensional object is generated?
1) cone
2) pyramid
3) prism
4) sphere

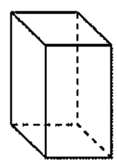
7. A student has a rectangular postcard that he folds in half lengthwise. Next, he rotates it continuously about the folded edge. Which three-dimensional object below is generated by this rotation?

1) 

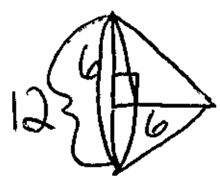
3) 



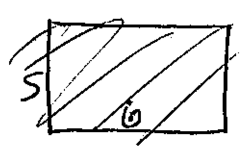
2) 

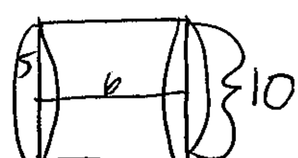
4) 

8. An isosceles right triangle whose legs measure 6 is continuously rotated about one of its legs to form a three-dimensional object. The three-dimensional object is a
1) cylinder with a diameter of 6
2) cylinder with a diameter of 12
3) cone with a diameter of 6
4) cone with a diameter of 12



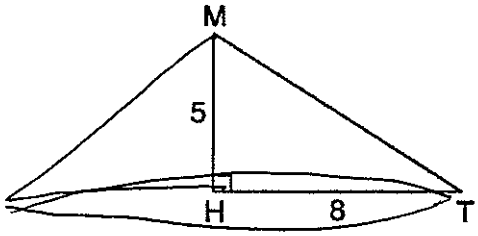
9. Which three-dimensional figure will result when a rectangle 6 inches long and 5 inches wide is continuously rotated about the longer side?
1) a rectangular prism with a length of 6 inches, width of 6 inches, and height of 5 inches
2) a rectangular prism with a length of 6 inches, width of 5 inches, and height of 5 inches
3) a cylinder with a radius of 5 inches and a height of 6 inches
4) a cylinder with a radius of 6 inches and a height of 5 inches





10. Square *MATH* has a side length of 7 inches. Which three-dimensional object will be formed by continuously rotating square *MATH* around side $\overline{AT}$?
1) a right cone with a base diameter of 7 inches
2) a right cylinder with a diameter of 7 inches
3) a right cone with a base radius of 7 inches
4) a right cylinder with a radius of 7 inches

11. In right triangle MTH shown below, $m\angle H = 90°$, $HT = 8$, and $HM = 5$. Determine and state, to the *nearest tenth*, the volume of the three-dimensional solid formed by rotating $\triangle MTH$ continuously around $\overline{MH}$.

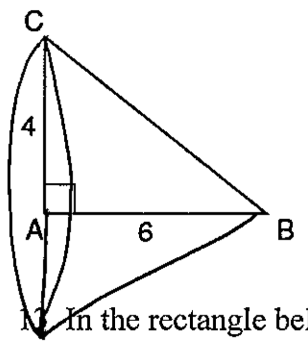


$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(8)^2(5)$

$V = 335.1$

12. In the diagram below, right triangle ABC has legs whose lengths are 4 and 6. What is the volume, in terms of π, of the three-dimensional object formed by continuously rotating the right triangle around $\overline{AB}$?

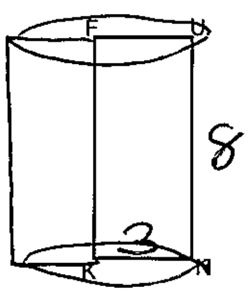


$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(4)^2(6)$

$V = 32\pi$

13. In the rectangle below, $\overline{UN} = 8\,in$ and $\overline{KN} = 3\,in$. Find the volume of the three dimensional object created by rotating rectangle FUNK continuously about side $\overline{FK}$ in terms of π.

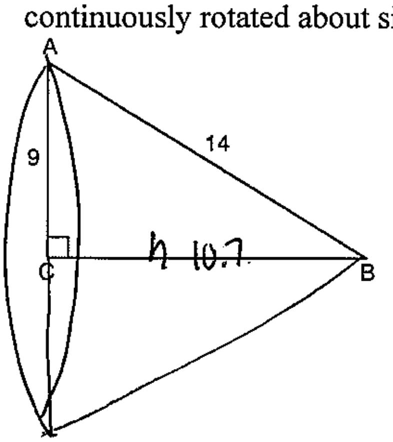


$V = \pi r^2 h$

$V = \pi(3)^2(8)$

$V = 72\pi$

14. In the diagram of right triangle ABC shown below, $AB = 14$ and $AC = 9$. What is the volume of the three dimensional object formed when the triangle is continuously rotated about side $\overline{BC}$ to the nearest tenth.



$a^2 + b^2 = c^2$

$9^2 + h^2 = 14^2$

$81 + h^2 = 196$

$-81 \quad -81$

$\sqrt{h^2} = \sqrt{115}$

$h = 10.7$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(9)^2(10.7)$

$V = 909.6$

CONVERSIONS

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilograms	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

Conversions

-Write the unit you are starting with

-Make a fraction:

Put the unit you are canceling on bottom

Put the unit you are changing to on top

1. 750 meter to kilometer

$750\cancel{m} \cdot \frac{1\ km}{100\ \cancel{m}} = \frac{750}{1000} = .75 km$

2. 1.2 kilometer to meter

$1.2\cancel{km} \cdot \frac{1000\ m}{1\ \cancel{km}} = 1.2(1000) = 1200m$

3. 220 centimeter to meter

$220\cancel{cm} \cdot \frac{1\ m}{100\ \cancel{cm}} = \frac{220}{100} = 2.2m$

4. 3.45 meter to centimeter

$3.45\cancel{m} \cdot \frac{100\ cm}{1\ \cancel{m}} = 3.45(100) = 345cm$

5. 45 minutes to hours

$45\cancel{min} \cdot \frac{1\ hr}{60\ \cancel{min}} = \frac{45}{60} = .75hr$

6. 1.2 hours to minutes

$1.2\cancel{hr} \cdot \frac{60\ min}{1\ \cancel{hr}} = 1.2(60) = 72 min$

7. 1.6 inches to centimeter

$1.6\cancel{in} \cdot \frac{2.54\ cm}{1\ \cancel{in}} = 1.6(2.54) = 4.1cm$

8. 3.2 centimeter to inches

$3.2\cancel{cm} \cdot \frac{1\ in}{2.54\ \cancel{cm}} = \frac{3.2}{2.54} = 1.3in$

9. 6.2 miles to feet

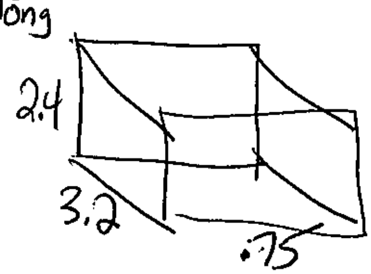
$6.2\cancel{mi} \cdot \frac{5280ft}{1\ \cancel{mi}} = 6.2(5280) = 32736ft$

10. 5000 feet to miles

$5000\cancel{ft} \cdot \frac{1\ mi}{5280\ \cancel{ft}} = \frac{5000}{5280} = .9 mi$

11. What is the volume, to the *nearest cubic foot*, of a rectangular prism that is 2.4 feet high, 3.2 feet wide, and 9 inches ~~high?~~ long

$9in \cdot \frac{1\ ft}{12\ in} = \frac{9}{12} = .75ft$



$V = lwh$

$V = .75(3.2)(2.4)$

$V = 5.76ft^3$

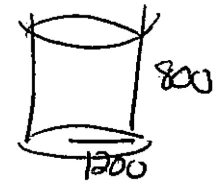
12. What is the volume of a cylinder, to the *nearest tenth of a cubic meter*, whose radius is 1200 meters and height is 0.8 kilometers?

$.8 km \cdot \frac{1000 m}{1 km} = .8(1000) = 800 m$

$V = \pi r^2 h$

$V = \pi(1200)^2(800)$

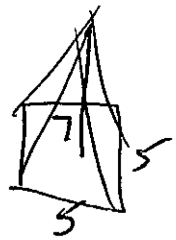
$V = 3619114737 m^3$



13. A child's tent can be modeled as a pyramid with a square base whose sides measure 60 inches and whose height measures 84 inches. What is the volume of the tent, to the *nearest cubic ~~inch~~?* foot

$60 in \cdot \frac{1 ft}{12 in} = \frac{60}{12} = 5 ft$

$84 in \cdot \frac{1 ft}{12 in} = \frac{84}{12} = 7 ft$



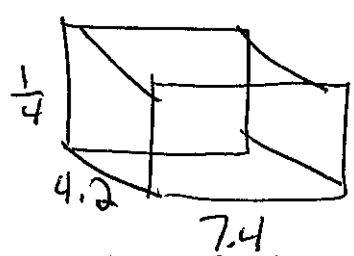
$V = \frac{1}{3} lwh$

$V = \frac{1}{3}(5)(5)(7)$

$V = 58 ft^3$

14. A rectangular table top has a length of 4.2 feet, a width of 7.1 feet, and a thickness of 3 inches. What is the volume of the rectangular table top to the *nearest cubic foot*?

$3 in \cdot \frac{1 ft}{12 in} = \frac{3}{12} = \frac{1}{4} ft$

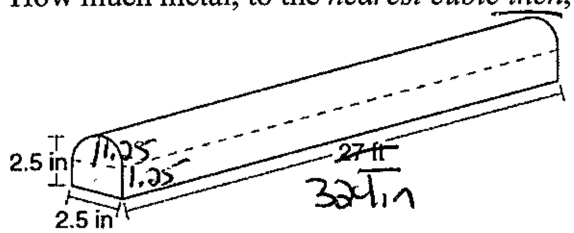


$V = lwh$

$V = 7.4(4.2)(\frac{1}{4})$

$V = 8 ft^3$

15. A fabricator is hired to make a 27-foot-long solid metal railing for the stairs at the local library. The railing is modeled by the diagram below. The railing is 2.5 inches high and 2.5 inches wide and is comprised of a rectangular prism and a half-cylinder. How much metal, to the *nearest cubic inch*, will the railing contain?



half cylinder

~~$V = \frac{1}{2}\pi(1.25$~~

$V = \frac{1}{2}\pi r^2 h$

$V = \frac{1}{2}\pi(1.25)^2(324)$

$V = 795..$

rectangular prism

$V = lwh$

$V = 2.5(1.25)(324)$

$V = 1012.5$

$795.. + 1012.5 = 1808 in^3$

$27 ft \cdot \frac{12 in}{1 ft}$

$27(12) = 324 in$

Name Schlansky
Mr. Schlansky

Date ____________
Geometry

Unit Analysis

1. A block of wood has a volume of 200 cm^3. The cost of the wood is \$.10 per gram and the density of the wood is 2.1 g/cm^3. What would be the cost of producing 15 of these blocks of wood.

$$200\ \cancel{cm^3} \times \frac{2.1\ \cancel{g}}{1\ \cancel{cm^3}} \times \frac{.10\ \$}{1\ \cancel{g}} \times 15 = \$630.00$$

2. A cylindrical test tube has a volume of 45 in^3. The liquid inside has weighs 4 ounces per cubic inch and the cost of the liquid is \$.12 per ounce. How much will it cost to fill the test tube to 80% of its capacity?

.8

$$45\ \cancel{in^3} \times \frac{4\ \cancel{oz}}{1\ \cancel{in^3}} \times \frac{.12\ \$}{1\ \cancel{oz}} \times .8 = \$17.28$$

3. The volume of a pool is 25,000 gallons. The cost of the water to fill the pool is \$120 per 8000 gallons. How much will it cost to fill the pool up 90%?

$$25000\ \cancel{gal} \times \frac{120\ \$}{8000\ \cancel{gal}} \times .9 = \frac{25000(120)(.9)}{8000} = \$337.50$$

4. An object made of steel has a volume of $24.1 cm^3$. The steel costs \$1.25 for 500 grams and has a density of $3.1 g/cm^3$. How much will it cost to make 25 of these objects?

$$24.1\ \cancel{cm^3} \times \frac{3.1\ \cancel{g}}{1\ \cancel{cm^3}} \times \frac{1.25\ \$}{500\ \cancel{g}} \times 25 = \frac{24.1(3.1)(1.25)(25)}{500}$$

\$4.67

5. A stone brick has a volume of 150 in^3. The stone weighs 5 grams per cubic inch and it costs \$4.52 for 500 grams of stone. How much will it cost to purchase enough stone to make 12 bricks?

$$150\ \cancel{in^3} \times \frac{5\ \cancel{g}}{1\ \cancel{in^3}} \times \frac{4.52\ \$}{500\ \cancel{g}} \times 12 = \frac{150(5)(4.52)(12)}{500}$$

\$81.36

5. A stone brick has a volume of 150 in^3. The stone weighs 5 grams per cubic inch and it costs \$4.52 for 500 grams of stone. How much will it cost to purchase enough stone to make 12 bricks?

$$150\,in^3 \cdot \frac{5\,g}{1\,in^3} \cdot \frac{4.52\,\$}{500\,g} \times 12 \qquad \frac{150(5)(4.52)(12)}{500} = \$81.36$$

6. A machinist creates a solid steel part for a wind turbine engine. The part has a volume of 1015 cubic centimeters. Steel can be purchased for \$0.29 per ~~kilogram~~ 1000g, and has a density of 7.95 g/cm³. If the machinist makes 500 of these parts, what is the cost of the steel, to the *nearest dollar*?

$$1015\,cm^3 \cdot \frac{7.95\,g}{1\,cm^3} \cdot \frac{.29\,\$}{1000\,g} \cdot \cancel{\times 500} \qquad \frac{1015(7.95)(.29)(500)}{1000}$$

\$1170

7. A water tower has a volume of 1000 liters and the cost of the water is \$250 per ~~cubic kiloliter.~~ 1000 l How much will it cost to fill the water tower up to 60% of its capacity?

$$1000\,l \cdot \frac{\$250}{1000\,l} \times .6 \qquad \frac{1000(250)(.6)}{1000} = \$150$$

8. A wax candle has a volume of 885 cubic centimeters. The wax costs \$1.24 per ~~kilogram~~ 1000 g and has a density of $1.9\,g/cm^3$. How much will it cost to make 80 candles?

$$885\,cm^3 \cdot \frac{1.9\,g}{1\,cm^3} \cdot \frac{1.24\,\$}{1000\,g} \times 80 \qquad \frac{885(1.9)(1.24)(80)}{1000} = \$166.80$$

9. An object has a volume of 12 cubic inches and the material it is made from has a density of $7.6\,g/in^3$. If the cost of the material is \$1.25 per ~~kilogram~~ 1000g, how much will it cost to make 50 of these objects?

$$12\,in^3 \cdot \frac{7.6\,g}{1\,in^3} \cdot \frac{1.25\,\$}{1000\,g} \times 50 \qquad \frac{12(7.6)(1.25)(50)}{1000} = \$5.70$$

Modeling Volume

1) Check units. Convert if necessary. To convert units: Multiply to get units to cancel out. Example: $3\ in \bullet \dfrac{2.54\ cm}{1\ in}$

2) FIND VOLUME (Likely to be compound volume (add) or displaced volume (subtract)

3) Begin unit analysis. Start with volume!
Example, a volume of 12 cubic inches has a density of $7.6\ g/in^3$, which costs \$1.25 per kilogram, and 50 are needed that are each filled up to 85%:

$$12\ in^3 \bullet \frac{7.6\ g}{1\ in^3} \bullet \frac{1 kg}{1000 g} \bullet \frac{\$1.25}{1\ kg} \bullet 50 \bullet .85$$

*If given volume, substitute for V and do Algebra!

1. Cylindrical bricks are needed to fill a hole in a homeowner's backyard. Each brick is to have a diameter of 4 cm and a height of 2 cm. The weight of the concrete that the brick is going to be made from is 2.1 ounces per cubic centimeter. If the concrete costs \$.14 per ounce, how much would it cost to purchase four bricks? Round your answer to the *nearest cent.*

$V=\pi r^2h$

$V=\pi(2)^2(2)$

$V=25\ldots cm^3$

$$25\ldots \cancel{cm^3} \cdot \frac{2.1\ \cancel{oz}}{1\ \cancel{cm^3}} \cdot \frac{.14\ \$}{1\ \cancel{oz}} \times 4 = \$29.56$$

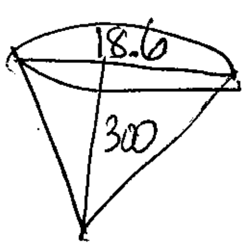
2. A town in upstate New York keeps sand in a silo that is in the shape of a cone. They use this sand to help de-ice the roads after a snowstorm. The silo has a diameter of 18.6 meters and a height of .3 kilometers. The weight of the sand is 1.2 ounces per cubic meter. If the sand costs \$.12 per ounce, how much will it cost the town to fill 80% of the silo?

$$.3\ \cancel{km} \cdot \frac{1000\ m}{1\ \cancel{km}} = 300m$$

$V=\frac{1}{3}\pi r^2h$

$V=\frac{1}{3}\pi(9.3)^2(300)$

$V=27171\ldots m^3$

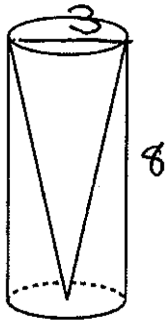


$$27171\ \cancel{m^3} \cdot \frac{1.2\ \cancel{oz}}{1\ \cancel{m^3}} \cdot \frac{.12\ \$}{1\ \cancel{oz}} \times .8$$

$3130.17

3. Walter wants to make 100 candles in the shape of a cone for his new candle business. The mold shown below will be used to make the candles. Each mold will have a height of 8 inches and a diameter of 3 inches.

Walter goes to a hobby store to buy the wax for his candles. The wax costs $0.10 per ounce. If the weight of the wax is 0.52 ounce per cubic inch, how much will it cost Walter to buy the wax for 100 candles?



$V = \frac{1}{3}\pi r^2 h$

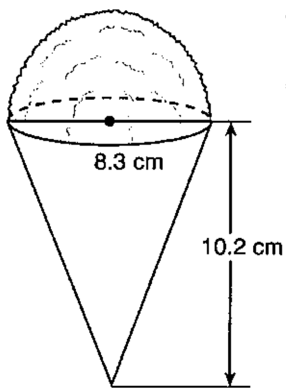
$V = \frac{1}{3}\pi (1.5)^2 (8)$

$V = 18... in^3$

$18... \cancel{in^3} \cdot \frac{.52 \; \cancel{oz}}{1 \; \cancel{in^3}} \cdot \frac{.10 \; \$}{1 \; \cancel{oz}} \times 100$

$18... (.52)(.10)(100) = \$98.02$

4. A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters. The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per ~~kilogram~~ 1000 g, of ice is $3.83. Determine and state the cost of the ice needed to make 50 snow cones.



hemisphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{1}{2}\left(\frac{4}{3}\pi (4.15)^3\right)$

$V = 149...$

cone

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (4.15)^2 (10.2)$

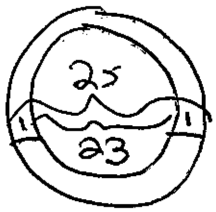
$V = 183...$

$149... + 183... = 333... \; cm^3$

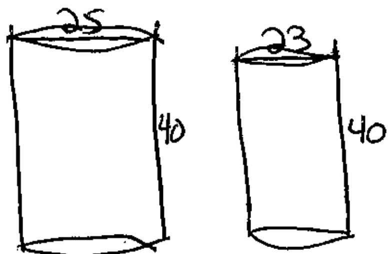
$333.. \cancel{cm^3} \cdot \frac{.697 \; g}{1 \; \cancel{cm^3}} \cdot \frac{3.83 \; \$}{1000 \; g} \times 50$

$\frac{333.. (.697)(3.83)(50)}{1000} = \$44.53$

5. A cylindrical casing is to be put around a garbage can in a busy street in Manhattan. The diameter is 25 inches. The height of the case will be 40 inches and the casing will be 1 inch thick. The density of the metal is .841 grams per cubic inch. What will be the mass of the casing?



Outer — Inner



$3015..in^3 \cdot \frac{.841\ g}{1\ in^3} = 2536\ g$

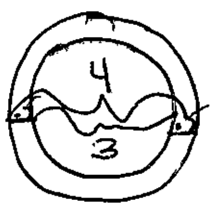
$V = \pi r^2 h$ — $V = \pi r^2 h$

$V = \pi (12.5)^2 (40)$ — $V = \pi (11.5)^2 (40)$

$V = 19634...$ — $V = 16619...$

$19634.. - 16619.. = 3015...in^3$

6. A bakery sells hollow chocolate spheres. The larger diameter of each sphere is 4 cm. The thickness of the chocolate of each sphere is 0.5 cm. Determine and state, to the *nearest tenth of a cubic centimeter*, the amount of chocolate in each hollow sphere. The bakery packages 8 of them into a box. If the density of the chocolate is 1.308 g/cm^3, determine and state, to the *nearest gram*, the total mass of the chocolate in the box.



Outer — Inner

$V = \frac{4}{3}\pi r^3$ — $V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (2)^3$ — $V = \frac{4}{3}\pi (1.5)^3$

$V = 33...$ — $V = 14...$

$33... - 14.. = 19...cm^3$

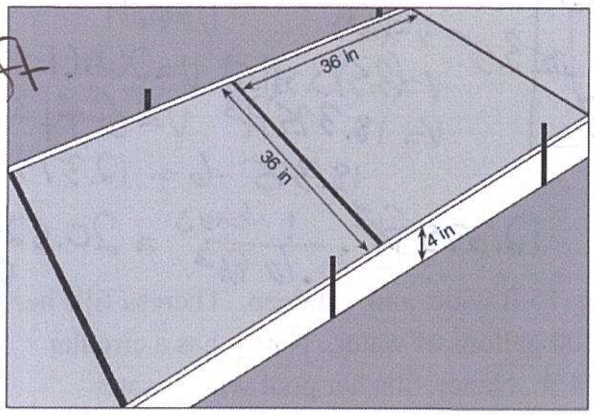
$19..cm^3 \cdot \frac{1.308\ g}{1\ cm^3} \times 8 = 203$ grams

7. Ian needs to replace two concrete sections in his sidewalk, as modeled below. Each section is 36 inches by 36 inches and 4 inches deep. He can mix his own concrete for \$3.25 per cubic foot. How much money will it cost Ian to replace the two concrete sections?

$36 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 3 \text{ ft}$

convert first

$4 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = \frac{1}{3} \text{ ft}$



$V = lwh$

$V = 3(3)(\frac{1}{3})$

$V = 3 \text{ ft}^3$

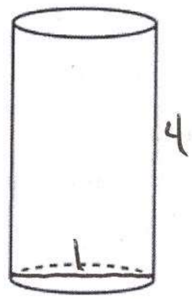
$3 \text{ ft}^3 \cdot \frac{3.25 \$}{1 \text{ ft}^3} \times 2 = \$19.50$

8. A concrete footing is a cylinder that is placed in the ground to support a building structure. The cylinder is 4 feet tall and 12 inches in diameter. A contractor is installing 10 footings. If a bag of concrete mix makes $\frac{2}{3}$ of a cubic foot of concrete, determine and state the minimum number of bags of concrete mix needed to make all 10 footings.

convert first

$12 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}}$

1 ft



$V = \pi r^2 h$

$V = \pi (.5)^2 (4)$

$V = 3.14.. \text{ ft}^3$

$\frac{3.14..(10)}{\frac{2}{3}} = 47.12..$ bags

48 bags

$3.14.. \text{ ft}^3 \cdot \frac{1 \text{ bag}}{\frac{2}{3} \text{ ft}^3} \times 10$

9. A gardener wants to buy enough mulch to cover a rectangular garden that is 3 feet by 10 feet. One bag contains 2 cubic feet of mulch and costs \$3.66. How much will the minimum number of bags cost to cover the garden with mulch 3 inches deep?

it's not asking about cost

$3 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = \frac{1}{4} \text{ ft}$

convert first

$V = lwh$

$V = 3(10)(\frac{1}{4})$

$V = 7.5 \text{ ft}^3$

$7.5 \text{ ft}^3 \cdot \frac{1 \text{ bag}}{2 \text{ ft}^3} \cdot \cancel{\frac{3.66 \$}{1 \text{ bag}}} = \cancel{\frac{7.5(3.66)}{2}} = \cancel{13.725}$

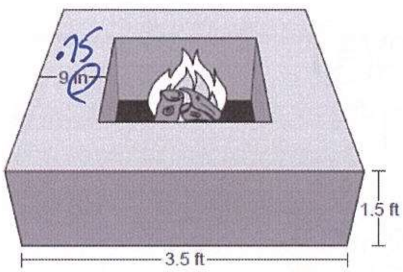
$\frac{7.5}{2} = 3.75$ bags

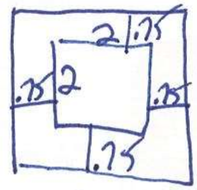
4 bags

10. Josh is making a square-based fire pit out of concrete for his backyard, as modeled by the right prism below. He plans to make the outside walls of the fire pit 3.5 feet on each side with a height of 1.5 feet. The concrete walls of the fire pit are going to be 9 inches thick. If a bag of concrete mix will fill 0.6 ft^3, determine and state the minimum number of bags needed to build the fire pit.

$9 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}}$

$.75 \text{ ft}$





outside

$V = lwh$

$V = (3.5)(3.5)(1.5)$

$V = 18.375 \text{ ft}^3$

inside

$V = lwh$

$V = 2(2)(1.5)$

$V = 6 \text{ ft}^3$

$18.375 - 6 = 12.375 \text{ ft}^3$

$12.375 \text{ ft}^3 \cdot \frac{1 \text{ bags}}{.6 \text{ ft}^3} = 20.625 \text{ bags}$

21 bags

11. Theresa has a rectangular pool 30 ft long, 15 ft wide, and 4 ft deep. Theresa fills her pool using city water at a rate of \$3.95 per 100 gallons of water. Nancy has a circular pool with a diameter of 24 ft and a depth of 4 ft. Nancy fills her pool with a water delivery service at a rate of \$200 per 6000 gallons. If Theresa and Nancy both fill their pools 6 inches from the top of the pool, determine and state who paid more to fill her pool. [1 ft^3 water = 7.48 gallons]

$6 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}}$

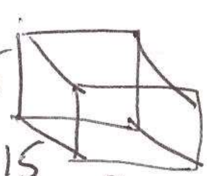
$.5 \text{ ft}$

$4 - .5 \text{ ft}$

3.5 ft high

Theresa paid more!

Theresa



$V = lwh$

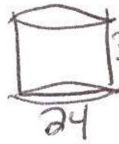
$V = 3.5(15)(30)$

$V = 1575 \text{ ft}^3$

$1575 \text{ ft}^3 \cdot \frac{7.48 \text{ gal}}{1 \text{ ft}^3} \cdot \frac{3.95 \$}{100 \text{ gal}}$

$\frac{1575(7.48)(3.95)}{100} = \$465..$

Nancy



$V = \pi r^2 h$

$V = \pi(12)^2(3.5)$

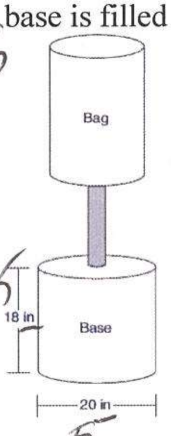
$V = 1583.. \text{ ft}^3$

$\frac{1583..(7.48)(200)}{6000} = \$394..$

$1583.. \text{ ft}^3 \cdot \frac{7.48 \text{ gal}}{1 \text{ ft}^3} \cdot \frac{200 \$}{6000 \text{ gal}}$

11. Shae has recently begun kickboxing and purchased training equipment as modeled in the diagram below. The total weight of the bag, pole, and unfilled base is 270 pounds. The cylindrical base is 18 inches tall with a diameter of 20 inches. The dry sand used to fill the base weighs 95.46 lbs per cubic foot.
To the *nearest pound*, determine and state the total weight of the training equipment if the base is filled to 85% of its capacity.

$18 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 1.5$



$20 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}}$ 1.5

$\frac{5}{3} \text{ ft}$

$\frac{5}{3}$

total weight = weight of equipment + weight of sand

$= 270 + 265.. = 536 \text{ pounds}$

$V = \pi r^2 h$

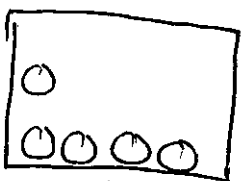
$V = \pi\left(\frac{5}{6}\right)^2(1.5)$

$V = 3.27.. \text{ ft}^3$

$3.27.. \text{ ft}^3 \cdot \frac{95.46 \text{ lb}}{1 \text{ ft}^3} \cdot .85 = 265.. \text{ lb}$

Shelf/Box Questions
-Draw a two dimensional diagram of the shelf/bottom of the box
-Find how many of each object with fit in each dimension by dividing the dimension by the diameter/width of the object and sketch that into the diagram
*For boxes, add in the third dimension
-Multiply the amount in each dimension by each other to come up with the total number.

1. Boxes of baseball cards are being put on a display shelf. Each box is a cube with edge length of 6 inches. The display shelf is 26 inches by 14 inches. The boxes must completely fit on the shelf and cannot be stacked on top of each other. What is the maximum number of boxes that can fit on the shelf?

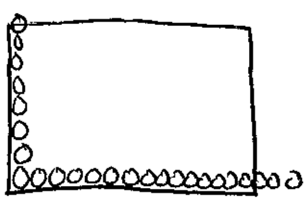
14

26

$4 \times 2 = \boxed{8}$

$\frac{26}{6} = 4.\overline{3} = 4$

$\frac{14}{6} = 2.\overline{3} = 2$

2. Cylindrical soup cans with a base diameter of 2.5 inches and a height of 4 inches are to be put on a display shelf. The display shelf measures 21 inches by 45 inches. The cans must completely fit on the shelf and cannot be stacked on top of each other. What is the maximum number of cans that can fit on the shelf?

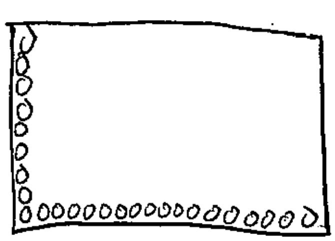
21

45

$18 \times 8 = \boxed{144}$

$\frac{21}{2.5} = 8.4 = 8$

$\frac{45}{2.5} = 18$

3. Lacrosse balls have a diameter of 6.47 centimeters and are to be put on a shelf that measures 120 centimeters by 60 centimeters. The balls must completely fit on the shelf and cannot be stacked on top of each other. What is the maximum number of balls that can fit on the shelf?

60

120

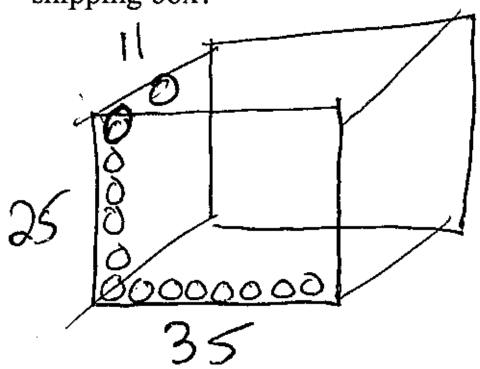
$18 \times 9 = \boxed{162}$

$\frac{60}{6.47}$ $\frac{120}{6.47}$

9.27 18.5

9 18

4. Funko Pops come in cubic packages with edge length of 4 inches. They are to be packed into a shipping box that is a rectangular prism that measures 35 inches by 25 inches by 11 inches. What are the maximum number of Funko Pops that can fit into the shipping box?



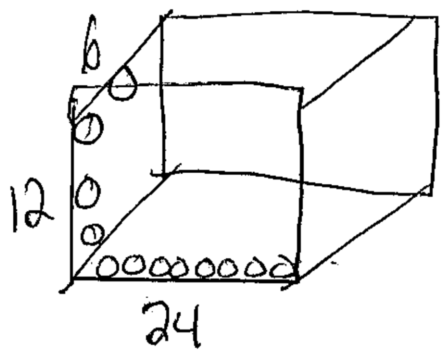
$8 \times 6 \times 2 = 96$

$\frac{35}{4}$ $\frac{25}{4}$ $\frac{11}{4}$

8.75 6.25 2.75

8 6 2

5. Baseballs that have a diameter of 2.8 inches are to be packed into a rectangular shipping box that has dimensions 24 inches by 12 inches by 6 inches. What is the maximum number of baseballs that can fit into the shipping box?



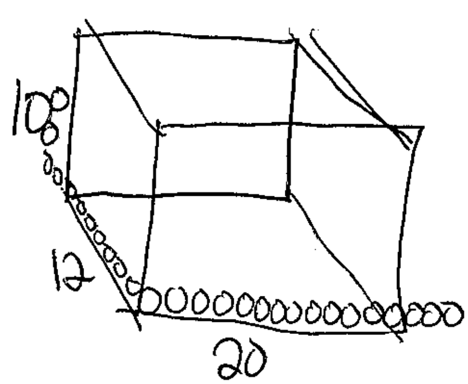
$\frac{24}{2.8}$ $\frac{12}{2.8}$ $\frac{6}{2.8}$

8.57- 4.28-- 2.14-

8 4 2

$8 \times 4 \times 2 = 64$

6. Ice cream cones are to be packed into a shipping box that has a base that measures 20 inches by 12 inches and has a height of 10 inches. The cones have a diameter of 1.2 inches and a height of 3.2 inches. How many cones can be packed into the box?



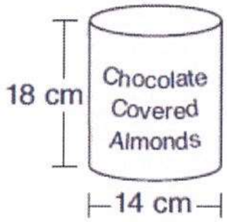
$\frac{20}{1.2}$ $\frac{12}{1.2}$ $\frac{10}{3.2}$

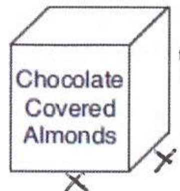
16.6-- 10 3.125

16 10 3

$16 \times 10 \times 3 = 480$

7. A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds. Same Volume
If the new container's height is 16 cm, determine and state, to the *nearest tenth of a centimeter*, the side length of the new container if both containers contain the same amount of almonds. A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.





$V = \pi r^2 h$

$V = \pi(7)^2(18)$

$V = 2770..\ cm^3$

$V = lwh$

$2770.. = x(x)(16)$

$\frac{2770..}{16} = \frac{16x^2}{16}$

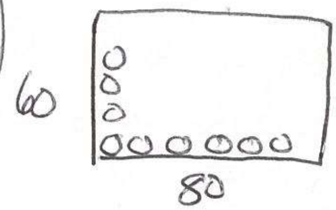
$\sqrt{173..} = \sqrt{x^2}$

$13.2 = x$

$\frac{80}{13.2}$ $\frac{60}{13.2}$

6.06.. 4.54..

6 4



$6 \times 4 = 24$

8. A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches.
Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs. The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the *nearest pound*, the total weight of all the baseballs in the fully packed box.

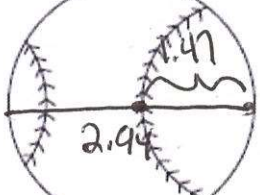
24in

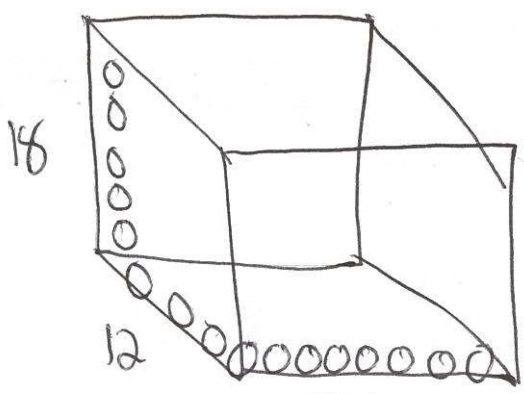
$2ft \cdot \frac{12in}{1ft}$

$1ft \cdot \frac{12in}{1ft}$

12 in

$8 \times 4 \times 6 = 192$ baseballs





$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(1.47)^3$

$V = 13.3..\ in^3$

$13.3..\ in^3 \cdot \frac{.025\ lb}{in^3} \times 192$

$\frac{24}{2.94}$ $\frac{12}{2.94}$ $\frac{18}{2.94}$

8.16.. 4.08.. 6.12..

8 4 6

64 pounds

Finding Center and Radius of a Circle Using Completing the Square

$(x-a)^2 + (y-b)^2 = r^2$ where (a,b) is the center and r is the radius
To put into center-radius form: COMPLETE THE SQUARE TWICE
To find center: Negate what is in the parenthesis. If there are no parentheses, the coordinate is 0.
Radius is the square root of the right hand side

Completing the Square

1) Write the x's together, y's together, and move constant to the other side

$$x^2 + bx + y^2 + by = c$$

2) Add $\left(\frac{b}{2}\right)^2$ to both sides for each variable
3) Factor each trinomial (Both factors must be the same)
4) Rewrite the factors as a binomial squared

1. $x^2 + y^2 + 16x + 6y + 9 = 0$ $\quad\left(\frac{16}{2}\right)^2 = 64$

$-9 \; -9$ $\quad\left(\frac{6}{2}\right)^2 = 9$

$x^2 + 16x + y^2 + 6y = -9$

$x^2 + 16x + \boxed{64} + y^2 + 6y + \boxed{9} = -9 + \boxed{64} + \boxed{9}$

$(x+8)^2 + (y+3)^2 = 64$

center: $(-8, -3)$ radius: 8

2. $x^2 + y^2 - 12x - 14y = 15$ $\quad\left(\frac{-12}{2}\right)^2 = 36 \quad \left(\frac{-14}{2}\right)^2 = 49$

$x^2 - 12x + y^2 - 14y = 15$

$x^2 - 12x + \boxed{36} + y^2 - 14y + \boxed{49} = 15 + \boxed{36} + \boxed{49}$

$(x-6)^2 + (y-7)^2 = 100$

center: $(6, 7)$ radius: 10

$\left(\frac{4}{2}\right)^2 = 4 \quad \left(\frac{-2}{2}\right)^2 = 1$

3. $x^2 + 4x + 12 + y^2 - 2y - 1 = 22$

$-12 \quad +1 \quad -12 + 1$

$x^2 + 4x + y^2 - 2y = 11$

$x^2 + 4x + \boxed{4} + y^2 - 2y + \boxed{1} = 11 + \boxed{4} + \boxed{1}$

$(x+2)^2 + (y-1)^2 = 16$

center $(-2, 1)$ radius: 4

*USE Conics app if multiple choice

4. What are the coordinates of the center of a circle whose equation is $\left(\frac{-16}{2}\right)^2$ $\left(\frac{6}{2}\right)^2$

$x^2+y^2-16x+6y+53=0$?

-53 -53

$x^2-16x+y^2+6y=-53$

64 9

1) $(-8,-3)$
2) $(-8,3)$
3) $(8,-3)$
4) $(8,3)$

$x^2-16x+\boxed{64}+y^2+6y+\boxed{9}=-53+\boxed{64}+\boxed{9}$

$(x-8)^2+(y+3)^2=20$

Center $(8,-3)$ $r=\sqrt{20}$

5. The equation of a circle is $x^2+y^2+6y=7$. What are the coordinates of the center and the length of the radius of the circle?

1) center $(0,3)$ and radius 4
2) center $(0,-3)$ and radius 4
3) center $(0,3)$ and radius 16
4) center $(0,-3)$ and radius 16

$x^2+y^2+6y+\boxed{9}=7+\boxed{9}$ $\left(\frac{6}{2}\right)^2=9$

$x^2+(y+3)^2=16$

$(0,-3)$ $r=4$

6. What are the coordinates of the center and length of the radius of the circle whose equation is $x^2+6x+y^2-4y=23$?

1) $(3,-2)$ and 36
2) $(3,-2)$ and 6
3) $(-3,2)$ and 36
4) $(-3,2)$ and 6

$\left(\frac{6}{2}\right)^2=9$ $\left(\frac{-4}{2}\right)^2=4$

$x^2+6x+\boxed{9}+y^2-4y+\boxed{4}=23+\boxed{9}+\boxed{4}$

$(x+3)^2+(y-2)^2=36$

$(-3,2)$ $r=6$

7. The equation of a circle is $x^2+y^2+12x=-27$. What are the coordinates of the center and the length of the radius of the circle?

1) center $(6,0)$ and radius 3
2) center $(6,0)$ and radius 9
3) center $(-6,0)$ and radius 3
4) center $(-6,0)$ and radius 9

$\left(\frac{12}{2}\right)^2=36$

$\cancel{x^2+y^2+12}$ $x^2+12x+\boxed{36}+y^2=-27+\boxed{36}$

$(x+6)^2+y^2=9$

$(-6,0)$ $r=6$

8. An equation of circle M is $x^2+y^2+6x-2y+1=0$. What are the coordinates of the center and the length of the radius of circle M?

-1 -1

1) center $(3,-1)$ and radius 9
2) center $(3,-1)$ and radius 3
3) center $(-3,1)$ and radius 9
4) center $(-3,1)$ and radius 3

$\left(\frac{6}{2}\right)^2$ $\left(\frac{-2}{2}\right)^2$

9 1

$x^2+6x+y^2-2y=-1$

$x^2+6x+\boxed{9}+y^2-2y+\boxed{1}=-1+\boxed{9}+\boxed{1}$

$(x+3)^2+(y-1)^2=9$

$(-3,1)$ $r=3$

$\left(\frac{2}{2}\right)^2$ $\left(\frac{-16}{2}\right)^2$

1 64

9. What are the coordinates of the center and length of the radius of the circle whose equation is $x^2 + y^2 + 2x - 16y + 49 = 0$?

-49 -49

1) center $(1,-8)$ and radius 4
2) center $(-1,8)$ and radius 4
3) center $(1,-8)$ and radius 16
4) center $(-1,8)$ and radius 16

$x^2+2x+y^2-16y=-49$

$x^2+2x+\boxed{1}+y^2-16y+\boxed{64}=-49+\boxed{1}+\boxed{64}$

$(x+1)^2+(y-8)^2=16$

$(-1,8)$ $r=4$

10. What are the coordinates of the center and the length of the radius of the circle whose equation is $x^2 + y^2 - 12y - 20.25 = 0$?

$+20.25$ $+20.25$

1) center $(0,6)$ and radius 7.5
2) center $(0,-6)$ and radius 7.5
3) center $(0,12)$ and radius 4.5
4) center $(0,-12)$ and radius 4.5

$x^2+y^2-12y=20.25$

$\left(\frac{-12}{2}\right)^2=36$

$x^2+y^2-12y+\boxed{36}=20.25+\boxed{36}$

$x^2+(y-6)^2=56.25$

center: $(0,6)$

$r=\sqrt{56.25}=7.5$

11. What is an equation of a circle whose center is $(1,4)$ and diameter is 10? $r=5$

$(1,4)$ $r=5$

1) $x^2 - 2x + y^2 - 8y = 8$
2) $x^2 + 2x + y^2 + 8y = 8$
3) $x^2 - 2x + y^2 - 8y = 83$
4) $x^2 + 2x + y^2 + 8y = 83$

Use conics app

or

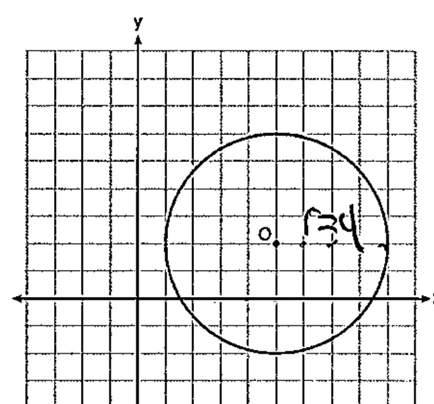
go through all 4 choices

$a=1$
$b=-2$
$c=-8$
$d=-8$

12. What is an equation of circle O shown in the graph below?

1) $x^2 + 10x + y^2 + 4y = -13$
2) $x^2 - 10x + y^2 - 4y = -13$ Center: $(5,2)$ radius: 4
3) $x^2 + 10x + y^2 + 4y = -25$
4) $x^2 - 10x + y^2 - 4y = -25$

$a=1$
$b=-10$
$c=-4$
$d=13$



Center: $(5,2)$

Line Dilations
THE IMAGE IS ALWAYS PARALLEL! SLOPE IS ALWAYS THE SAME!
Conceptual:
Determine if the point is on the line by substituting the x and y coordinates into the equation of the line.
If the point is on the line: Same y intercept (Exact same equation).
If the point is on the line: Different y intercept.

Writing the equation:
If center is origin: Multiply scale factor and original b to find new b
If center is on the line: The image is the same equation as the original.

If the center or scale factor is not given, all we know is that they are parallel (same slope).

1. The line $y=-5x-1$ is dilated by a scale factor of 2 and centered at the origin. Write an equation that represents the image of the line after the dilation.

multiply scale factor and b

$m=-5$
$b=2(-1)=-2$
$y=-5x-2$

2. The line $y=-2x+4$ is dilated by a scale factor of $\frac{5}{2}$ and centered at the origin. Write an equation that represents the image of the line after the dilation.

multiply scale factor and b

$m=-2$
$b=\frac{5}{2}(4)=10$ $\quad y=-2x+10$

3. The line $y = 2x - 4$ is dilated by a scale factor of $\frac{3}{2}$ and centered at the origin. Which equation represents the image of the line after the dilation?

multiply scale factor and b

1) $y = 2x - 4$
2) $y = 2x - 6$
3) $y = 3x - 4$
4) $y = 3x - 6$

$m=2$
$b=\frac{3}{2}(-4)$
$b=-6$
~~y~~ $y=2x-6$

4. What is an equation of the image of the line $y=\frac{3}{2}x-4$ after a dilation of a scale factor of $\frac{3}{4}$ centered at the origin?

multiply scale factor and b

1) $y=\frac{9}{8}x-4$
2) $y=\frac{9}{8}x-3$
3) $y=\frac{3}{2}x-4$
4) $y=\frac{3}{2}x-3$

$m=\frac{3}{2}$
$b=\frac{3}{4}(-4)=-3$
$y=\frac{3}{2}x-3$

5. Line $y = 3x - 1$ is transformed by a dilation with a scale factor of 2 and centered at (3, 8). The line's image is $\rightarrow$ Same equation

$8=3(3)-1$

$8=8$ ✓

1) $y = 3x - 8$
2) $y = 3x - 4$
3) $y = 3x - 2$
4) $y = 3x - 1$

6. Line MN is dilated by a scale factor of 2 centered at the point (0, 6). If $\overleftrightarrow{MN}$ is represented by $y = -3x + 6$, which equation can represent $\overleftrightarrow{M'N'}$, the image of $\overleftrightarrow{MN}$? $\rightarrow$ Same equation

1) $y = -3x + 12$
2) $y = -3x + 6$
3) $y = -6x + 12$
4) $y = -6x + 6$

$6=-3(0)+6$

$6=6$ ✓

7. The line $y = 4x - 2$ is dilated by a scale factor of 3 and centered at the point (-1,-6). Which equation represents the image of the line after the dilation? $\rightarrow$ Same equation

1) $y = 4x - 2$
2) $y = 4x - 6$
3) $y = 12x - 2$
4) $y = 12x - 6$

$-6=4(-1)-2$

$-6=-6$ ✓

8. The line $y = \frac{1}{2}x + 5$ is dilated by a scale factor of 4 and centered at the point (4,7). Which equation represents the image of the line after the dilation? $\rightarrow$ Same equation

1) $y = \frac{1}{2}x + 20$
2) $y = \frac{1}{2}x + 5$
3) $y = 2x + 20$
4) $y = 2x + 5$

$7=\frac{1}{2}(4)+5$

$7=7$ ✓

9. The equation of line h is $2x + y = 1$. Line m is the image of line h after a dilation of scale factor 4 with respect to the origin. What is the equation of the line m?

$-2x \quad -2x$

1) $y = -2x + 1$
2) $y = -2x + 4$
3) $y = 2x + 4$
4) $y = 2x + 1$

$y=-2x+1$ $\rightarrow$ multiply scale factor and b

$m=-2$

$b=4(1)=4$

$y=-2x+4$

10. The line $2x + 3y = 8$ is dilated by a scale factor of 3 and centered at the point (1,2). Which equation represents the image of the line after the dilation? $\rightarrow$ Same equation

1) $y = -\frac{2}{3}x + \frac{8}{3}$
2) $y = -\frac{2}{3}x + 8$
3) $y = -2x + \frac{8}{3}$
4) $y = -2x + 8$

$2(1)+3(2)=8$

$8=8$ ✓

$2x+3y=8$

$-2x \quad -2x$

$\frac{3y}{3}=\frac{-2x+8}{3}$

$y=-\frac{2}{3}x+\frac{8}{3}$

11. The line $3y = -2x + 8$ is transformed by a dilation centered at the origin. Which linear equation could be its image?

$+2x \quad +2x$

1) $2x + 3y = 5$
2) $2x - 3y = 5$
3) $3x + 2y = 5$
4) $3x - 2y = 5$

$2x+3y=8$

12. The line represented by the equation $4y = 3x + 7$ is transformed by a dilation centered at the origin. Which linear equation could represent its image?

1) $3x - 4y = 9$
2) $3x + 4y = 9$
3) $4x - 3y = 9$
4) $4x + 3y = 9$

$4y=3x+7$
$-3x \quad -3x$
$\frac{-3x+4y}{-1}=\frac{7}{-1}$
$3x-4y=-7$

13. The line $-3x + 4y = 8$ is transformed by a dilation centered at the origin. Which linear equation could represent its image?

1) $y = \frac{4}{3}x + 8$
2) $y = \frac{3}{4}x + 8$
3) $y = -\frac{3}{4}x - 8$
4) $y = -\frac{4}{3}x - 8$

$-\cancel{3x}+4y=8$
$+\cancel{3x} \quad +3x$
$\frac{4y}{4}=\frac{3x+8}{4}$
$y=\frac{3}{4}x+2$

since we don't know scale factor, all we know is parallel (same slope)

14. Line n is represented by the equation $3x + 4y = 20$. Determine and state the equation of line p, the image of line n, after a dilation of scale factor $\frac{1}{3}$ centered at the point $(4, 2)$. Explain your answer.

$3(4)+4(2)=20$
$20=20$ ✓

$3x+4y=20$
the center of dilation is on the line so the equation remains the same.

15. Aliyah says that when the line $4x + 3y = 24$ is dilated by a scale factor of 2 centered at the point $(3,4)$, the equation of the dilated line is $y = -\frac{4}{3}x + 16$. Is Aliyah correct? Explain why.

$4(3)+3(4)=24$
$24=24$ ✓
Since the center is on the line, the equation must be the same.

$\cancel{4x}+3y=24$
$-\cancel{4x} \quad -4x$
$\frac{3y}{3}=\frac{-4x+24}{3}$
$y=-\frac{4}{3}x+8$

No, the equations are not the same.

Dilating Segments with Perimeter and Area
Multiply the original segment and scale factor to find the image.
Multiply the original perimeter and scale factor to find the image perimeter.
Multiply the original area and the $(scale\ \ factor)^2$ to find the image area.
*You may have to use distance formula to find original segment.
*The center of dilation does not effect the size of the image
*When the midpoints are joined, the scale factor is $\frac{1}{2}$

1. A line segment with a length of 5 is dilated by a scale factor of 4. What is the length of its image?

$5(4) = 20$

2. A line segment has a length of 12 and is dilated by $\frac{1}{2}$. What is the length of its image?

$12(\frac{1}{2}) = 6$

3. A three-inch line segment is dilated by a scale factor of 6 and centered at its midpoint. What is the length of its image?
1) 9 inches
2) 2 inches
3) 15 inches
4) 18 inches

$3(6) = 18$

4. Triangle JOY has a perimeter of 10 and an area of 12. What is the perimeter and area of triangle JOY after a dilation by a scale factor of 2?

Perimeter image ~~area~~ = scale factor(perimeter original)
perimeter = 2(10) = 20

area image = area original(scale factor)^2
area = $12(2)^2 = 48$

5. Quadrilateral CAMI has a perimeter of 20 and an area of 15. What is the perimeter and area of quadrilateral CAMI after a dilation by a scale factor of 4?

perimeter image = perimeter original(scale factor)
perimeter = 20(4) = 80

area image = area original (scale factor)^2
area = $15(4)^2 = 240$

Perimeter RSTV = 4(9) = 36

6. Given square *RSTV*, where *RS* = 9 cm. If square *RSTV* is dilated by a scale factor of 3 about a given center, what is the perimeter, in centimeters, of the image of *RSTV* after the dilation?

Perimeter image = perimeter original (scale factor)

Perimeter = 36(3) = 108

1) 12
2) 27
3) 36
4) 108

7. Triangle *RJM* has an area of 6 and a perimeter of 12. If the triangle is dilated by a scale factor of 3 centered at the origin, what are the area and perimeter of its image, triangle *R'J'M'*?

P image = P original (scale factor)

P = 12(3) = 36

A image = A original (scale factor)^2

A = 6(3)^2 = 54

1) area of 9 and perimeter of 15
2) area of 18 and perimeter of 36
3) area of 54 and perimeter of 36
4) area of 54 and perimeter of 108

8. Rectangle *A'B'C'D'* is the image of rectangle *ABCD* after a dilation centered at point *A* by a scale factor of $\frac{2}{3}$. Which statement is correct?

P image = $\frac{2}{3}$ P original

1) Rectangle *A'B'C'D'* has a perimeter that is $\frac{2}{3}$ the perimeter of rectangle *ABCD*.
2) Rectangle *A'B'C'D'* has a perimeter that is $\frac{3}{2}$ the perimeter of rectangle *ABCD*.
3) Rectangle *A'B'C'D'* has an area that is $\frac{2}{3}$ the area of rectangle *ABCD*.
4) Rectangle *A'B'C'D'* has an area that is $\frac{3}{2}$ the area of rectangle *ABCD*.

1:2 ratio

9. In the diagram below of $\triangle ABC$, *D*, *E*, and *F* are the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively.

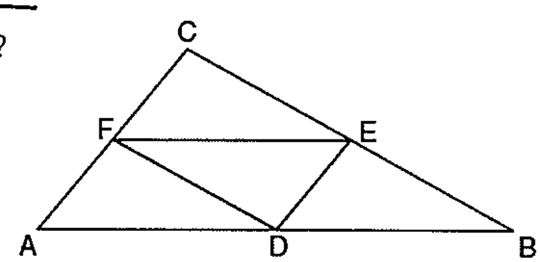
What is the ratio of the area of $\triangle CFE$ to the area of $\triangle CAB$?

1) 1:1
2) 1:2
3) 1:3
4) 1:4

(scale factor/ratio)^2

$(1:2)^2 = 1^2:2^2$

1:4



10. The area of $\triangle TAP$ is 36 cm^2. A second triangle, *JOE*, is formed by connecting the midpoints of each side of $\triangle TAP$. What is the area of *JOE*, in square centimeters?

1) 9
2) 12
3) 18
4) 27

1:2 scale factor = $\frac{1}{2}$

area image = area original (scale factor)^2

$= 36(\frac{1}{2})^2$

$= 9$

Equation of a line through a point flip and negate

1) Find m using parallel (same slope) or perpendicular (negative reciprocal slopes).
2) Substitute into $y - y_1 = m(x - x_1)$. Don't forget to negate x_1 and y_1.
3) If it's multiple choice, you may have to distribute and isolate y.

1. What is the equation of a line that passes through the point $(-3, -11)$ and is parallel to the line whose equation is $2x - y = 4$?

x_1 y_1 — same slope

1) $y = 2x + 5$
2) $y = 2x - 5$
3) $y = \frac{1}{2}x + \frac{25}{2}$
4) $y = -\frac{1}{2}x - \frac{25}{2}$

$-2x$ $-2x$

$\frac{-y}{-1} = \frac{-2x+4}{-1}$

$y = 2x - 4$

$m = 2$

$m\parallel = 2$

$x_1 = -3$

$y_1 = -11$

$y - y_1 = m(x - x_1)$

$y + 11 = 2(x + 3)$

$y + 11 = 2x + 6$

-11 -11

$y = 2x - 5$

2. What is an equation of the line that passes through the point $(-2, 5)$ and is perpendicular to the line whose equation is $y = \frac{1}{2}x + 5$?

x_1 y_1 — negative reciprocal slopes

$m = \frac{1}{2}$

1) $y - 5 = \frac{1}{2}(x + 2)$
2) $y - 5 = -2(x + 2)$
3) $y + 5 = \frac{1}{2}(x - 2)$
4) $y + 5 = -2(x - 2)$

$y - y_1 = m(x - x_1)$

$y - 5 = -2(x + 2)$

$m\perp = -2$

$x_1 = -2$

$y_1 = 5$

3. What is an equation of the line that contains the point $(3, -1)$ and is perpendicular to the line whose equation is $y = -3x + 2$?

x_1 y_1 — negative reciprocal slopes

1) $y = -3x + 8$
2) $y = -3x$
3) $y = \frac{1}{3}x$
4) $y = \frac{1}{3}x - 2$

$m = -3$

$m\perp = \frac{1}{3}$

$x_1 = 3$

$y_1 = -1$

$y - y_1 = m(x - x_1)$

$y + 1 = \frac{1}{3}(x - 3)$

$y + 1 = \frac{1}{3}x - 1$

-1 -1

$y = \frac{1}{3}x - 2$

4. An equation of the line that passes through $(2, -1)$ and is parallel to the line $2y + 3x = 8$ is

x_1 y_1 — same slope

1) $y + 1 = -\frac{3}{2}(x - 2)$
2) $y + 1 = \frac{2}{3}(x - 2)$
3) $y - 1 = -\frac{3}{2}(x + 2)$
4) $y - 1 = \frac{2}{3}(x + 2)$

$-3x$ $-3x$

$\frac{2y}{2} = \frac{-3x + 8}{2}$

$y = -\frac{3}{2}x + 4$

$m = -\frac{3}{2}$

$m\parallel = -\frac{3}{2}$

$x_1 = 2$

$y_1 = -1$

$y - y_1 = m(x - x_1)$

$y + 1 = -\frac{3}{2}(x - 2)$

$= 3/5$

negative reciprocal slopes

5. What is an equation of the line that is perpendicular to the line whose equation is $y = \frac{3}{5}x - 2$ and that passes through the point $(3, -6)$?

$x_1 \; y_1$

1) $y = \frac{5}{3}x - 11$
2) $y = -\frac{5}{3}x + 11$
3) $y = -\frac{5}{3}x - 1$
4) $y = \frac{5}{3}x + 1$

$m\perp = -\frac{5}{3}$

$x_1 = 3$

$y_1 = -6$

$y - y_1 = m(x - x_1)$

$y + 6 = -\frac{5}{3}(x - 3)$

$y + 6 = -\frac{5}{3}x + 5$

$\;\;\; -6 \qquad\qquad -6$

$y = -\frac{5}{3}x - 1$

6. The equation of a line is $y = \frac{2}{3}x + 5$. What is an equation of the line that is perpendicular to the given line and that passes through the point $(4, 2)$?

$x_1 \; y_1$

negative reciprocal slopes

1) $y = \frac{2}{3}x - \frac{2}{3}$
2) $y = \frac{3}{2}x - 4$
3) $y = -\frac{3}{2}x + 7$
4) $y = -\frac{3}{2}x + 8$

$\boxed{m = \frac{2}{3}}$

$y - y_1 = m(x - x_1)$

$m\perp = -\frac{3}{2}$

$x_1 = 4$

$y_1 = 2$

$y - 2 = -\frac{3}{2}(x - 4)$

$y - 2 = -\frac{3}{2}x + 6$

$\;\;\; +2 \qquad\qquad +2$

$y = -\frac{3}{2}x + 8$

7. What is an equation of the line that passes through the point $(6, 8)$ and is perpendicular to a line with equation $y = \frac{3}{2}x + 5$?

$x_1 \; y_1$

1) $y - 8 = \frac{3}{2}(x - 6)$
2) $y - 8 = -\frac{2}{3}(x - 6)$
3) $y + 8 = \frac{3}{2}(x + 6)$
4) $y + 8 = -\frac{2}{3}(x + 6)$

$m = \frac{3}{2}$

$y - y_1 = m(x - x_1)$

$y - 8 = -\frac{2}{3}(x - 6)$

negative reciprocal slopes

$m\perp = -\frac{2}{3}$

$x_1 = 6$

$y_1 = 8$

8. What is an equation of a line which passes through $(6, 9)$ and is perpendicular to the line whose equation is $4x - 6y = 15$?

$x_1 \; y_1$

$-4x \qquad -4x$

1) $y - 9 = -\frac{3}{2}(x - 6)$
2) $y - 9 = \frac{2}{3}(x - 6)$
3) $y + 9 = -\frac{3}{2}(x + 6)$
4) $y + 9 = \frac{2}{3}(x + 6)$

$\frac{-6y}{-6} = \frac{-4x + 15}{-6}$

$y = \frac{2}{3}x - \frac{5}{2}$

negative reciprocal slopes

$m\perp = -\frac{3}{2}$

$x_1 = 6$

$y_1 = 9$

$y - y_1 = m(x - x_1)$

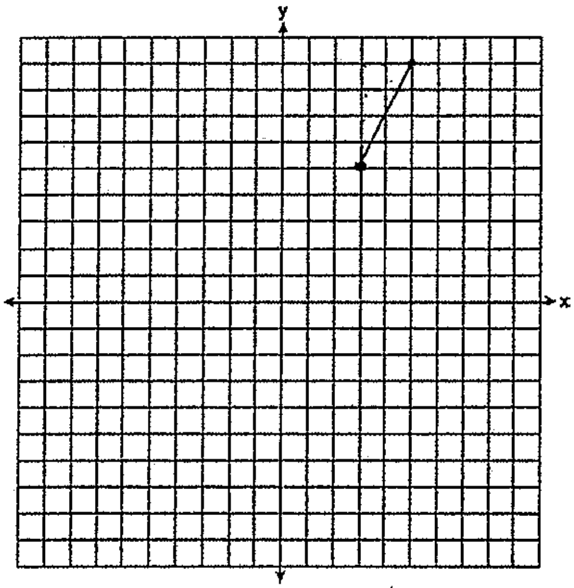
$y - 9 = -\frac{3}{2}(x - 6)$

Name Schlansky
Mr. Schlansky

Date ____________
Geometry

Perpendicular Bisector

1. Write an equation of the perpendicular bisector of the line segment whose endpoints are (3,5) and (5,9).



$m = \frac{\Delta y}{\Delta x}$

$m = \frac{4}{2}$

$m = 2$

$mp = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$mp = \left(\frac{3+5}{2}, \frac{5+9}{2}\right)$

$mp = \left(\frac{8}{2}, \frac{14}{2}\right)$

$mp = (4,7)$

$m\perp = -\frac{1}{2}$

$x_1 = 4$

$y_1 = 7$

$y - y_1 = m(x - x_1)$

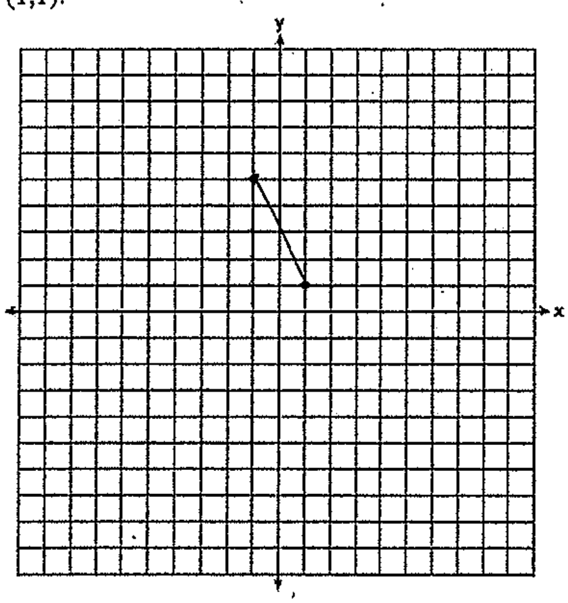
$y - 7 = -\frac{1}{2}(x - 4)$

$y - 7 = -\frac{1}{2}x + 2$

$+7 \qquad +7$

$y = -\frac{1}{2}x + 9$

2. Write an equation of the perpendicular bisector of the line segment whose endpoints are (-1,5) and (1,1).



$m = \frac{\Delta y}{\Delta x}$

$m = \frac{-4}{2}$

$m = -2$

$mp = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$mp = \left(\frac{-1+1}{2}, \frac{5+1}{2}\right)$

$mp = \left(\frac{0}{2}, \frac{6}{2}\right)$

$mp = (0,3)$

$m\perp = \frac{1}{2}$

$x_1 = 0$

$y_1 = 3$

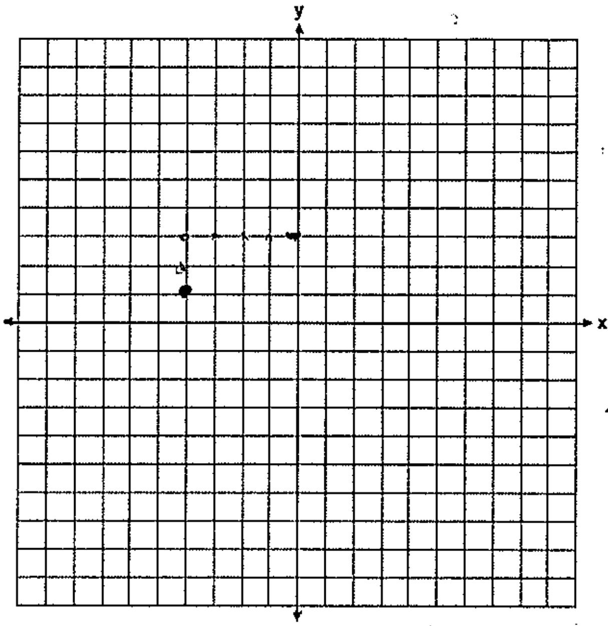
$y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{2}x$

$+3 \qquad +3$

$y = \frac{1}{2}x + 3$

3. Write an equation of the perpendicular bisector of the line segment whose endpoints are (-4,1) and (0,3) in both point slope and slope intercept form.



$m = \frac{\Delta y}{\Delta x}$ $mp = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$m = \cancel{\;}$ $= \left(\frac{-4+0}{2}, \frac{1+3}{2}\right)$

$m = \frac{2}{4}$ $(-2,2)$

$m = \frac{1}{2}$

$m\perp = 2$ $y - y_1 = m(x - x_1)$

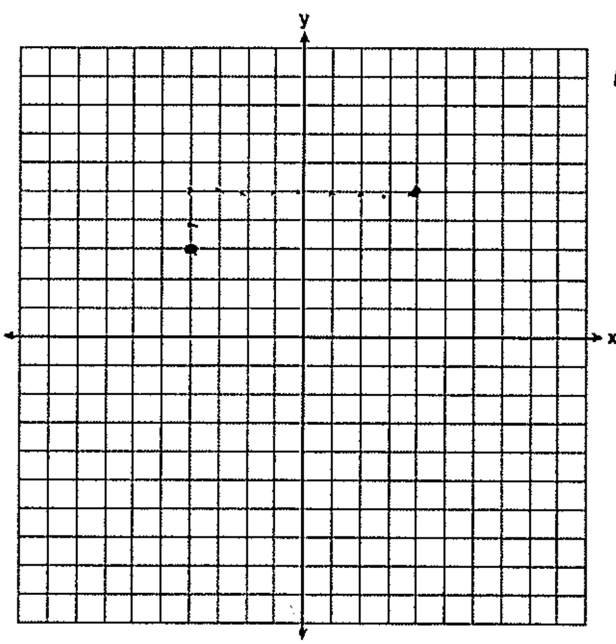
$x_1 = -2$ $y - 2 = 2(x+2)$

$y_1 = 2$ $y - 2 = 2x + 4$

$+2 \quad +2$

$y = 2x+6$

4. Write an equation of the perpendicular bisector of the line segment whose endpoints are (-4,3) and (4,5) in both point slope and slope intercept form.



$m = \frac{\Delta y}{\Delta x}$ $mp = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-4+4}{2}, \frac{3+5}{2}\right)$

$m = \frac{2}{8}$

$m = \frac{1}{4}$ $(0,4)$

$m\perp = -4$ $y - y_1 = m(x - x_1)$

$x_1 = 0$ $\cancel{y=4}$

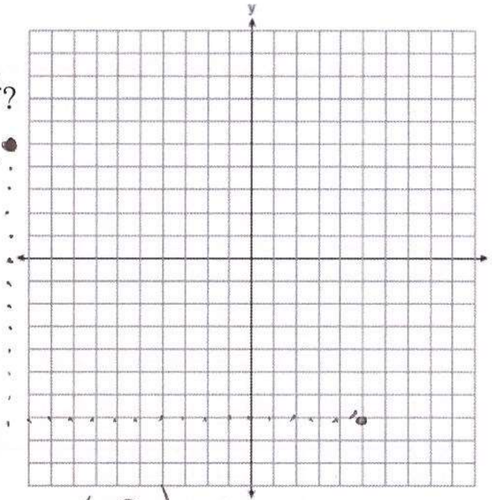
$y_1 = 4$ $y - 4 = -4(x - 0)$

$y - 4 = -4x + 0$

$+4 \quad +4$

$y = -4x+4$

5. Line segment NY has endpoints $N(-11,5)$ and $Y(5,-7)$. What is the equation of the perpendicular bisector of $\overline{NY}$?



1) $y+1=\frac{4}{3}(x+3)$

2) $y+1=-\frac{3}{4}(x+3)$

3) $y-6=\frac{4}{3}(x-8)$

4) $y-6=-\frac{3}{4}(x-8)$

$m=\frac{\Delta y}{\Delta x}$ $\quad mp=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$m=\frac{-12}{16}$ $\quad =\frac{-11+5}{2},\frac{5+-7}{2}$

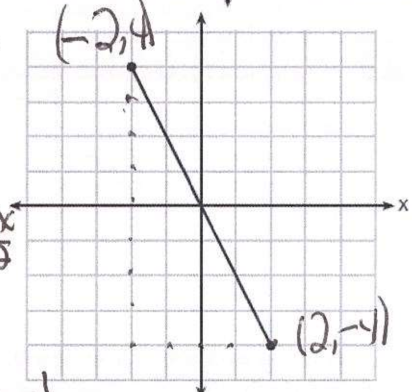
$m=-\frac{3}{4}$ $\quad (-3,-1)$

$m\perp=\frac{4}{3}$ $\quad y-y_1=m(x-x_1)$

$x_1=-3$ $\quad y+1=\frac{4}{3}(x+3)$

$y_1=-1$

6. What is an equation of the perpendicular bisector of the line segment shown in the diagram below?



1) $y+2x=0$

2) $y-2x=0$

3) $2y+x=0$

4) $2y-x=0$

$+x \quad +x$

$m=\frac{\Delta y}{\Delta x}$ $\quad mp=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ $\quad m\perp=\frac{1}{2}$ $\quad \frac{2y}{2}=\frac{x}{2}$

$=\left(\frac{-2+2}{2},\frac{4+-4}{2}\right)$ $\quad x_1=0$

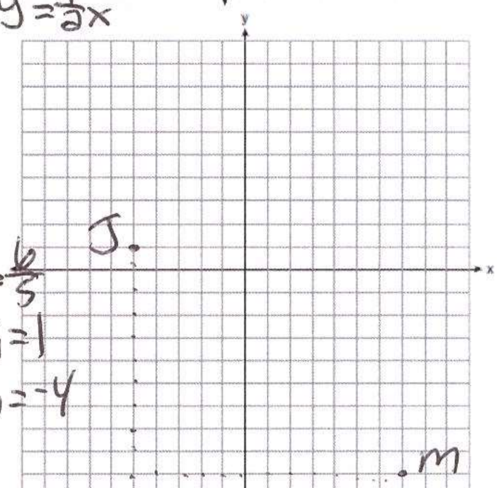
$m=\frac{-8}{4}$ $\quad y_1=0$

$(0,0)$ $\quad y-y_1=m(x-x_1)$

$m=-2$

$y-0=\frac{1}{2}(x-0)$ $\quad y=\frac{1}{2}x$

7. Segment JM has endpoints $J(-5,1)$ and $M(7,-9)$. An equation of the perpendicular bisector of $\overline{JM}$ is



1) $y-4=\frac{5}{6}(x+1)$

2) $y+4=\frac{5}{6}(x-1)$

3) $y-4=\frac{6}{5}(x+1)$

4) $y+4=\frac{6}{5}(x-1)$

$m=\frac{\Delta y}{\Delta x}$ $\quad mp=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ $\quad y-y_1=m(x-x_1)$ $\quad m\perp=\frac{6}{5}$

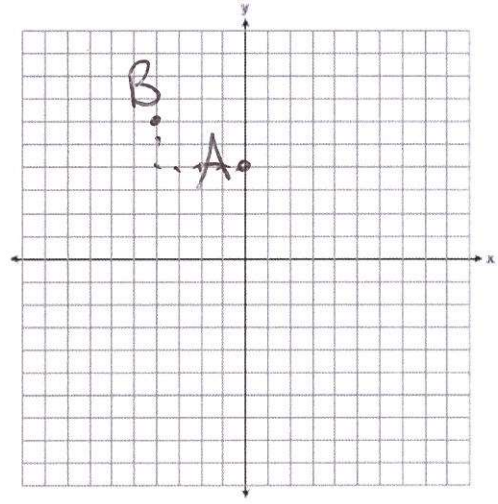
$=\left(\frac{-5+7}{2},\frac{1+-9}{2}\right)$ $\quad y+4=\frac{6}{5}(x-1)$ $\quad x_1=1$

$m=\frac{-10}{12}$ $\quad y_1=-4$

$(1,-4)$

$m=\frac{-5}{6}$

8. The endpoints of $\overline{AB}$ are $A(0,4)$ and $B(-4,6)$. Which equation of a line represents the perpendicular bisector of $\overline{AB}$?



1) $y=-\frac{1}{2}x+4$

2) $y=-2x+1$

3) $y=2x+8$

4) $y=2x+9$

$m=\frac{-2}{4}=-\frac{1}{2}$

$mp=\left(\frac{0+-4}{2},\frac{4+6}{2}\right)$

$(-2,5)$

$m\perp=2$ $\quad y-y_1=m(x-x_1)$

$x_1=-2$ $\quad y-5=2(x+2)$

$y_1=5$ $\quad y-5=2x+4$

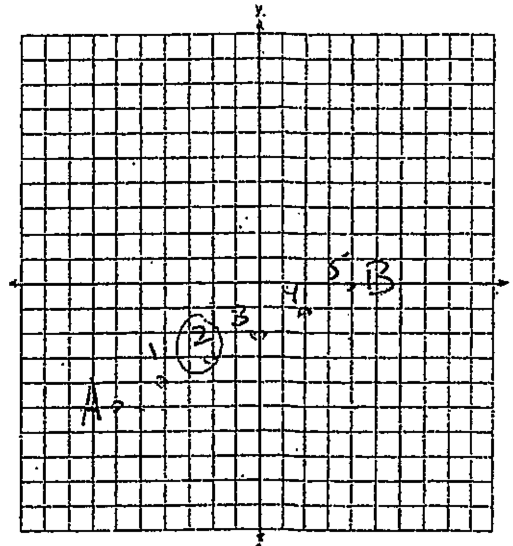
$+5 \quad +5$

$y=2x+9$

Partitions

1) **Find** $\frac{\Delta x}{p}$ *and* $\frac{\Delta y}{p}$ where p is the number of partitions.
2) **Count those values out on the graph between the two endpoints**
3) **Circle and state the point that matches the given ratio.**
BE CAREFUL WHICH POINT YOU START FROM!

1. The coordinates of the endpoints of $\overline{AB}$ are $A(-6,-5)$ and $B(4,0)$. Point P is on $\overline{AB}$. Determine and state the coordinates of point P, such that $AP{:}PB$ is $2{:}3$. p=5



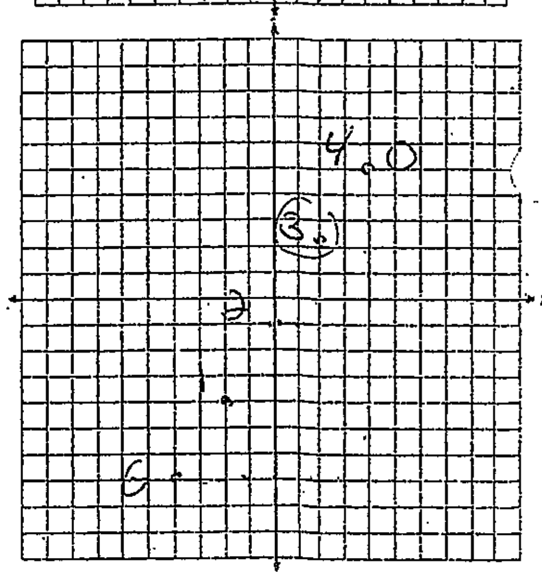
$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$

$\frac{10}{5}$ $\frac{5}{5}$

2 1

(-2,-3)

2. What are the coordinates of the point on the directed line segment from $G(-4,-7)$ to $O(4,5)$ that partitions the segment into a ratio of 3 to 1? p=4



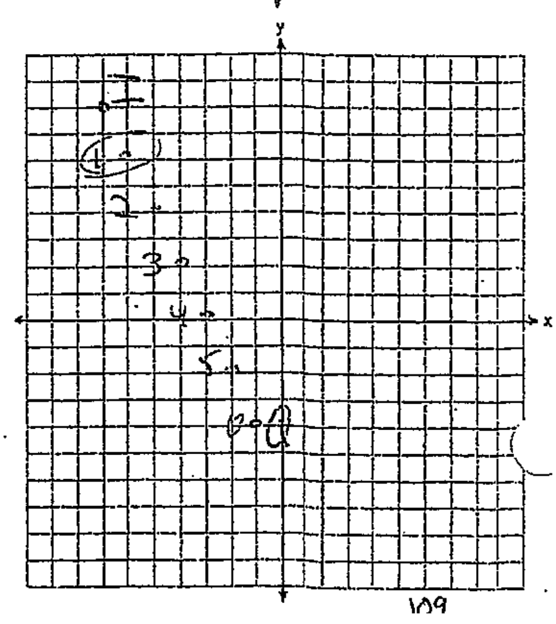
$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$

$\frac{8}{4}$ $\frac{12}{4}$

2 3

(2,2)

3. Directed line segment IQ has endpoints whose coordinates are $I(-7,8)$ and $Q(-1,-4)$. Determine the coordinates of point J that divides the segment in the ratio 1 to 5. p=6



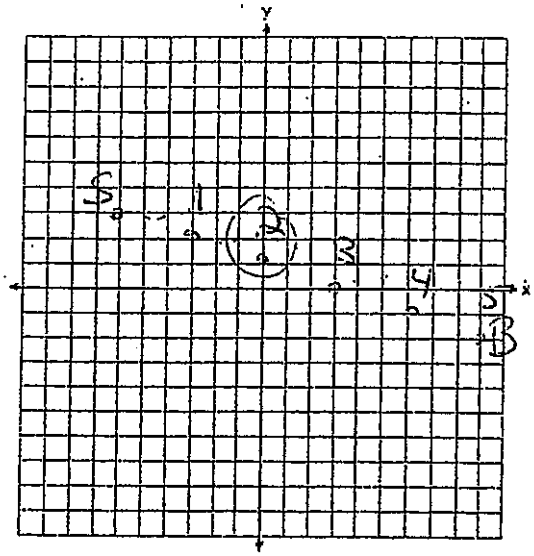
$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$

$\frac{6}{6}$ $\frac{12}{6}$

1 2

(-6,6)

4. Directed line segment SB has endpoints whose coordinates are $S(-6,3)$ and $B(9,-2)$. Determine the coordinates of point J that divides the segment in the ratio 2 to 3. $p=5$

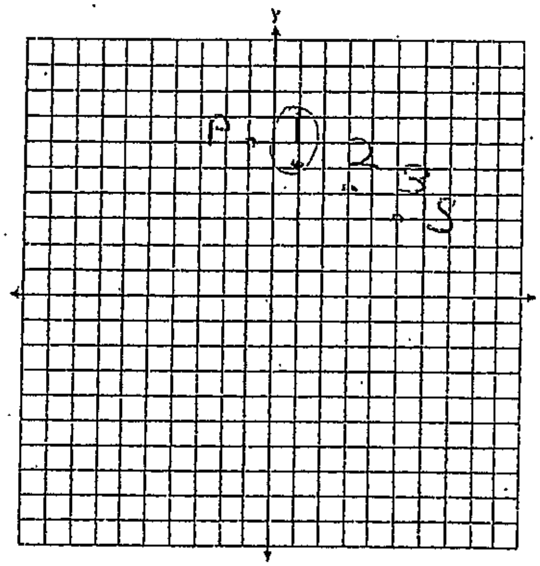


$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$ $(0,1)$

$\frac{15}{5}$ $\frac{5}{5}$

3 1

5. What are the coordinates of the point on the directed line segment from $P(-1,6)$ to $S(5,3)$ that partitions the segment into a ratio of 1 to 2? $p=3$

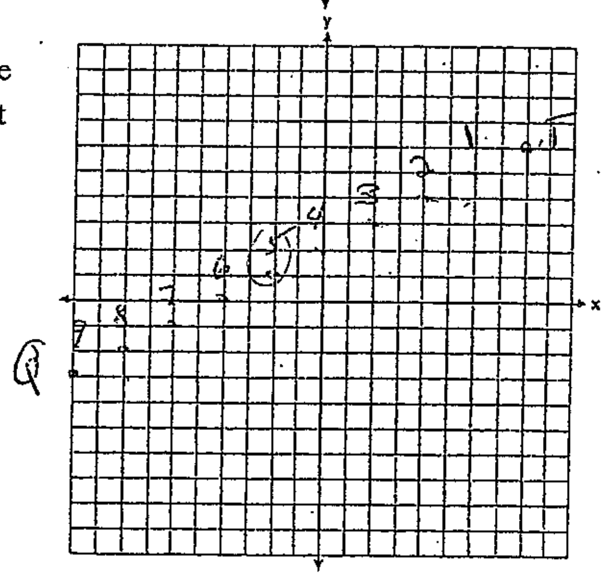


$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$ $(1,5)$

$\frac{6}{3}$ $\frac{3}{3}$

2 1

6. Directed line segment JK has endpoints whose coordinates are $J(8,6)$ and $Q(-10,-3)$. Determine the coordinates of point O that divides the segment in the ratio 5 to 4. $p=9$



$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$ $(-2,1)$

$\frac{18}{9}$ $\frac{9}{9}$

2 1

Area with Coordinate Geometry
Box Method

1) Build a rectangle around the shape
2) Find the area of the rectangle ($A=lw$)
3) Find the area of the triangles outside of the shape ($A=.5lw$)
4) Subtract the triangle areas from the rectangle area

Find the area of the following shapes

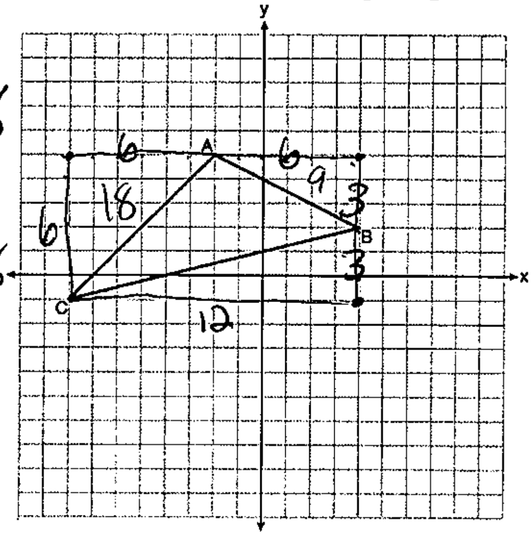
1.

$A_R = 6(12) = 72$

$A_{T1} = \frac{1}{2}(6)(6) = 18$

$A_{T2} = \frac{1}{2}(6)(3) = 9$

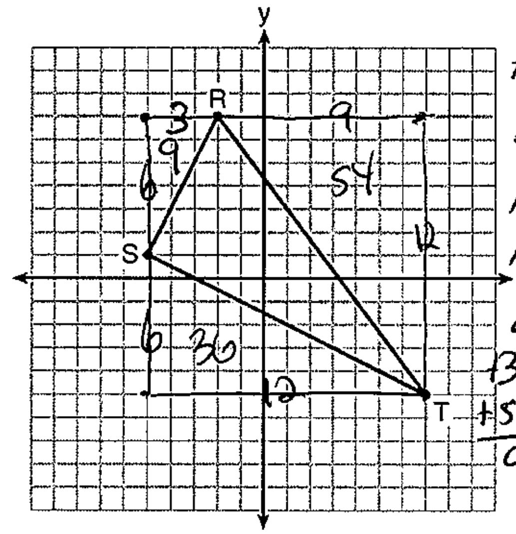
$A_{T3} = \frac{1}{2}(12)(3) = 18$



$18 + 9 + 18 = 45$

$72 - 45 = \boxed{27}$

2.



$A_R = 12(12) = 144$

$A_{T1} = \frac{1}{2}(3)(6) = 9$

$A_{T2} = \frac{1}{2}(6)(12) = 36$

$A_{T3} = \frac{1}{2}(9)(12) = 54$

$9 + 36 + 54 = 99$

$144 - 99 = \boxed{45}$

3.

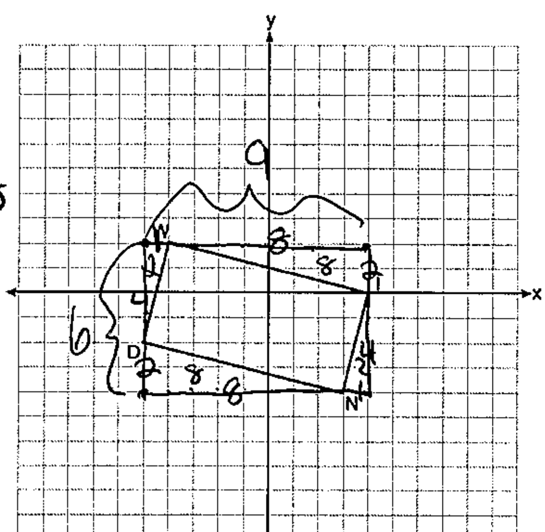
$A_R = 9(6) = 54$

$A_{T1} = \frac{1}{2}(1)(4) = 2$

$A_{T2} = \frac{1}{2}(8)(2) = 8$

$A_{T3} = \frac{1}{2}(4)(1) = 2$

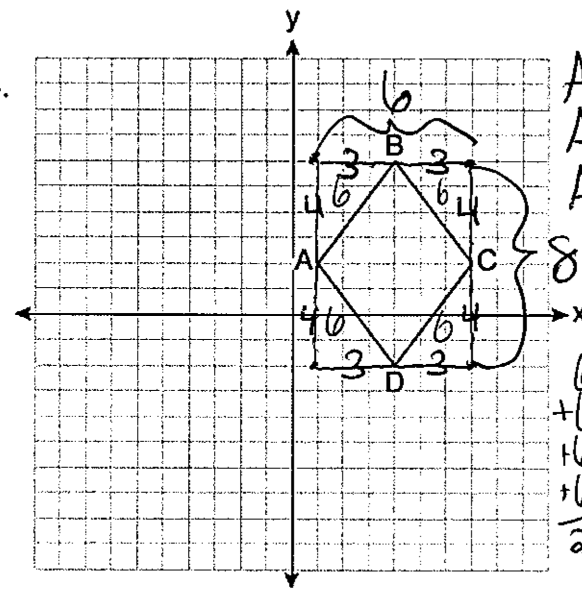
$A_{T4} = \frac{1}{2}(2)(8) = 8$



$2 + 8 + 2 + 8 = 20$

$54 - 20 = \boxed{34}$

4.



$A_R = 6(8) = 48$

$A_{T1} = \frac{1}{2}(3)(4) = ($

$A_{T2} = \frac{1}{2}(3)(4) =$

$A_{T3} = \frac{1}{2}(3)(4) =$

$A_{T4} = \frac{1}{2}(3)(4) =$

$6 + 6 + 6 + 6 = 24$

$48 - 24 = \boxed{24}$

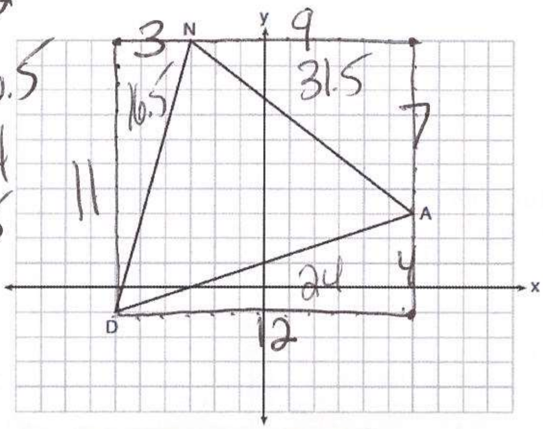
5.

$A_R = 11(12) = 132$

$A_{T1} = \frac{1}{2}(3)(11) = 16.5$

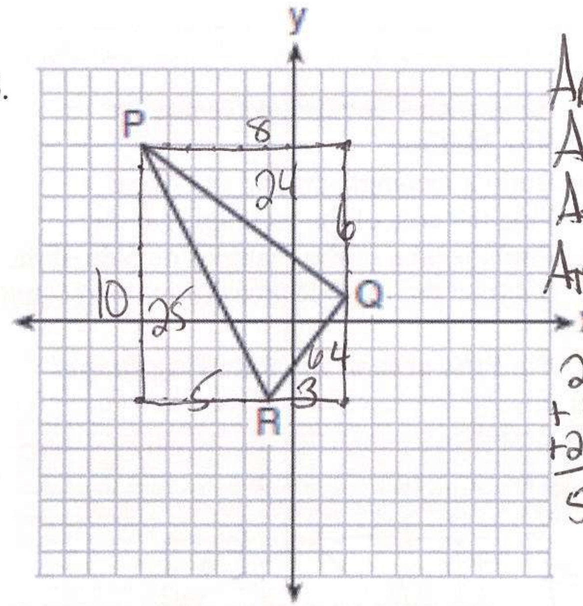
$A_{T2} = \frac{1}{2}(12)(4) = 24$

$A_{T3} = \frac{1}{2}(9)(7) = 31.5$



$$\begin{array}{r} 16.5 \\ +31.5 \\ +24 \\ \hline 72 \end{array} \qquad \begin{array}{r} 132 \\ -72 \\ \hline \boxed{60} \end{array}$$

6.



$A_R = 8(10) = 80$

$A_{T1} = \frac{1}{2}(8)(6) = 24$

$A_{T2} = \frac{1}{2}(4)(3) = 6$

$A_{T3} = \frac{1}{2}(10)(5) = 25$

$$\begin{array}{r} 24 \\ +6 \\ +25 \\ \hline 55 \end{array} \qquad \begin{array}{r} 80 \\ -55 \\ \hline \boxed{25} \end{array}$$

7.

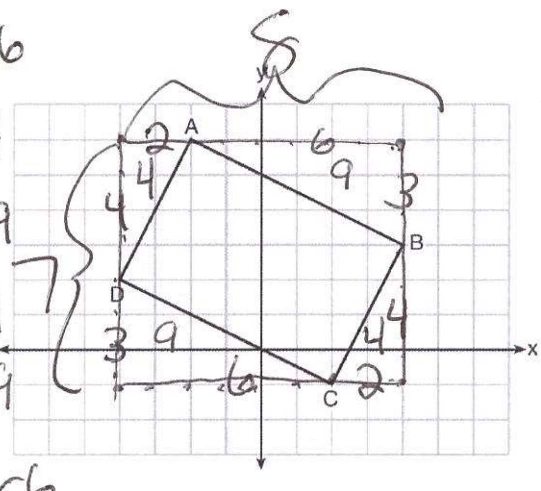
$A_R = 8(7) = 56$

$A_{T1} = \frac{1}{2}(2)(4) = 4$

$A_{T2} = \frac{1}{2}(6)(3) = 9$

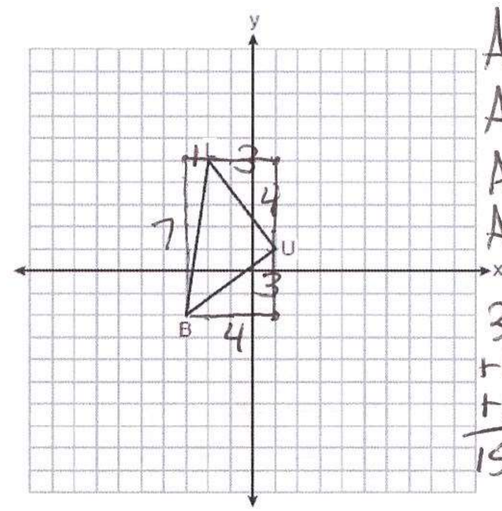
$A_{T3} = \frac{1}{2}(4)(2) = 4$

$A_{T4} = \frac{1}{2}(3)(6) = 9$



$$\begin{array}{r} 4 \\ +9 \\ +4 \\ +9 \\ \hline 26 \end{array} \qquad \begin{array}{r} 56 \\ -26 \\ \hline \boxed{30} \end{array}$$

8.



$A_R = 7(4) = 28$

$A_{T1} = \frac{1}{2}(1)(7) = 3.5$

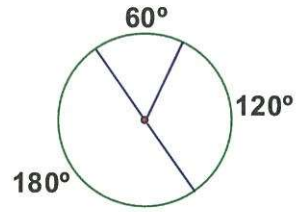
$A_{T2} = \frac{1}{2}(3)(4) = 6$

$A_{T3} = \frac{1}{2}(3)(4) = 6$

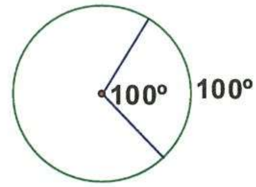
$$\begin{array}{r} 3.5 \\ +6 \\ +6 \\ \hline 15.5 \end{array} \qquad \begin{array}{r} 28 \\ -15.5 \\ \hline \boxed{12.5} \end{array}$$

Circle Angle and Segment Rules:

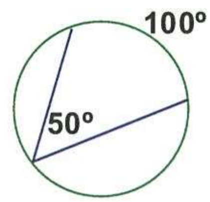
The arcs of a circle add to 360°
A diameter cuts a circle into 2 halves of 180° each



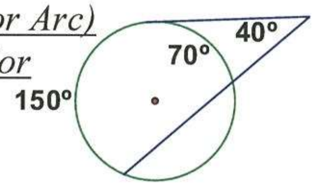
Central Angle: Has its vertex at the center of the circle
Central angle is equal to the measure of the intercepted arc

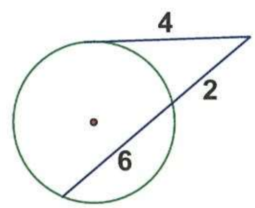


Inscribed Angle: Has its vertex on the circle
Inscribed angle is half of the measure of the intercepted arc

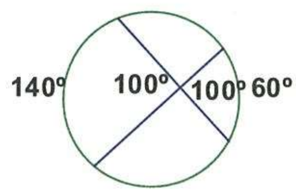


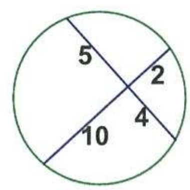
Exterior Segments/Angles:
Angles: *2(Exterior Angle) = (Major Arc – Minor Arc)*
Segments: *Whole • Exterior = Whole • Exterior*



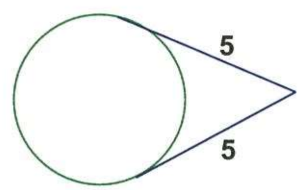


Intersecting Segments/Angles:
Angles: *2(Vertical Angle) = Arc + Arc*
Segments: *Part • Part = Part • Part*

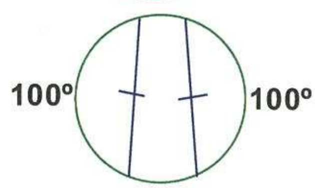




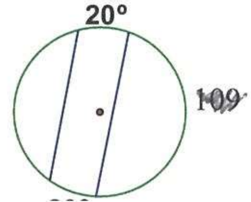
Two tangents drawn from the same point are congruent



Congruent chords intercept congruent arcs



Parallel chords intercept congruent arcs

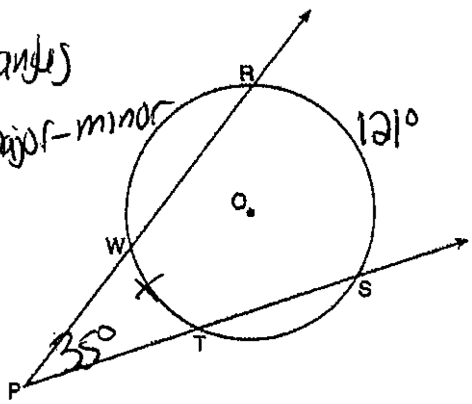


2(exterior angle) = major − minor

1. As shown in the diagram below, secants $\overrightarrow{PWR}$ and $\overrightarrow{PTS}$ are drawn to circle O from external point P.
If $m\angle RPS = 35°$ and $m\overparen{RS} = 121°$, determine and state $m\overparen{WT}$.

exterior arcs and angles

2(EA) = major − minor



$2(EA) = \text{major} - \text{minor}$

$2(35) = 121 - x$

$70 = 121 - x$

$-121 \quad -121$

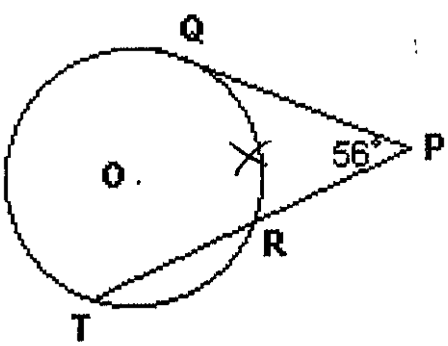
$\frac{-51}{-1} = \frac{-x}{-1}$

$51 = x$

2. In the diagram of circle O, $\overline{PQ}$ is tangent to O at Q and $\overline{PRT}$ is a secant. If $m\angle P = 56$ and $m\overparen{QT} = 192$, find $m\overparen{QR}$

exterior arcs and angles

A) = major − minor



$2(EA) = \text{major} - \text{minor}$

$2(56) = 192 - x$

$112 = 192 - x$

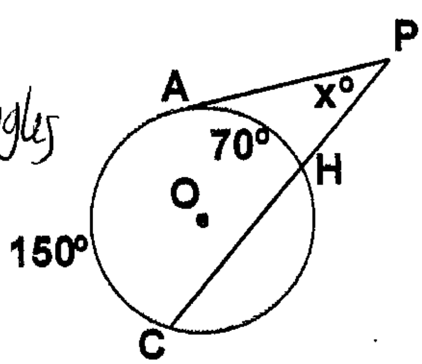
$-192 \quad -192$

$\frac{-80}{-1} = \frac{-x}{-1}$

$80 = x$

3. $\overparen{AC} = 150°$, $\overparen{AH} = 70°$, find $m\angle APH$

- exterior arcs and angles



$2(EA) = \text{major} - \text{minor}$

$2x = 150 - 70$

$\frac{2x}{2} = \frac{80}{2}$

$x = 40$

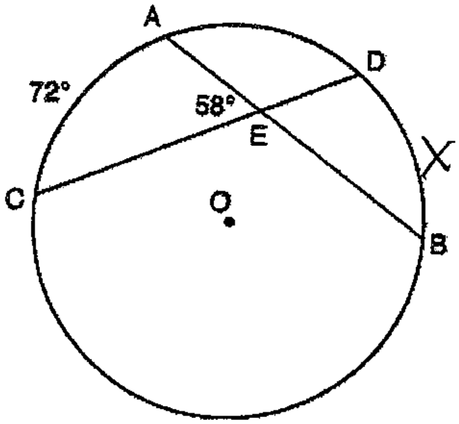
(EA) = major − minor

2(vertical angle) = arc + arc

4. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $m\widehat{AC} = 72°$ and $m\angle AEC = 58°$, how many degrees are in $m\widehat{DB}$?

- interior
- arcs and angles

2(VA) = arc + arc



2(VA) = arc + arc

$2(58) = x + 72$

$116 = x + 72$

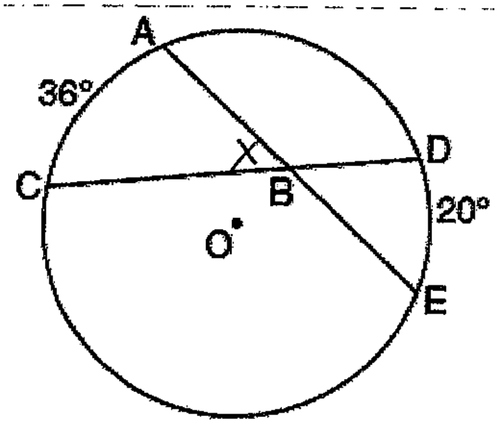
$-72 \quad -72$

$44 = x$

5. In the diagram below of circle O, chords $\overline{AE}$ and $\overline{DC}$ intersect at point B, such that $m\widehat{AC} = 36$ and $m\widehat{DE} = 20$. What is $m\angle ABC$?

- interior
- arcs and angles

2(VA) = arc + arc



2(VA) = arc + arc

$2x = 36 + 20$

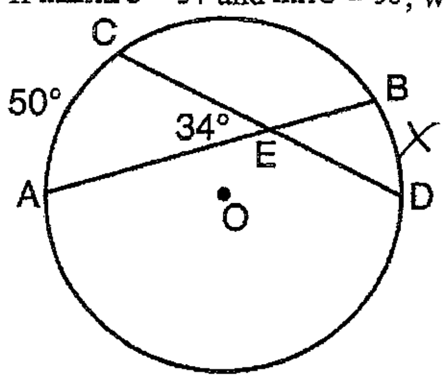
$\frac{2x}{2} = \frac{56}{2}$

$x = 28$

6. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $m\angle AEC = 34$ and $m\widehat{AC} = 50$, what is $m\widehat{DB}$?

- interior
- arcs and angles

2(VA) = arc + arc



2(VA) = arc + arc

$2(34) = x + 50$

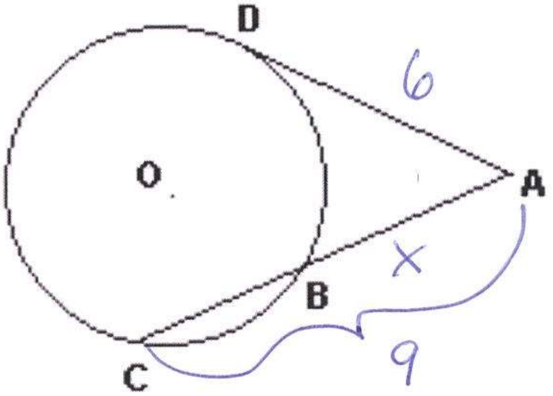
$68 = x + 50$

$-50 \quad -50$

$18 = x$

Whole · exterior = whole · exterior

7. In the diagram, $\overline{AD}$ is tangent to circle O at D, and $\overline{CBA}$ is a secant. If $AD = 6$ and $AC = 9$, what is AB?



$w \cdot e = w \cdot e$

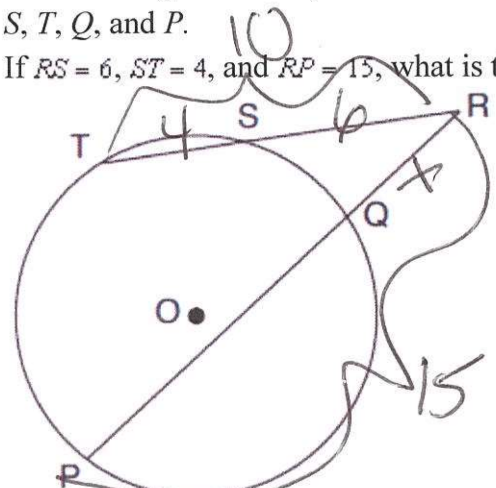
$6 \cdot 6 = 9 \cdot x$

$\frac{36}{9} = \frac{9x}{9}$

$4 = x$

8. In the diagram below, secants $\overline{RST}$ and $\overline{RQP}$, drawn from point R, intersect circle O at S, T, Q, and P.

If $RS = 6$, $ST = 4$, and $RP = 15$, what is the length of $\overline{RQ}$?



$w \cdot e = w \cdot e$

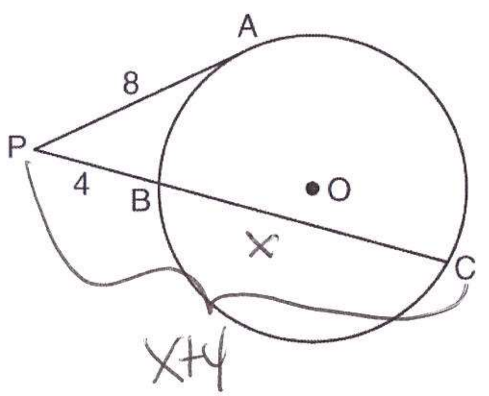
$10 \cdot 6 = 15 \cdot x$

$\frac{60}{15} = \frac{15x}{15}$

$4 = x$

9. In the diagram below of circle O, $\overline{PA}$ is tangent to circle O at A, and $\overline{PBC}$ is a secant with points B and C on the circle.

If $PA = 8$ and $PB = 4$, what is the length of $\overline{BC}$?



$w \cdot e = w \cdot e$

$8 \cdot 8 = (x+4)(4)$

$64 = 4x + 16$

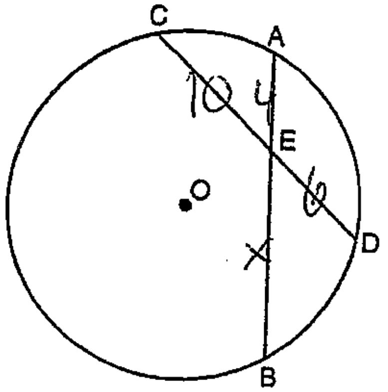
$-16 \quad -16$

$\frac{48}{4} = \frac{4x}{4}$

$12 = x$

Part · Part = Part · Part

10. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $CE = 10$, $ED = 6$, and $AE = 4$, what is the length of $\overline{EB}$?



$P \cdot P = P \cdot P$

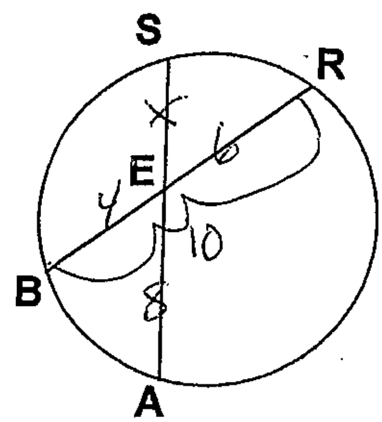
$10 \cdot 6 = 4 \cdot x$

$\frac{60}{4} = \frac{\cancel{4}x}{\cancel{4}}$

$15 = x$

– Segments
– interior
$P \cdot P = P \cdot P$

11. If $\overline{BR} = 10$, $\overline{BE} = 4$, $\overline{AE} = 8$, find $\overline{ES}$



$P \cdot P = P \cdot P$

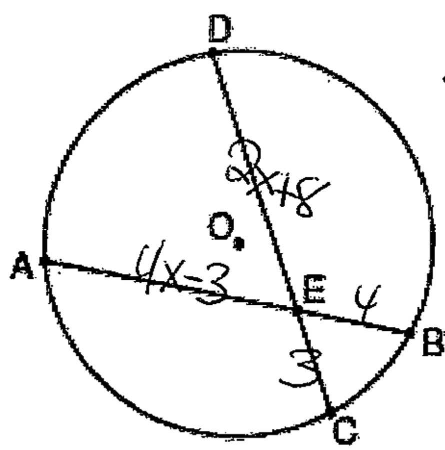
$6 \cdot 4 = 8 \cdot x$

$\frac{24}{8} = \frac{8x}{8}$

$3 = x$

– Segments
– interior
$P \cdot P = P \cdot P$
*10 is not a part

12. In the diagram of circle O below, chord $\overline{AB}$ intersects chord $\overline{CD}$ at E, $DE = 2x + 8$, $EC = 3$, $AE = 4x - 3$, and $EB = 4$.
What is the value of x?



$P \cdot P = P \cdot P$

$3(2x+8) = 4(4x-3)$

$6x + 24 = 16x - 12$

$-6x \quad\quad -6x$

$24 = 10x - 12$

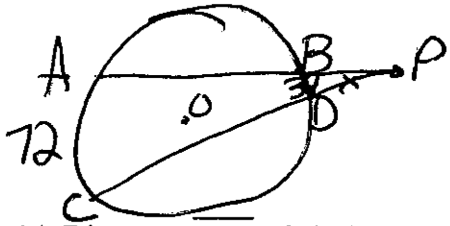
$+12 \quad\quad +12$

$\frac{36}{10} = \frac{10x}{10}$

$3.6 = x$

– Segments
– interior
$P \cdot P = P \cdot P$

13. In circle O two secants, $\overline{ABP}$ and $\overline{CDP}$, are drawn to external point P. If $m\overset{\frown}{AC} = 72°$, and $m\overset{\frown}{BD} = 34°$, what is the measure of $\angle P$?



$2(EA) = \text{major} - \text{minor}$

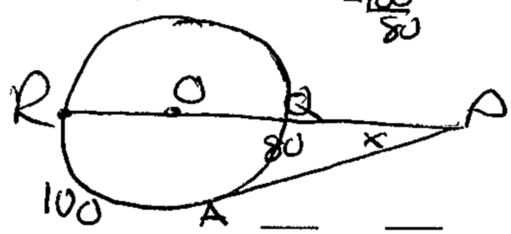
$2x = 72 - 34$

$\frac{2x}{2} = \frac{38}{2}$

$x = 19$

14. Diameter $\overline{ROQ}$ of circle O is extended through Q to point P, and tangent $\overline{PA}$ is drawn. If $m\overset{\frown}{RA} = 100°$, what is $m\angle P$?

$180 - 100 = 80$



$2(EA) = \text{major} - \text{minor}$

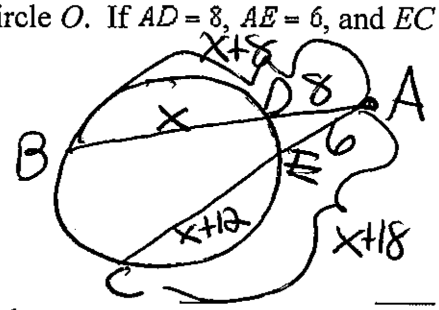
$2x = 100 - 80$

$\frac{2x}{2} = \frac{20}{2}$

$x = 10$

15. In circle O, secants $\overline{ADB}$ and $\overline{AEC}$ are drawn from external point A such that points D, B, E, and C are on circle O. If $AD = 8$, $AE = 6$, and EC is 12 more than BD, the length of $\overline{BD}$ is

1) 6
2) 22
3) 36
4) 48



$w \cdot e = w \cdot e$

$(x+8)8 = (x+18)(6)$

$8x + 64 = 6x + 108$

$-6x \quad -6x$

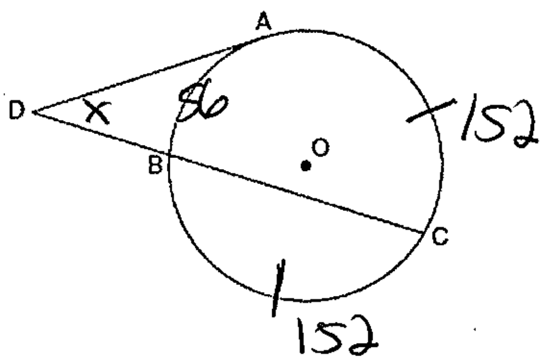
$2x + 64 = 108$

$-64 \quad -64$

$\frac{2x}{2} = \frac{44}{2}$

$x = 22$

16. In the diagram below, tangent $\overline{DA}$ and secant $\overline{DBC}$ are drawn to circle O from external point D, such that $\overset{\frown}{AC} \cong \overset{\frown}{BC}$. If $m\overset{\frown}{BC} = 152°$, determine and state $m\angle D$.



$2(EA) = \text{major} - \text{minor}$

$2x = 152 - 56$

$\frac{2x}{2} = \frac{96}{2}$

$x = 48$

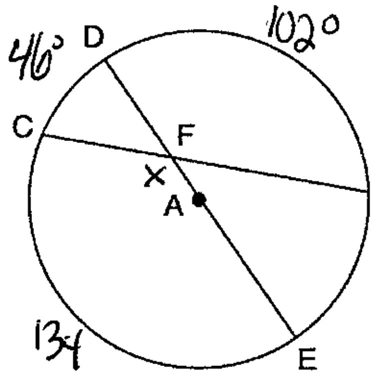
$152 + 152 + x = 360$

$304 + x = 360$

$-304 \quad -304$

$x = 56$

17. In circle A below, chord $\overline{BC}$ and diameter $\overline{DAE}$ intersect at F. If $m\widehat{CD} = 46°$ and $m\widehat{DB} = 102°$, what is $m\angle CFE$?



$$\begin{array}{r} 180 \\ -46 \\ \hline 134 \end{array}$$

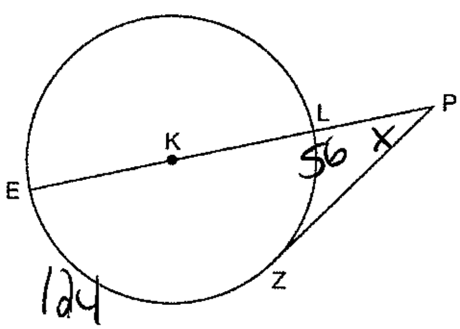
$2(VA) = arc + arc$

$2x = 134 + 102$

$\frac{2x}{2} = \frac{236}{2}$

$x = 118$

18. In the diagram below of circle K, secant $\overline{PLKE}$ and tangent $\overline{PZ}$ are drawn from external point P. If $m\widehat{LZ} = 56°$, determine and state the degree measure of angle P.



$$\begin{array}{r} 180 \\ -56 \\ \hline 124 \end{array}$$

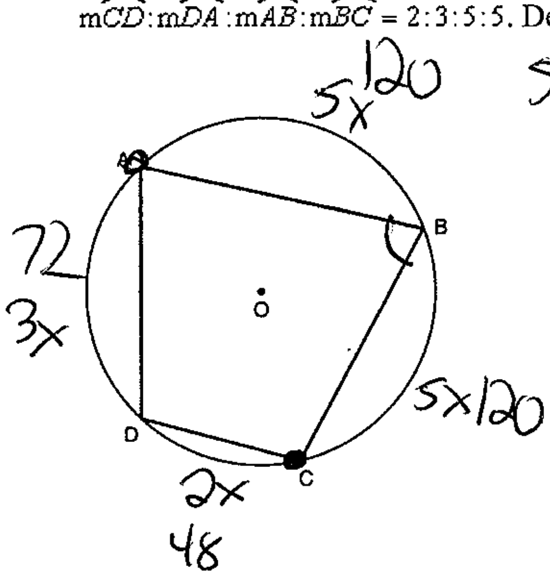
$2(EA) = major - minor$

$2x = 124 - 56$

$\frac{2x}{2} = \frac{68}{2}$

$x = 34$

19. In the diagram below, quadrilateral $ABCD$ is inscribed in circle O, and $m\widehat{CD} : m\widehat{DA} : m\widehat{AB} : m\widehat{BC} = 2:3:5:5$. Determine and state $m\angle B$.



$5x + 5x + 2x + 3x = 360$

$\frac{15x}{15} = \frac{360}{15}$

$x = 24$

$5(24) = 120$

$3(24) = 72$

$2(24) = 48$

$\angle B = \frac{1}{2}(AC)$

$\angle B = \frac{1}{2}(120)$

$\angle B = 60°$

The opposite angles add to 180°

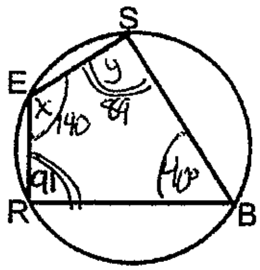
Name Schlansky
Mr. Schlansky

Date ____________
Geometry

Quadrilaterals Inscribed In a Circle

1. In the diagram below, quadrilateral $SBRE$ is inscribed in the circle. If $m\angle BRE = 91°$ and $m\angle SBR = 40°$, find $m\angle BSE$ and $m\angle SER$

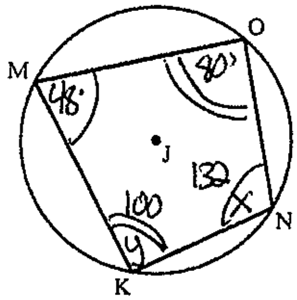
89 140°



$40 + x = 180$
$-40 \quad -40$
$x = 140$

$91 + y = 180$
$-91 \quad -91$
$y = 89$

2. In the diagram below, quadrilateral MONK is inscribed in circle J, $m\angle KMO = 48°$ and $m\angle MON = 80°$. Find the measures of $m\angle KNO$ and $m\angle MKN$.



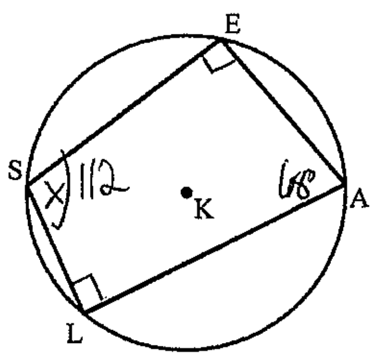
$x + 48 = 180$
$-48 \quad -48$
$x = 132$

$80 + y = 180$
$-80 \quad -80$
$y = 100$

90°

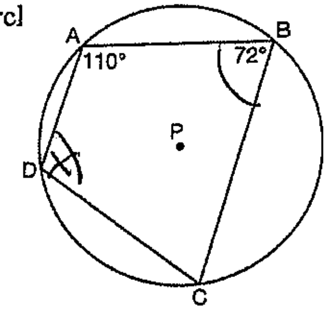
3. In the diagram below, quadrilateral SEAL is inscribed in circle K, $\overline{SE} \perp \overline{EA}$ and $m\angle EAL = 68°$. Find the measures of $m\angle SLA$ and $m\angle ESL$.

90° 112



$x + 68 = 180$
$-68 \quad -68$
$x = 112$

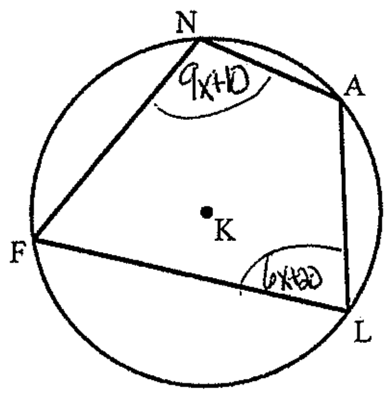
4. In the diagram below, quadrilateral $ABCD$ is inscribed in circl



What is $m\angle ADC$?
1) 70°
2) 72°
3) 108°
4) 110°

$x + 72 = 180$
$-72 \ -72$
$x = 108$

5. In the diagram below, quadrilateral FLAN is inscribed in circle K, $m\angle FNA = 9x + 10$ and $m\angle FLA = 6x + 20$. Find the measures of $m\angle FLA$.

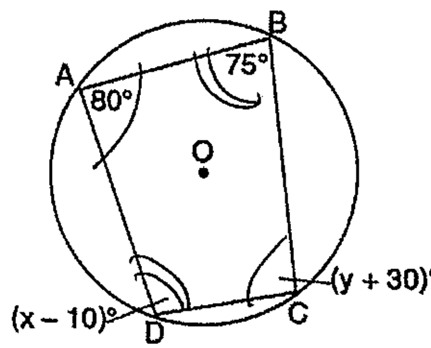


$9x + 10 + 6x + 20 = 180$
$15x + 30 = 180$
$-30 \ -30$
$\frac{15x}{15} = \frac{150}{15}$
$x = 10$

$6x + 20$
$6(10) + 20$
$80°$

6. Quadrilateral $ABCD$ is inscribed in circle O, as shown below.

If $m\angle A = 80°$, $m\angle B = 75°$, $m\angle C = (y + 30)°$, and $m\angle D = (x - 10)°$, which statement is true?
1) $x = 85$ and $y = 50$
2) $x = 90$ and $y = 45$
3) $x = 110$ and $y = 75$
4) $x = 115$ and $y = 70$



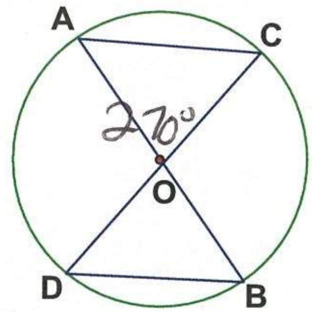
$80 + y + 30 = 180$
$y + 110 = 180$
$-110 \ -110$
$y = 70$

$75 + x - 10 = 180$
$x + 65 = 180$
$-65 \ -65$
$x = 115$

Area of a Sector = $\frac{\theta \pi r^2}{360}$

If given area of a sector, use algebra to solve for missing variable

1. In circle O, $m\angle AOC = 70$ and $\overline{AO} = 2\ in$. Find the area of sector COA to the nearest square inch. → type π in

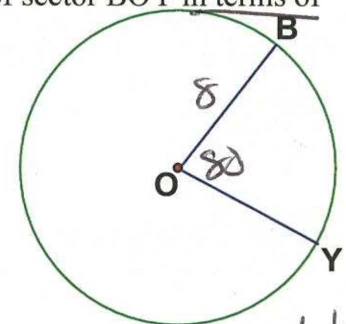


$A = \frac{\theta \pi r^2}{360}$

$A = \frac{70\pi(2)^2}{360}$

$A = 2\ in^2$

2. In circle O, if $\angle$ BOY = 80° and $\overline{BO} = 8\ cm$, find the area of sector BOY in terms of π. → don't type π in



$A = \frac{\theta \pi r^2}{360}$

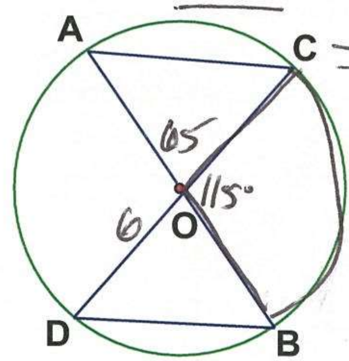
$A = \frac{80\pi(8)^2}{360}$

$A = \frac{128}{9}\pi$

→ don't type π in

3. In circle O, $m\angle AOC = 65$ and $\overline{DO} = 6\ in$. Find the area of sector COB in terms of π

$180 - 65 = 115$



$A = \frac{\theta \pi r^2}{360}$

$A = \frac{115\pi(6)^2}{360}$

$A = \frac{23}{2}\pi$

4. Determine and state, in terms of π, the area of a sector that intercepts a 40° arc of a circle with a radius of 4.5. → don't type π in

$A = \frac{\theta \pi r^2}{360}$

$A = \frac{40\pi(4.5)^2}{360}$

$A = 2.25\pi$

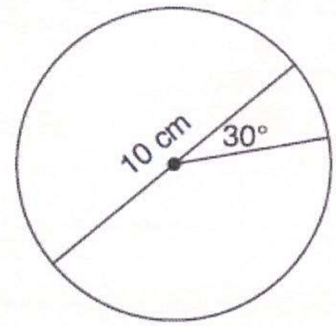
5. A circle with a diameter of 10 cm and a central angle of 30° is drawn below. What is the area, to the *nearest tenth of a square centimeter*, of the sector formed by the 30° angle?

1) 5.2
2) 6.5
3) 13.1
4) 26.2

$A = \frac{\theta \pi r^2}{360}$

$A = \frac{30\pi(5)^2}{360}$

$A = 6.5$



6. A circle has a radius of 6.4 inches. Determine and state, to the *nearest square inch*, the area of a sector whose arc measures 80°.

$A = \frac{\theta \pi r^2}{360}$

$A = \frac{80\pi(6.4)^2}{360} = \boxed{29}$

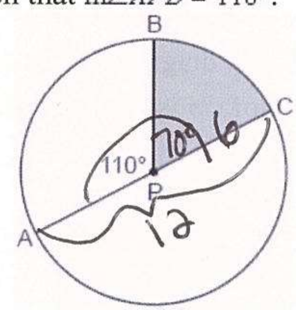
7. In circle P below, diameter $\overline{AC}$ and radius $\overline{BP}$ are drawn such that $m\angle APB = 110°$. If $AC = 12$, what is the area of shaded sector BPC?

1) $\frac{7}{6}\pi$
2) 7π
3) 11π
4) 28π

don't type π in

$A = \frac{\theta \pi r^2}{360}$

$A = \frac{70\pi(6)^2}{360} = 7\pi$



$\begin{array}{r} 180 \\ -110 \\ \hline 70 \end{array}$

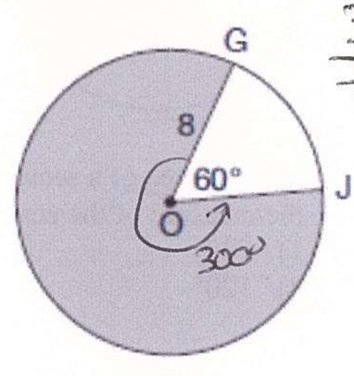
8. In the diagram below of circle O, $GO = 8$ and $m\angle GOJ = 60°$. What is the area, in terms of π, of the shaded region?

1) $\frac{4\pi}{3}$
2) $\frac{20\pi}{3}$
3) $\frac{32\pi}{3}$
4) $\frac{160\pi}{3}$

$A = \frac{\theta \pi r^2}{360}$

$A = \frac{300\pi(8)^2}{360}$

$A = \frac{160}{3}\pi$



$\begin{array}{r} 360 \\ -60 \\ \hline 300 \end{array}$

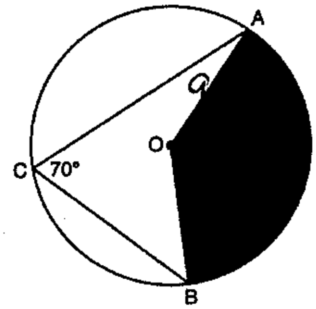
9. In the diagram below of circle O, $\overline{AC}$ and $\overline{BC}$ are chords, and $m\angle ACB = 70°$. If $OA = 9$, the area of the shaded sector AOB is

1) 3.5π
2) 7π
3) 15.75π
4) 31.5π

$A=\frac{\theta\pi r^2}{360}$

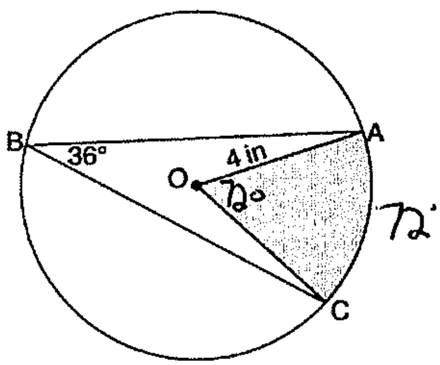
$A=\frac{140\pi(9)^2}{360}$

$A=\frac{63}{2}\pi$



140°

10. In the diagram below of circle O, the measure of inscribed angle ABC is 36° and the length of $\overline{OA}$ is 4 inches. Determine and state, to the *nearest tenth of a square inch,* the area of the shaded sector.



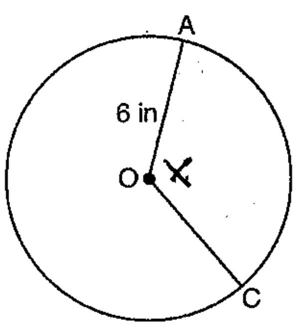
72°

$A=\frac{\theta\pi r^2}{360}$

$A=\frac{72\pi(4)^2}{360}$

$A= 10.1$

11. In the diagram below of circle O, the area of the shaded sector AOC is 12π in^2 and the length of $\overline{OA}$ is 6 inches. Determine and state $m\angle AOC$.



$A=\frac{\theta\pi r^2}{360}$

$\frac{12\cancel{\pi}}{1}=\frac{x\cancel{\pi}(6)^2}{360}$

Use equation solver if necessary

$\frac{36x}{36}=\frac{4320}{36}$

$x=120$

12. The area of a sector of a circle with a radius measuring 15 cm is 75π cm^2. What is the measure of the central angle that forms the sector?

1) 72°
2) 120°
3) 144°
4) 180°

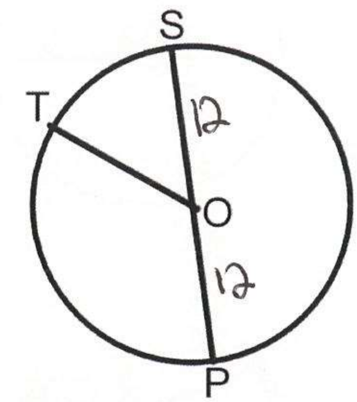
$A=\frac{\theta\pi r^2}{360}$

$\frac{75\cancel{\pi}}{1}=\frac{x\cancel{\pi}(15)^2}{360}$

$\frac{225x}{225}=\frac{27000}{225}$

$x=120°$

13. In the diagram below of circle *O*, the area of sector *STO* is $48\pi\ in^2$ and the length of $\overline{OP}$ is 12 inches. Determine and state $m\angle SOT$



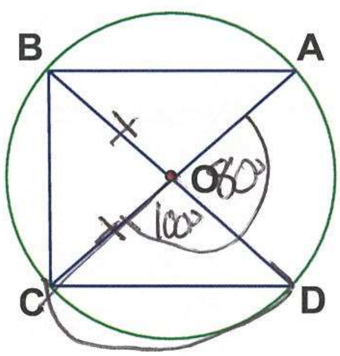
$A = \frac{\theta \pi r^2}{360}$

$\frac{48\pi}{1} = \frac{x\pi(12)^2}{360}$

$\frac{144x}{144} = \frac{17280}{144}$

$x = 120°$

14. In circle O, diameters $\overline{BOD}$ and $\overline{COA}$ intersect at the center of the circle O. If the area of sector OCD = 240π square inches and $m\angle AOD = 80$, find the measure of $\overline{OB}$ to the nearest tenth of an inch.



$A = \frac{\theta \pi r^2}{360}$

$\frac{240\pi}{1} = \frac{100\pi x^2}{360}$

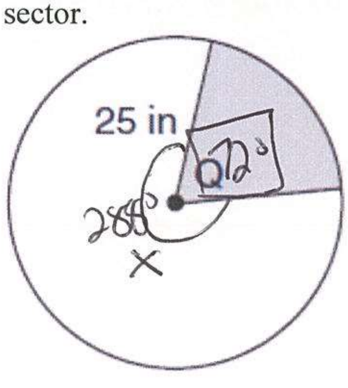
$\frac{100x^2}{100} = \frac{86400}{100}$

$\sqrt{x^2} = \sqrt{864}$

$x = 29.4$

15. In the diagram below, the circle has a radius of 25 inches. The area of the *unshaded* sector is 500π in^2.
Determine and state the degree measure of angle *Q*, the central angle of the shaded sector.

$360 - 288 = 72°$



$A = \frac{\theta \pi r^2}{360}$

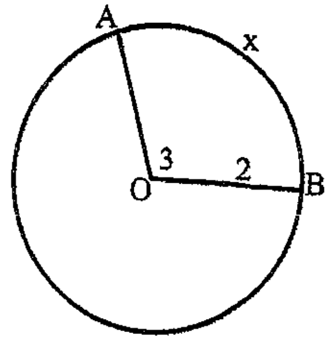
$\frac{500\pi}{1} = \frac{x\pi(25)^2}{360}$

$\frac{625x}{625} = \frac{180000}{625}$

$x = 288°$

Arc Length: $s = \theta r$, where s = arc length, θ = central angle (in radians), r = radius

1. In circle O, the measure of central angle AOB is 3 radians and the length of $\overline{OB}$ is 2 cm. What is the measure of arc AB?

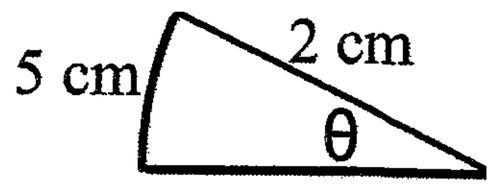


$S = \theta r$

$x = 3(2)$

$x = 6$

2. What is the measure of the central angle below?



$S = \theta r$

$\frac{5}{2} = \frac{x(\cancel{2})}{\cancel{2}}$

$\frac{5}{2} = x$

3. What is the measure of the radius of a sector whose arc length is 12 inches and has a central angle of 4 radians?

$S = \theta r$

$\frac{12}{4} = \frac{\cancel{4}r}{\cancel{4}}$

$3 = r$

4. A wheel has a radius of 18 inches. Which distance, to the *nearest inch*, does the wheel travel when it rotates through an angle of $\frac{2\pi}{5}$ radians?

$S = \theta r$

$x = \frac{2\pi}{5}(18)$

$x = 22.6$

5. What is the measure of a central angle in degrees whose arc length is 6 meters and whose radius measures 8 meters?

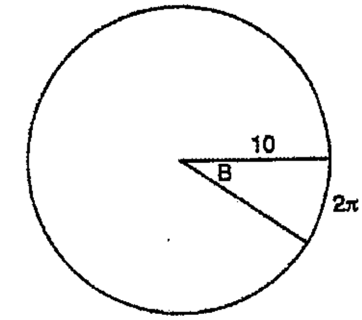
$S = \theta r$

$\frac{6}{8} = \frac{x8}{8}$

$x = \frac{3}{4}$

6. In the diagram below, the circle shown has radius 10. Angle B intercepts an arc with a length of 2π.
What is the measure of angle B, in radians?

1) $10 + 2\pi$
2) 20π
3) $\frac{\pi}{5}$
4) $\frac{5}{\pi}$

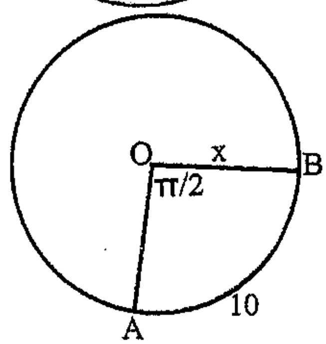


$S = \theta r$

$\frac{2\pi}{10} = \frac{x(10)}{10}$

$\frac{\pi}{5} = x$

7. In circle O, the measure of central angle AOB is $\frac{\pi}{2}$ radians and the length of arc AB is 10 cm. What is the measure of radius $\overline{OB}$ to the *nearest tenth of a cm*?



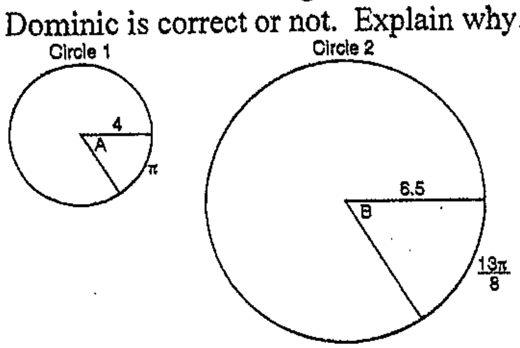
$S = \theta r$

$10 = \frac{\pi}{2}(x)$

$\frac{20}{\pi} = \frac{\pi x}{\pi}$

$6.4 = x$

8. In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle A intercepts an arc of length π, and angle B intercepts an arc of length $\frac{13\pi}{8}$.
Dominic thinks that angles A and B have the same radian measure. State whether Dominic is correct or not. Explain why.



$S = \theta r$

$\frac{\pi}{4} = \frac{x(4)}{4}$

$\frac{\pi}{4} = x$

Yes

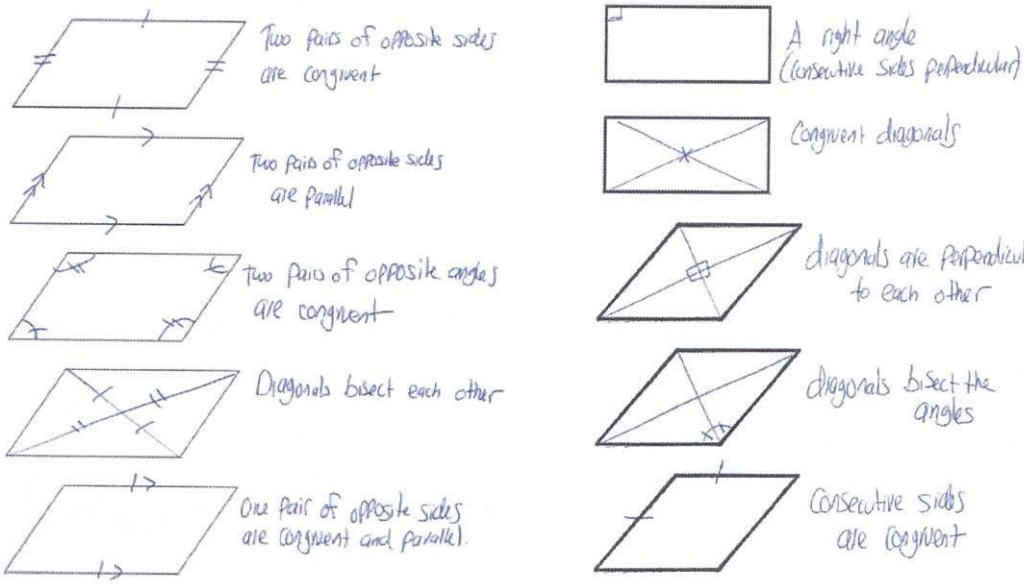
$S = \theta r$

$\frac{13\pi}{8} = x(6.5)$

$\frac{13\pi}{52} = \frac{52x}{52}$

$\frac{\pi}{4} = x$

Parallelogram Properties



A rectangle and rhombus have all of the properties of the parallelogram.
A square has all of the properties of the parallelogram, rectangle, and rhombus.
A trapezoid has one pair of opposite sides parallel and one pair of opposite sides not parallel.
An isosceles trapezoid is a trapezoid that has congruent legs and congruent diagonals.
For properties questions, draw the shape!

1. Which of the following is not true of all rectangles?
1) Consecutive sides are perpendicular
2) Opposite sides are parallel
3) Diagonals are perpendicular to each other rhombus property
4) Diagonals bisect each other

2. Which of the following is true about rhombuses?
1) Consecutive sides are perpendicular
2) Opposite sides are congruent P-gram property
3) Consecutive angles are congruent
4) Diagonals are congruent

3. Which of the following is *not* true about all parallelograms?
1) Diagonals bisect each other
2) Diagonals are perpendicular to each other rhombus property
3) Opposite angles are congruent
4) Consecutive angles are supplementary

4. A quadrilateral whose diagonals bisect each other and are perpendicular is a
1) rhombus
2) rectangle
3) trapezoid
4) parallelogram

5. If the diagonals of a quadrilateral do *not* bisect each other, then the quadrilateral could be a

not a parallelogram

1) rectangle
2) rhombus
3) square
4) trapezoid

6. Which statement is true about every parallelogram?
1) All four sides are congruent.
2) The interior angles are all congruent.
3) Two pairs of opposite sides are congruent.
4) The diagonals are perpendicular to each other.

7. Which quadrilateral has diagonals that always bisect its angles and also bisect each other?
1) rhombus
2) rectangle
3) parallelogram
4) isosceles trapezoid

8. The diagonals of a quadrilateral are congruent but do not bisect each other. This quadrilateral is

not a parallelogram

1) an isosceles trapezoid
2) a parallelogram
3) a rectangle
4) a rhombus

9. Given three distinct quadrilaterals, a square, a rectangle, and a rhombus, which quadrilaterals must have perpendicular diagonals?
1) the rhombus, only
2) the rectangle and the square
3) the rhombus and the square Square has all rectangle and rhombus properties
4) the rectangle, the rhombus, and the square

10. A parallelogram must be a rhombus when its
1) Diagonals are congruent.
2) Opposite sides are parallel.
3) Diagonals are perpendicular.
4) Opposite angles are congruent.

11. A parallelogram must be a rectangle when its
1) diagonals are perpendicular
2) diagonals are congruent
3) opposite sides are parallel
4) opposite sides are congruent

12. A rectangle must be a square when its need a rhombus property
1) consecutive sides are perpendicular
2) diagonals are congruent
3) diagonals are perpendicular to each other
4) opposite sides are parallel

13. A rhombus must be a square when its need a rectangle property
1) consecutive sides are congruent
2) diagonals are congruent
3) opposite angles are congruent
4) diagonals are perpendicular to each other

14. A parallelogram must be a rectangle when its
1) consecutive sides are congruent
2) opposite angles are congruent
3) consecutive sides are perpendicular (perpendicular lines form right angles)
4) opposite sides are parallel

15. Which of the following properties does not make a parallelogram a rhombus?
1) diagonals bisect the angles
2) diagonals are perpendicular to each other
3) opposite angles are congruent not one of the 3 ways to prove a rhombus
4) consecutive sides are congruent

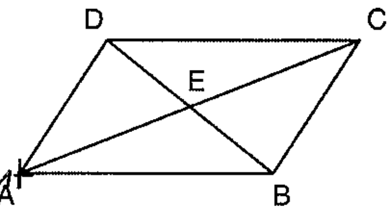
16. Which of the following properties does not make a rhombus a square? need a rectangle property
1) Diagonals are congruent ✓
2) Diagonals are perpendicular to each other ✗
3) Consecutive sides are perpendicular ✓
4) Consecutive angles are congruent ✓
(all right angles)

17. Which set of statements would describe a parallelogram that can always be classified as a rhombus?
I. Diagonals are perpendicular bisectors of each other. ✓
II. Diagonals bisect the angles from which they are drawn. ✓
III. Diagonals form four congruent isosceles right triangles. ✓ must be a square which must be a rhombus

1) I and II
2) I and III
3) II and III
4) I, II, and III

rectangle has ≅ diagonals
rhombus has ⊥ diagonals
therefore, it must be a square

18. In the diagram below, parallelogram *ABCD* has diagonals $\overline{AC}$ and $\overline{BD}$ that intersect at point *E*.

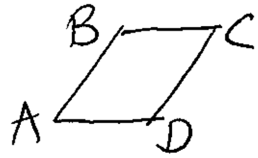


Which expression is *not* always true?

1) $\angle DAE \cong \angle BCE$ alt. int ∡s ✓
2) $\angle DEC \cong \angle BEA$ vertical ∡s ✓
3) $\overline{AC} \cong \overline{DB}$ ✗ diagonals are not always congruent
4) $\overline{DE} \cong \overline{EB}$ ✓ diagonals bisect each other

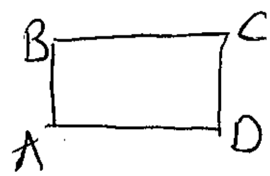
19. If *ABCD* is a parallelogram, which statement would prove that *ABCD* is a rhombus?

1) $\angle ABC \cong \angle CDA$
2) $\overline{AC} \cong \overline{BD}$
3) $\overline{AC} \perp \overline{BD}$ ⊥ diagonals
4) $\overline{AB} \perp \overline{CD}$



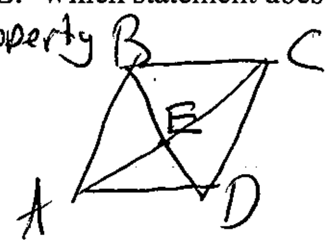
20. If *ABCD* is a parallelogram, which statement would prove that *ABCD* is a rectangle?

1) $\angle ABC \cong \angle CDA$
2) $\overline{AC} \cong \overline{BD}$ congruent diagonals
3) $\overline{AC} \perp \overline{BD}$
4) $\overline{AB} \perp \overline{CD}$

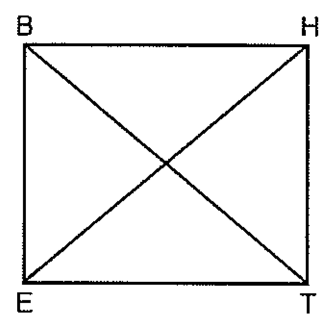


21. In rectangle *ABCD*, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at *E*. Which statement does *not* prove rectangle *ABCD* is a square? need a rhombus property

1) $\overline{AC} \cong \overline{DB}$ congruent diagonals
2) $\overline{AB} \cong \overline{BC}$ consecutive sides congruent
3) $\overline{AC} \perp \overline{DB}$ perpendicular diagonals
4) $\overline{AC}$ bisects $\angle DCB$ diagonals bisect the angles



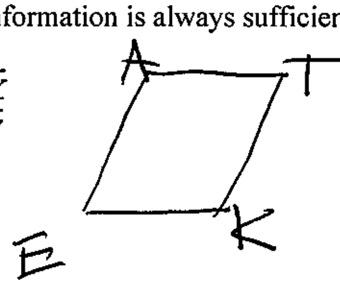
22. Parallelogram *BETH*, with diagonals $\overline{BT}$ and $\overline{HE}$, is drawn below. What additional information is sufficient to prove that *BETH* is a rectangle?



1) $\overline{BT} \perp \overline{HE}$ ⊥ diagonals
2) $\overline{BE} \parallel \overline{HT}$ 1 pair of opposite sides ∥
3) $\overline{BT} \cong \overline{HE}$ congruent diagonals
4) $\overline{BE} \cong \overline{ET}$ consecutive sides ≅

23. Parallelogram *EATK* has diagonals $\overline{ET}$ and $\overline{AK}$. Which information is always sufficient to prove *EATK* is a rhombus?

1) $\overline{EA} \perp \overline{AT}$
2) $\overline{EA} \cong \overline{AT}$ consecutive sides ≅
3) $\overline{ET} \cong \overline{AK}$
4) $\overline{ET} \cong \overline{AT}$



Triangles/Parallel Lines Cut By a Transversal/Angles of Parallelograms

1) The three angles of a triangle add to equal 180°. **Look for triangles.**
   *The four angles of a quadrilateral add to 360°.
2) Linear pairs add to 180°. **Look for linear pairs.**
3) Vertical angles are congruent. Look for an X (intersecting lines).
4) **Given congruent sides:** Isosceles triangle has congruent angles opposite congruent sides.
5) **Given equilateral triangle:** Equilateral triangle has angles 60, 60, 60.
6) **Given angle bisector:** An angle bisector cuts an angle into two congruent halves.
7) **Given parallel**: Extend parallel lines and transversal. Follow the transversal and fill in all 8 angles. If angles are the same (both acute or both obtuse), the angles are congruent. If the angles are different (one acute and one obtuse), the angles are supplementary (add to 180).
8) **Given parallelogram**: Opposite angles are congruent and consecutive angles are supplementary (add to 180)

1. In the diagram below, $\overleftrightarrow{RCBT}$ and $\triangle ABC$ are shown with $m\angle A = 60$ and $m\angle ABT = 125$.

What is $m\angle ACR$?

1) 125
2) 115
3) 65
4) 55

$\triangle ABC$

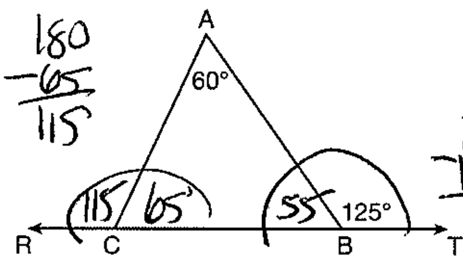
$x+60+55=180$

$115+x=180$

$-115 \quad -115$

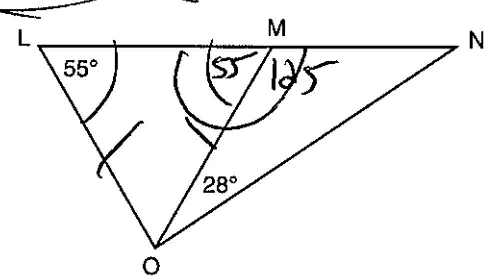
$x=65$

$180 - 65 = 115$



$180 - 125 = 55$

2. In the diagram below, $\triangle LMO$ is isosceles with $LO = MO$.



$180 - 55 = 125$

If $m\angle L = 55$ and $m\angle NOM = 28$, what is $m\angle N$?

1) 27
2) 28
3) 42
4) 70

$\triangle OMN$

$28+125+x=180$

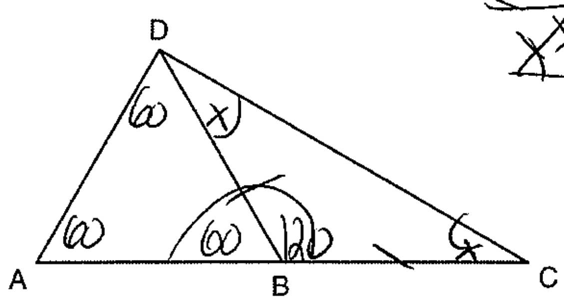
$153+x=180$

$-153 \quad -153$

$x=27$

3. In the diagram below of $\triangle ACD$, B is a point on $\overline{AC}$ such that $\triangle ADB$ is an equilateral triangle, and $\triangle DBC$ is an isosceles triangle with $\overline{DB} \cong \overline{BC}$. Find $m\angle C$.

60/60/60



$180 - 60 = 120$

$\triangle DBC$

$x+x+120=180$

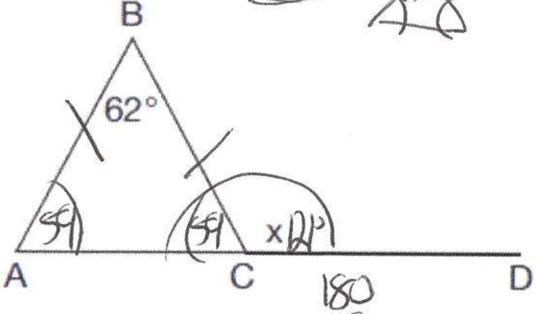
$2x+120=180$

$-120 \quad -120$

$\frac{2x}{2}=\frac{60}{2}$

$x=30$

4. Given $\triangle ABC$ with ~~$m\angle B = 62°$~~ and side $\overline{AC}$ extended to D, as shown below. Which value of x makes $\overline{AB} \cong \overline{CB}$?



$\triangle ABC$

$x+x+62=180$

$2x+62=180$

$-62 \quad -62$

$\frac{2x}{2}=\frac{118}{2}$

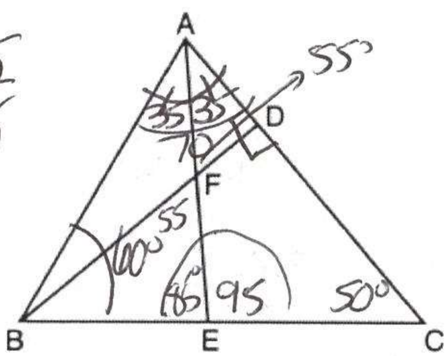
$x=59$

$180 - 59 = 121$

$x=121°$

5. In the diagram of $\triangle ABC$ below, $\overline{AE}$ bisects angle BAC, and altitude $\overline{BD}$ is drawn. If $m\angle C = 50°$ and $m\angle ABC = 60°$, what is $m\angle FEB$?

$180 - 95 = 85$



$\triangle BAC$

$60+50+x=180$

$110+x=180$

$-110 \quad -110$

$x=70$

$\triangle ADF$

$35+90+x=180$

$125+x=180$

$-125 \quad -125$

$x=55$

$\triangle ACE$

$35+50+x=180$

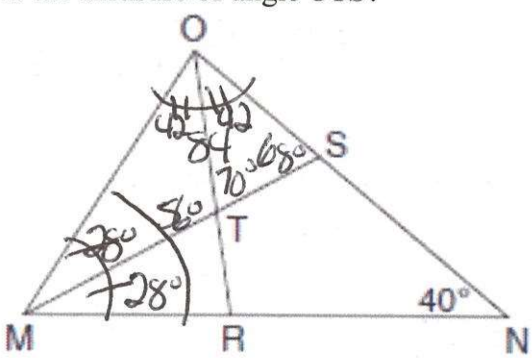
$85+x=180$

$-85 \quad -85$

$x=95$

$\angle FEB=85°$

6. In the diagram below of triangle MNO, $\angle M$ and $\angle O$ are bisected by $\overline{MS}$ and $\overline{OR}$, respectively. Segments MS and OR intersect at T, and $m\angle N = 40°$. If $m\angle TMR = 28°$, what is the measure of angle OTS?



$\triangle OMN$

$56+40+x=180$

$96+x=180$

$-96 \quad -96$

$x=84$

$\triangle MOS$

$28+84+x=180$

$112+x=180$

$-112 \quad -112$

$x=68$

$\triangle OTS$

$42+68+x=180$

$110+x=180$

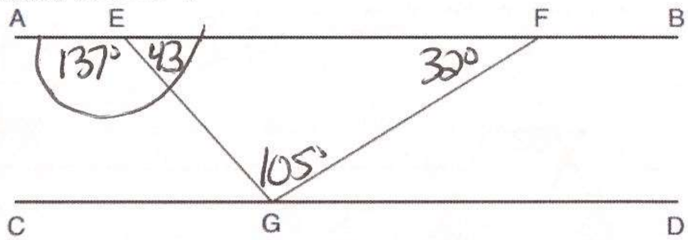
$-110 \quad -110$

$x=70$

$\angle OTS=70°$

7. In the diagram below, $\overline{AEFB} \parallel \overline{CGD}$, and $\overline{GE}$ and $\overline{GF}$ are drawn.
If $m\angle EFG = 32°$ and $m\angle AEG = 137°$, what is $m\angle EGF$?

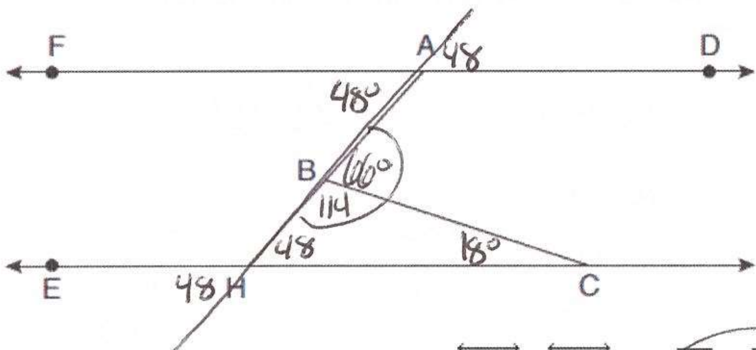
1) 11°
2) 43°
3) 75°
4) 105°



$180 - 137 = 43$

△EFG
$43+32+x=180$
$75+x=180$
$-75 \quad -75$
$x=105$

8. In the diagram below, $\overline{FAD} \parallel \overline{EHC}$, and $\overline{ABH}$ and $\overline{BC}$ are drawn. If $m\angle FAB = 48°$ and $m\angle ECB = 18°$, what is $m\angle ABC$?

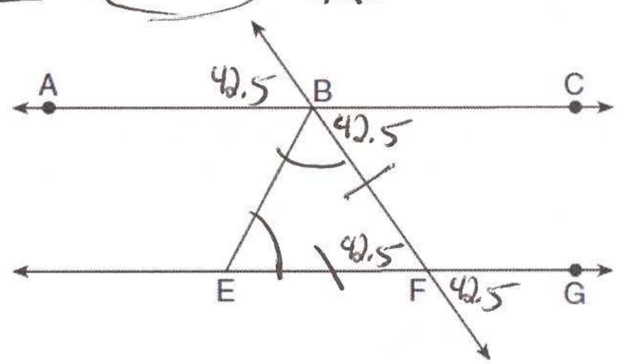


△HBC
$48+18+x=180$
$66+x=180$
$-66 \quad -66$
$x=114$
$\angle ABC=66°$

$180 - 114 = 66$

9. As shown in the diagram below, $\overleftrightarrow{ABC} \parallel \overleftrightarrow{EFG}$ and $\overline{BF} \cong \overline{EF}$.
If $m\angle CBF = 42.5°$, then $m\angle EBF$ is

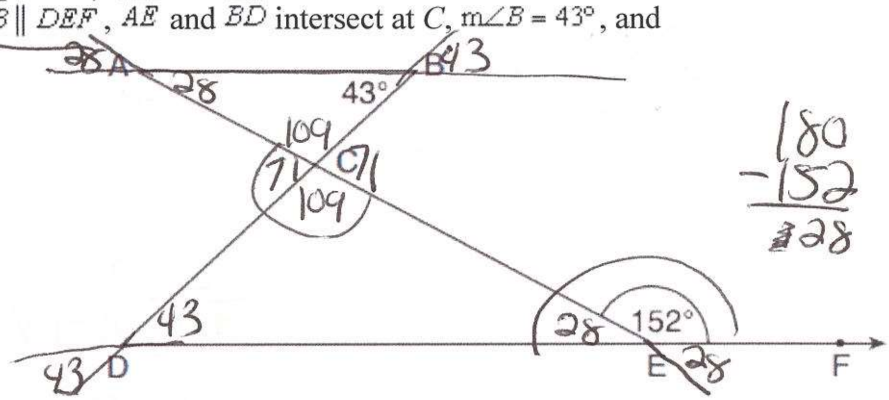
1) 42.5°
2) 68.75°
3) 95°
4) 137.5°



△FBE
$x+x+42.5=180$
$2x+42.5=180$
$-42.5 \quad -42.5$
$\frac{2x}{2}=\frac{137.5}{2}$
$x=68.75$

10. In the diagram below, $\overline{AB} \parallel \overrightarrow{DEF}$, $\overline{AE}$ and $\overline{BD}$ intersect at C, $m\angle B = 43°$, and $m\angle CEF = 152°$.
Which statement is true?

1) $m\angle D = 28°$
2) $m\angle A = 43°$
3) $m\angle ACD = 71°$
4) $m\angle BCE = 109°$



$180 - 152 = 28$

△ABC
$28+43+x=180$
$71+x=180$
$-71 \quad -71$
$x=109$

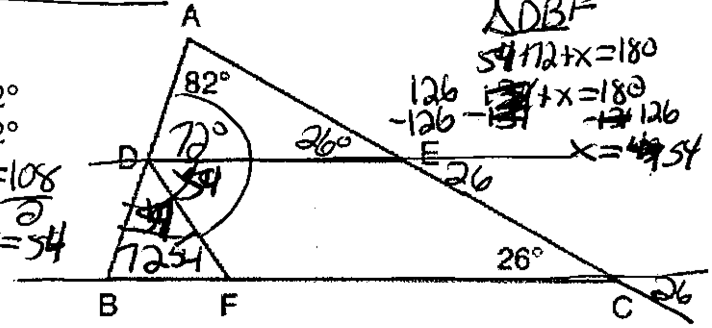
parallel

$\triangle ABC$
$82+26+x=180$
$108+x=180$
$-108 \quad -108$
$x=72$

$\triangle DBF$
$54+72+x=180$
$126+x=180$
$-126 \quad -126$
$x=54$

11. In the diagram below, $\overline{DE}$ divides $\overline{AB}$ and $\overline{AC}$ proportionally, m$\angle C$ = 26°, m$\angle A$ = 82°, and $\overline{DF}$ bisects $\angle BDE$.
The measure of angle DFB is

1) 36°
2) 54°
3) 72°
4) 82°



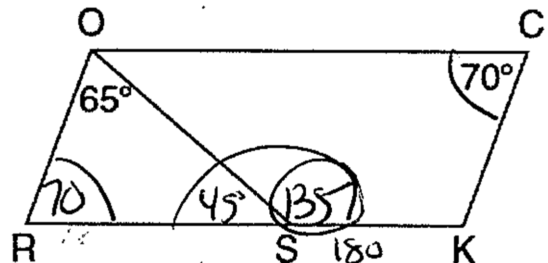
$\triangle ADE$
$82+26+x=180$
$108+x=180$
$-108 \quad -108$
$x=72$

$\overline{ADB}$
$72+x+x=180$
$2x+72=180$
$-72 \quad -72$
$\frac{2x}{2}=\frac{108}{2}$
$x=54$

12. In the diagram below of parallelogram $ROCK$, m$\angle C$ is 70° and m$\angle ROS$ is 65°.
What is m$\angle KSO$?

1) 45°
2) 110°
3) 115°
4) 135°

opposite ∡s ≅



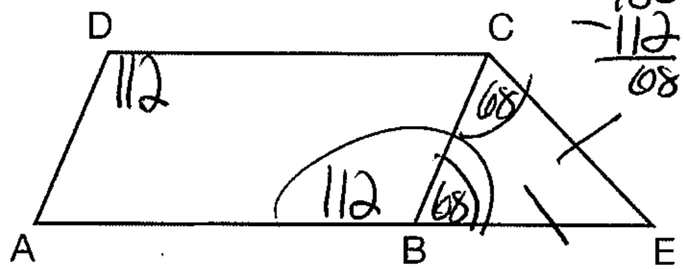
$\triangle ROS$
$70+65+x=180$
$135+x=180$
$-135 \quad -135$
$x=45$

180
-45
135

13. In the diagram below, $ABCD$ is a parallelogram, $\overline{AB}$ is extended through B to E, and $\overline{CE}$ is drawn.

If $\overline{CE} \cong \overline{BE}$ and m$\angle D$ = 112°, what is m$\angle E$?

1) 44°
2) 56°
3) 68°
4) 112°



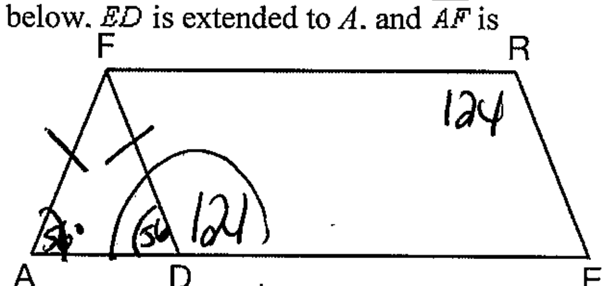
180
-112
68

$\triangle CBE$
$68+68+x=180$
$136+x=180$
$-136 \quad -136$
$x=44$

14. In the diagram of parallelogram $FRED$ shown below. $\overline{ED}$ is extended to A. and $\overline{AF}$ is drawn such that $\overline{AF} \cong \overline{DF}$.

If m$\angle R$ = 124°, what is m$\angle AFD$?

1) 124°
2) 112°
3) 68°
4) 56°



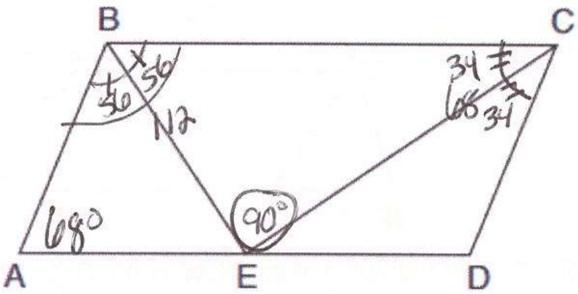
180
-124
56

$\triangle AFD$
$56+56+x=180$
$112+x=180$
$-112 \quad -112$
$x=68$

15. In parallelogram *ABCD* shown below, the bisectors of $\angle ABC$ and $\angle DCB$ meet at *E*, a point on $\overline{AD}$.
If $m\angle A = 68°$, determine and state $m\angle BEC$.

$$\begin{array}{r} 56 \\ +34 \\ \hline 90 \end{array} \quad \begin{array}{r} 180 \\ -90 \\ \hline 90 \end{array}$$

$$\begin{array}{r} 180 \\ -68 \\ \hline 112 \end{array}$$

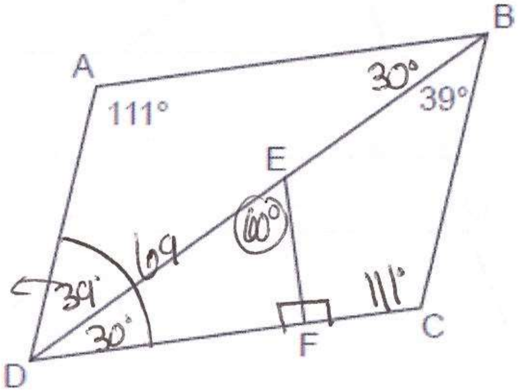


16. In the diagram below of parallelogram *ABCD*, diagonal $\overline{BED}$ and $\overline{EF}$ are drawn, $\overline{EF} \perp \overline{DFC}$, $m\angle DAB = 111°$, and $m\angle DBC = 39°$. What is $m\angle DEF$?

$$\begin{array}{r} 180 \\ -111 \\ \hline 69 \end{array}$$

$$\begin{array}{r} 90 \\ +30 \\ \hline 120 \end{array} \quad \begin{array}{r} 180 \\ -120 \\ \hline 60 \end{array}$$

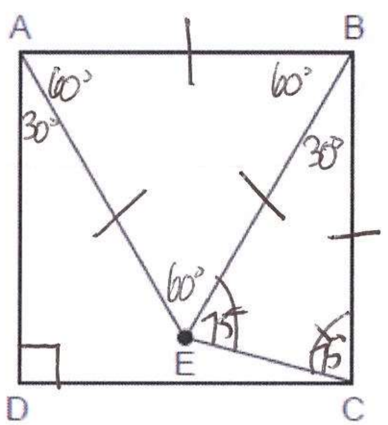
alt int ∠



17. In the diagram below, point *E* is located inside square *ABCD* such that $\triangle ABE$ is equilateral, and $\overline{CE}$ is drawn. What is $m\angle BEC$? 75°

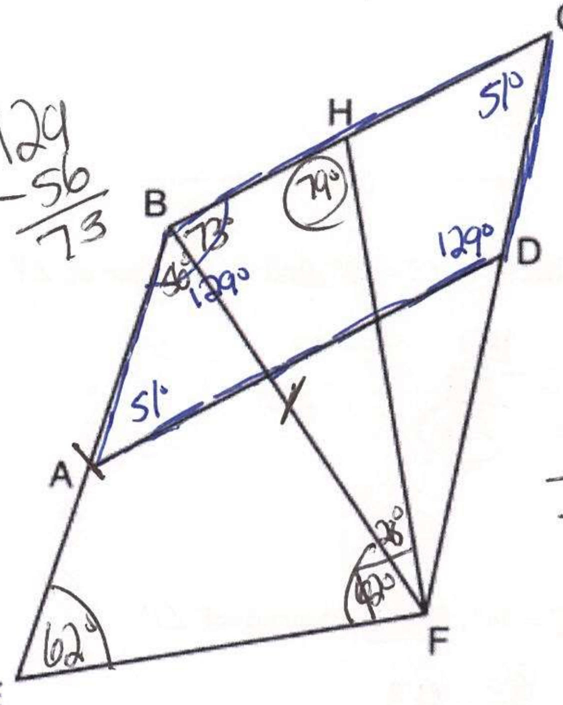
$$\begin{array}{r} 90 \\ -60 \\ \hline 30 \end{array}$$

$$\begin{array}{r} 180 \\ -30 \\ \hline 150 \end{array} \quad \frac{150}{2} = 75$$



18. Quadrilateral $EBCF$ and $\overline{AD}$ are drawn below, such that $ABCD$ is a parallelogram, $\overline{EB} \cong \overline{FB}$, and $\overline{EF} \perp \overline{FH}$. If $m\angle E = 62°$ and $m\angle C = 51°$, what is $m\angle FHB$?

$$\begin{array}{r} 129 \\ -56 \\ \hline 73 \end{array}$$



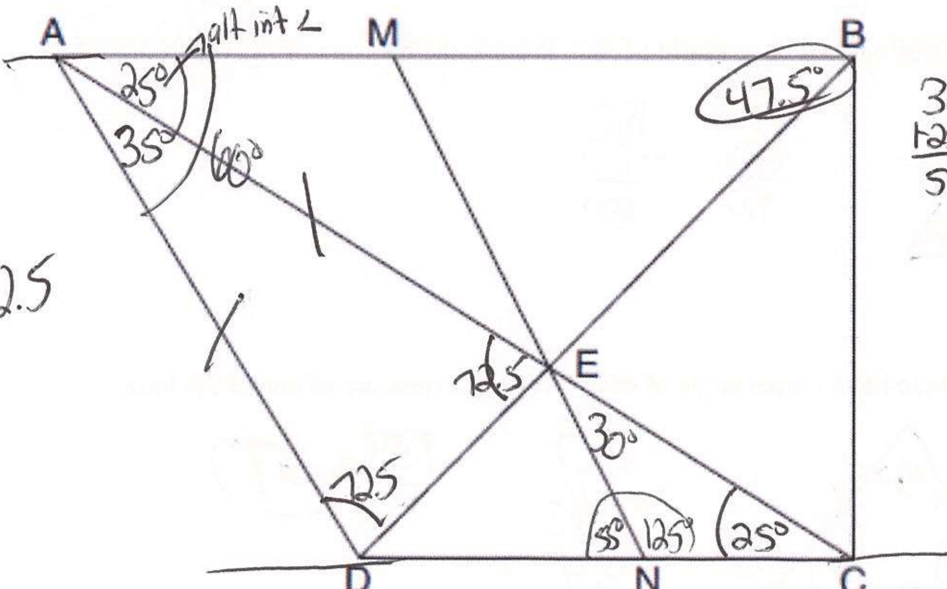
$$\begin{array}{r} 180 \\ -51 \\ \hline 129 \end{array} \quad \begin{array}{r} 73 \\ +28 \\ \hline 101 \end{array} \quad \begin{array}{r} 180 \\ -101 \\ \hline 79 \end{array}$$

$$\begin{array}{r} 90 \\ -62 \\ \hline 28 \end{array} \quad \begin{array}{r} 62 \\ +62 \\ \hline 124 \end{array} \quad \begin{array}{r} 180 \\ -124 \\ \hline 56 \end{array}$$

19. Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at E, and $\overline{AD} \cong \overline{AE}$. If $m\angle DAE = 35°$, $m\angle DCE = 25°$, and $m\angle NEC = 30°$, determine and state $m\angle ABD$.

$$\begin{array}{r} 180 \\ -35 \\ \hline 145 \end{array}$$

$$\frac{145}{2} = 72.5$$



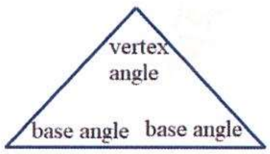
$$\begin{array}{r} 30 \\ +25 \\ \hline 55 \end{array} \quad \begin{array}{r} 180 \\ -55 \\ \hline 125 \end{array} \quad \begin{array}{r} 60 \\ +72.5 \\ \hline 132.5 \end{array} \quad \begin{array}{r} 180 \\ -132.5 \\ \hline 47.5 \end{array}$$

$$\begin{array}{r} 180 \\ -125 \\ \hline 55 \end{array}$$

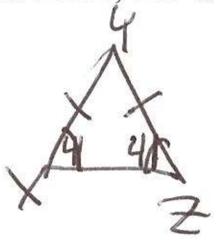
Isosceles Triangles with Vocabulary

Isosceles triangles have congruent sides opposite congruent angles. The congruent angles are called base angles and the non-congruent angle is called the vertex angle.

The angles of a triangle add to equal 180°.

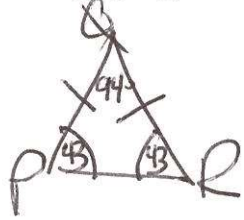


1. In ΔXYZ, $\angle X$ and $\angle Z$ are the base angles. If $m\angle Z = 41°$, find the measure of $\angle Y$.



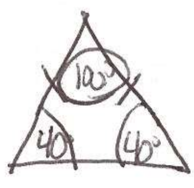
$41 + 41 = 82$ $\quad$ $180 - 82 = 98$

2. In ΔPQR, $\angle Q$ is the vertex angle. If $\angle Q = 94°$, find the measure of $\angle P$.



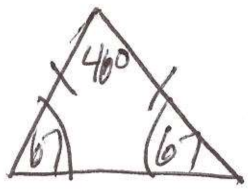
$180 - 94 = 86$ $\quad$ $\frac{86}{2} = 43$

3. An isosceles triangle has a base angle of 40°. What is the measure of the vertex angle?



$40 + 40 = 80$ $\quad$ $180 - 80 = 100$

4. An isosceles triangle has a vertex angle of 46°. What is the measure of one of the base angles?

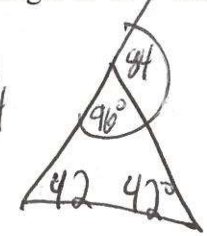


$180 - 46 = 134$ $\quad$ $\frac{134}{2} = 67°$

5. The measure of one of the base angles of an isosceles triangle is 42°. The measure of an exterior angle at the vertex of the triangle is

1) 42°
2) 84°
3) 96°
4) 138°

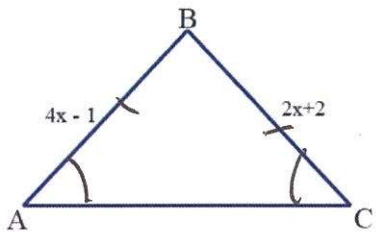
$180 - 96 = 84$



$42 + 42 = 84$

$180 - 84 = 96$

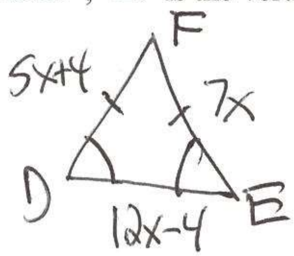
6. In ΔABC, $\angle A$ and $\angle C$ are the base angles. Find $\overline{BC}$



$4x-1=2x+2$
$-2x \quad -2x$
$2x-1=2$
$+1 \quad +1$
$\frac{2x}{2}=\frac{3}{2}$

$x=1.5$
$\overline{BC}=2x+2$
$\overline{BC}=2(1.5)+2$
$BC=5$

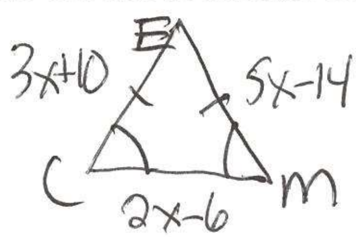
7. In ΔDEF, $\angle F$ is the vertex angle. If $\overline{DF}=5x+4$, $\overline{DE}=12x-4$, and $\overline{EF}=7x$, find $\overline{DE}$.



$5x+4=7x$
$-5x \quad -5x$
$\frac{4}{2}=\frac{2x}{2}$
$2=x$

$DE=12x-4$
$DE=12(2)-4$
$DE=20$

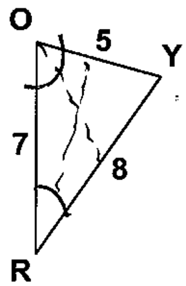
8. In triangle *CEM*, $CE = 3x + 10$, $ME = 5x - 14$, and $CM = 2x - 6$. Determine and state the value of *x* that would make *CEM* an isosceles triangle with the vertex angle at *E*.



$3x+10=5x-14$
$-3x \quad -3x$
$10=2x-14$
$+14 \quad +14$
$\frac{24}{2}=\frac{2x}{2}$
$12=x$

Largest/Smallest Sides/Angles in a Triangle
The largest side is opposite the largest angle
The smallest side is opposite the smallest angle

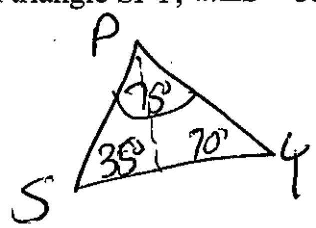
1. What is the largest angle of ΔROY? What is the smallest angle of ΔROY?



largest angle is $\angle O$

smallest angle is $\angle R$

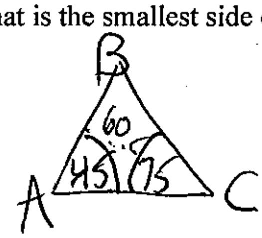
2. In triangle SPY, $m\angle S = 35°$ and $m\angle Y = 70°$. What is the largest side of triangle SPY?



$$\begin{array}{r} 35 \\ +70 \\ \hline 105 \end{array} \qquad \begin{array}{r} 180 \\ -105 \\ \hline 75 \end{array}$$

$\overline{SY}$

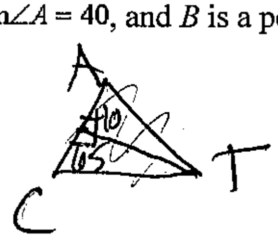
3. In ΔABC, $m\angle A = 45$ $m\angle B = 60$ and $m\angle C = 75$. What is the largest side of ΔABC?
What is the smallest side of ΔABC?

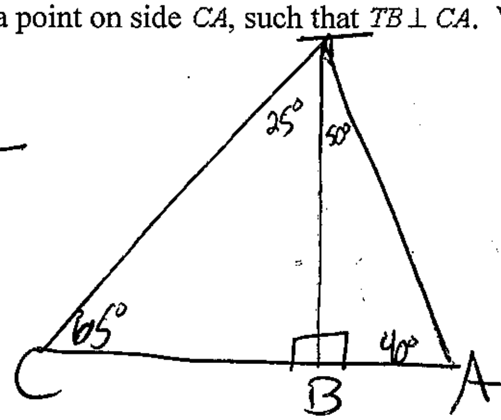


$\overline{AB}$ is largest
$\overline{BC}$ is smallest

4. In ΔCAT, $m\angle C = 65$, $m\angle A = 40$, and B is a point on side $\overline{CA}$, such that $\overline{TB} \perp \overline{CA}$. Which line segment is shortest?

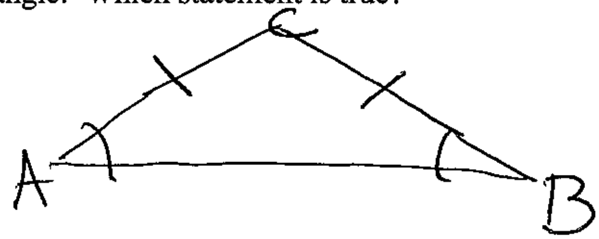
~~1)~~ $\overline{CT}$ largest of ΔCBT
2) $\overline{BC}$ smallest of ΔCBT
~~3)~~ $\overline{TB}$ middle of ΔCBT
~~4)~~ $\overline{AT}$ $\overline{AT} > \overline{TB}$
because of ΔABT





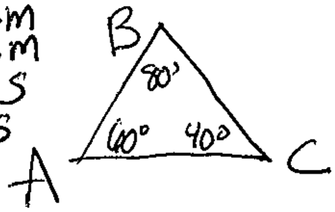
5. In $\triangle ABC$, $\angle A \cong \angle B$ and $\angle C$ is an obtuse angle. Which statement is true?

~~1)~~ $\overline{AC} \cong \overline{AB}$ and $\overline{BC}$ is the longest side.
✓2) $\overline{AC} \cong \overline{BC}$ and $\overline{AB}$ is the longest side.
~~3)~~ $\overline{AC} \cong \overline{AB}$ and $\overline{BC}$ is the shortest side.
4) $\overline{AC} \cong \overline{BC}$ and $\overline{AB}$ is the shortest side.



6. In $\triangle ABC$, $m\angle A = 60$, $m\angle B = 80$, and $m\angle C = 40$. Which inequality is true?

~~1)~~ $AB > BC$ S>M
✓2) $AC > BC$ L>M
x 3) $AC < BA$ L<S
x 4) $BC < BA$ M<S

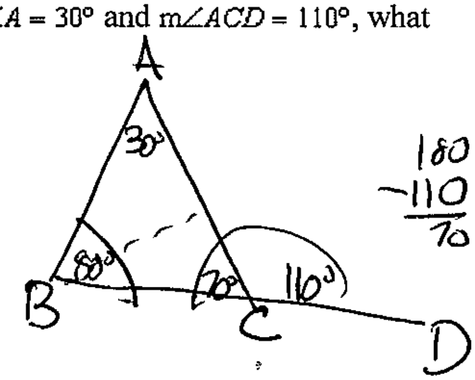


AB = small
BC = ~~median~~ middle
AC = largest

7. In $\triangle ABC$, side $\overline{BC}$ is extended through C to D. If $m\angle A = 30°$ and $m\angle ACD = 110°$, what is the longest side of $\triangle ABC$?

1) $\overline{AC}$
2) $\overline{BC}$
3) $\overline{AB}$
4) $\overline{CD}$

70 + 30 = 100
180 − 100 = 80°

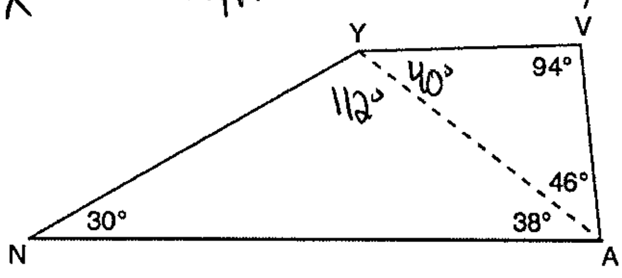


180 − 110 = 70

8. In the diagram of quadrilateral $NAVY$ below, $m\angle YNA = 30°$, $m\angle YAN = 38°$, $m\angle AVY = 94°$, and $m\angle VAY = 46°$.
Which segment has the shortest length?

~~1)~~ $\overline{AY}$ ~~BA~~ S△AYN (∠△AYV)
~~2)~~ $\overline{NY}$ M△AYN
3) $\overline{VA}$ S△AYV
~~4)~~ $\overline{VY}$ M△AYV

30 + 38 = 68
180 − 68 = 112



94 + 46 = 140
180 − 140 = 40

Euclidean Proofs:

If it is not specified, prove triangles are congruent
To prove triangles are congruent, prove 3 pairs of sides/angles are congruent
To prove segments or angles, use CPCTC
<u>*If you get stuck, make something up and keep on going!</u>

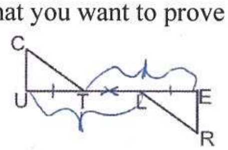
1) Do a mini proof with your givens

Altitude creates two congruent right angles
Median creates two congruent segments
Line bisector creates two congruent segments
Midpoint creates two congruent segments
Angle bisector creates two congruent angles
Perpendicular lines create two congruent right angles
Parallel lines cut by a transversal create
Congruent corresponding angles (1 in, 1 out) OR congruent alternate interior angles (2 out) OR congruent alternate exterior angles (2 out)
*Perpendicular bisector is perpendicular and line bisector (1 pair of congruent right angles, 1 pair of congruent segs)
*If segments bisect each other, they are both cut in half (2 pairs of congruent segments)

2) Use additional tools:

Vertical Angles are congruent (Look for an X)
Reflexive Property (A side/angle is in both triangles and is congruent to itself)
Isosceles Triangles (In a triangle, congruent angles are opposite congruent sides)
Addition and Subtraction Property (If you need more or less of a shared side)
*You must use three congruent statements to get one congruent statement for the triangles. The two that you are adding/subtracting and the one that you want to prove in the triangle.

7. Given: $\overline{UL} \cong \overline{TE}$
Prove: $\overline{UT} \cong \overline{LE}$



statements	Reasons
(1) $\overline{UL} \cong \overline{TE}$	(1) given
(2) $\overline{TL} \cong \overline{TL}$	(2) reflexive property
(3) $\overline{UT} \cong \overline{LE}$	(3) subtraction property

or
$\overline{UL} + \overline{TL} \cong \overline{TE} + \overline{TL}$

Parallelogram Theorems	Circle Theorems (Look for inscribed angles)
A parallelogram/rectangle/rhombus/square has: Two pairs of opposite sides congruent Two pairs of opposite sides parallel Diagonals that bisect each other Opposite angles congruent	**Angles inscribed to the same arc are congruent** **An angle inscribed to a semicircle is a right angle** **A tangent and a radius/diameter form a right angles** All radii/diameters of a circle are congruent Congruent arcs have congruent chords have congruent central angles Parallel Lines intercept congruent arcs Tangents drawn from the same point are congruent
A rectangle/square has: Congruent right angles Congruent diagonals	
A rhombus/square has: All sides congruent Perpendicular diagonals Diagonals that bisect the angles	

To prove triangles are SIMILAR, prove AA ≅ AA
If asked to prove a proportion/multiplication:

1) Prove triangles are similar
2) Corresponding Sides of Similar Triangle are In Proportion (CSSTIP)
3) Cross Products are Equal

Work Backwards!

(3) $\triangle AED \sim \triangle CEB$	(3) $AA \cong AA$
(4) $\frac{AE}{ED} = \frac{CE}{EB}$	(4) CSSTIP
(5) $AE \cdot EB = CE \cdot ED$	(5) cross products are equal

Euclidean Proofs (Basic)

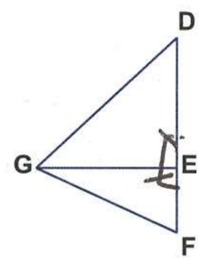
If it is not specified, prove triangles are congruent
To prove triangles are congruent, prove 3 pairs of sides/angles are congruent
To prove segments or angles, use CPCTC
<u>*If you get stuck, make something up and keep on going!</u>

1) Do a mini proof with your givens
Altitude creates congruent right angles
Median creates congruent segments
Line bisector creates congruent segments
Midpoint creates congruent segments
Angle bisector creates congruent angles
Perpendicular lines create congruent right angles
When given parallel lines:
Corresponding angles are congruent OR Alternate interior angles are congruent OR Alternate exterior angles are congruent

2) Use additional tools:
Vertical Angles are congruent (Look for an X)
Reflexive Property (A side/angle is congruent to itself)

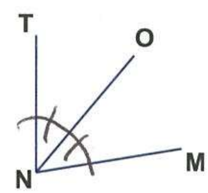
Mini Proofs

1. Given: $\overline{GE}$ is an altitude



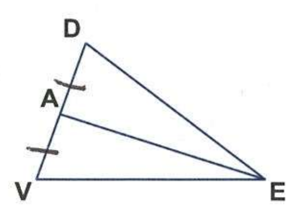
Statements	Reasons
(1) $\overline{GE}$ is an altitude	(1) given
(2) $\angle FEG \cong \angle DEG$	(2) An altitude creates congruent right angle

2. Given: $\overline{ON}$ bisects $\angle$TNM



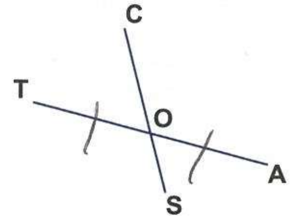
statements	Reasons
(2) $\angle TNO \cong \angle MNO$	(2) An angle bisector creates congruent angles
(1) $\overline{ON}$ bisects $\angle TNM$	(1) given

3. Given: A is the midpoint of $\overline{DV}$



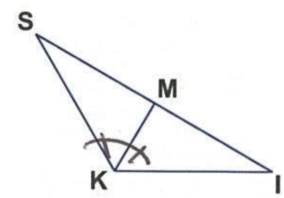
statements	Reasons
(1) A is the midpoint of $\overline{DV}$	(1) given
(2) $\overline{DA} \cong \overline{AV}$	(2) A midpoint creates congruent segments

4. Given: $\overline{CS}$ bisects $\overline{TA}$



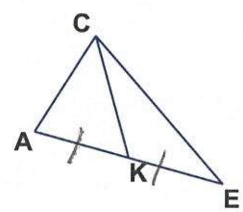
Statements	Reasons
① $\overline{CS}$ bisects $\overline{TA}$	① given
② $\overline{TO} \cong \overline{OA}$	② A line bisector creates congruent segments

5. Given: $\overline{KM}$ bisects $\angle$ SKI



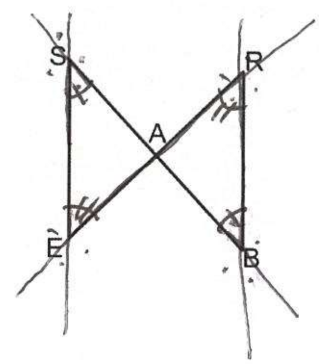
Statements	Reasons
① $\overline{KM}$ bisects $\angle SKI$	① given
② $\angle SKM \cong \angle IKM$	② An angle bisector creates two congruent angles.

6. $\overline{CK}$ is a median



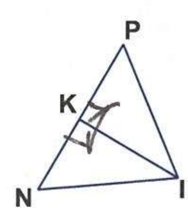
Statements	Reasons
① $\overline{CK}$ is a median	① given
② $\overline{AK} \cong \overline{KE}$	② A median creates congruent segments

7. Given: $\overline{SE} \parallel \overline{RB}$



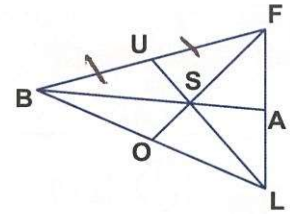
Statements	Reasons
① $\overline{SE} \parallel \overline{RB}$	① given
② $\angle S \cong \angle B$, $\angle R \cong \angle E$	② Parallel lines cut by a transversal create congruent alternate interior angles

8. Given: $\overline{IK} \perp \overline{PN}$



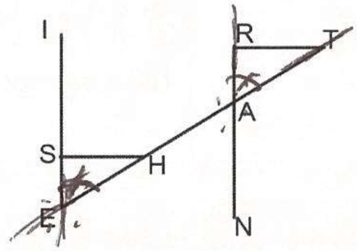
Statements	Reasons
② $\angle PKI \cong \angle NKI$	② Perpendicular lines form congruent right angles
① $\overline{IK} \perp \overline{PN}$	① given

9. Given: U is the midpoint of $\overline{BF}$



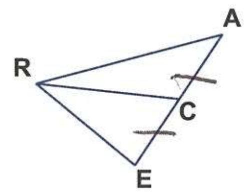
Statements	Reasons
① U is the midpoint of $\overline{BF}$	① given
② $\overline{BU} \cong \overline{UF}$	② A midpoint creates congruent segments

10. Given: $\overline{IE} \parallel \overline{RN}$



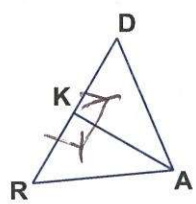
Statements	Reasons
① $\overline{IE} \parallel \overline{RN}$	① given
② $\angle SEH \cong \angle RAT$	② Parallel lines cut by a transversal create congruent corresponding angles

11. Given: C is the midpoint of $\overline{AE}$



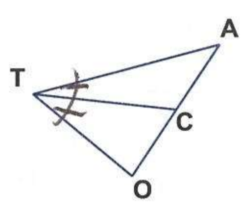
Statements	Reasons
① C is the midpoint of $\overline{AE}$	① given
② $\overline{AC} \cong \overline{CE}$	② A midpoint creates congruent segments

12. Given: $\overline{AK} \perp \overline{DR}$



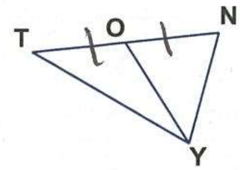
Statements	Reasons
① $\overline{AK} \perp \overline{DR}$	① given
② $\angle AKD \cong \angle AKR$	② Perpendicular lines form congruent right angles

13. Given: $\overline{CT}$ bisects $\angle$ATO



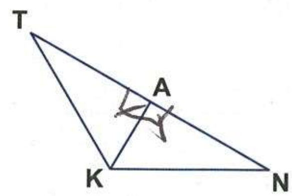
Statements	Reasons
① $\overline{CT}$ bisects $\angle ATO$	① given
② $\angle ATC \cong \angle OTC$	② An angle bisector creates congruent angles

14. Given: $\overline{YO}$ is a median

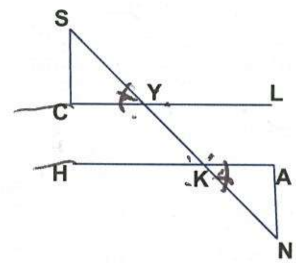


Statements	Reasons
① YO is a median	① given
② $\overline{TO} \cong \overline{ON}$	② A median creates congruent segments

15. Given: $\overline{KA}$ is an altitude



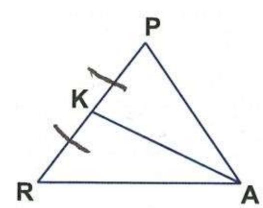
Statements	Reasons
① $\overline{KA}$ is an altitude	① given
② $\angle TAK \cong \angle NAK$	② An altitude creates congruent right angles



16. Given: $\overline{CL} \parallel \overline{HA}$

Statements	Reasons
① $\overline{CL} \parallel \overline{HA}$	① given
② $\angle SYC \cong \angle AKN$	② parallel lines cut by a transversal create congruent alternate exterior angles

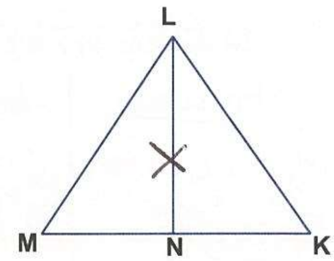
17. Given: $\overline{KA}$ bisects $\overline{PR}$



Statements	Reasons
① $\overline{KA}$ bisects $\overline{PR}$	① given
② $\overline{PK} \cong \overline{KR}$	② A line bisector creates congruent segments

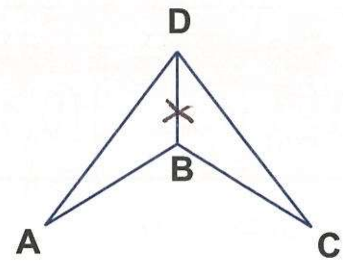
Reflexive Property and Vertical Angles

1. Given: None
   Prove: $\Delta LNM \cong \Delta LNK$



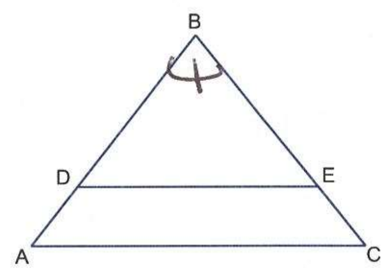
Statements	Reasons
(1) $\overline{LN} \cong \overline{LN}$	(1) Reflexive Property

2. Given: None
   Prove: $\Delta DBA \cong \Delta DBC$



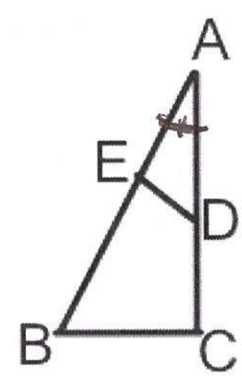
Statements	Reasons
(1) $\overline{DB} \cong \overline{DB}$	(1) Reflexive Property

3. Given: None
   Prove: $\Delta BDE \sim \Delta BAC$



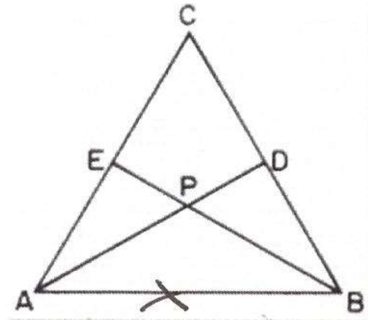
Statements	Reasons
(1) $\angle B \cong \angle B$	(1) Reflexive Property

4. Given: None
   Prove: $\Delta ABC \sim \Delta ADE$



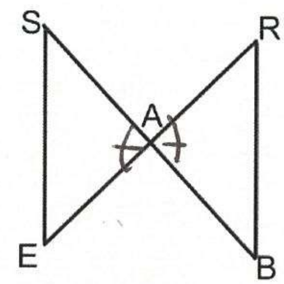
Statements	Reasons
(1) $\angle A \cong \angle A$	(1) Reflexive Property

5. Given: None
Prove: $\Delta AEB \cong \Delta BDA$



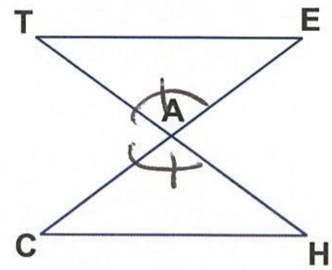
Statements	Reasons
① $\overline{AB} \cong \overline{AB}$	① Reflexive Property

6. Given: None
Prove: $\Delta SAE \cong \Delta RAB$



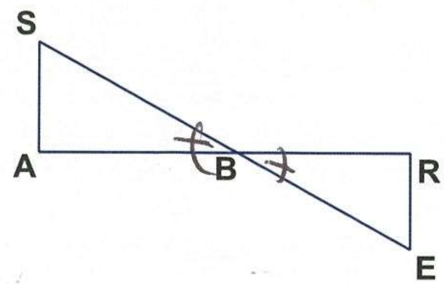
Statements	Reasons
① $\angle SAE \cong \angle RAB$	① vertical angles are congruent

7. Given: None
Prove: $\Delta TAE \cong \Delta CAH$



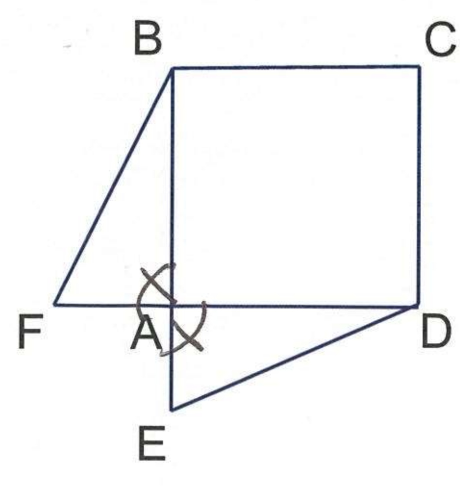
Statements	Reasons
① $\angle TAE \cong \angle CAH$	① vertical angles are congruent

8. Given: None
Prove: $\Delta SBA \cong \Delta EBR$



Statements	Reasons
① $\angle SBA \cong \angle RBE$	① vertical angles are congruent

9. Given: None
Prove: $\Delta BAF \cong \Delta DAE$



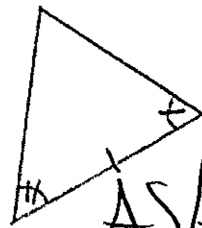
Statements	Reasons
① $\angle BAF \cong \angle DAE$	① vertical angles are congruent

Methods for Proving Triangles are Congruent

(6)

AAS

(7)



ASA

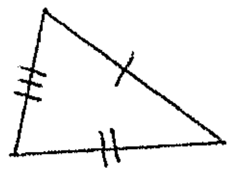
(8)

HL

(9)

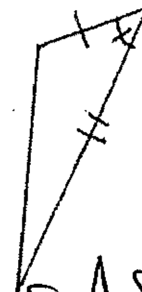
SAS

(10)



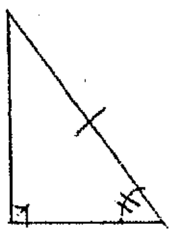
SSS

(11)



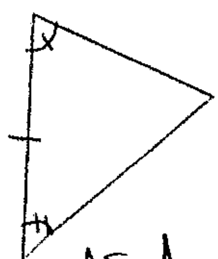
SAS

(12)



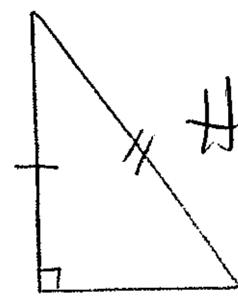
AAS

(13)



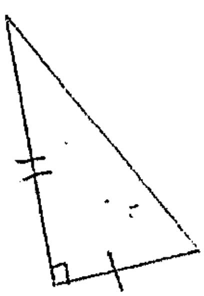
ASA

(14)



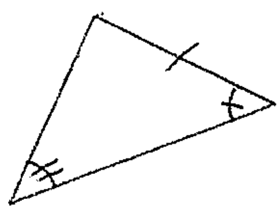
HL

(15)



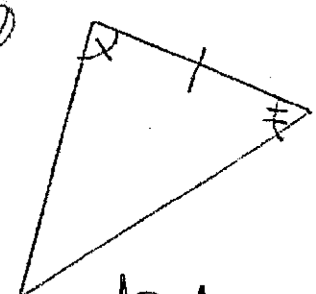
SAS

(16)



AAS

(17)

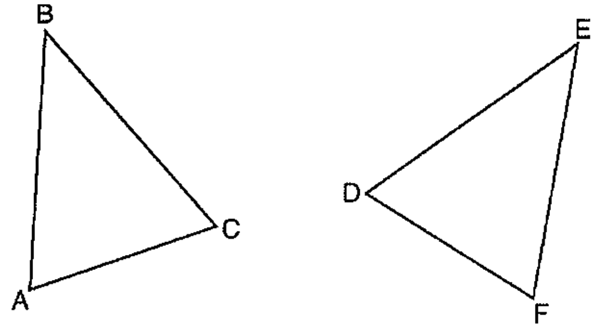


ASA

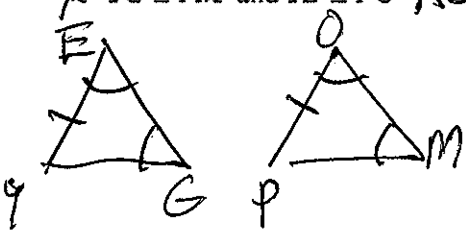
Congruent Triangle Methods with Sequences of Rigid Motions
If a sequence of rigid motions is performed, the image is CONGRUENT to the original!

1. Which statement is sufficient evidence that ΔDEF is congruent to ΔABC?
~~1)~~ $AB = DE$ and $BC = EF$ SS
~~2)~~ $\angle D \cong \angle A, \angle B \cong \angle E, \angle C \cong \angle F$ AAA
✓ 3) There is a sequence of rigid motions that maps $\overline{AB}$ onto $\overline{DE}$, $\overline{BC}$ onto $\overline{EF}$, and $\overline{AC}$ onto $\overline{DF}$. SSS
~~4)~~ There is a sequence of rigid motions that maps ~~point A~~ onto ~~point~~ D, $\overline{AB}$ onto $\overline{DE}$, and $\angle B$ onto $\angle E$. SA

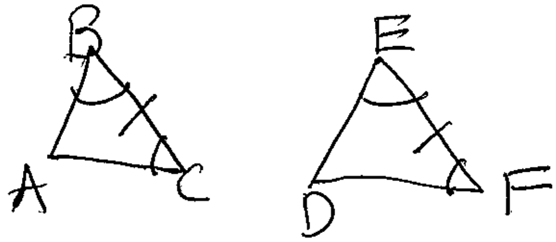
this is nothing



2. Triangles *YEG* and *POM* are two distinct non-right triangles such that $\angle G \cong \angle M$. Which statement is sufficient to prove $\triangle YEG$ is always congruent to $\triangle POM$?
~~1)~~ $\angle E \cong \angle O$ and $\angle Y \cong \angle P$ AAA
~~2)~~ $\overline{YG} \cong \overline{PM}$ and $\overline{YE} \cong \overline{PO}$ ASS
3) There is a sequence of rigid motions that maps $\angle E$ onto $\angle O$ and $\overline{YE}$ onto $\overline{PO}$. AAS
~~4)~~ There is a sequence of rigid motions that maps ~~point Y onto point~~ P and $\overline{YG}$ onto $\overline{PM}$. SA

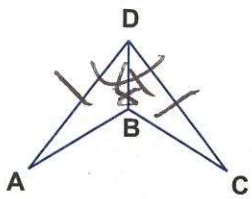


3. In the two distinct acute triangles *ABC* and *DEF*, $\angle B \cong \angle E$. Triangles *ABC* and *DEF* are congruent when there is a sequence of rigid motions that maps
1) $\angle A$ onto $\angle D$, and $\angle C$ onto $\angle F$ AAA
2) $\overline{AC}$ onto $\overline{DF}$, and $\overline{BC}$ onto $\overline{EF}$ ASS
3) $\angle C$ onto $\angle F$, and $\overline{BC}$ onto $\overline{EF}$ ASA
4) poi~~nt A onto point~~ D, and $\overline{AB}$ onto $\overline{DE}$ AS



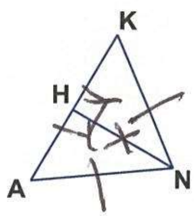
Proving Triangles are Congruent

1. Given: $\overline{BD}$ bisects $\angle ADC$
$\overline{AD} \cong \overline{DC}$
Prove: $\overline{AB} \cong \overline{BC}$



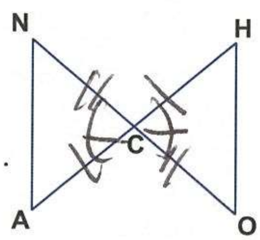
Statements	Reasons
(1) $\overline{BD}$ bisects ∠ADC	(1) given
(2) ∠ADB ≅ ∠CDB	(2) An angle bisector creates congruent angles
(3) $\overline{AD} \cong \overline{DC}$	(3) given
(4) $\overline{DB} \cong \overline{DB}$	(4) Reflexive Property
(5) △ADB ≅ △CDB	(5) SAS
(6) AB ≅ BC	(6) CPCTC

2. Given: $\overline{HN} \perp \overline{KA}$, $\overline{KN} \cong \overline{AN}$
Prove: $\angle HAN \cong \angle HKN$



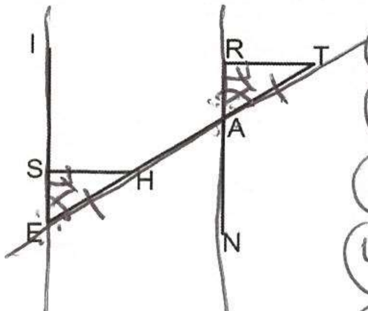
Statements	Reasons
(1) $\overline{HN} \perp \overline{KA}$	(1) given
(2) ∠KHN ≅ ∠AHN	(2) Perpendicular lines form congruent right angles
(3) $\overline{KN} \cong \overline{AN}$	(3) given
(4) $\overline{HN} \cong \overline{HN}$	(4) Reflexive Property
(5) △AHN ≅ △KHN	(5) HL
(6) ∠HAN ≅ ∠HKN	(6) CPCTC

3. Given: $\overline{NO}$ and $\overline{HA}$ bisect each other
Prove: $\overline{NA} \cong \overline{HO}$



Statements	Reasons
(1) $\overline{NO}$ and $\overline{HA}$ bisect each other	(1) given
(2) $\overline{AC} \cong \overline{CH}$, $\overline{NC} \cong \overline{CO}$	(2) A line bisector creates congruent segments
(3) ∠NCA ≅ ∠HCO	(3) vertical angles are congruent
(4) △ANC ≅ △HOC	(4) SAS
(5) $\overline{NA} \cong \overline{HO}$	(5) CPCTC

4. Given: $\overline{IE} \parallel \overline{RN}$, $\overline{TR} \perp \overline{RN}$, $\overline{HS} \perp \overline{IE}$, $\overline{EH} \cong \overline{AT}$
Prove: $\overline{SH} \cong \overline{RT}$



Statements	Reasons
(1) $\overline{IE} \parallel \overline{RN}$	(1) given
(2) ∠TAR ≅ ∠HES	(2) Parallel lines cut by a transversal create congruent corresponding angles
(3) $\overline{TR} \perp \overline{RN}$, $\overline{HS} \perp \overline{IE}$	(3) given
(4) ∠TRA ≅ ∠HSE	(4) Perpendicular lines form congruent right angles
(5) ~~△HS~~ $\overline{EH} \cong \overline{AT}$	(5) given
(6) △HSE ≅ △TRA	(6) AAS
(7) SH ≅ RT	(7) CPCTC

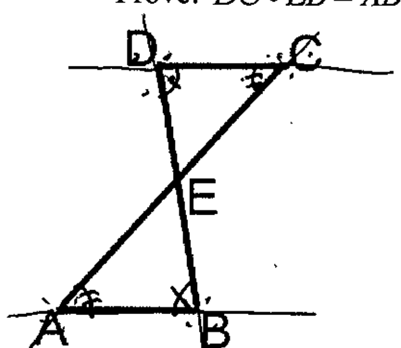
To prove triangles are SIMILAR, prove AA $\cong$ AA
If asked to prove a proportion/multiplication:

1) **Prove triangles are similar**
2) **Corresponding Sides of Similar Triangle are In Proportion (CSSTIP)**
3) **Cross Products are Equal**

Work Backwards!

1. Given $\overline{AB} \parallel \overline{DC}$

   Prove: $\overline{DC} \bullet \overline{EB} = \overline{AB} \bullet \overline{DE}$

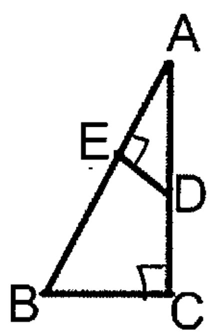


Statements	Reasons
(1) $\overline{AB} \parallel \overline{DC}$	(1) given
(2) $\angle D \cong \angle B$, $\angle C \cong \angle A$	(2) Parallel lines cut by a transversal create congruent alternate interior angles
(3) $\triangle DCE \sim \triangle BAE$	(3) AA $\cong$ AA
(4) $\frac{\overline{DC}}{\overline{DE}} = \frac{\overline{AB}}{\overline{EB}}$	(4) CSSTIP
(5) $\overline{DC} \cdot \overline{EB} = \overline{AB} \cdot \overline{DE}$	(5) Cross products are equal

2. Given: $\overline{BC} \perp \overline{AC}$

   $\overline{DE} \perp \overline{AB}$

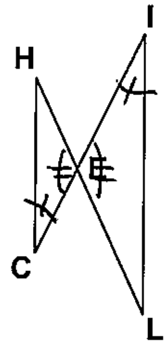
   Prove: $\overline{AC} \bullet \overline{AD} = \overline{AE} \bullet \overline{AB}$



Statements	Reasons
(1) $\overline{BC} \perp \overline{AC}$, $\overline{DE} \perp \overline{AB}$	(1) given
(2) $\angle ACB \cong \angle AED$	(2) Perpendicular lines form congruent right angles
(3) $\angle A \cong \angle A$	(3) Reflexive Property
(4) $\triangle ACB \sim \triangle AED$	(4) AA $\cong$ AA
(5) $\frac{\overline{AC}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AD}}$	(5) CSSTIP
(6) $\overline{AC} \cdot \overline{AD} = \overline{AE} \cdot \overline{AB}$	(6) Cross products are equal

3. Given: $\angle HCE \cong \angle LIE$

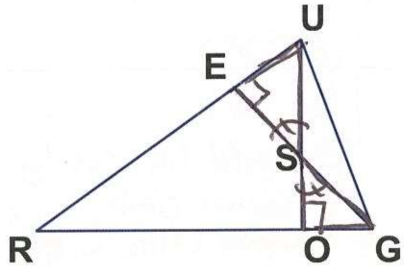
   Prove: $\overline{CE} \bullet \overline{IL} = \overline{CH} \bullet \overline{EI}$



Statements	Reasons
(1) $\angle HCE \cong \angle LIE$	(1) given
(2) $\angle HEC \cong \angle IEL$	(2) vertical angles are congruent
(3) $\triangle HCE \sim \triangle LIE$	(3) AA $\cong$ AA
(4) $\frac{\overline{CE}}{\overline{EI}} = \frac{\overline{CH}}{\overline{IL}}$	(4) CSSTIP
(5) $\overline{CE} \cdot \overline{IL} = \overline{CH} \cdot \overline{EI}$	(5) Cross products are equal

4. Given: $\overline{UO} \perp \overline{RG}$, $\overline{UR} \perp \overline{EG}$

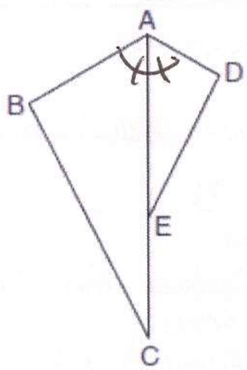
Prove: $\dfrac{\overline{US}}{\overline{SO}} = \dfrac{\overline{EU}}{\overline{OG}}$



Statements	Reasons
① $\overline{UO} \perp \overline{RG}, \overline{UR} \perp \overline{EG}$	① given
② $\angle UES \cong \angle SOG$	② Perpendicular lines form congruent right angles
③ $\angle USE \cong \angle GSO$	③ vertical angles are congruent
④ $\triangle USE \sim \triangle GSO$	④ AA≅AA
⑤ $\dfrac{\overline{US}}{\overline{SO}} = \dfrac{\overline{EU}}{\overline{OG}}$	⑤ CSSTIP

5. Given: $\overline{CA}$ bisects $\angle BAD$, $\angle ABC \cong \angle ADE$

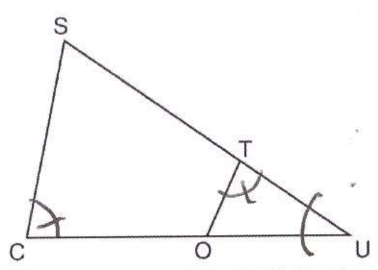
Prove: $\overline{BC} \bullet \overline{AE} = \overline{DE} \bullet \overline{AC}$



Statements	Reasons
① $\overline{CA}$ bisects $\angle BAD$	① given
② $\angle BAC \cong \angle DAE$	② An angle bisector creates 2 ≅ angles
③ $\angle ABC \cong \angle ADE$	③ given
④ $\triangle BAC \sim \triangle DAE$	④ AA≅AA
⑤ $\dfrac{\overline{BC}}{\overline{AC}} = \dfrac{\overline{DE}}{\overline{AE}}$	⑤ CSSTIP
⑥ $\overline{BC} \cdot \overline{AE} = \overline{DE} \cdot \overline{AC}$	⑥ cross products are equal

6. In $\triangle SCU$ shown below, points T and O are on $\overline{SU}$ and $\overline{CU}$, respectively. Segment OT is drawn so that $\angle C \cong \angle OTU$.

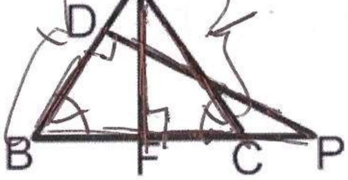
Prove: $\overline{SC} \bullet \overline{OU} = \overline{OT} \bullet \overline{SU}$



Statements	Reasons
① $\angle C \cong \angle OTU$	① given
② $\angle U \cong \angle U$	② Reflexive Property
③ $\triangle SCU \sim \triangle OTU$	③ AA≅AA
④ $\dfrac{\overline{SC}}{\overline{SU}} = \dfrac{\overline{OT}}{\overline{OU}}$	④ CSSTIP
⑤ $\overline{SC} \cdot \overline{OU} = \overline{OT} \cdot \overline{SU}$	⑤ cross products are equal

7. Given: $\overline{AB} \cong \overline{AC}$, $\overline{AF} \perp \overline{BC}$, $\overline{PD} \perp \overline{AB}$

Prove: $\overline{FC} \bullet \overline{PB} = \overline{DB} \bullet \overline{AC}$



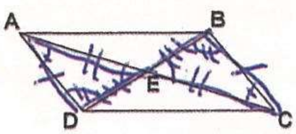
Statements	Reasons
① $\overline{AB} \cong \overline{AC}$	① given
② $\angle B \cong \angle C$	② In a triangle, congruent angles are opposite congruent sides
③ $\overline{AF} \perp \overline{BC}, \overline{PD} \perp \overline{AB}$	③ given
④ $\angle AFC \cong \angle PDB$	④ Perpendicular lines form congruent right angles
⑤ $\triangle FCA \sim \triangle DBP$	⑤ AA≅AA
⑥ $\dfrac{\overline{FC}}{\overline{AC}} = \dfrac{\overline{DB}}{\overline{PB}}$	⑥ CSSTIP
⑦ $\overline{FC} \cdot \overline{PB} = \overline{DB} \cdot \overline{AC}$	⑦ cross products are equal

Euclidean Proofs with Parallelogram and Circle Theorems

Parallelogram Theorems	**Circle Theorems**
A parallelogram/rectangle/rhombus/square has: Two pairs of opposite sides congruent Two pairs of opposite sides parallel Diagonals that bisect each other Opposite angles congruent **A rectangle/square has:** A right angle Congruent diagonals **A rhombus/square has:** All sides congruent Perpendicular diagonals Diagonals that bisect the angles	**All radii/diameters of a circle are congruent** **Angles inscribed to the same arc are congruent** **An angle inscribed to a semicircle is a right angle** **A tangent and a radius/diameter form a right angles** Congruent arcs have congruent chords have congruent central angles Parallel Lines intercept congruent arcs Tangents drawn from the same point are congruent

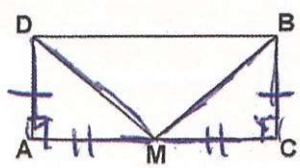
I'm giving you all 6 even though you only need 3

1. Given: ABCD is a parallelogram
Prove: $\Delta AED \cong \Delta CEB$



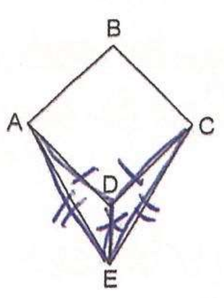
Statements	Reasons
① ABCD is a parallelogram	① given
② $\overline{AD} \cong \overline{BC}$	② A p-gram has opposite sides ≅
③ $\overline{AE} \cong \overline{EC}$, $\overline{BE} \cong \overline{ED}$	③ A p-gram has diagonals that bisect each other
④ $\angle EAD \cong \angle BCE$, $\angle ADE \cong \angle CBE$	④ A p-gram has ≅ alternate interior angles
⑤ $\angle AED \cong \angle BEC$	⑤ vertical angles are congruent
⑥ $\Delta AED \cong \Delta CEB$	⑥ SSS/SAS/ASA/AAS depending on which three you did

2. Given: ABCD is a rectangle, M is the midpoint of $\overline{AC}$
Prove: $\overline{DM} \cong \overline{BM}$



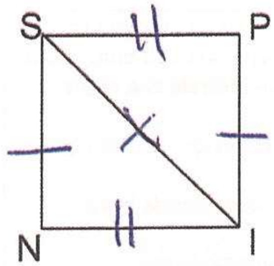
Statements	Reasons
① ABCD is a rectangle	① given
② $\overline{DA} \cong \overline{BC}$	② A rectangle has 2 pairs of opp sides ≅
③ $\angle DAM \cong \angle BCM$	③ A rectangle has ≅ right angles
④ M is midpoint of $\overline{AC}$	④ given
⑤ $\overline{AM} \cong \overline{MC}$	⑤ A midpoint creates 2 ≅ segments
⑥ $\Delta DAM \cong \Delta BCM$	⑥ SAS ≅ SAS
⑦ $\overline{DM} \cong \overline{BM}$	⑦ CPCTC

3. Given: ABCD is a rhombus, $\overline{AE} \cong \overline{CE}$
Prove: $\angle ADE \cong \angle CDE$



Statements	Reasons
① ABCD is a rhombus	① given
② $\overline{AD} \cong \overline{DC}$	② A rhombus has all sides ≅
③ $\overline{AE} \cong \overline{CE}$	③ given
④ $\overline{DE} \cong \overline{DE}$	④ Reflexive Property
⑤ $\Delta ADE \cong \Delta CDE$	⑤ SSS ≅ SSS
⑥ $\angle ADE \cong \angle CDE$	⑥ CPCTC

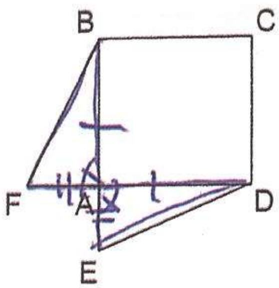
4. Given: SPIN is a square
Prove: $\Delta SNI \cong \Delta SPI$



Statements	Reasons
① SPIN is a square	① given
② $\overline{SN} \cong \overline{PI}$, $\overline{SP} \cong \overline{NI}$	② A square has ≅ opposite sides
③ $\overline{SI} \cong \overline{SI}$	③ Reflexive Property
④ $\Delta SNI \cong \Delta SPI$	④ SSS ≅ SSS

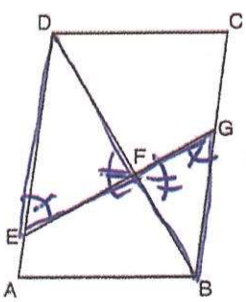
*Also right angles and alternate interior angles

5. Given: ABCD is a square, $\overline{FA} \cong \overline{AE}$
Prove: $\overline{BF} \cong \overline{DE}$



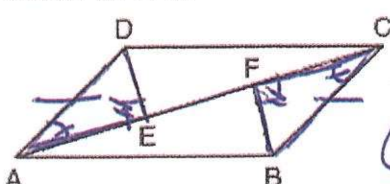
Statements	Reasons
① ABCD is a square	① given
② $\overline{BA} \cong \overline{AD}$	② A square has all sides ≅
③ $\overline{FA} \cong \overline{AE}$	③ given
④ $\angle FAB \cong \angle DAE$	④ vertical angles are congruent
⑤ $\Delta BAF \cong \Delta DAE$	⑤ SAS ≅ SAS
⑥ $\overline{BF} \cong \overline{DE}$	⑥ CPCTC

6. Given: Parallelogram *ABCD*, $\overline{EFG}$, and diagonal $\overline{DFB}$
Prove: $\triangle DEF \sim \triangle BGF$



Statements	Reasons
① Parallelogram ABCD	① given
② $\angle DEF \cong \angle BGF$	② A parallelogram has ≅ alternate interior angles
③ $\angle DFE \cong \angle BFG$	③ vertical angles are congruent
④ $\Delta DEF \sim \Delta BGF$	④ AA ≅ AA

7. In parallelogram *ABCD*, $\overline{BF}$ and $\overline{DE}$ are perpendicular to diagonal $\overline{AC}$ at points *F* and *E*.
Prove: $\overline{AE} \cong \overline{CF}$



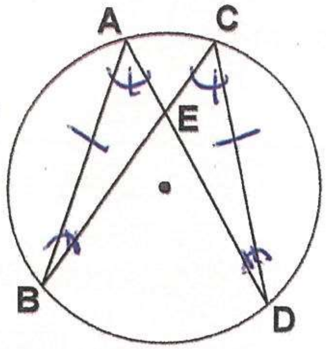
Statements	Reasons
① Parallelogram ABCD	① given
② $\overline{AD} \cong \overline{BC}$	② A parallelogram has opposite sides ≅
③ $\angle DAE \cong \angle BCF$	③ A parallelogram has congruent alternate interior angles
④ $\overline{BF}$ and $\overline{DE}$ are perpendicular to $\overline{AC}$	④ given
⑤ $\angle DEA \cong \angle CFB$	⑤ perpendicular lines create ≅ right angles
⑥ $\Delta DEA \cong \Delta CEB$	⑥ AAS ≅ AAS
⑦ $\overline{AE} \cong \overline{CF}$	⑦ CPCTC

*Look for inscribed angles

8. Given: Chords $\overline{AD}$ and $\overline{BC}$ of circle O intersect at E, $\overline{AB} \cong \overline{CD}$

Prove: $\overline{BC} \cong \overline{AD}$

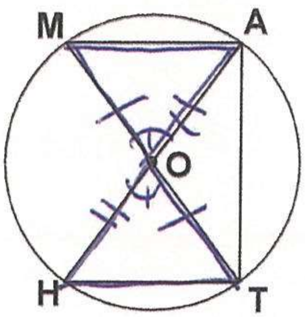
*You could have also used vertical angles



Statements	Reasons
① $\overline{AB} \cong \overline{CD}$	① given
② $\angle BAE \cong \angle DCE$, $\angle ABE \cong \angle CDE$	② Angles inscribed to the same arc are congruent
③ $\triangle BAE \cong \triangle DCE$	③ ASA ≅ ASA
④ $\overline{BC} \cong \overline{AD}$	④ CPCTC

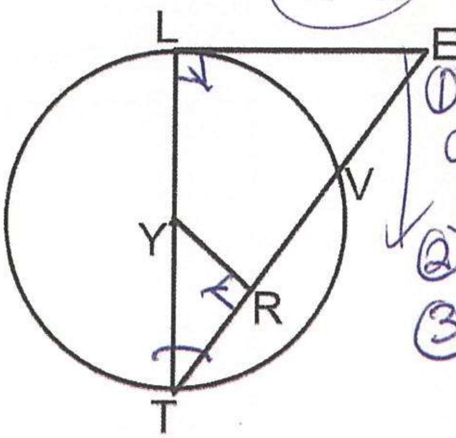
9. Given: Circle O with diameters $\overline{MOT}$ and $\overline{AOH}$.

Prove: $\overline{MA} \cong \overline{HT}$



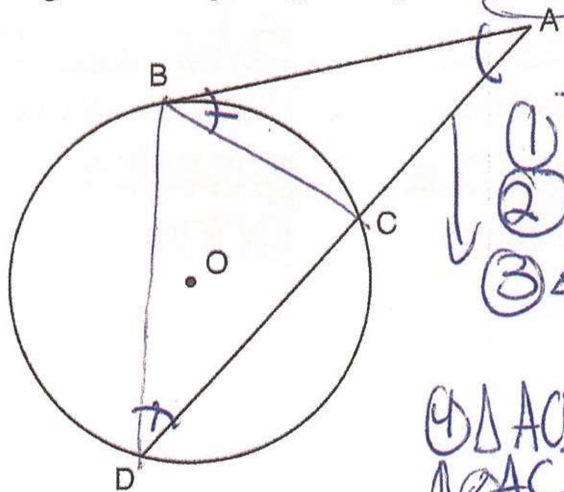
Statements	Reasons
① $\overline{MO} \cong \overline{OT}$, $\overline{AO} \cong \overline{OH}$	① All radii of a circle are ≅
② $\angle MOA \cong \angle HOT$	② vertical angles are ≅
③ $\triangle MOA \cong \triangle TOH$	③ SAS ≅ SAS
④ $\overline{MA} \cong \overline{HT}$	④ CPCTC

10. In circle Y, tangent $\overline{LE}$ is drawn to diameter $\overline{TYL}$ and $\overline{YR} \perp \overline{TE}$. Prove that $\frac{TE}{TY} = \frac{TL}{TR}$. work backwards



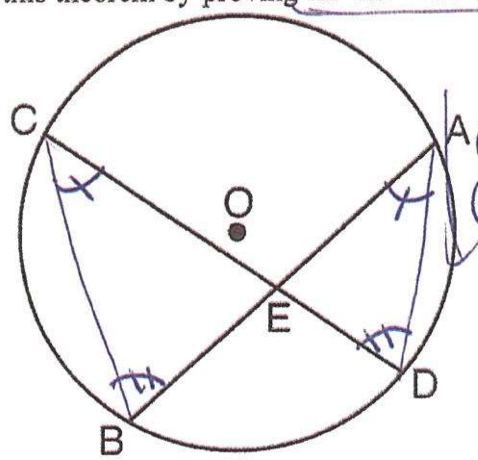
Statements	Reasons
① Tangent LE is drawn to diameter TYL, $YR \perp TE$	① given
② $\angle TLE \cong \angle TRY$	② An angle formed by a tangent and diameter and perpendicular lines form congruent right angles
③ $\angle RTY \cong \angle RTY$	③ Reflexive Property
④ $\triangle TLE \sim \triangle TRY$	④ AA ≅ AA
⑤ $\frac{TE}{TY} = \frac{TL}{TR}$	⑤ CSSTIP

11. In the diagram below, secant $\overline{ACD}$ and tangent $\overline{AB}$ are drawn from external point A to circle O.
Prove the theorem: If a secant and a tangent are drawn to a circle from an external point, the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared. ($AC \cdot AD = AB^2$) → work backwards



Statements	Reasons
(1) $\overline{BC}, \overline{BD}$	(1) Auxilary lines can be drawn
(2) $\angle BAC \cong \angle BAC$	(2) Reflexive Property
(3) $\angle BDC \cong \angle ABC$	(3) Angles inscribed to the same arc are $\cong$
(4) $\triangle ACB \sim \triangle ABD$	(4) AA≅AA
(5) $\frac{AC}{AB} = \frac{AB}{AD}$	(5) CSSTP
(6) $AC \cdot AD = AB^2$	(6) Cross products are equal

12. Given: Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E
Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. Prove this theorem by proving $AE \cdot EB = CE \cdot ED$. → work backwards



Statements	Reasons
(1) $\overline{CB}, \overline{AD}$	(1) Auxilary lines can be drawn
(2) $\angle BCE \cong \angle DAE$, $\angle CBE \cong \angle ADE$	(2) Angles inscribed to the same arc are $\cong$
(3) $\triangle AED \sim \triangle CEB$	(3) AA≅AA
(4) $\frac{AE}{ED} = \frac{CE}{EB}$	(4) CSSTP
(5) $AE \cdot EB = CE \cdot ED$	(5) Cross products are equal

*you could have also used vertical angles

Isosceles Triangle Theorem Mini Proofs

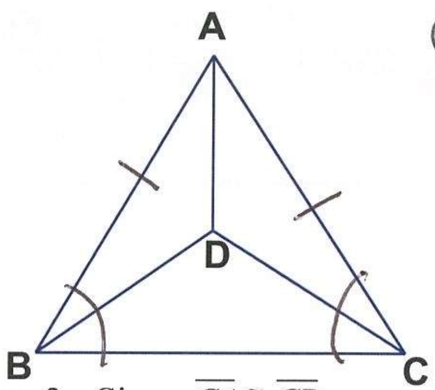
In a triangle, congruent angles are opposite congruent sides

If the given sides/angles are not sides/angles of the triangles you are trying to prove, check to see if they make an isosceles triangle. Conclude the sides/angles opposite the ones you are given.

1. Given: $\angle ABC \cong \angle ACB$

   Prove: $\Delta ADB \cong \Delta ADC$

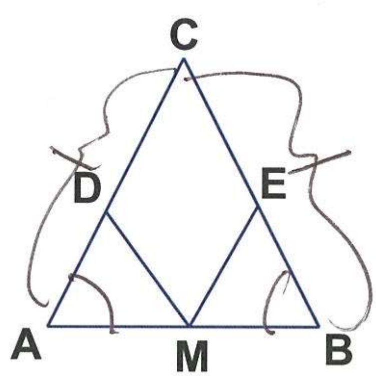
statements	Reasons
① $\angle ABC \cong \angle ACB$	① given
② $\overline{AB} \cong \overline{AC}$	② In a triangle, congruent angles are opposite congruent sides



2. Given: $\overline{CA} \cong \overline{CB}$

   Prove: $\Delta ADM \cong \Delta BEM$

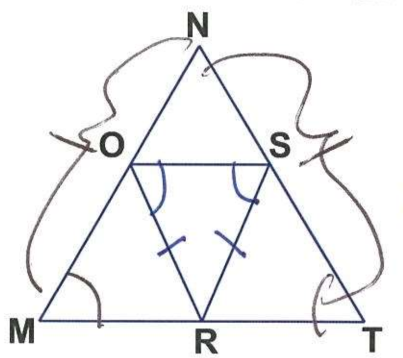
statements	Reasons
① $\overline{CA} \cong \overline{CB}$	① given
② $\angle A \cong \angle B$	② In a triangle, congruent angles are opposite congruent sides



3. Given: $\overline{MN} \cong \overline{NT}$, $\angle ROS \cong \angle RSO$

   Prove: $\Delta MOR \cong \Delta TSR$

statements	Reasons
① $\overline{MN} \cong \overline{NT}$	① given
② $\angle M \cong \angle T$	② In a triangle, congruent angles are opposite congruent sides
③ $\angle ROS \cong \angle RSO$	③ given
④ $\overline{RO} \cong \overline{RS}$	④ In a triangle, congruent angles are opposite congruent sides

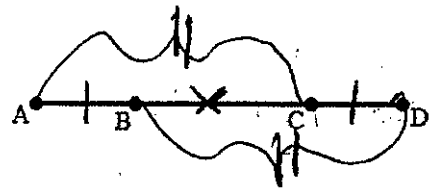


Name Schlansky
Mr. Schlansky

Date ______________
Geometry

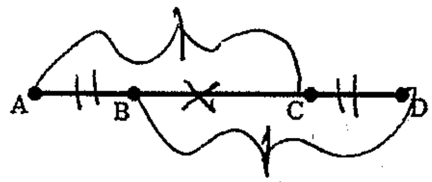
Addition and Subtraction Property Mini Proofs

1. Given: $\overline{AB} \cong \overline{CD}$
Prove: $\overline{AC} \cong \overline{BD}$



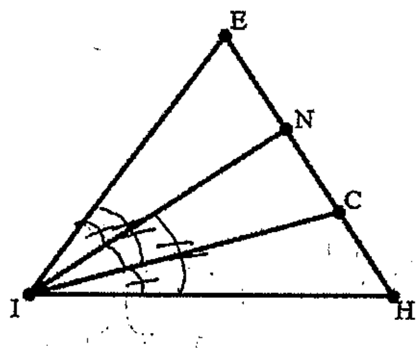
statements	Reasons
① $\overline{AB} \cong \overline{CD}$	① given
② $\overline{BC} \cong \overline{BC}$	② reflexive property
③ $\overline{AC} \cong \overline{BD}$ or $\overline{AB}+\overline{BC}=\overline{CD}+\overline{BC}$	③ Addition Property

2. Given: $\overline{AC} \cong \overline{BD}$
Prove: $\overline{AB} \cong \overline{CD}$



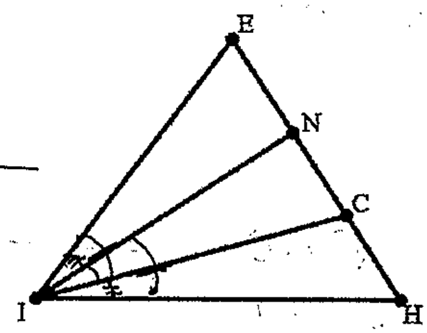
statements	Reasons
① $\overline{AC} \cong \overline{BD}$	① given
② $\overline{BC} \cong \overline{BC}$	② reflexive property
③ $\overline{AB} \cong \overline{CD}$ or $\overline{AC}-\overline{BC}=\overline{BD}-\overline{BC}$	③ Subtraction property

3. Given: $\angle EIN \cong \angle HIC$
Prove: $\angle EIC \cong \angle HIN$



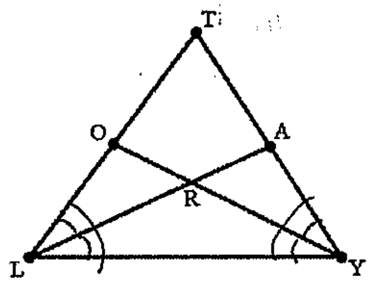
Statements	Reasons
① $\angle EIN \cong \angle HIC$	① given
② $\angle NIC \cong \angle NIC$	② reflexive property
③ $\angle EIC \cong \angle HIN$ or $\angle EIN+\angle NIC \cong \angle HIC+\angle NIC$	③ Addition Property

4. Given: $\angle EIC \cong \angle HIN$
Prove: $\angle EIN \cong \angle HIC$



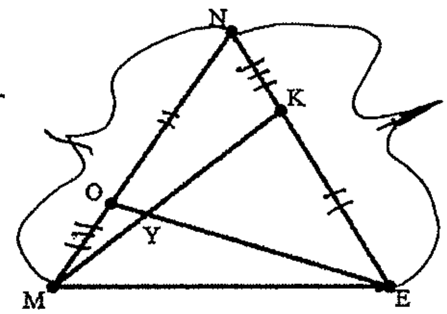
Statements	Reasons
① $\angle EIC \cong \angle HIN$	① given
② $\angle NIC \cong \angle NIC$	② reflexive property
③ $\angle EIN \cong \angle HIC$ or $\angle EIC-\angle NIC=\angle HIN-\angle NIC$	③ Subtraction property

5. Given: $\angle TLA \cong \angle TYO$, $\angle ALY \cong \angle OYL$
Prove: $\angle TLY \cong \angle TYL$



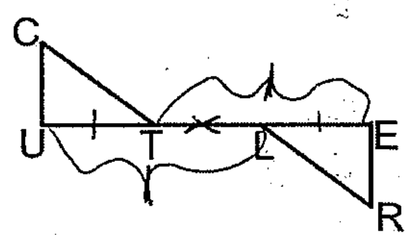
Statements	Reasons
1 $\angle TLA \cong \angle TYO$	1 given
2 $\angle ALY \cong \angle OYL$	2 given
3 $\angle TLY \cong \angle TYL$ or $\angle TLA + \angle ALY \cong \angle TYO + \angle OYL$	3 Addition Property

6. Given: $\overline{MN} \cong \overline{NE}$, $\overline{ON} \cong \overline{KE}$
Prove: $\overline{MO} \cong \overline{KN}$



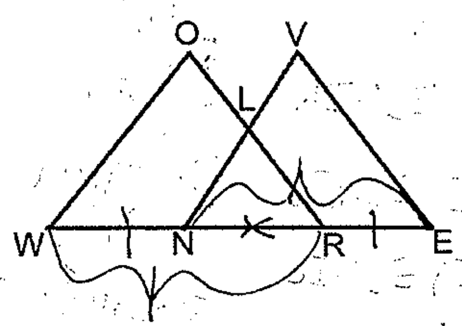
statements	Reasons
1 $\overline{MN} \cong \overline{NE}$	1 given
2 $\overline{ON} \cong \overline{KE}$	2 given
3 $\overline{MO} \cong \overline{KN}$ or $\overline{MN} - \overline{ON} = \overline{NE} - \overline{KE}$	3 subtraction property

7. Given: $\overline{UL} \cong \overline{TE}$
Prove: $\overline{UT} \cong \overline{LE}$



statements	Reasons
1 $\overline{UL} \cong \overline{TE}$	1 given
2 $\overline{TL} \cong \overline{TL}$	2 reflexive property
3 $\overline{UT} \cong \overline{LE}$ or $\overline{UL} - \overline{TL} = \overline{TE} - \overline{TL}$	3 subtraction property

8. Given: $\overline{WN} \cong \overline{RE}$
Prove: $\overline{WR} \cong \overline{NE}$



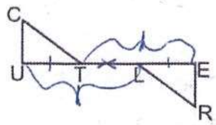
statements	Reasons
1 $\overline{WN} \cong \overline{RE}$	1 given
2 $NR \cong NR$	2 reflexive property
3 $\overline{WR} \cong \overline{NE}$ or $\overline{WN} + \overline{NR} = \overline{RE} + \overline{NR}$	3 addition property

Euclidean Triangle Proofs with Additional Tools

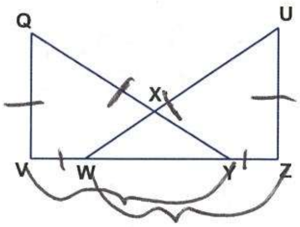
Vertical Angles are congruent (Look for an X)
Reflexive Property (A side/angle is congruent to itself)
Isosceles Triangles (In a triangle, congruent angles are opposite congruent sides)
Addition and Subtraction Property (If you need more or less of a shared side)
*You must use three congruent statements to get one congruent statement for the triangles. The two that you are adding/subtracting and the one that you want to prove in the triangle.

7. Given: $\overline{UL} \cong \overline{TE}$
Prove: $\overline{UT} \cong \overline{LE}$

statements	Reasons
① $\overline{UL} \cong \overline{TE}$	① given
② $\overline{TL} \cong \overline{TL}$	② reflexive property
③ $\overline{UT} \cong \overline{LE}$ or $\overline{UL}-\overline{TL} = \overline{TE}-\overline{TL}$	③ subtraction property



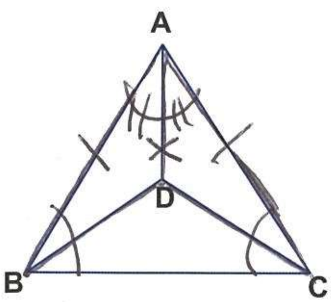
1. Given: $\overline{QV} \cong \overline{UZ}$, $\overline{VW} \cong \overline{YZ}$, $\overline{YQ} \cong \overline{WU}$
Prove: $\angle Q \cong \angle U$



Statements	Reasons
① $\overline{QV} \cong \overline{UZ}$	① given
② $\overline{VW} \cong \overline{YZ}$	② given
③ $\overline{WY} \cong \overline{WY}$	③ reflexive property
④ $\overline{VY} \cong \overline{WZ}$ or $\overline{VW}+\overline{WY} = \overline{YZ}+\overline{WY}$	④ Addition Property
⑤ $\overline{YQ} \cong \overline{WU}$	⑤ given
⑥ $\triangle QVY \cong \triangle UZW$	⑥ SSS≅SSS
⑦ $\angle Q \cong \angle U$	⑦ CPCTC

2. Given: $\angle ABC \cong \angle ACB$, $\overline{AD}$ bisects $\angle BAC$

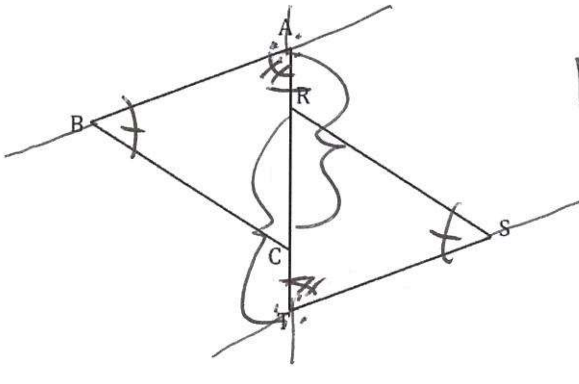
Prove: $\overline{BD} \cong \overline{DC}$



Statements	Reasons
① $\angle ABC \cong \angle ACB$	① given
② $\overline{AB} \cong \overline{AC}$	② In a triangle, congruent angles are opposite congruent sides
③ $\overline{AD}$ bisects $\angle BAC$	③ given
④ $\angle BAD \cong \angle CAD$	④ An angle bisector creates 2 ≅ angles
⑤ $\overline{AD} \cong \overline{AD}$	⑤ Reflexive Property
⑥ $\triangle BAD \cong \triangle CAD$	⑥ SAS≅SAS
⑦ $\overline{BD} \cong \overline{DC}$	⑦ CPCTC

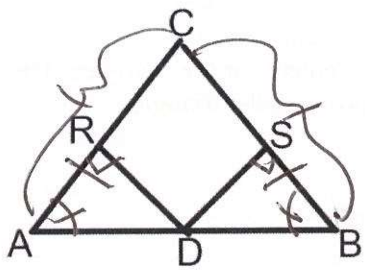
3. Given: $\angle B \cong \angle S$, $\overline{AB} \parallel \overline{ST}$, $\overline{AR} \cong \overline{TC}$

Prove: $\overline{BC} \cong \overline{SR}$



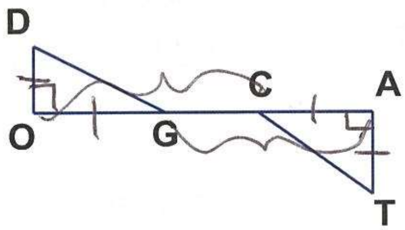
Statements	Reasons
① $\angle B \cong \angle S$	① given
② $\overline{AB} \parallel \overline{ST}$	② given
③ $\angle BAC \cong \angle STR$	③ parallel lines cut by a transversal create congruent alternate interior angles
④ $\overline{AR} \cong \overline{TC}$	④ given
⑤ $\overline{RC} \cong \overline{RC}$	⑤ reflexive property
⑥ $\overline{AC} \cong \overline{RT}$ or $\overline{AR}+\overline{RC} = \overline{TC}+\overline{RC}$	⑥ addition property
⑦ $\triangle BAC \cong \triangle STR$	⑦ AAS≅AAS
⑧ $\overline{BC} \cong \overline{SR}$	⑧ CPCTC

4. Given: In ΔABC, $\overline{CA} \cong \overline{CB}$, $\overline{AR} \cong \overline{BS}$, $\overline{DR} \perp \overline{AC}$, and $\overline{DS} \perp \overline{BC}$
Prove: $\overline{DR} \cong \overline{DS}$



Statements	Reasons
① $\overline{CA} \cong \overline{CB}$	① given
② $\angle A \cong \angle B$	② In a triangle, congruent angles are opposite congruent sides
③ $\overline{AR} \cong \overline{BS}$	③ given
④ $\overline{DR} \perp \overline{AC}$, $\overline{DS} \perp \overline{BC}$	④ given
⑤ $\angle DRA \cong \angle BSD$	⑤ Perpendicular lines form congruent right angles.
⑥ $\Delta ARD \cong \Delta BSD$	⑥ ASA ≅ ASA
⑦ $\overline{DR} \cong \overline{DS}$	⑦ CPCTC

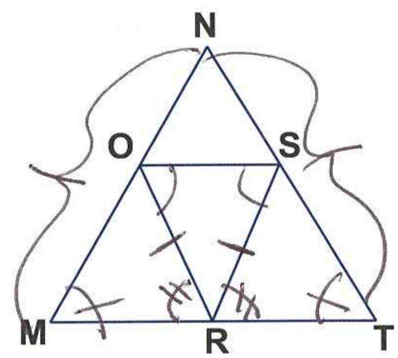
5. Given: $\overline{DO} \perp \overline{OA}$, $\overline{TA} \perp \overline{OA}$, $\overline{DO} \cong \overline{TA}$, $\overline{OC} \cong \overline{AG}$
Prove: $\overline{DG} \cong \overline{TC}$



Statements	Reasons
① $\overline{DO} \perp \overline{OA}$, $\overline{TA} \perp \overline{OA}$	① given
② $\angle DOG \cong \angle TAC$	② Perpendicular lines form congruent right angles
③ $\overline{DO} \cong \overline{TA}$	③ given
④ $\overline{OC} \cong \overline{AG}$	④ given
⑤ $\overline{GC} \cong \overline{GC}$	⑤ reflexive property
⑥ $\overline{OG} \cong \overline{CA}$ or $\overline{OC} - \overline{GC} = \overline{AG} - \overline{GC}$	⑥ subtraction property
⑦ $\Delta DOG \cong \Delta TAC$	⑦ SAS ≅ SAS
⑧ $\overline{DG} \cong \overline{TC}$	⑧ CPCTC

6. Given: $\overline{MN} \cong \overline{NT}$, $\angle ROS \cong \angle RSO$, $\angle ORM \cong \angle SRT$

Prove: $\Delta MOR \cong \Delta TSR$



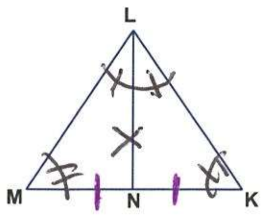
Statements	Reasons
① $MN \cong TN$	① given
② $\angle M \cong \angle T$	② In a triangle, congruent angles are opposite congruent sides
③ $\angle ROS \cong \angle RSO$	③ given
④ $\overline{OR} \cong \overline{SR}$	④ In a triangle, congruent angles are opposite congruent sides
⑤ $\angle ORM \cong \angle SRT$	⑤ given
⑥ $\Delta MOR \cong \Delta TSR$	⑥ AAS ≅ AAS

Triangle Proofs Using CPCTC

Prove the triangles are congruent

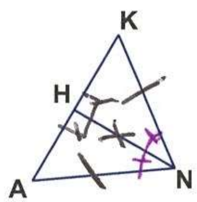
Use CPCTC in order to get what you need in order to prove what you are being asked to prove.

1. Given: $\overline{LN}$ bisects $\angle KLM$
   $\angle LKM \cong \angle LMK$
   Prove: N is the midpoint of $\overline{MK}$



Statements	Reasons
1) $\overline{LN}$ bisects $\angle KLM$	1) given
2) $\angle MLN \cong \angle KLN$	2) An angle bisector creates two congruent angles
3) $\angle LKM \cong \angle LMK$	3) given
4) $\overline{LN} \cong \overline{LN}$	4) reflexive property
5) $\triangle MLN \cong \triangle KLN$	5) AAS ≅ AAS
6) $\overline{MN} \cong \overline{NK}$	6) CPCTC
7) N is the midpoint of $\overline{MK}$	7) A midpoint creates two congruent segments

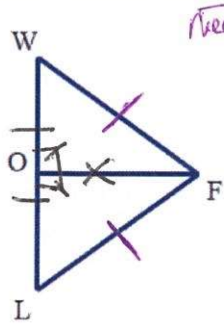
2. Given: $\overline{HN} \perp \overline{KA}$, $\overline{KN} \cong \overline{AN}$
   Prove: $\overline{HN}$ bisects $\angle KNA$



Statements	Reasons
1) $\overline{HN} \perp \overline{KA}$	1) given
2) $\angle KHN \cong \angle AHN$	2) Perpendicular lines form congruent right angles
3) $\overline{KN} \cong \overline{AN}$	3) given
4) $\overline{HN} \cong \overline{HN}$	4) Reflexive Property
5) $\triangle KHN \cong \triangle AHN$	5) HL ≅ HL
6) $\angle KNH \cong \angle ANH$	6) CPCTC
7) $\overline{HN}$ bisects $\angle KNA$	7) An angle bisector creates two congruent angles

3. Given: $\overline{OF}$ is the perpendicular bisector of $\overline{WL}$
Prove: ΔWFL is isosceles

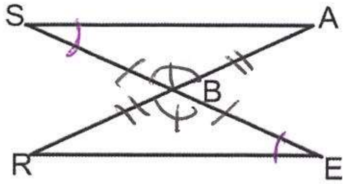
need 2 ≅ sides or angles



Statements	Reasons
1) $\overline{OF}$ is the perpendicular bisector of $\overline{WL}$	1) given
2) $\overline{WO} \cong \overline{OL}$	2) A line bisector creates two congruent segments
3) $\angle WOF \cong \angle LOF$	3) Perpendicular lines form congruent right angles
4) $\overline{OF} \cong \overline{OF}$	4) Reflexive Property
5) $\triangle WOF \cong \triangle LOF$	5) SAS ≅ SAS
6) $\overline{WF} \cong \overline{LF}$	6) CPCTC
7) $\triangle WFL$ is isosceles	7) An isosceles triangle has two congruent sides

4. Given: $\overline{SE}$ and $\overline{AR}$ bisect each other.

Prove that $\overline{SA} \parallel \overline{RE}$ need ≅ alternate interior angles



Statements	Reasons
1) $\overline{SE}$ and $\overline{AR}$ bisect each other	1) given
2) $\overline{SB} \cong \overline{BE}$, $\overline{RB} \cong \overline{BA}$	2) A line bisector creates 2 ≅ segments
3) $\angle SBA \cong \angle RBE$	3) vertical angles are congruent
4) $\triangle SBA \cong \triangle EBR$	4) SAS ≅ SAS
5) $\angle S \cong \angle E$	5) CPCTC
6) $\overline{SA} \parallel \overline{RE}$	6) Parallel lines cut by a transversal create congruent ~~alt.~~ alternate interior angles

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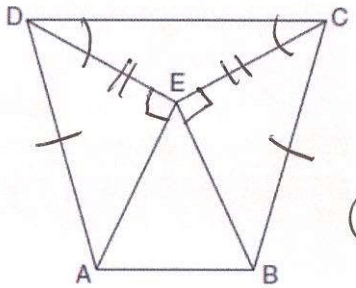
5. Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.

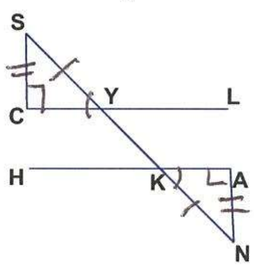
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.



Statements	Reasons
① Isosceles trapezoid ABCD	① given
② $\overline{AD} \cong \overline{BC}$	② ~~AD~~ An isosceles trapezoid has congruent legs
③ $\angle CDE \cong \angle DCE$	③ given
④ $\overline{DE} \cong \overline{CE}$	④ In a triangle, congruent angles are opposite congruent sides
⑤ $\overline{AE} \perp \overline{DE}$, $\overline{BE} \perp \overline{CE}$	⑤ given
⑥ $\angle DEA \cong \angle CEB$	⑥ Perpendicular lines form congruent right angles
⑦ $\triangle ADE \cong \triangle BCE$	⑦ HL
⑧ ~~AA~~ $\overline{AE} \cong \overline{BE}$	⑧ CPCTC
⑨ $\triangle AEB$ is isosceles	⑨ An isosceles triangle has two $\cong$ sides

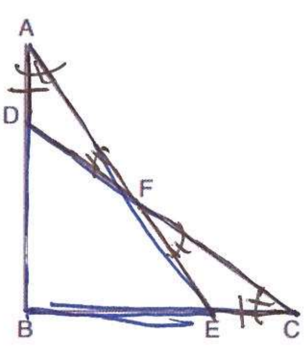
6. Given: $\overline{SC} \perp \overline{CL}$, $\overline{HA} \perp \overline{AN}$, $\overline{SY} \cong \overline{KN}$, and $\overline{SC} \cong \overline{AN}$.

Prove $\overline{CL} \parallel \overline{HA}$



statements	Reasons
① $\overline{SC} \perp \overline{CL}$, $\overline{HA} \perp \overline{AN}$	① given
② $\angle SCY \cong \angle NAK$	② Perpendicular lines form congruent right $\angle$s.
③ $\overline{SY} \cong \overline{KN}$	③ given
④ $\overline{SC} \cong \overline{AN}$	④ given
⑤ $\triangle SCY \cong \triangle NAK$	⑤ HL
⑥ $\angle SYC \cong \angle NKA$	⑥ CPCTC
⑦ $\overline{CL} \parallel \overline{HA}$	⑦ Parallel lines cut by a transversal form congruent ~~...~~ alternate exterior angles.

7. In the diagram below, $\triangle ABE \cong \triangle CBD$. Prove: $\triangle AFD \cong \triangle CFE$



Statements	Reasons
① $\triangle ABE \cong \triangle CBD$	① given
② $\overline{AB} \cong \overline{BC}$, $\overline{DB} \cong \overline{BE}$	② CPCTC
③ $\overline{AD} \cong \overline{EC}$ or $\overline{AB} - \overline{DB} = \overline{BC} - \overline{BE}$	③ Subtraction Property
④ $\angle A \cong \angle C$	④ CPCTC
⑤ $\angle AFD \cong \angle CFE$	⑤ Vertical angles are congruent
⑥ $\triangle AFD \cong \triangle CFE$	⑥ AAS

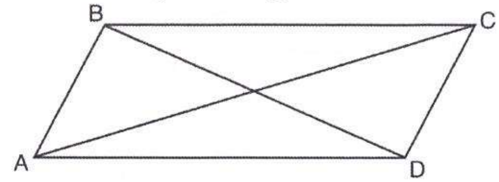
Euclidean Parallelogram Proofs/Parallelogram Properties

To prove parallelograms: Always prove parallelogram first. You will probably have to use congruent triangles with CPCTC to get at least one of the properties.

1. Quadrilateral *ABCD* with diagonals $\overline{AC}$ and $\overline{BD}$ is shown in the diagram below.

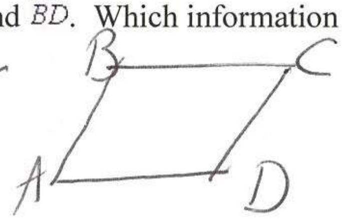
Which information is *not* enough to prove *ABCD* is a parallelogram?

1) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{DC}$ ✓
2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$ ✓
3) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$ X
4) $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$



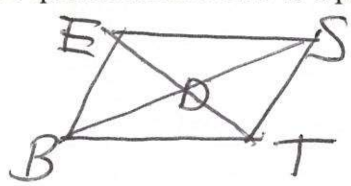
2. Quadrilateral *ABCD* has diagonals $\overline{AC}$ and $\overline{BD}$. Which information is *not* sufficient to prove *ABCD* is a parallelogram?

1) $\overline{AC}$ and $\overline{BD}$ bisect each other. ✓
2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$ ✓
3) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$ ✓
4) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$ X

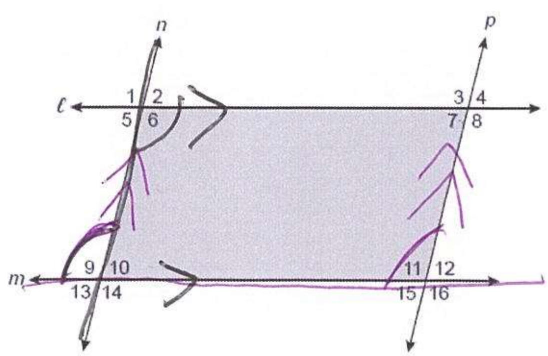


18 Quadrilateral *BEST* has diagonals that intersect at point *D*. Which statement would *not* be sufficient to prove quadrilateral *BEST* is a parallelogram?

1) $\overline{BD} \cong \overline{SD}$ and $\overline{ED} \cong \overline{TD}$ ✓
2) $\overline{BE} \cong \overline{ST}$ and $\overline{ES} \cong \overline{TB}$ ✓
3) $\overline{ES} \cong \overline{TB}$ and $\overline{BE} \parallel \overline{TS}$ X
4) $\overline{ES} \parallel \overline{BT}$ and $\overline{BE} \parallel \overline{TS}$ ✓



4. In the diagram below, lines ℓ and m intersect lines n and p to create the shaded quadrilateral as shown.



Which congruence statement would be sufficient to prove the quadrilateral is a parallelogram?

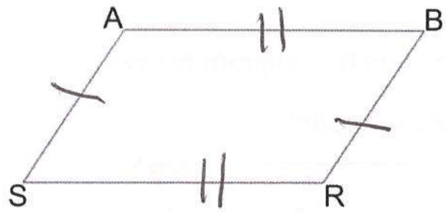
1) $\angle 1 \cong \angle 6$ and $\angle 9 \cong \angle 14$
2) $\angle 5 \cong \angle 10$ and $\angle 6 \cong \angle 9$
3) $\angle 5 \cong \angle 7$ and $\angle 10 \cong \angle 15$
4) $\angle 6 \cong \angle 9$ and $\angle 9 \cong \angle 11$

2) ~~$\angle 5 \cong \angle 10$ and $\angle 6 \cong \angle 9$~~

4) ~~$\angle 6 \cong \angle 9$ and $\angle 9 \cong \angle 11$~~

5. Given: $\overline{SA} \cong \overline{BR}$, $\overline{AB} \cong \overline{SR}$

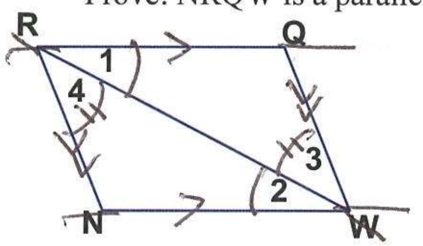
Prove: SABR is a parallelogram



Statements	Reasons
① $\overline{SA} \cong \overline{BR}$, $\overline{AB} \cong \overline{SR}$	① given
② SABR is a parallelogram	② A parallelogram has 2 pairs of opposite sides congruent

6. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

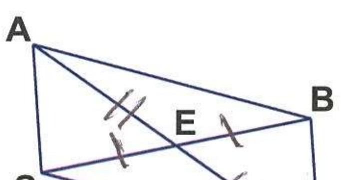
Prove: NRQW is a parallelogram



Statements	Reasons
① $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	① given
② $\overline{RQ} \parallel \overline{NW}$, $\overline{RN} \parallel \overline{QW}$	② A parallelogram has 2 pairs of opposite sides parallel

7. Given: E is the midpoint of $\overline{SB}$, $\overline{AE} \cong \overline{ER}$

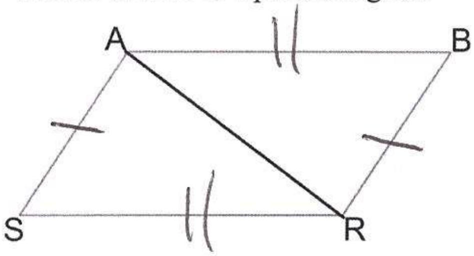
Prove: SABR is a parallelogram



Statements	Reasons
① E is midpoint of $\overline{SB}$	① given
② $\overline{SE} \cong \overline{EB}$	② a midpoint creates 2 ≅ segments
③ $\overline{AE} \cong \overline{ER}$	③ given
④ SABR is a parallelogram	④ A parallelogram has diagonals that bisect each other

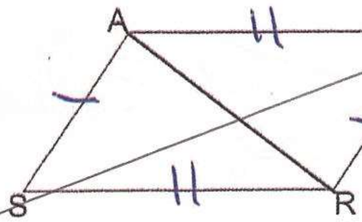
8. Given: $\Delta ASR \cong \Delta RBA$

Prove: SABR is a parallelogram



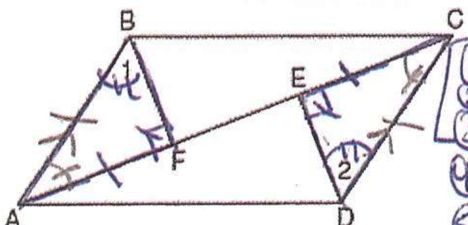
Statements	Reasons
① ~~SA≅~~ $\Delta ASR \cong \Delta RBA$	① given
② $\overline{SA} \cong \overline{BR}$, $\overline{AB} \cong \overline{SR}$	② CPCTC
③ SABR is a parallelogram	③ A parallelogram has 2 pairs of opposite sides ≅

29. Given: $\Delta ASR \cong \Delta RBA$
Prove: SABR is a parallelogram



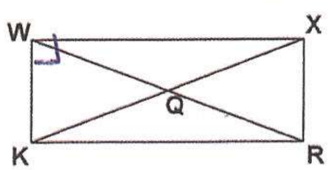
Statements	Reasons
① $\Delta ASR \cong \Delta RBA$	① given
② $\overline{AS} \cong \overline{BR}$, $\overline{AB} \cong \overline{SR}$	② CPCTC
③ SABR is a P-gram	③ A p-gram has 2 pairs of opposite sides congruent

9 30. Given: Quadrilateral $ABCD$, diagonal $\overline{AFEC}$, $\overline{AE} \cong \overline{FC}$, $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$, $\angle 1 \cong \angle 2$
Prove: $ABCD$ is a parallelogram.



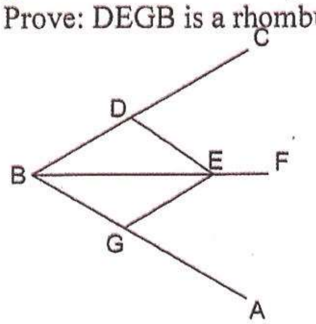
Statements	Reasons
① $\overline{AE} \cong \overline{FC}$	① given
② $\overline{FE} \cong \overline{FE}$	② Reflexive Property
③ $\overline{AF} \cong \overline{EC}$	③ Subtraction property
④ $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$	④ given
⑤ $\angle AFB \cong \angle CED$	⑤ perpendicular lines create $\cong$ right angles
⑥ $\angle 1 \cong \angle 2$	⑥ given
⑦ $\Delta AFB \cong \Delta CED$	⑦ AAS $\cong$ AAS
⑧ $\overline{AB} \cong \overline{CD}$, $\angle FAB \cong \angle DCE$	⑧ CPCTC
⑨ $\overline{AB} \parallel \overline{DC}$	⑨ If alternate interior angles are $\cong$, then the lines are $\parallel$
⑩ ABCD is a P-gram	⑩ A P-gram has 1 pair of opposite sides $\cong$ and $\parallel$.

10 31. Given: WXRK is a parallelogram, $\overline{KW} \perp \overline{WX}$
Prove: WXRK is a rectangle



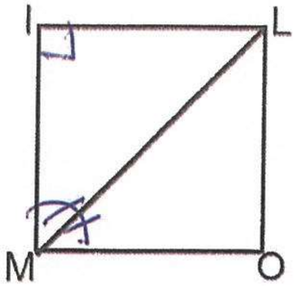
Statements	Reasons
① WXRK is a P-gram	① given
② $\overline{KW} \perp \overline{WX}$	② given
③ $\angle KWX$ is a right angle	③ perpendicular lines form right angles
④ WXRK is a rectangle	④ A rectangle is a parallelogram with a right angle

11 32. Given: BDEG is a parallelogram, $\overline{BF}$ bisects $\angle$ CBA
Prove: DEGB is a rhombus



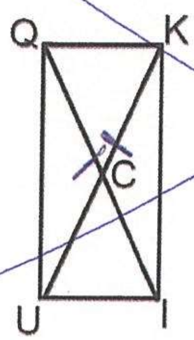
Statements	Reasons
① BDEG is a parallelogram	① given
② BF bisects $\angle$ CBA	② given
③ DEGB is a rhombus	③ A rhombus is a parallelogram whose diagonals bisect their angles

12 33. Given: MILO is a parallelogram, $\angle IML \cong \angle OML$, $\overline{MI} \perp \overline{IL}$
Prove: MILO is a square



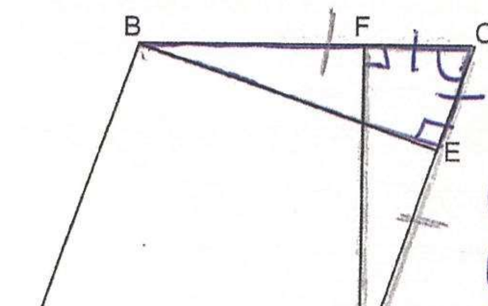
Statements	Reasons
① MILO is a parallelogram	① given
② ∠IML ≅ ∠OML	② given
③ $\overline{LM}$ bisects ∠IMO	③ An angle bisector creates 2 ≅ angles
④ $\overline{MI} \perp \overline{IL}$	④ given
⑤ ∠MIL is a right angle	⑤ Perpendicular lines form right angles
⑥ MILO is a square	⑥ A square is a P-gram that has diagonals that bisect the angles and a right angle

34. Given: QUIK is a parallelogram, $\overline{QI} \cong \overline{KU}$
Prove: QUIK is a rectangle



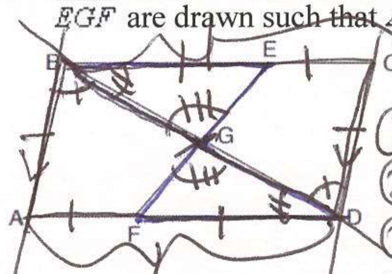
Statements	Reasons
① QUIK is a P-gram	① given
② $\overline{QI} \cong \overline{KU}$	② given
③ QUIK is a rectangle	③ A parallelogram with congruent diagonals is a rectangle.

13 35. In the diagram of parallelogram *ABCD* below, $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$, $\overline{CE} \cong \overline{CF}$.
Prove *ABCD* is a rhombus.



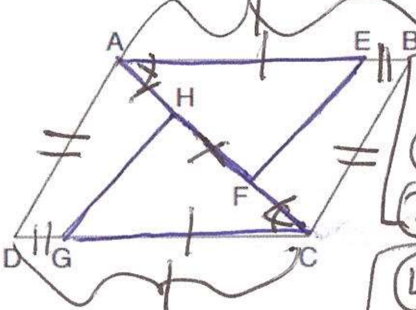
Statements	Reasons
① Parallelogram ABCD	① given
② $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$	② given
③ ∠DFC ≅ ∠BEC	③ Perpendicular lines create 2 ≅ right angles
④ $\overline{CE} \cong \overline{CF}$	④ given
⑤ ∠BCD ≅ ∠BCD	⑤ Reflexive Property
⑥ △BCE ≅ △DCF	⑥ ASA ≅ ASA
⑦ $\overline{BC} \cong \overline{CD}$	⑦ CPCTC
⑧ ABCD is a rhombus	⑧ A rhombus is a parallelogram with consecutive sides ≅

14. In quadrilateral $ABCD$, E and F are points on $\overline{BC}$ and $\overline{AD}$, respectively, and $\overline{BGD}$ and $\overline{EGF}$ are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$. Prove: $\overline{FG} \cong \overline{EG}$ $\Delta BEG \cong \Delta DFG$



Statements	Reasons
1) $\angle ABG \cong \angle CDG$	1) given
2) $\overline{AB} \cong \overline{CD}$	2) given
3) $\overline{BD} \cong \overline{BD}$	3) Reflexive Property
4) $\Delta ABD \cong \Delta CDB$	4) SAS
5) $BA \parallel CD$	5) Parallel lines cut by a transversal create congruent alternate interior angles
6) ABCD is a parallelogram	6) A parallelogram has 1 pair of opposite sides congruent and parallel
7) $\overline{BC} \cong \overline{AD}$	7) A parallelogram has opposite sides congruent
8) $\overline{CE} \cong \overline{AF}$	8) given
9) $\overline{BE} \cong \overline{FD}$	9) Subtraction Property
10) $\angle CBD \cong \angle ADB$	10) CPCTC
11) $\angle BGE \cong \angle DGF$	11) vertical angles are congruent
12) $\Delta BEG \cong \Delta DFG$	12) AAS
13) $\overline{FG} \cong \overline{EG}$	13) CPCTC

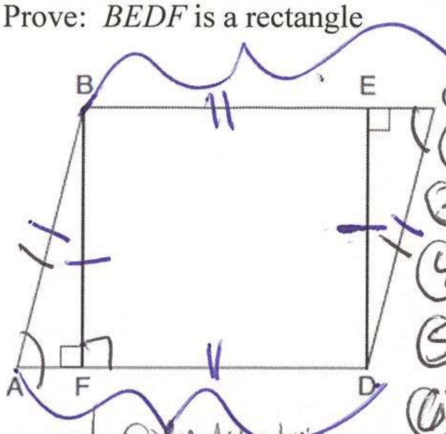
15. In the diagram of quadrilateral $ABCD$ with diagonal $\overline{AC}$ shown below, segments GH and EF are drawn, $\overline{AE} \cong \overline{CG}$, $\overline{BE} \cong \overline{DG}$, $\overline{AH} \cong \overline{CF}$, and $\overline{AD} \cong \overline{CB}$. Prove: $\overline{EF} \cong \overline{GH}$ $\Delta AEG \cong \Delta CGH$



Statements	Reasons
1) $\overline{AE} \cong \overline{CG}$	1) given
2) $\overline{BE} \cong \overline{DG}$	2) given
3) $\overline{AB} \cong \overline{DC}$	3) Addition Property
4) $\overline{AH} \cong \overline{CF}$	4) given
5) $\overline{HF} \cong \overline{HF}$	5) Reflexive Property
6) $\overline{AF} \cong \overline{HC}$	6) Addition Property
7) $\overline{AD} \cong \overline{CB}$	7) given
8) ABCD is a parallelogram	8) A parallelogram has 2 pairs of opposite sides congruent
9) $\overline{AB} \parallel \overline{DC}$	9) A parallelogram has opposite sides parallel.
10) $\angle EAF \cong \angle HCG$	10) Parallel lines cut by a transversal create congruent alternate interior angles
11) $\Delta AEG \cong \Delta CGH$	11) SAS
12) $\overline{EF} \cong \overline{GH}$	12) CPCTC

16. ~~14~~. Given: Parallelogram $ABCD$, $\overline{BF} \perp \overline{AFD}$, and $\overline{DE} \perp \overline{BEC}$

Prove: $BEDF$ is a rectangle

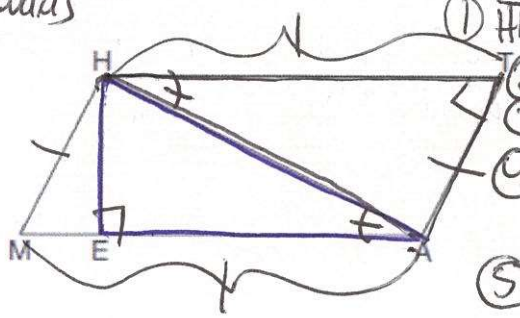


Statements	Reasons
(1) Parallelogram ABCD	(1) given
(2) $\angle BAF \cong \angle BCD$	(2) A P-gram has opposite ∡ congruent
(3) $\overline{BA} \cong \overline{CD}$	(3) A P-gram has opposite sides ≅
(4) $\overline{BF} \perp \overline{AFD}$, $\overline{DE} \perp \overline{BEC}$	(4) given
(5) $\angle BFA \cong \angle CED$	(5) Perpendicular lines create ≅ right ∡
(6) $\triangle BFA \cong \triangle DEC$	(6) AAS
(7) $\overline{BC} \cong \overline{AD}$	(7) A P-gram has opposite sides ≅
(8) $\overline{EC} \cong \overline{AF}$	(8) CPCTC
(9) $\overline{BE} \cong \overline{ED}$	(9) Subtraction Property
(10) $\overline{BF} \cong \overline{ED}$	(10) CPCTC
(11) BEDF is a P-gram	(11) A Parallelogram has 2 pairs of opposite sides congruent
(12) $\angle BFD$ is a right angle	(12) Perpendicular lines form right angles
(13) BEDF is a rectangle	(13) A rectangle is a Parallelogram with a right angle

17 ~~15~~. Given: Quadrilateral $MATH$, $\overline{HM} \cong \overline{AT}$, $\overline{HT} \cong \overline{AM}$, $\overline{HE} \perp \overline{MEA}$, and $\overline{HA} \perp \overline{AT}$.

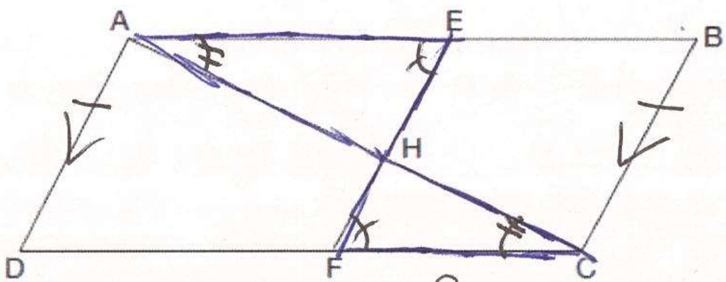
Prove: $TA \bullet HA = HE \bullet TH$

work backwards



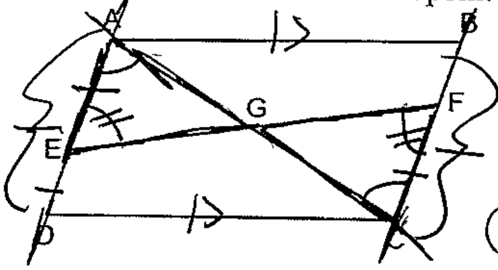
Statements	Reasons
(1) $\overline{HM} \cong \overline{AT}$, $\overline{HT} \cong \overline{AM}$	(1) given
(2) MATH is a P-gram	(2) A P-gram has 2 pairs of opp sides ≅
(3) $\overline{HT} \parallel \overline{MA}$	(3) A P-gram has opposite sides ∥
(4) $\angle THA \cong \angle HAE$	(4) parallel lines cut by a transversal create congruent alternate interior angles
(5) $\overline{HE} \perp \overline{MEA}$, $\overline{HA} \perp \overline{AT}$	(5) given
(6) $\angle HEA \cong \angle HTA$	(6) Perpendicular lines form congruent right angles
(7) $\triangle THA \sim \triangle EAH$	(7) AA
(8) $\dfrac{TA}{TH} = \dfrac{HE}{HA}$	(8) CSSTP
(9) $TA \cdot HA = HE \cdot TH$	(9) cross products are equal

18.16. Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$. Prove: $(EH)(CH) = (FH)(AH)$ work backwards



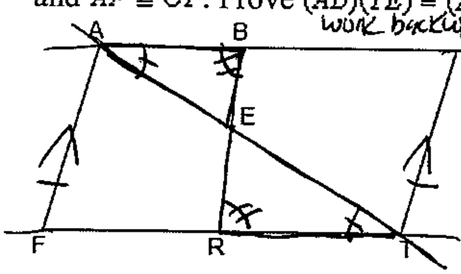
Statements	Reasons
1) $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$	1) given
2) $\overline{AD} \parallel \overline{BC}$	2) Transitive Poperty
3) $\overline{AD} \cong BC$	3) given
4) ABCD is a parallelogram	4) A parallelogram has 1 pair of opposite sides congruent and parallel.
5) $\overline{AB} \parallel \overline{DC}$	5) A parallelogram has opposite sides parallel
6) $\angle AEH \cong \angle CFH$ $\angle EAH \cong \angle HCF$	6) Parallel lines cut by a transversal create congruent alternate interior angles
7) $\triangle EHA \sim \triangle FHC$	7) AA
8) $\dfrac{EH}{AH} = \dfrac{FH}{CH}$	8) CSSTP
9) $(EH)(CH) = (FH)(AH)$	9) cross products are equal

19. Given: Quadrilateral $ABCD$, $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$, diagonal $\overline{AC}$ intersects $\overline{EF}$ at G, and $\overline{DE} \cong \overline{BF}$. Prove: G is the midpoint of $\overline{EF}$.



Statements	Reasons
① $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$	① given
② ABCD is a parallelogram	② A parallelogram has 1 pair of opposite sides congruent and parallel
③ $\overline{AD} \parallel \overline{BC}$	③ A parallelogram has opposite sides ∥
④ $\angle EAG \cong \angle FCG$, $\angle AEG \cong \angle GFC$	④ Parallel lines cut by a transversal create congruent alternate interior angles
⑤ $\overline{DE} \cong \overline{BF}$	⑤ given
⑥ $\overline{AD} \cong \overline{BC}$	⑥ A parallelogram has opposite sides ≅
⑦ $\overline{AE} \cong \overline{FC}$ or $\overline{AD} - \overline{DE} = \overline{BC} - \overline{BF}$	⑦ Subtraction Property
⑧ $\triangle AEG \cong \triangle CFG$	⑧ ASA
⑨ $\overline{EG} \cong \overline{GF}$	⑨ CPCTC
⑩ G is the midpoint of $\overline{EF}$	⑩ A midpoint creates two congruent segments.

20. In the diagram below of quadrilateral $FACT$, $\overline{BR}$ intersects diagonal $\overline{AT}$ at E, $\overline{AF} \parallel \overline{CT}$, and $\overline{AF} \cong \overline{CT}$. Prove $(AB)(TE) = (AE)(TR)$ work backward



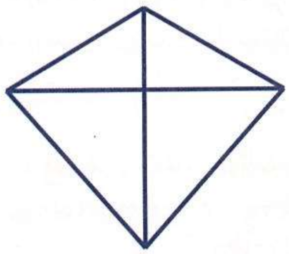
Statements	Reasons
① $\overline{AF} \parallel \overline{CT}$, $\overline{AF} \cong \overline{CT}$	① given
② FACT is a parallelogram	② A parallelogram has 1 pair of opposite sides ≅ and ∥.
③ $\overline{AC} \parallel \overline{FT}$	③ A parallelogram has opposite sides parallel.
④ $\angle BAE \cong \angle RTE$, $\angle ABE \cong \angle TRE$	④ Parallel lines cut by a transversal create ≅ alternate interior angles.
⑤ $\triangle ABE \sim \triangle TRE$	⑤ AA ≅ AA
⑥ $\frac{AB}{TR} = \frac{AE}{TE}$	⑥ CSSTIP
⑦ $(AB)(TE) = (AE)(TR)$	⑦ cross products are equal

Perpendicular Bisector Proofs Multiple Choice

Perpendicular bisector creates

-two pairs of congruent triangles so all of their corresponding parts are congruent due to CPCTC

-two isosceles triangles

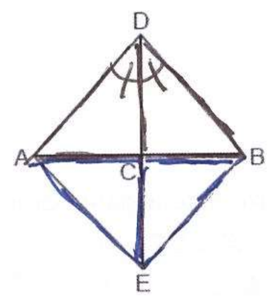


The top 2 small triangles are congruent and the top big triangle is isosceles

The bottom 2 small triangles are congruent and the bottom big triangle is isosceles

1. In the diagram below of quadrilateral *ADBE*, $\overline{DE}$ is the perpendicular bisector of $\overline{AB}$. Which statement is always true?

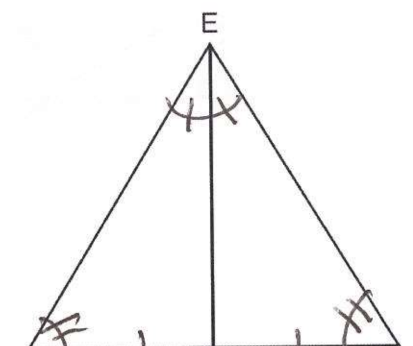
1) $\angle ADC \cong \angle BDC$
2) $\angle EAC \cong \angle DAC$
3) $\overline{AD} \cong \overline{BE}$
4) $\overline{AE} \cong \overline{AD}$



2. Line segment *EA* is the perpendicular bisector of $\overline{ZT}$, and $\overline{ZE}$ and $\overline{TE}$ are drawn.

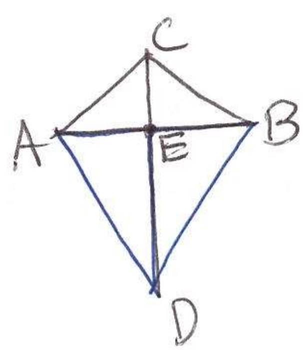
Which conclusion can *not* be proven?

1) $\overline{EA}$ bisects angle *ZET*. ✓
2) Triangle *EZT* is equilateral. X
3) $\overline{EA}$ is a median of triangle *EZT*. ✓
4) Angle *Z* is congruent to angle *T*. ✓

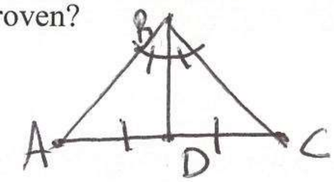


3. Segment *CD* is the perpendicular bisector of $\overline{AB}$ at *E*. Which pair of segments does *not* have to be congruent?

1) $\overline{AD}, \overline{BD}$ ✓
2) $\overline{AC}, \overline{BC}$ ✓
3) $\overline{AE}, \overline{BE}$ ✓
4) $\overline{DE}, \overline{CE}$ X



4. In $\triangle ABC$, $\overline{BD}$ is the perpendicular bisector of $\overline{ADC}$. Based upon this information, which statements below can be proven?



I. $\overline{BD}$ is a median.

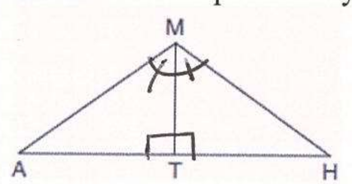
II. $\overline{BD}$ bisects $\angle ABC$.

III. $\triangle ABC$ is isosceles.

1) I and II, only
2) I and III, only
3) II and III, only
4) I, II, and III

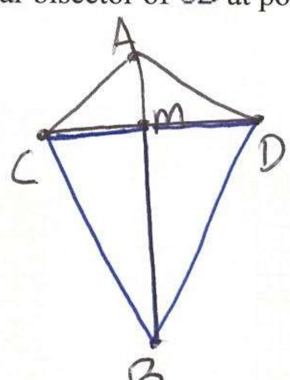
5. In triangle *MAH* below, $\overline{MT}$ is the perpendicular bisector of $\overline{AH}$.
Which statement is *not* always true?

1) $\triangle MAH$ is isosceles. 2) $\triangle MAT$ is isosceles. 3) $\overline{MT}$ bisects $\angle AMH$. 4) $\angle A$ and $\angle TMH$ are complementary.



6. Segment *AB* is the perpendicular bisector of $\overline{CD}$ at point *M*. Which statement is always true?

1) $\overline{CB} \cong \overline{DB}$
2) $\overline{CD} \cong \overline{AB}$
3) $\triangle ACD \sim \triangle BCD$
4) $\triangle ACM \sim \triangle BCM$



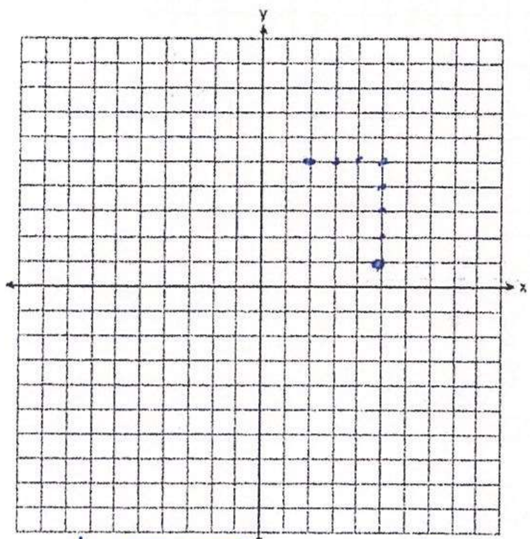
Name Schlansky $d=\sqrt{\Delta x^2+\Delta y^2}$ Date ____________

Mr. Schlansky Geometry

Calculating Distance

Calculate the distance between the following sets of points. Express in simplest radical form

1. (5,1) and (2,5)



$d=\sqrt{\Delta x^2+\Delta y^2}$

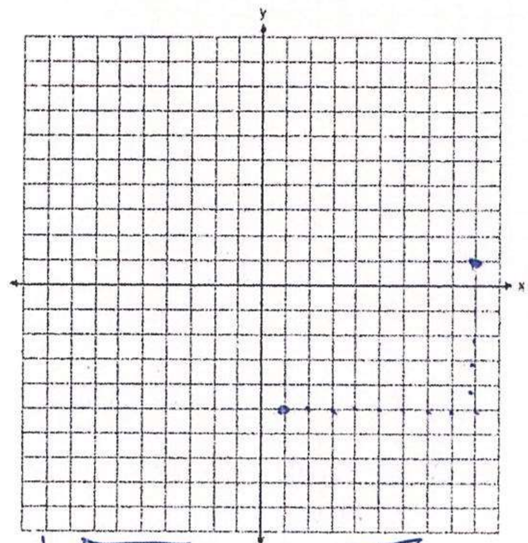
$d=\sqrt{3^2+4^2}$

$d=\sqrt{9+16}$

$d=\sqrt{25}$

$d=5$

2. (9,1) and (1,-5)



$d=\sqrt{\Delta x^2+\Delta y^2}$

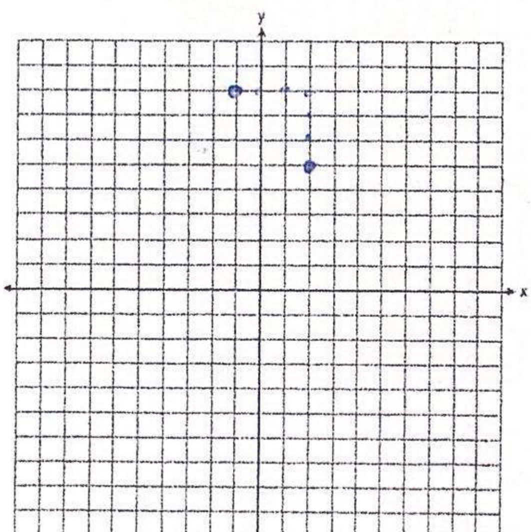
$d=\sqrt{8^2+6^2}$

$d=\sqrt{64+36}$

$d=\sqrt{100}$

$d=10$

3. (2,5) and (-1,8)



$d=\sqrt{\Delta x^2+\Delta y^2}$

$d=\sqrt{3^2+3^2}$

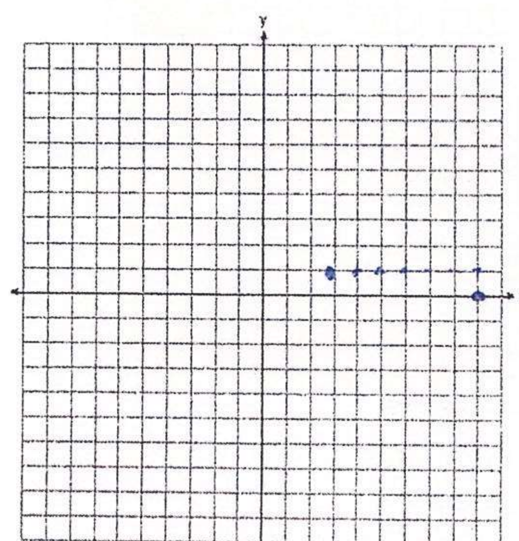
$d=\sqrt{9+9}$

$d=\sqrt{18}$

$\sqrt{9}\ \sqrt{2}$

$3\sqrt{2}$

4. (3,1) and (9,0)



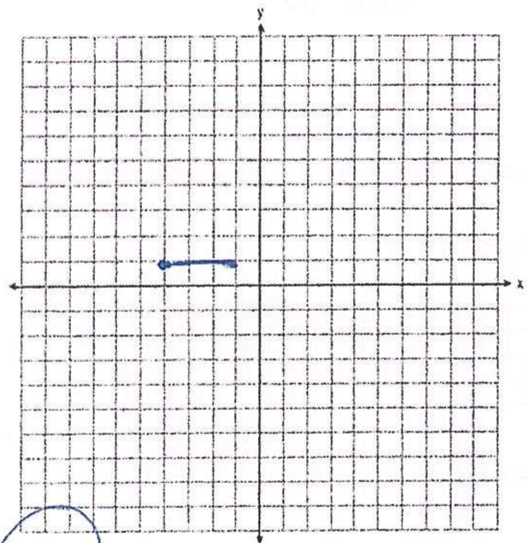
$d=\sqrt{\Delta x^2+\Delta y^2}$

$d=\sqrt{6^2+1^2}$

$d=\sqrt{36+1}$

$d=\sqrt{37}$

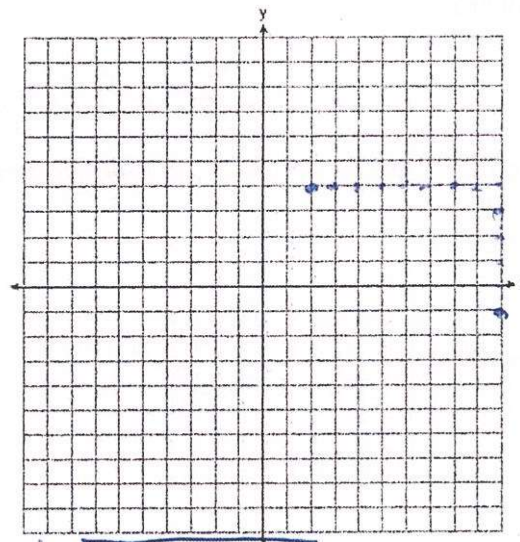
5. (-4,1) and (-1, 1)



3

If it is a straight line, you can just count without doing distance formula.

6. (10,-1) and (2, 4)



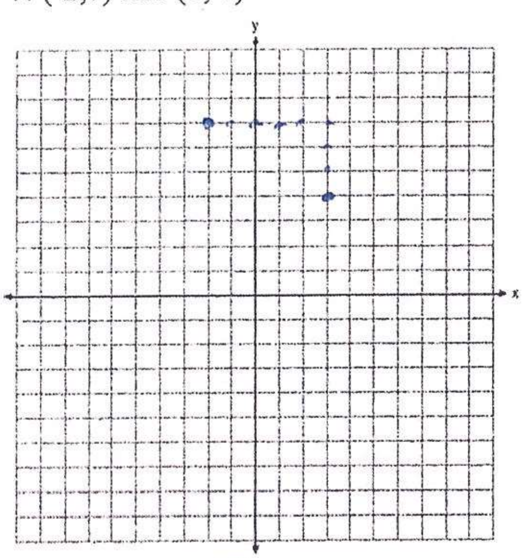
$d=\sqrt{\Delta x^2+\Delta y^2}$

$d=\sqrt{8^2+5^2}$

$d=\sqrt{64+25}$

$d=\sqrt{89}$

7. (-2,7) and (3, 4)



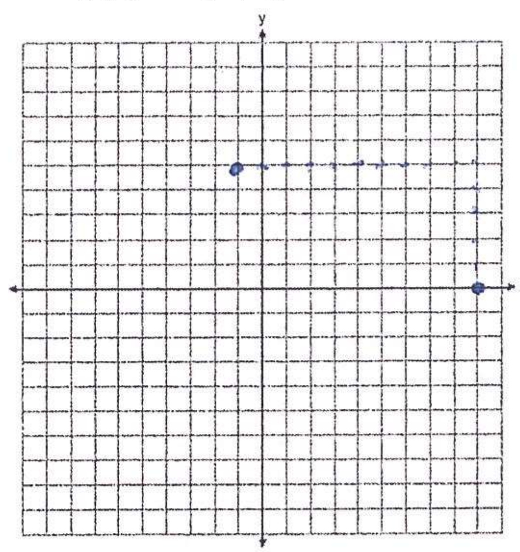
$d=\sqrt{\Delta x^2+\Delta y^2}$

$d=\sqrt{5^2+3^2}$

$d=\sqrt{25+9}$

$d=\sqrt{34}$

8. (9,0) and (-1, 5)



$d=\sqrt{\Delta x^2+\Delta y^2}$

$d=\sqrt{10^2+5^2}$

$d=\sqrt{100+25}$

$d=\sqrt{125}$

$\sqrt{25}\sqrt{5}$

$d=5\sqrt{5}$

$m = \frac{\Delta y}{\Delta x}$

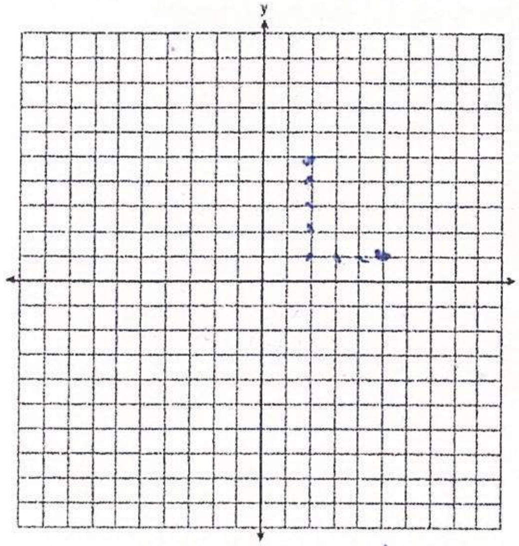
Name Schlansky
Mr. Schlansky

Date ____________
Geometry

Calculating Slope

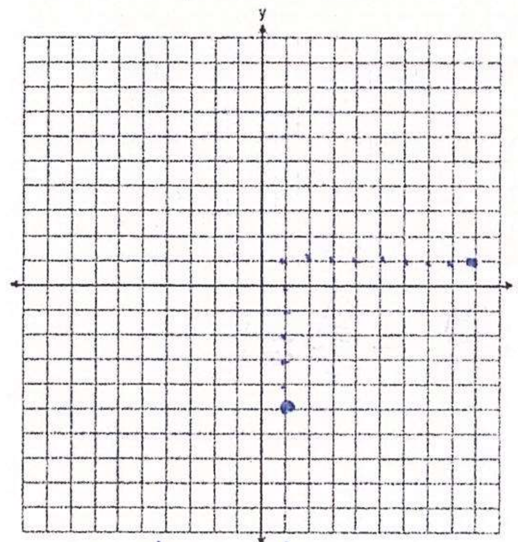
Calculate the slopes between the following sets of points. Express in simplest terms

1. (5,1) and (2,5)



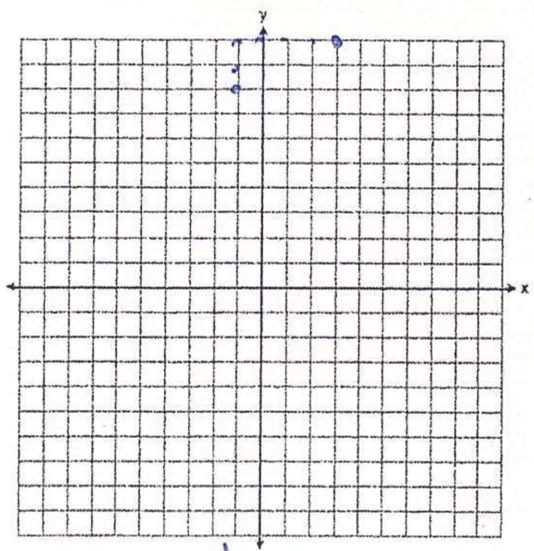
$m = \frac{\Delta y}{\Delta x} = \frac{-4}{3}$

2. (9,1) and (1,-5)



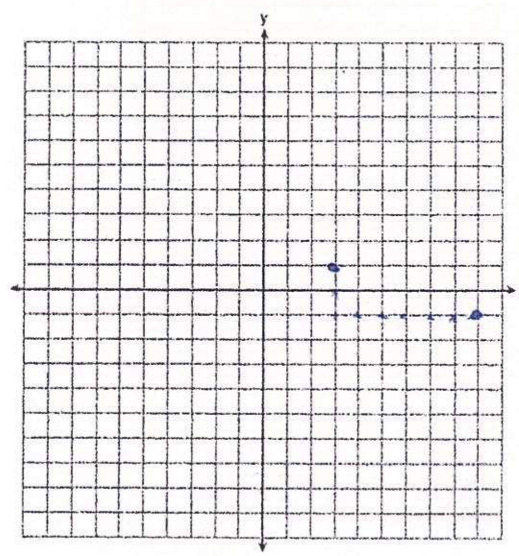
$m = \frac{\Delta y}{\Delta x} = \frac{6}{8} = \frac{3}{4}$

3. (3,10) and (-1,8)



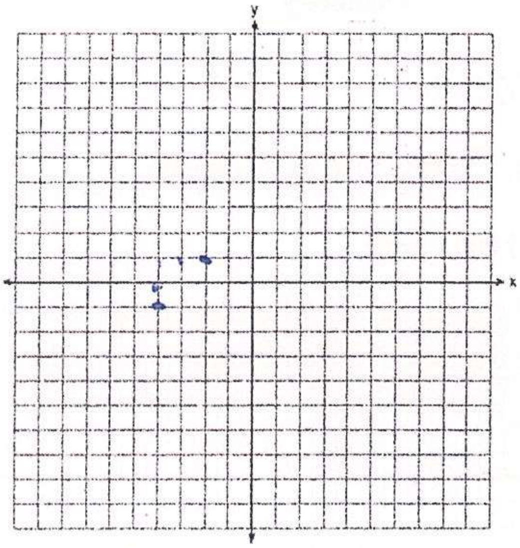
$m = \frac{\Delta y}{\Delta x} = \frac{2}{4} = \frac{1}{2}$

4. (3,1) and (9,-1)



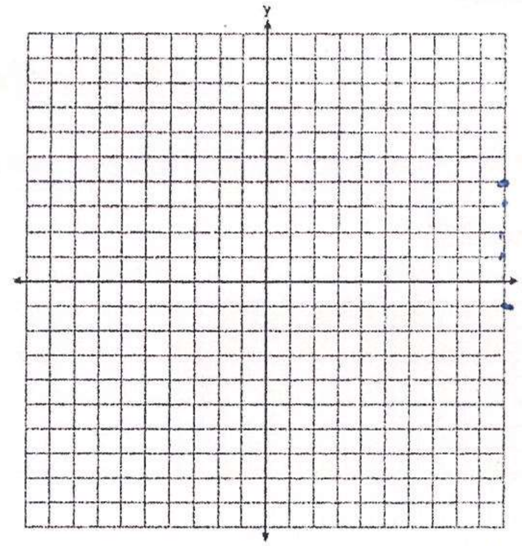
$m = \frac{\Delta y}{\Delta x} = \frac{-2}{6} = \frac{-1}{3}$

5. (-2,1) and (-4, -1)



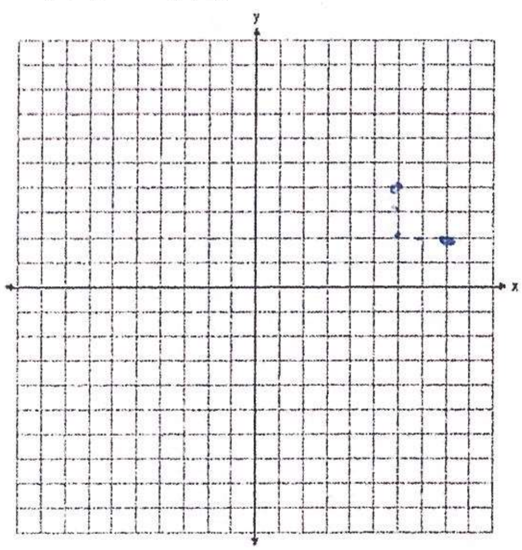
$m = \frac{\Delta y}{\Delta x} = \frac{2}{2} = 1$

6. (10,-1) and (10, 4)



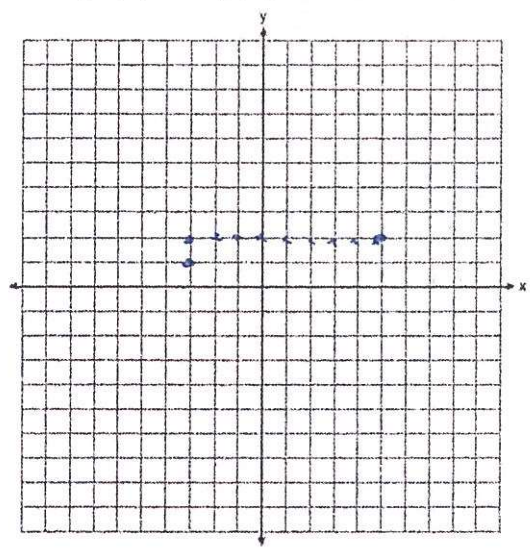
$m = \frac{\Delta y}{\Delta x} = \frac{5}{0} =$ No slope

7. (8,2) and (6,4)



$m = \frac{\Delta y}{\Delta x} = \frac{-2}{2} = -1$

8. (-3,1) and (5,2)



$m = \frac{\Delta y}{\Delta x}$

$m = \frac{1}{8}$

Coordinate Geometry Proofs

Distance (Length) = $\sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Slope = $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

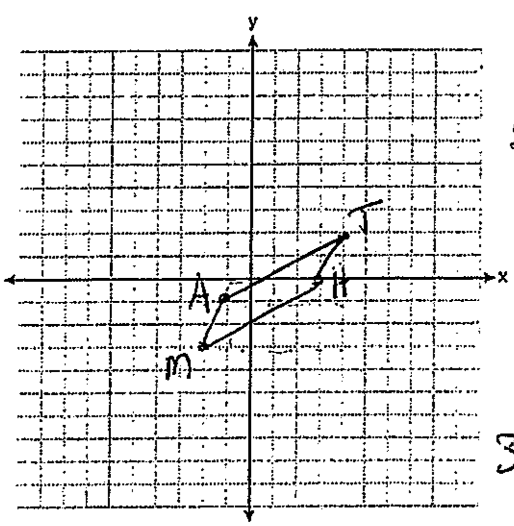
Midpoint = (*average x, average y*) = $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

How do you prove...?

...an **isosceles triangle**? (2 Distances)
Two Congruent Sides
. ... a **right triangle**? (3 Distances)
Show the sides fit into Pythagorean Theorem
... a **parallelogram**? (4 Distances)
Two Pairs of Opposite Sides Congruent
... a **rhombus**? (4 Distances)
All Sides Congruent
... a **rectangle**? (6 Distances)
1) Two Pairs of Opposite Sides Congruent
2) Diagonals Congruent
... a **square**? (6 Distances)
1) All Sides Congruent
2) Diagonals Congruent
...a **trapezoid**? (4 Slopes)
1) 1 pair of opposite sides parallel
2) 1 pair of opposite sides not parallel
...an **isosceles trapezoid**? (4 Slopes, 2 Distances)
1) 1 pair of opposite sides parallel
2) 1 pair of opposite sides not parallel
3) Congruent Legs

1. Graph the quadrilateral MATH: M(-2, -3) A(-1, -1) T(4, 2) H(3, 0). Prove that MATH **IS** a parallelogram but is **NOT** a rectangle.

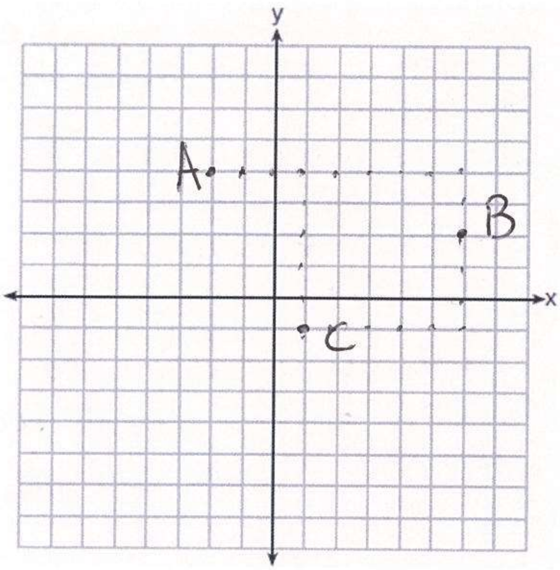
1) Math is a parallelogram because it has 2 pairs of opposite sides $\cong$
It is not a rectangle because diagonals are not $\cong$



2) $d\overline{mA} = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$
$d\overline{TH} = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$
$d\overline{AT} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$
$d\overline{mH} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$
$d\overline{AH} = \sqrt{4^2+1^2} = \sqrt{16+1} = \sqrt{17}$
$d\overline{mT} = \sqrt{6^2+5^2} = \sqrt{36+25} = \sqrt{61}$

3) $\overline{mA} \cong \overline{TH}$, $\overline{AT} \cong \overline{mH}$ because they have the Same distance
$\overline{AH} \ncong \overline{mT}$ because they don't have the Same distance

2. A triangle has vertices $A(-2,4)$, $B(6,2)$, and $C(1,-1)$. Prove that $\triangle ABC$ is an isosceles right triangle. [The use of the set of axes below is optional.]

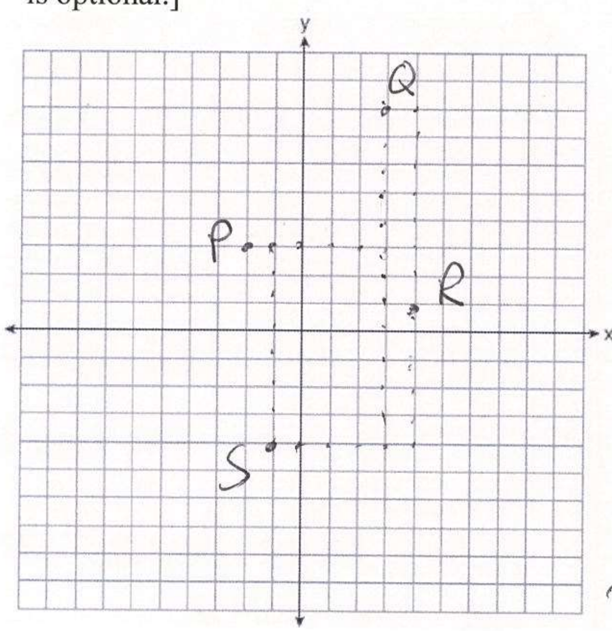


1) $\triangle ABC$ is an isosceles right triangle because it has two congruent sides and its sides fit into Pythagolean Theorem.

2) $d\overline{AC} = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$
$d\overline{CB} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$
$d\overline{BA} = \sqrt{8^2+2^2} = \sqrt{64+4} = \sqrt{68}$

3) $\overline{AC} \cong \overline{CB}$ because they have the same distance.
$a^2+b^2=c^2$
$\sqrt{34}^2+\sqrt{34}^2=\sqrt{68}^2$
$34+34=68$
$68=68$ ✓

3. Quadrilateral $PQRS$ has vertices $P(-2,3)$, $Q(3,8)$, $R(4,1)$, and $S(-1,-4)$. Prove that $PQRS$ is a rhombus. Prove that $PQRS$ is *not* a square. [The use of the set of axes below is optional.]

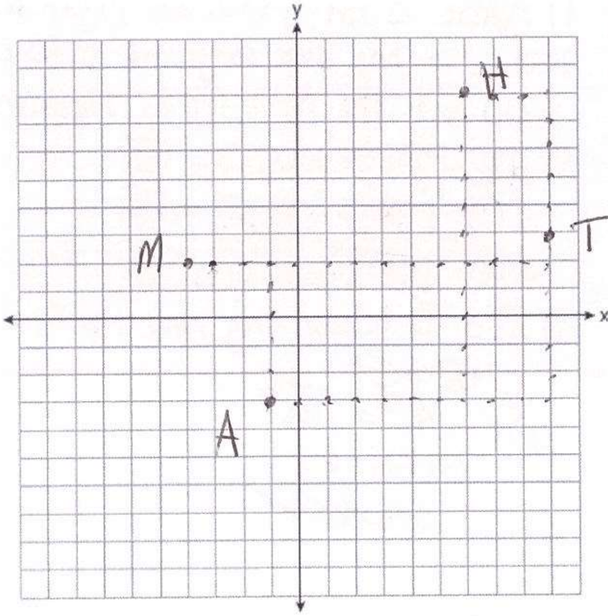


1) PQRS is a rhombus because all sides ale congruent. It is not a squale because diagonals ale not congruent.

2) $d\overline{PQ} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$
$d\overline{QR} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$
$d\overline{RS} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$
$d\overline{SP} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$
$d\overline{PR} = \sqrt{6^2+2^2} = \sqrt{36+4} = \sqrt{40}$
$d\overline{QS} = \sqrt{4^2+12^2} = \sqrt{16+144} = \sqrt{160}$

3) $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$ because they have the same distance.
$\overline{PR} \not\cong \overline{QS}$ because they don't have the same distane.

4. The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$. Prove that quadrilateral *MATH* is a parallelogram. Prove that quadrilateral *MATH* is a rectangle. [The use of the set of axes below is optional.]



1) MATH is a parallelogram because it has 2 pairs of opposite sides congruent. Parallelogram MATH is a rectangle because the diagonals are congruent.

2) $d\,\overline{MH} = \sqrt{10^2+6^2} = \sqrt{100+36} = \sqrt{136}$

$d\,\overline{AT} = \sqrt{10^2+6^2} = \sqrt{100+36} = \sqrt{136}$

$d\,\overline{MA} = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$

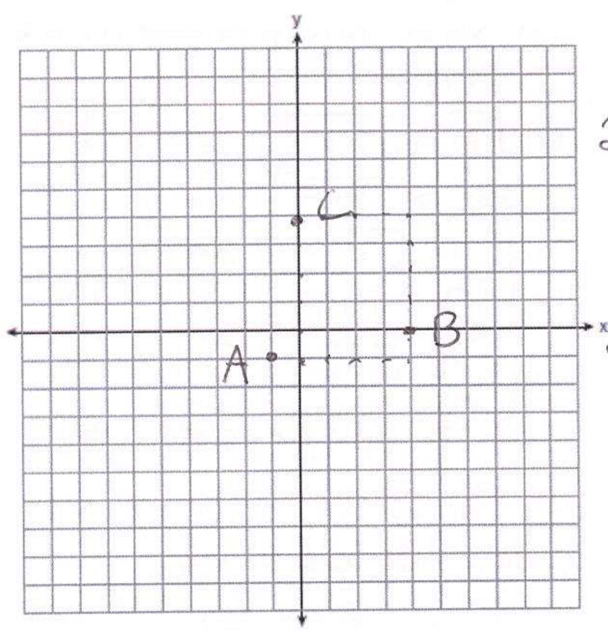
$d\,\overline{HT} = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$

$d\,\overline{MT} = \sqrt{13^2+1^2} = \sqrt{169+1} = \sqrt{170}$

$d\,\overline{AH} = \sqrt{7^2+11^2}\ \sqrt{49+121} = \sqrt{170}$

3) $\overline{MH} \cong \overline{AT}$, $\overline{MA} \cong \overline{HT}$, $\overline{MT} \cong \overline{AH}$ because they have the same distance.

5. Triangle *ABC* has vertices with coordinates $A(-1,-1)$, $B(4,0)$, and $C(0,4)$. Prove that $\triangle ABC$ is an isosceles triangle but *not* an equilateral triangle. [The use of the set of axes below is optional.]



1) $\triangle ABC$ is isosceles because it has two congruent sides. It is not ~~isosceles~~ equilateral because not all sides are congruent.

2) $d\,\overline{AC} = \sqrt{1^2+5^2} = \sqrt{1+25} = \sqrt{26}$

$d\,\overline{AB} = \sqrt{5^2+1^2} = \sqrt{25+1} = \sqrt{26}$

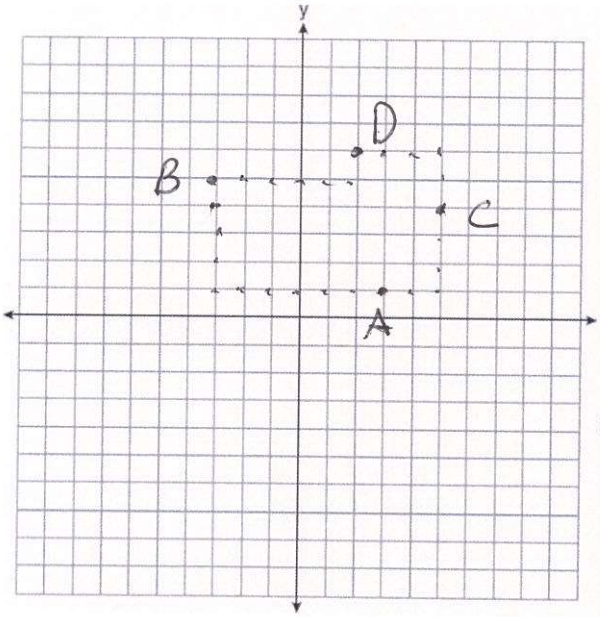
$d\,\overline{CB} = \sqrt{4^2+4^2} = \sqrt{16+16} = \sqrt{32}$

3) $\overline{AC} \cong \overline{AB}$ because they have the same distance.

$\overline{AB} \not\cong \overline{CB}$ because they don't have the same distance.

6. Quadrilateral ABCD has vertices A(3,1) B(-3,5) C(5,4) and D(2,6). Prove quadrilateral ABCD is a trapezoid but *not* an isosceles trapezoid.

1) ABCD is a trapezoid because it has 1 pair of opposite sides parallel. It is not isosceles because it does not have congruent legs.



2) $m\overline{BA} = \frac{-4}{6} = \frac{-2}{3}$

$m\overline{DC} = \frac{-2}{3}\ \frac{-2}{3}$

$d\overline{BD} = \sqrt{5^2+1^2} = \sqrt{25+1} = \sqrt{26}$

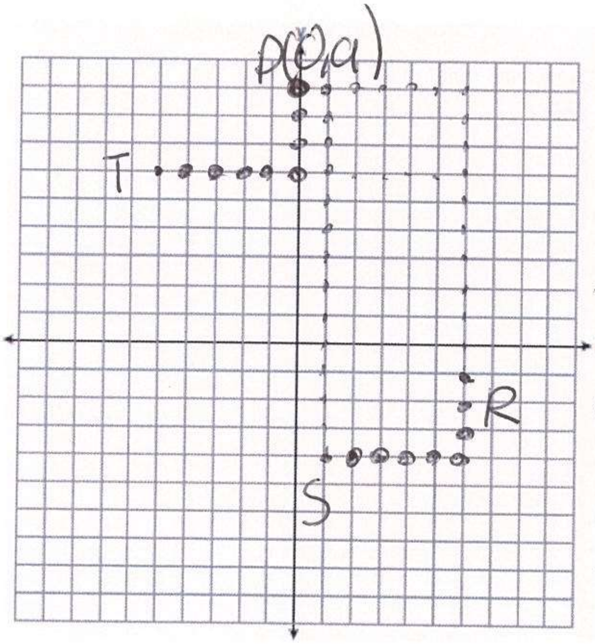
$d\overline{AC} = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$

3) $\overline{BA} \parallel \overline{DC}$ because they have the same slope

$\overline{BD} \not\cong \overline{AC}$ because they don't have the same distance.

7. In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle. State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle. Prove that your quadrilateral $RSTP$ is a rectangle. [The use of the set of axes below is optional.]

1) $\triangle RST$ is a right triangle because its sides fit into Pythagorean theorem



2) $d\overline{RS} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\overline{ST} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\overline{TR} = \sqrt{11^2+7^2} = \sqrt{121+49} = \sqrt{170}$

3) $a^2+b^2=c^2$ $\quad \sqrt{34}^2 + \sqrt{136}^2 = \sqrt{170}^2$

$34+136=170 \quad 170=170$ ✓

1) RSTP is a rectangle because it has 2 pairs of opposite sides congruent and diagonals congruent.

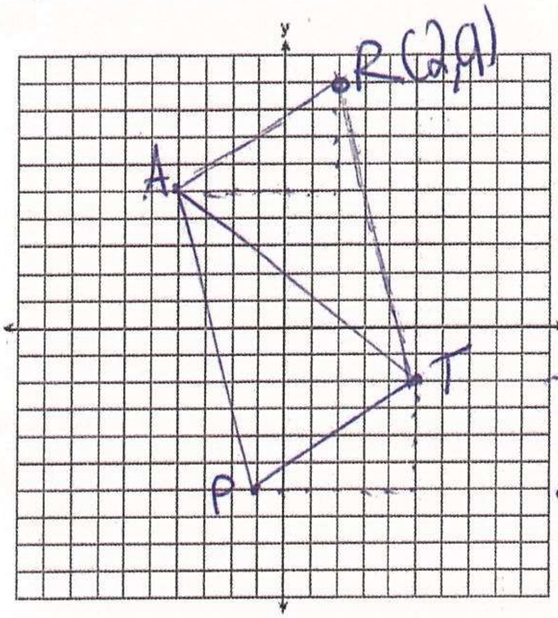
2) $d\overline{TP} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\overline{PR} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\overline{PS} = \sqrt{1^2+13^2} = \sqrt{1+169} = \sqrt{170}$

3) $\overline{RS} \cong \overline{TP}$, $\overline{ST} \cong \overline{PR}$, $\overline{TR} \cong \overline{PS}$ because they have the same distance

8~~4~~. In the coordinate plane, the vertices of triangle PAT are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes below is optional.] State the coordinates of R so that quadrilateral $PART$ is a parallelogram. Prove that quadrilateral $PART$ is a parallelogram.



1) $\triangle PAT$ is an isosceles triangle because it has two congruent sides

2) $dPA = \sqrt{3^2+11^2} = \sqrt{9+121} = \sqrt{130}$

$dTA = \sqrt{9^2+7^2} = \sqrt{81+49} = \sqrt{130}$

3) $\overline{PA} \cong \overline{TA}$ because they have the same distance

1) PART is a parallelogram because it has 2 pairs of opposite sides congruent

2) $m\overline{AR} = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

$mRT = \sqrt{3^2+11^2} = \sqrt{9+121} = \sqrt{130}$

$mPT = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

3) $\overline{AR} \cong \overline{PT}$, $\overline{PA} \cong \overline{RT}$ because they have the same distance

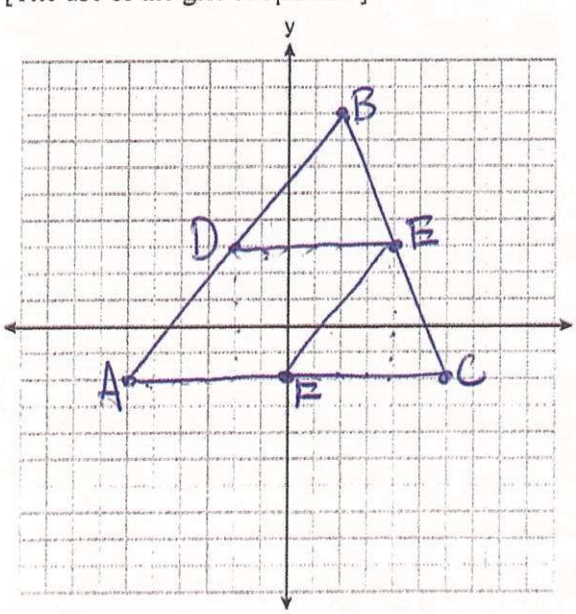
9~~5~~. Given: $\triangle ABC$ with vertices $A(-6,-2)$, $B(2,8)$, and $C(6,-2)$. $\overline{AB}$ has midpoint D, $\overline{BC}$ has midpoint E, and $\overline{AC}$ has midpoint F.

Prove: $ADEF$ is a parallelogram

$ADEF$ is *not* a rhombus

[The use of the grid is optional.]

midpoint = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



F	E	D
midpoint AC	midpoint BC	midpoint AB
$\left(\frac{-6+6}{2}, \frac{-2+-2}{2}\right)$	$\left(\frac{2+6}{2}, \frac{8+-2}{2}\right)$	$\left(\frac{-6+2}{2}, \frac{-2+8}{2}\right)$
$\left(\frac{0}{2}, \frac{-4}{2}\right)$	$\left(\frac{8}{2}, \frac{6}{2}\right)$	$\left(\frac{-4}{2}, \frac{6}{2}\right)$
$(0,-2)$	$(4,3)$	$(-2,3)$

1) ADEF is a parallelogram because it has 2 pairs of opposite sides congruent. It is not a rhombus because not all sides are congruent.

2) $d\overline{AD} = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$

$d\overline{DE} = 6$

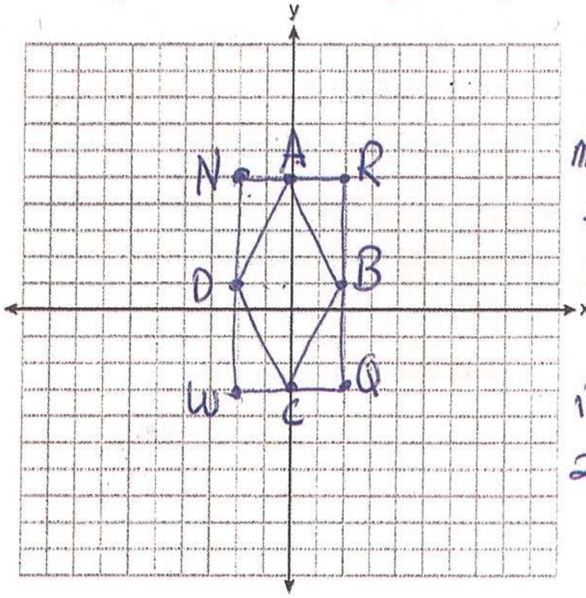
$d\overline{EF} = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$

$d\overline{FA} = 6$

3) $\overline{AD} \cong \overline{EF}$, $\overline{DE} \cong \overline{FA}$ because they have the same distance

$\overline{AD} \not\cong \overline{DE}$ because they don't have the same distance

10 ~~6~~. The vertices of rectangle NRQW are N(-2,5), R(2,5), Q(2,-3), and W(-2,-3). If A is the midpoint $\overline{NR}$, B is the midpoint of $\overline{RQ}$, C is the midpoint of $\overline{QW}$, and D is the midpoint of $\overline{WN}$, prove that ABCD is a ~~parallelogram but *not* a~~ rhombus.



~~Statements~~

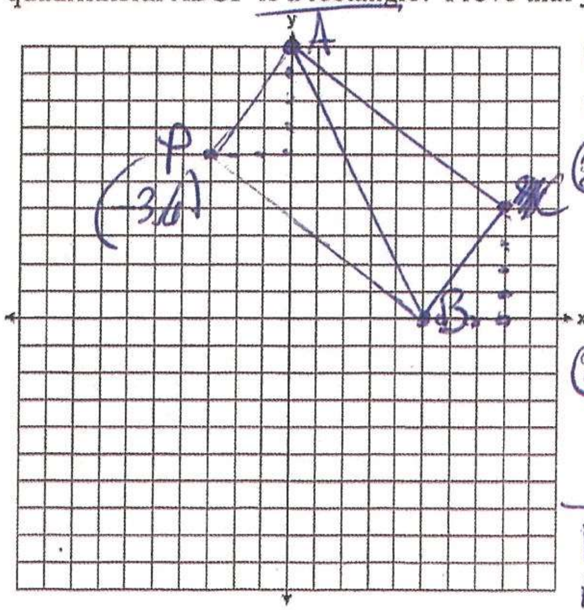
A	B	C	D
midpoint $\overline{NR}$	midpoint $\overline{RQ}$	midpoint $\overline{QW}$	midpoint
$\frac{-2+2}{2}, \frac{5+5}{2}$	$\frac{2+2}{2}, \frac{5+-3}{2}$	$\frac{2+-2}{2}, \frac{-3+-3}{2}$	$\frac{-2+-2}{2}, \frac{5 \ldots}{}$
$\frac{0}{2}, \frac{10}{2}$	$\frac{4}{2}, \frac{2}{2}$	$\frac{0}{2}\ \frac{-6}{2}$	$\frac{-4}{2}, \frac{2}{2}$
(0,5)	(2,1)	(0,-3)	(-2,1)

1) ABCD is a rhombus because all sides are congruent

2) $d\overline{DA} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\overline{AB} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\overline{BC} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\overline{CD} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$

3) $\overline{DA} \cong \overline{AB} \cong \overline{BC} \cong \overline{CD}$ because they have the same distance

11 ~~7~~. In the coordinate plane, the vertices of triangle ABC are A(0,10) B(5,0) and C(8,4). Prove that Triangle ABC is a right triangle. State the coordinates of point *P* such that quadrilateral *ABCP* is a rectangle. Prove that your quadrilateral *ABCP* is a rectangle.



1) $\triangle ABC$ is a right triangle because its sides fit into Pythagorean Theorem

2) $d\overline{BC} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$
$d\overline{CA} = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$
$d\overline{AB} = \sqrt{5^2+10^2} = \sqrt{25+100} = \sqrt{125}$

3) $a^2+b^2=c^2$
$\sqrt{25}^2+\sqrt{100}^2=\sqrt{125}^2$
$125=125$ ✓

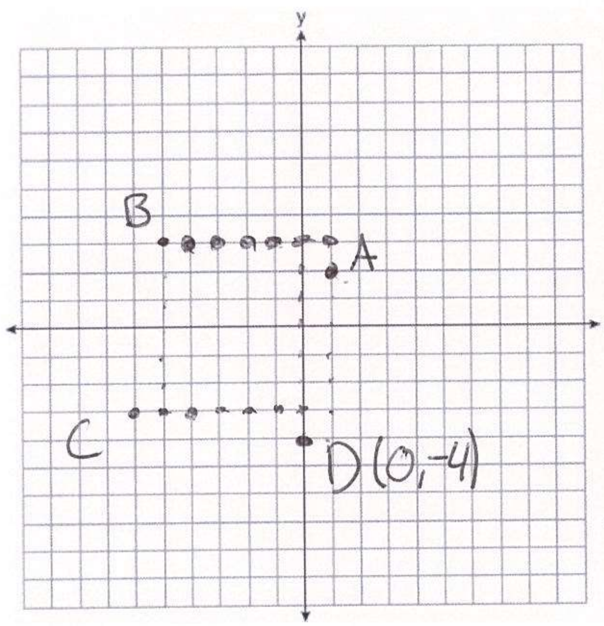
1) ABCP is a rectangle because it has 2 pairs of opposite sides congruent and diagonals congruent

2) $d\overline{PA} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$
$d\overline{PB} = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$
$d\overline{PC} = \sqrt{11^2+2^2} = \sqrt{121+4} = \sqrt{125}$

3) $\overline{PA} \cong \overline{BC}$, $\overline{PB} \cong \overline{AC}$, $\overline{PC} \cong \overline{AB}$ because they have the same distance

D(0,-4)

12. The coordinates of the vertices of $\triangle ABC$ are $A(1,2)$, $B(-5,3)$, and $C(-6,-3)$. Prove that $\triangle ABC$ is isosceles. State the coordinates of point D such that quadrilateral $ABCD$ is a square. Prove that your quadrilateral $ABCD$ is a square. [The use of the set of axes below is optional.]



1) $\triangle ABC$ is isosceles because two sides are congruent.

2) $d\overline{CB} = \sqrt{1^2+6^2} = \sqrt{1+36} = \sqrt{37}$

$d\overline{AB} = \sqrt{6^2+1^2} = \sqrt{36+1} = \sqrt{37}$

3) $\overline{CB} \cong \overline{AB}$ because they have the same distance.

---

1) ABCD is a square because all sides are congruent and diagonals are congruent.

2) $d\overline{AD} = \sqrt{1^2+6^2} = \sqrt{1+36} = \sqrt{37}$

$d\overline{DC} = \sqrt{6^2+1^2} = \sqrt{36+1} = \sqrt{37}$

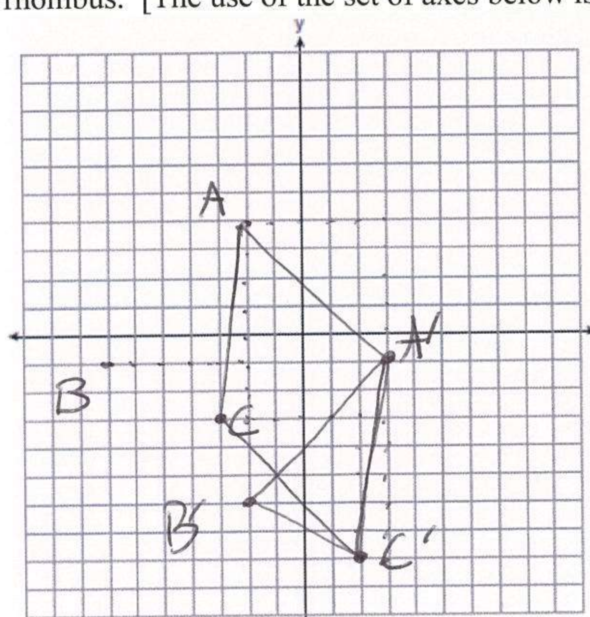
$d\overline{BD} = \sqrt{5^2+7^2} = \sqrt{25+49} = \sqrt{74}$

$d\overline{AC} = \sqrt{7^2+5^2} = \sqrt{49+25} = \sqrt{74}$

3) $\overline{CB} \cong \overline{AB} \cong \overline{AD} \cong \overline{DC}$ because they have the same distance.

$\overline{BD} \cong \overline{AC}$ because they have the same distance.

13. The coordinates of the vertices of $\triangle ABC$ are $A(-2,4)$, $B(-7,-1)$, and $C(-3,-3)$. Prove that $\triangle ABC$ is isosceles. State the coordinates of $\triangle A'B'C'$, the image of $\triangle ABC$, after a translation 5 units to the right and 5 units down. Prove that quadrilateral $AA'C'C$ is a rhombus. [The use of the set of axes below is optional.]



A'(3,-1)
B'(-2,-6)
C'(2,-8)

1) $\triangle ABC$ is isosceles because it has 2 $\cong$ sides.

2) $d\overline{AB} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$d\overline{AC} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$

3) $\overline{AB} \cong \overline{AC}$ because they have the same distance.

---

1) AA'C'C is a rhombus because all sides are congruent

2) $d\overline{AA'} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$d\overline{A'C'} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$

$d\overline{C'C} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$d\overline{CA} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$

3) $\overline{AA'} \cong \overline{A'C'} \cong \overline{C'C} \cong \overline{CA}$ because they have the same distance

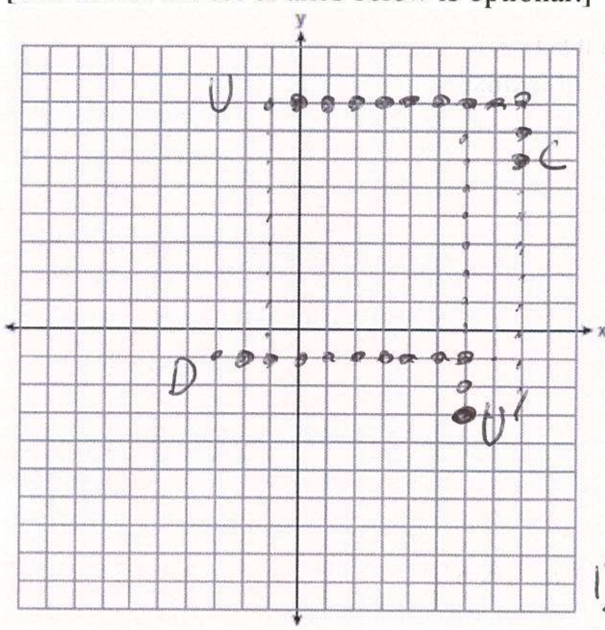
14. Given: Triangle *DUC* with coordinates $D(-3,-1)$, $U(-1,8)$, and $C(8,6)$ U'(6,-3)

Prove: $\triangle DUC$ is a right triangle

Point *U* is reflected over $\overline{DC}$ to locate its image point, *U'*, forming quadrilateral *DUCU'*.

Prove quadrilateral *DUCU'* is a square.

[The use of the set of axes below is optional.]



1) $\triangle DUC$ is a right triangle because its sides fit into Pythagorean Theorem.

2) $d\overline{DU} = \sqrt{2^2+9^2} = \sqrt{4+81} = \sqrt{85}$

$d\overline{UC} = \sqrt{9^2+2^2} = \sqrt{81+4} = \sqrt{85}$

$d\overline{CD} = \sqrt{11^2+7^2} = \sqrt{121+49} = \sqrt{170}$

3) $a^2+b^2=c^2$

$\sqrt{85}^2+\sqrt{85}^2=\sqrt{170}^2$

$85+85=170$

$170=170$ ✓

---

1) DUCU' is a square because all sides are congruent and diagonals are congruent.

2) $d\overline{CU'} = \sqrt{2^2+9^2} = \sqrt{4+81} = \sqrt{85}$

$d\overline{U'D} = \sqrt{9^2+2^2} = \sqrt{81+4} = \sqrt{85}$

$d\overline{UU'} = \sqrt{7^2+11^2} = \sqrt{49+121} = \sqrt{170}$

3) $\overline{UC} \cong \overline{CU'} \cong \overline{U'D} \cong \overline{DU}$ and $\overline{UU'} \cong \overline{DC}$ because they have the same distance.

Coordinate Geometry Applications

Slope: $m = \dfrac{\Delta y}{\Delta x}$

Distance: $d = \sqrt{\Delta x^2 + \Delta y^2}$

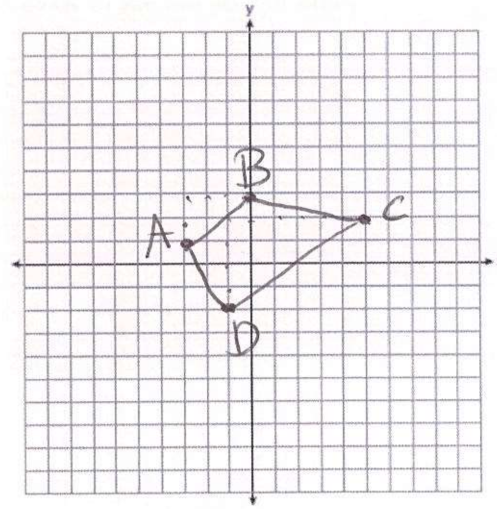
1. A quadrilateral has vertices with coordinates $(-3, 1)$, $(0, 3)$, $(5, 2)$, and $(-1, -2)$. Which type of quadrilateral is this?

1) rhombus
2) rectangle
3) square
4) trapezoid

$m\overline{AB} = \frac{2}{3}$

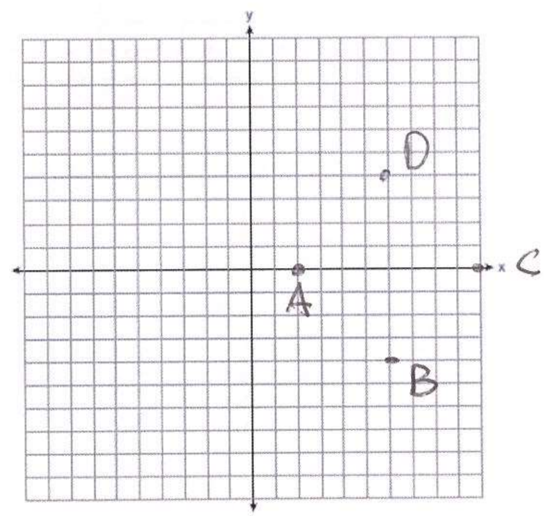
$m\overline{DC} = \frac{4}{6} = \frac{2}{3}$

$\overline{AB} \parallel \overline{DC}$



2. Quadrilateral ABCD has coordinates A(2,0), B(6,-4), C(10,0), and D(6,4). ABCD *cannot* be

1) rhombus
2) rectangle
3) square
4) trapezoid

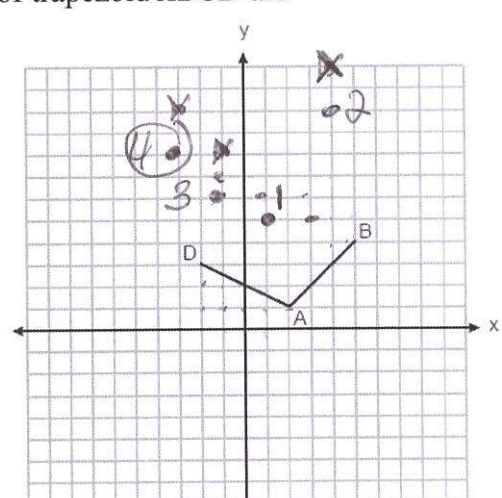


3. On the set of axes below, the coordinates of three vertices of trapezoid *ABCD* are $A(2, 1)$, $B(5, 4)$, and $D(-2, 3)$.
Which point could be vertex *C*?

1) $(1, 5)$
2) $(4, 10)$
3) $(-1, 6)$
4) $(-3, 8)$

$\overline{DA}$ must be $\parallel$ to $\overline{BC}$ (same slope)

$mDA = \frac{-2}{4} = \frac{-1}{2}$



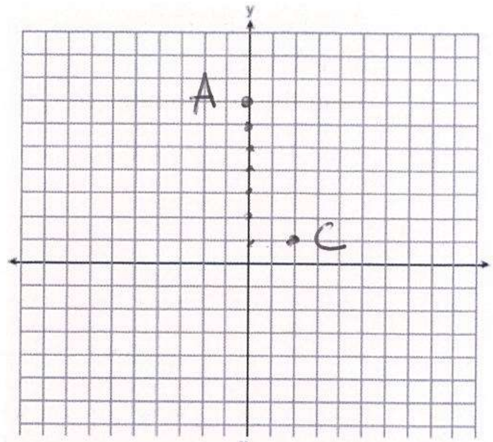
4. Parallelogram $ABCD$ has coordinates $A(0,7)$ and $C(2,1)$. Which statement would prove that $ABCD$ is a rhombus?

1) The midpoint of $\overline{AC}$ is $(1,4)$.
2) The length of $\overline{BD}$ is $\sqrt{40}$.
3) The slope of $\overline{BD}$ is $\frac{1}{3}$.
4) The slope of $\overline{AB}$ is $\frac{1}{3}$.

$\perp$ diagonals
negative reciprocal slopes

$m\overline{AC} = \frac{-6}{2} = -3$

$m\perp = \frac{1}{3}$



5. Parallelogram QRST has coordinates Q(-3,2) and S(6,0). Which statement would prove that QRST is a rectangle?

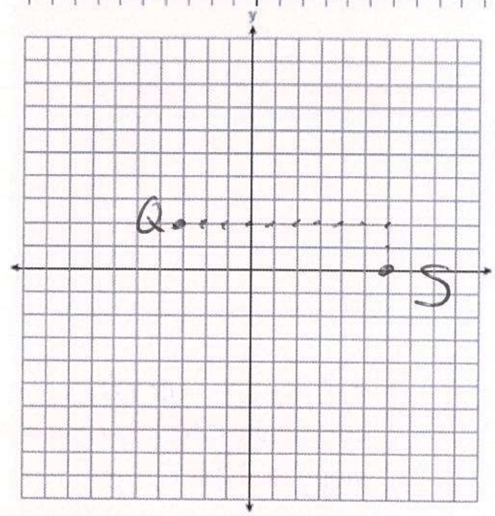
1) The slope of $\overline{RT}$ is $\frac{9}{2}$
2) The length of $\overline{RT}$ is $\sqrt{85}$
3) The midpoint of $\overline{RT}$ is (1.5,1)
4) $\overline{QR} \cong \overline{ST}$

$\cong$ diagonals
same distance

$d\overline{QS} = \sqrt{9^2+2^2}$

$d\overline{QS} = \sqrt{81+4}$

$d\overline{QS} = \sqrt{85}$



6. The diagonals of rhombus $TEAM$ intersect at $P(2,1)$. If the equation of the line that contains diagonal $\overline{TA}$ is $y = -x + 3$, what is the equation of a line that contains diagonal EM?

1) $y = x - 1$
2) $y = x - 3$
3) $y = -x - 1$
4) $y = -x - 3$

$\perp$ diagonals
negative reciprocal slopes

$mTA = -1$

$mEM = 1$

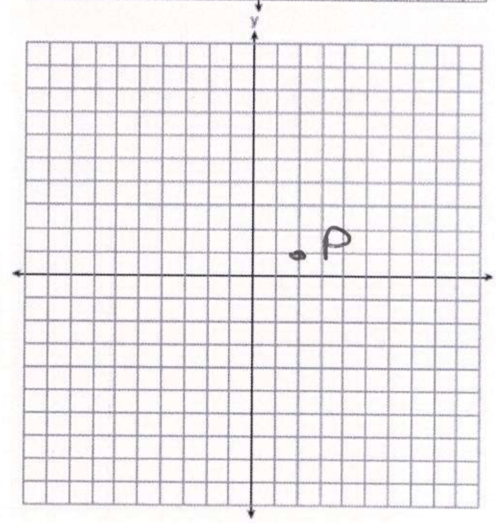
$m\perp = 1$
$x_1 = 2$
$y_1 = 1$

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x-2)$

$y - \cancel{1} = x - 2$
$\quad +1 \qquad +1$

$y = x - 1$

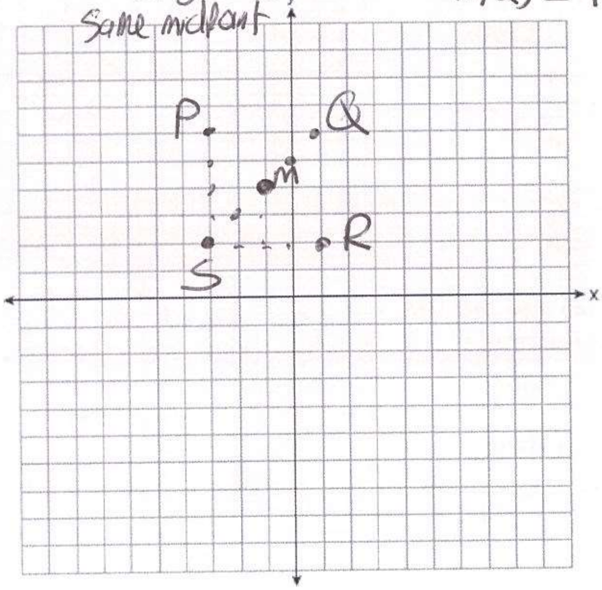


7. Square PQRS has diagonal $\overline{PR}$ with P(-3,6) and R(1,2).
Find the coordinates of Q and S.

(1,6) (-3,2) $m\overline{PR} = \frac{-4}{4} = -1$

$m\overline{QS} = 1$

⊥ diagonals
Same midpoint

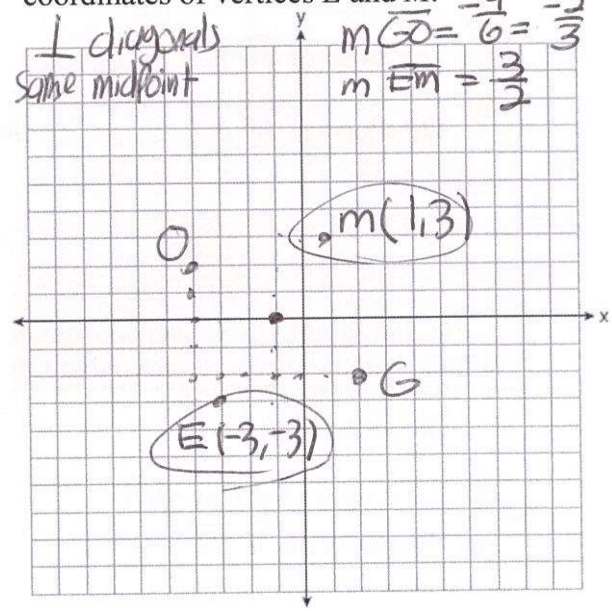


8. In square *GEOM*, the coordinates of G are $(2,-2)$ and the coordinates of O are $(-4,2)$. Determine and state the coordinates of vertices E and M.

⊥ diagonals
Same midpoint

$m\overline{GO} = \frac{-4}{6} = \frac{-2}{3}$

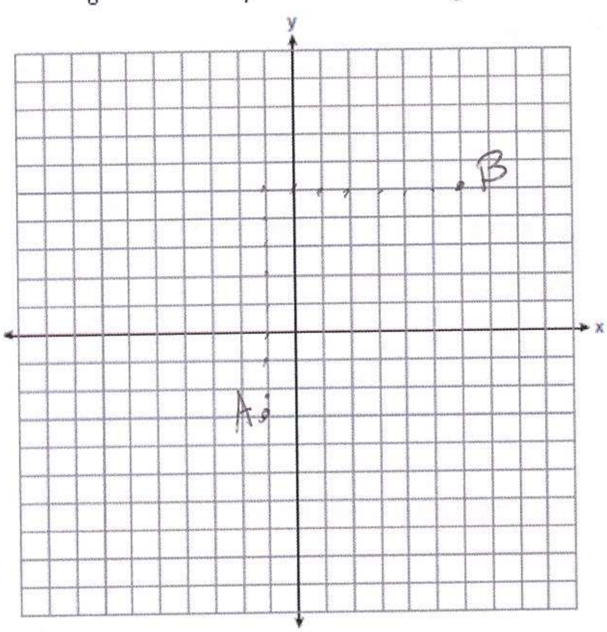
$m\,\overline{Em} = \frac{3}{2}$



9. Rectangle *ABCD* has two vertices at coordinates $A(-1,-3)$ and $B(6,5)$. The slope of $\overline{BC}$ is Consecutive sides ⊥
negative reciprocal slopes

1) $-\frac{7}{8}$ 3) $-\frac{8}{7}$ $m\overline{AB} = \frac{8}{7}$

2) $\frac{7}{8}$ 4) $\frac{8}{7}$ $m\perp = -\frac{7}{8}$



10. Triangle *RST* has vertices with coordinates $R(-3,-2)$, $S(3,2)$ and $T(4,-4)$. Determine and state an equation of the line parallel to $\overline{RT}$ that passes through point S.

(3,2)

Same slope

$m\overline{RT} = \frac{-2}{7}$

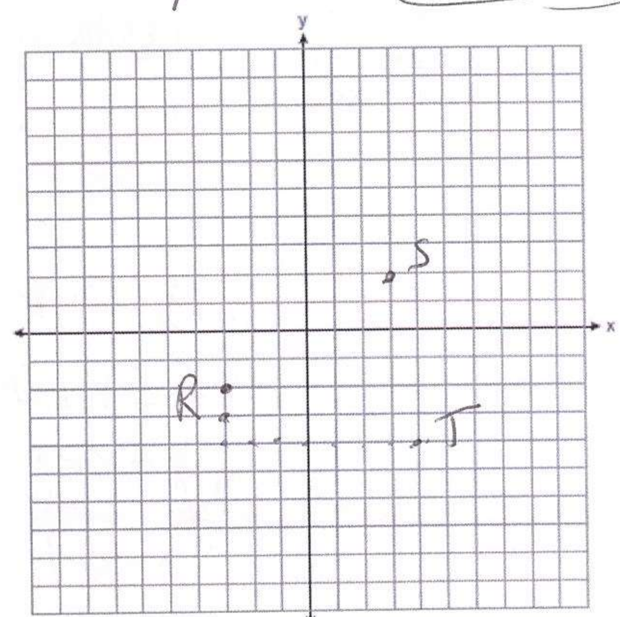
$m\parallel = \frac{-2}{7}$

$x_1 = 3$

$y_1 = 2$

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{-2}{7}(x - 3)$



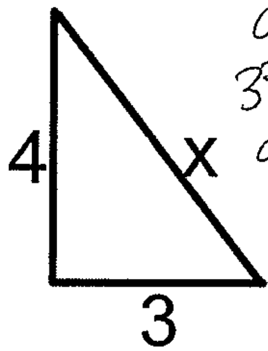
Pythagorean Theorem

Look out for hidden right triangles where you may need to use $a^2 + b^2 = c^2$

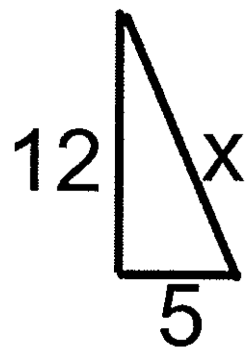
a and *b* are the legs

c is the hypotenuse

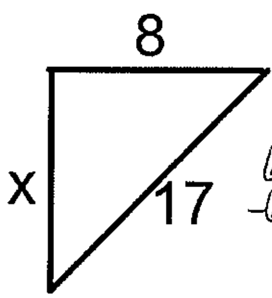
Find the missing side of each right triangle rounding to the nearest tenth if necessary

1. 

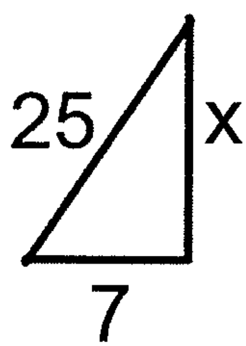
$a^2+b^2=c^2$
$3^2+4^2=x^2$
$9+16=x^2$
$\sqrt{25}=\sqrt{x^2}$
$5=x$

2. 

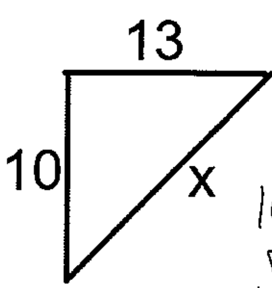
$a^2+b^2=c^2$
$5^2+12^2=x^2$
$25+144=x^2$
$\sqrt{169}=\sqrt{x^2}$
$13=x$

3. 

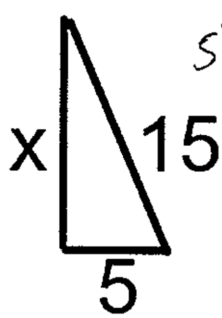
$a^2+b^2=c^2$
$8^2+x^2=17^2$
$64+x^2=289$
$-64 \quad -64$
$\sqrt{x^2}=\sqrt{225}$
$x=15$

4. 

$a^2+b^2=c^2$
$7^2+x^2=25^2$
$49+x^2=625$
$-49 \quad -49$
$\sqrt{x^2}=\sqrt{576}$
$x=24$

5. 

$a^2+b^2=c^2$
$10^2+13^2=x^2$
$100+169=x^2$
$\sqrt{269}=\sqrt{x^2}$
$16.4=x$

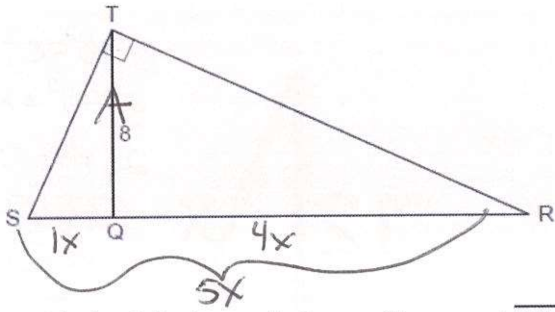
6. 

$a^2+b^2=c^2$
$5^2+x^2=15^2$
$25+x^2=225$
$-25 \quad -25$
$\sqrt{x^2}=\sqrt{200}$
$x=14.1$

Ratios

If you see a ratio, put an x behind each number!

1. Right triangle STR is shown below, with $m\angle T = 90°$. Altitude $\overline{TQ}$ is drawn to $\overline{SQR}$, and $TQ = 8$. If the ratio $SQ:QR$ is 1:4, determine and state the length of $\overline{SR}$.



$\frac{S}{A} = \frac{A}{S}$

$\frac{1x}{8} = \frac{8}{4x}$

$4x^2 = 64$

$\frac{4x^2}{4} = \frac{64}{4}$

$\sqrt{x^2} = \sqrt{16}$

$x = 4$

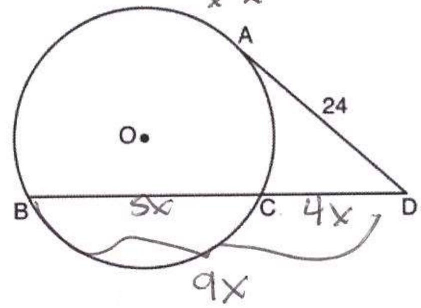
$\overline{SR} = 5x$

$SR = 5(4)$

$SR = 20$

2. Circle O is drawn below with secant $\overline{BCD}$. The length of tangent $\overline{AD}$ is 24. If the ratio of $DC:CB$ is 4:5, what is the length of $\overline{CB}$?

1) 36
2) 20
3) 16
4) 4



$W \cdot E = W \cdot E$

$24 \cdot 24 = 9x \cdot 4x$

$\frac{576}{36} = \frac{36x^2}{36}$

$\sqrt{16} = \sqrt{x^2}$

$4 = x$

$\overline{CB} = 5x$

$\overline{CB} = 5(4)$

$\overline{CB} = 20$

3. The ratio of the measures of the angles of a triangle is 2:3:5. Find the measure of the *smallest* angle of the triangle.

$2x + 3x + 5x = 180$

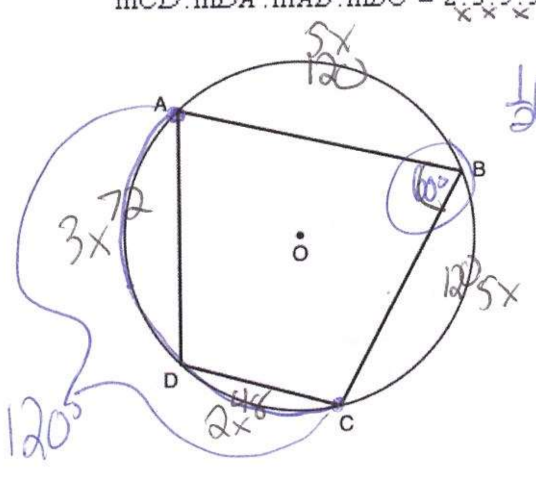
$\frac{10x}{10} = \frac{180}{10}$

$x = 18$

$2x$

$2(18) = 36$

4. In the diagram below, quadrilateral $ABCD$ is inscribed in circle O, and $m\overset{\frown}{CD} : m\overset{\frown}{DA} : m\overset{\frown}{AB} : m\overset{\frown}{BC} = 2:3:5:5$. Determine and state $m\angle B$.



$\frac{1}{2}(120) = 60$

$5x + 5x + 3x + 2x = 360$

$\frac{15x}{15} = \frac{360}{15}$

$x = 24$

$5(24) = 120$

$3(24) = 72$

$2(24) = 48$

Common Core High School Math Reference Sheet
(Algebra I, Geometry, Algebra II)

CONVERSIONS

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilograms	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

FORMULAS

Triangle	$A = \frac{1}{2}bh$	Pythagorean Theorem	$a^2 + b^2 = c^2$
Parallelogram	$A = bh$	Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Circle	$A = \pi r^2$	Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Circle	$C = \pi d$ or $C = 2\pi r$	Geometric Sequence	$a_n = a_1 r^{n-1}$
General Prisms	$V = Bh$	Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1-r}$ where $r \neq 1$
Cylinder	$V = \pi r^2 h$	Radians	$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$
Sphere	$V = \frac{4}{3}\pi r^3$	Degrees	$1 \text{ degree} = \frac{\pi}{180} \text{ radians}$
Cone	$V = \frac{1}{3}\pi r^2 h$	Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$
Pyramid	$V = \frac{1}{3}Bh$		