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Common Core Geometry Regents Review Packet!

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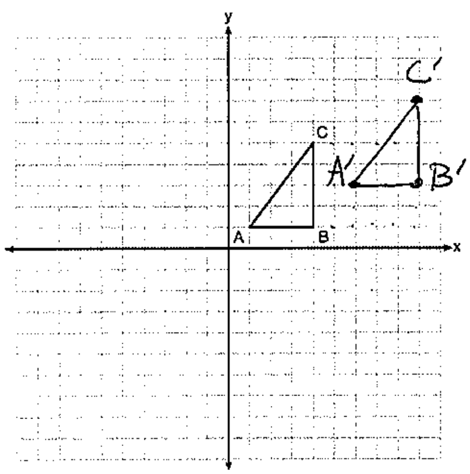
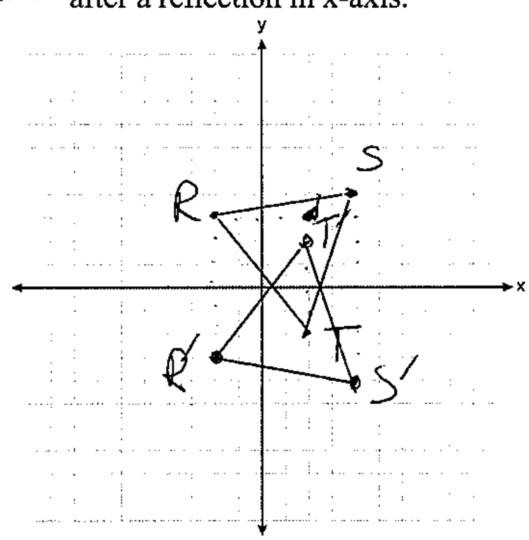
2023

Performing Transformations
Reflections
Flip (Count to what you are reflecting over)
*Switch the coordinates for reflection over $y = x$
y = # is horizontal line, x = # is vertical line. You must graph these lines before you can reflect over them.
Rotations
$R_{90} = (-y, x)$
$R_{180} = (-x, -y)$
$R_{270} = (y, -x)$
Translation
Slide. Count out the translation on the grid
Dilations
If centered at the origin: multiply the coordinates by the scale factor
If centered at a point: Count from the center to each point the number of times of the scale factor.

1. In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. Graph and the image of $\triangle ABC$ after the translation five units to the right and two units up.

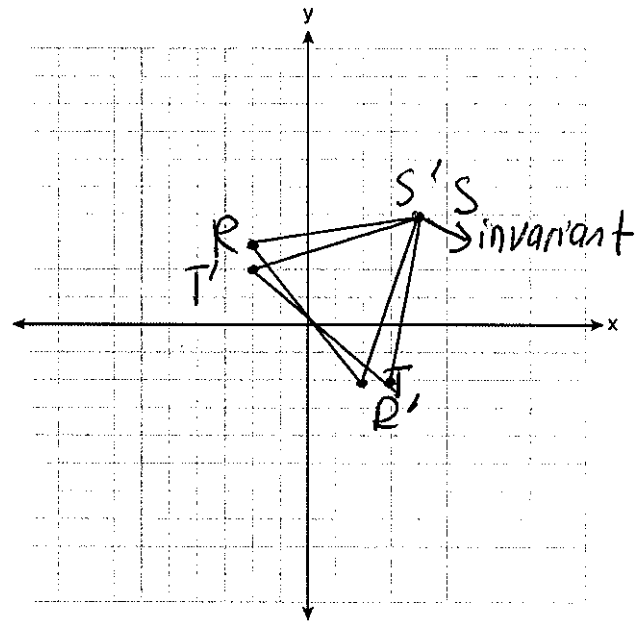


2. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a reflection in x-axis.

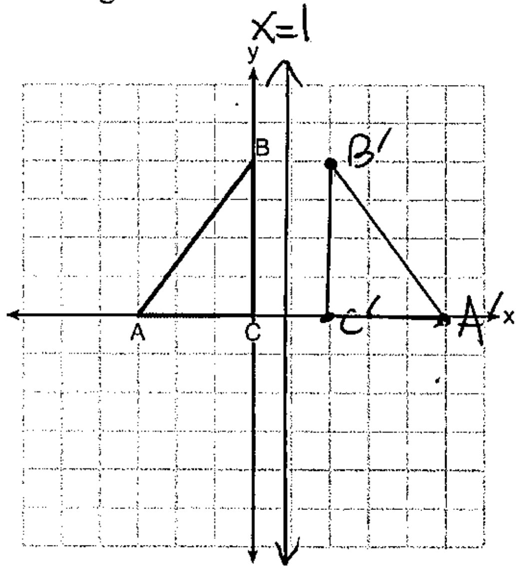


3. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a reflection in the line $y = x$.

$R(-2,3) \xrightarrow{r_{y=x}} (3,-2)$
$S(4,4) \rightarrow (4,4)$
$T(2,-2) \rightarrow (-2,2)$



4. Triangle ABC is graphed on the set of axes below. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection over the line $x = 1$.

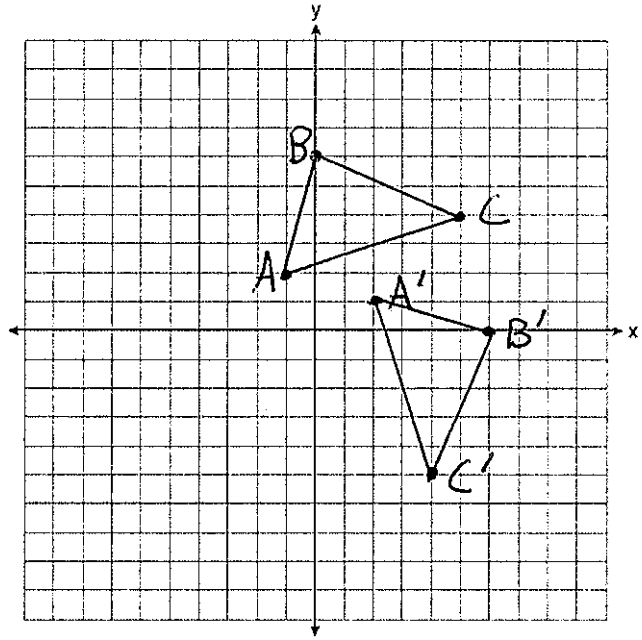


5. On the accompanying set of axes, graph $\triangle ABC$ with coordinates $A(-1,2)$, $B(0,6)$, and $C(5,4)$. Then graph $\triangle A'B'C'$, the image of $\triangle ABC$ after a counter-clockwise rotation of 270 centered at the origin.

$A(-1,2) \xrightarrow{y,-x} (2,1)$

$B(0,6) \rightarrow (6,0)$

$C(5,4) \rightarrow (4,-5)$

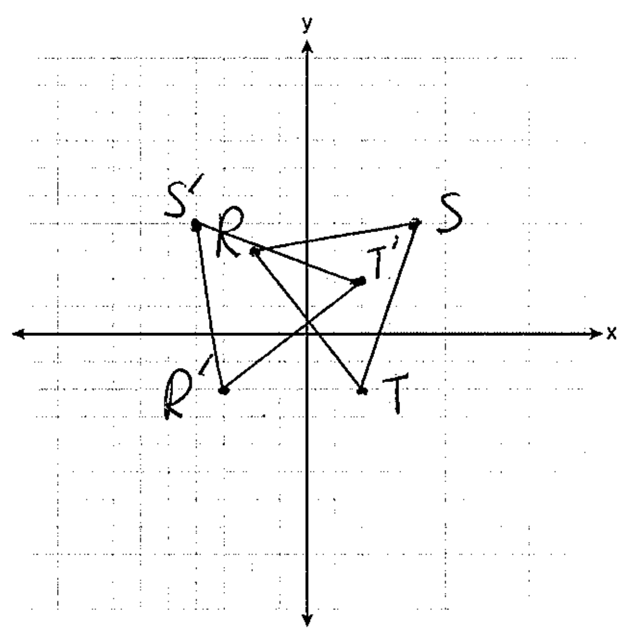


6. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a counter-clockwise rotation of 90 centered at the origin.

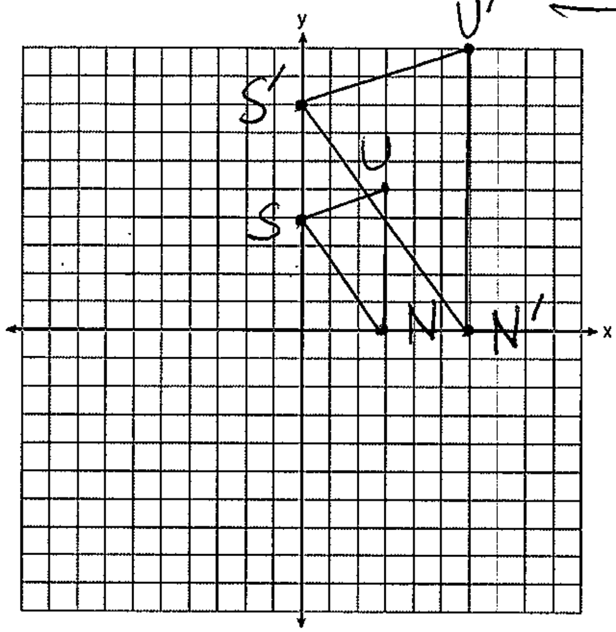
$R(-2,3) \xrightarrow{-y,x} (-3,-2)$

$S(4,4) \rightarrow (-4,4)$

$T(2,-2) \rightarrow (2,2)$



7. Triangle SUN has coordinates $S(0,4)$, $U(3,5)$, and $N(3,0)$. On the accompanying grid, draw and label ΔSUN. Then, graph and state the coordinates of $\Delta S'U'N'$, the image of ΔSUN after a dilation of 2 centered at the origin. multiply



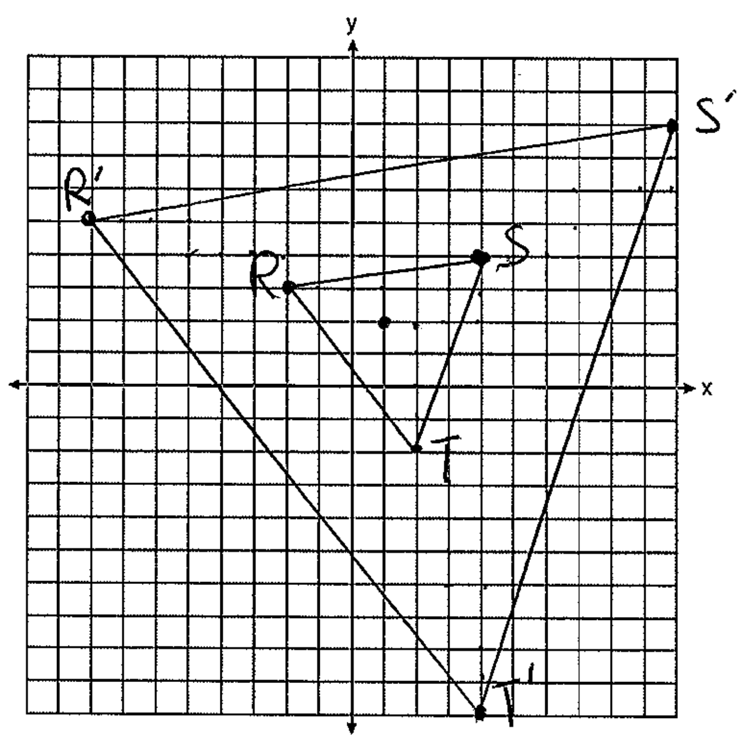
$S(0,4) \xrightarrow{\cdot 2} (0,8)\ S'$

$U(3,5) \rightarrow (6,10)\ U'$

$N(3,0) \rightarrow (6,0)\ N'$

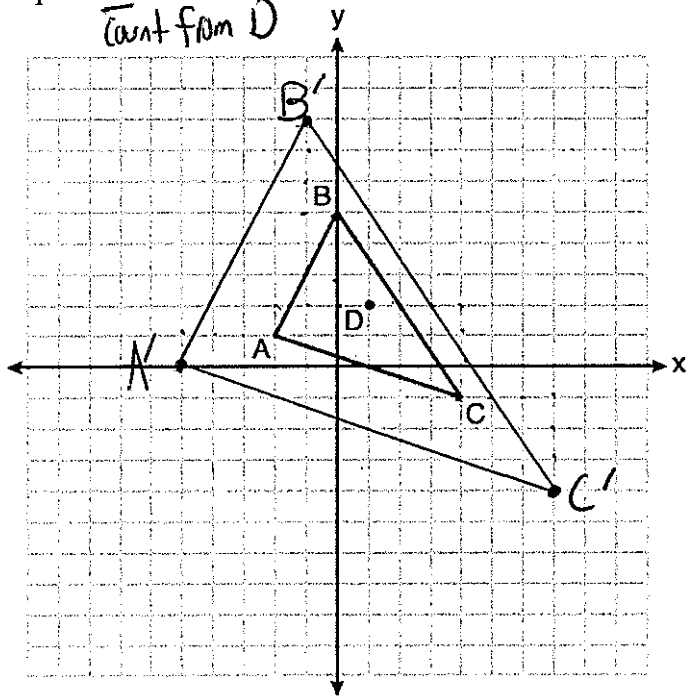
8. The coordinates of the vertices of ΔRST are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph ΔRST and $\Delta R'S'T'$, the image of ΔRST after a dilation of 3 centered at (1,2).

Count from the center

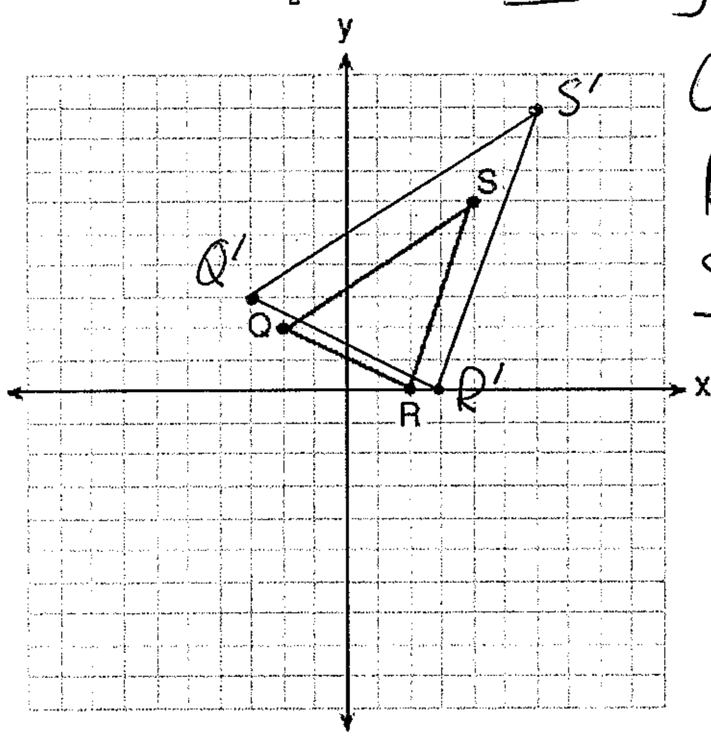


9. Triangle ABC and point $D(1, 2)$ are graphed on the set of axes below.
Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point D.

Count from D



10. Triangle QRS is graphed on the set of axes below.
On the same set of axes, graph and label $\triangle Q'R'S'$, the image of $\triangle QRS$ after a dilation with a scale factor of $\frac{3}{2}$ centered at the origin. multiply



$Q(-2,2) \xrightarrow{\cdot\frac{3}{2}} (-3,3)$

$R(2,0) \rightarrow (3,0)$

$S(4,6) \rightarrow (6,9)$

Rigid Motion Properties

A rigid motion preserves size and angle measure producing a congruent figure
They all produce a congruent figure except dilation.

1. Which transformation would *not* always produce an image that would be congruent to the original figure?
1) translation ~~2)~~ dilation 3) rotation 4) reflection

2. The vertices of ΔJKL have coordinates $J(5,1)$, $K(-2,-3)$, and $L(-4,1)$. Under which transformation is the image $\Delta J'K'L'$ *not* congruent to ΔJKL?
1) a translation of two units to the right and two units down
2) a counterclockwise rotation of 180 degrees around the origin
3) a reflection over the x-axis
~~4)~~ a dilation with a scale factor of 2 and centered at the origin

3. If $\triangle A'B'C'$ is the image of $\triangle ABC$, under which transformation will the triangles *not* be congruent?
1) reflection over the x-axis
2) translation to the left 5 and down 4
~~3)~~ dilation centered at the origin with scale factor 2
4) rotation of 270° counterclockwise about the origin

4. Under which transformation would $\triangle A'B'C'$, the image of $\triangle ABC$, *not* be congruent to $\triangle ABC$?
1) reflection over the y-axis
2) rotation of 90° clockwise about the origin
3) translation of 3 units right and 2 units down
~~4)~~ dilation with a scale factor of 2 centered at the origin

5. The image of $\triangle DEF$ is $\triangle D'E'F'$. Under which transformation will he triangles *not* be congruent?
1) a reflection through the origin
2) a reflection over the line $y = x$
3) a dilation with a scale factor of 1 centered at $(2,3)$
~~4)~~ a dilation with a scale factor of $\frac{3}{2}$ centered at the origin

6. The vertices of $\triangle PQR$ have coordinates $P(2,3)$, $Q(3,8)$, and $R(7,3)$. Under which transformation of $\triangle PQR$ are distance and angle measure preserved?
1) $(x,y) \rightarrow (2x, 3y)$ dilations
2) $(x,y) \rightarrow (x+2, 3y)$ dilation
3) $(x,y) \rightarrow (2x, y+3)$ dilation
~~4)~~ $(x,y) \rightarrow (x+2, y+3)$

7. Which transformation would result in the perimeter of a triangle being different from the perimeter of its image?
1) $(x,y) \rightarrow (y,x)$
2) $(x,y) \rightarrow (x,-y)$
~~3)~~ $(x,y) \rightarrow (4x,4y)$
4) $(x,y) \rightarrow (x+2, y-5)$

dilations

Identifying Transformations

Check for orientation!!! (The direction of the letters)

The only transformation that changes orientation is a line reflection (an even amount of reflections will preserve orientation).

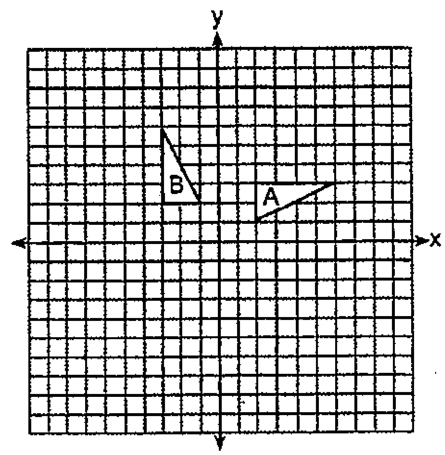
Translation = slide

Rotation = turn

Reflection = flip

Dilation = change size (enlarge or shrink)

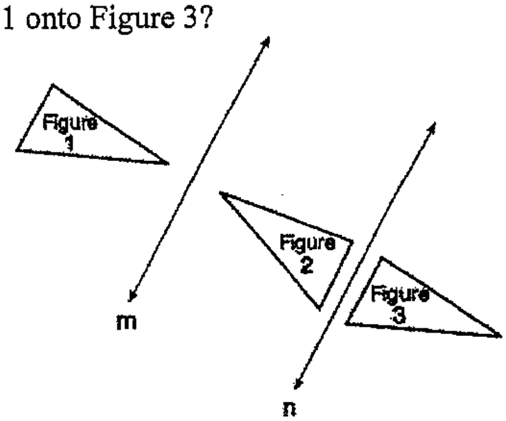
1. In the diagram below, which single transformation was used to map triangle *A* onto triangle *B*?



1) line reflection
2) rotation
3) dilation
4) translation

2. In the diagram below, line *m* is parallel to line *n*. Figure 2 is the image of Figure 1 after a reflection over line *m*. Figure 3 is the image of Figure 2 after a reflection over line *n*. Which single transformation would carry Figure 1 onto Figure 3?

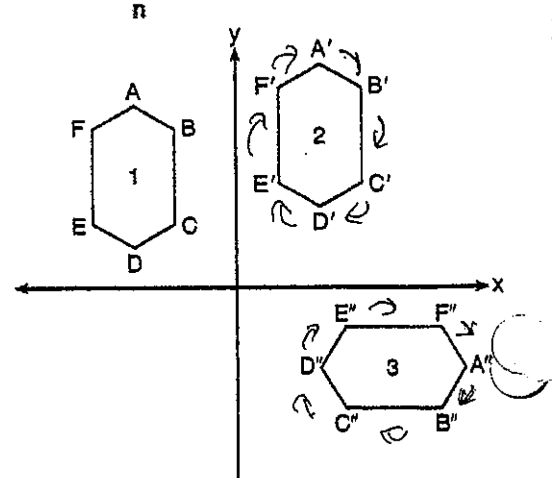
1) a dilation
2) a rotation
3) a reflection
4) a translation



3. In the diagram below, congruent figures 1, 2, and 3 are drawn.

Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?

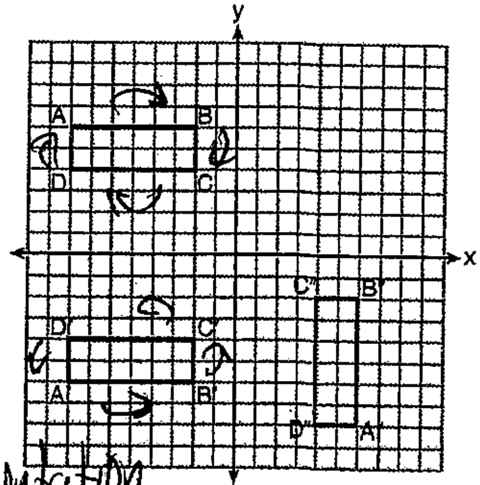
1) a reflection followed by a translation
2) a rotation followed by a translation
3) a translation followed by a reflection
4) a translation followed by a rotation



same orientation

4. A sequence of transformations maps rectangle $ABCD$ onto rectangle $A''B''C''D''$, as shown in the diagram below.
Which sequence of transformations maps $ABCD$ onto $A'B'C'D'$ and then maps $A'B'C'D'$ onto $A''B''C''D''$?

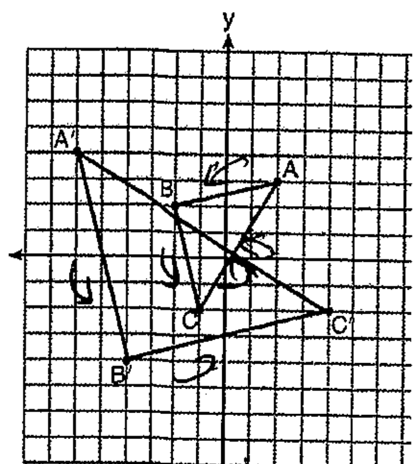
1) a reflection followed by a rotation
2) a reflection followed by a translation
3) a translation followed by a rotation
4) a translation followed by a reflection



opposite orientation

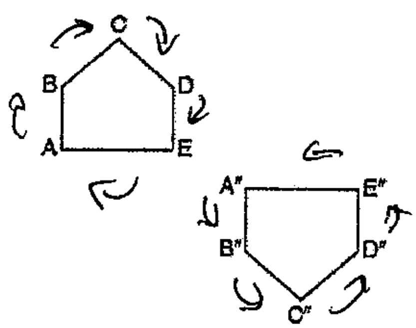
5. Which sequence of transformations will map $\triangle ABC$ onto $\triangle A'B'C'$?

1) reflection and translation
2) rotation and reflection
3) translation and dilation
4) dilation and rotation



same orientation

6. Identify which sequence of transformations could map pentagon $ABCDE$ onto pentagon $A''B''C''D''E''$, as shown below.

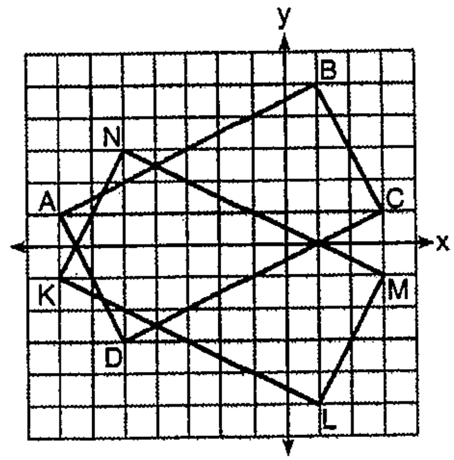


opposite orientation
reflection

1) dilation followed by a rotation
2) translation followed by a rotation
3) line reflection followed by a translation
4) line reflection followed by a line reflection

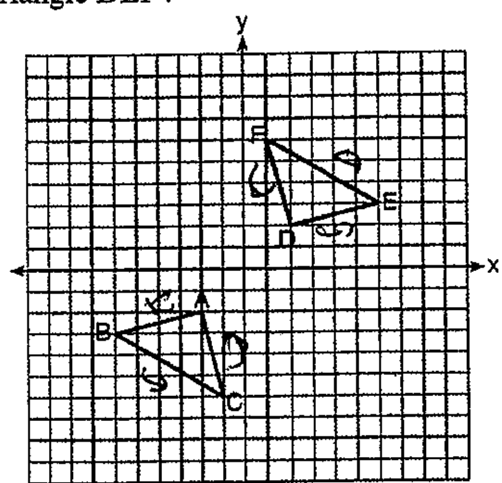
7. On the set of axes below, rectangle *ABCD* can be proven congruent to rectangle *KLMN* using which transformation?

1) rotation
2) translation
3) reflection over the x-axis
4) reflection over the y-axis



8. Triangle *ABC* and triangle *DEF* are graphed on the set of axes below. Which sequence of transformations maps triangle *ABC* onto triangle *DEF*?

1) a reflection over the x-axis followed by a reflection over the y-axis
2) a 180° rotation about the origin followed by a reflection over the line $y = x$
3) a 90° clockwise rotation about the origin followed by a reflection over the y-axis
4) a translation 8 units to the right and 1 unit up followed by a 90° counterclockwise rotation about the origin

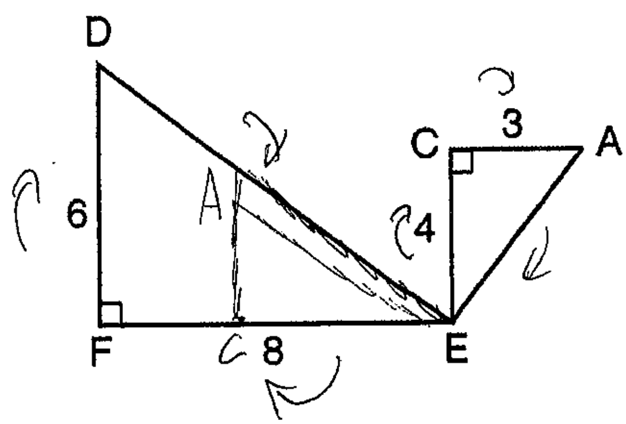


Same orientation

9. Given: $\triangle AEC$, $\triangle DEF$, and $\overline{FE} \perp \overline{CE}$

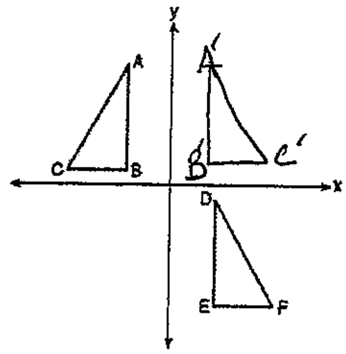
What is a correct sequence of similarity transformations that shows $\triangle AEC \sim \triangle DEF$?

1) a rotation of 180 degrees about point E followed by a horizontal translation
2) a counterclockwise rotation of 90 degrees about point E followed by a horizontal translation
3) a rotation of 180 degrees about point E followed by a dilation with a scale factor of 2 centered at point E
4) a counterclockwise rotation of 90 degrees about point E followed by a dilation with a scale factor of 2 centered at point E



Same orientation

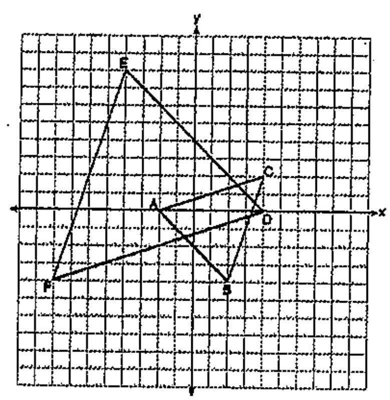
10. In the diagram below, $\triangle ABC \cong \triangle DEF$.



Which sequence of transformations maps $\triangle ABC$ onto $\triangle DEF$?

1) a reflection over the x-axis followed by a translation
2) a reflection over the y-axis followed by a translation
3) a rotation of 180° about the origin followed by a translation
4) a counterclockwise rotation of 90° about the origin followed by a translation

11. On the set of axes below, $\triangle ABC$ has vertices at $A(-2,0)$, $B(2,-4)$, $C(4,2)$, and $\triangle DEF$ has vertices at $D(4,0)$, $E(-4,8)$, $F(-8,-4)$.



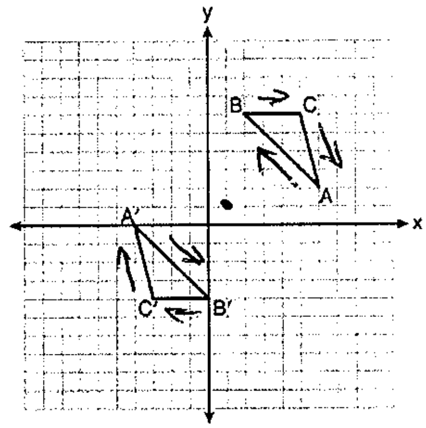
Which sequence of transformations will map $\triangle ABC$ onto $\triangle DEF$?

1) a dilation of $\triangle ABC$ by a scale factor of 2 centered at point A
2) a dilation of $\triangle ABC$ by a scale factor of $\frac{1}{2}$ centered at point A
3) a dilation of $\triangle ABC$ by a scale factor of 2 centered at the origin, followed by a rotation of 180° about the origin
4) a dilation of $\triangle ABC$ by a scale factor of $\frac{1}{2}$ centered at the origin, followed by a rotation of 180° about the origin

Same orientation

12. On the set of axes below, $\triangle ABC \cong \triangle A'B'C'$.
Triangle ABC maps onto $\triangle A'B'C'$ after a

1) ~~reflection over the line $y = -x$~~
2) ~~reflection over the line $y = -x + 2$~~
3) rotation of 180° centered at (1,1)
4) rotation of 180° centered at the origin



13. On the set of axes below, pentagon $ABCDE$ is congruent to $A''B''C''D''E''$.
Which describes a sequence of rigid motions that maps $ABCDE$ onto $A''B''C''D''E''$?

Same orientation

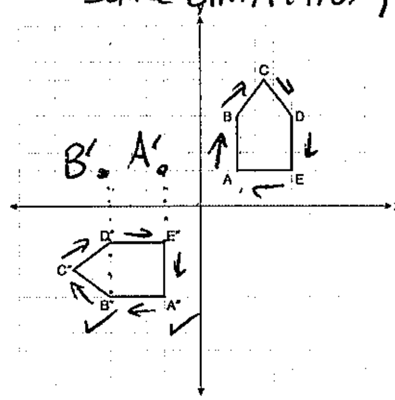
Perform the rotation

$-y, x$

$A(2,2) \to (-2,2)$

$B(2,5) \to (-5,2)$

1) ~~a rotation of 90° counterclockwise about the origin followed by a reflection over the x-axis~~
2) a rotation of 90° counterclockwise about the origin followed by a translation down 7 units
3) a reflection over the y-axis followed by a reflection over the x-axis
4) ~~a reflection over the x-axis followed by a rotation of 90° counterclockwise about the origin~~



14. On the set of axes below, $\triangle LET$ and $\triangle L''E''T''$ are graphed in the coordinate plane where $\triangle LET \cong \triangle L''E''T''$.
Which sequence of rigid motions maps $\triangle LET$ onto $\triangle L''E''T''$?

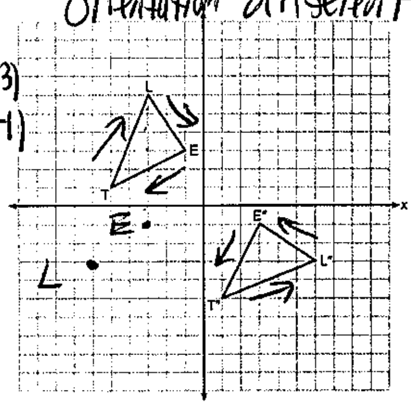
Orientation different

$-y, x$

$L(-3,6) \to (-6,-3)$

$E(-1,3) \to (-3,-1)$

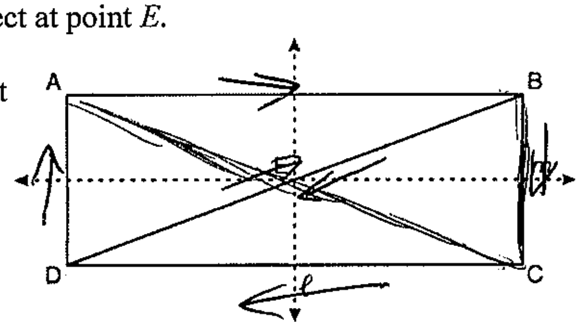
1) ~~a reflection over the y-axis followed by a reflection over the x-axis~~
2) ~~a rotation of 180° about the origin~~
3) a rotation of 90° counterclockwise about the origin followed by a reflection over the y-axis
4) a reflection over the x-axis followed by a rotation of 90° clockwise about the origin



15. In the diagram below, $ABCD$ is a rectangle, and diagonal $\overline{BD}$ is drawn. Line ℓ, a vertical line of symmetry, and line m, a horizontal line of symmetry, intersect at point E.
Which sequence of transformations will map $\triangle ABD$ onto $\triangle CDB$?

1) ~~a reflection over line ℓ followed by a 180° rotation about point E~~
2) a reflection over line ℓ followed by a reflection over line m
3) ~~a 180° rotation about point B~~
4) ~~a reflection over $\overline{DB}$~~

Orientation ~~different~~ Same



Rigid Motion Proofs

To prove triangles are congruent/similar using rigid motions/transformations

1) Identify the transformations (**Check for orientation to determine if reflection**)
On the grid: reflect/rotate/dilate first
Off the grid: translate first
Translate ________ to ________
Reflect Δ________ over
Rotate Δ________ about point __until it maps onto Δ________
Dilation Δ________ centered at point ____ by a scale factor of $\frac{image}{original}$

Congruence	Similarity
2) A _______ and _______ are rigid motions. 3) **A rigid motion preserves size and angle measure producing a congruent figure.**	2) A dilation and ________ preserve angle measure producing a similar figure.

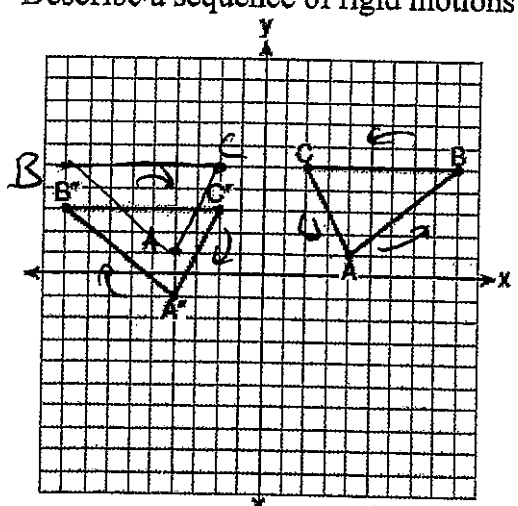
1. Triangle $A'B'C'$ is the image of triangle ABC after a translation of 2 units to the right and 3 units up. Is triangle ABC congruent to triangle $A'B'C'$? Explain why.

2) Yes, a translation is a rigid motion.
3) A rigid motion preserves size and angle measure producing a congruent figure.

2. After a reflection over a line, $\Delta A'B'C'$ is the image of ΔABC. Explain why triangle ABC is congruent to triangle $\Delta A'B'C'$.

2) A reflection is a rigid motion.
3) A rigid motion preserves size and angle measure producing a congruent figure.

3. The graph below shows ΔABC and its image, $\Delta A''B''C''$.
Describe a sequence of rigid motions which would map ΔABC onto $\Delta A''B''C''$.

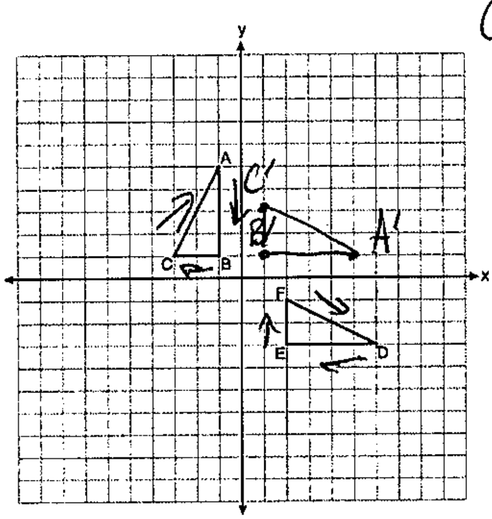


1) Reflect Δ ABC over the y axis followed by a translation 2 units down.

orientation opposite
line reflection.

4. On the set of axes below, $\triangle ABC$ and $\triangle DEF$ are graphed. Describe a sequence of rigid motions that would map $\triangle ABC$ onto $\triangle DEF$.

Orientation same
rotation $y, -x$

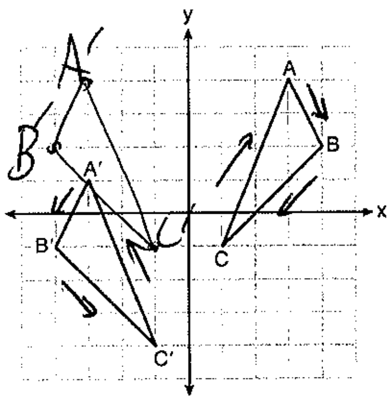


$A(-1,5) \rightarrow (5,1)$
$B(-1,1) \rightarrow (1,1)$
$C(-3,1) \rightarrow (1,3)$

Rotate $\triangle ABC$ counter-clockwise 270° centered at the origin followed by a translation right 1 and down 4.

5. As graphed on the set of axes below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a sequence of transformations.
Is $\triangle A'B'C'$ congruent to $\triangle ABC$? Use the properties of rigid motion to explain your answer.

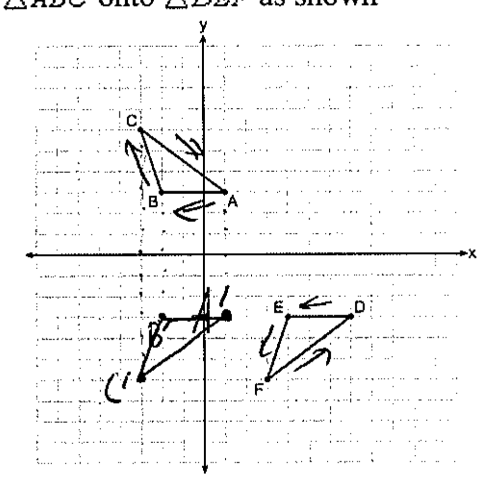
Orientation different
reflection



1) Reflect $\triangle ABC$ over the y-axis followed by a translation down 3 units.
2) Yes, a reflection and translation are rigid motions.
3) A rigid motion preserves size and angle measure producing a congruent figure.

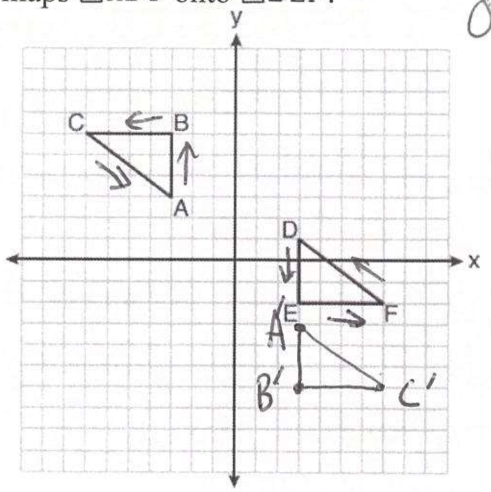
6. Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.

Orientation different
reflection



1) Reflect $\triangle ABC$ over the x-axis followed by a translation 6 units to the right.

7. On the set of axes below, $\triangle ABC \cong \triangle DEF$. Describe a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle DEF$.

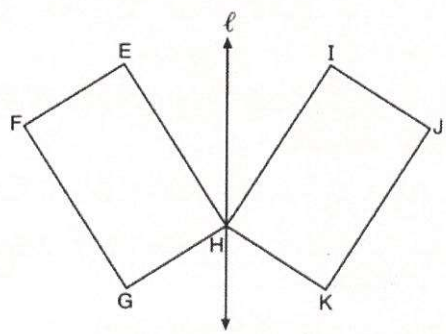


Orientation same
rotation

$A(-3,3) \xrightarrow{-x,-y} (3,-3)$
$B(-3,6) \rightarrow (3,-6)$
$C(-7,6) \rightarrow (7,-6)$

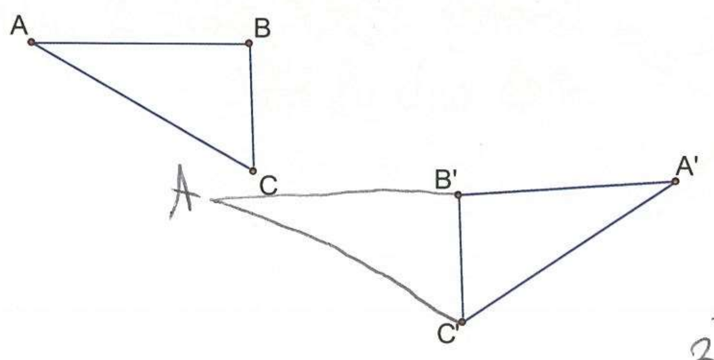
1) rotate $\triangle ABC$ 180° counter-clockwise centered at the origin followed by a translation 4 units up.

8. In the diagram below, parallelogram *EFGH* is mapped onto parallelogram *IJKH* after a reflection over line ℓ. Use the properties of rigid motions to explain why parallelogram *EFGH* is congruent to parallelogram *IJKH*.



A reflection is a rigid motion.
A rigid motion preserves size and angle measure producing a congruent figure.

9. Prove that $\Delta ABC \cong \Delta A'B'C'$ using rigid motions.



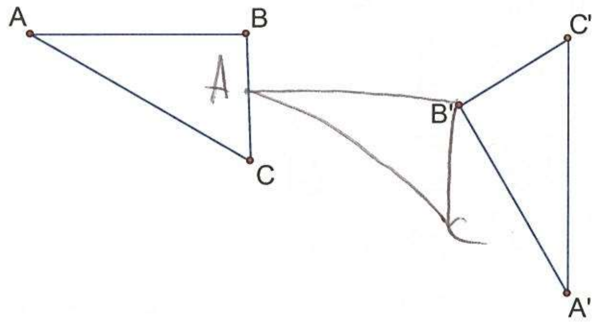
1) Translate $\overline{BC}$ to $\overline{B'C'}$ followed by reflecting $\triangle ABC$ over $\overline{BC}$

2) A translation and reflection are rigid motions.

3) A rigid motion preserves size and angle measure producing a congruent figure.

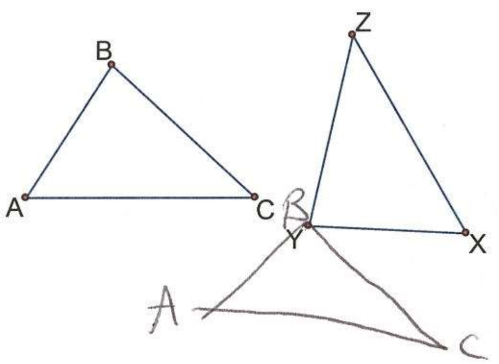
10. Prove that $\Delta ABC \cong \Delta A'B'C'$ using rigid motions.

1) Translate B to B' followed by rotating ΔABC about B until it maps onto $\Delta A''B''C''$.
2) A translation and rotation are rigid motions.
3) A rigid motion preserves size and angle measure producing a congruent figure.



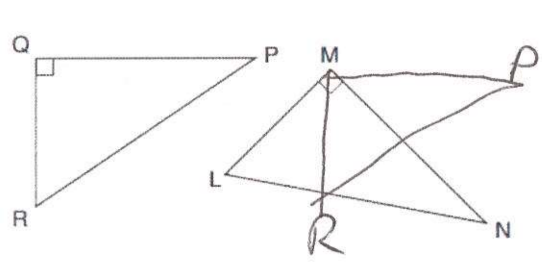
11. Prove that $\Delta ABC \cong \Delta XYZ$ using rigid motions.

1) Translate B to Y followed by rotating ΔABC about B until it maps onto ΔXYZ.
2) A translation and rotation are rigid motions.
3) A rigid motion preserves size and angle measure producing a congruent figure.



12. In the diagram below, right triangle *PQR* is transformed by a sequence of rigid motions that maps it onto right triangle *NML*. Identify the sequence of rigid motions that was performed.

1) Translate Q to M followed by rotating ΔPQR about Q until it maps onto ΔNML.



Regular Polygon Rotations

To determine the minimum number of degrees a regular polygon must be rotated to be mapped onto itself:

1) The minimum rotation is $\frac{360}{n}$.
2) **Any multiple of that will also map the regular polygon onto itself!**

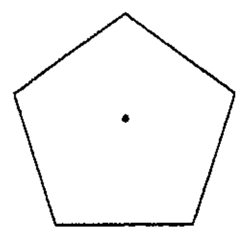
1. What is the minimum number of degrees a regular decagon must be rotated to be mapped onto itself?

$\frac{360}{n}$ $\frac{360}{10} = 36°$

2. What is the minimum number of degrees a regular hexagon must be rotated to be carried onto itself?

$\frac{360}{n}$ $\frac{360}{6} = 60°$

3. A regular pentagon is shown in the diagram below.



If the pentagon is rotated clockwise around its center, the minimum number of degrees it must be rotated to carry the pentagon onto itself is

1) 54°
~~2) 72°~~
3) 108°
4) 360°

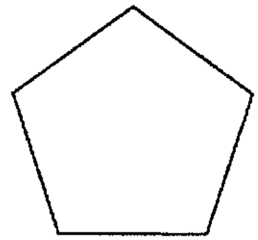
$\frac{360}{n}$ $\frac{360}{5} = 72°$

4. Which regular polygon has a minimum rotation of 45° to carry the polygon onto itself?

~~(1)~~ octagon $\frac{360}{8} = 45$ (3) hexagon $\frac{360}{6} = 60$
(2) decagon $\frac{360}{10} = 36$ (4) pentagon $\frac{360}{5} = 72$

5. The regular polygon below is rotated about its center. Which angle of rotation will carry the figure onto itself?

1) 60°
2) 108°
3) 216° $\frac{216}{72}=3$ $\frac{360}{5}=72$
4) 540°



6. Which rotation would map a regular hexagon onto itself?

1) 45°
2) 150°
3) 240° $\frac{240}{60}=4$
4) 315°

$\frac{360}{6}=60$

7. Which rotation about its center will carry a regular decagon onto itself?

1) 54°
2) 162°
3) 198°
4) 252°

$\frac{360}{10}=36$ $\frac{252}{36}=7$

8. Which rotation about its center will carry a regular octagon onto itself?

1) 80°
2) 315° $\frac{315}{45}=7$
3) 280°
4) 120°

$\frac{360}{8}=45$

9. Which of the following rotations would <u>not</u> map a regular pentagon onto itself?

1) 144
2) 120
3) 216
4) 720

$\frac{360}{5}=72$

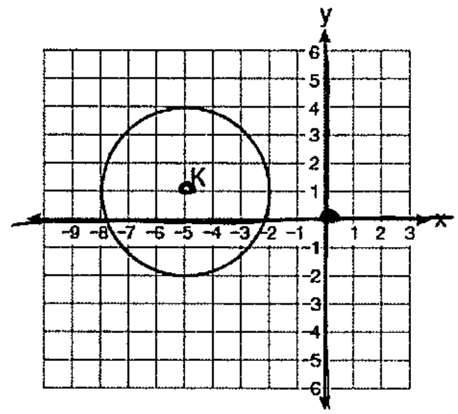
10. Which of the following rotations would not map an equilateral triangle onto itself?

1) 120°
2) 240°
3) 180°
4) 480°

$\frac{360}{3}=120$

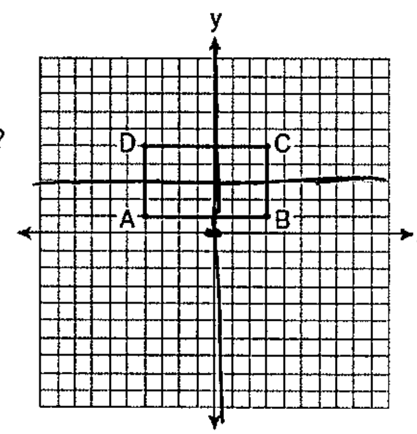
To map a shape onto itself:
Translation/Dilation: Never.
Reflection: **The line of reflection must be a line of symmetry** (cuts shape in half).
Rotation: **Center of rotation must be the center of the shape.** Use common sense for degree measure.

1. Circle *K* is shown in the graph below.
Which of the following transformations map circle K onto itself?



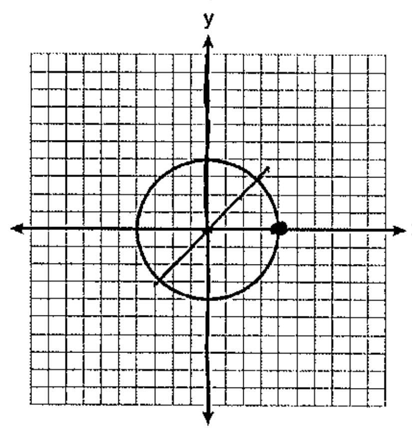
1) Reflection over the line $x-axis$
2) Reflection over the $y-axis$
3) Rotation of 90 centered at the origin
4) Rotation of 90 centered at K

2. On the set of axes below, Geoff drew rectangle *ABCD*.



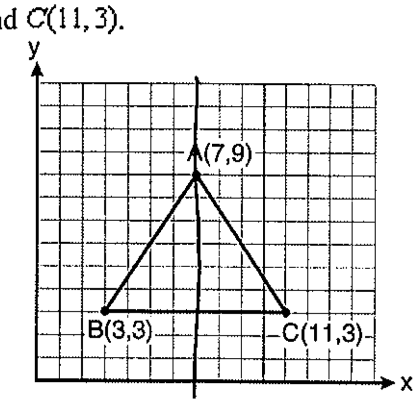
What of the following transformations would map the rectangle onto itself?
1) Reflection over the y axis
2) Reflection over the line $y=3$
3) Rotation of 180 centered at the origin
4) Translation one unit to the right

3. In the diagram below, which transformation does *not* map the circle onto itself?



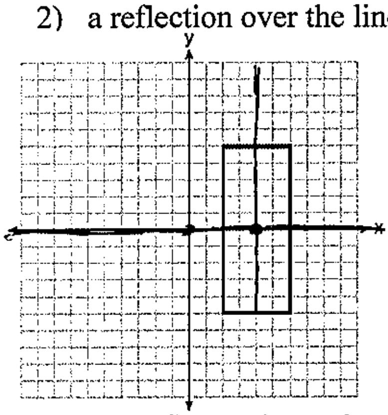
1) Rotation of 80 centered at the origin
2) Reflection over the line $y=x$
3) Rotation of 180 centered at (4,0)
4) Reflection over the line $x=0$

4. The vertices of the triangle in the diagram below are $A(7,9)$, $B(3,3)$, and $C(11,3)$.
Which transformation will map ΔABC onto itself?
1) Rotation of 60 centered at (3,3)
2) Reflection over the line $y=5$
3) Reflection over the line $x=7$
4) Translation 3 units up



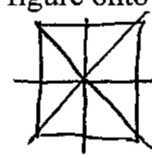
5. As shown in the graph below, the quadrilateral is a rectangle.
Which transformation would *not* map the rectangle onto itself?

1) a reflection over the x-axis
2) a reflection over the line $x = 4$
3) a rotation of 180° about the origin
4) a rotation of 180° about the point $(4, 0)$



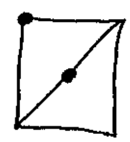
6. Which figure always has exactly four lines of reflection that map the figure onto itself?

1) square
2) rectangle
3) regular octagon
4) equilateral triangle



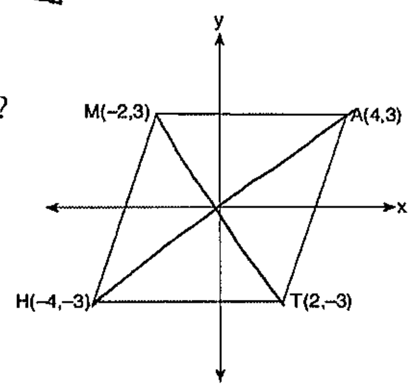
7. Which transformation would *not* carry a square onto itself?

1) a reflection over one of its diagonals
2) a 90° rotation clockwise about its center
3) a 180° rotation about one of its vertices
4) a reflection over the perpendicular bisector of one side



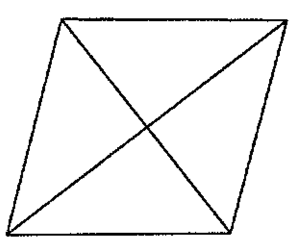
8. Which transformation carries the parallelogram below onto itself?

1) a reflection over $y = x$
2) a reflection over $y = -x$
3) a rotation of 90° counterclockwise about the origin
4) a rotation of 180° counterclockwise about the origin



9. The figure below shows a rhombus with noncongruent diagonals.
Which transformation would *not* carry this rhombus onto itself?

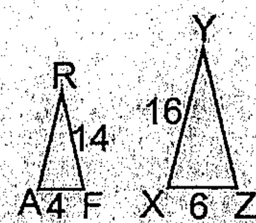
1) a reflection over the shorter diagonal
2) a reflection over the longer diagonal
3) a clockwise rotation of 90° about the intersection of the diagonals
4) a counterclockwise rotation of 180° about the intersection of the diagonals



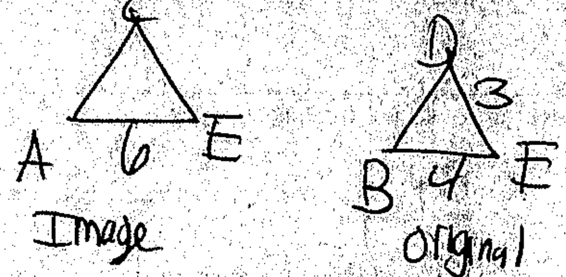
Scale factor = $\frac{image}{original}$

1. In the diagram below, ΔXYZ is the image of ΔARF after a dilation.

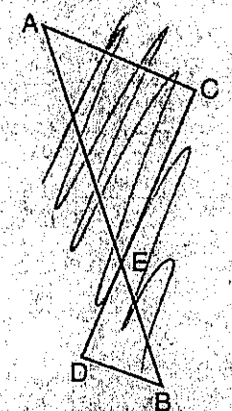
What is the scale factor of the dilation? $\frac{6}{4} = \frac{3}{2}$



2. In the diagram below, ΔACE is the image of ΔBDE after a sequence of transformations. If $\overline{AE} = 6$, $\overline{DE} = 3$, and $\overline{EB} = 4$, what is the scale factor?

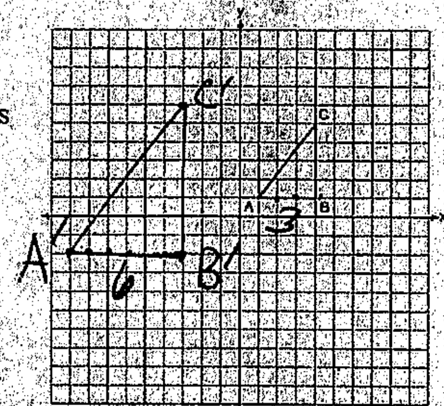


$\frac{6}{4} = 3$



3. In the diagram below, ΔABC has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. The coordinates of its image after a sequence of transformations is $A'(-9,-2)$, $B'(-3,-2)$, *and* $C'(-3,6)$. What is the scale factor?

$\frac{6}{3} = 2$



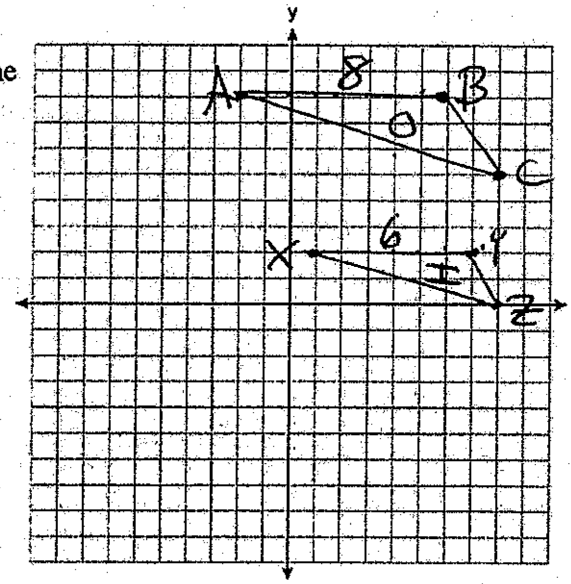
4. After a dilation with center $(0, 0)$, the image of $\overline{DB}$ is $\overline{D'B'}$. If $DB = 4.5$ and $D'B' = 18$, the scale factor of this dilation is

1) $\frac{1}{5}$
2) 5
3) $\frac{1}{4}$
4) 4

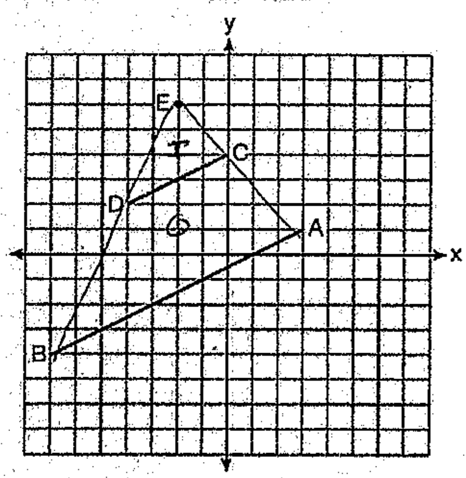
$\frac{18}{4.5} = 4$

5. $\triangle ABC$ has coordinates $A(-2,8), B(6,8),$ *and* $C(8,5)$. The coordinates of $\triangle XYZ$, the image of $\triangle ABC$ after a sequence of transformations is $X(1,2), Y(7,2),$ *and* $Z(8,0)$. What is the scale factor?

$\frac{6}{8} = \frac{3}{4}$



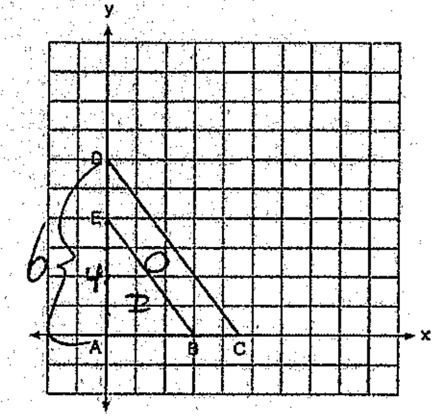
6. In the diagram below, $\overline{CD}$ is the image of $\overline{AB}$ after a dilation of scale factor k with center E.



Which ratio is equal to the scale factor k of the dilation?

1) $\frac{EC}{EA}$ $\frac{I}{O}$
2) $\frac{BA}{EA}$ $\frac{O}{O}$
3) $\frac{EA}{BA}$ $\frac{O}{O}$
4) $\frac{EA}{EC}$ $\frac{O}{I}$

7. In the diagram below, $\triangle ABE$ is the image of $\triangle ACD$ after a dilation centered at the origin. The coordinates of the vertices are $A(0,0)$, $B(3,0)$, $C(4.5,0)$, $D(0,6)$, and $E(0,4)$.



The scale factor of dilation is

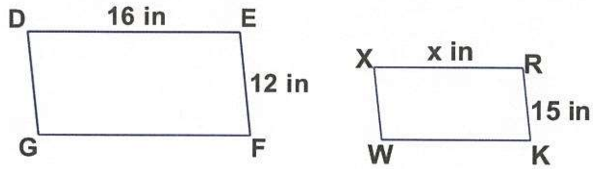
1) $\frac{2}{3}$
2) $\frac{3}{2}$
3) $\frac{3}{4}$
4) $\frac{4}{3}$

$\frac{4}{6} = \frac{2}{3}$

Similar Triangles with Parallel Lines

If the lines are parallel, the triangles are similar and the sides are in proportion.

1. Parallelogram DEFG is similar to parallelogram XRKW. Find x.

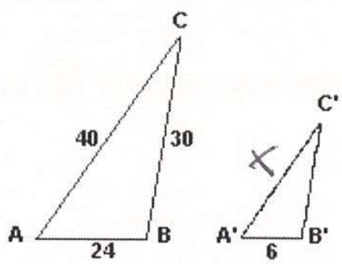


$$\frac{16}{x} = \frac{12}{15}$$

$$\frac{12x}{12} = \frac{240}{12}$$

$$x = 20$$

2. In the diagram, ΔABC is similar to $\Delta A'B'C'$, $AB = 24$, $BC = 30$, and $CA = 40$. If the shortest side of $\Delta A'B'C'$ is 6, find the length of the longest side of $\Delta A'B'C'$.

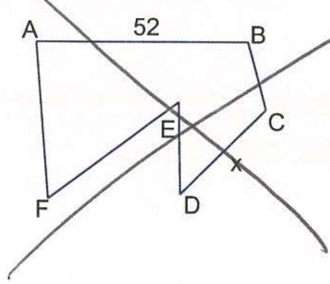


$$\frac{40}{x} = \frac{24}{6}$$

$$\frac{24x}{24} = \frac{240}{24}$$

$$x = 10$$

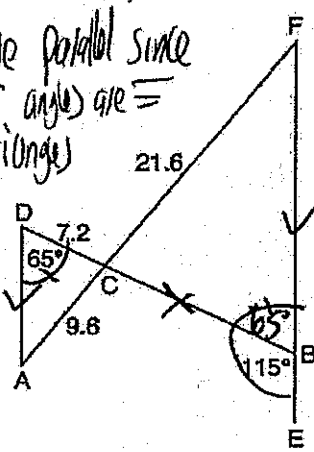
3. Polygon ABCDEF is similar to polygon XRKQMG. Find x.



Similarity with Parallel Lines

1. If the lines are parallel, the triangles are similar and the sides are in proportion.
In the diagram below, $\overline{AF}$, and $\overline{DB}$ intersect at C, and $\overline{AD}$ and $\overleftrightarrow{FBE}$ are drawn such that $m\angle D = 65°$, $m\angle CBE = 115°$, $DC = 7.2$, $AC = 9.6$, and $FC = 21.6$. What is the length of $\overline{CB}$?

the lines are parallel since alternate interior angles are = making the triangles similar



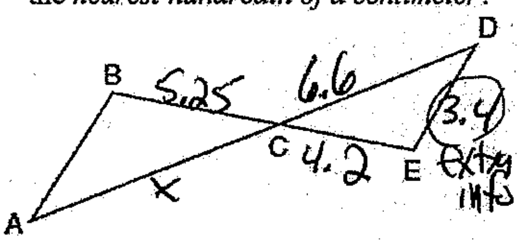
$$\begin{array}{r}180\\-115\\\hline 65\end{array}$$

$$\cancel{\frac{7.2}{x} = \frac{9.6}{21.6}}$$

$$\frac{9.6x}{9.6} = \frac{155.52}{9.6}$$

$$\boxed{x = 16.2}$$

2. In the diagram below, $\overline{AD}$ intersects $\overline{BE}$ at C, and $\overline{AB} \parallel \overline{DE}$.
If $CD = 6.6$ cm, $DE = 3.4$ cm, $CE = 4.2$ cm, and $BC = 5.25$ cm, what is the length of $\overline{AC}$, to the *nearest hundredth of a centimeter*?



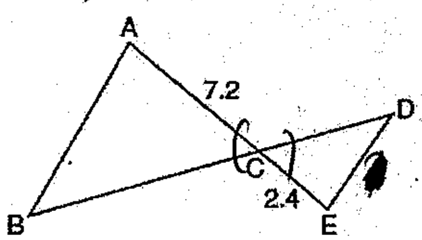
$$\frac{6.6}{x} = \frac{4.2}{5.25}$$

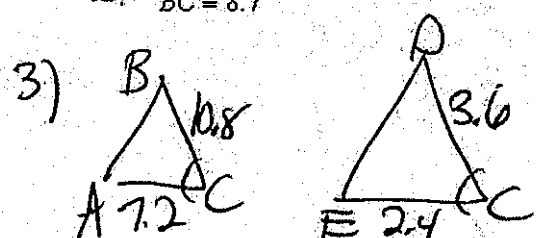
$$\frac{4.2x}{4.2} = \frac{34.65}{4.2}$$

$$x = 8.25$$

3. In the diagram below, $AC = 7.2$ and $CE = 2.4$.
Which statement is *not* sufficient to prove $\triangle ABC \sim \triangle EDC$?

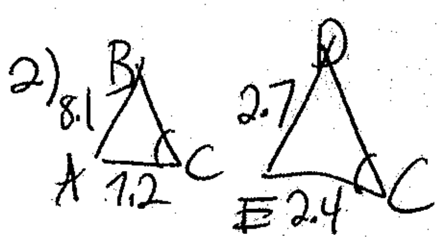
1) $\overline{AB} \parallel \overline{ED}$ ✓ If lines are ∥, △s are similar
2) $DE = 2.7$ and $AB = 8.1$
3) $CD = 3.6$ and $BC = 10.8$
4) $DE = 3.0$, $AB = 9.0$, $CD = 2.9$, and $BC = 8.7$



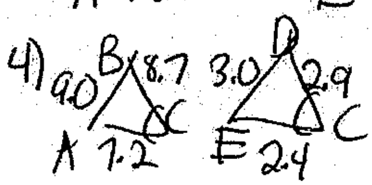


$$\frac{10.8}{3} = \frac{7.2}{2.4}$$

$21.6 = 25.92$ ✗
Not in proportion



✗ Not AA, SAS, or SSS



$$\frac{9}{3} = \frac{8.7}{2.9} = \frac{7.2}{2.4}$$

$3 = 3 = 3$ ✓

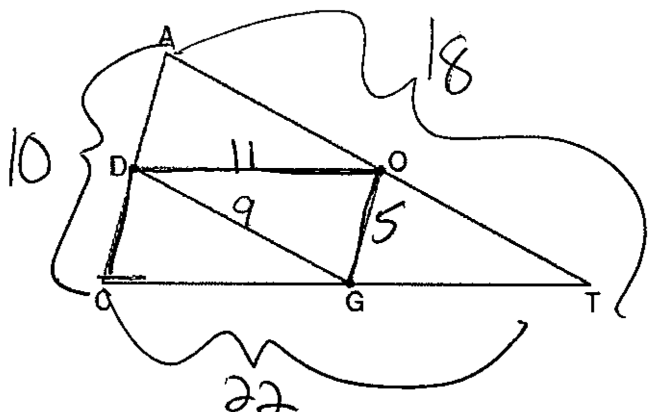
SSS

Joining the Midpoints of a Triangle

The midsegments are half of the opposite parallel sides

$2(\textit{midsegment}) = \textit{opposite side}$

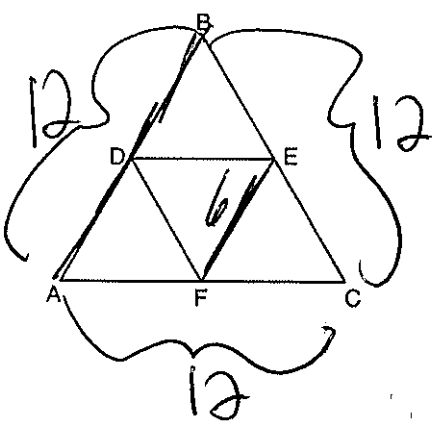
1. In the diagram below of $\triangle ACT$, D is the midpoint of $\overline{AC}$, O is the midpoint of $\overline{AT}$, and G is the midpoint of $\overline{CT}$. If $AC = 10$, $AT = 18$, and $CT = 22$, what is the perimeter of parallelogram $CDOG$?



$$\begin{array}{r} 11 \\ +11 \\ +5 \\ +5 \\ \hline 32 \end{array}$$

2. In the diagram below, the vertices of $\triangle DEF$ are the midpoints of the sides of equilateral triangle ABC, and the perimeter of $\triangle ABC$ is 36 cm. What is the length, in centimeters, of $\overline{EF}$?

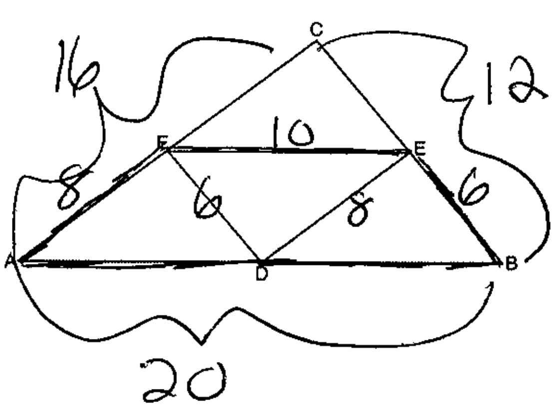
3 = 12



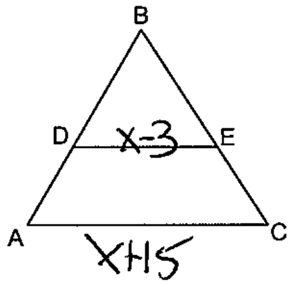
$\overline{EF} = 6$

3. In the diagram of $\triangle ABC$ shown below, D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$, and F is the midpoint of $\overline{AC}$. If $AB = 20$, $BC = 12$, and $AC = 16$, what is the perimeter of trapezoid $ABEF$?

$20 + 10 + 8 + 6 = 44$



4. D and E are midpoints of $\overline{AB}$ and $\overline{BC}$ respectively. If $\overline{AC} = x + 15$ and $\overline{DE} = x - 3$, find the measure of $\overline{DE}$.



2(midsegment) = opposite side

$2(x-3) = x+15$

$2x - 6 = x + 15$

$-x \quad -x$

$x - 6 = 15$

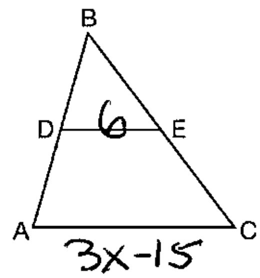
$+6 \quad +6$

$x = 21$

$\overline{DE} = 21 - 3$

$\boxed{\overline{DE} = 18}$

5. In $\triangle ABC$, D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{BC}$. If $AC = 3x - 15$ and $DE = 6$, what is the value of x?



1) 6
2) 7
3) 9
4) 12

2(midsegment) = opposite side

$2(6) = 3x - 15$

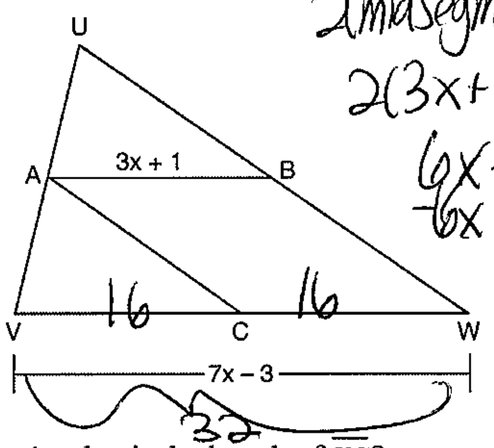
$12 = 3x - 15$

$+15 \quad +15$

$\frac{27}{3} = \frac{3x}{3}$

$9 = x$

6. In the diagram of $\triangle UVW$ below, A is the midpoint of $\overline{UV}$, B is the midpoint of $\overline{UW}$, C is the midpoint of $\overline{VW}$, and $\overline{AB}$ and $\overline{AC}$ are drawn.



2(midsegment) = opposite side

$2(3x+1) = 7x - 3$

$6x + 2 = 7x - 3$

$-6x \quad -6x$

$2 = x - 3$

$+3 \quad +3$

$5 = x$

If $VW = 7x - 3$ and $AB = 3x + 1$, what is the length of $\overline{VC}$?

1) 5
2) 13
3) 16
4) 32

$\overline{VW} = 7(5) - 3$

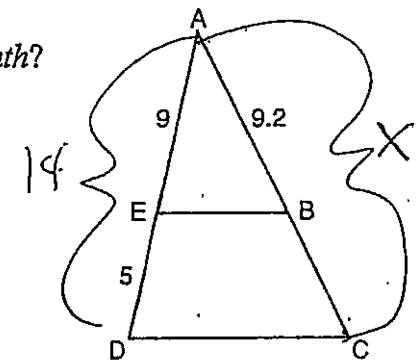
$\overline{VW} = 32$

Candy Corn Problems

If the bases are not involved: $\frac{top}{top} = \frac{bottom}{bottom} = \frac{side}{side}$

If bases are involved: separate your triangles!

1. In the diagram of $\triangle ADC$ below, $\overline{EB} \parallel \overline{DC}$, $AE = 9$, $ED = 5$, and $AB = 9.2$.



What is the length of $\overline{AC}$, to the *nearest tenth*?

1) 5.1
2) 5.2
3) 14.3
4) 14.4

bases not involved

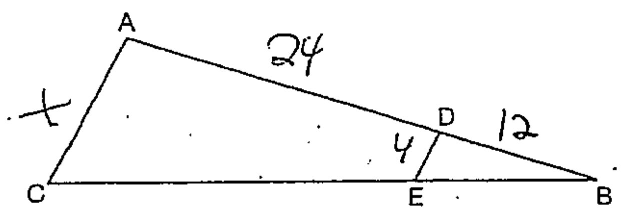
$\frac{top}{top} = \frac{side}{side}$

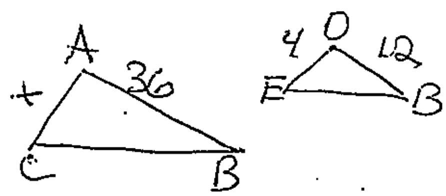
$\frac{9}{9.2} = \frac{14}{x}$

$\frac{9x}{9} = \frac{128.8}{9}$ → $x = 14.3$

2. In the diagram of $\triangle ABC$, points D and E are on $\overline{AB}$ and $\overline{CB}$, respectively, such that $\overline{AC} \parallel \overline{DE}$.

bases involved
separate





If $AD = 24$, $DB = 12$, and $DE = 4$, what is the length of $\overline{AC}$?

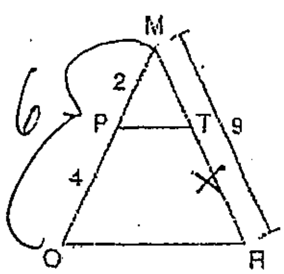
1) 8
2) 12
3) 16
4) 72

$\frac{x}{4} = \frac{36}{12}$

$\frac{12x}{12} = \frac{144}{12}$

$x = 12$

3. Given $\triangle MRO$ shown below, with trapezoid $PTRO$, $MR = 9$, $MP = 2$, and $PO = 4$.



bases not involved

$\frac{bottom}{bottom} = \frac{side}{side}$

$\frac{9}{x} = \frac{6}{9}$

What is the length of $\overline{TR}$?

$\frac{6x}{6} = \frac{36}{6}$

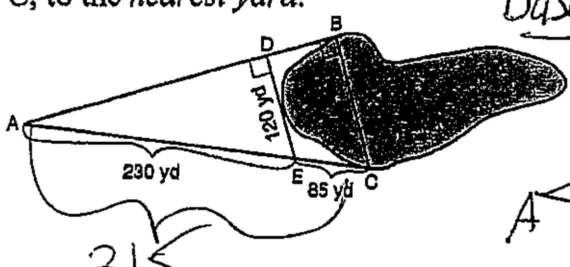
$x = 6$

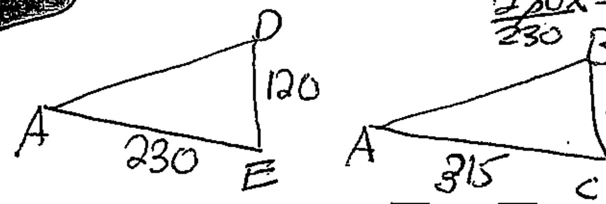
1) 4.5
2) 5
3) 3
4) 6

4. To find the distance across a pond from point B to point C, a surveyor drew the diagram below. The measurements he made are indicated on his diagram. Use the surveyor's information to determine and state the distance from point B to point C, to the *nearest yard.*

bases involved

separate



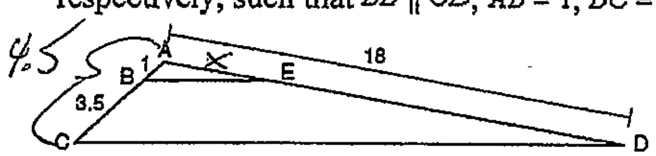


$\frac{230}{315} = \frac{120}{x}$

$\frac{230x}{230} = \frac{37800}{230}$

$x = 164$

5. In the diagram below, triangle ACD has points B and E on sides $\overline{AC}$ and $\overline{AD}$, respectively, such that $\overline{BE} \parallel \overline{CD}$, $AB = 1$, $BC = 3.5$, and $AD = 18$.



What is the length of $\overline{AE}$, to the *nearest tenth*?

bases not involved

$\frac{\text{top}}{\text{top}} = \frac{\text{side}}{\text{side}}$

$\frac{1}{x} = \frac{4.5}{18}$

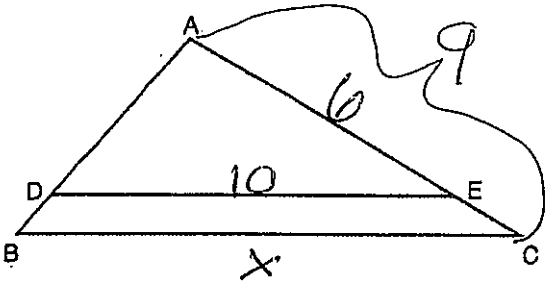
$\frac{4.5x}{4.5} = \frac{18}{4.5}$

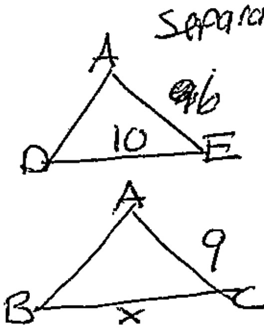
$x = 4.0$

6. In the diagram of $\triangle ABC$ shown below, $\overline{DE} \parallel \overline{BC}$. If $\overline{AE} = 6$, $\overline{DE} = 10$, and $\overline{AC} = 9$, find $\overline{BC}$

bases involved

separate





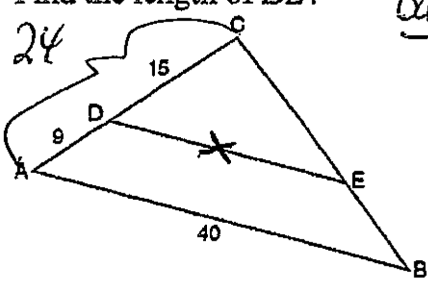
$\frac{10}{x} = \frac{6}{9}$

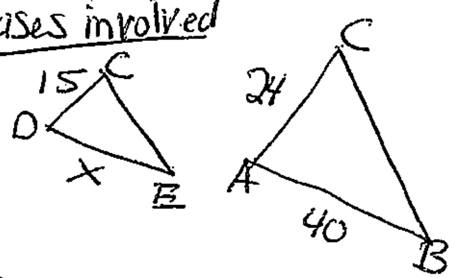
$\frac{6x}{6} = \frac{90}{6}$

$x = 15$

7. In the diagram of $\triangle ABC$ below, $\overline{DE}$ is parallel to $\overline{AB}$, $CD = 15$, $AD = 9$, and $AB = 40$. Find the length of $\overline{DE}$.

bases involved



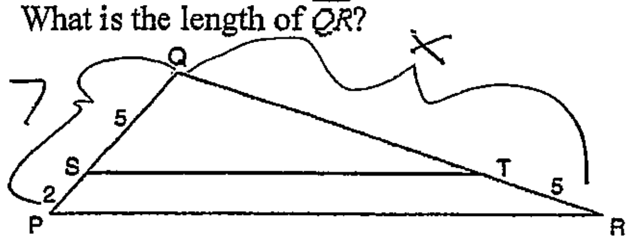


$\frac{15}{24} = \frac{x}{40}$

$\frac{24x}{24} = \frac{600}{24}$

$x = 25$

8. In the diagram below of $\triangle PQR$, $\overline{ST}$ is drawn parallel to $\overline{PR}$, $PS = 2$, $SQ = 5$, and $TR = 5$ What is the length of $\overline{QR}$?



bases not involved

$\frac{\text{side}}{\text{side}} = \frac{\text{bottom}}{\text{bottom}}$

$\frac{7}{x} = \frac{2}{5}$

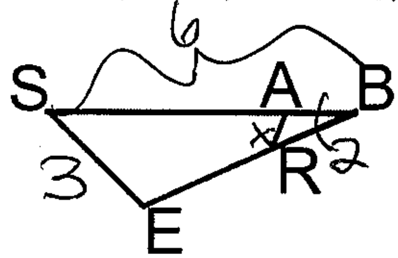
$\frac{2x}{2} = \frac{35}{2}$

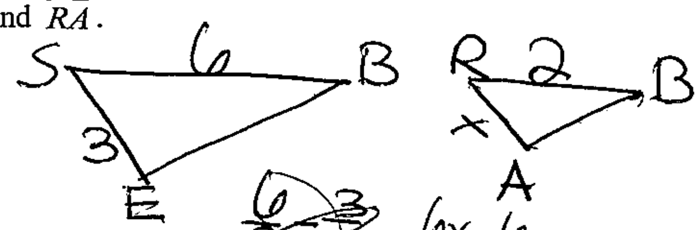
$x = 17.5$

Overlapping Similar Triangles

1) Separate the triangles and draw them with the same orientation
2) Match up the corresponding letters (use reflexive property)
3) Create a proportion and solve

1. In triangle SEB, A is on $\overline{SB}$, and E is on $\overline{EB}$ so that $\angle E \cong \angle BAR$.
If $\overline{SB} = 6$, $\overline{RB} = 2$, and $\overline{SE} = 3$, find $\overline{RA}$.



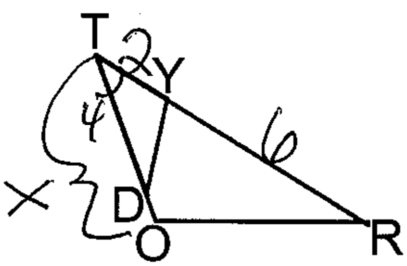


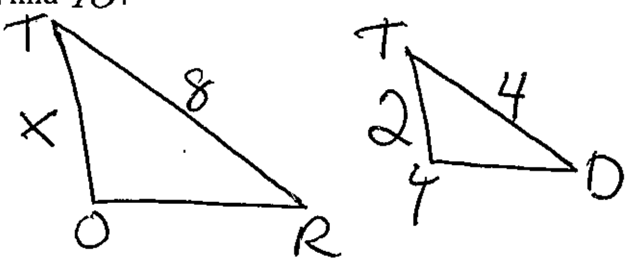
$\cancel{\frac{6}{2} = \frac{3}{x}}$

$$\frac{6x}{6} = \frac{6}{6}$$

$x = 1$

2. In triangle TOR, Y is on $\overline{TR}$, and D is on $\overline{TO}$ so that $\angle TYD \cong \angle ROT$.
If $\overline{TY} = 2$, $\overline{YR} = 6$, and $\overline{TD} = 4$, find $\overline{TO}$.



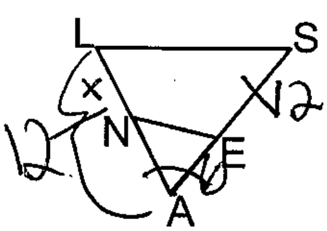


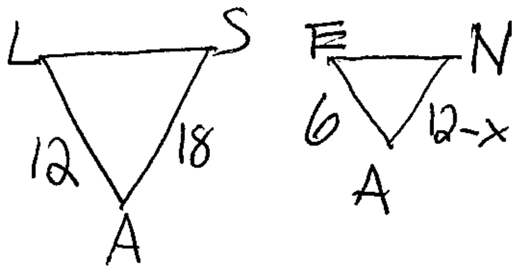
$\cancel{\frac{x}{2} = \frac{8}{4}}$

$$\frac{4x}{4} = \frac{16}{4}$$

$x = 4$

3. In triangle SAL, N is on $\overline{LA}$, and E is on $\overline{AS}$ so that $\angle AEN \cong \angle L$.
If $\overline{AE} = 6$, $\overline{ES} = 12$, and $\overline{ES} \cong \overline{AL}$, find $\overline{NL}$.





$$\frac{12}{6} = \frac{18}{12-x}$$

$12(12-x) = 108$

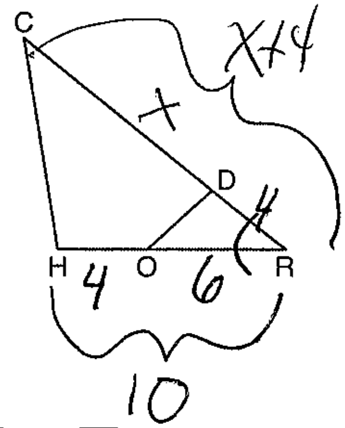
$144 - 12x = 108$

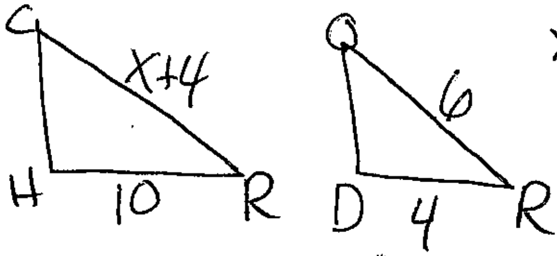
$-144 \qquad -144$

$$\frac{-12x}{-12} = \frac{-36}{-12}$$

$x = 3$

4. In triangle *CHR*, *O* is on $\overline{HR}$, and *D* is on $\overline{CR}$ so that $\angle H \cong RDO$. If $RD = 4$, $RO = 6$, and $OH = 4$, what is the length of $\overline{CD}$?





$$\frac{x+4}{6} = \frac{10}{4}$$

$$4(x+4) = 60$$

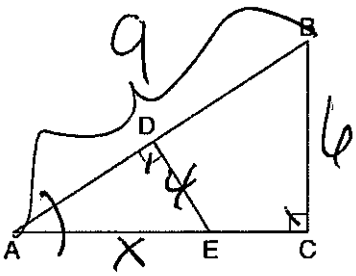
$$4x + 16 = 60$$

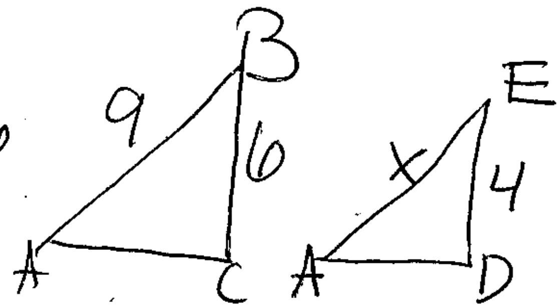
$$-16 \quad -16$$

$$\frac{4x}{4} = \frac{44}{4}$$

$$x = 11$$

5. In $\triangle ABC$ shown below, $\angle ACB$ is a right angle, *E* is a point on $\overline{AC}$, and $\overline{ED}$ is drawn perpendicular to hypotenuse $\overline{AB}$. If $AB = 9$, $BC = 6$, and $DE = 4$, what is the length of $\overline{AE}$?



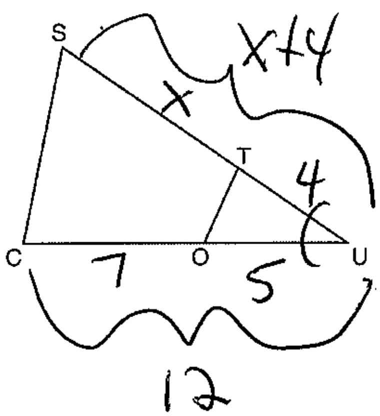


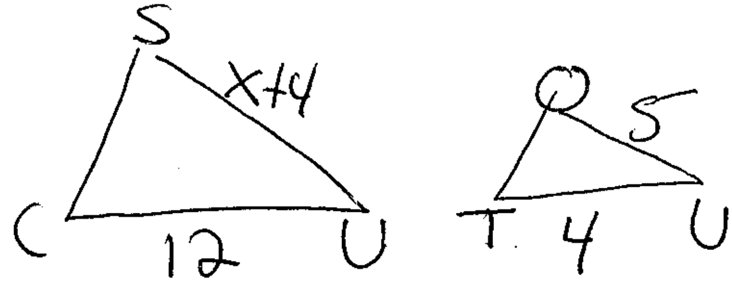
$$\frac{9}{x} = \frac{6}{4}$$

$$\frac{6x}{6} = \frac{36}{6}$$

$$x = 6$$

6. In $\triangle SCU$ shown below, points *T* and *O* are on $\overline{SU}$ and $\overline{CU}$, respectively. Segment $\overline{OT}$ is drawn so that $\angle C \cong \angle OTU$.
If $TU = 4$, $OU = 5$, and $OC = 7$, what is the length of $\overline{ST}$?





$$\frac{x+4}{5} = \frac{12}{4}$$

$$4(x+4) = 60$$

$$4x + 16 = 60$$

$$-16 \quad -16$$

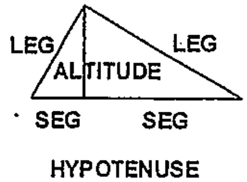
$$\frac{4x}{4} = \frac{44}{4}$$

$$x = 11$$

When an altitude is drawn to a right triangle
HLLS and SAAS

$$\frac{H}{L}=\frac{L}{S}\quad \frac{S}{A}=\frac{A}{S}$$

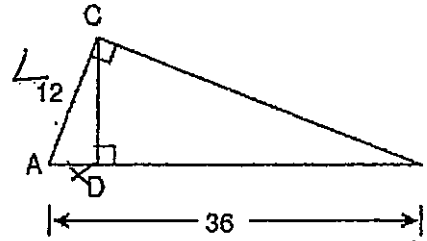
If L is involved, use HLLS
If A is involved, use SAAS



Know how to reduce radicals:

1) Separate into perfect square and non perfect square
2) Take the square root of the perfect square

1. In the diagram below of right triangle ACB, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.



If $AB = 36$ and $AC = 12$, what is the length of $\overline{AD}$?

1) 32
2) 6
3) 3
4) 4

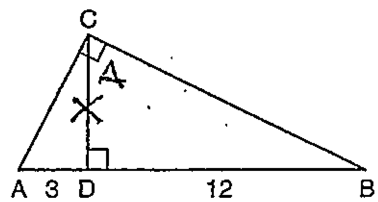
$\frac{H}{L}=\frac{L}{S}$

~~$\frac{36}{12}=\frac{12}{x}$~~

$\frac{36x}{36}=\frac{144}{36}$

$x=4$

2. In the diagram below of right triangle ABC, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.



If $AD = 3$ and $DB = 12$, what is the length of altitude $\overline{CD}$?

1) 6
2) $6\sqrt{5}$
3) 3
4) $3\sqrt{5}$

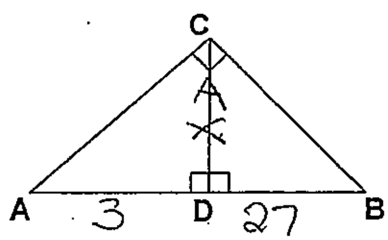
$\frac{S}{A}=\frac{A}{S}$

~~$\frac{3}{x}=\frac{x}{12}$~~

$\sqrt{x^2}=\sqrt{36}$

$x=6$

3. If $\overline{AD} = 3$ and $\overline{DB} = 27$, find $\overline{CD}$



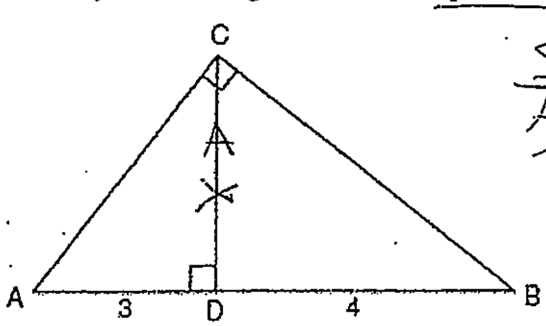
$\frac{S}{A}=\frac{A}{S}$

$\frac{3}{x}=\frac{x}{27}$

$\sqrt{x^2}=\sqrt{81}$

$x=9$

4. In the diagram below of right triangle ACB, altitude $\overline{CD}$ intersects $\overline{AB}$ at D. If $AD = 3$ and $DB = 4$, find the length of $\overline{CD}$ in simplest radical form.



$\frac{S}{A}=\frac{A}{S}$

~~$\frac{3}{x}=\frac{x}{4}$~~

$\sqrt{x^2}=\sqrt{12}$

$\sqrt{4}\ \sqrt{3}$

$x=2\sqrt{3}$

5. Triangle ABC shown below is a right triangle with altitude $\overline{AD}$ drawn to the hypotenuse $\overline{BC}$.

If $BD = 2$ and $DC = 10$, what is the length of $\overline{AB}$?

1) $2\sqrt{2}$
2) $2\sqrt{5}$
3) $2\sqrt{6}$
4) $2\sqrt{30}$

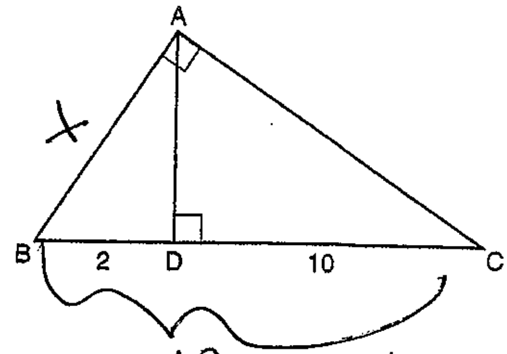
$\frac{H}{L} = \frac{L}{S}$

$\cancel{\frac{12}{x} = \frac{x}{2}}$

$\sqrt{x^2} = \sqrt{24}$

$\sqrt{4}\sqrt{6}$

$x = 2\sqrt{6}$



6. In right triangle ABC shown in the diagram below, altitude $\overline{BD}$ is drawn to hypotenuse $\overline{AC}$, $CD = 12$, and $AD = 3$.

What is the length of $\overline{AB}$?

1) $5\sqrt{3}$
2) 6
3) $3\sqrt{5}$
4) 9

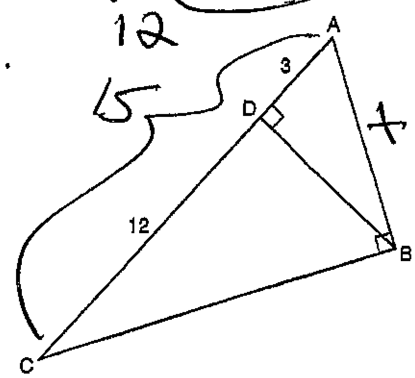
$\frac{H}{L} = \frac{L}{S}$

$\frac{15}{x} = \frac{x}{3}$

$\sqrt{x^2} = \sqrt{45}$

$\sqrt{9}\sqrt{5}$

$x = 3\sqrt{5}$



7. In the diagram below of right triangle ABC, altitude $\overline{BD}$ is drawn to hypotenuse $\overline{AC}$, $AC = 16$, and $CD = 7$.

What is the length of $\overline{BD}$?

1) $3\sqrt{7}$
2) $4\sqrt{7}$
3) $7\sqrt{3}$
4) 12

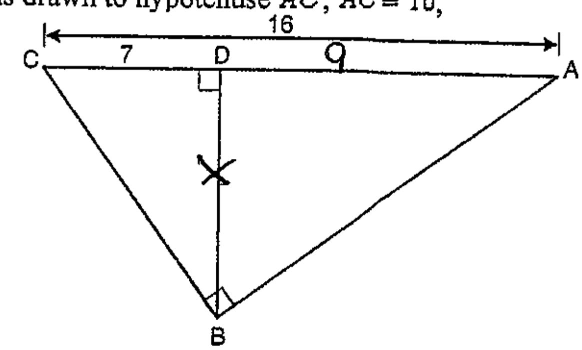
$\frac{S}{A} = \frac{A}{S}$

$\frac{7}{x} = \frac{x}{9}$

$\sqrt{x^2} = \sqrt{63}$

$\sqrt{9}\sqrt{7}$

$x = 3\sqrt{7}$



8. In the diagram below of $\triangle ABC$, $\angle ABC$ is a right angle, $AC = 12$, $AD = 8$, and altitude $\overline{BD}$ is drawn.

What is the length of $\overline{BC}$?

1) $4\sqrt{2}$
2) $4\sqrt{3}$
3) $4\sqrt{5}$
4) $4\sqrt{6}$

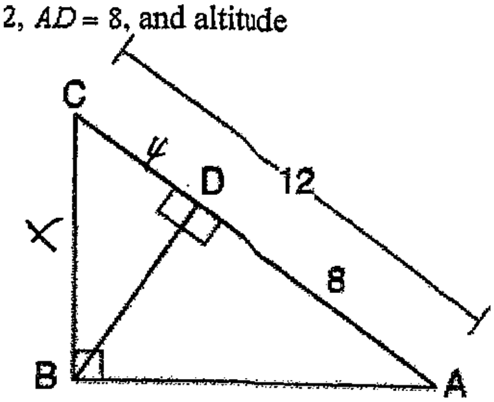
$\frac{H}{L} = \frac{L}{S}$

$\frac{12}{x} = \frac{x}{4}$

$\sqrt{x^2} = \sqrt{48}$

$\sqrt{16}\sqrt{3}$

$4\sqrt{3}$



9. In $\triangle RST$ shown below, altitude $\overline{SU}$ is drawn to $\overline{RT}$ at U.
If $SU = h$, $UT = 12$, and $RT = 42$, which value of h will make $\triangle RST$ a right triangle with $\angle RST$ as a right angle?

1) $6\sqrt{3}$
2) $6\sqrt{10}$
3) $6\sqrt{14}$
4) $6\sqrt{35}$

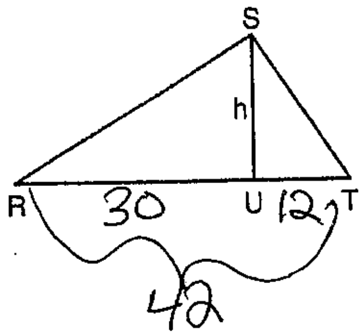
$\frac{S}{A} = \frac{A}{S}$

$\frac{30}{h} = \frac{h}{12}$

$\sqrt{h^2} = \sqrt{360}$

$\sqrt{36}\sqrt{10}$

$6\sqrt{10}$



10. In the diagram of right triangle ABC, $\overline{CD}$ intersects hypotenuse $\overline{AB}$ at D.
If $AD = 4$ and $DB = 6$, which length of $\overline{AC}$ makes $\overline{CD} \perp \overline{AB}$?

1) $2\sqrt{6}$
2) $2\sqrt{10}$
3) $2\sqrt{15}$
4) $4\sqrt{2}$

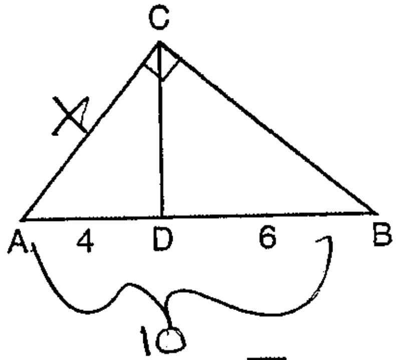
$\frac{H}{L} = \frac{L}{S}$

$\cancel{\frac{10}{x} = \frac{x}{4}}$

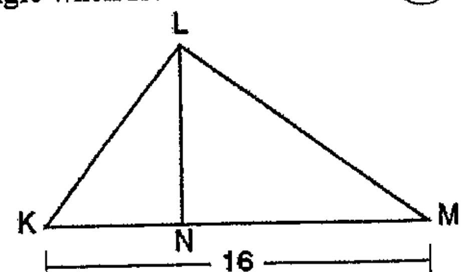
$\sqrt{x^2} = \sqrt{40}$

$\sqrt{4}\sqrt{10}$

$2\sqrt{10}$



11. Kirstie is testing values that would make triangle KLM a right triangle when $\overline{LN}$ is an altitude, and $KM = 16$, as shown below.



Which lengths would make triangle KLM a right triangle?

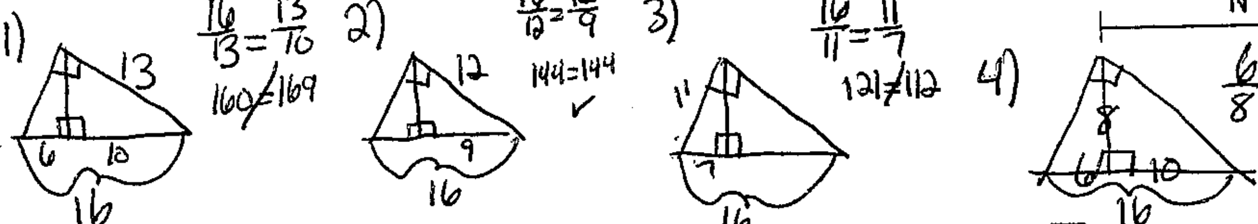
1) $LM = 13$ and $KN = 6$
2) $LM = 12$ and $NM = 9$
3) $KL = 11$ and $KN = 7$
4) $LN = 8$ and $NM = 10$

1) $\frac{16}{13} = \frac{13}{10}$ $160 \neq 169$

2) $\frac{16}{12} = \frac{12}{9}$ $144 = 144$ ✓

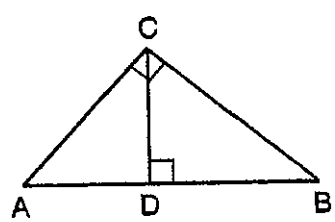
3) $\frac{16}{11} = \frac{11}{7}$ $121 \neq 112$

4) $\frac{6}{8} = \frac{8}{10}$ $64 \neq 60$



12. In the diagram below, $\overline{CD}$ is the altitude drawn to the hypotenuse $\overline{AB}$ of right triangle ABC.
Which lengths would *not* produce an altitude that measures $6\sqrt{2}$?

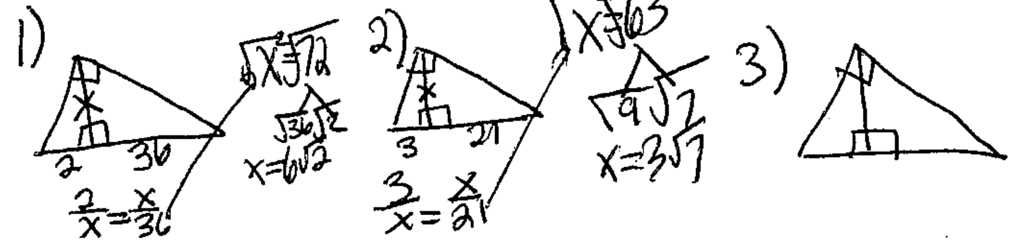
1) $AD = 2$ and $DB = 36$
2) $AD = 3$ and $AB = 24$
3) $AD = 6$ and $DB = 12$
4) $AD = 8$ and $AB = 17$



1) $\sqrt{x^2} = \sqrt{72}$ $\sqrt{36}\sqrt{2}$ $x = 6\sqrt{2}$ $\frac{2}{x} = \frac{x}{36}$

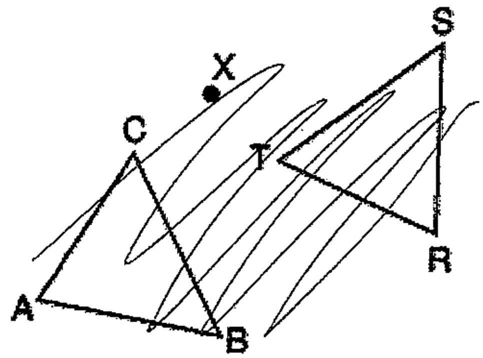
2) $\sqrt{x^2} = \sqrt{63}$ $\sqrt{9}\sqrt{7}$ $x = 3\sqrt{7}$ $\frac{3}{x} = \frac{x}{21}$

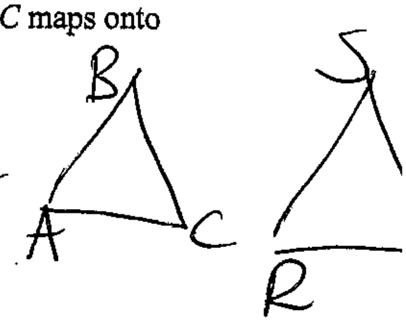
3)



Corresponding Parts of Congruent Triangles are Congruent
Redraw the shapes so it is more clear to see what parts correspond to each other

1. After a counterclockwise rotation about point X, scalene triangle ABC maps onto $\triangle RST$, as shown in the diagram below.





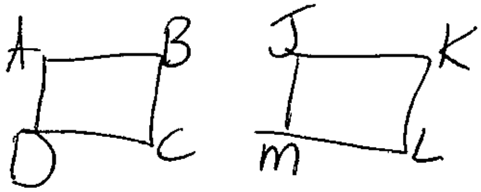
Which statement must be true?

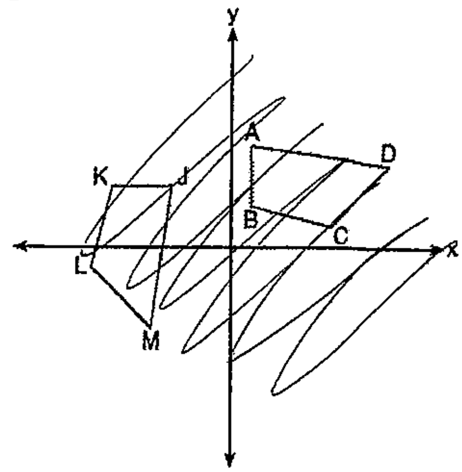
1) $\angle A \cong \angle R$
2) $\angle A \cong \angle S$
3) $\overline{CB} \cong \overline{TR}$
4) $\overline{CA} \cong \overline{TS}$

2. In the diagram below, a sequence of rigid motions maps $ABCD$ onto $JKLM$.

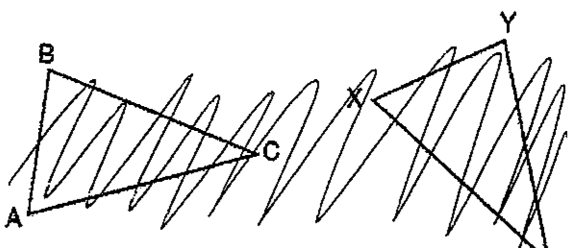
Which of the following statements must be true?

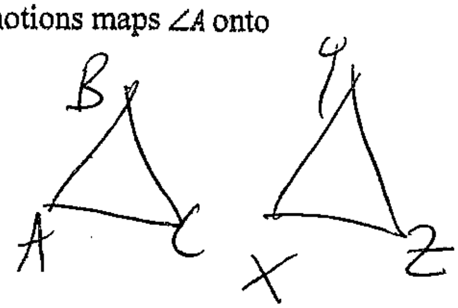
1) $\angle L \cong \angle B$ 3) $\overline{JK} \cong \overline{AC}$
2) $\angle A \cong \angle J$ 4) $\overline{JM} \cong \overline{AB}$





3. In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$,
$\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$.

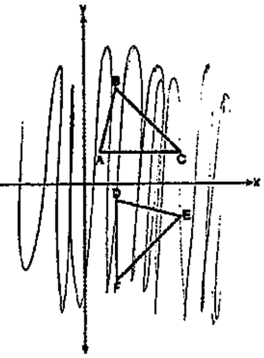




Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why. Yes, corresponding sides of congruent triangles are congruent.

Determine and state whether $\angle A \cong \angle Y$. Explain why. No, those angles don't correspond to each other.

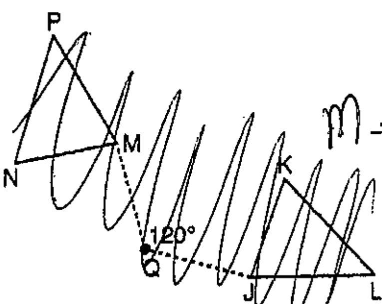
4. The image of ΔABC after a rotation of 90° clockwise about the origin is ΔDEF, as shown below.



Which statement is true?

1) $\overline{BC} \cong \overline{DE}$ X
2) $\overline{AB} \cong \overline{DF}$ X
3) $\angle C \cong \angle E$ X
4) $\angle A \cong \angle D$ ✓

5. Triangle MNP is the image of triangle JKL after a 120° counterclockwise rotation about point Q. If the measure of angle L is 47° and the measure of angle N is 57°, determine the measure of angle M. Explain how you arrived at your answer.



$47+57+x=180$

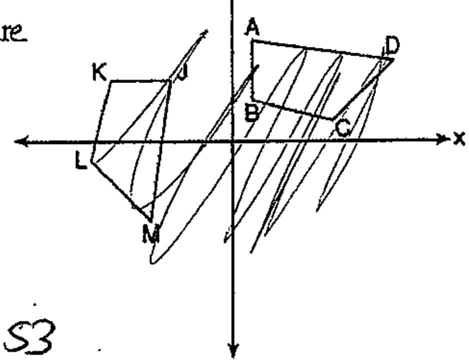
$104+x=180$

$-104 \quad -104$

$x=76$

Corresponding angles of congruent triangles are congruent.
A rotation is a rigid motion.
A rigid motion preserves size and angle measure producing a congruent figure

6. In the diagram below, a sequence of rigid motions maps $ABCD$ onto $JKLM$.
If $m\angle A = 82°$, $m\angle B = 104°$, and $m\angle L = 121°$, the measure of $\angle M$ is



1) 53°
2) 82°
3) 104°
4) 121°

angles of a quad add to 360

$104+82+121+x=360$

$307+x=360$

$-307 \quad -307$

$x=53$

7. In the diagram below, ΔABC with sides 13, 15, and 16, is mapped onto ΔDEF after a clockwise rotation of 90° about point P.
If $DE = 2x - 1$, what is the value of x?

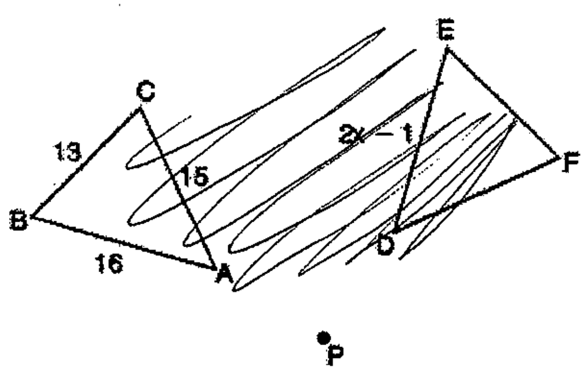
$2x-1=16$

$+1 \quad +1$

$2x=17$

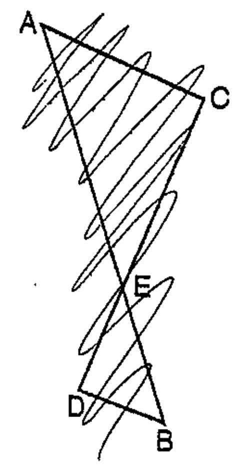
$\frac{2x}{2}=\frac{17}{2}$

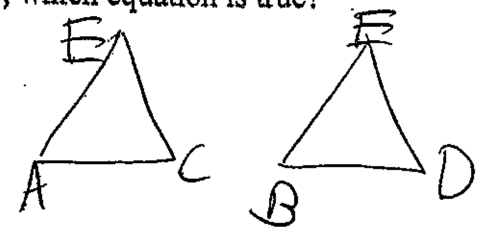
$x=8.5$



To determine if a proportion is correct
Look at the letters vertically and horizontally
One direction, the letters should correspond
Second direction, the letters should be in the same triangle
*It does not matter which direction does which

1. As shown in the diagram below, $\overline{AB}$ and $\overline{CD}$ intersect at E, and $\overline{AC} \parallel \overline{BD}$. Given $\Delta AEC \sim \Delta BED$, which equation is true?





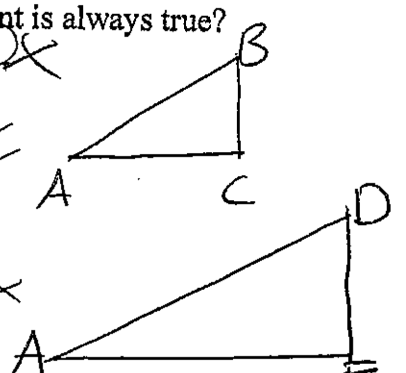
1) $\frac{\overline{CE}}{\overline{DE}} = \frac{\overline{EB}}{\overline{EA}}$

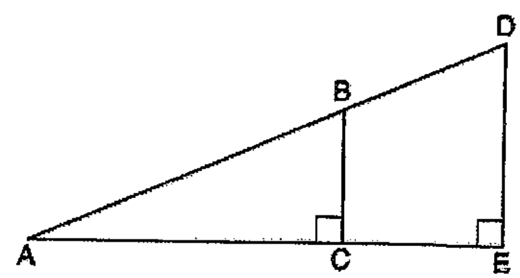
2) $\frac{\overline{AE}}{\overline{BE}} = \frac{\overline{AC}}{\overline{BD}}$

3) $\frac{\overline{EC}}{\overline{AE}} = \frac{\overline{BE}}{\overline{ED}}$

4) $\frac{\overline{ED}}{\overline{EC}} = \frac{\overline{AC}}{\overline{BD}}$

2. In the diagram below of right triangle AED, $\overline{BC} \parallel \overline{DE}$. Which statement is always true?





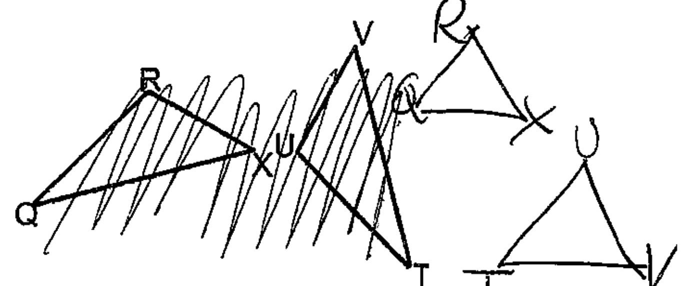
1) $\frac{AC}{BC} = \frac{DE}{AE}$

2) $\frac{AB}{AD} = \frac{BC}{DE}$

3) $\frac{AC}{CE} = \frac{BC}{DE}$

4) $\frac{DE}{BC} = \frac{DB}{AB}$

3. In the diagram below, $\Delta QRX \sim \Delta TUV$. Which of the following statements is *not* true?



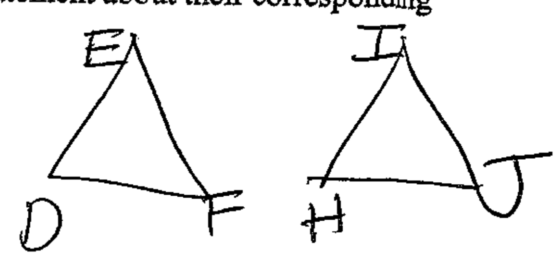
1) $\frac{\overline{QR}}{\overline{TU}} = \frac{\overline{QX}}{\overline{TV}}$

2) $\frac{\angle X}{\angle V} = \frac{\angle Q}{\angle T}$

3) $\frac{\overline{RX}}{\overline{UV}} = \frac{\overline{VT}}{\overline{XQ}}$

4) $\frac{\overline{QX}}{\overline{QR}} = \frac{\overline{TV}}{\overline{TU}}$

4. Given that $\Delta DEF \sim \Delta HIJ$, which is the correct statement about their corresponding sides?



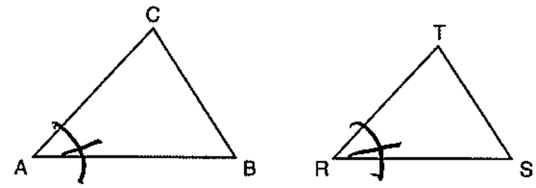
1) $\frac{\overline{EF}}{\overline{IJ}} = \frac{\overline{DE}}{\overline{HI}} = \frac{\overline{DF}}{\overline{HJ}}$

2) $\frac{\overline{EF}}{\overline{HI}} = \frac{\overline{IJ}}{\overline{DE}} = \frac{\overline{DF}}{\overline{HJ}}$

3) $\frac{\overline{DE}}{\overline{HI}} = \frac{\overline{EF}}{\overline{HJ}} = \frac{\overline{DF}}{\overline{IJ}}$

4) $\frac{\overline{DE}}{\overline{JI}} = \frac{\overline{EF}}{\overline{HJ}} = \frac{\overline{FD}}{\overline{HI}}$

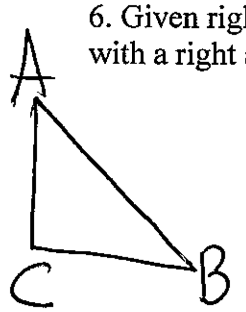
5. In the diagram below, $\triangle ABC \sim \triangle RST$.

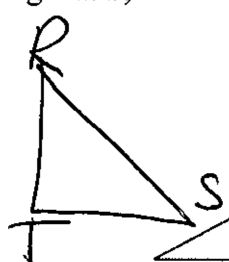


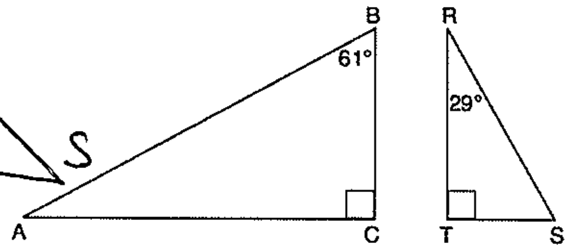
Which statement is *not* true?

1) $\angle A \cong \angle R$
2) $\frac{AB}{RS} = \frac{BC}{ST}$
3) $\frac{AB}{BC} = \frac{ST}{RS}$
4) $\frac{AB + BC + AC}{RS + ST + RT} = \frac{AB}{RS}$

6. Given right triangle ABC with a right angle at C, $m\angle B = 61°$. Given right triangle RST with a right angle at T, $m\angle R = 29°$.



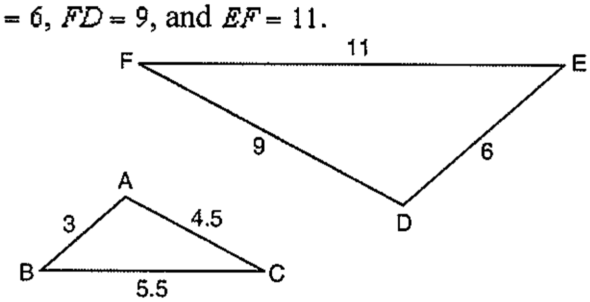




Which proportion in relation to $\triangle ABC$ and $\triangle RST$ is *not* correct?

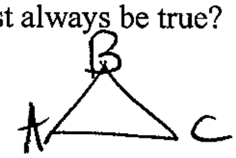
1) $\frac{AB}{RS} = \frac{RT}{AC}$
2) $\frac{BC}{ST} = \frac{AB}{RS}$
3) $\frac{BC}{ST} = \frac{AC}{RT}$
4) $\frac{AB}{AC} = \frac{RS}{RT}$

7. In the diagram below, $\triangle DEF$ is the image of $\triangle ABC$ after a clockwise rotation of 180° and a dilation where $AB = 3$, $BC = 5.5$, $AC = 4.5$, $DE = 6$, $FD = 9$, and $EF = 11$.



Which relationship must always be true?

1) $\frac{m\angle A}{m\angle D} = \frac{1}{2}$ $\quad \frac{1}{1}$
2) $\frac{m\angle C}{m\angle F} = \frac{2}{1}$ $\quad \frac{1}{1}$
3) $\frac{m\angle A}{m\angle C} = \frac{m\angle F}{m\angle D}$
4) $\frac{m\angle B}{m\angle E} = \frac{m\angle C}{m\angle F}$

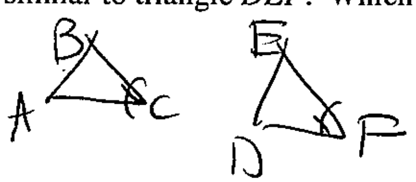


$\frac{AB}{BC} = \frac{DE}{EF}$

$\frac{AC}{DF} = \frac{BC}{EF}$

8. Scalene triangle ABC is similar to triangle DEF. Which statement is *false*?

1) $AB:BC = DE:EF$
2) $AC:DF = BC:EF$
3) $\angle ACB \cong \angle DFE$
4) $\angle ABC \cong \angle EDF$



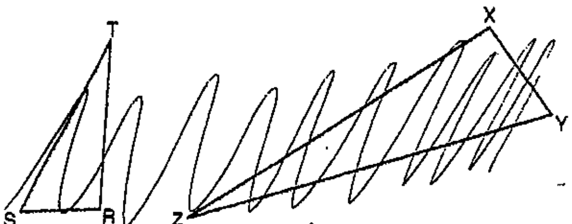
To show triangles are similar:
The ANGLES of similar triangles are congruent
The SIDES of similar triangles are in proportion

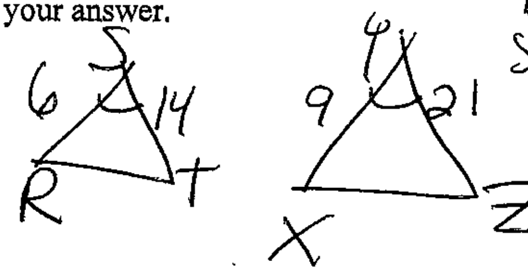
1) AA (2 pairs of corresponding angles are congruent)
2) SAS (2 pairs of corresponding sides are in proportion and the corresponding angles between them are congruent)
3) SSS (3 pairs of corresponding sides are in proportion)

*Congruent triangles must be similar. Similar triangles are not necessarily congruent.

1. Triangles RST and XYZ are drawn below. If $RS = 6$, $ST = 14$, $XY = 9$, $YZ = 21$, and $\angle S \cong \angle Y$, is $\triangle RST$ similar to $\triangle XYZ$? Justify your answer.

Determine if sides are in proportion.





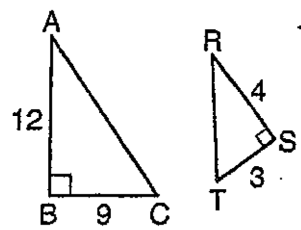
$\frac{6}{9} = \frac{14}{21}$

$126 = 126$ ✓

Yes, SAS. Two pairs of corresponding sides are in proportion and the angle between them is congruent.

2. Using the information given below, which set of triangles can *not* be proven similar?

1)

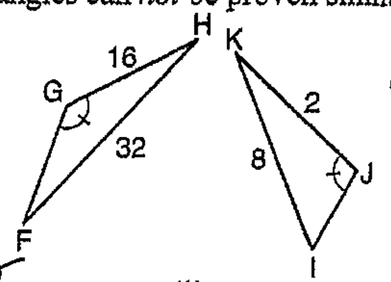


$\frac{3}{9} = \frac{4}{12}$

$36 = 36$ ✓

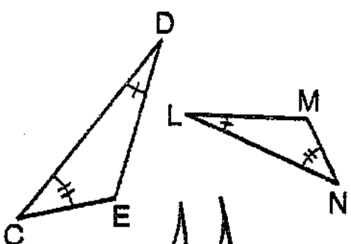
SAS

~~3)~~



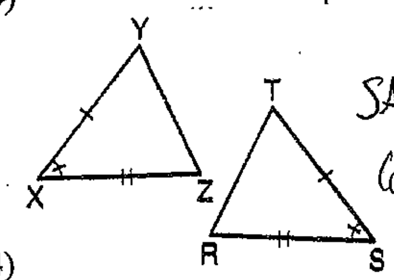
not SAS, SSS, or AA

2)



AA

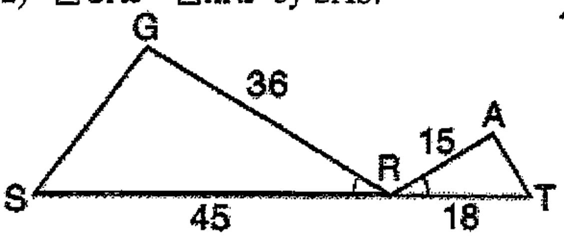
4)



SAS congruence. Congruent triangles are similar

3. In the diagram below, $\angle GRS \cong \angle ART$, $GR = 36$, $SR = 45$, $AR = 15$, and $RT = 18$. Which triangle similarity statement is correct?

1) $\triangle GRS \sim \triangle ART$ by AA.
2) $\triangle GRS \sim \triangle ART$ by SAS.
3) $\triangle GRS \sim \triangle ART$ by SSS.
~~4)~~ $\triangle GRS$ is not similar to $\triangle ART$.



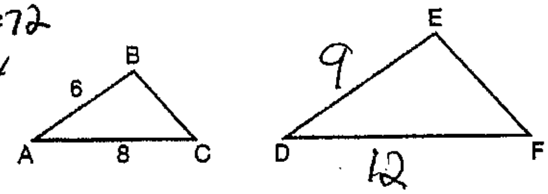
$\frac{15}{36} = \frac{18}{45}$

$675 \neq 648$

the sides are not in proportion.

4. In the diagram below, $\triangle ABC \sim \triangle DEF$.
If $AB = 6$ and $AC = 8$, which statement will justify similarity by SAS?

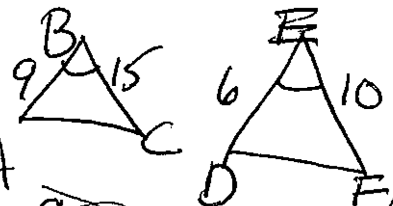
1) $DE = 9$, $DF = 12$, and $\angle A \cong \angle D$ $\frac{6}{9} = \frac{8}{12}$ $72 = 72$
2) $DE = 8$, $DF = 10$, and $\angle A \cong \angle D$ $\frac{6}{8} = \frac{8}{10}$ $60 \neq 64$
3) $DE = 36$, $DF = 64$, and $\angle C \cong \angle F$
4) $DE = 15$, $DF = 20$, and $\angle C \cong \angle F$ Not SAS



5. Triangles ABC and DEF are drawn below.

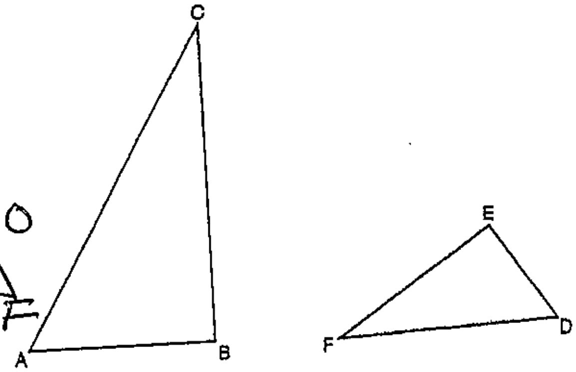
If $AB = 9$, $BC = 15$, $DE = 6$, $EF = 10$, and $\angle B \cong \angle E$, which statement is true?

1) $\angle CAB \cong \angle DEF$ X
2) $\frac{AB}{CB} = \frac{FE}{DE}$ X
3) $\triangle ABC \sim \triangle DEF$ ✓ SAS
4) $\frac{AB}{DE} = \frac{FE}{CB}$ X



$\frac{9}{6} = \frac{15}{10}$

$90 = 90$ ✓



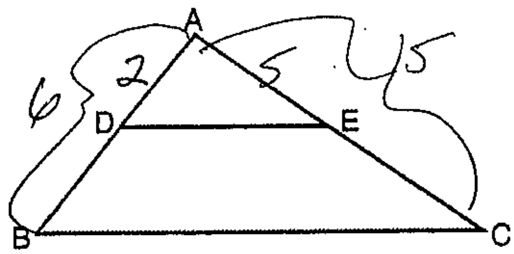
6. In the diagram below, $\triangle ABC \sim \triangle ADE$.

The sides must be in proportion

Which measurements are justified by this similarity?

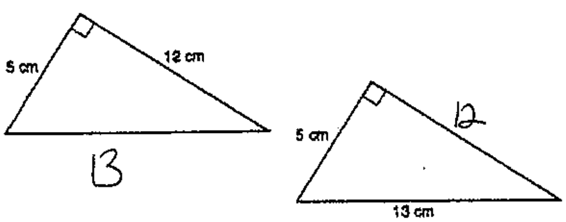
1) $AD = 3$, $AB = 6$, $AE = 4$, and $AC = 12$
2) $AD = 5$, $AB = 8$, $AE = 7$, and $AC = 10$
3) $AD = 3$, $AB = 9$, $AE = 5$, and $AC = 10$
4) $AD = 2$, $AB = 6$, $AE = 5$, and $AC = 15$ $\frac{2}{5} = \frac{6}{15}$

$30 = 30$ ✓



7. Skye says that the two triangles below are congruent. Margaret says that the two triangles are similar. Are Skye and Margaret both correct? Explain why.

$a^2 + b^2 = c^2$
$5^2 + 12^2 = x^2$
$25 + 144 = x^2$
$\sqrt{169} = \sqrt{x^2}$
$13 = x$



They are congruent by SSS. Congruent figures are similar.

8. If $\triangle ABC$ is mapped onto $\triangle DEF$ after a line reflection and $\triangle DEF$ is mapped onto $\triangle XYZ$ after a translation, the relationship between $\triangle ABC$ and $\triangle XYZ$ is that they are always

1) congruent and similar
2) congruent but not similar
3) similar but not congruent
4) neither similar nor congruent

A rigid motion preserves size and angle measure producing a congruent figure.
congruent figures are similar

Right Triangles

If only sides are involved, use Pythagorean theorem! ($a^2 + b^2 = c^2$)

If an angle is involved, use SOHCAHTOA

1) Label each side with O, A, and H

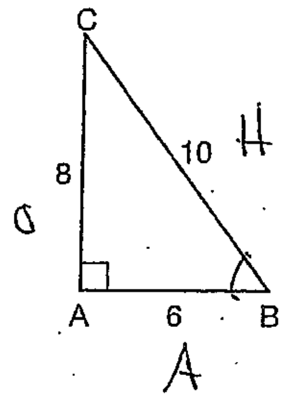
2) Determine whether to use sine, cosine, or tangent (Which two are involved?)

3) Substitute into appropriate formula

***If finding a side, cross multiply and solve**

***If finding an angle, use $\sin^{-1}$, $\cos^{-1}$, *or* $\tan^{-1}$**

1. In $\triangle ABC$ below, the measure of $\angle A = 90°$, $AB = 6$, $AC = 8$, and $BC = 10$.



Which ratio represents the cosine of $\angle B$?

1) $\frac{10}{8}$

2) $\frac{8}{6}$

3) $\frac{6}{10}$

4) $\frac{8}{10}$

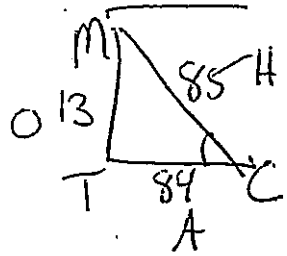
$\cos\theta = \frac{A}{H}$

$\cos B = \frac{6}{10}$

2. In triangle MCT, the measure of $\angle T = 90°$, $MC = 85$ cm, $CT = 84$ cm, and $TM = 13$cm. Which ratio represents the sine of $\angle C$?

1) $\frac{13}{85}$

2) $\frac{84}{85}$



3) $\frac{13}{84}$

4) $\frac{84}{13}$

$\sin\theta = \frac{O}{H}$

$\sin C = \frac{13}{85}$

3. As shown in the diagram below, a ladder 12 feet long leans against a wall and makes an angle of 72° with the ground.

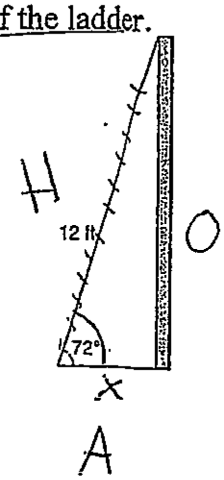
Find, to the *nearest tenth of a foot,* the distance from the wall to the base of the ladder.

$\cos\theta = \frac{A}{H}$

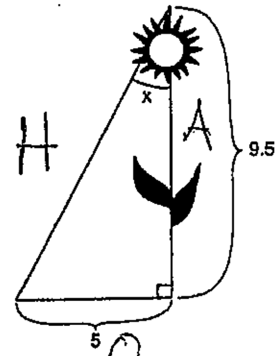
$\cos 72 = \frac{x}{12}$

$\cancel{\frac{.3090}{1} = \frac{x}{12}}$

$x = 3.7$



4. The diagram below shows the path a bird flies from the top of a 9.5-foot-tall sunflower to a point on the ground 5 feet from the base of the sunflower.



To the *nearest tenth of a degree*, what is the measure of angle x?
1) 27.8
2) 31.8
3) 58.2
4) 62.2

$\tan\theta = \frac{O}{A}$

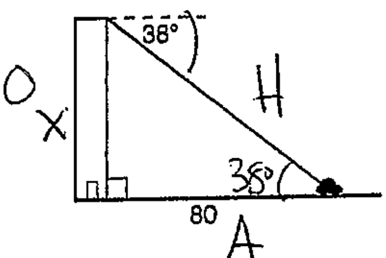
$\cancel{\tan^{-1}} \cancel{\tan} x = \tan^{-1}\frac{5}{9.5}$

$x = \tan^{-1}\left(\frac{5}{9.5}\right)$

$x = 27.8$

5. From the top of an apartment building, the angle of depression to a car parked on the street below is 38 degrees, as shown in the diagram below. The car is parked 80 feet from the base of the building. Find the height of the building, to the *nearest tenth of a foot.*

*angle of depression = angle of elevation



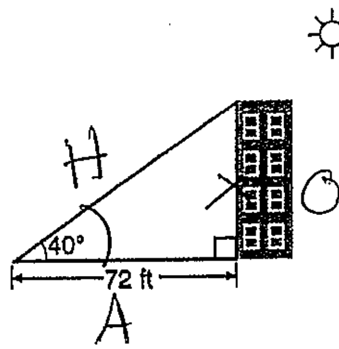
$\tan\theta = \frac{O}{A}$

$\tan 38 = \frac{x}{80}$

$\frac{.7813}{1} = \frac{x}{80}$

$x = 62.5$

6. As shown in the diagram below, a building casts a 72-foot shadow on the ground when the angle of elevation of the Sun is 40°.
How tall is the building, to the *nearest foot*?
1) 46
2) 60
3) 86
4) 94



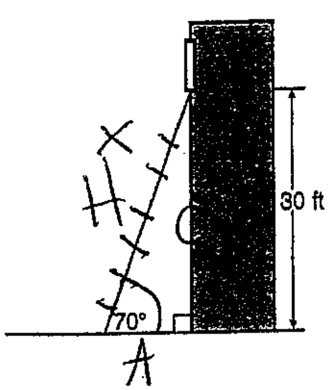
$\tan\theta = \frac{O}{A}$

$\tan 40 = \frac{x}{72}$

$\frac{.8391}{1} = \frac{x}{72}$

$x = 60$

7. A carpenter leans an extension ladder against a house to reach the bottom of a window 30 feet above the ground. As shown in the diagram below, the ladder makes a 70° angle with the ground. To the *nearest foot*, determine and state the length of the ladder.



$\sin\theta = \frac{O}{H}$

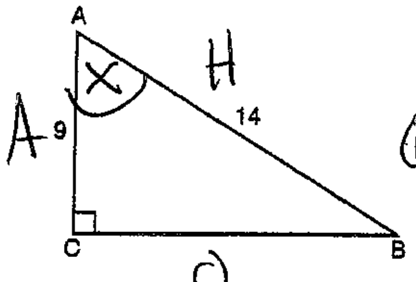
$\sin 70 = \frac{30}{x}$

$\frac{.9397}{1} = \frac{30}{x}$

$\frac{.9397x}{.9397} = \frac{30}{.9397}$

$x = 32$

8. In the diagram of right triangle *ABC* shown below, *AB* = 14 and *AC* = 9. What is the measure of ∠*A*, to the *nearest degree*?



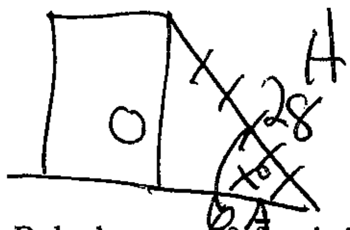
$\cos\theta = \frac{A}{H}$

$\cos^{-1}\cos x = \frac{9}{14}$

$x = \cos^{-1}\left(\frac{9}{14}\right)$

$x =$

9. A 28-foot ladder is leaning against a house. The bottom of the ladder is 6 feet from the base of the house. Find the measure of the angle formed by the ladder and the ground, to the *nearest degree*.



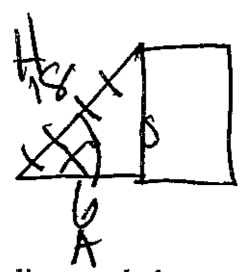
$\cos\theta = \frac{A}{H}$

$\cos^{-1}\cos x = \frac{6}{28}$

$x = \cos^{-1}\left(\frac{6}{28}\right)$

$x = 78°$

10. Bob places an 18-foot ladder 6 feet from the base of his house and leans it up against the side of his house. Find, to the *nearest degree*, the measure of the angle the bottom of the ladder makes with the ground.



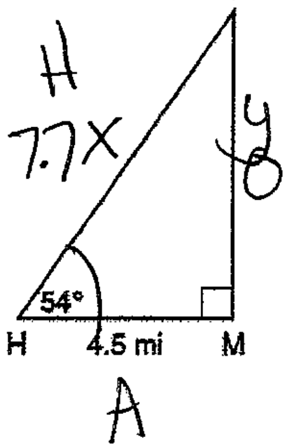
$\cos\theta = \frac{A}{H}$

$\cos^{-1}\cos x = \frac{6}{18}$

$x = \cos^{-1}\left(\frac{6}{18}\right)$

$x = 71°$

11. As shown in the diagram below, an island (*I*) is due north of a marina (*M*). A boat house (*H*) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.
Determine and state, to the *nearest tenth of a mile*, the distance from the boat house (*H*) to the island (*I*). Determine and state, to the *nearest tenth of a mile*, the distance from the island (*I*) to the marina (*M*).



$\cos\theta = \frac{A}{H}$

$\cos 54 = \frac{4.5}{x}$

$\cancel{\frac{.5878}{1} = \frac{4.5}{x}}$

$\cancel{.5878}x = 4.5$

$x = 7.7$

$\tan\theta = \frac{O}{A}$

$\tan 54 = \frac{x}{4.5}$

$\frac{1.3764}{1} = \frac{x}{4.5}$

$a^2 + b^2 = c^2$

$4.5^2 + x^2 = 7.$

$20.25 + x^2 =$

-20.25

$\sqrt{x^2} = \sqrt{39}$

$x = 6.$

Name Schlansky
Mr. Schlansky

Date ____________
Geometry

Common Core Geometry Common Part IIIs

Compound Right Triangle Problems

Procedure 1: Subtraction: Find corresponding parts of the two triangles and subtract them.

Procedure 2: Reflexive: Find a side/angle that's in both triangles. Use that new side/angle to find what you are looking for.

1. As shown in the diagram below, a ship is heading directly toward a lighthouse whose beacon is 125 feet above sea level. At the first sighting, point A, the angle of elevation from the ship to the light was 7°. A short time later, at point D, the angle of elevation was 16°. To the *nearest foot*, determine and state how far the ship traveled from point A to point D.

$\tan 7 = \frac{125}{x}$

$\frac{.1228}{1} = \frac{125}{x}$

$\frac{.1228x}{.1228} = \frac{125}{.1228}$

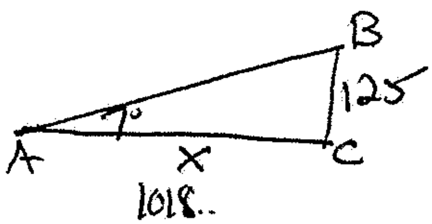
$x = 1018..$

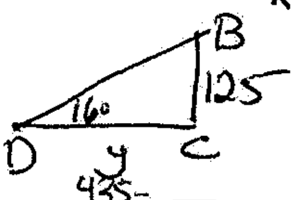
$\tan 16 = \frac{125}{y}$

$\frac{.2867}{1} = \frac{125}{y}$

$\frac{.2867y}{.2867} = \frac{125}{.2867}$

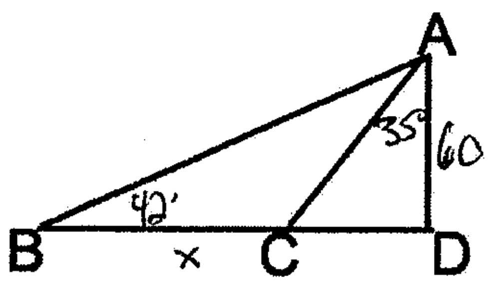
$y = 435..$





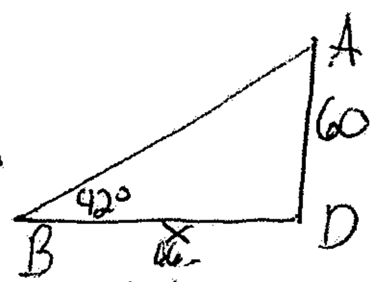
$1018.. - 435.. = 582$ ft

2. In the diagram below, $m\angle CAD = 35$, $m\angle ABD = 42$, and $m\overline{AD} = 60$. Find to the nearest tenth, $m\overline{BC}$.



$66.. - 42..$

24.6

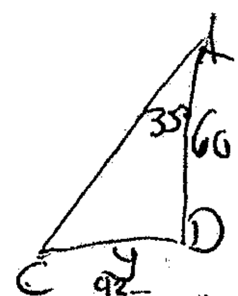


$\tan 42 = \frac{60}{x}$

$\frac{.9004}{1} = \frac{60}{x}$

$\frac{.9004x}{.9004} = \frac{60}{.9004}$

$x = 66..$



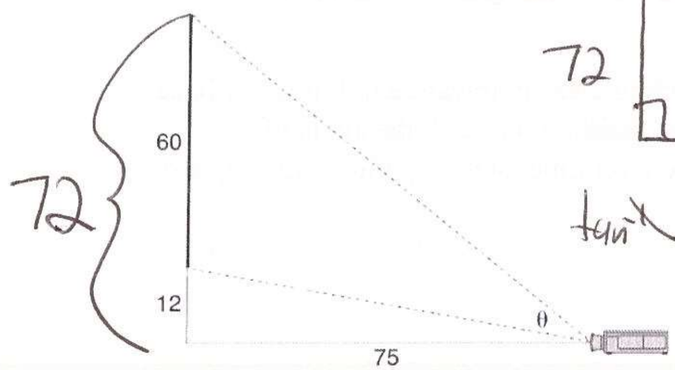
$\tan 35 = \frac{y}{60}$

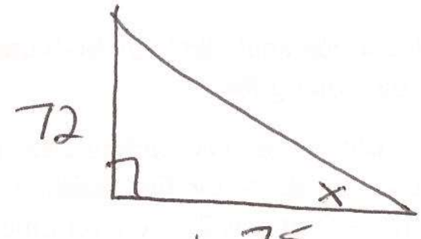
$\frac{.7002}{1} = \frac{y}{60}$

$y = 42..$

3. As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 feet off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the screen.
Determine and state, to the *nearest tenth of a degree*, the measure of θ, the projection angle.

$43.83\ldots - 9.09\ldots = 34.7$

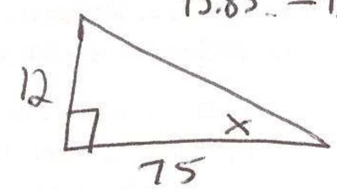




$\tan^{-1} \tan x = \tan^{-1} \frac{72}{75}$

$x = \tan^{-1}\left(\frac{72}{75}\right)$

$x = 43.83\ldots$



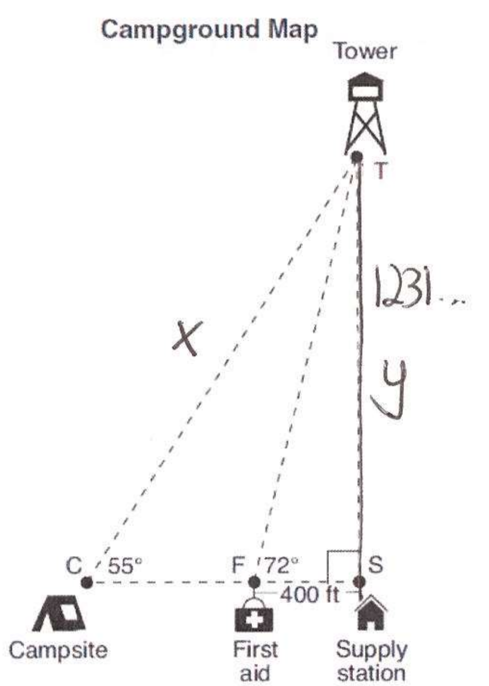
$\tan^{-1} \tan x = \tan^{-1} \frac{12}{75}$

$x = \tan^{-1}\left(\frac{12}{75}\right)$

$x = 9.09\ldots$

4. The map of a campground is shown below. Campsite *C*, first aid station *F*, and supply station *S* lie along a straight path. The path from the supply station to the tower, *T*, is perpendicular to the path from the supply station to the campsite. The length of path $\overline{FS}$ is 400 feet. The angle formed by path $\overline{TF}$ and path $\overline{FS}$ is 72°. The angle formed by path $\overline{TC}$ and path $\overline{CS}$ is 55°. Determine and state, to the *nearest foot*, the distance from the campsite to the tower.

ST is in both triangles



$\tan 72 = \frac{y}{400}$

$\frac{3.0777}{1} = \frac{y}{400}$

$y = 1231\ldots$

$\sin 55 = \frac{1231\ldots}{x}$

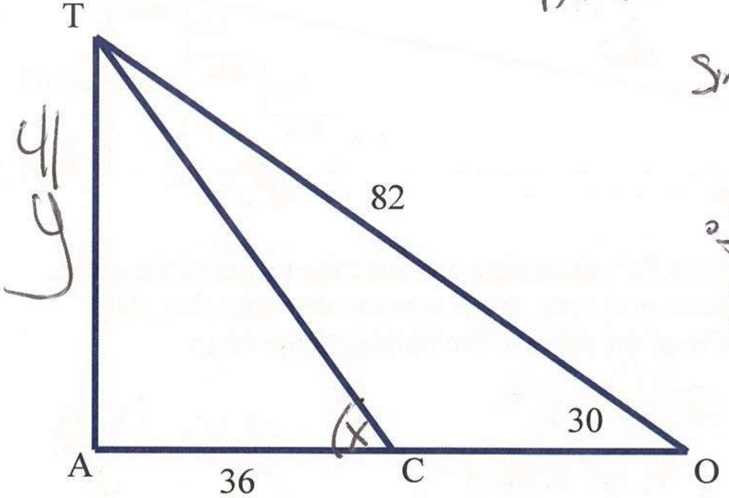
$.8192 = \frac{1231\ldots}{x}$

$\frac{.8192x}{.8192} = \frac{1231\ldots}{.8192\ldots}$

$x = 1503$

5. Find the measure of $\angle TCA$ in the diagram of right triangle TAO below to the nearest tenth of a degree.

$\overline{TA}$ is in both triangles



$\sin 30 = \frac{y}{82}$

$\tan^{-1} \tan x = \frac{41}{36}$

$\frac{.5}{1} = \frac{y}{82}$

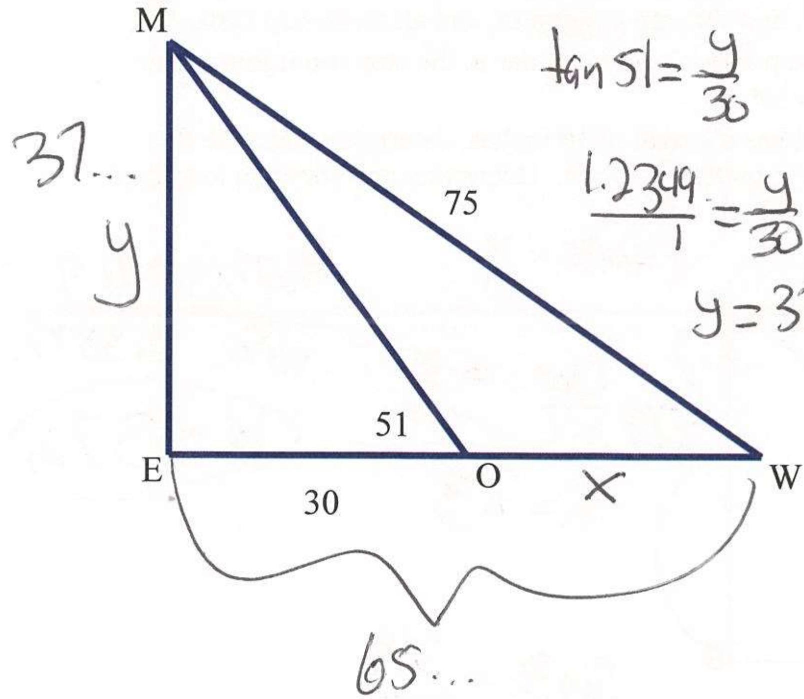
$x = \tan^{-1}\left(\frac{41}{36}\right)$

$y = 41.$

$x = 48.7°$

6. Find the measure of $\overline{OW}$ in the diagram of right triangle MEW below to the nearest unit.

$\overline{ME}$ is in both triangles



$\tan 51 = \frac{y}{30}$

$a^2 + b^2 = c^2$

$\frac{1.2349}{1} = \frac{y}{30}$

$37..^2 + b^2 = 75^2$

$1372.. + b^2 = 5625$

$y = 37..$

$-1372.. \quad -1372..$

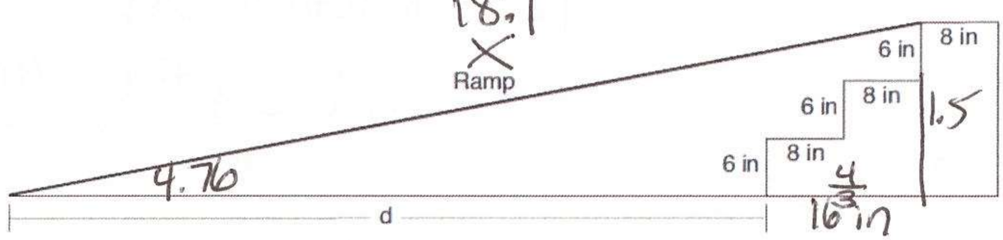
$\sqrt{b^2} = \sqrt{4252..}$

$b = 65..$

$65.. - 30 = 35$

7. As modeled in the diagram below, an access ramp starts on flat ground and ends at the beginning of the top step. Each step is 6 inches tall and 8 inches deep.

$18\text{ in} \cdot \frac{1\text{ ft}}{12\text{ in}} = 1.5\text{ ft}$



18 in

$16\text{ in} \cdot \frac{1\text{ ft}}{12\text{ in}} = \frac{4}{3}\text{ ft}$

If the angle of elevation of the ramp is 4.76°, determine and state the length of the ramp, to the *nearest tenth of a foot*. Determine and state, to the *nearest tenth of a foot*, the horizontal distance, *d*, from the bottom of the stairs to the bottom of the ramp.

$\sin 4.76 = \frac{1.5}{x}$

$.0830\ldots = \frac{1.5}{x}$

$\frac{.0830\ldots x}{.0830} = \frac{1.5}{.0830\ldots}$

$x = 18.1\text{ ft}$

$a^2+b^2=c^2$

$x^2+1.5^2=18.1^2$

$x^2+2.25=327.61$

$-2.25 \quad -2.25$

$\sqrt{x^2}=\sqrt{325.36}$

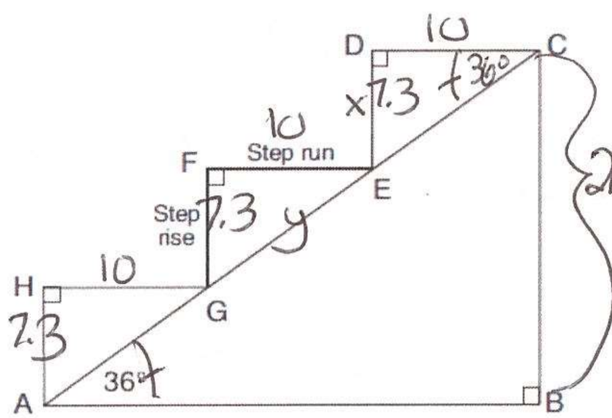
$x = 18.0\ldots$

$-\frac{4}{3}$

16.7 ft

8. A homeowner is building three steps leading to a deck, as modeled by the diagram below. All three step rises, $\overline{HA}$, $\overline{FG}$, and $\overline{DE}$, are congruent, and all three step runs, $\overline{HG}$, $\overline{FE}$, and $\overline{DC}$, are congruent. Each step rise is perpendicular to the step run it joins. The measure of $\angle CAB = 36°$ and $\angle CBA = 90°$.

If each step run is parallel to $\overline{AB}$ and has a length of 10 inches, determine and state the length of each step rise, to the *nearest tenth of an inch*. Determine and state the length of $\overline{AC}$, to the *nearest inch*.



$\tan 36 = \frac{x}{10}$

$\frac{.7265\ldots}{1} = \frac{x}{10}$

$x = 7.3$

$\sin 36 = \frac{21.9}{y}$

$\frac{.5877}{1} = \frac{21.9}{y}$

$\frac{.5877y}{.5877} = \frac{21.9}{.5877\ldots}$

$y = 37\text{ in}$

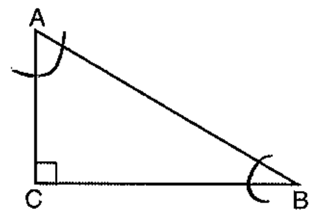
Acute Angles in a Right Triangle

$\sin A = \cos B$ **: In a right triangle, the sine of one acute angle is equal to the cosine of the other acute angle**

$A + B = 90$ **: The two acute angles in a right triangle are complementary**

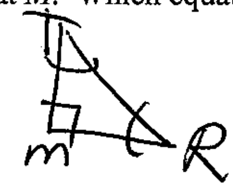
1. In scalene triangle ABC shown in the diagram below, $m\angle C = 90°$.

Which equation is always true?



1) $\sin A = \sin B$
2) $\cos A = \cos B$
3) $\cos A = \sin C$
4) $\sin A = \cos B$

2. Right triangle TMR is a scalene triangle with the right angle at M. Which equation is true?



1) $\sin M = \cos T$
2) $\sin R = \cos R$
3) $\sin T = \cos R$
4) $\sin T = \cos M$

3. Given: Right triangle ABC with right angle at C. If $\sin A$ increases, does $\cos B$ increase or decrease? Explain why.

inclease because sin A = cos B

4. In right triangle ABC, $m\angle C = 90°$. If $\cos B = \frac{5}{13}$, which function also equals $\frac{5}{13}$?

1) $\tan A$
2) $\tan B$
3) $\sin A$
4) $\sin B$

5. In right triangle ABC, $m\angle C = 90°$ and $AC \neq BC$. Which trigonometric ratio is equivalent to $\sin B$?

1) $\cos A$
2) $\cos B$
3) $\tan A$
4) $\tan B$

6. In right triangle ABC with the right angle at C, $\sin A = 2x + 0.1$ and $\cos B = 4x - 0.7$. Determine and state the value of x. Explain your answer.

$\cancel{2x} + 0.1 = 4x - 0.7$
$-\cancel{2x} \quad -2x$
$0.1 = 2x - 0.7$
$+0.7 \quad +0.7$

$\frac{0.8}{2} = \frac{2x}{2}$
$0.4 = x$

They ale equal because sin A = cos B

$\sin A = \cos B \qquad A+B=90$

7. If $\sin(3x+2)^\circ = \cos(4x-10)^\circ$, what is the value of x to the *nearest tenth*?

(1) 7.6 (2) 12.0 ~~(3)~~ 14.0 (4) 26.9

$3x+2+4x-10=90$

$7x-8=90$

$+8 \quad +8$

$\frac{7x}{7}=\frac{98}{7}$

$x=14$

$\sin A = \cos B$

8. If $\sin(2x+7)^\circ = \cos(4x-7)^\circ$, what is the value of x?

1) 7
~~2)~~ 15
3) 21
4) 30

$A+B=90$

$2x+7+4x-7=90$

$\frac{6x}{6}=\frac{90}{6}$

$x=15$

9. In a right triangle, $\sin(40-x)^\circ = \cos(3x)^\circ$. What is the value of x?

$\sin A = \cos B$

1) 10
2) 15
3) 20
~~4)~~ 25

$A+B=90$

$40-x+3x=90$

$2x+40=90$

$-40 \quad -40$

$\frac{2x}{2}=\frac{50}{2}$

$x=25$

$\sin A = \cos B$

10. In a right triangle, the acute angles have the relationship $\sin(2x+4) = \cos(46)$. What is the value of x?

~~1)~~ 20
2) 21
3) 24
4) 25

$A+B=90$

$2x+4+46=90$

$2x+50=90$

$-50 \quad -50$

$\frac{2x}{2}=\frac{40}{2}$

$x=20$

11. Find the value of R that will make the equation $\sin 73^\circ = \cos R$ true when $0^\circ < R < 90^\circ$. Explain your answer.

$\sin A = \cos B$

$A+B=90$

$73+R=90$

$-73 \quad -73$

$R=17$

the acute angles of a right triangle are complementary.

12. Which expression is always equivalent to $\sin x$ when $0^\circ < x < 90^\circ$?

~~1)~~ $\cos(90^\circ - x)$
2) $\cos(45^\circ - x)$
3) $\cos(2x)$
4) $\cos x$

$\sin x = \cos y$

$x+y=90$

$-x \quad -x$

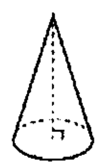
$y=90-x$

Cross Sections (2 dimensional slice of a 3 dimensional object):
The base of the shape is always one of its cross sections
Rectangular Prism: Rectangle, triangle
Cylinder: Circle, ellipse, rectangle
Cone: Circle, ellipse, triangle, "curved" rectangle
Pyramid: Rectangle, triangle
Sphere: Circle

1. Which type of shape can represent a two-dimensional cross-section of a sphere?
   (1) circular (2) triangular (3) square (4) rectangular

2. Which is *not* a possible two-dimensional cross section of a three-dimensional cylinder?
   (1) circle (2) rectangle (3) ellipes (4) triangle

3. William is drawing pictures of cross sections of the right circular cone below.



Which drawing can *not* be a cross section of a cone?


(1)

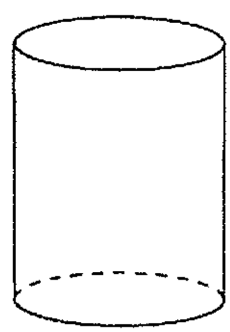

(3)


(2)


(4)

vertical

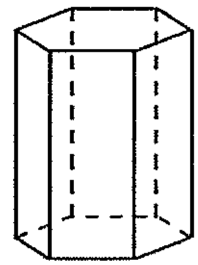
4. A plane intersects a cylinder perpendicular to its bases.



This cross section can be described as a
1) rectangle
2) parabola
3) triangle
4) circle

5. A right hexagonal prism is shown below. A two-dimensional cross section that is perpendicular to the base is taken from the prism.

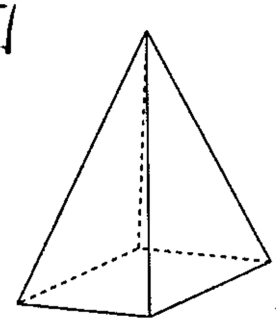
vertical



Which figure describes the two-dimensional cross section?
1) triangle
2) rectangle
3) pentagon
4) hexagon

6. In the diagram below, a plane intersects a square pyramid parallel to its base.

horizontal

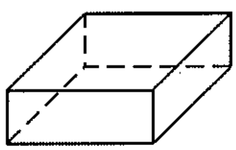


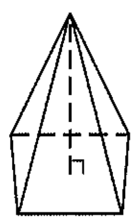
Which two-dimensional shape describes this cross section?

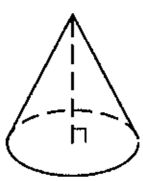
1) circle
2) square
3) triangle
4) pentagon

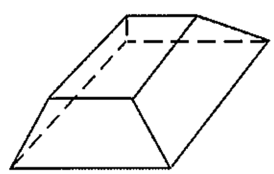
7. Which figure can have the same cross section as a sphere?

circle

1) 

3) 

2) 

4) 

vertical

8. A plane intersects a hexagonal prism. The plane is perpendicular to the base of the prism. Which two-dimensional figure is the cross section of the plane intersecting the prism?

1) triangle
2) trapezoid
3) hexagon
4) rectangle

9. The cross section of a regular pyramid contains the altitude of the pyramid. The shape of this cross section is a

vertical

1) circle
2) square
3) triangle
4) rectangle

Volume

Volume = (<u>Area of the base</u>)(height), if it comes to a point, multiply by $\frac{1}{3}$.

Area of the base is USUALLY $A = lw$ (rectangle/square) or $A = \pi r^2$ (circle)

Most volume formulas are on the reference sheet. Be careful. B = area of the base
General Prism: $V = (area\ base)(height)$
Rectangular prism: $V = lwh$
Cylinder: $V = \pi r^2 h$
Pyramid: $V = \frac{1}{3}lwh$
Cone: $V = \frac{1}{3}\pi r^2 h$
Sphere: $V = \frac{4}{3}\pi r^3$

1. A cylinder has a diameter of 10 inches and a height of 2.3 inches. What is the volume of this cylinder, to the *nearest tenth of a cubic inch*?

$V=\pi r^2 h$
$V=\pi(5)^2(2.3)$
$V=180.6\ in^3$

2. What is the volume of a rectangular prism whose length is 4 cm, width is 6 cm, and height is 5 cm?

$V=lwh$
$V=4(6)(5)$
$V=120\ cm^3$

3. What is the volume of a cube if each side of the cube measures 8 in?

$V=lwh$
$V=8(8)(8)$
$V=512\ in^3$

4. What is the volume of a cylinder whose height is 12 inches and whose diameter is 20 inches in terms of π?

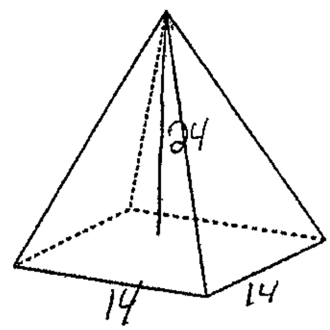
$V=\pi r^2 h$
$V=\pi(10)^2(12)$
$V=1200\pi$

5. Find the volume of a sphere that has a diameter of 12 in in terms of π.

$V=\frac{4}{3}\pi r^3$
$V=\frac{4}{3}\pi(6)^3$
$V=288\pi$

6. A regular pyramid has a square base with an edge length of 14 and an altitude of 24. Find its volume.

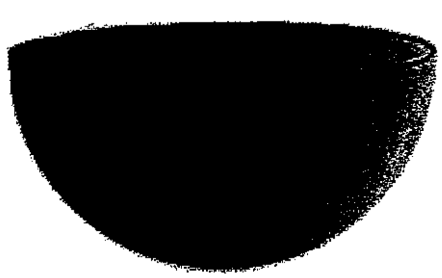
$V=\frac{1}{3}lwh$
$V=\frac{1}{3}(14)(14)(24)$
$V=1568$ units^3



7. Find the volume of a cone with a height of 12 in and a diameter of 8 in rounded to the nearest hundredth.

$V=\frac{1}{3}\pi r^2 h$
$V=\frac{1}{3}\pi(4)^2(12)$
$V=201.06$ in^3

8. Find the volume of the object below if the diameter is 18.2 meters. Round your answer to the *nearest cubic meter*.



hemisphere
$V=\frac{1}{2}(\frac{4}{3}\pi r^3)$
$V=\frac{1}{2}(\frac{4}{3}\pi(9.1)^3)$
$V=1578$ m^3

$density = \frac{mass}{volume}$ Population density $= \frac{Population}{area}$

Name Schlansky
Mr. Schlansky

Date ____________
Geometry

Density

1. Farmer John has a farm with a chicken pen in it. The chicken pen is rectangular measuring 5 yards by 7 yards. If there are 48 chickens in the pen, what is the population density to the nearest tenth of a chicken?

$pd = \frac{p}{a}$

$pd = \frac{48 \text{ chickens}}{35 \text{ yd}^2}$

$A = lw$
$A = 5(7)$
$A = 35 \text{ yd}^2$

2. Jennifer is having her Sweet 16 party on a giant circular patio that has a radius of 7.2 meters. If there are 83 people at the party, to the nearest tenth, what is the population density?

$pd = \frac{p}{a}$

$pd = \frac{83 \text{ ppl}}{162... \text{ yd}^2}$

$pd = .5 \text{ ppl/yd}^2$

$A = \pi r^2$
$A = \pi (7.2)^2$
$A = 162... \text{ m}^2$

3. For a music festival, a stage was built in the shape of a right triangle whose sides measure 6 yards, 8 yards, and 10 yards. At the end of the concert, all of the performers came out an performed together. There were a total of 62 performers on the stage. To the nearest tenth of a person, what was the population density on the stage?

$pd = \frac{p}{a}$

$pd = \frac{62 \text{ ppl}}{24 \text{ yd}^2}$

$pd = 2.6 \text{ ppl/yd}^2$

$A = \frac{1}{2}bh$
$A = \frac{1}{2}(6)(8)$
$A = 24 \text{ yd}^2$

4. Town A has an area of 12 square miles. Town B has an area of 10 square miles. If town A has a population of 8,198 people and town B has a population of 7,384 people, which town has a greater population density? Justify your answer.

Town A

$pd = \frac{p}{a}$

$pd = \frac{8198 \text{ ppl}}{12 \text{ mi}^2}$

$pd = 683... \text{ ppl/mi}^2$

Town B

$pd = \frac{p}{a}$

$pd = \frac{7384 \text{ ppl}}{10 \text{ mi}^2}$

$pd = 738.4 \text{ ppl/mi}^2$

Town B has a greater population density

5. A brick that weighs 1824 grams has dimensions that measure 4 cm by 3 cm by 8 cm. To the nearest tenth, what is the density of the brick?

$d = \frac{m}{V}$

$d = \frac{1824g}{96\ cm^3}$

$d = 19\ g/cm^3$

$V = lwh$

$V = 4(3)(8)$

$V = 96\ cm^3$

6. A cylindrical candleholder has a diameter of 4.5 cm and a height of 20 cm. If the candleholder has a mass of 2900 g, rounded to the nearest whole number, what is its density?

$d = \frac{m}{V}$

$d = \frac{2900g}{318...cm^3}$

$d = 9\ g/cm^3$

Type π

$V = \pi r^2 h$

$V = \pi(2.25)^2(20)$

$V = 318...\ cm^3$

7. What is the density of a solid sphere of clay that has a diameter of 3.2 inches and has a mass of 552 grams? Round your answer to the nearest tenth.

$d = \frac{m}{V}$

$d = \frac{552\ grams}{17...\ in^3}$

$d = 32.2\ g/in^3$

Type π in

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(1.6)^3$

$V = 17...\ in^3$

l,w,h all equal

8. A wooden cube has an edge length of 6 centimeters and a mass of 137.8 grams. Determine the density of the cube, to the *nearest thousandth*. State which type of wood the cube is made of, using the density table below.

Type of Wood	Density (g/cm^3)
Pine	0.373
Hemlock	0.431
Elm	0.554
Birch	0.601
Ash	0.638
Maple	0.676
Oak	0.711

$d = \frac{m}{V}$

$d = \frac{137.8g}{216\ cm^3}$

$d = .638\ g/cm^3$

Ash

$V = lwh$

$V = 6(6)(6)$

$V = 216\ cm^3$

Volume with Algebra
Substitute into appropriate volume formula
Solve the equation
***To get rid of a fraction, multiply by the denominator**
***To get rid of cubed, take the cubed root (final step)**

1. A brick in the shape of a rectangular prism has a base that measures 3 inches by 5 inches. If the volume of the brick is 90 cubic inches, what is the height of the brick?

$V = lwh$

$90 = 3(5)(x)$

$\frac{90}{15} = \frac{15x}{15}$

$6 = x$

2. A right circular cylinder has a volume of 1,000 cubic inches and a height of 8 inches. What is the radius of the cylinder to the *nearest tenth of an inch*?

$V = \pi r^2 h$

$\frac{1000}{8\pi} = \frac{\pi r^2 (8)}{8\pi}$

$\sqrt{39.7..} = \sqrt{r^2}$

$6.3 = r$

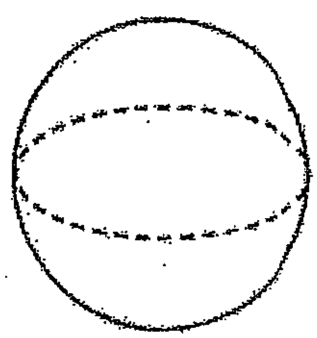
3. The base of a pyramid is a rectangle with a width of 6 cm and a length of 8 cm. Find, in centimeters, the height of the pyramid if the volume is 288 cm^3.

$V = \frac{1}{3} lwh$ $x = 18$

$288 = \frac{1}{3}(6)(8)(x)$

$\frac{288}{16} = \frac{16x}{16}$

4. Find the radius of a sphere with a volume of 576π cubic units. Find the answer to the *nearest tenth of a unit*. $r = 7.6$



$V = \frac{4}{3}\pi r^3$

$3(576\pi) = (\frac{4}{3}\pi r^3)3$

$\frac{1728\pi}{4\pi} = \frac{4\pi r^3}{4\pi}$

$\sqrt[3]{432} = \sqrt[3]{r^3}$

5. The volume of a cylinder is 12,566.4 cm^3. The height of the cylinder is 8 cm. Find the radius of the cylinder to the *nearest tenth of a centimeter.*

$V = \pi r^2 h$

$\frac{12566.4}{8\pi} = \frac{\pi r^2(8)}{8\pi}$

$\sqrt{500..} = \sqrt{r^2}$

$22.4 = r$

6. The Parkside Packing Company needs a rectangular shipping box. The box must have a length of 11 inches and a width of 8 inches. Find, to the *nearest tenth of an inch*, the minimum height of the box such that the volume is *at least* 800 cubic inches.

$V = lwh$

$800 = 11(8)(x)$

$\frac{800}{88} = \frac{88x}{88}$

$9.1 = x$

7. If the volume of a sphere is 36π, what is the radius of the sphere?

~~(1)~~ 3 (2) 6 (3) 12 (4) 24

$V = \frac{4}{3}\pi r^3$

$3(36\pi) = (\frac{4}{3}\pi r^3)3$

$\frac{108\pi}{4\pi} = \frac{4\pi r^3}{4\pi}$

$\sqrt[3]{27} = \sqrt[3]{r^3}$

$3 = r$

8. Find the length of the radius of a cylinder to the *nearest tenth* if it has a volume of 60 cm^3 and a height of 10 cm.

$V = \pi r^2 h$

$\frac{60}{10\pi} = \frac{\pi r^2(10)}{10\pi}$

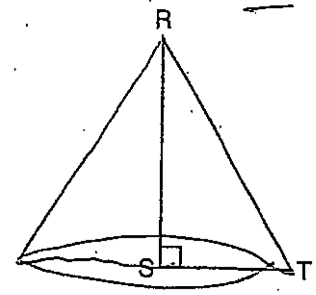
$\sqrt{1.9..} = \sqrt{r^2}$

$1.4 = r$

3 dimensional rotations ALMOST ALWAYS form a cylinder or cone

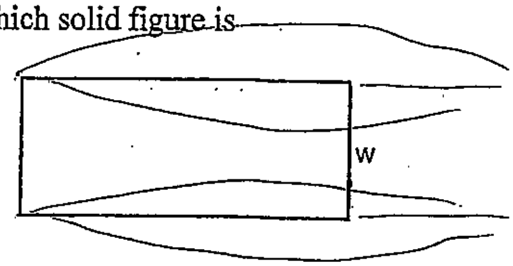
Reflect the shape in 2 dimensions and connect the images with curves

1. Which object is formed when right triangle *RST* shown below is rotated around leg $\overline{RS}$?



1) a pyramid with a square base
2) an isosceles triangle
3) a right triangle
4) a cone

2. If the rectangle below is continuously rotated about side *w*, which solid figure is formed?

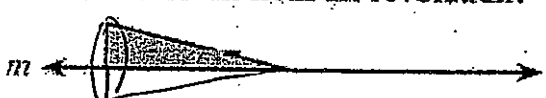


1) pyramid
2) rectangular prism
3) cone
4) cylinder

3. If you rotated the shaded figure below about line *m*, which solid would result from the revolution?

(1) cylinder (2) cone (3) cube (4) sphere

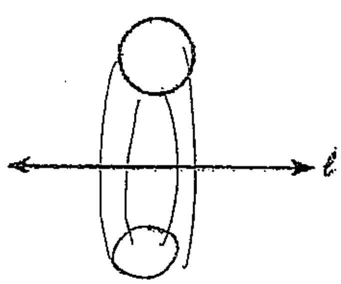
4. If you rotated the triangular region of the figure below about line *m*, what solid would result from the revolution?



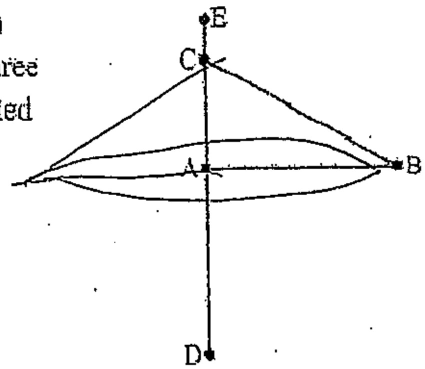
(1) cylinder (2) cone (3) cube (4) sphere

5. What shape will be formed if the circle in the graph is rotated continuously about the line ℓ?

(1) a sphere
(2) a donut
(3) a cylinder
(4) a cone

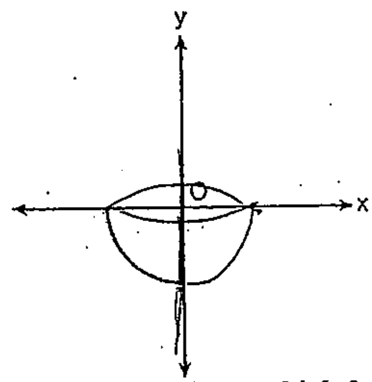


6. ΔABC is shown in the diagram to the right. If $m\angle A = 90$, what three dimensional object will be generated if ΔABC is rotated about $\overleftrightarrow{ED}$?
(1) another right triangle
(2) a cylinder
(3) a cone
(4) a pyramid

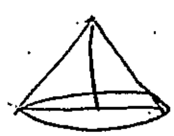


7. Circle O is centered at the origin. In the diagram below, a quarter of circle O is graphed.

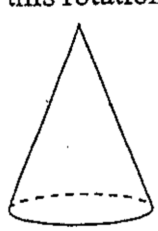
Which three-dimensional figure is generated when the quarter circle is continuously rotated about the y-axis?
1) cone
2) sphere
3) cylinder
4) hemisphere

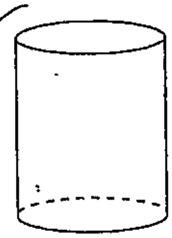


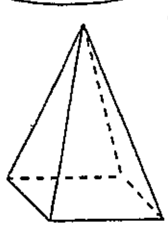
8. If an equilateral triangle is continuously rotated around one of its medians, which 3-dimensional object is generated?
1) cone
2) pyramid
3) prism
4) sphere

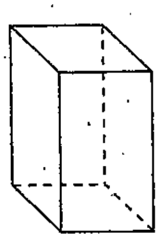


9. A student has a rectangular postcard that he folds in half lengthwise. Next, he rotates it continuously about the folded edge. Which three-dimensional object below is generated by this rotation?

1)


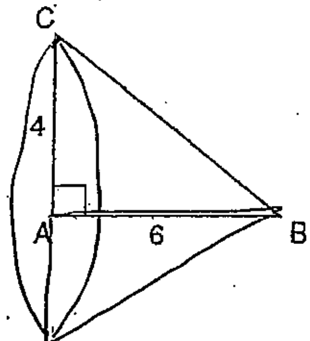
3)


2)


4)


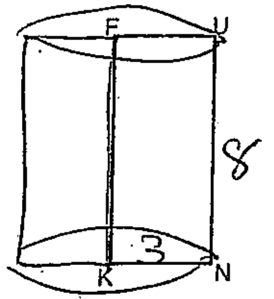
10. In the diagram below, right triangle ABC has legs whose lengths are 4 and 6. What is the volume of the three-dimensional object formed by continuously rotating the right triangle around $\overline{AB}$?

~~1)~~ 32π 3) 96π
2) 48π 4) 144π



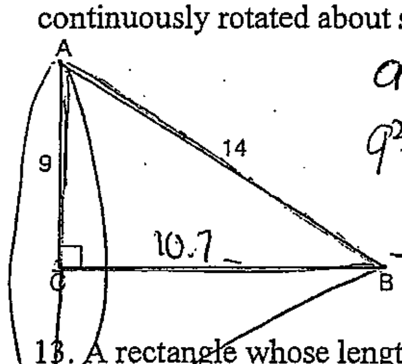
$V=\frac{1}{3}\pi r^2 h$
$V=\frac{1}{3}\pi(4)^2(6)$
$V=32\pi$

11. In the rectangle below, $\overline{UN}=8\,in$ and $\overline{KN}=3\,in$. Find the volume of the three dimensional object created by rotating rectangle FUNK continuously about side $\overline{FK}$



$V=\pi r^2 h$
$V=\pi(3)^2(8)$
$V=72\pi$

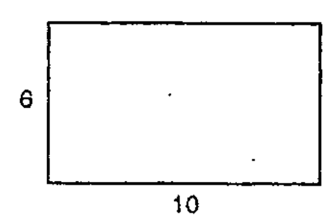
12. In the diagram of right triangle ABC shown below, $AB=14$ and $AC=9$. What is the volume of the three dimensional object formed when the triangle is continuously rotated about side $\overline{BC}$



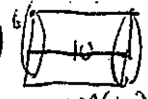
$a^2+b^2=c^2$
$9^2+x^2=14^2$
$81+x^2=196$
$-81 \quad -81$
$\sqrt{x^2}=\sqrt{115}$
$x=10.7..$

$V=\frac{1}{3}\pi r^2 h$
$V=\frac{1}{3}\pi(9)^2(10.7..)$
$V\approx 910$

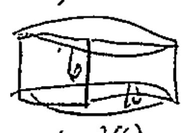
13. A rectangle whose length and width are 10 and 6, respectively, is shown below. The rectangle is continuously rotated around a straight line to form an object whose volume is 150π.



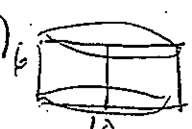
Which line could the rectangle be rotated around?
1) a long side ~~3)~~ the vertical line of symmetry
2) a short side 4) the horizontal line of symmetry

1) 

$V=\pi(6)^2(10)$
$V=360\pi$

2) 

$V=\pi(10)^2(6)$
$V=600\pi$

3) 

$V=\pi(5)^2(6)$
$V=150\pi$

4) 

$V=\pi(3)^2(10)$
$V=90\pi$

Name Schlansky
Mr. Schlansky

Date ____________
Geometry

Unit Analysis

1. A block of wood has a volume of 200 cm^3. The cost of the wood is $.10 per gram and the density of the wood is 2.1 g/cm^3. What would be the cost of producing 15 of these blocks of wood.

$$200\ \cancel{cm^3} \times \frac{2.1\ \cancel{g}}{1\ \cancel{cm^3}} \times \frac{.10\ \$}{1\ \cancel{g}} \times 15 = \$630.00$$

2. A cylindrical test tube has a volume of 45 in^3. The liquid inside has weighs 4 ounces per cubic inch and the cost of the liquid is $.12 per ounce. How much will it cost to fill the test tube to 80% of its capacity?

.8

$$45\ \cancel{in^3} \times \frac{4\ \cancel{oz}}{1\ \cancel{in^3}} \times \frac{.12\ \$}{1\ \cancel{oz}} \times .8 = \$17.28$$

3. The volume of a pool is 25,000 gallons. The cost of the water to fill the pool is $120 per 8000 gallons. How much will it cost to fill the pool up 90%?

$$25000\ \cancel{gal} \times \frac{120\ \$}{8000\ \cancel{gal}} \times .9 = \frac{25000(120)(.9)}{8000} = \$337.50$$

4. An object made of steel has a volume of $24.1 cm^3$. The steel costs $1.25 for 500 grams and has a density of $3.1 g/cm^3$. How much will it cost to make 25 of these objects?

$$24.1\ \cancel{cm^3} \times \frac{3.1\ \cancel{g}}{1\ \cancel{cm^3}} \times \frac{1.25\ \$}{500\ \cancel{g}} \times 25 = \frac{24.1(3.1)(1.25)(25)}{500}$$

$4.67

5. A stone brick has a volume of 150 in^3. The stone weighs 5 grams per cubic inch and it costs $4.52 for 500 grams of stone. How much will it cost to purchase enough stone to make 12 bricks?

$$150\ \cancel{in^3} \times \frac{5\ \cancel{g}}{1\ \cancel{in^3}} \times \frac{4.52\ \$}{500\ \cancel{g}} \times 12 = \frac{150(5)(4.52)(12)}{500}$$

$81.36

6. A machinist creates a solid steel part for a wind turbine engine. The part has a volume of 1015 cubic centimeters. Steel can be purchased for $0.29 per kilogram, and has a density of 7.95 g/cm^3. If the machinist makes 500 of these parts, what is the cost of the steel, to the *nearest dollar*?

convert g to Kg

$$1015\,\cancel{cm^3} \times \frac{7.95\,\cancel{g}}{1\,\cancel{cm^3}} \times \frac{1\,\cancel{Kg}}{1000\,\cancel{g}} \times \frac{.29\,\$}{1\,\cancel{Kg}} \times 500 = \frac{1015(7.95)(.29)(500)}{1000}$$

\$1170

7. A water tower has a volume of 1000 liters and the cost of the water is $250 per cubic kiloliter. How much will it cost to fill the water tower up to 60% of its capacity?

Convert l to Kl

$$1000\,\cancel{l} \times \frac{1\,\cancel{Kl}}{1000\,\cancel{l}} \times \frac{250\,\$}{1\,\cancel{Kl}} \times .6 = \frac{1000(250)(.6)}{1000} = \$150$$

8. A wax candle has a volume of 885 cubic centimeters. The wax costs $1.24 per kilogram and has a density of $1.9\,g/cm^3$. How much will it cost to make 80 candles?

Convert g to Kg

$$885\,\cancel{cm^3} \times \frac{1.9\,\cancel{g}}{1\,\cancel{cm^3}} \times \frac{1\,\cancel{Kg}}{1000\,\cancel{g}} \times \frac{1.24\,\$}{1\,\cancel{Kg}} \times 80 = \frac{885(1.9)(1.24)(80)}{1000}$$

\$166.80

9. An object has a volume of 12 cubic inches and the material it is made from has a density of $7.6\ g/in^3$. If the cost of the material is $1.25 per kilogram, how much will it cost to make 50 of these objects?

Convert g to Kg

$$12\,\cancel{in^3} \times \frac{7.6\,\cancel{g}}{1\,\cancel{in^3}} \times \frac{1\,\cancel{Kg}}{1000\,\cancel{g}} \times \frac{1.25\,\$}{1\,\cancel{Kg}} \times 50 = \frac{12(7.6)(1.25)(50)}{1000}$$

\$5.70

10. An object has a volume of 1200 cubic feet. The material it is made of weighs 3.2 pounds per cubic foot and it costs $2.50 per ounce. If a company has to pay 75% of the cost, how much will the company have to pay for 15 of these objects?

Convert Pounds to ounce

$$1200\,\cancel{ft^3} \times \frac{3.2\,\cancel{Pounds}}{1\,\cancel{ft^3}} \times \frac{16\,\cancel{ounces}}{1\,\cancel{Pounds}} \times \frac{2.50\,\$}{1\,\cancel{ounces}} \times .75 \times 15$$

\$1,728,000

Modeling Volume

1) Check units. Convert if necessary. To convert units: Multiply to get units to cancel out. Example: $3\ in \bullet \frac{2.54\ cm}{1\ in}$

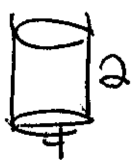
2) FIND VOLUME (Likely to be compound volume (add) or displaced volume (subtract)

3) Begin unit analysis. Start with volume!
Example, a volume of 12 cubic inches has a density of $7.6\ g/in^3$, which costs \$1.25 per kilogram, and 50 are needed that are each filled up to 85%:

$$12\ in^3 \bullet \frac{7.6\ g}{1\ in^3} \bullet \frac{1 kg}{1000 g} \bullet \frac{\$1.25}{1\ kg} \bullet 50 \bullet .85$$

*If given volume, substitute for V and do Algebra!

1. Cylindrical bricks are needed to fill a hole in a homeowner's backyard. Each brick is to have a diameter of 4 cm and a height of 2 cm. The weight of the concrete that the brick is going to be made from is 2.1 ounces per cubic centimeter. If the concrete costs \$.14 per ounce, how much would it cost to purchase four bricks? Round your answer to the *nearest cent.*



$V=\pi r^2 h$

$V=\pi(2)^2(2)$

$V=25...cm^3$

$25...\cancel{cm^3} \cdot \frac{2.1\ \cancel{oz}}{1\ \cancel{cm^3}} \cdot \frac{.14\ \$}{1\ \cancel{oz}} \times 4 = \$29.56$

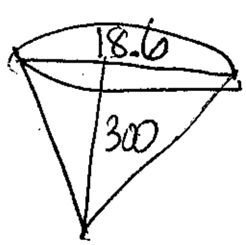
2. A town in upstate New York keeps sand in a silo that is in the shape of a cone. They use this sand to help de-ice the roads after a snowstorm. The silo has a diameter of 18.6 meters and a height of .3 kilometers. The weight of the sand is 1.2 ounces per cubic meter. If the sand costs \$.12 per ounce, how much will it cost the town to fill 80% of the silo?

$.3\cancel{Km} \cdot \frac{1000\ m}{1\ \cancel{Km}} = 300m$

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi(9.3)^2(300)$

$V=27171...m^3$



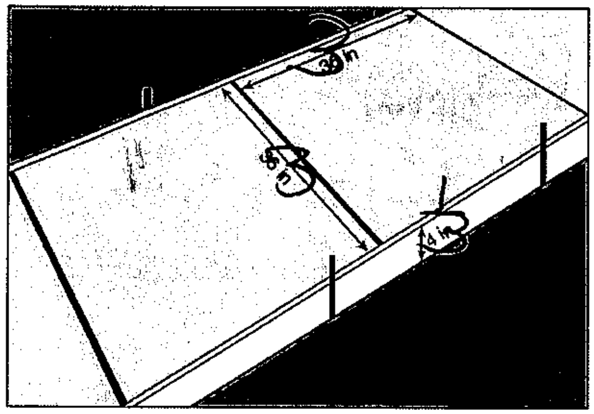
$27171\cancel{m^3} \cdot \frac{1.2\ \cancel{oz}}{1\ \cancel{m^3}} \cdot \frac{.12\ \$}{1\ \cancel{oz}} \times .8$

$\$3130.17$

3. Ian needs to replace two concrete sections in his sidewalk, as modeled below. Each section is 36 inches by 36 inches and 4 inches deep. He can mix his own concrete for \$3.25 per cubic foot.

$36 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 3 \text{ ft}$

$4 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = \frac{1}{3} \text{ ft}$



How much money will it cost Ian to replace the two concrete sections?

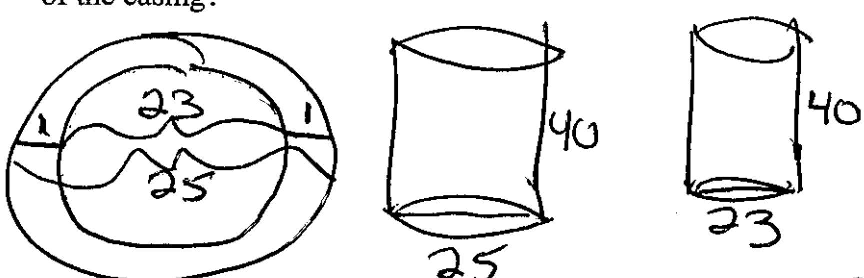
$V = lwh$

$V = 3(3)(\frac{1}{3})$

$V = 3 \text{ ft}^3$

$3 \cancel{\text{ft}^3} \cdot \frac{3.25 \$}{1 \cancel{\text{ft}^3}} \times 2 = \$19.50$

4. A cylindrical casing is to be put around a garbage can in a busy street in Manhattan. The diameter is 25 inches. The height of the case will be 40 inches and the casing will be 1 inch thick. The density of the metal is .841 grams per cubic inch. What will be the mass of the casing?



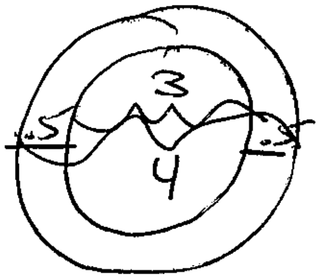
$V = \pi(12.5)^2(40)$ $\quad V = \pi(11.5)^2(40)$

$V = 19634... - V = 16619...$

$V = 3015... \text{ in}^3$

$3015.. \cancel{\text{in}^3} \cdot \frac{.841 \text{ g}}{1 \cancel{\text{in}^3}} = 2536 \text{ g}$

5. A bakery sells hollow chocolate spheres. The larger diameter of each sphere is 4 cm. The thickness of the chocolate of each sphere is 0.5 cm. Determine and state, to the *nearest tenth of a cubic centimeter*, the amount of chocolate in each hollow sphere. The bakery packages 8 of them into a box. If the density of the chocolate is 1.308 g/cm^3, determine and state, to the *nearest gram*, the total mass of the chocolate in the box.



$V=\frac{4}{3}\pi(2)^3$ $\quad V=\frac{4}{3}\pi(1.5)^3$

$V=33\ldots - V=19\ldots = 19\ldots\ cm^3$

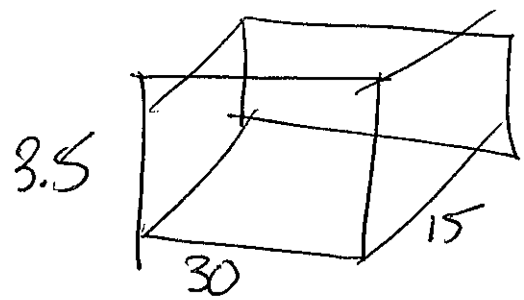
$19\ldots\ \cancel{cm^3} \cdot \frac{1.308\ g}{1\ \cancel{cm^3}} \times 8 = 203$ grams

height of water=3.5 ft

6. Theresa has a rectangular pool 30 ft long, 15 ft wide, and 4 ft deep. Theresa fills her pool using city water at a rate of \$3.95 per 100 gallons of water. Nancy has a circular pool with a diameter of 24 ft and a depth of 4 ft. Nancy fills her pool with a water delivery service at a rate of \$200 per 6000 gallons. If Theresa and Nancy both fill their pools 6 inches from the top of the pool, determine and state who paid more to fill her pool. [1ft^3 water = 7.48 gallons]

Nancy

Theresa



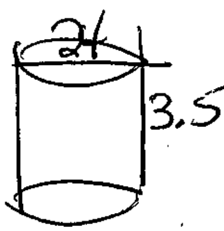
Theresa paid more

$V=lwh$

$V=30(15)(3.5)$

$V=1575\ ft^3$

$1575\ \cancel{ft^3} \cdot \frac{7.48\ gal}{1\ \cancel{ft^3}} \cdot \frac{3.95\$}{100\ gal} = \$465\ldots$



$V=\pi r^2 h$

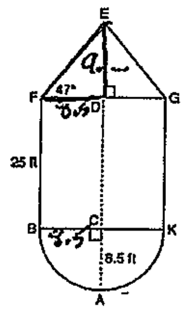
$V=\pi(12)^2(3.5)$

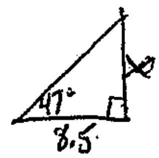
$V=1583\ldots\ ft^3$

$1583\ldots\ \cancel{ft^3} \cdot \frac{7.48\ gal}{1\ ft^3} \cdot \frac{200\$}{6000\ gal}$

\$394...

1 ~~14~~. The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.





$\tan 47 = \frac{x}{8.5}$

$\frac{1.0724}{1} = \frac{x}{8.5}$

$x = 9.1\ldots$

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47°$, determine and state, to the *nearest cubic foot*, the volume of the water tower. The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer. don't care

Cone

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(8.5)^2(9.1\ldots)$

$V = 689\ldots$ +

Cylinder

$V = \pi r^2 h$

$V = \pi(8.5)^2(25)$

$V = 5674\ldots$ +

hemisphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{1}{2}\left(\frac{4}{3}\pi(8.5)^3\right)$

$V = 1286\ldots = 7650 \text{ ft}^3$

$7650 \text{ ft}^3 \cdot \frac{62.4 \text{ pounds}}{1 \text{ ft}^3} \times .85 = 405756$ pounds

$405756 > 400000$

No!

8. Walter wants to make 100 candles in the shape of a cone for his new candle business. The mold shown below will be used to make the candles. Each mold will have a height of 8 inches and a diameter of 3 inches. To the *nearest cubic inch*, what will be the total volume of 100 candles?

Walter goes to a hobby store to buy the wax for his candles. The wax costs \$0.10 per ounce. If the weight of the wax is 0.52 ounce per cubic inch, how much will it cost Walter to buy the wax for 100 candles? If Walter spent a total of \$37.83 for the molds and charges \$1.95 for each candle, what is Walter's profit after selling 100 candles?

$$1885\ in^3 \cdot \frac{.52\ oz}{1\ in^3} \cdot \frac{.10\ \$}{1\ oz} = 98.02$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi (1.5)^2 (8)$$

$$V = 18...$$

$$18..(100) = 1885\ in^3$$

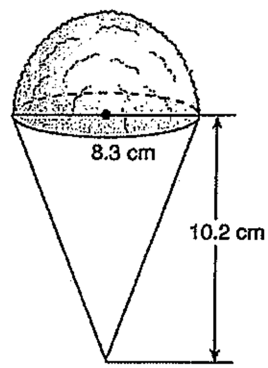
Profit = amount made − amount spent

amount made = 1.95(100) = 195

amount spent = 98.02 + 37.83 = 135.85

$$195 - 135.85 = \$59.15$$

10. A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.



The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per kilogram, of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.

cone / hemisphere

$V=\frac{1}{3}\pi r^2 h$ $\quad V=\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V=\frac{1}{3}\pi(4.15)^2(10.2)$ $\quad V=\frac{1}{2}\left(\frac{4}{3}\pi(4.15)^3\right)$

$V=183...$ $\quad V=149..$

$183... + 149.. = 333... cm^3$

$$333...\,\cancel{cm^3}\cdot\frac{.697\,\cancel{g}}{1\,\cancel{cm^3}}\cdot\frac{1\,\cancel{kg}}{1000\,\cancel{g}}\cdot\frac{3.83\,\$}{1\,\cancel{kg}}\times 50 = \$44.53$$

9 ~~10~~. Jasmine and Nicole are third grade teachers and decided they were going to throw their classes an ice cream party. Jasmine is going to get her students cones while Nicole is going to get her students cups in the shape of cylinders.

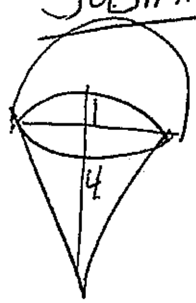
The cones have a height of 4 inches and a diameter of 1 inch. The cones will be completely full of ice cream with a hemispherical scoop on top, which has the same diameter as the cone. The ice cream weighs 0.7 ounces per cubic inch and costs \$.20 per ounce. She must also pay \$.20 for each cone. Jasmine has 24 students in her class.

The cups have a height of 2 inches and a diameter of 8 centimeters. (must convert) The cups will be 90% full of ice cream and there is no cost for the actual cup. This ice cream also weights 0.7 ounces per cubic inch and costs \$.22 per ounce. Nicole has 21 students in her class.

Assuming that every student in the class gets ice cream, which teacher will spend more money and by how much. Round your answer to the *nearest cent.*

$8\,cm \cdot \frac{1\,in}{2.54\,cm} = 3..\,in$

Jasmine



$V=\frac{1}{3}\pi r^2 h \quad V=\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V=\frac{1}{3}\pi(.5)^2(4) \quad V=\frac{1}{2}\left(\frac{4}{3}\pi(.5)^3\right)$

$V=1.. \quad V=.2..$

$1.. + .2.. = 1.3...\,in^3$

$1.3..\,in^3 \cdot \frac{0.7\,oz}{1\,in^3} \cdot \frac{.20\,\$}{1\,oz} \times 24$

$4.. + .20(24) = 9.20$

the cones

Nicole

$V=\pi r^2 h$

$V=\pi(1.57..)^2(2)$

$V=15....\,in^3$

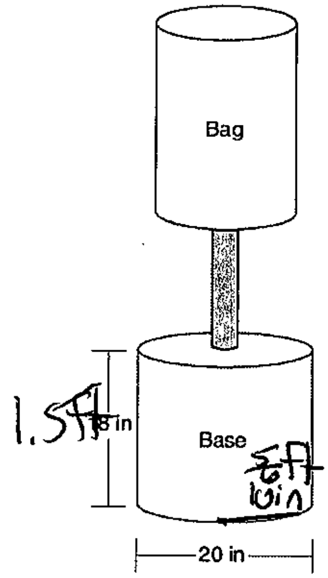
$15..\,in^3 \cdot \frac{0.7\,oz}{1\,in^3} \cdot \frac{.22\,\$}{1\,oz} \times .9 \times 21$

\$45.35

Nicole will spend more money. \$36.15

11. Shae has recently begun kickboxing and purchased training equipment as modeled in the diagram below. The total weight of the bag, pole, and unfilled base is 270 pounds. → weight of unit
The cylindrical base is 18 inches tall with a diameter of 20 inches. The dry sand used to fill the base weighs 95.46 lbs per cubic foot.
To the *nearest pound*, determine and state the total weight of the training equipment if the base is filled to 85% of its capacity.

Weight of training equipment = weight of unit + weight of sand
$= 270 + 265... = 536$ pounds



$V = \pi r^2 h$

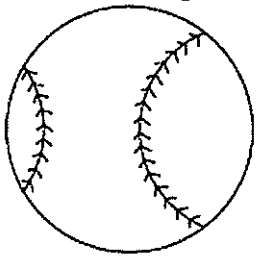
$V = \pi (\frac{5}{6})^2 (1.5)$

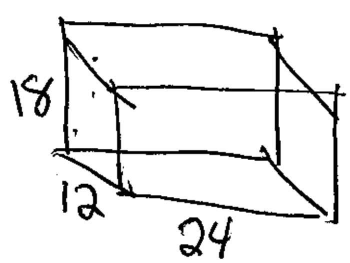
$V = 3.27..ft^3$

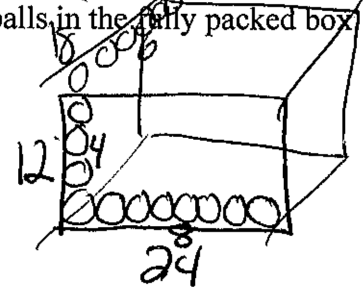
$3.27..\cancel{ft^3} \cdot \frac{95.46\ lb}{1\ \cancel{ft^3}} \times .85 = 265...lb$

$18\cancel{in} \cdot \frac{1\ ft}{12\ \cancel{in}} = 1.5ft \quad 10\cancel{in} \cdot \frac{1\ ft}{12\ \cancel{in}} = \frac{5}{6}ft$

12. A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches.
Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs. The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the *nearest pound*, the total weight of all the baseballs in the fully packed box.







\# of baseballs

8(4)(6)

192 baseballs

$2\cancel{ft} \cdot \frac{12 in}{1 \cancel{ft}} = 24in$

$1\cancel{ft} \cdot \frac{12 in}{1 ft} = 12in$

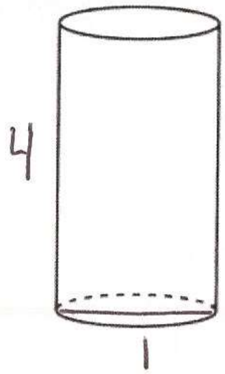
$V = \frac{4}{3}\pi(1.47)^3$

$V = 13.3...in^3$

$13.3..\cancel{in^3} \cdot \frac{.025\ lb}{1\ \cancel{in^3}} \times 192 = 64$ pounds

13. A concrete footing is a cylinder that is placed in the ground to support a building structure. The cylinder is 4 feet tall and 12 inches in diameter. A contractor is installing 10 footings. If a bag of concrete mix makes $\frac{2}{3}$ of a cubic foot of concrete, determine and state the minimum number of bags of concrete mix needed to make all 10 footings.

$12\,in \cdot \frac{1\,ft}{12\,in} = 1\,ft$



$V=\pi r^2 h$

$V=\pi(.5)^2(4)$

$V=3.1415...\,ft^3$

$3.1415..\,ft^3 \cdot \frac{1\ bag}{\frac{2}{3}\,ft^3} \cdot \times 10$

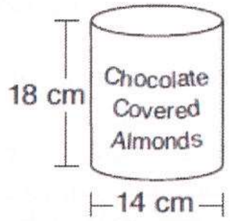
$47.1..$ 48 bags

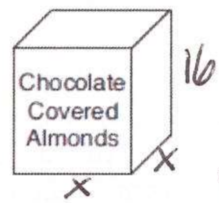
14. A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.

$V=\pi r^2 h$

$V=\pi(7)^2(18)$

$V=2770...\,cm^3$





$V=lwh$

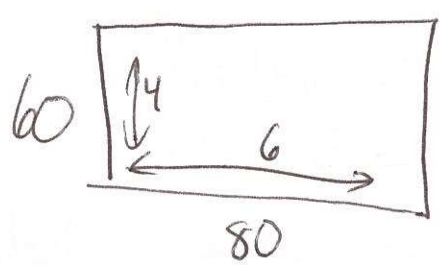
$2770.. = x(x)(16)$

$\frac{2770...}{16} = \frac{16x^2}{16}$

$\sqrt{173} = \sqrt{x^2}$

$13.2 = x$

If the new container's height is 16 cm, determine and state, to the *nearest tenth of a centimeter*, the side length of the new container if both containers contain the same amount of almonds. A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.



$\frac{80}{13.2} = 6.06...$ $\quad 6(4)=24$

$\frac{60}{13.2} = 4.54..$

Finding Center and Radius of a Circle Using Completing the Square

$(x-a)^2 + (y-b)^2 = r^2$ where (a,b) is the center and r is the radius

To put into center-radius form: COMPLETE THE SQUARE TWICE

To find center: Negate what is in the parenthesis. If there are no parentheses, the coordinate is 0.

Radius is the square root of the right hand side

Completing the Square

1) Write the x's together, y's together, and move constant to the other side

$$x^2 + bx + y^2 + by = c$$

2) Add $\left(\frac{b}{2}\right)^2$ to both sides for each variable

3) Factor each trinomial (Both factors must be the same)

4) Rewrite the factors as a binomial squared

$\left(\frac{-12}{2}\right)^2 = 36$ $\left(\frac{-14}{2}\right)^2 = 49$

$\left(\frac{16}{2}\right)^2 = 64$

1. $x^2 + y^2 + 16x + 6y + \cancel{9} = 0$ $\left(\frac{6}{2}\right)^2 = 9$

-9 -9

$x^2 + 16x + y^2 + 6y = -9$

$x^2 + 16x + \boxed{64} + y^2 + 6y + \boxed{9} = -9 + \boxed{64} + \boxed{9}$

$(x+8)(x+8) + (y+3)(y+3) = 64$

$(x+8)^2 + (y+3)^2 = 64$

center: $(-8, -3)$

$r = 8$

2. $x^2 + y^2 - 12x - 14y = 15$

$x^2 - 12x + y^2 - 14y = 15$

$x^2 - 12x + \boxed{36} + y^2 - 14y + \boxed{49} = 15 + \boxed{36} + \boxed{49}$

$(x-6)(x-6) + (y-7)(y-7) = 100$

$(x-6)^2 + (y-7)^2 = 100$

Center: $(6, 7)$

$r = 10$

$\left(\frac{-4}{2}\right)^2 = 4$ $\left(\frac{8}{2}\right)^2 = 16$

3. $x^2 + 8y + \cancel{10} + y^2 - 4x = 6$

-10 -10

$x^2 - 4x + y^2 + 8y = -4$

$x^2 - 4x + \boxed{4} + y^2 + 8y + \boxed{16} = -4 + \boxed{4} + \boxed{16}$

$(x-2)(x-2) + (y+4)(y+4) = 16$

$(x-2)^2 + (y+4)^2 = 16$

center: $(2, -4)$

$r = 4$

$\left(\frac{4}{2}\right)^2 = 4$ $\left(\frac{-2}{2}\right)^2 = 1$

4. $x^2 + 4x + 12 + y^2 - 2y - 1 = 22$

$x^2 + 4x + y^2 - 2y + \cancel{11} = 22$

-11 -11

$x^2 + 4x + y^2 - 2y = 11$

$x^2 + 4x + \boxed{4} + y^2 - 2y + \boxed{1} = 11 + \boxed{4} + \boxed{1}$

$(x+2)(x+2) + (y-1)(y-1) = 16$

$(x+2)^2 + (y-1)^2 = 16$

center: $(-2, 1)$

$r = 4$

$(\frac{-16}{2})^2=64$ $(\frac{6}{2})^2=9$

5. What are the coordinates of the center of a circle whose equation is $x^2+y^2-16x+6y+53=0$?

1) $(-8,-3)$
2) $(-8,3)$
3) $(8,-3)$
4) $(8,3)$

-53

$x^2-16x+y^2+6y=-53$

$x^2-16x+\boxed{64}+y^2+6y+\boxed{9}=-53+\boxed{64}+\boxed{9}$

$(x-8)(x-8)+(y+3)(y+3)=20$

$(x-8)^2+(y+3)^2=20$

$(8,-3)$ $r=\sqrt{20}$

6. The equation of a circle is $x^2+y^2+6y=7$. What are the coordinates of the center and the length of the radius of the circle?

1) center $(0,3)$ and radius 4
2) center $(0,-3)$ and radius 4
3) center $(0,3)$ and radius 16
4) center $(0,-3)$ and radius 16

$(\frac{6}{2})^2=9$

$x^2+y^2+6y+\boxed{9}=7+\boxed{9}$

$x^2+(y+3)(y+3)=16$

$x^2+(y+3)^2=16$

$(0,-3)$ $r=4$

7. What are the coordinates of the center and length of the radius of the circle whose equation is $x^2+6x+y^2-4y=23$?

1) $(3,-2)$ and 36
2) $(3,-2)$ and 6
3) $(-3,2)$ and 36
4) $(-3,2)$ and 6

$(\frac{6}{2})^2=9$

$(\frac{-4}{2})^2=4$

$x^2+6x+\boxed{9}+y^2-4y+\boxed{4}=23+\boxed{9}+\boxed{4}$

$(x+3)(x+3)+(y-2)(y-2)=36$

$(x+3)^2+(y-2)^2=36$

$(-3,2)$ $r=6$

8. What is an equation of a circle whose center is $(1,4)$ and diameter is 10? $r=5$

Since the coordinates are both positive, the x and y coefficients must both be negative.

1) $x^2-2x+y^2-8y=8$
2) ~~$x^2+2x+y^2+8y=8$~~
3) $x^2-2x+y^2-8y=83$
4) ~~$x^2+2x+y^2+8y=83$~~

$(\frac{-2}{2})^2=1$

$(\frac{-8}{2})^2=16$

1) $x^2-2x+\boxed{1}+y^2-8y+\boxed{16}=8+\boxed{1}+\boxed{16}$

$(x-1)^2+(y-4)^2=25$

$(1,4)$ $r=5$ ✓

3) $x^2-2x+\boxed{1}+y^2-8y+\boxed{16}=83+\boxed{1}+$

$(x-1)^2+(y-4)^2=100$

$(1,4)$ $r=10$ ✗

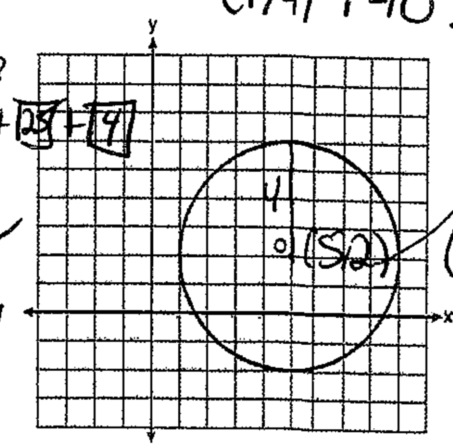
9. What is an equation of circle O shown in the graph below?

1) ~~$x^2+10x+y^2+4y=-13$~~
2) $x^2-10x+y^2-4y=-13$
3) ~~$x^2+10x+y^2+4y=-25$~~
4) $x^2-10x+y^2-4y=-25$

2) $x^2-10x+\boxed{25}+y^2-4y+\boxed{4}=-13+\boxed{25}+\boxed{4}$

$(x-5)^2+(y-2)^2=16$

$(5,2)$ $r=4$ ✓



$(\frac{-10}{2})^2=$

$(\frac{-4}{2})^2=$

4) $x^2-10x+\boxed{25}+y^2-4y+\boxed{4}=-25+\boxed{25}+\boxed{4}$

$(x-5)^2+(y-2)^2=4$

$(5,2)$ $r=2$ ✗

Line Dilations

THE IMAGE IS ALWAYS PARALLEL! SLOPE IS ALWAYS THE SAME!

Conceptual:

Determine if the point is on the line by substituting the x and y coordinates into the equation of the line.

If the point is on the line: Same y intercept (Exact same equation).

If the point is on the line: Different y intercept.

Writing the equation:

If center is origin: Multiply scale factor and original b to find new b

If center is on the line: The image is the same equation as the original.

If the center or scale factor is not given, all we know is that they are parallel (same slope).

1. The line $y = -5x - 1$ is dilated by a scale factor of 2 and centered at the origin. Write an equation that represents the image of the line after the dilation.

multiply scale factor and b

$m = -5$

$b = 2(-1) = -2$

$y = -5x - 2$

2. The line $y = -2x + 4$ is dilated by a scale factor of $\frac{5}{2}$ and centered at the origin. Write an equation that represents the image of the line after the dilation.

multiply scale factor and b

$m = -2$

$b = \frac{5}{2}(4) = 10$

$y = -2x + 10$

3. The line $y = 2x - 4$ is dilated by a scale factor of $\frac{3}{2}$ and centered at the origin. Which equation represents the image of the line after the dilation?

multiply scale factor and b

1) $y = 2x - 4$
2) $y = 2x - 6$
3) $y = 3x - 4$
4) $y = 3x - 6$

$m = 2$

$b = \frac{3}{2}(-4)$

$b = -6$

$y = 2x - 6$

4. What is an equation of the image of the line $y = \frac{3}{2}x - 4$ after a dilation of a scale factor of $\frac{3}{4}$ centered at the origin?

multiply scale factor and b

1) $y = \frac{9}{8}x - 4$
2) $y = \frac{9}{8}x - 3$
3) $y = \frac{3}{2}x - 4$
4) $y = \frac{3}{2}x - 3$

$m = \frac{3}{2}$

$b = \frac{3}{4}(-4) = -3$

$y = \frac{3}{2}x - 3$

5. Line $y = 3x - 1$ is transformed by a dilation with a scale factor of 2 and centered at (3, 8). The line's image is → Same equation

1) $y = 3x - 8$
2) $y = 3x - 4$
3) $y = 3x - 2$
4) $y = 3x - 1$

$8 = 3(3) - 1$

$8 = 8$ ✓

6. Line MN is dilated by a scale factor of 2 centered at the point $(0, 6)$. If $\overleftrightarrow{MN}$ is represented by $y = -3x + 6$, which equation can represent $\overleftrightarrow{M'N'}$, the image of $\overleftrightarrow{MN}$? → Same equation

1) $y = -3x + 12$
2) $y = -3x + 6$
3) $y = -6x + 12$
4) $y = -6x + 6$

$6 = -3(0) + 6$

$6 = 6$ ✓

7. The line $y = 4x - 2$ is dilated by a scale factor of 3 and centered at the point (-1,-6). Which equation represents the image of the line after the dilation? → Same equation

1) $y = 4x - 2$ 3) $y = 12x - 2$
2) $y = 4x - 6$ 4) $y = 12x - 6$

$-6 = 4(-1) - 2$

$-6 = -6$ ✓

8. The line $y = \frac{1}{2}x + 5$ is dilated by a scale factor of 4 and centered at the point (4,7). Which equation represents the image of the line after the dilation? → Same equation

1) $y = \frac{1}{2}x + 20$ 3) $y = 2x + 20$
2) $y = \frac{1}{2}x + 5$ 4) $y = 2x + 5$

$7 = \frac{1}{2}(4) + 5$

$7 = 7$ ✓

9. The equation of line h is $2x + y = 1$. Line m is the image of line h after a dilation of scale factor 4 with respect to the origin. What is the equation of the line m? → multiply scale factor and b

$-2x \quad -2x$

1) $y = -2x + 1$
2) $y = -2x + 4$
3) $y = 2x + 4$
4) $y = 2x + 1$

$y = -2x + 1$

$m = -2$

$b = 4(1) = 4$

$y = -2x + 4$

10. The line $2x + 3y = 8$ is dilated by a scale factor of 3 and centered at the point (1,2). Which equation represents the image of the line after the dilation? → Same equation

1) $y = -\frac{2}{3}x + \frac{8}{3}$ 3) $y = -2x + \frac{8}{3}$
2) $y = -\frac{2}{3}x + 8$ 4) $y = -2x + 8$

$2(1) + 3(2) = 8$

$8 = 8$ ✓

$\cancel{2x}$

$2x + 3y = 8$

$-2x \quad -2x$

$\frac{3y}{3} = \frac{-2x + 8}{3}$

$y = -\frac{2}{3}x + \frac{8}{3}$

since we don't know scale factor, all we know is parallel (same slope)

11. The line $3y = -2x + 8$ is transformed by a dilation centered at the origin. Which linear equation could be its image?

$+2x \quad +2x$

1) $2x + 3y = 5$
2) $2x - 3y = 5$
3) $3x + 2y = 5$
4) $3x - 2y = 5$

$2x+3y=8$

12. The line represented by the equation $4y = 3x + 7$ is transformed by a dilation centered at the origin. Which linear equation could represent its image?

1) $3x - 4y = 9$
2) $3x + 4y = 9$
3) $4x - 3y = 9$
4) $4x + 3y = 9$

$4y=3x+7$
$-3x \quad -3x$
$\frac{-3x+4y}{-1}=\frac{7}{-1}$
$3x-4y=-7$

13. The line $-3x + 4y = 8$ is transformed by a dilation centered at the origin. Which linear equation could represent its image?

1) $y = \frac{4}{3}x + 8$
2) $y = \frac{3}{4}x + 8$
3) $y = -\frac{3}{4}x - 8$
4) $y = -\frac{4}{3}x - 8$

$-3x+4y=8$
$+3x \quad +3x$
$\frac{4y}{4}=\frac{3x+8}{4}$
$y=\frac{3}{4}x+2$

14. Line n is represented by the equation $3x + 4y = 20$. Determine and state the equation of line p, the image of line n, after a dilation of scale factor $\frac{1}{3}$ centered at the point $(4, 2)$. Explain your answer.

$3(4)+4(2)=20$
$20=20$ ✓

$3x+4y=20$
the center of dilation is on the line so the equation remains the same.

15. Aliyah says that when the line $4x + 3y = 24$ is dilated by a scale factor of 2 centered at the point $(3, 4)$, the equation of the dilated line is $y = -\frac{4}{3}x + 16$. Is Aliyah correct? Explain why.

$4(3)+3(4)=24$
$24=24$ ✓
Since the center is on the line, the equation must be the same.

$4x+3y=24$
$-4x \quad -4x$
$\frac{3y}{3}=\frac{-4x+24}{3}$
$y=-\frac{4}{3}x+8$

No, the equations are not the same.

Dilating Segments with Perimeter and Area

Multiply the original segment and scale factor to find the image.

Multiply the original perimeter and scale factor to find the image perimeter.

Multiply the original area and the $(scale\ \ factor)^2$ to find the image area.

*You may have to use distance formula to find original segment.

*The center of dilation does not effect the size of the image

1. A line segment with a length of 5 is dilated by a scale factor of 4. What is the length of its image?

multiply

$5(4)=20$

2. A line segment has a length of 12 and is dilated by $\frac{1}{2}$. What is the length of its image?

multiply $\quad 12(\frac{1}{2})=6$

3. A three-inch line segment is dilated by a scale factor of 6 and centered at its midpoint. What is the length of its image? multiply $\quad 3(6)=18$ $\quad$ irrelevant for length

1) 9 inches
2) 2 inches
3) 15 inches
4) 18 inches

4. The coordinates of the endpoints of $\overline{AB}$ are $A(2,3)$ and $B(5,-1)$. Determine the length of $\overline{A'B'}$, the image of $\overline{AB}$, after a dilation of $\frac{1}{2}$ centered at the origin.

multiply $\quad$ Irrelevant for length.

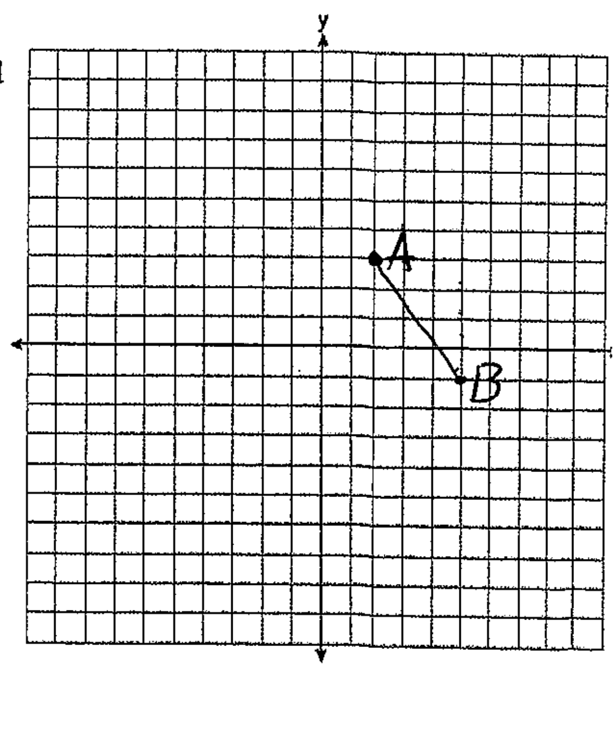
$d\overline{AB}=\sqrt{3^2+4^2}$

$d\overline{AB}=\sqrt{9+16}$

$d\overline{AB}=\sqrt{25}$

$d\overline{AB}=5$

$5(\frac{1}{2})=\frac{5}{2}$



scale factor = perimeter factor
$(\text{scale factor})^2$ = area factor

5. Triangle JOY has a perimeter of 10 and an area of 12. What is the perimeter and area of triangle JOY after a dilation by a scale factor of 2?

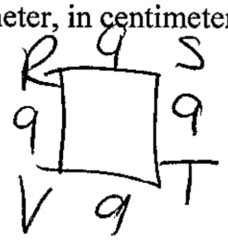
perimeter: $10(2) = 20$
area: $12(2)^2 = 48$

6. Quadrilateral CAMI has a perimeter of 20 and an area of 15. What is the perimeter and area of quadrilateral CAMI after a dilation by a scale factor of 4?

perimeter: $20(4) = 80$
area: $15(4)^2 = 240$

7. Given square *RSTV*, where $RS = 9$ cm. If square *RSTV* is dilated by a scale factor of 3 about a given center, what is the perimeter, in centimeters, of the image of *RSTV* after the dilation?
1) 12
2) 27
3) 36
4) 108

~~9(3) = 27~~



$36(3) = 108$

P=36

8. Triangle *RJM* has an area of 6 and a perimeter of 12. If the triangle is dilated by a scale factor of 3 centered at the origin, what are the area and perimeter of its image, triangle *R'J'M'*?
1) area of 9 and perimeter of 15
2) area of 18 and perimeter of 36
3) area of 54 and perimeter of 36
4) area of 54 and perimeter of 108

perimeter: $12(3) = 36$
area: $6(3)^2 = 54$

9. Rectangle *A'B'C'D'* is the image of rectangle *ABCD* after a dilation centered at point *A* by a scale factor of $\frac{2}{3}$. Which statement is correct?

1) Rectangle *A'B'C'D'* has a perimeter that is $\frac{2}{3}$ the perimeter of rectangle *ABCD*. ✓
2) Rectangle *A'B'C'D'* has a perimeter that is $\frac{3}{2}$ the perimeter of rectangle *ABCD*. ✗
3) Rectangle *A'B'C'D'* has an area that is $\left(\frac{2}{3}\right)^2$ the area of rectangle *ABCD*. ✗
4) Rectangle *A'B'C'D'* has an area that is $\frac{3}{2}$ the area of rectangle *ABCD*. ✗

Equation of a line through a point flip and negate

1) Find m using parallel (same slope) or perpendicular (negative reciprocal slopes).
2) Substitute into $y - y_1 = m(x - x_1)$. Don't forget to negate x_1 and y_1.
3) If it's multiple choice, you may have to distribute and isolate y.

1. What is the equation of a line that passes through the point $(-3, -11)$ and is parallel to the line whose equation is $2x - y = 4$?

x_1 y_1 same slope

1) $y = 2x + 5$
2) ~~2)~~ $y = 2x - 5$
3) $y = \frac{1}{2}x + \frac{25}{2}$
4) $y = -\frac{1}{2}x - \frac{25}{2}$

$-2x \quad -2x$

$\frac{-y}{-1} = \frac{-2x+4}{-1}$

$y = 2x - 4$

$m = 2$

$m\parallel = 2$

$x_1 = -3$

$y_1 = -11$

$y - y_1 = m(x - x_1)$

$y + 11 = 2(x + 3)$

$y + \cancel{11} = 2x + 6$

$-11 \quad -11$

$y = 2x - 5$

2. What is an equation of the line that passes through the point $(-2, 5)$ and is perpendicular to the line whose equation is $y = \frac{1}{2}x + 5$?

x_1 y_1 negative reciprocal slopes

1) $y - 5 = \frac{1}{2}(x + 2)$
2) ~~2)~~ $y - 5 = -2(x + 2)$
3) $y + 5 = \frac{1}{2}(x - 2)$
4) $y + 5 = -2(x - 2)$

$m = \frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 5 = -2(x + 2)$

$m\perp = -2$

$x_1 = -2$

$y_1 = 5$

3. What is an equation of the line that contains the point $(3, -1)$ and is perpendicular to the line whose equation is $y = -3x + 2$?

x_1 y_1 negative reciprocal slopes

1) $y = -3x + 8$
2) $y = -3x$
3) $y = \frac{1}{3}x$
4) ~~4)~~ $y = \frac{1}{3}x - 2$

$m = -3$

$m\perp = \frac{1}{3}$

$x_1 = 3$

$y_1 = -1$

$y - y_1 = m(x - x_1)$

$y + 1 = \frac{1}{3}(x - 3)$

$y + \cancel{1} = \frac{1}{3}x - 1$

$-1 \quad -1$

$y = \frac{1}{3}x - 2$

4. An equation of the line that passes through $(2, -1)$ and is parallel to the line $2y + 3x = 8$ is

x_1 y_1 same slope

1) ~~1)~~ $y + 1 = -\frac{3}{2}(x - 2)$
2) $y + 1 = \frac{2}{3}(x - 2)$
3) $y - 1 = -\frac{3}{2}(x + 2)$
4) $y - 1 = \frac{2}{3}(x + 2)$

$-3x \quad -3x$

$\frac{2y}{2} = \frac{-3x + 8}{2}$

$y = -\frac{3}{2}x + 4$

$m = -\frac{3}{2}$

$m\parallel = -\frac{3}{2}$

$x_1 = 2$

$y_1 = -1$

$y - y_1 = m(x - x_1)$

$y + 1 = -\frac{3}{2}(x - 2)$

$=3/5$

negative reciprocal slopes

5. What is an equation of the line that is perpendicular to the line whose equation is $y=\frac{3}{5}x-2$ and that passes through the point $(3,-6)$?
$x_1\ y_1$

$m\perp=-\frac{5}{3}$
$x_1=3$
$y_1=-6$

1) $y=\frac{5}{3}x-11$
2) $y=-\frac{5}{3}x+11$
3) $y=-\frac{5}{3}x-1$
4) $y=\frac{5}{3}x+1$

$y-y_1=m(x-x_1)$
$y+6=-\frac{5}{3}(x-3)$
$y+6=-\frac{5}{3}x+5$
$-6 \qquad\qquad -6$
$y=-\frac{5}{3}x-1$

6. The equation of a line is $y=\frac{2}{3}x+5$. What is an equation of the line that is perpendicular to the given line and that passes through the point $(4,2)$?
$x_1\ y_1$

negative reciprocal slopes

1) $y=\frac{2}{3}x-\frac{2}{3}$
2) $y=\frac{3}{2}x-4$
3) $y=-\frac{3}{2}x+7$
4) $y=-\frac{3}{2}x+8$

$\boxed{m=\frac{2}{3}}$

$y-y_1=m(x-x_1)$ $\quad m\perp=-\frac{3}{2}$
$y-2=-\frac{3}{2}(x-4)$ $\quad x_1=4$
$y_1=2$
$y-2=-\frac{3}{2}x+6$
$+2 \qquad\qquad +2$
$y=-\frac{3}{2}x+8$

7. What is an equation of the line that passes through the point $(6,8)$ and is perpendicular to a line with equation $y=\frac{3}{2}x+5$?
$x_1\ y_1$

$m=\frac{3}{2}$

negative reciprocal slopes

$m\perp=-\frac{2}{3}$
$x_1=6$
$y_1=8$

1) $y-8=\frac{3}{2}(x-6)$
2) $y-8=-\frac{2}{3}(x-6)$
3) $y+8=\frac{3}{2}(x+6)$
4) $y+8=-\frac{2}{3}(x+6)$

$y-y_1=m(x-x_1)$
$y-8=-\frac{2}{3}(x-6)$

8. What is an equation of a line which passes through $(6,9)$ and is perpendicular to the line whose equation is $4x-6y=15$?
$x_1\ y_1$

$-4x \qquad -4x$

negative reciprocal slopes

1) $y-9=-\frac{3}{2}(x-6)$
2) $y-9=\frac{2}{3}(x-6)$
3) $y+9=-\frac{3}{2}(x+6)$
4) $y+9=\frac{2}{3}(x+6)$

$\frac{-6y}{-6}=\frac{-4x+15}{-6}$
$y=\frac{2}{3}x-\frac{5}{2}$

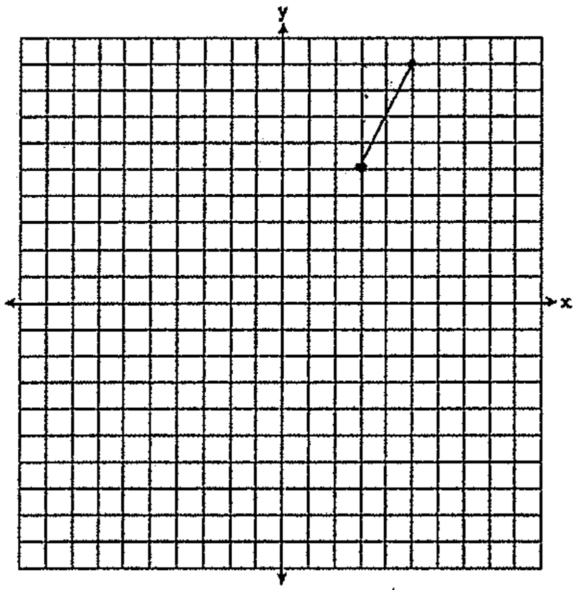
$m\perp=-\frac{3}{2}$
$x_1=6$
$y_1=9$
$y-y_1=m(x-x_1)$
$y-9=-\frac{3}{2}(x-6)$

Name Schlansky
Mr. Schlansky

Date ____________
Geometry

Perpendicular Bisector

1. Write an equation of the perpendicular bisector of the line segment whose endpoints are (3,5) and (5,9).



$m=\frac{\Delta y}{\Delta x}$

$m=\frac{4}{2}$

$m=2$

$mp=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$mp=\left(\frac{3+5}{2},\frac{5+9}{2}\right)$

$mp=\left(\frac{8}{2},\frac{14}{2}\right)$

$mp=(4,7)$

$m\perp=-\frac{1}{2}$

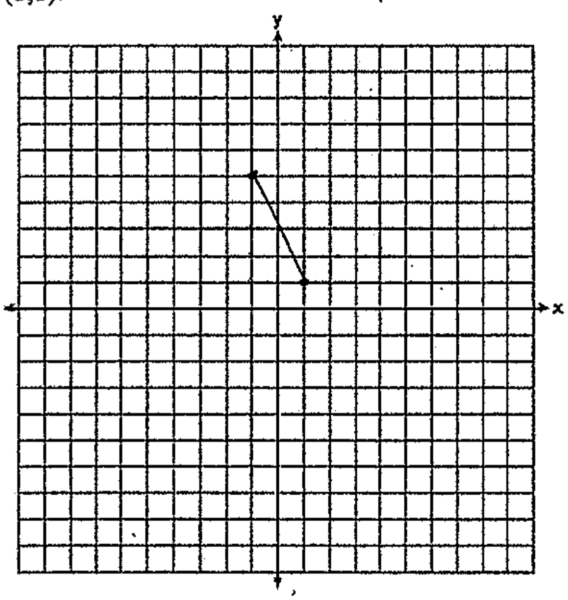
$x_1=4$

$y_1=7$

$y-y_1=m(x-x_1)$

$y-7=-\frac{1}{2}(x-4)$

2. Write an equation of the perpendicular bisector of the line segment whose endpoints are (-1,5) and (1,1).



$m=\frac{\Delta y}{\Delta x}$

$m=\frac{-4}{2}$

$m=-2$

$mp=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$mp=\left(\frac{-1+1}{2},\frac{5+1}{2}\right)$

$mp=\left(\frac{0}{2},\frac{6}{2}\right)$

$mp=(0,3)$

$m\perp=\frac{1}{2}$

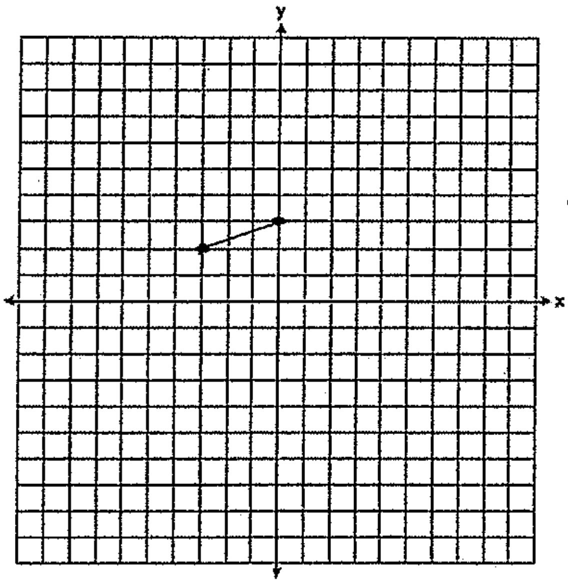
$x_1=0$

$y_1=3$

$y-y_1=m(x-x_1)$

$y-3=\frac{1}{2}x$

3. Write an equation of the perpendicular bisector of the line segment whose endpoints are (-3,2) and (0,3).



$m=\frac{\Delta y}{\Delta x}$ $mp=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$m=\frac{1}{3}$ $mp=\left(\frac{-3+0}{2},\frac{2+3}{2}\right)$ $m\perp=-3$

$x_1=-1.5$

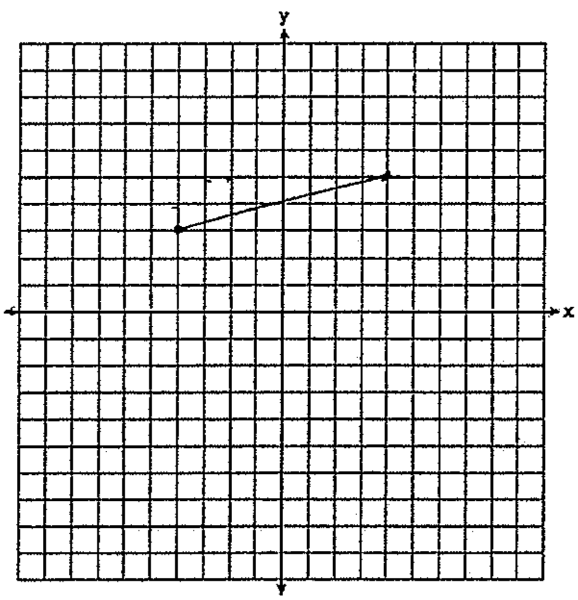
$y_1=2.5$

$mp=\left(-\frac{3}{2},\frac{5}{2}\right)$

$mp=(-1.5,2.5)$ $y-y_1=m(x-x_1)$

$y-2.5=-3(x+1.5)$

4. Write an equation of the perpendicular bisector of the line segment whose endpoints are (-4,3) and (4,5)



$m=\frac{\Delta y}{\Delta x}$ $mp=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$m=\frac{2}{8}$ $mp=\left(\frac{-4+4}{2},\frac{3+5}{2}\right)$

$mp=\left(\frac{0}{2},\frac{8}{2}\right)$

$m=\frac{1}{4}$ $mp=(0,4)$

$y-y_1=m(x-x_1)$ $m\perp=-4$

$y-4=-4(x-0)$ $x_1=0$

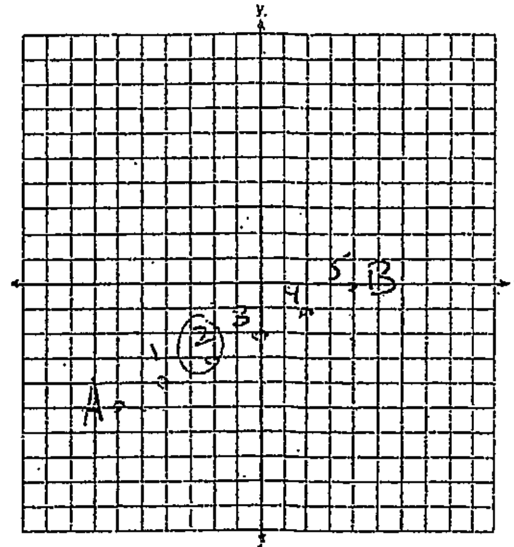
$y_1=4$

$y-4=-4x$

Partitions

1) **Find** $\frac{\Delta x}{p}$ *and* $\frac{\Delta y}{p}$ where p is the number of partitions.
2) **Count those values out on the graph between the two endpoints**
3) **Circle and state the point that matches the given ratio.**
BE CAREFUL WHICH POINT YOU START FROM!

1. The coordinates of the endpoints of $\overline{AB}$ are $A(-6,-5)$ and $B(4,0)$. Point P is on $\overline{AB}$. Determine and state the coordinates of point P, such that $AP:PB$ is $2:3$. $p=5$



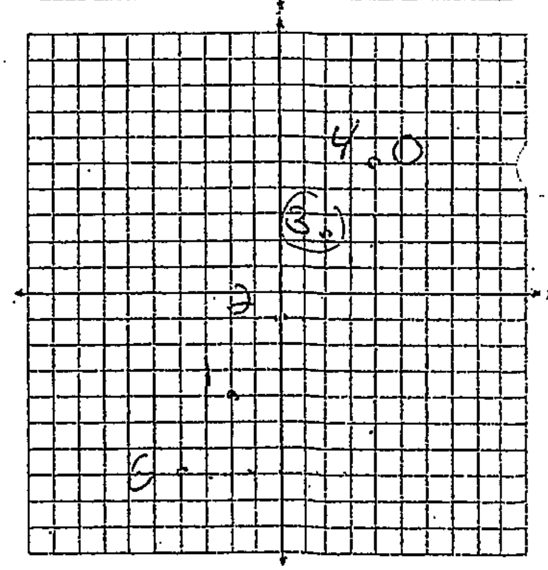
$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$

$\frac{10}{5}$ $\frac{5}{5}$

2 1

$(-2,-3)$

2. What are the coordinates of the point on the directed line segment from $G(-4,-7)$ to $O(4,5)$ that partitions the segment into a ratio of 3 to 1? $p=4$



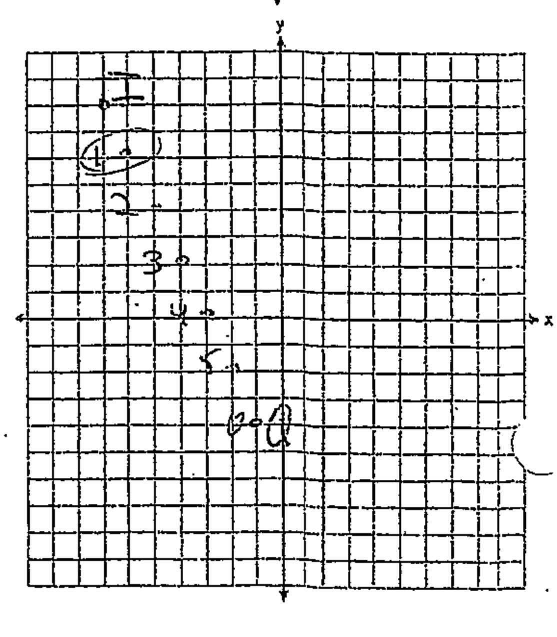
$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$

$\frac{8}{4}$ $\frac{12}{4}$

2 3

$(2,2)$

3. Directed line segment IQ has endpoints whose coordinates are $I(-7,8)$ and $Q(-1,-4)$. Determine the coordinates of point J that divides the segment in the ratio 1 to 5. $p=6$



$(-6,6)$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$

$\frac{6}{6}$ $\frac{12}{6}$

1 2

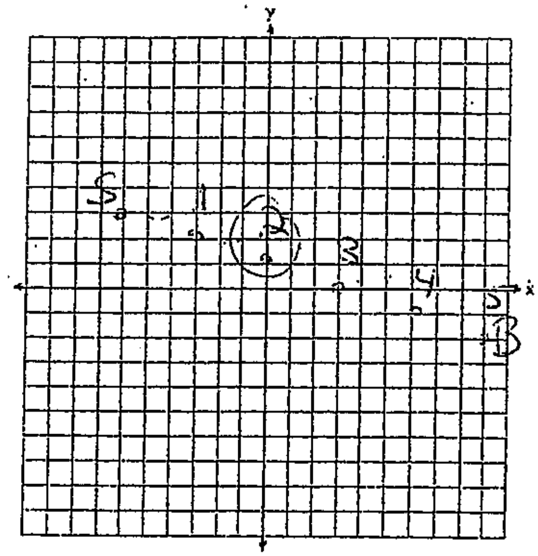
4. Directed line segment SB has endpoints whose coordinates are $S(-6,3)$ and $B(9,-2)$. Determine the coordinates of point J that divides the segment in the ratio 2 to 3. $p=5$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$

$\frac{15}{5}$ $\frac{5}{5}$

3 1

$(0,1)$



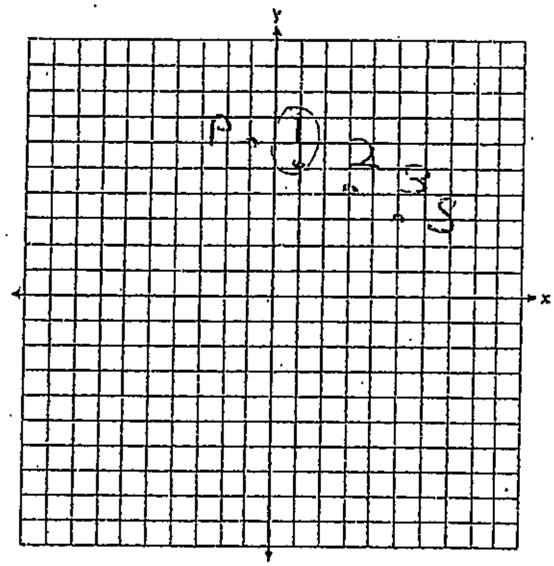
5. What are the coordinates of the point on the directed line segment from $P(-1,6)$ to $S(5,3)$ that partitions the segment into a ratio of 1 to 2? $P=3$

$\frac{\Delta x}{P}$ $\frac{\Delta y}{P}$

$\frac{6}{3}$ $\frac{3}{3}$

2 1

$(1,5)$



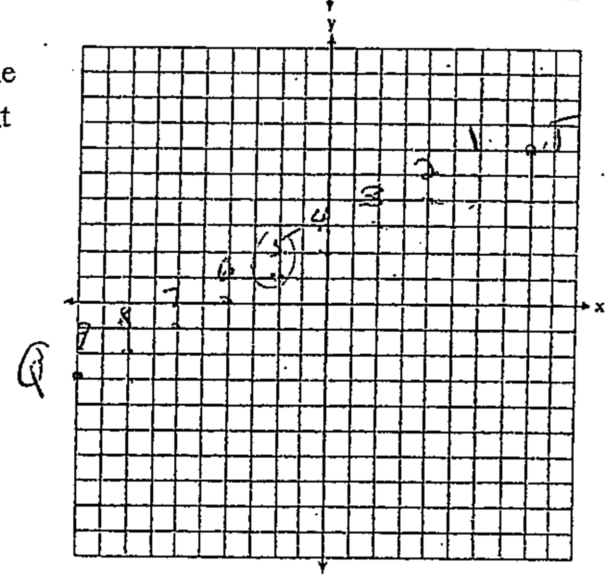
6. Directed line segment JK has endpoints whose coordinates are $J(8,6)$ and $Q(-10,-3)$. Determine the coordinates of point O that divides the segment in the ratio 5 to 4. $p=9$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$

$\frac{18}{9}$ $\frac{9}{9}$

2 1

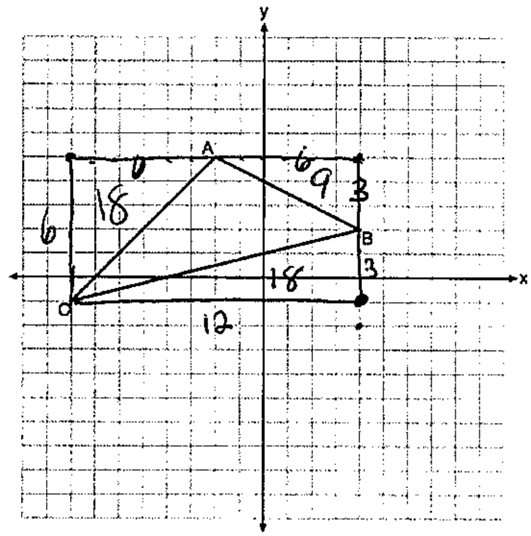
$(-2,1)$



Area with Coordinate Geometry
Box Method

1) Build a rectangle around the shape
2) Find the area of the rectangle $(A = lw)$
3) Find the area of the triangles outside of the shape $(A = \frac{1}{2}lw)$
4) Subtract the triangle areas from the rectangle area

1. Triangle ABC with coordinates $A(-2,5)$, $B(4,2)$, and $C(-8,-1)$ is graphed on the set of axes below. Determine and state the area of $\triangle ABC$.

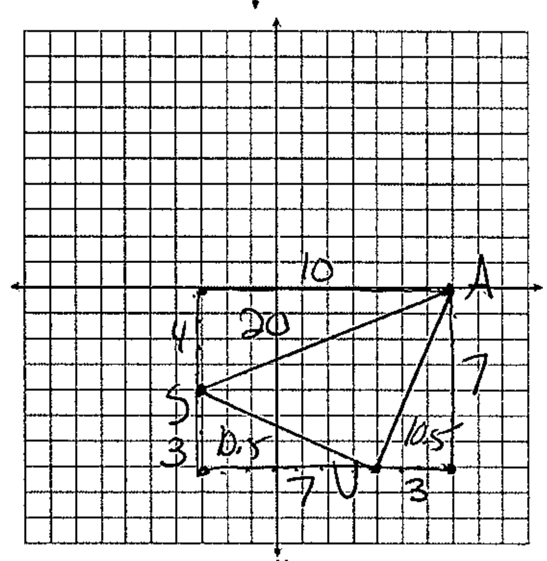


$A_R = lw$
$A_R = 12(6) = 72$

$A_{t1} = \frac{1}{2}(6)(6) = 18$
$A_{t2} = \frac{1}{2}(6)(3) = 9$
$A_{t3} = \frac{1}{2}(12)(3) = 18$
45

72
−45
$\boxed{27}$

2. Triangle USA has vertices U(4,-7), S(-3,-4), and A(7,0). Find the area of triangle USA.



$A_R = lw$
$A_R = 10(7) = 70$

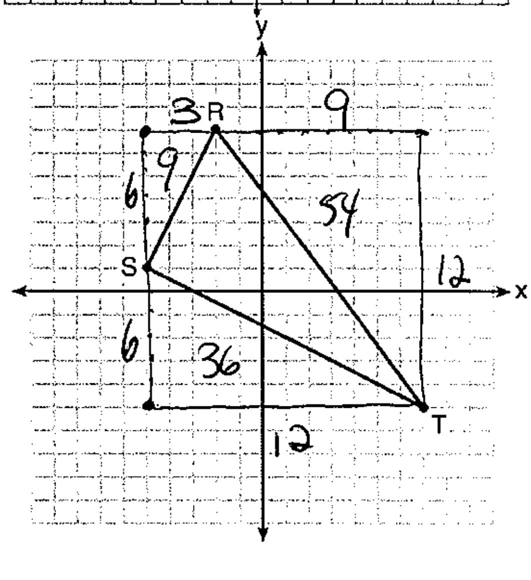
$A_{t1} = \frac{1}{2}(10)(4) = 20$
$A_{t2} = \frac{1}{2}(3)(7) = 10.5$
$A_{t3} = \frac{1}{2}(3)(7) = 10.5$
41

70
−41
$\boxed{29}$

3. Triangle RST is graphed on the set of axes below.

How many square units are in the area of $\triangle RST$?

1) $9\sqrt{3} + 15$
2) $9\sqrt{5} + 15$
3) 45
4) 90

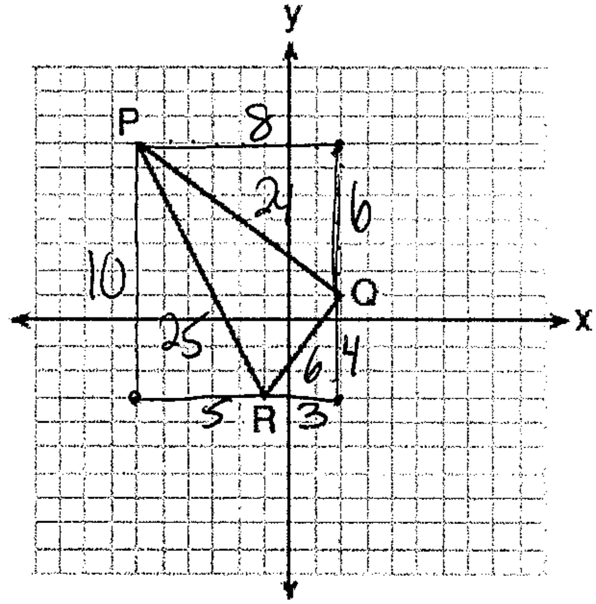


$A_R = 12(12) = 144$
$A_{t1} = \frac{1}{2}(6)(3) = 9$
$A_{t2} = \frac{1}{2}(9)(12) = 54$
$A_{t3} = \frac{1}{2}(6)(12) = 36$
99

144
−99
45

4. On the set of axes below, the vertices of $\triangle PQR$ have coordinates $P(-6,7)$, $Q(2,1)$, and $R(-1,-3)$. What is the area of $\triangle PQR$?

1) 10 3) 25
2) 20 4) 50



$A_R = 10(8) = 80$ $A_{T1} = \frac{1}{2}(10)(5) = 25$

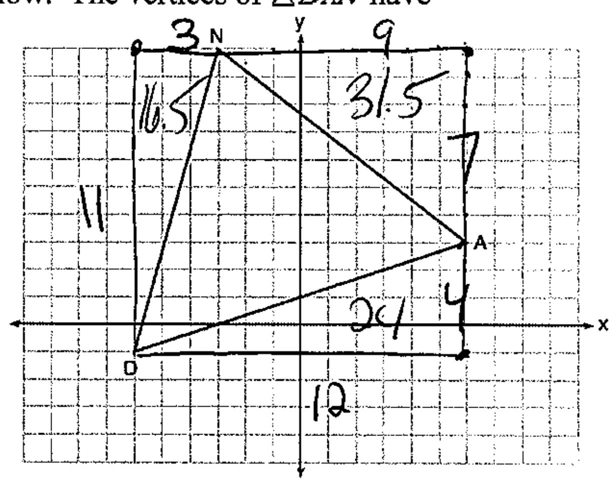
$A_{T2} = \frac{1}{2}(8)(6) = 24$

$A_{T3} = \frac{1}{2}(4)(3) = 6$

55

$80 - 55 = 25$

5. Triangle DAN is graphed on the set of axes below. The vertices of $\triangle DAN$ have coordinates $D(-6,-1)$, $A(6,3)$, and $N(-3,10)$.



What is the area of $\triangle DAN$?

1) 60
2) 120
3) $20\sqrt{13}$
4) $40\sqrt{13}$

$A_R = 11(12) = 132$

$A_{T1} = \frac{1}{2}(3)(11) = 16.5$

$A_{T2} = \frac{1}{2}(9)(7) = 31.5$

$A_{T3} = \frac{1}{2}(12)(4) = 24$

72

$132 - 72 = 60$

6. On the set of axes below, rectangle $WIND$ has vertices with coordinates $W(-4,2)$, $I(4,0)$, $N(3,-4)$, and $D(-5,-2)$. What is the area of rectangle $WIND$?

$A_R = (9)(6) = 54$ $A_{T1} = \frac{1}{2}(4)(1) = 2$

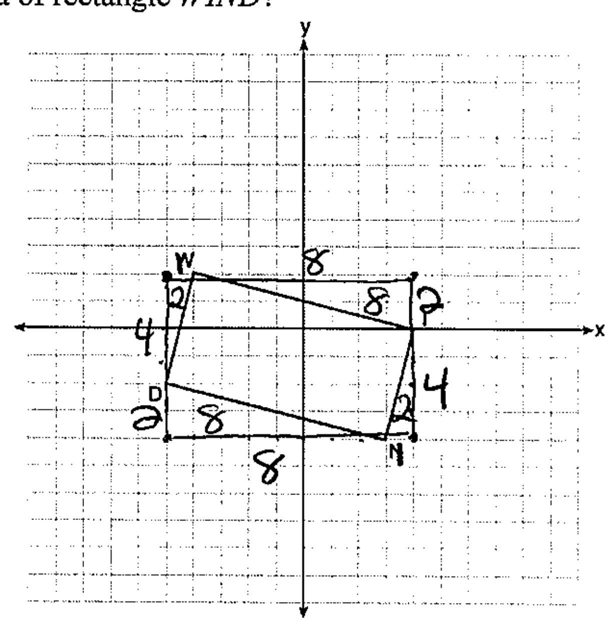
$A_{T2} = \frac{1}{2}(4)(1) = 2$

$A_{T3} = \frac{1}{2}(8)(2) = 8$

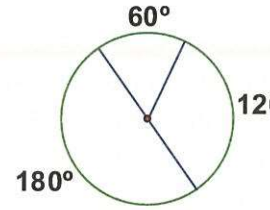
$A_{T4} = \frac{1}{2}(8)(2) = 8$

20

$54 - 20 = 34$

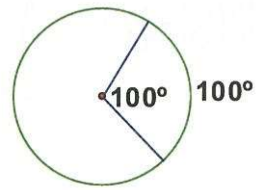


Circle Angle and Segment Rules:

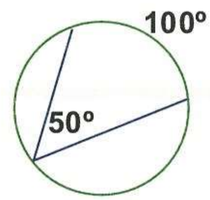


The arcs of a circle add to 360°
A diameter cuts a circle into 2 halves of 180° each

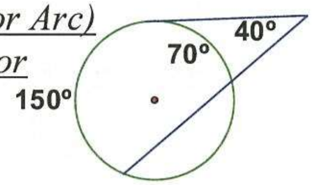
Central Angle: Has its vertex at the center of the circle
Central angle is equal to the measure of the intercepted arc

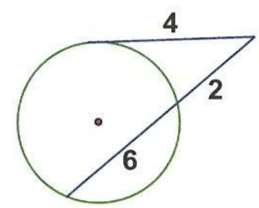


Inscribed Angle: Has its vertex on the circle
Inscribed angle is half of the measure of the intercepted arc

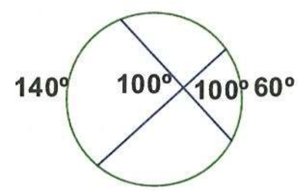


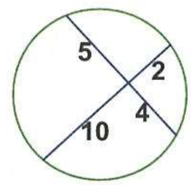
Exterior Segments/Angles:
Angles: *2(Exterior Angle) = (Major Arc – Minor Arc)*
Segments: *Whole • Exterior = Whole • Exterior*



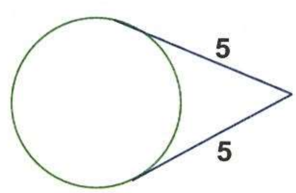


Intersecting Segments/Angles:
Angles: *2(Vertical Angle) = Arc + Arc*
Segments: *Part • Part = Part • Part*

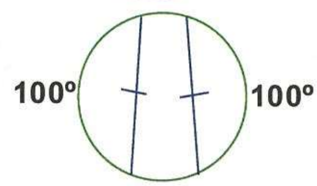




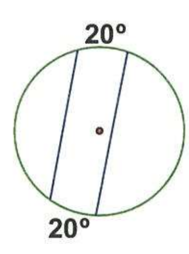
Two tangents drawn from the same point are congruent



Congruent chords intercept congruent arcs



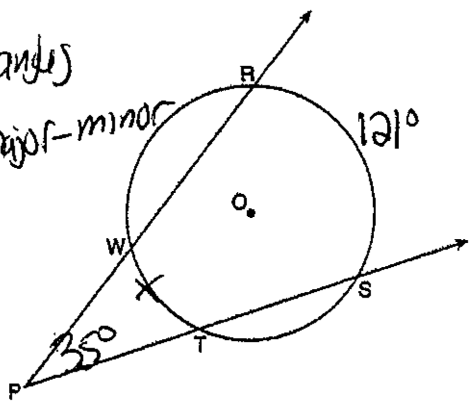
Parallel chords intercept congruent arcs



2(exterior angle) = major − minor

1. As shown in the diagram below, secants $\overrightarrow{PWR}$ and $\overrightarrow{PTS}$ are drawn to circle O from external point P.
If $m\angle RPS = 35°$ and $m\widehat{RS} = 121°$, determine and state $m\widehat{WT}$.

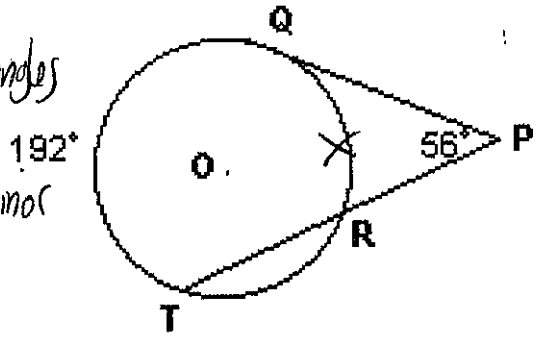
exterior
arcs and angles
2(EA) = major − minor



$2(EA) = \text{major} - \text{minor}$
$2(35) = 121 - x$
$70 = 121 - x$
$-121 \quad -121$
$\frac{-51}{-1} = \frac{-x}{-1}$
$51 = x$

2. In the diagram of circle O, $\overline{PQ}$ is tangent to O at Q and $\overline{PRT}$ is a secant. If $m\angle P = 56$ and $m\widehat{QT} = 192$, find $m\widehat{QR}$

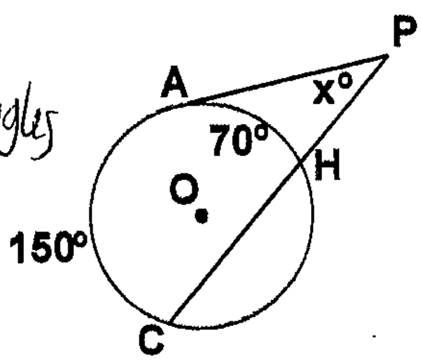
exterior
arcs and angles
A) = major − minor



$2(EA) = \text{major} - \text{minor}$
$2(56) = 192 - x$
$112 = 192 - x$
$-192 \quad -192$
$\frac{-80}{-1} = \frac{-x}{-1}$
$80 = x$

3. $\widehat{AC} = 150°$, $\widehat{AH} = 70°$, find $m\angle APH$

exterior
arcs and angles



$2(EA) = \text{major} - \text{minor}$
$2x = 150 - 70$
$\frac{2x}{2} = \frac{80}{2}$
$x = 40$

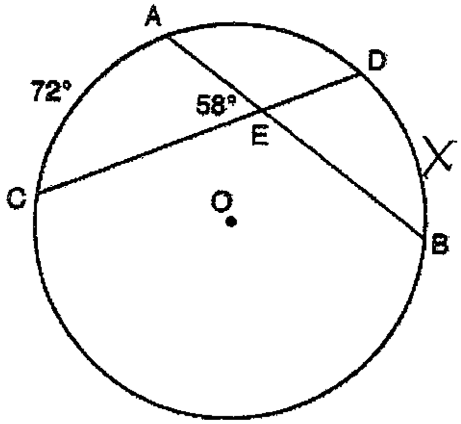
EA) = major − minor

2(vertical angle) = arc + arc

- interior
- arcs and angles

2(VA)=arc+arc

4. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $m\widehat{AC} = 72°$ and $m\angle AEC = 58°$, how many degrees are in $m\widehat{DB}$?



2(VA)=arc+arc

2(58) = x+72

116 = x+72

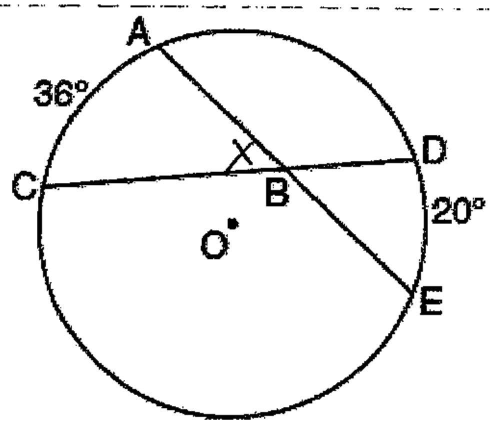
-72 -72

44 = x

5. In the diagram below of circle O, chords $\overline{AE}$ and $\overline{DC}$ intersect at point B, such that $m\widehat{AC} = 36$ and $m\widehat{DE} = 20$. What is $m\angle ABC$?

- interior
- arcs and angles

2(VA)=arc+arc



2(VA)=arc+arc

2x = 36+20

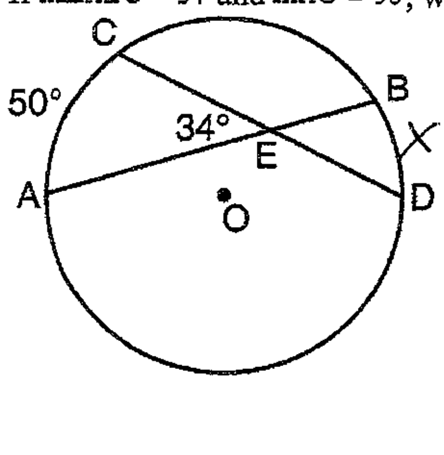
$\frac{2x}{2} = \frac{56}{2}$

x = 28

6. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $m\angle AEC = 34$ and $m\widehat{AC} = 50$, what is $m\widehat{DB}$?

- interior
- arcs and angles

2(VA)=arc+arc



2(VA)=arc+arc

2(34) = x+50

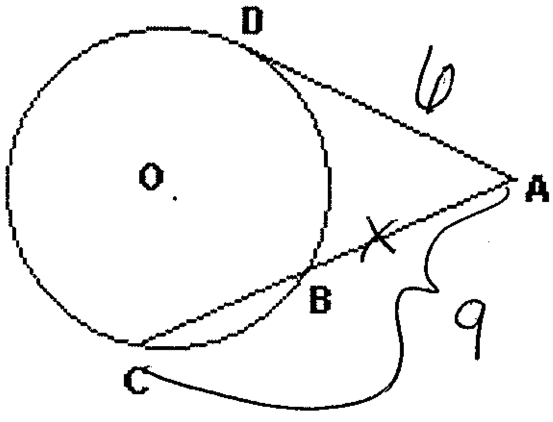
68 = x+50

-50 -50

18 = x

whole · exterior = whole · exterior

7. In the diagram, $\overline{AD}$ is tangent to circle O at D, and $\overline{CBA}$ is a secant. If $AD = 6$ and $AC = 9$, what is AB?



whole · exterior = whole · exterior

$6 \cdot 6 = 9 \cdot x$

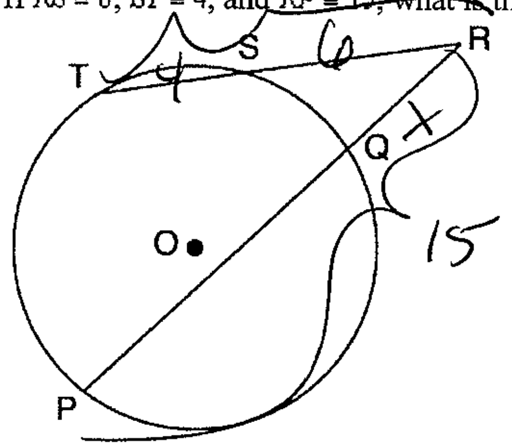
$\frac{36}{9} = \frac{9x}{9}$

$4 = x$

8. In the diagram below, secants $\overline{RST}$ and $\overline{RQP}$, drawn from point R, intersect circle O at S, T, Q, and P.

10

If $RS = 6$, $ST = 4$, and $RP = 15$, what is the length of $\overline{RQ}$?



$w \cdot e = w \cdot e$

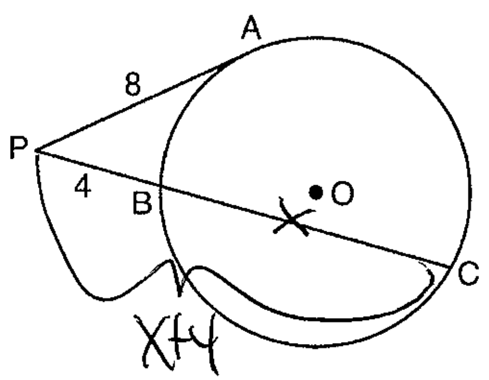
$10 \cdot 6 = 15 \cdot x$

$\frac{60}{15} = \frac{15x}{15}$

$4 = x$

9. In the diagram below of circle O, $\overline{PA}$ is tangent to circle O at A, and $\overline{PBC}$ is a secant with points B and C on the circle.

If $PA = 8$ and $PB = 4$, what is the length of $\overline{BC}$?



$w \cdot e = w \cdot e$

$8 \cdot 8 = (x+4)(4)$

$64 = 4x + 16$

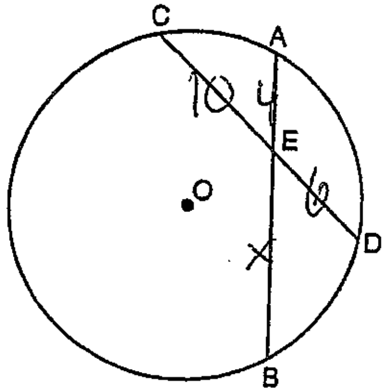
$-16 \quad -16$

$\frac{48}{4} = \frac{4x}{4}$

$12 = x$

Part · Part = Part · Part

10. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $CE = 10$, $ED = 6$, and $AE = 4$, what is the length of $\overline{EB}$?



$P \cdot P = P \cdot P$

$10 \cdot 6 = 4 \cdot x$

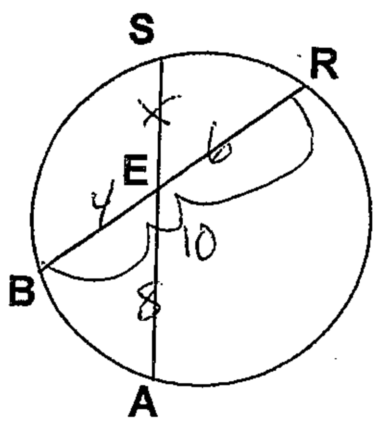
$\frac{60}{4} = \frac{\cancel{4}x}{\cancel{4}}$

$15 = x$

- Segments
- interior

$P \cdot P = P \cdot P$

11. If $\overline{BR} = 10$, $\overline{BE} = 4$, $\overline{AE} = 8$, find $\overline{ES}$



$P \cdot P = P \cdot P$

$6 \cdot 4 = 8 \cdot x$

$\frac{24}{8} = \frac{8x}{8}$

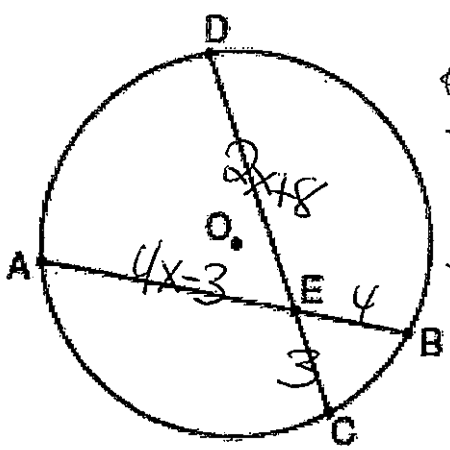
$3 = x$

- Segments
- interior

$P \cdot P = P \cdot P$

*10 is not a part

12. In the diagram of circle O below, chord $\overline{AB}$ intersects chord $\overline{CD}$ at E, $DE = 2x + 8$, $EC = 3$, $AE = 4x - 3$, and $EB = 4$.
What is the value of x?



$P \cdot P = P \cdot P$

$3(2x+8) = 4(4x-3)$

$6x + 24 = 16x - 12$

$-6x \quad -6x$

$24 = 10x - \cancel{12}$

$+12 \quad \cancel{+12}$

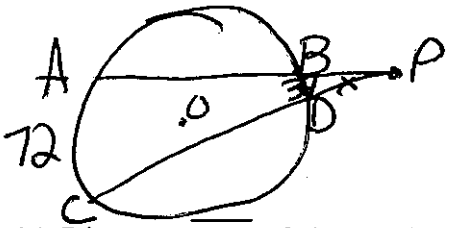
$\frac{36}{10} = \frac{\cancel{10}x}{\cancel{10}}$

$3.6 = x$

- Segments
- interior

$P \cdot P = P \cdot P$

13. In circle O two secants, $\overline{ABP}$ and $\overline{CDP}$, are drawn to external point P. If $m\widehat{AC} = 72°$, and $m\widehat{BD} = 34°$, what is the measure of $\angle P$?



$2(EA) = \text{major} - \text{minor}$

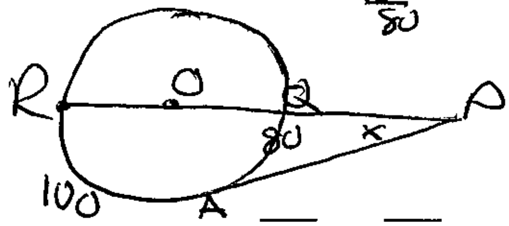
$2x = 72 - 34$

$\frac{2x}{2} = \frac{38}{2}$

$x = 19$

14. Diameter $\overline{ROQ}$ of circle O is extended through Q to point P, and tangent $\overline{PA}$ is drawn. If $m\widehat{RA} = 100°$, what is $m\angle P$?

$180 - 100 = 80$



$2(EA) = \text{major} - \text{minor}$

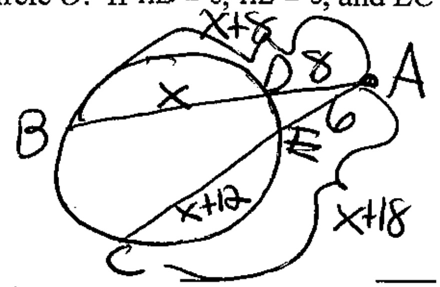
$2x = 100 - 80$

$\frac{2x}{2} = \frac{20}{2}$

$x = 10$

15. In circle O, secants $\overline{ADB}$ and $\overline{AEC}$ are drawn from external point A such that points D, B, E, and C are on circle O. If $AD = 8$, $AE = 6$, and EC is 12 more than BD, the length of $\overline{BD}$ is

1) 6
2) 22
3) 36
4) 48



$w \cdot e = w \cdot e$

$(x+8)8 = (x+18)(6)$

$8x + 64 = 6x + 108$

$-6x \quad -6x$

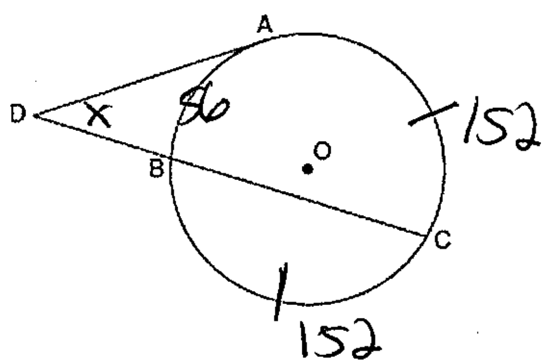
$2x + 64 = 108$

$-64 \quad -64$

$\frac{2x}{2} = \frac{44}{2}$

$x = 22$

16. In the diagram below, tangent $\overline{DA}$ and secant $\overline{DBC}$ are drawn to circle O from external point D, such that $\widehat{AC} \cong \widehat{BC}$. If $m\widehat{BC} = 152°$, determine and state $m\angle D$.



$2(EA) = \text{major} - \text{minor}$

$2x = 152 - 56$

$\frac{2x}{2} = \frac{96}{2}$

$x = 48$

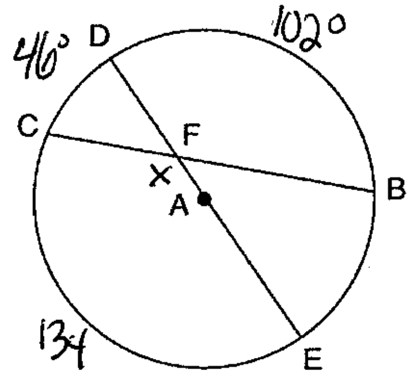
$152 + 152 + x = 360$

$304 + x = 360$

$-304 \quad -304$

$x = 56$

17. In circle A below, chord $\overline{BC}$ and diameter $\overline{DAE}$ intersect at F. If $m\overarc{CD} = 46°$ and $m\overarc{DB} = 102°$, what is $m\angle CFE$?



$$\begin{array}{r} 180 \\ -46 \\ \hline 134 \end{array}$$

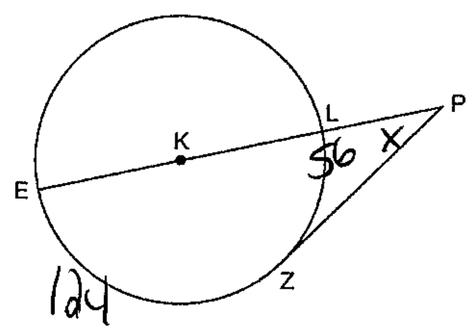
2(VA) = arc + arc

$2x = 134 + 102$

$\frac{2x}{2} = \frac{236}{2}$

$x = 118$

18. In the diagram below of circle K, secant $\overline{PLKE}$ and tangent $\overline{PZ}$ are drawn from external point P. If $m\overarc{LZ} = 56°$, determine and state the degree measure of angle P.



$$\begin{array}{r} 180 \\ -56 \\ \hline 124 \end{array}$$

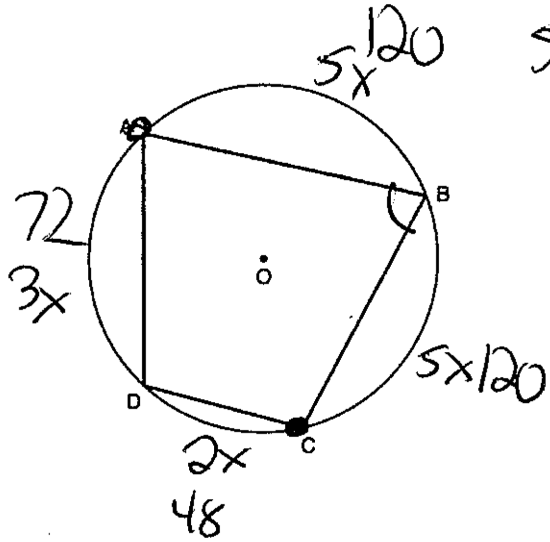
2(EA) = major − minor

$2x = 124 - 56$

$\frac{2x}{2} = \frac{68}{2}$

$x = 34$

19. In the diagram below, quadrilateral $ABCD$ is inscribed in circle O, and $m\overarc{CD}:m\overarc{DA}:m\overarc{AB}:m\overarc{BC} = 2:3:5:5$. Determine and state $m\angle B$.



$5x + 5x + 2x + 3x = 360$

$\frac{15x}{15} = \frac{360}{15}$

$x = 24$

$5(24) = 120$

$3(24) = 72$

$2(24) = 48$

$\angle B = \frac{1}{2}(AC)$

$\angle B = \frac{1}{2}(120)$

$\angle B = 60°$

The opposite angles add to 180°

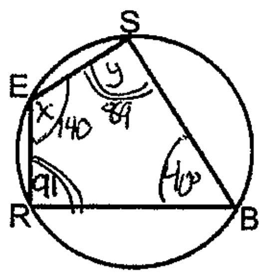
Name Schlansky
Mr. Schlansky

Date ____________
Geometry

Quadrilaterals Inscribed In a Circle

1. In the diagram below, quadrilateral *SBRE* is inscribed in the circle. If $m\angle BRE = 91°$ and $m\angle SBR = 40°$, find $m\angle BSE$ and $m\angle SER$

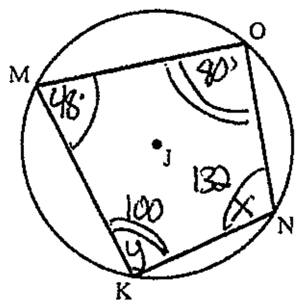
89 140°



$40 + x = 180$
$-40 \quad -40$
$x = 140$

$91 + y = 180$
$-91 \quad -91$
$y = 89$

2. In the diagram below, quadrilateral MONK is inscribed in circle J, $m\angle KMO = 48°$ and $m\angle MON = 80°$. Find the measures of $m\angle KNO$ and $m\angle MKN$.

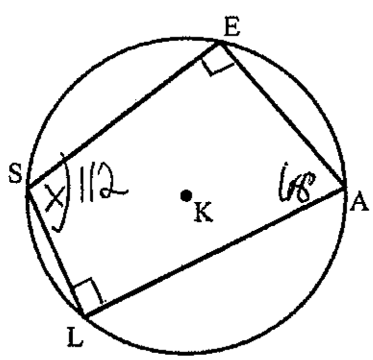


$x + 48 = 180$
$-48 \quad -48$
$x = 132$

$80 + y = 180$
$-80 \quad -80$
$y = 100$

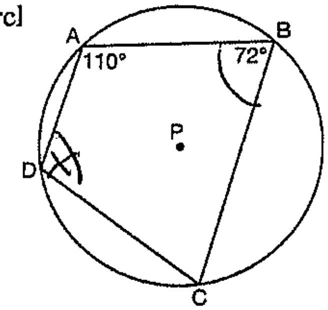
3. In the diagram below, quadrilateral SEAL is inscribed in circle K, $\overline{SE} \perp \overline{EA}$ (90°) and $m\angle EAL = 68°$. Find the measures of $m\angle SLA$ and $m\angle ESL$.

90° 112



$x + 68 = 180$
$-68 \quad -68$
$x = 112$

4. In the diagram below, quadrilateral *ABCD* is inscribed in circl

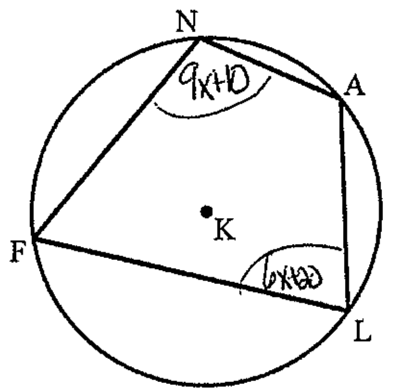


What is m∠*ADC*?

1) 70°
2) 72°
3) 108°
4) 110°

$x+72=180$
$-72\ -72$
$x=108$

5. In the diagram below, quadrilateral FLAN is inscribed in circle K, $m\angle FNA = 9x+10$ and $m\angle FLA = 6x+20$. Find the measures of $m\angle FLA$.



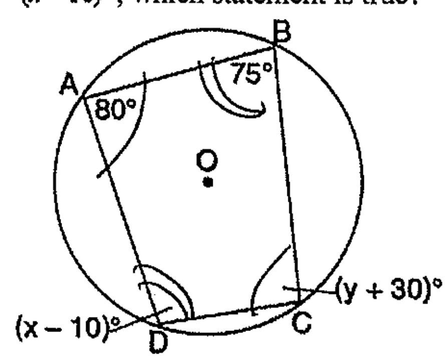
$9x+10+6x+20=180$
$15x+30=180$
$-30\ -30$
$\frac{15x}{15}=\frac{150}{15}$
$x=10$

$6x+20$
$6(10)+20$
$80°$

6. Quadrilateral *ABCD* is inscribed in circle *O*, as shown below.

If m∠*A* = 80°, m∠*B* = 75°, m∠*C* = $(y+30)°$, and m∠*D* = $(x-10)°$, which statement is true?

1) $x = 85$ and $y = 50$
2) $x = 90$ and $y = 45$
3) $x = 110$ and $y = 75$
4) $x = 115$ and $y = 70$



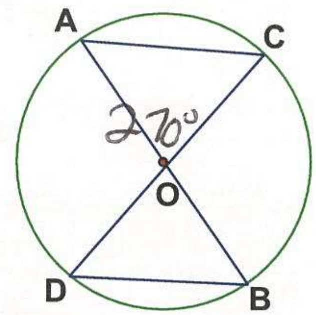
$80+y+30=180$
$y+110=180$
$-110\ -110$
$y=70$

$75+x-10=180$
$x+65=180$
$-65\ -65$
$x=115$

Area of a Sector = $\frac{\theta \pi r^2}{360}$

If given area of a sector, use algebra to solve for missing variable

1. In circle O, $m\angle AOC = 70$ and $\overline{AO} = 2\ in$. Find the area of sector COA to the nearest square inch. → type π in

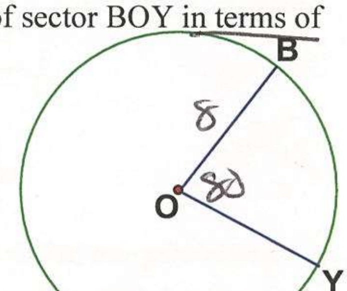


$A = \frac{\theta \pi r^2}{360}$

$A = \frac{70\pi(2)^2}{360}$

$A = 2\ in^2$

2. In circle O, if $\angle$BOY = 80° and $\overline{BO} = 8\ cm$, find the area of sector BOY in terms of π: → don't type π in



$A = \frac{\theta \pi r^2}{360}$

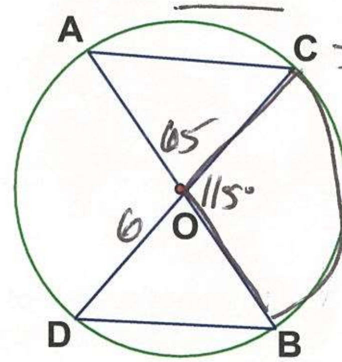
$A = \frac{80\pi(8)^2}{360}$

$A = \frac{128}{9}\pi$

→ don't type π in

3. In circle O, $m\angle AOC = 65$ and $\overline{DO} = 6\ in$. Find the area of sector COB in terms of π

$180 - 65 = 115$



$A = \frac{\theta \pi r^2}{360}$

$A = \frac{115\pi(6)^2}{360}$

$A = \frac{23}{2}\pi$

4. Determine and state, in terms of π, the area of a sector that intercepts a 40° arc of a circle with a radius of 4.5. → don't type π in

$A = \frac{\theta \pi r^2}{360}$

$A = \frac{40\pi(4.5)^2}{360}$ → $A = 2.25\pi$

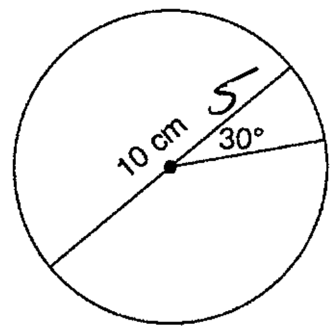
5. A circle with a diameter of 10 cm and a central angle of 30° is drawn below. What is the area, to the *nearest tenth of a square centimeter*, of the sector formed by the 30° angle?

1) 5.2
2) 6.5
3) 13.1
4) 26.2

$A = \frac{\theta \pi r^2}{360}$ type π in

$A = \frac{30\pi(5)^2}{360}$

$A = 6.5$



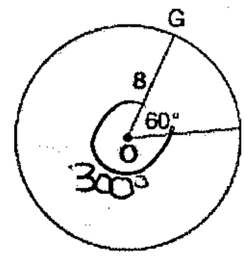
6. In the diagram below of circle O, $GO = 8$ and $m\angle GOJ = 60°$. What is the area, in terms of π, of the shaded region?

1) $\frac{4\pi}{3}$
2) $\frac{20\pi}{3}$
3) $\frac{32\pi}{3}$
4) $\frac{160\pi}{3}$

$A = \frac{\theta \pi r^2}{360}$ don't type π in

$A = \frac{300\pi(8)^2}{360}$

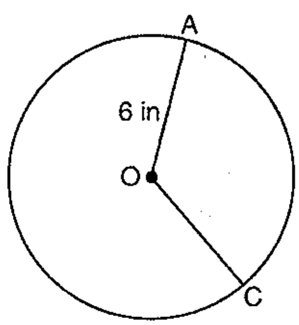
$A = \frac{160}{3}\pi$



$$\begin{array}{r} 360 \\ -60 \\ \hline 300 \end{array}$$

A

7. In the diagram below of circle O, the area of the shaded sector AOC is 12π in^2 and the length of $\overline{OA}$ is 6 inches. Determine and state $m\angle AOC$.



$A = \frac{\theta \pi r^2}{360}$

$\frac{12\pi}{1} = \frac{x\pi(6)^2}{360}$

$\frac{36x}{36} = \frac{4320}{36}$

$x = 120°$

A

8. The area of a sector of a circle with a radius measuring 15 cm is 75π cm^2. What is the measure of the central angle that forms the sector?

1) 72°
2) 120°
3) 144°
4) 180°

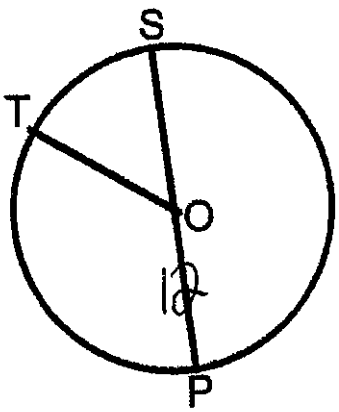
$A = \frac{\theta \pi r^2}{360}$

$\frac{75\pi}{1} = \frac{x\pi(15)^2}{360}$

$\frac{225x}{225} = \frac{27000}{225}$

$x = 120°$

9. In the diagram below of circle O, the area of sector STO is $48\pi\ in^2$ and the length of $\overline{OP}$ is 12 inches. Determine and state $m\angle SOT$



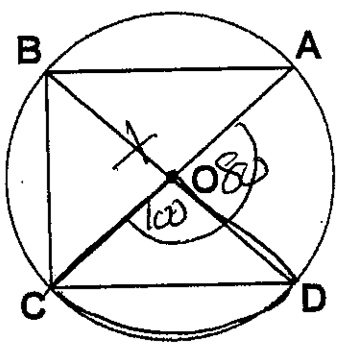
$A = \frac{\theta \pi r^2}{360}$

$\frac{48\pi}{1} = \frac{x\pi(12)^2}{360}$

$\frac{17280}{144} = \frac{144x}{144}$

$x = 120°$

10. In circle O, diameters $\overline{BOD}$ and $\overline{COA}$ intersect at the center of the circle O. If the area of sector OCD = 240π square inches and $m\angle AOD = 80$, find the measure of $\overline{OB}$ to the nearest tenth of an inch.



$180 - 80 = 100$

$A = \frac{\theta \pi r^2}{360}$

$\frac{240\pi}{1} = \frac{100\pi(x)^2}{360}$

$\frac{86400}{100} = \frac{100x^2}{100}$

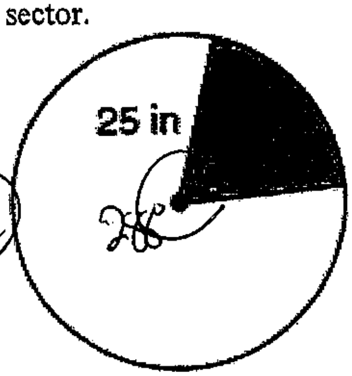
$\sqrt{x^2} = \sqrt{864}$

$x = 29.4$

11. In the diagram below, the circle has a radius of 25 inches. The area of the *unshaded* sector is 500π in^2.

Determine and state the degree measure of angle Q, the central angle of the shaded sector.

$360 - 288 = 72°$



$A = \frac{\theta \pi r^2}{360}$

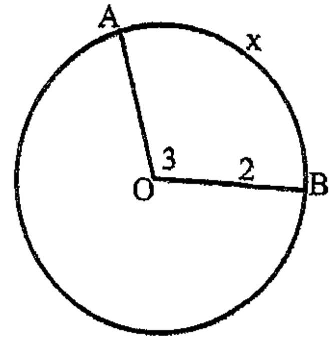
$\frac{500\pi}{1} = \frac{x\pi(25)^2}{360}$

$\frac{625x}{625} = \frac{180000}{625}$

$x = 288$

Arc Length: $s = \theta r$, where s = arc length, θ = central angle (in radians), r = radius

1. In circle O, the measure of central angle AOB is 3 radians and the length of $\overline{OB}$ is 2 cm. What is the measure of arc AB?

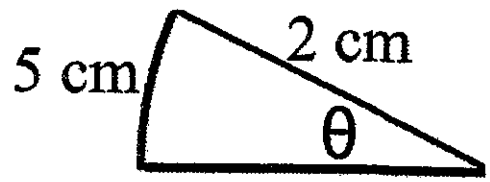


$s = \theta r$

$x = 3(2)$

$x = 6$

2. What is the measure of the central angle below?



$s = \theta r$

$\frac{s}{2} = \frac{x(\cancel{2})}{\cancel{2}}$

$\frac{5}{2} = x$

3. What is the measure of the radius of a sector whose arc length is 12 inches and has a central angle of 4 radians?

$s = \theta r$

$\frac{12}{4} = \frac{\cancel{4}r}{\cancel{4}}$

$3 = r$

4. A wheel has a radius of 18 inches. Which distance, to the *nearest inch*, does the wheel travel when it rotates through an angle of $\frac{2\pi}{5}$ radians?

$s = \theta r$

$x = \frac{2\pi}{5}(18)$

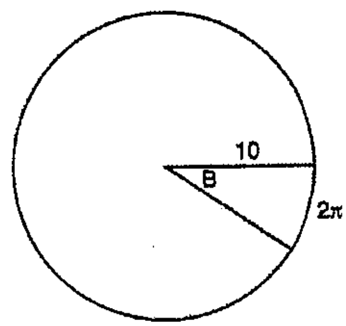
$x = 22.6$

5. What is the measure of a central angle in degrees whose arc length is 6 meters and whose radius measures 8 meters?

$S = \theta r$ $\quad x = \frac{3}{4}$

$\frac{6}{8} = \frac{x8}{8}$

6. In the diagram below, the circle shown has radius 10. Angle B intercepts an arc with a length of 2π.
What is the measure of angle B, in radians?



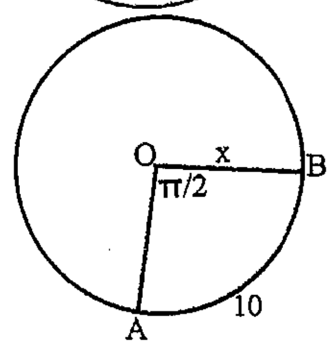
1) $10 + 2\pi$
2) 20π
3) $\frac{\pi}{5}$
4) $\frac{5}{\pi}$

$S = \theta r$

$\frac{2\pi}{10} = \frac{x(10)}{10}$

$\frac{\pi}{5} = x$

7. In circle O, the measure of central angle AOB is $\frac{\pi}{2}$ radians and the length of arc AB is 10 cm. What is the measure of radius $\overline{OB}$ to the *nearest tenth of a cm*?



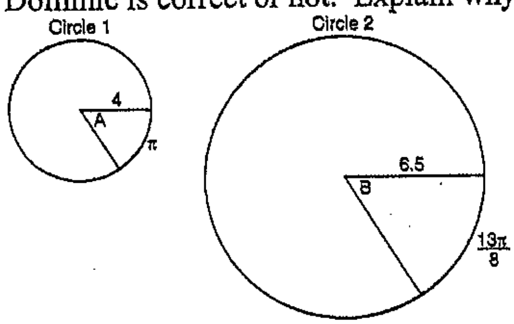
$S = \theta r$

$\cancel{10 = \frac{\pi}{2}(x)}$

$\frac{20}{\pi} = \frac{\pi x}{\pi}$

$6.4 = x$

8. In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle A intercepts an arc of length π, and angle B intercepts an arc of length $\frac{13\pi}{8}$.
Dominic thinks that angles A and B have the same radian measure. State whether Dominic is correct or not. Explain why.



$S = \theta r$

$\frac{\pi}{4} = \frac{x(4)}{4}$

$\frac{\pi}{4} = x$

Yes

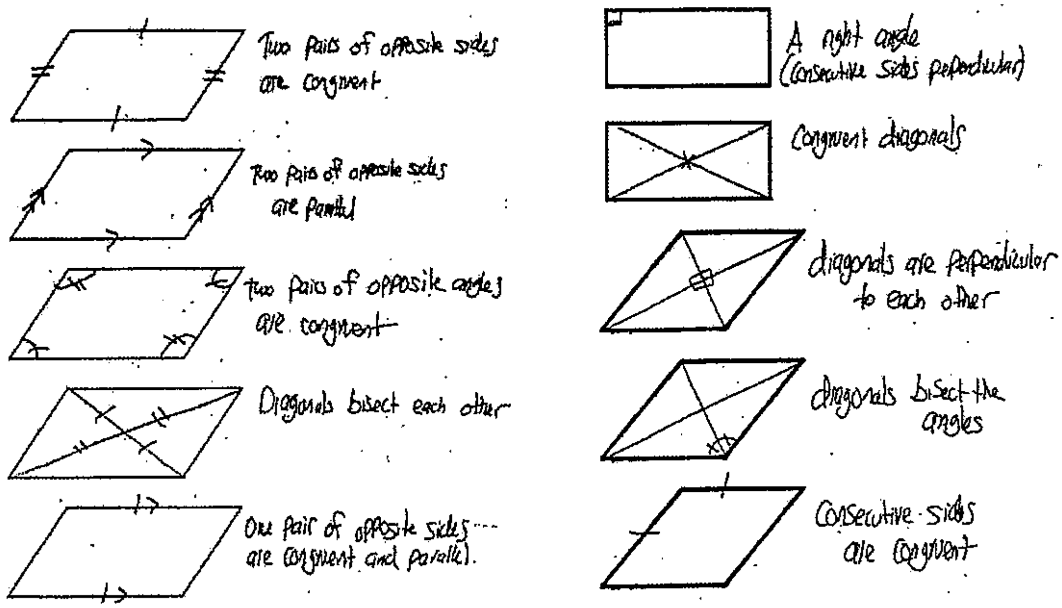
$S = \theta r$

$\cancel{\frac{13\pi}{8} = \frac{x(6.5)}{1}}$

$\frac{13\pi}{52} = \frac{52x}{52}$

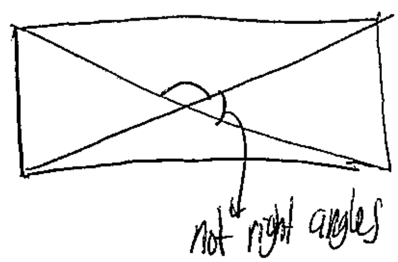
$\frac{\pi}{4} = x$

Parallelogram Properties

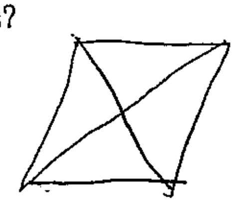


A rectangle and rhombus have all of the properties of the parallelogram.
A square has all of the properties of the parallelogram, rectangle, and rhombus.
A trapezoid has one pair of opposite sides parallel and one pair of opposite sides not parallel.
An isosceles trapezoid is a trapezoid that has congruent legs and congruent diagonals.
For properties questions, draw the shape!

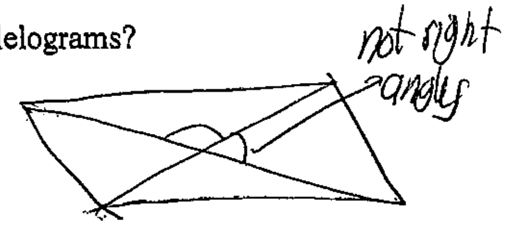
1. Which of the following is not true of all rectangles?
1) Consecutive sides are perpendicular
2) Opposite sides are parallel
3) Diagonals are perpendicular to each other
4) Diagonals bisect each other



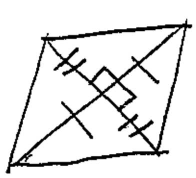
2. Which of the following is true about rhombuses?
1) Consecutive sides are perpendicular
2) Opposite sides are congruent
3) Consecutive angles are congruent
4) Diagonals are congruent



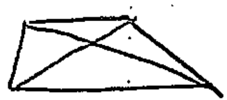
3. Which of the following is *not* true about all parallelograms?
1) Diagonals bisect each other
2) Diagonals are perpendicular to each other
3) Opposite angles are congruent
4) Consecutive angles are supplementary



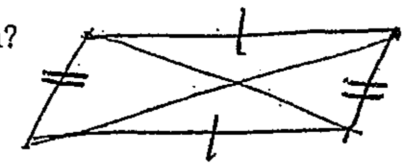
4. A quadrilateral whose diagonals bisect each other and are perpendicular is a
1) rhombus
2) rectangle
3) trapezoid
4) parallelogram



5. If the diagonals of a quadrilateral do *not* bisect each other, then the quadrilateral could be a
1) rectangle
2) rhombus
3) square
4) trapezoid

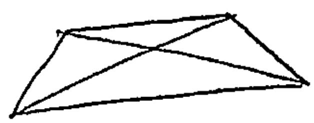


6. Which statement is true about every parallelogram?
1) All four sides are congruent.
2) The interior angles are all congruent.
3) Two pairs of opposite sides are congruent.
4) The diagonals are perpendicular to each other.

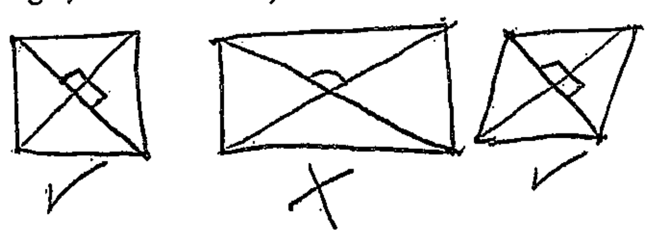


7. Which quadrilateral has diagonals that always bisect its angles and also bisect each other?
1) rhombus
2) rectangle
3) parallelogram
4) isosceles trapezoid

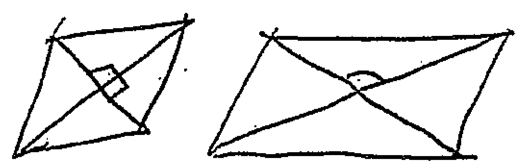
8. The diagonals of a quadrilateral are congruent but do not bisect each other. This quadrilateral is
1) an isosceles trapezoid
2) a parallelogram
3) a rectangle
4) a rhombus



9. Given three distinct quadrilaterals, a square, a rectangle, and a rhombus, which quadrilaterals must have perpendicular diagonals?
1) the rhombus, only
2) the rectangle and the square
3) the rhombus and the square
4) the rectangle, the rhombus, and the square



10. A parallelogram must be a rhombus when its Which property proves a rhombus?
1) Diagonals are congruent.
2) Opposite sides are parallel.
3) Diagonals are perpendicular.
4) Opposite angles are congruent.

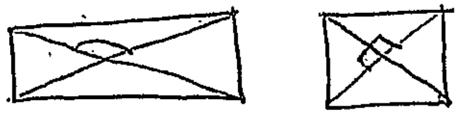


which property proves a rectangle

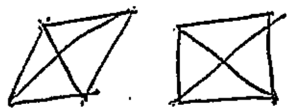
11. A parallelogram must be a rectangle when its
1) diagonals are perpendicular
2) diagonals are congruent
3) opposite sides are parallel
4) opposite sides are congruent

which property proves a square

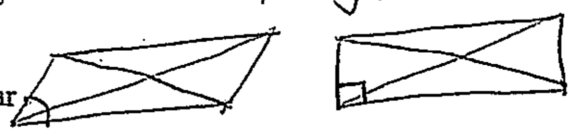
12. A rectangle must be a square when its
1) consecutive sides are perpendicular
2) diagonals are congruent
3) diagonals are perpendicular to each other
4) opposite sides are parallel



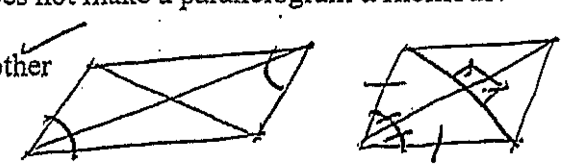
13. A rhombus must be a square when its which property proves a square
1) consecutive sides are congruent
2) diagonals are congruent
3) opposite angles are congruent
4) diagonals are perpendicular to each other



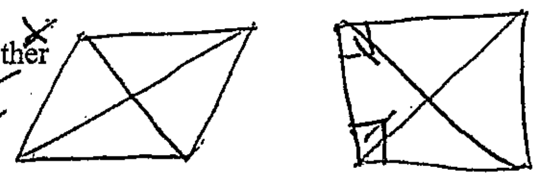
14. A parallelogram must be a rectangle when its which property proves a rectangle
1) consecutive sides are congruent
2) opposite angles are congruent
3) consecutive sides are perpendicular
4) opposite sides are parallel



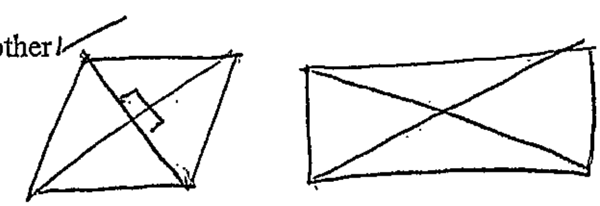
15. Which of the following properties does not make a parallelogram a rhombus?
1) diagonals bisect the angles
2) diagonals are perpendicular to each other
3) opposite angles are congruent
4) consecutive sides are congruent



16. Which of the following properties does not make a rhombus a square?
1) Diagonals are congruent
2) Diagonals are perpendicular to each other
3) Consecutive sides are perpendicular
4) Consecutive angles are congruent



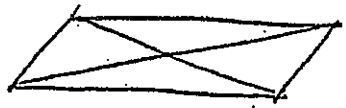
17. Which property is true of all rhombuses but not of all rectangles?
1) opposite sides are parallel
2) diagonals are perpendicular to each other
3) diagonals bisect each other
4) opposite angles are congruent



18. Which set of statements would describe a parallelogram that can always be classified as a rhombus?

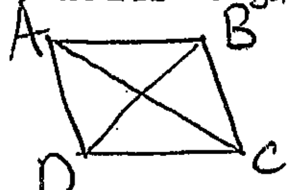
I. Diagonals are perpendicular bisectors of each other. ✓
II. Diagonals bisect the angles from which they are drawn. ✓
III. Diagonals form four congruent isosceles right triangles. ✓ → must be a square all squares are rhombuses

1) I and II
2) I and III
3) II and III
~~4)~~ I, II, and III



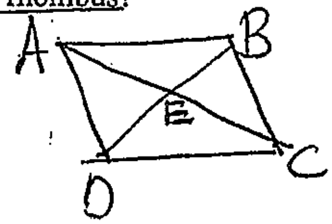

19. If $ABCD$ is a parallelogram, which statement would prove that $ABCD$ is a rhombus?

1) $\angle ABC \cong \angle CDA$ opposite angles $\cong$
2) $\overline{AC} \cong \overline{BD}$ diagonals $\cong$
~~3)~~ $\overline{AC} \perp \overline{BD}$ diagonals perpendicular to each other
4) $\overline{AB} \perp \overline{CD}$ opposite sides $\perp$

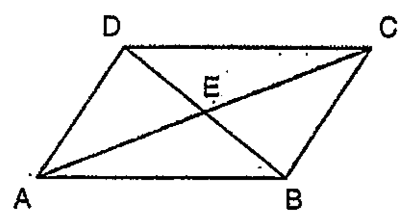


20. In parallelogram $ABCD$, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E. Which statement does *not* prove parallelogram $ABCD$ is a rhombus?

~~1)~~ $\overline{AC} \cong \overline{DB}$ $\cong$ diagonals
2) $\overline{AB} \cong \overline{BC}$ consecutive sides $\cong$
3) $\overline{AC} \perp \overline{DB}$ $\perp$ diagonals
4) $\overline{AC}$ bisects $\angle DCB$ diagonals bisect the angles



21. In the diagram below, parallelogram $ABCD$ has diagonals $\overline{AC}$ and $\overline{BD}$ that intersect at point E.



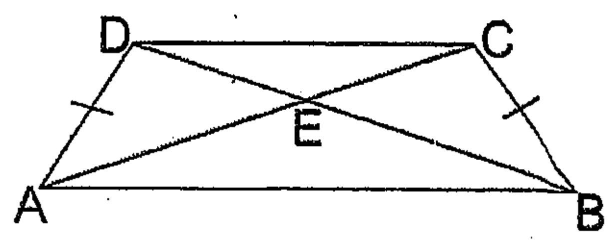
Which expression is *not* always true?

1) $\angle DAE \cong \angle BCE$ $\cong$ alternate interior angles
2) $\angle DEC \cong \angle BEA$ $\cong$ vertical angles
~~3)~~ $\overline{AC} \cong \overline{DB}$ $\cong$ diagonals
4) $\overline{DE} \cong \overline{EB}$ diagonals bisect each other

22. In the diagram below, isosceles trapezoid ABCD has diagonals $\overline{AC}$ and $\overline{BD}$ that intersect at point E.

Which expression is *not* always true?

1) $\overline{AC} \cong \overline{DB}$ $\cong$ diagonals
2) $\overline{DC} \parallel \overline{AB}$ opposite sides $\parallel$
~~3)~~ $\overline{DE} \cong \overline{AE}$ diagonals bisect each other and $\cong$
4) $\overline{AD} \cong \overline{CB}$ opposite $\cong$ legs



Triangles/Parallel Lines Cut By a Transversal/Angles of Parallelograms

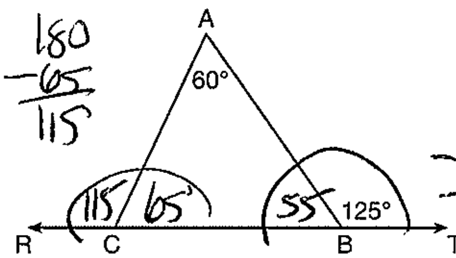
1) The three angles of a triangle add to equal 180°. **Look for triangles.**
   *The four angles of a quadrilateral add to 360°.
2) Linear pairs add to 180°. **Look for linear pairs.**
3) Vertical angles are congruent. Look for an X (intersecting lines).
4) **Given congruent sides:** Isosceles triangle has congruent angles opposite congruent sides.
5) **Given equilateral triangle:** Equilateral triangle has angles 60, 60, 60.
6) **Given angle bisector:** An angle bisector cuts an angle into two congruent halves.
7) **Given parallel**: Extend parallel lines and transversal. Follow the transversal and fill in all 8 angles. If angles are the same (both acute or both obtuse), the angles are congruent. If the angles are different (one acute and one obtuse), the angles are supplementary (add to 180).
8) **Given parallelogram**: Opposite angles are congruent and consecutive angles are supplementary (add to 180)

1. In the diagram below, $\overleftrightarrow{RCBT}$ and $\triangle ABC$ are shown with $m\angle A = 60$ and $m\angle ABT = 125$.

What is $m\angle ACR$?
1) 125
2) 115
3) 65
4) 55

$\triangle ABC$
$x+60+55=180$
$115+x=180$
$-115 \quad -115$
$x=65$

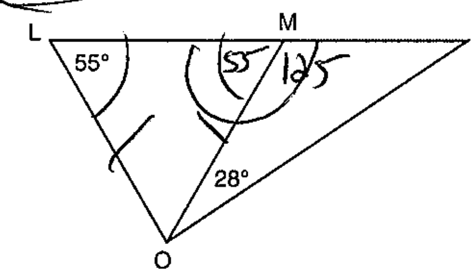
180
-65
115



180
-125
55

2. In the diagram below, $\triangle LMO$ is isosceles with $LO = MO$.

180
-55
125

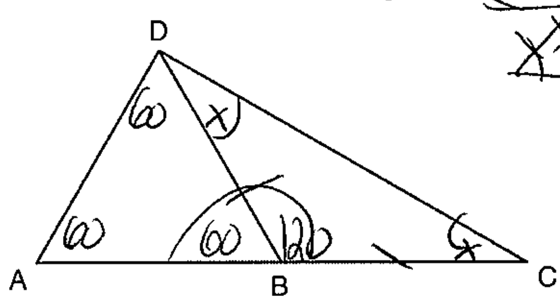


If $m\angle L = 55$ and $m\angle NOM = 28$, what is $m\angle N$?
1) 27
2) 28
3) 42
4) 70

$\triangle OMN$
$28+125+x=180$
$153+x=180$
$-153 \quad -153$
$x=27$

3. In the diagram below of $\triangle ACD$, B is a point on $\overline{AC}$ such that $\triangle ADB$ is an equilateral triangle, and $\triangle DBC$ is an isosceles triangle with $\overline{DB} \cong \overline{BC}$. Find $m\angle C$.

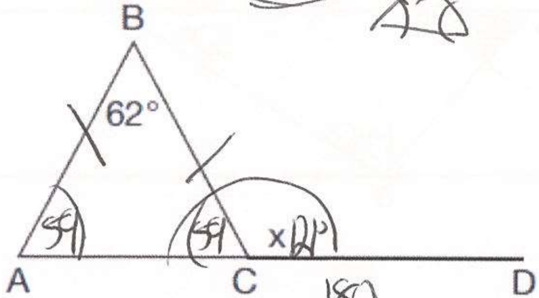
60, 60, 60



180
-60
120

$\triangle DBC$
$x+x+120=180$
$2x+120=180$
$-120 \quad -120$
$\frac{2x}{2}=\frac{60}{2}$
$x=30$

4. Given $\triangle ABC$ with $m\angle B = 62°$ and side $\overline{AC}$ extended to D, as shown below. Which value of x makes $\overline{AB} \cong \overline{CB}$?



180
−59
121

$\triangle ABC$

$x+x+62=180$

$2x+62=180$

$-62 \quad -62$

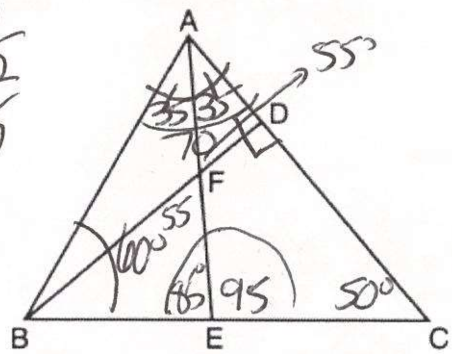
$\frac{2x}{2}=\frac{118}{2}$

$x=59$

$x=121°$

5. In the diagram of $\triangle ABC$ below, $\overline{AE}$ bisects angle BAC, and altitude $\overline{BD}$ is drawn. If $m\angle C = 50°$ and $m\angle ABC = 60°$, what is $m\angle FEB$?

180
−95
85



$\triangle BAC$

$60+50+x=180$

$110+x=180$

$-110 \quad -110$

$x=70$

$\triangle ADF$

$35+90+x=180$

$125+x=180$

$-125 \quad -125$

$x=55$

$\triangle ACE$

$35+50+x=180$

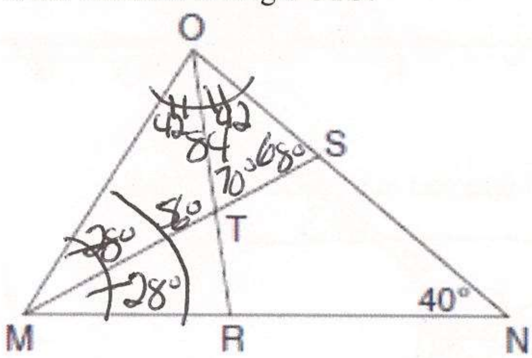
$85+x=180$

$-85 \quad -85$

$x=95$

$\angle FEB=85°$

6. In the diagram below of triangle MNO, $\angle M$ and $\angle O$ are bisected by $\overline{MS}$ and $\overline{OR}$, respectively. Segments MS and OR intersect at T, and $m\angle N = 40°$. If $m\angle TMR = 28°$, what is the measure of angle OTS?



$\triangle OMN$

$56+40+x=180$

$96+x=180$

$-96 \quad -96$

$x=84$

$\triangle MOS$

$28+84+x=180$

$112+x=180$

$-112 \quad -112$

$x=68$

$\triangle OTS$

$42+68+x=180$

$110+x=180$

$-110 \quad -110$

$x=70$

$\angle OTS=70°$

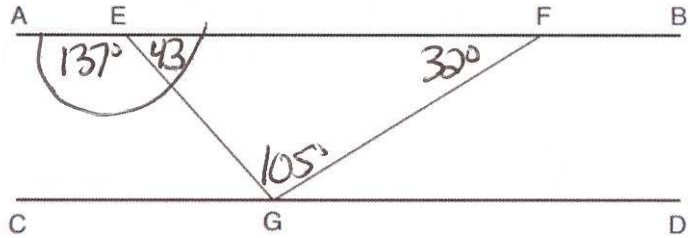
7. In the diagram below, $\overline{AEFB} \parallel \overline{CGD}$, and $\overline{GE}$ and $\overline{GF}$ are drawn.
If $m\angle EFG = 32°$ and $m\angle AEG = 137°$, what is $m\angle EGF$?

1) 11°
2) 43°
3) 75°
4) 105°

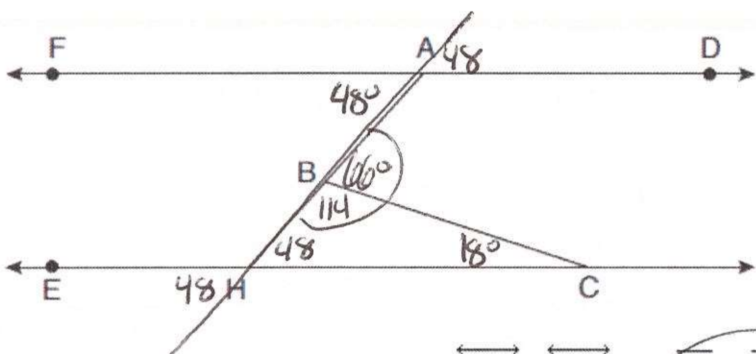
$$\begin{array}{r} 180 \\ -137 \\ \hline 43 \end{array}$$

$\triangle EFG$

$43 + 32 + x = 180$
$75 + x = 180$
$-75 \quad -75$
$x = 105$



8. In the diagram below, $\overline{FAD} \parallel \overline{EHC}$, and $\overline{ABH}$ and $\overline{BC}$ are drawn. If $m\angle FAB = 48°$ and $m\angle ECB = 18°$, what is $m\angle ABC$?



$\triangle HBC$

$48 + 18 + x = 180$
$66 + x = 180$
$-66 \quad -66$
$x = 114$

$$\begin{array}{r} 180 \\ -114 \\ \hline 66 \end{array}$$

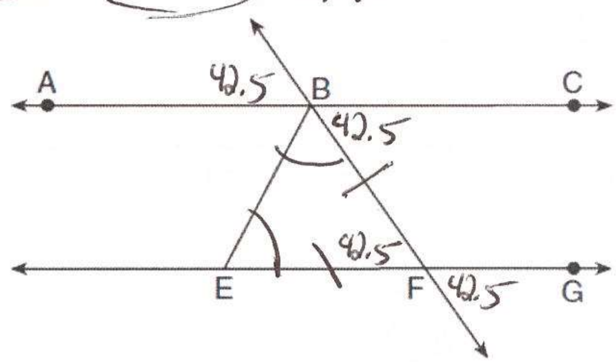
$\angle ABC = 66°$

9. As shown in the diagram below, $\overleftrightarrow{ABC} \parallel \overleftrightarrow{EFG}$ and $\overline{BF} \cong \overline{EF}$.
If $m\angle CBF = 42.5°$, then $m\angle EBF$ is

1) 42.5°
2) 68.75°
3) 95°
4) 137.5°

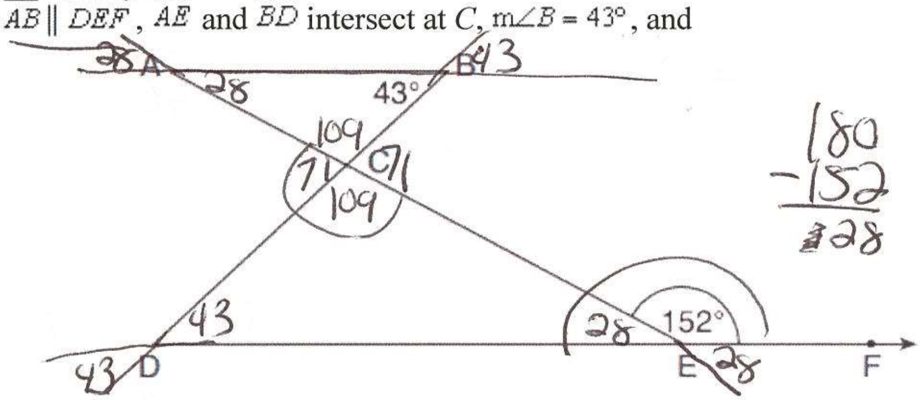
$\triangle FBE$

$x + x + 42.5 = 180$
$2x + 42.5 = 180$
$-42.5 \quad -42.5$
$\frac{2x}{2} = \frac{137.5}{2}$
$x = 68.75$



10. In the diagram below, $\overline{AB} \parallel \overrightarrow{DEF}$, $\overline{AE}$ and $\overline{BD}$ intersect at C, $m\angle B = 43°$, and $m\angle CEF = 152°$.
Which statement is true?

1) $m\angle D = 28°$
2) $m\angle A = 43°$
3) $m\angle ACD = 71°$
4) $m\angle BCE = 109°$



$$\begin{array}{r} 180 \\ -152 \\ \hline 28 \end{array}$$

$\triangle ABC$

$28 + 43 + x = 180$
$71 + x = 180$
$-71 \quad -71$
$x = 109$

parallel

$\triangle ABC$
$82+26+x=180$
$108+x=180$
$-108 \quad -108$
$x=72$

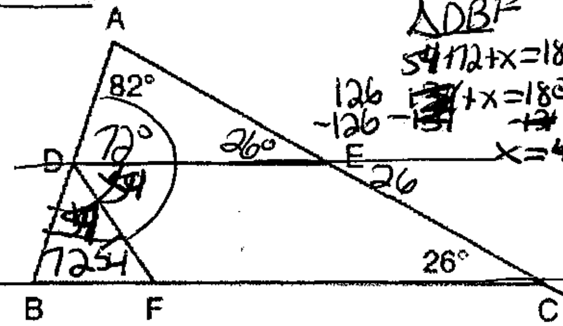
11. In the diagram below, $\overline{DE}$ divides $\overline{AB}$ and $\overline{AC}$ proportionally, $m\angle C = 26°$, $m\angle A = 82°$, and $\overline{DF}$ bisects $\angle BDE$.
The measure of angle DFB is

1) 36°
2) 54°
3) 72°
4) 82°

$\triangle ADE$
$82+26+x=180$
$108+x=180$
$-108 \quad -108$
$x=72$

$\overline{ADB}$
$72+x+x=180$
$2x+72=180$
$-72 \quad -72$
$\frac{2x}{2}=\frac{108}{2}$
$x=54$

$\triangle DBF$
$54+72+x=180$
$126+x=180$
$-126 \quad -126$
$x=54$

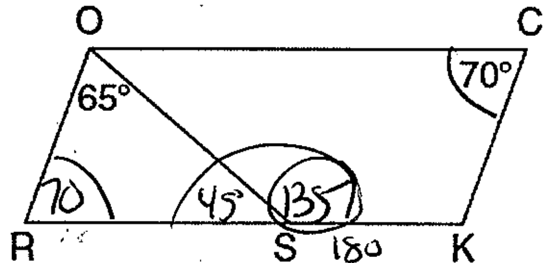


12. In the diagram below of parallelogram $ROCK$, $m\angle C$ is 70° and $m\angle ROS$ is 65°.
What is $m\angle KSO$?

opposite ∠s ≅

1) 45°
2) 110°
3) 115°
4) 135°

$\triangle ROS$
$70+65+x=180$
$135+x=180$
$-135 \quad -135$
$x=45$



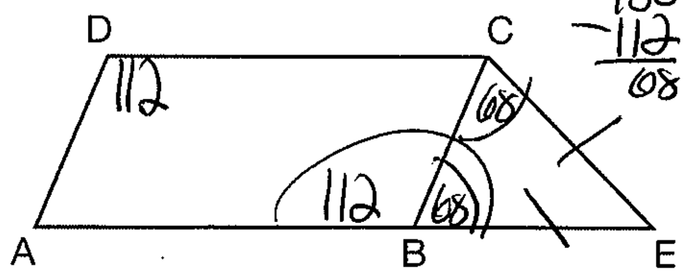
13. In the diagram below, $ABCD$ is a parallelogram, $\overline{AB}$ is extended through B to E, and $\overline{CE}$ is drawn.

If $\overline{CE} \cong \overline{BE}$ and $m\angle D = 112°$, what is $m\angle E$?

1) 44°
2) 56°
3) 68°
4) 112°

$\triangle CBE$
$68+68+x=180$
$136+x=180$
$-136 \quad -136$
$x=44$

180
-112
68

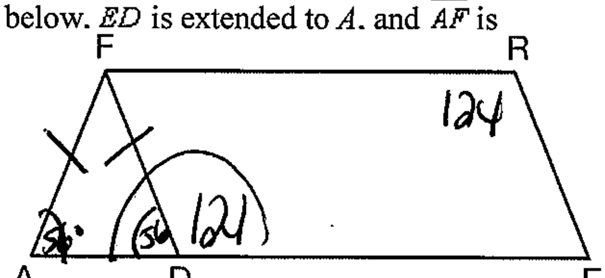


14. In the diagram of parallelogram $FRED$ shown below. $\overline{ED}$ is extended to A. and $\overline{AF}$ is drawn such that $\overline{AF} \cong \overline{DF}$.

If $m\angle R = 124°$, what is $m\angle AFD$?

1) 124°
2) 112°
3) 68°
4) 56°

$\triangle AFD$
$56+56+x=180$
$112+x=180$
$-112 \quad -112$
$x=68$

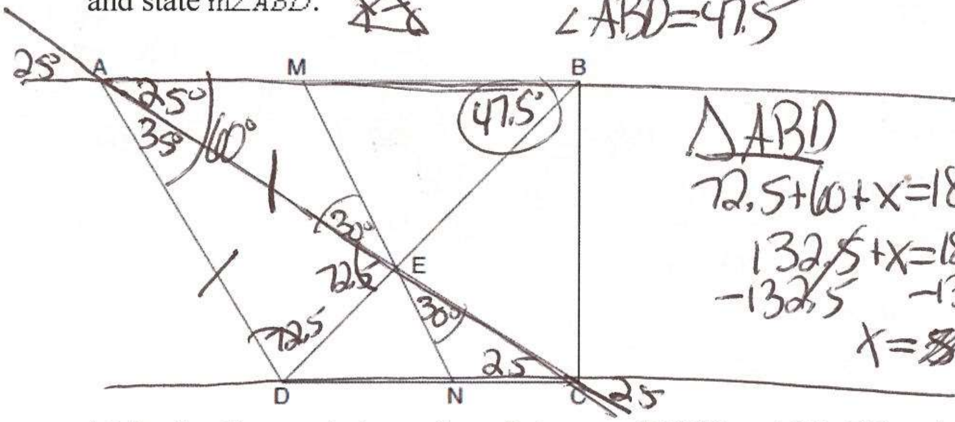


180
-124
56

11

15. Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at E, and $\overline{AD} \cong \overline{AE}$. If $m\angle DAE = 35°$, $m\angle DCE = 25°$, and $m\angle NEC = 30°$, determine and state $m\angle ABD$.

$\angle ABD = 47.5$

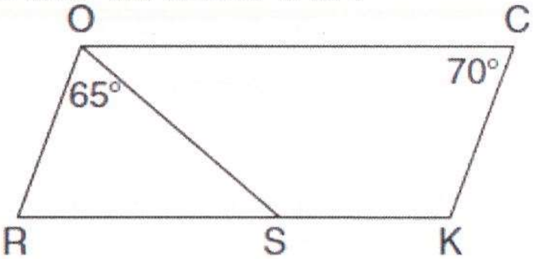


ΔABD
$72.5 + 60 + x = 180$
$132.5 + x = 180$
$-132.5 \quad -132.5$
$x = 47.5$

ΔADE
$x + x + 35 = 180$
$2x + 35 = 180$
$-35 \quad -35$
$2x = 145$
$\frac{2x}{2} = \frac{145}{2}$
$x = 72.5$

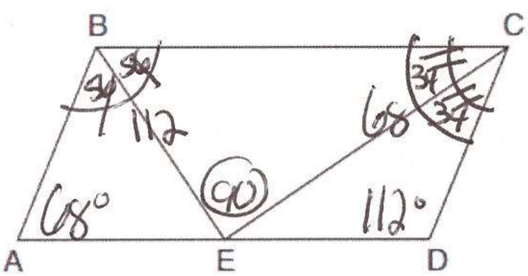
16. In the diagram below of parallelogram $ROCK$, $m\angle C$ is 70° and $m\angle ROS$ is 65°.

Same as 12



What is $m\angle KSO$?
1) 45°
2) 110°
3) 115°
4) 135°

17. In parallelogram $ABCD$ shown below, the bisectors of $\angle ABC$ and $\angle DCB$ meet at E, a point on $\overline{AD}$.
If $m\angle A = 68°$, determine and state $m\angle BEC$.



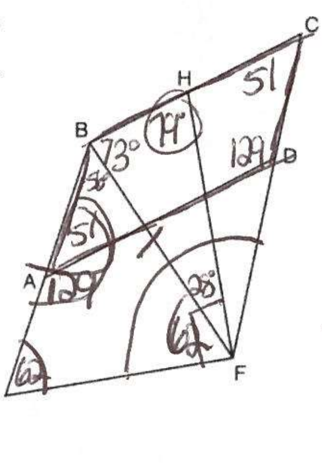
180
-68
112

consecutive angles of a parallelogram are supplementary

ΔBEC
$56 + 34 + x = 180$
$90 + x = 180$
$-90 \quad -90$
$x = 90$

18. Quadrilateral $EBCF$ and $\overline{AD}$ are drawn below, such that $ABCD$ is a parallelogram, $\overline{EB} \cong \overline{FB}$, and $\overline{EF} \perp \overline{FH}$. If $m\angle E = 62°$ and $m\angle C = 51°$, what is $m\angle FHB$?

180
-51
129



90
-62
28

ΔEBF
$62 + 62 + x = 180$
$124 + x = 180$
$-124 \quad -124$
$x = 56$

$\angle ABC$
180
-51
129
-56
73

ΔFHB
$73 + 28 + x = 180$
$101 + x = 180$
$-101 \quad -101$
$x = 79$

$\angle FHB = 79°$

Euclidean Proofs:

If it is not specified, prove triangles are congruent
To prove triangles are congruent, prove 3 pairs of sides/angles are congruent
To prove segments or angles, use CPCTC
If you get stuck, make something up and keep on going!

1) **Do a mini proof with your givens**

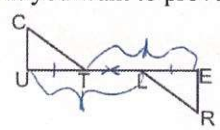
Altitude creates two congruent right angles
Median creates two congruent segments
Line bisector creates two congruent segments
Midpoint creates two congruent segments
Angle bisector creates two congruent angles
Perpendicular lines create two congruent right angles
Parallel lines cut by a transversal create
Congruent corresponding angles (1 in, 1 out) OR congruent alternate interior angles (2 out) OR congruent alternate exterior angles (2 out)
*Perpendicular bisector is perpendicular and line bisector (1 pair of congruent right angles, 1 pair of congruent segs)
*If segments bisect each other, they are both cut in half (2 pairs of congruent segments)

2) **Use additional tools:**

Vertical Angles are congruent (Look for an X)
Reflexive Property (A side/angle is in both triangles and is congruent to itself)
Isosceles Triangles (In a triangle, congruent angles are opposite congruent sides)
Addition and Subtraction Property (If you need more or less of a shared side)
*You must use three congruent statements to get one congruent statement for the triangles. The two that you are adding/subtracting and the one that you want to prove in the triangle.

7. Given: $\overline{UL} \cong \overline{TE}$
Prove: $\overline{UT} \cong \overline{LE}$

statements	reasons
① $\overline{UL} \cong \overline{TE}$	① given
② $\overline{TL} \cong \overline{TL}$	② reflexive property
③ $\overline{UT} \cong \overline{LE}$	③ subtraction property



Parallelogram Theorems	Circle Theorems (Look for inscribed angles)
A parallelogram/rectangle/rhombus/square has: Two pairs of opposite sides congruent Two pairs of opposite sides parallel Diagonals that bisect each other Opposite angles congruent	**Angles inscribed to the same arc are congruent** **An angle inscribed to a semicircle is a right angle** **A tangent and a radius/diameter form a right angles** All radii/diameters of a circle are congruent Congruent arcs have congruent chords have congruent central angles Parallel Lines intercept congruent arcs Tangents drawn from the same point are congruent
A rectangle/square has: Congruent right angles Congruent diagonals	
A rhombus/square has: All sides congruent Perpendicular diagonals Diagonals that bisect the angles	

To prove triangles are SIMILAR, prove AA ≅ AA
If asked to prove a proportion/multiplication:
1) Prove triangles are similar
2) Corresponding Sides of Similar Triangle are In Proportion (CSSTIP)
3) Cross Products are Equal

Work Backwards!

③ $\triangle AED \sim \triangle CEB$	③ AA≅AA
④ $\frac{AE}{ED} = \frac{CE}{EB}$	④ CSSTIP
⑤ $AE \cdot EB = CE \cdot ED$	⑤ cross products are equal

Euclidean Proofs (Basic)

If it is not specified, prove triangles are congruent
To prove triangles are congruent, prove 3 pairs of sides/angles are congruent
To prove segments or angles, use CPCTC
***If you get stuck, make something up and keep on going!**

1) Do a mini proof with your givens

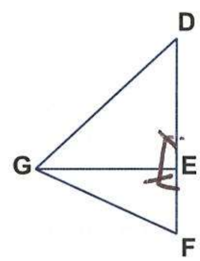
Altitude creates congruent right angles
Median creates congruent segments
Line bisector creates congruent segments
Midpoint creates congruent segments
Angle bisector creates congruent angles
Perpendicular lines create congruent right angles
When given parallel lines:
Corresponding angles are congruent OR Alternate interior angles are congruent OR Alternate exterior angles are congruent

2) Use additional tools:

Vertical Angles are congruent (Look for an X)
Reflexive Property (A side/angle is congruent to itself)

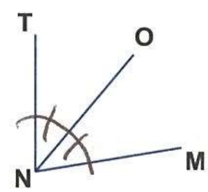
Mini Proofs

1. Given: $\overline{GE}$ is an altitude



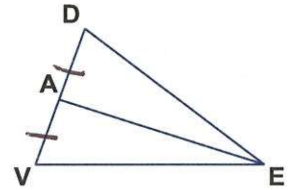
Statements	Reasons
① $\overline{GE}$ is an altitude	① given
② $\angle FEG \cong \angle DEG$	② An altitude creates congruent right angle

2. Given: $\overline{ON}$ bisects $\angle$TNM



Statements	Reasons
② $\angle TNO \cong \angle MNO$	② An angle bisector creates congruent angles
① $\overline{ON}$ bisects $\angle TNM$	① given

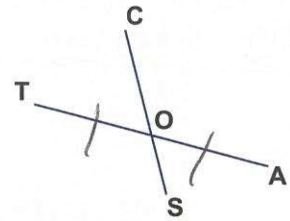
3. Given: A is the midpoint of $\overline{DV}$



Statements	Reasons
① A is the midpoint of $\overline{DV}$	① given
② $\overline{DA} \cong \overline{AV}$	② A midpoint creates congruent segments

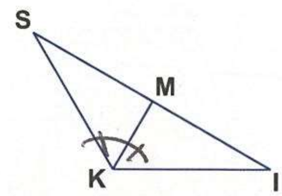
4. Given: $\overline{CS}$ bisects $\overline{TA}$

Statements	Reasons
(1) $\overline{CS}$ bisects $\overline{TA}$	(1) given
(2) $\overline{TO} \cong \overline{OA}$	(2) A line bisector creates congruent segments



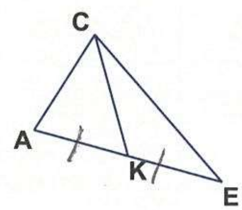
5. Given: $\overline{KM}$ bisects $\angle$ SKI

Statements	Reasons
(1) $\overline{Km}$ bisects $\angle SKI$	(1) given
(2) $\angle SKM \cong \angle IKM$	(2) An angle bisector creates two congruent angles.



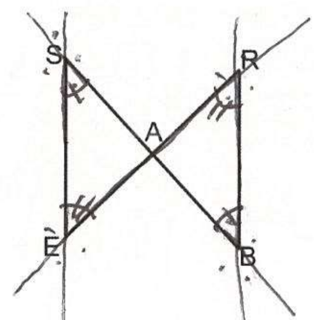
6. $\overline{CK}$ is a median

Statements	Reasons
(1) $\overline{CK}$ is a median	(1) given
(2) $\overline{AK} \cong \overline{KE}$	(2) A median creates congruent segments



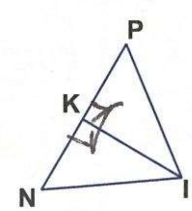
7. Given: $\overline{SE} \parallel \overline{RB}$

Statements	Reasons
(1) $\overline{SE} \parallel \overline{RB}$	(1) given
(2) $\angle S \cong \angle B$, $\angle R \cong \angle E$	(2) Parallel lines cut by a transversal create congruent alternate interior angles

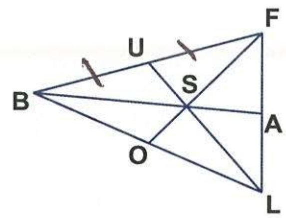


8. Given: $\overline{IK} \perp \overline{PN}$

Statements	Reasons
(2) $\angle PKI \cong \angle NKI$	(2) Perpendicular lines form congruent right angles
(1) $\overline{IK} \perp \overline{PN}$	(1) given

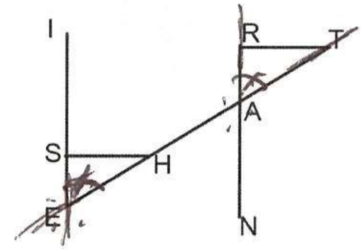


9. Given: U is the midpoint of $\overline{BF}$



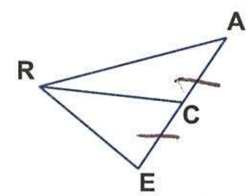
statements	Reasons
① U is the midpoint of $\overline{BF}$	① given
② $\overline{BU} \cong \overline{UF}$	② A midpoint creates congruent segments

10. Given: $\overline{IE} \parallel \overline{RN}$



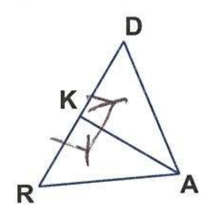
statements	Reasons
① $\overline{IE} \parallel \overline{RN}$	① given
② $\angle SEH \cong \angle RAT$	② Parallel lines cut by a transversal create congruent corresponding angles

11. Given: C is the midpoint of $\overline{AE}$



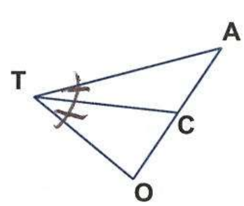
Statements	Reasons
① C is the midpoint of $\overline{AE}$	① given
② $\overline{AC} \cong \overline{CE}$	② A midpoint creates congruent segments

12. Given: $\overline{AK} \perp \overline{DR}$



statements	Reasons
① $\overline{AK} \perp \overline{DR}$	① given
② $\angle AKD \cong \angle AKR$	② Perpendicular lines form congruent right angles

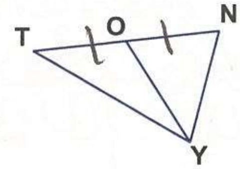
13. Given: $\overline{CT}$ bisects $\angle$ATO



statements	Reasons
① $\overline{CT}$ bisects $\angle ATO$	① given
② $\angle ATC \cong \angle OTC$	② An angle bisector creates congruent angles

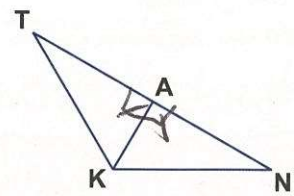
14. Given: $\overline{YO}$ is a median

Statements	Reasons
① YO is a median	① given
② $\overline{TO} \cong \overline{ON}$	② A median creates congruent segments



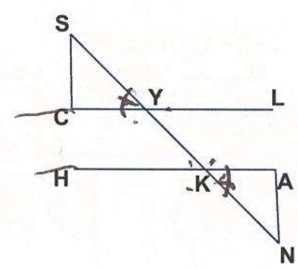
15. Given: $\overline{KA}$ is an altitude

Statements	Reasons
① $\overline{KA}$ is an altitude	① given
② $\angle TAK \cong \angle NAK$	② An altitude creates congruent right angles



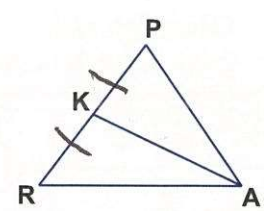
16. Given: $\overline{CL} \parallel \overline{HA}$

Statements	Reasons
① CL ∥ HA	① given
② $\angle SYC \cong \angle AKN$	② parallel lines cut by a transversal create congruent alternate exterior angles



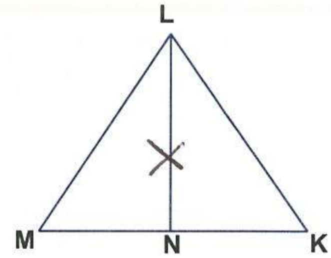
17. Given: $\overline{KA}$ bisects $\overline{PR}$

Statements	Reasons
① KA bisects PR	① given
② $PK \cong KR$	② A line bisector creates congruent segments



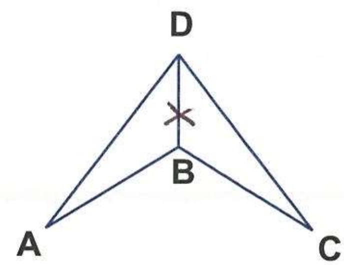
Reflexive Property and Vertical Angles

1. Given: None
   Prove: $\Delta LNM \cong \Delta LNK$



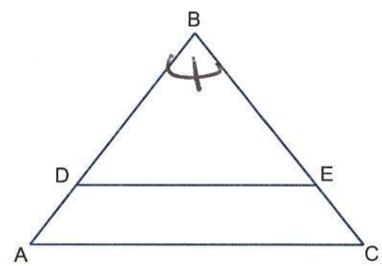
Statements	Reasons
① $\overline{LN} \cong \overline{LN}$	① Reflexive Property

2. Given: None
   Prove: $\Delta DBA \cong \Delta DBC$



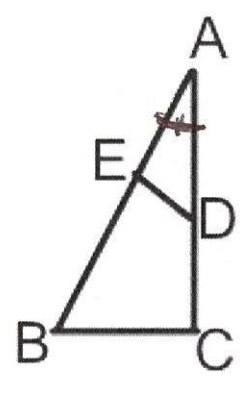
Statements	Reasons
① $\overline{DB} \cong \overline{DB}$	① Reflexive Property

3. Given: None
   Prove: $\Delta BDE \sim \Delta BAC$



Statements	Reasons
① $\angle B \cong \angle B$	① Reflexive Property

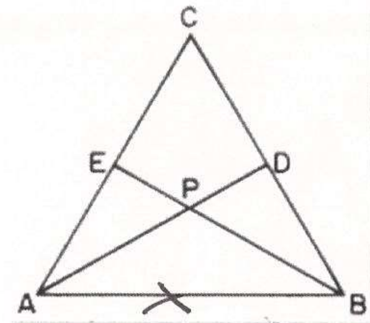
4. Given: None
   Prove: $\Delta ABC \sim \Delta ADE$



Statements	Reasons
① $\angle A \cong \angle A$	① Reflexive Property

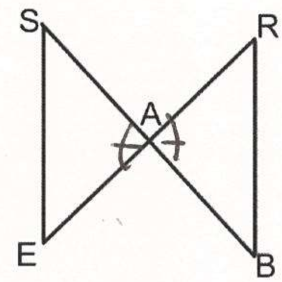
5. Given: None
Prove: $\Delta AEB \cong \Delta BDA$

Statements	Reasons
(1) $\overline{AB} \cong \overline{AB}$	(1) Reflexive Property



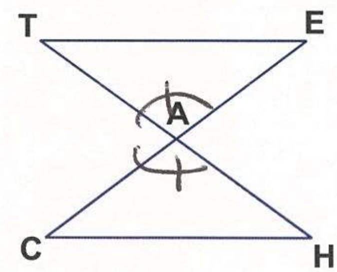
6. Given: None
Prove: $\Delta SAE \cong \Delta RAB$

Statements	Reasons
(1) $\angle SAE \cong \angle RAB$	(1) vertical angles are congruent



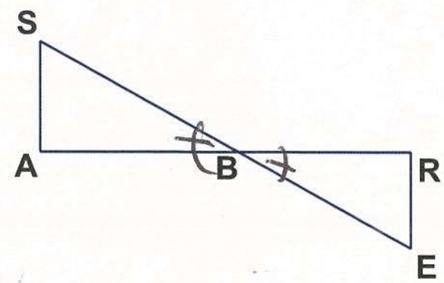
7. Given: None
Prove: $\Delta TAE \cong \Delta CAH$

Statements	Reasons
(1) $\angle TAE \cong \angle CAH$	(1) vertical angles are congruent



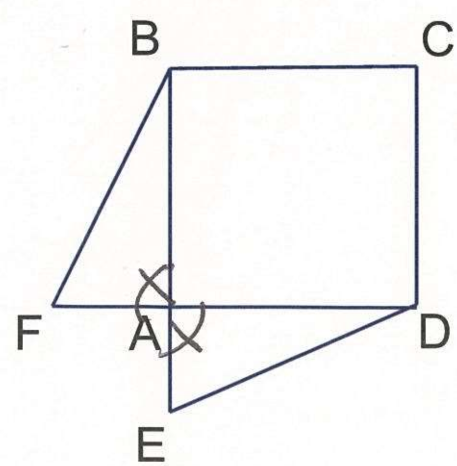
8. Given: None
Prove: $\Delta SBA \cong \Delta EBR$

Statements	Reasons
(1) $\angle SBA \cong \angle RBE$	(1) vertical angles are congruent



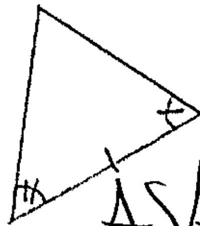
9. Given: None
Prove: $\Delta BAF \cong \Delta DAE$

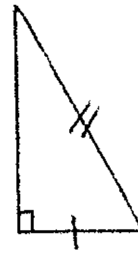
Statements	Reasons
(1) $\angle BAF \cong \angle DAE$	(1) vertical angles are congruent



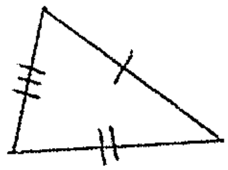
Methods for Proving Triangles are Congruent

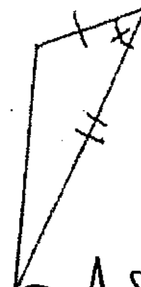
6  AAS

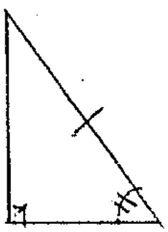
7  ASA

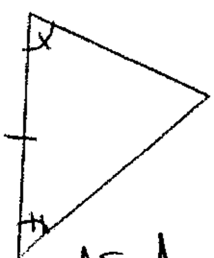
8  HL

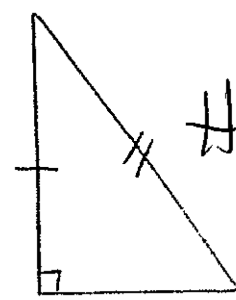
9  SAS

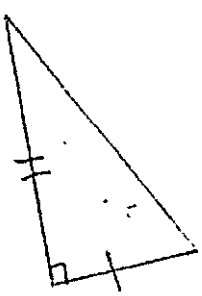
10  SSS

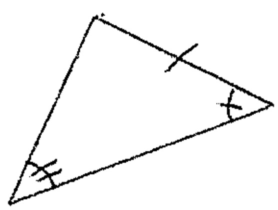
11  SAS

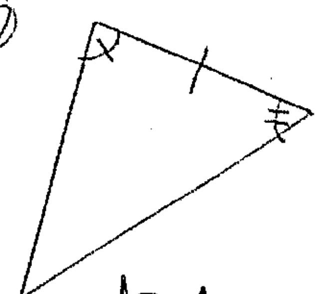
12  AAS

13  ASA

14  HL

15  SAS

16  AAS

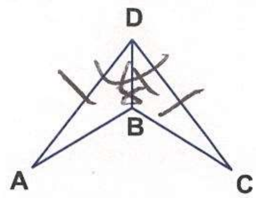
17  ASA

Proving Triangles are Congruent

1. Given: $\overline{BD}$ bisects $\angle ADC$

$\overline{AD} \cong \overline{DC}$

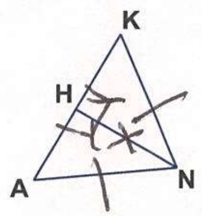
Prove: $\overline{AB} \cong \overline{BC}$



Statements	Reasons
(1) $\overline{BD}$ bisects $\angle ADC$	(1) given
(2) $\angle ADB \cong \angle CDB$	(2) An angle bisector creates congruent angles
(3) $\overline{AD} \cong \overline{DC}$	(3) given
(4) $\overline{DB} \cong \overline{DB}$	(4) Reflexive Property
(5) $\triangle ADB \cong \triangle CDB$	(5) SAS
(6) $AB \cong BC$	(6) CPCTC

2. Given: $\overline{HN} \perp \overline{KA}$, $\overline{KN} \cong \overline{AN}$

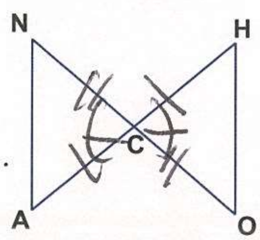
Prove: $\angle HAN \cong \angle HKN$



Statements	Reasons
(1) $\overline{HN} \perp \overline{KA}$	(1) given
(2) $\angle KHN \cong \angle AHN$	(2) Perpendicular lines form congruent right angles
(3) $\overline{KN} \cong \overline{AN}$	(3) given
(4) $\overline{HN} \cong \overline{HN}$	(4) Reflexive Property
(5) $\triangle AHN \cong \triangle KHN$	(5) HL
(6) $\angle HAN \cong \angle HKN$	(6) CPCTC

3. Given: $\overline{NO}$ and $\overline{HA}$ bisect each other

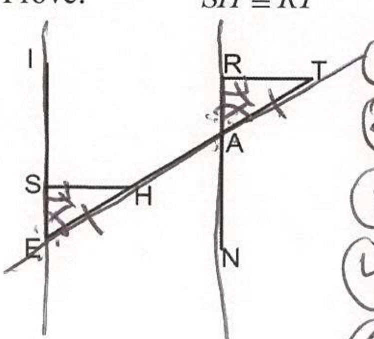
Prove: $\overline{NA} \cong \overline{HO}$



Statements	Reasons
(1) $\overline{NO}$ and $\overline{HA}$ bisect each other	(1) given
(2) $\overline{AC} \cong \overline{CH}$, $\overline{NC} \cong \overline{CO}$	(2) A line bisector creates congruent segments
(3) $\angle NCA \cong \angle HCO$	(3) vertical angles are congruent
(4) $\triangle ANC \cong \triangle HOC$	(4) SAS
(5) $\overline{NA} \cong \overline{HO}$	(5) CPCTC

4. Given: $\overline{IE} \parallel \overline{RN}$, $\overline{TR} \perp \overline{RN}$, $\overline{HS} \perp \overline{IE}$, $\overline{EH} \cong \overline{AT}$

Prove: $\overline{SH} \cong \overline{RT}$



Statements	Reasons
(1) $\overline{IE} \parallel \overline{RN}$	(1) given
(2) $\angle TAR \cong \angle HES$	(2) Parallel lines cut by a transversal create congruent corresponding angles
(3) $\overline{TR} \perp \overline{RN}$, $\overline{HS} \perp \overline{IE}$	(3) given
(4) $\angle TRA \cong \angle HSE$	(4) Perpendicular lines form congruent right angles
(5) ~~$\triangle HS$~~ $\overline{EH} \cong \overline{AT}$	(5) given
(6) $\triangle HSE \cong \triangle TRA$	(6) AAS
(7) $\overline{SH} \cong \overline{RT}$	(7) CPCTC

Euclidean Similar Triangle Proofs

To prove triangles are SIMILAR, prove AA ≅ AA

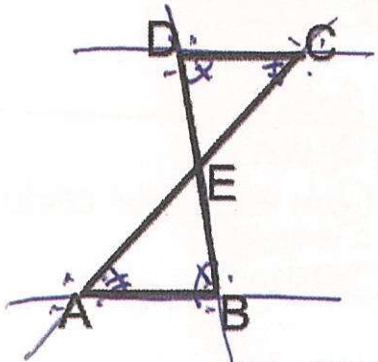
If asked to prove a proportion/multiplication:

1) **Prove triangles are similar**
2) **Corresponding Sides of Similar Triangle are In Proportion (CSSTIP)**
3) **Cross Products are Equal**

Work Backwards!

1. Given $\overline{AB} \parallel \overline{DC}$

Prove: $\Delta ABE \sim \Delta CDE$



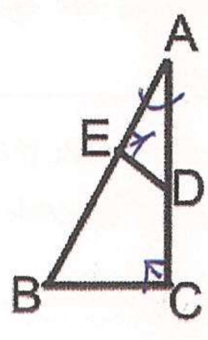
Statements	Reasons
① $\overline{AB} \parallel \overline{DC}$	① given
② $\angle CDE \cong \angle ABE$, $\angle DCE \cong \angle EAB$	② Parallel lines cut by a transversal create ≅ alternate interior angles
③ $\Delta ABE \sim \Delta CDE$	③ AA ≅ AA

*you could have also used vertical angles

2. Given: $\overline{BC} \perp \overline{AC}$

$\overline{DE} \perp \overline{AB}$

Prove: $\Delta ABC \sim \Delta ADE$

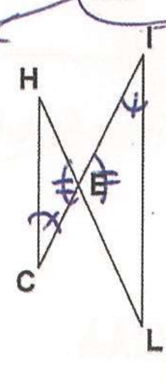


Statements	Reasons
① $\overline{BC} \perp \overline{AC}$, $\overline{DE} \perp \overline{AB}$	① given
② $\angle ABC \cong \angle AED$	② Perpendicular lines create ≅ right angles
③ $\angle EAD \cong \angle EAD$	③ Reflexive Property
④ $\Delta ABC \sim \Delta ADE$	④ AA ≅ AA

3. Given: $\angle HCE \cong \angle LIE$

Prove: $\dfrac{CE}{CH} = \dfrac{EI}{IL}$

start backwards

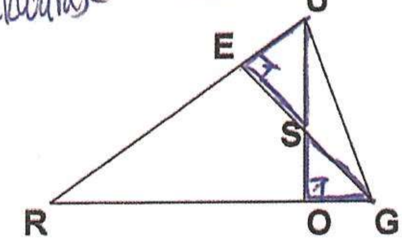


Statements	Reasons
① $\angle HCE \cong \angle LIE$	① given
② $\angle HEC \cong \angle IEL$	② vertical angles are congruent
③ $\Delta CEH \sim \Delta IEL$	③ AA ≅ AA
④ $\dfrac{CE}{CH} = \dfrac{EI}{IL}$	④ CSSTIP

4. Given: $\overline{UO} \perp \overline{RG}$, $\overline{UR} \perp \overline{EG}$

Prove: $\dfrac{\overline{US}}{\overline{SO}} = \dfrac{\overline{EU}}{\overline{OG}}$

work backwards

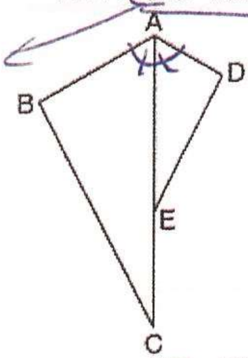


Statements	Reasons
① $\overline{UO} \perp \overline{RG}, \overline{UR} \perp \overline{EG}$	① given
② $\angle UES \cong \angle SOG$	② perpendicular lines create congruent right angles
③ $\angle USE \cong \angle OSG$	③ vertical angles are congruent
④ $\triangle USE \sim \triangle GSO$	④ AA ≅ AA
⑤ $\frac{US}{SO} = \frac{EU}{OG}$	⑤ CSSTIP

5. Given: $\overline{CA}$ bisects $\angle CAD$, $\angle ABC \cong \angle ADE$

Prove: $\overline{BC} \cdot \overline{AE} = \overline{DE} \cdot \overline{AC}$

work backwards

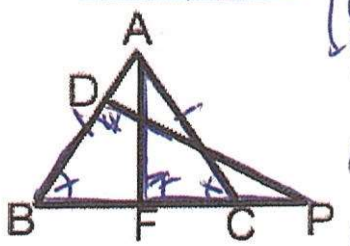


Statements	Reasons
① CA bisects $\angle CAD$	① given
② $\angle BAC \cong \angle DAE$	② An angle bisector creates 2 ≅ angles
③ $\angle ABC \cong \angle ADE$	③ given
④ $\triangle BCA \sim \triangle DEA$	④ AA ≅ AA
⑤ $\frac{BC}{AC} = \frac{DE}{AE}$	⑤ CSSTIP
⑥ $BC \cdot AE = DE \cdot AC$	⑥ cross products are equal

6. Given: $\overline{AB} \cong \overline{AC}$, $\overline{AF} \perp \overline{BC}$, $\overline{PD} \perp \overline{AB}$

Prove: $\overline{FC} \cdot \overline{PB} = \overline{DB} \cdot \overline{AC}$

work backwards

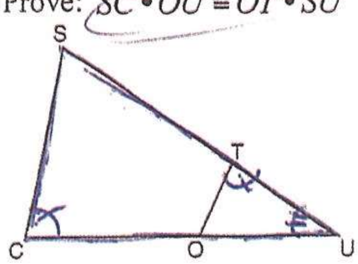


Statements	Reasons
① $AB \cong AC$	① given
② $\angle ABF \cong \angle ACF$	② Isosceles Triangle theorem
③ $\overline{AF} \perp \overline{BC}, \overline{PD} \perp \overline{AB}$	③ given
④ $\angle PDB \cong \angle AFC$	④ perpendicular lines create ≅ right angles
⑤ $\triangle FCA \sim \triangle DBP$	⑤ AA ≅ AA
⑥ $\frac{FC}{AC} = \frac{DB}{PB}$	⑥ CSSTIP
⑦ $FC \cdot PB = DB \cdot AC$	⑦ cross products are equal

7. In $\triangle SCU$ shown below, points T and O are on $\overline{SU}$ and $\overline{CU}$, respectively. Segment OT is drawn so that $\angle C \cong \angle OTU$.

Prove: $\overline{SC} \cdot \overline{OU} = \overline{OT} \cdot \overline{SU}$

work backwards



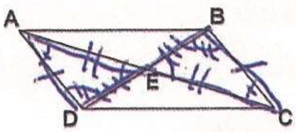
Statements	Reasons
① $\angle C \cong \angle OTU$	① given
② $\angle OUT \cong \angle OUT$	② Reflexive Property
③ $\triangle SCU \sim \triangle OTU$	③ AA ≅ AA
④ $\frac{SC}{SU} = \frac{OT}{OU}$	④ CSSTIP
⑤ $SC \cdot OU = OT \cdot SU$	⑤ cross products are equal

Euclidean Proofs with Parallelogram and Circle Theorems

Parallelogram Theorems	Circle Theorems
A parallelogram/rectangle/rhombus/square has: Two pairs of opposite sides congruent Two pairs of opposite sides parallel Diagonals that bisect each other Opposite angles congruent	**All radii/diameters of a circle are congruent** **Angles inscribed to the same arc are congruent** **An angle inscribed to a semicircle is a right angle** **A tangent and a radius/diameter form a right angles** Congruent arcs have congruent chords have congruent central angles Parallel Lines intercept congruent arcs Tangents drawn from the same point are congruent
A rectangle/square has: A right angle Congruent diagonals	
A rhombus/square has: All sides congruent Perpendicular diagonals Diagonals that bisect the angles	

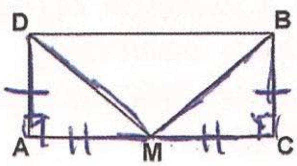
I'm giving you all 6 even though you only need 3

1. Given: ABCD is a parallelogram
Prove: $\Delta AED \cong \Delta CEB$



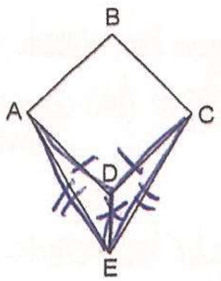
Statements	Reasons
① ABCD is a parallelogram	① given
② $\overline{AD} \cong \overline{BC}$	② A p-gram has opposite sides ≅
③ $\overline{AE} \cong \overline{EC}$, $\overline{BE} \cong \overline{ED}$	③ A p-gram has diagonals that bisect each other
④ $\angle EAD \cong \angle BCE$, $\angle ADE \cong \angle CBE$	④ A p-gram has ≅ alternate interior angles
⑤ $\angle AED \cong \angle BEC$	⑤ vertical angles are congruent
⑥ $\Delta AED \cong \Delta CEB$	⑥ SSS/SAS/ASA/AAS depending on which three you did

2. Given: ABCD is a rectangle, M is the midpoint of $\overline{AC}$
Prove: $\overline{DM} \cong \overline{BM}$



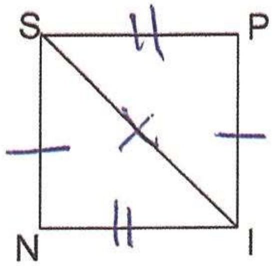
Statements	Reasons
① ABCD is a rectangle	① given
② $\overline{DA} \cong \overline{BC}$	② A ~~[illegible]~~ rectangle has 2 pairs of opp sides ≅
③ $\angle DAM \cong \angle BCM$	③ A rectangle has ≅ right angles
④ M is midpoint of $\overline{AC}$	④ given
⑤ $\overline{AM} \cong \overline{MC}$	⑤ A midpoint creates 2 ≅ segments
⑥ $\Delta DAM \cong \Delta BCM$	⑥ SAS ≅ SAS
⑦ $\overline{DM} \cong \overline{BM}$	⑦ CPCTC

3. Given: ABCD is a rhombus, $\overline{AE} \cong \overline{CE}$
Prove: $\angle ADE \cong \angle CDE$



Statements	Reasons
① ABCD is a rhombus	① given
② $\overline{AD} \cong \overline{DC}$	② A rhombus has all sides ≅
③ $\overline{AE} \cong \overline{CE}$	③ given
④ $\overline{DE} \cong \overline{DE}$	④ Reflexive Property
⑤ $\Delta ADE \cong \Delta CDE$	⑤ SSS ≅ SSS
⑥ $\angle ADE \cong \angle CDE$	⑥ CPCTC

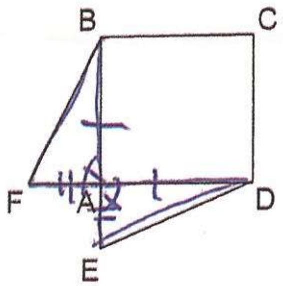
4. Given: SPIN is a square
Prove: $\Delta SNI \cong \Delta SPI$



Statements	Reasons
① SPIN is a square	① given
② $\overline{SN} \cong \overline{PI}$, $\overline{SP} \cong \overline{NI}$	② A square has $\cong$ ~~[illegible]~~ opposite sides
③ $\overline{SI} \cong \overline{SI}$	③ Reflexive Property
④ $\Delta SNI \cong \Delta SPI$	④ SSS $\cong$ SSS

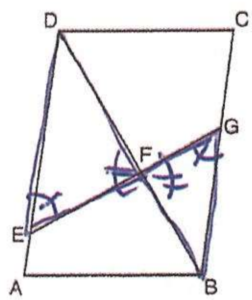
*Also right angles and alternate interior angles

5. Given: ABCD is a square, $\overline{FA} \cong \overline{AE}$
Prove: $\overline{BF} \cong \overline{DE}$



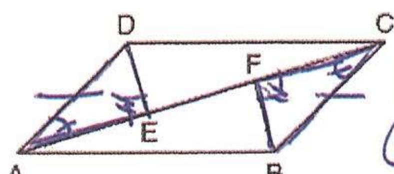
Statements	Reasons
① ABCD is a square	① given
② $\overline{BA} \cong \overline{AD}$	② A square has all sides $\cong$
③ $\overline{FA} \cong \overline{AE}$	③ given
④ $\angle FAB \cong \angle DAE$	④ vertical angles are congruent
⑤ $\Delta BAF \cong \Delta DAE$	⑤ SAS $\cong$ SAS
⑥ $\overline{BF} \cong \overline{DE}$	⑥ CPCTC

6. Given: Parallelogram $ABCD$, $\overline{EFG}$, and diagonal $\overline{DFB}$
Prove: $\Delta DEF \sim \Delta BGF$



Statements	Reasons
① Parallelogram ABCD	① given
② $\angle DEF \cong \angle BGF$	② A parallelogram has $\cong$ alternate interior angles
③ $\angle DFE \cong \angle BFG$	③ vertical angles are congruent
④ $\Delta DEF \sim \Delta BGF$	④ AA $\cong$ AA

7. In parallelogram $ABCD$, $\overline{BF}$ and $\overline{DE}$ are perpendicular to diagonal $\overline{AC}$ at points F and E.
Prove: $\overline{AE} \cong \overline{CF}$



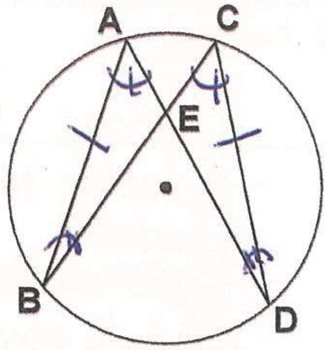
Statements	Reasons
① Parallelogram ABCD	① given
② $\overline{AD} \cong \overline{BC}$	② A parallelogram has opposite sides $\cong$
③ $\angle DAE \cong \angle BCF$	③ A parallelogram has congruent alternate interior angles
④ $\overline{BF}$ and $\overline{DE}$ are perpendicular to $\overline{AC}$	④ given
⑤ $\angle DEA \cong \angle CFB$	⑤ Perpendicular lines create $\cong$ right angles
⑥ $\Delta DEA \cong \Delta CFB$	⑥ AAS $\cong$ AAS
⑦ $\overline{AE} \cong \overline{CF}$	⑦ CPCTC

*Look for inscribed angles

8. Given: Chords $\overline{AD}$ and $\overline{BC}$ of circle O intersect at E, $\overline{AB} \cong \overline{CD}$

Prove: $\overline{BC} \cong \overline{AD}$

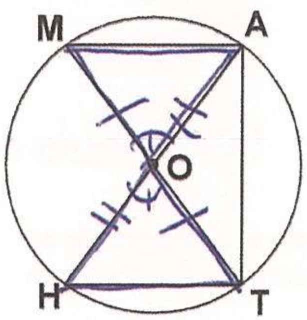
*you could have also used vertical angles



Statements	Reasons
① $\overline{AB} \cong \overline{CD}$	① given
② $\angle BAE \cong \angle DCE$, $\angle ABE \cong \angle CDE$	② Angles inscribed to the same arc are congruent
③ $\triangle BAE \cong \triangle DCE$	③ ASA≅ASA
④ $\overline{BC} \cong \overline{AD}$	④ CPCTC

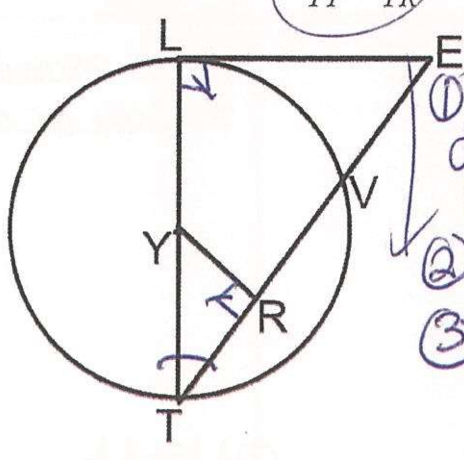
9. Given: Circle O with diameters $\overline{MOT}$ and $\overline{AOH}$.

Prove: $\overline{MA} \cong \overline{HT}$



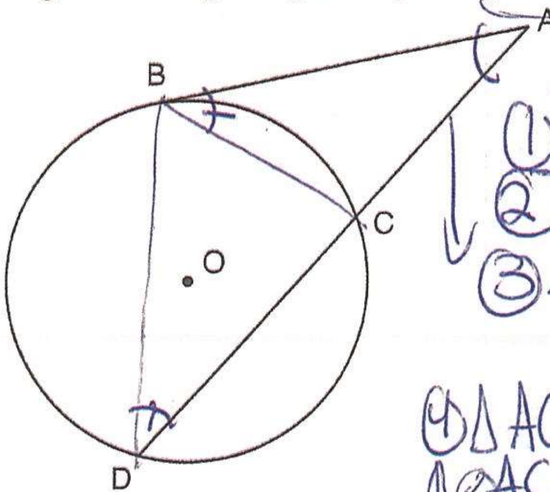
Statements	Reasons
① $\overline{MO} \cong \overline{OT}$, $\overline{AO} \cong \overline{OH}$	① All radii of a circle are ≅
② $\angle MOA \cong \angle HOT$	② vertical angles are ≅
③ $\triangle MOA \cong \triangle TOH$	③ SAS≅SAS
④ $\overline{MA} \cong \overline{HT}$	④ CPCTC

10. In circle Y, tangent $\overline{LE}$ is drawn to diameter $\overline{TYL}$ and $\overline{YR} \perp \overline{TE}$. Prove that $\frac{TE}{TY} = \frac{TL}{TR}$. work backwards



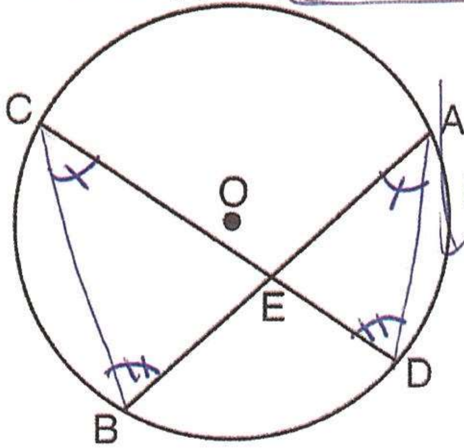
Statements	Reasons
① Tangent LE is drawn to diameter TYL, $\overline{YR} \perp \overline{TE}$	① given
② $\angle TLE \cong \angle TRY$	② An angle formed by a tangent and diameter and perpendicular lines form congruent right angles
③ $\angle RTY \cong \angle RTY$	③ Reflexive Property
④ $\triangle TLE \sim \triangle TRY$	④ AA≅AA
⑤ $\frac{TE}{TY} = \frac{TL}{TR}$	⑤ CSSTIP

11. In the diagram below, secant $\overline{ACD}$ and tangent $\overline{AB}$ are drawn from external point A to circle O.
Prove the theorem: If a secant and a tangent are drawn to a circle from an external point, the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared. ($AC \cdot AD = AB^2$) → work backwards



Statements	Reasons
(1) $\overline{BC}, \overline{BD}$	(1) Auxiliary lines can be drawn
(2) $\angle BAC \cong \angle BAC$	(2) Reflexive Property
(3) $\angle BDC \cong \angle ABC$	(3) Angles inscribed to the same arc are $\cong$
(4) $\triangle ACB \sim \triangle ABD$	(4) $AA \cong AA$
(5) $\frac{AC}{AB} = \frac{AB}{AD}$	(5) CSSTPP
(6) $AC \cdot AD = AB^2$	(6) Cross products are equal

12. Given: Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E
Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. Prove this theorem by proving $AE \cdot EB = CE \cdot ED$. → work backwards



Statements	Reasons
(1) $\overline{CB}, \overline{AD}$	(1) Auxiliary lines can be drawn
(2) $\angle BCE \cong \angle DAE$, $\angle CBE \cong \angle ADE$	(2) Angles inscribed to the same arc are $\cong$
(3) $\triangle AED \sim \triangle CEB$	(3) $AA \cong AA$
(4) $\frac{AE}{ED} = \frac{CE}{EB}$	(4) CSSTPP
(5) $AE \cdot EB = CE \cdot ED$	(5) Cross products are equal

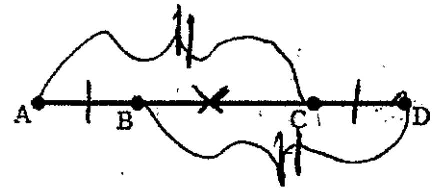
*you could have also used vertical angles

Name Schlansky
Mr. Schlansky

Date ______________
Geometry

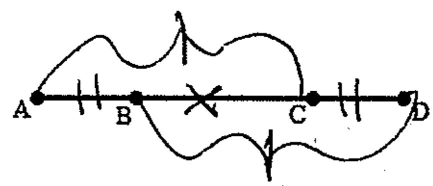
Addition and Subtraction Property Mini Proofs

1. Given: $\overline{AB} \cong \overline{CD}$
Prove: $\overline{AC} \cong \overline{BD}$



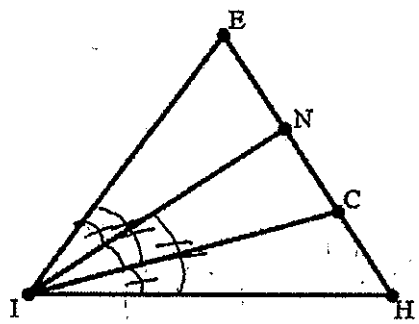
statements	Reasons
① $\overline{AB} \cong \overline{CD}$	① given
② $\overline{BC} \cong \overline{BC}$	② reflexive property
③ $\overline{AC} \cong \overline{BD}$	③ Addition Property

2. Given: $\overline{AC} \cong \overline{BD}$
Prove: $\overline{AB} \cong \overline{CD}$



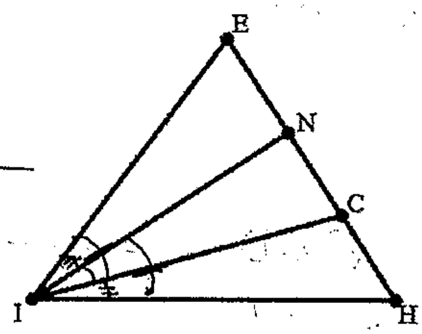
statements	Reasons
① $\overline{AC} \cong \overline{BD}$	① given
② $\overline{BC} \cong \overline{BC}$	② reflexive property
③ $\overline{AB} \cong \overline{CD}$	③ subtraction property

3. Given: $\angle EIN \cong \angle HIC$
Prove: $\angle EIC \cong \angle HIN$



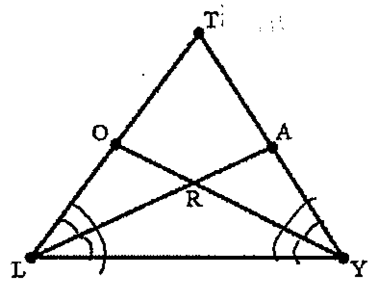
statements	Reasons
① $\angle EIN \cong \angle HIC$	① given
② $\angle NIC \cong \angle NIC$	② reflexive property
③ $\angle EIC \cong \angle HIN$	③ Addition Property

4. Given: $\angle EIC \cong \angle HIN$
Prove: $\angle EIN \cong \angle HIC$



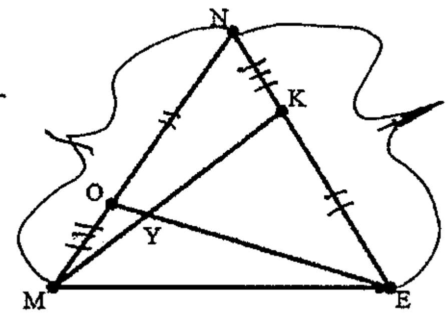
Statements	Reasons
① $\angle EIC \cong \angle HIN$	① given
② $\angle NIC \cong \angle NIC$	② reflexive property
③ $\angle EIN \cong \angle HIC$	③ subtraction property

5. Given: $\angle TLA \cong \angle TYO$, $\angle ALY \cong \angle OYL$
Prove: $\angle TLY \cong \angle TYL$



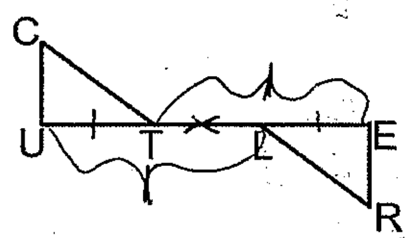
Statements	Reasons
① $\angle TLA \cong \angle TYO$	① given
② $\angle ALY \cong \angle OYL$	② given
③ $\angle TLY \cong \angle TYL$	③ Addition Property

6. Given: $\overline{MN} \cong \overline{NE}$, $\overline{ON} \cong \overline{KE}$
Prove: $\overline{MO} \cong \overline{KN}$



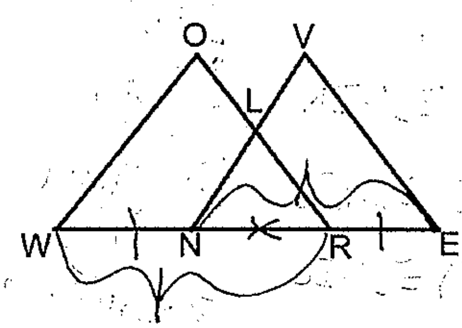
statements	Reasons
① $\overline{MN} \cong \overline{NE}$	① given
② $\overline{ON} \cong \overline{KE}$	② given
③ $\overline{MO} \cong \overline{KN}$	③ subtraction property

7. Given: $\overline{UL} \cong \overline{TE}$
Prove: $\overline{UT} \cong \overline{LE}$



statements	Reasons
① $\overline{UL} \cong \overline{TE}$	① given
② $\overline{TL} \cong \overline{TL}$	② reflexive property
③ $\overline{UT} \cong \overline{LE}$	③ subtraction property

8. Given: $\overline{WN} \cong \overline{RE}$
Prove: $\overline{WR} \cong \overline{NE}$

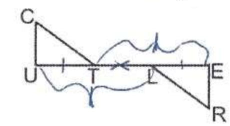


statements	Reasons
① $\overline{WN} \cong \overline{RE}$	① given
② $\overline{NR} \cong \overline{NR}$	② reflexive property
③ $\overline{WR} \cong \overline{NE}$	③ addition property

Euclidean Triangle Proofs with Additional Tools

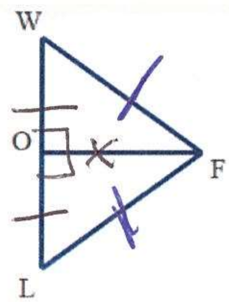
Vertical Angles are congruent (Look for an X)
Reflexive Property (A side/angle is congruent to itself)
Isosceles Triangles (In a triangle, congruent angles are opposite congruent sides)
Addition and Subtraction Property (If you need more or less of a shared side)
*You must use three congruent statements to get one congruent statement for the triangles. The two that you are adding/subtracting and the one that you want to prove in the triangle.

7. Given: $\overline{UL} \cong \overline{TE}$
Prove: $\overline{UT} \cong \overline{LE}$



statements	Reasons
① $\overline{UL} \cong \overline{TE}$	① given
② $\overline{TL} \cong \overline{TL}$	② reflexive property
③ $\overline{UT} \cong \overline{LE}$	③ subtraction property

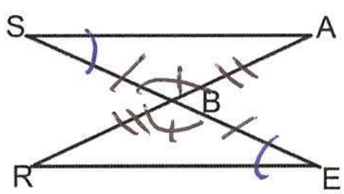
1. Given: $\overline{OF}$ is the perpendicular bisector of $\overline{WL}$
Prove: ΔWFL is isosceles



statements	Reasons
① $\overline{OF}$ is perpendicular bisector of $\overline{WL}$	① given
② $\overline{WO} \cong \overline{OL}$	② A line bisector creates congruent segments
③ $\angle WOF \cong \angle LOF$	③ perpendicular lines form congruent right angles
④ $\overline{OF} \cong \overline{OF}$	④ Reflexive Property
⑤ $\Delta WOF \cong \Delta LOF$	⑤ SAS
⑥ $\overline{WF} \cong \overline{LF}$	⑥ CPCTC
⑦ ΔWFL is isosceles	⑦ Isosceles Triangle Theorem

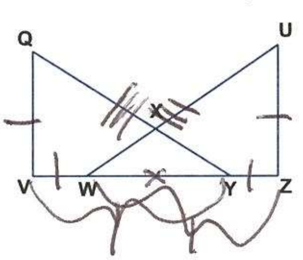
2. Given: $\overline{SE}$ and $\overline{AR}$ bisect each other.

Prove that $\overline{SA} \parallel \overline{RE}$



Statements	Reasons
① $\overline{SE}$ and $\overline{AR}$ bisect each other	① given
② $\overline{SB} \cong \overline{BE}$, $\overline{RB} \cong \overline{BA}$	② A line bisector creates congruent segments
③ $\angle SBA \cong \angle RBE$	③ vertical angles are congruent
④ $\Delta SBA \cong \Delta EBR$	④ SAS
⑤ $\angle ASB \cong \angle BER$	⑤ CPCTC
⑥ $\overline{SA} \parallel \overline{RE}$	⑥ Parallel lines cut by a transversal create congruent alternate interior angles

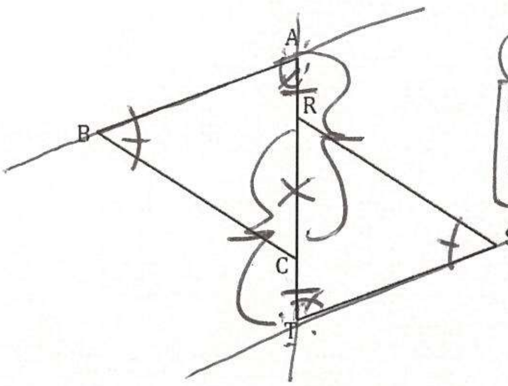
3. Given: $\overline{QV} \cong \overline{UZ}$
$\overline{VW} \cong \overline{YZ}$
$\overline{YQ} \cong \overline{WU}$
Prove: $\angle Q \cong \angle U$



Statements	Reasons
① $\overline{QV} \cong \overline{UZ}$	① given
② $\overline{VW} \cong \overline{YZ}$	② given
③ $\overline{WY} \cong \overline{WY}$	③ Reflexive Property
④ $\overline{VY} \cong \overline{WZ}$	④ Addition Property
⑤ $\overline{YQ} \cong \overline{UW}$	⑤ given
⑥ $\Delta QVY \cong \Delta UZW$	⑥ SSS
⑦ $\angle Q \cong \angle U$	⑦ CPCTC

4. Given: $\angle B \cong \angle S$, $\overline{AB} \parallel \overline{ST}$, $\overline{AR} \cong \overline{TC}$

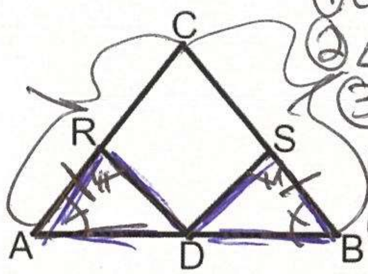
Prove: $\overline{BC} \cong \overline{SR}$



Statements	Reasons
① $\angle B \cong \angle S$	① given
② $\overline{AB} \parallel \overline{ST}$	② given
③ $\angle BAC \cong \angle STR$	③ Parallel lines cut by a transversal create congruent alternate interior angles
④ $\overline{AR} \cong \overline{TC}$	④ given
⑤ $RC \cong RC$	⑤ Reflexive Property
⑥ $AC \cong RT$	⑥ Addition Property
⑦ $\triangle BAC \cong \triangle STR$	⑦ AAS
⑧ $\overline{BC} \cong \overline{SR}$	⑧ CPCTC

5. Given: In ΔABC, $\overline{CA} \cong \overline{CB}$, $\overline{AR} \cong \overline{BS}$, $\overline{DR} \perp \overline{AC}$, and $\overline{DS} \perp \overline{BC}$

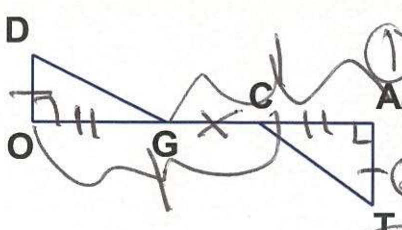
Prove: $\overline{DR} \cong \overline{DS}$



Statements	Reasons
① $\overline{CA} \cong \overline{CB}$	① given
② $\angle CAD \cong \angle CBD$	② Isosceles Triangle theorem
③ $\overline{AR} \cong \overline{BS}$	③ given
④ $\overline{DR} \perp \overline{AC}$, $\overline{DS} \perp \overline{BC}$	④ given
⑤ $\angle ARD \cong \angle BSD$	⑤ Perpendicular lines create congruent right angles
⑥ $\triangle ARD \cong \triangle BSD$	⑥ ASA
⑦ $\overline{DR} \cong \overline{DS}$	⑦ CPCTC

6. Given: $\overline{DO} \perp \overline{OA}$, $\overline{TA} \perp \overline{OA}$, $\overline{DO} \cong \overline{TA}$, $\overline{OC} \cong \overline{AG}$

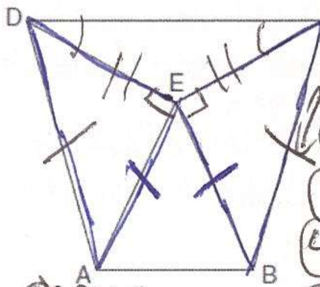
Prove: $\overline{DG} \cong \overline{TC}$



Statements	Reasons
① $\overline{DO} \perp \overline{OA}$, $\overline{TA} \perp \overline{OA}$	① given
② $\angle DOG \cong \angle CAT$	② Perpendicular lines form congruent right angles
③ $\overline{DO} \cong \overline{TA}$	③ given
④ $\overline{OC} \cong \overline{AG}$	④ given
⑤ $\overline{GC} \cong \overline{GC}$	⑤ reflexive Property
⑥ $\overline{OG} \cong \overline{CA}$	⑥ Subtraction Property
⑦ $\triangle DOG \cong \triangle TAC$	⑦ SAS
⑧ $\overline{DG} \cong \overline{TC}$	⑧ CPCTC

7. Isosceles trapezoid *ABCD* has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments *AE*, *BE*, *CE*, and *DE* are drawn in trapezoid *ABCD* such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.

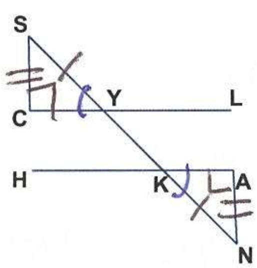
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.



Statements	Reasons
(2) $\overline{AD} \cong \overline{BC}$	(1) given
(1) Isosceles trapezoid ABCD	(2) An isosceles trapezoid has congruent legs
(3) $\angle CDE \cong \angle DCE$	(3) given
(4) $\overline{DE} \cong \overline{EC}$	(4) Isosceles Triangle Theorem
(5) $\overline{AE} \perp \overline{DE}$, $\overline{BE} \perp \overline{CE}$	(5) given
(6) $\angle DEA \cong \angle CEB$	(6) Perpendicular lines form congruent right angles
(7) $\triangle ADE \cong \triangle BCE$	(7) HL
(8) $\overline{AE} \cong \overline{EB}$	(8) CPCTC
(9) $\triangle AEB$ is isosceles	(9) Isosceles Triangle theolem

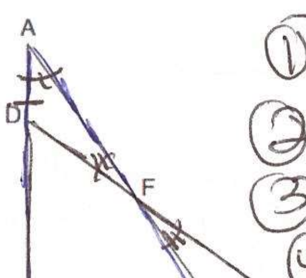
8. Given: $\overline{SC} \perp \overline{CL}$, $\overline{HA} \perp \overline{AN}$, $\overline{SY} \cong \overline{KN}$, and $\overline{SC} \cong \overline{AN}$.

Prove $\overline{CL} \parallel \overline{HA}$

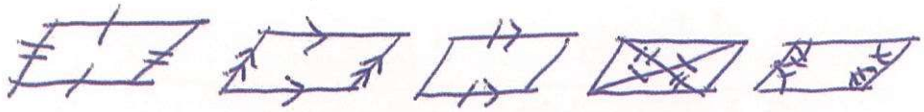


Statements	Reasons
(1) $\overline{SC} \perp \overline{CL}$, $\overline{HA} \perp \overline{AN}$	(1) given
(2) $\angle SCY \cong \angle NAK$	(2) Perpendicular lines form congruent right angles
(3) $\overline{SY} \cong \overline{KN}$, $\overline{SC} \cong \overline{AN}$	(3) Given
(4) $\triangle SCY \cong \triangle NAK$	(4) HL
(5) $\angle SYC \cong \angle AKN$	(5) CPCTC
(6) $\overline{CL} \parallel \overline{HA}$	(6) Palalel lines cut by a transversal create congruent alternate exterior angles

9. In the diagram below, $\triangle ABE \cong \triangle CBD$. Prove: $\triangle AFD \cong \triangle CFE$



Statements	Reasons
(1) $\triangle ABE \cong \triangle CBD$	(1) given
(2) $\angle DAF \cong \angle FCE$	(2) CPCTC
(3) $\overline{AB} \cong \overline{CB}$	(3) CPCTC
(4) $\overline{DB} \cong \overline{BE}$	(4) CPCTC
(5) $\overline{AD} \cong \overline{CE}$	(5) subtraction Property
(6) $\angle AFD \cong \angle CFE$	(6) vertical angles are congruent
(7) $\triangle AFD \cong \triangle CFE$	(7) AAS

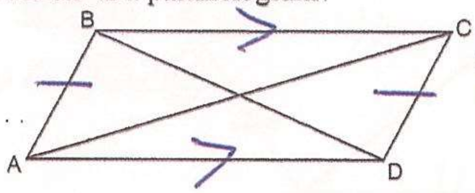


5 methods

1 ~~24~~. Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$ is shown in the diagram below.

Which information is *not* enough to prove $ABCD$ is a parallelogram?

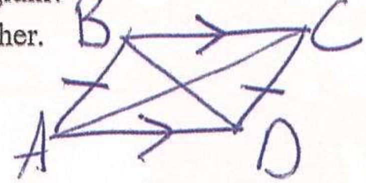
1) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{DC}$
2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$
3) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$
4) $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$



not one of the 5 methods

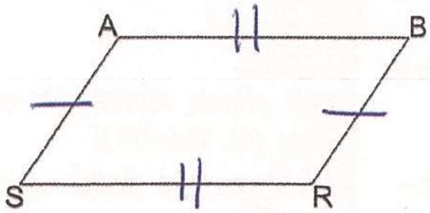
2 ~~25~~. Quadrilateral $ABCD$ has diagonals $\overline{AC}$ and $\overline{BD}$. Which information is *not* sufficient to prove $ABCD$ is a parallelogram?

1) $\overline{AC}$ and $\overline{BD}$ bisect each other.
2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$
3) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$
4) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$



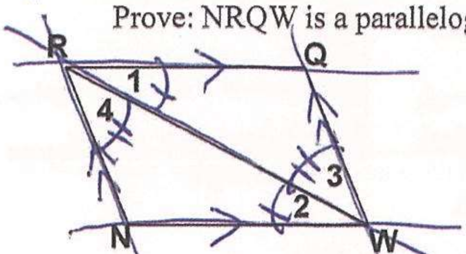
not one of the 5 methods

3 ~~26~~. Given: $\overline{SA} \cong \overline{BR}$, $\overline{AB} \cong \overline{SR}$
Prove: SABR is a parallelogram



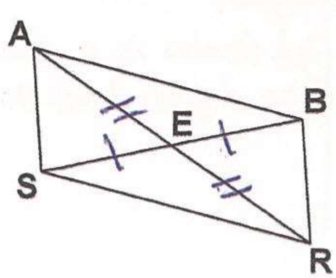
Statements	Reasons
① $\overline{SA} \cong \overline{BR}$, $\overline{AB} \cong \overline{SR}$	① given
② SABR is a parallelogram	② A parallelogram has 2 pairs of opposite sides $\cong$

4 ~~27~~. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$
Prove: NRQW is a parallelogram



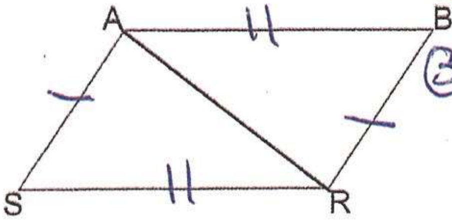
Statements	Reasons
① $\angle 1 \cong \angle 2$	① given
② $\overline{RQ} \parallel \overline{NW}$	② If alt int $\angle$s are $\cong$, then the lines are $\parallel$
③ $\angle 3 \cong \angle 4$	③ given
④ $\overline{RN} \parallel \overline{QW}$	④ If alt int $\angle$s are $\cong$, then the lines are
⑤ NRQW is a parallelogram	⑤ A parallelogram has 2 pairs of opposite sides parallel.

5 ~~28~~. Given: E is the midpoint of $\overline{SB}$, $\overline{AE} \cong \overline{ER}$
Prove: SABR is a parallelogram



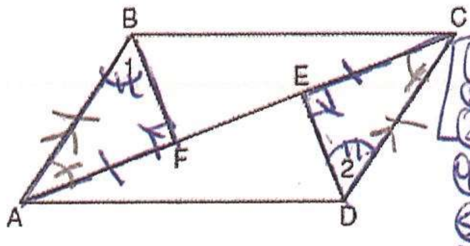
Statements	Reasons
① E is the midpoint of $\overline{SB}$	① given
② $\overline{SE} \cong \overline{EB}$	② A midpoint creates 2 $\cong$ segments
③ $\overline{AE} \cong \overline{ER}$	③ given
④ SABR is a parallelogram	④ A parallelogram has diagonals that bisect each other

6 29. Given: $\Delta ASR \cong \Delta RBA$
Prove: SABR is a parallelogram



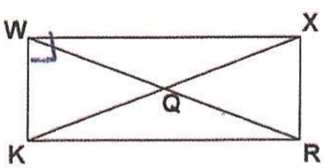
Statements	Reasons
① $\Delta ASR \cong \Delta RBA$	① given
② $\overline{AS} \cong \overline{BR}$, $\overline{AB} \cong \overline{SR}$	② CPCTC
③ SABR is a P-gram	③ A p-gram has 2 pairs of opposite sides congruent

1 30. Given: Quadrilateral $ABCD$, diagonal $\overline{AFEC}$, $\overline{AE} \cong \overline{FC}$, $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$, $\angle 1 \cong \angle 2$
Prove: $ABCD$ is a parallelogram.



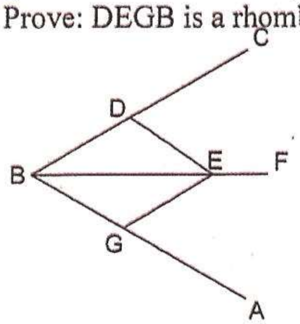
Statements	Reasons
① $\overline{AE} \cong \overline{FC}$	① given
② $\overline{FE} \cong \overline{FE}$	② Reflexive Property
③ $\overline{AF} \cong \overline{EC}$	③ Subtraction property
④ $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$	④ given
⑤ $\angle AFB \cong \angle CED$	⑤ perpendicular lines create ≅ right angles
⑥ $\angle 1 \cong \angle 2$	⑥ given
⑦ $\Delta AFB \cong \Delta CED$	⑦ AAS ≅ AAS
⑧ $\overline{AB} \cong \overline{CD}$, $\angle FAB \cong \angle DCE$	⑧ CPCTC
⑨ $\overline{AB} \parallel \overline{DC}$	⑨ If alternate interior angles are ≅, then the lines are ∥
⑩ ABCD is a P-gram	⑩ A P-gram has 1 pair of opposite sides ≅ and ∥.

8 31. Given: WXRK is a parallelogram, $\overline{KW} \perp \overline{WX}$
Prove: WXRK is a rectangle



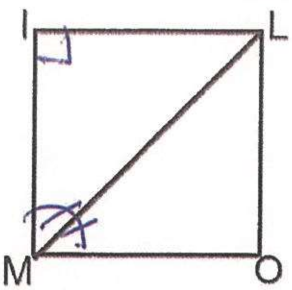
Statements	Reasons
① WXRK is a P-gram	① given
② $\overline{KW} \perp \overline{WX}$	② given
③ $\angle KWX$ is a right angle	③ perpendicular lines form right angles
④ WXRK is a rectangle	④ A rectangle is a parallelogram with a right angle

9 32. Given: BDEG is a parallelogram, $\overline{BF}$ bisects $\angle$ CBA
Prove: DEGB is a rhombus



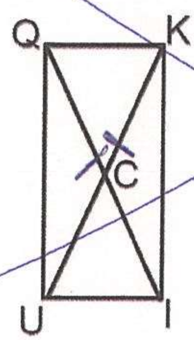
Statements	Reasons
① BDEG is a parallelogram	① given
② $\overline{BF}$ bisects $\angle CBA$	② given
③ DEGB is a rhombus	③ A rhombus is a parallelogram whose diagonals bisect their angles

10 33. Given: MILO is a parallelogram, $\angle IML \cong \angle OML$, $\overline{MI} \perp \overline{IL}$
Prove: MILO is a square



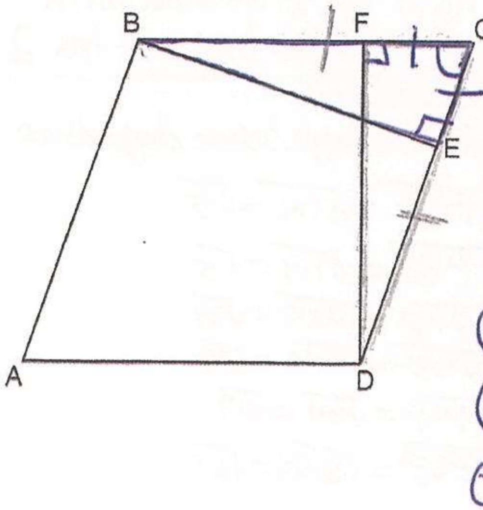
Statements	Reasons
① MILO is a parallelogram	① given
② $\angle IML \cong \angle OML$	② given
③ $\overline{LM}$ bisects $\angle IMO$	③ An angle bisector creates 2 ≅ angles
④ $\overline{MI} \perp \overline{IL}$	④ given
⑤ $\angle MIL$ is a right angle	⑤ Perpendicular lines form right angles
⑥ MILO is a square	⑥ A square is a p-gram that has diagonals that bisect the angles and a right angle

34. Given: QUIK is a parallelogram, $\overline{QI} \cong \overline{KU}$
Prove: QUIK is a rectangle



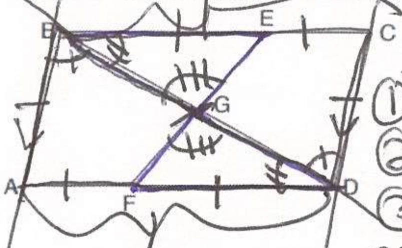
Statements	Reasons
① QUIK is a p-gram	① given
② $\overline{QI} \cong \overline{KU}$	② given
③ QUIK is a rectangle	③ A parallelogram with congruent diagonals is a rectangle.

11 35. In the diagram of parallelogram $ABCD$ below, $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$, $\overline{CE} \cong \overline{CF}$.
Prove $ABCD$ is a rhombus.



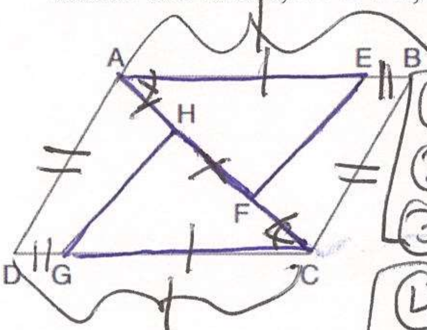
Statements	Reasons
① Parallelogram ABCD	① given
② $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$	② given
③ $\angle DFC \cong \angle BEC$	③ Perpendicular lines create 2 ≅ right angles
④ $\overline{CE} \cong \overline{CF}$	④ given
⑤ $\angle BCD \cong \angle BCD$	⑤ Reflexive Property
⑥ $\triangle BCE \cong \triangle DCF$	⑥ ASA ≅ ASA
⑦ $\overline{BC} \cong \overline{CD}$	⑦ CPCTC
⑧ ABCD is a rhombus	⑧ A rhombus is a parallelogram with consecutive sides ≅

12. In quadrilateral $ABCD$, E and F are points on $\overline{BC}$ and $\overline{AD}$, respectively, and $\overline{BGD}$ and $\overline{EGF}$ are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$. Prove: $\overline{FG} \cong \overline{EG}$ $\triangle BEG \cong \triangle DFG$



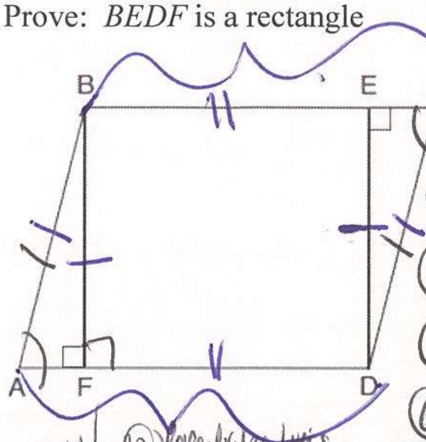
Statements	Reasons
① $\angle ABG \cong \angle CDG$	① given
② $\overline{AB} \cong \overline{CD}$	② given
③ $\overline{BD} \cong \overline{BD}$	③ Reflexive Property
④ $\triangle ABD \cong \triangle CDB$	④ SAS
⑤ $BA \parallel CD$	⑤ Parallel lines cut by a transversal create congruent alternate interior angles
⑥ ABCD is a parallelogram	⑥ A parallelogram has 1 pair of opposite sides congruent and parallel
⑦ $\overline{BC} \cong \overline{AD}$	⑦ A parallelogram has opposite sides congruent
⑧ $\overline{CE} \cong \overline{AF}$	⑧ given
⑨ $\overline{BE} \cong \overline{FD}$	⑨ subtraction property
⑩ $\angle CBD \cong \angle ADB$	⑩ CPCTC
⑪ $\angle BGE \cong \angle DGF$	⑪ vertical angles are congruent
⑫ $\triangle BEG \cong \triangle DFG$	⑫ AAS
⑬ $\overline{FG} \cong \overline{EG}$	⑬ CPCTC

13. In the diagram of quadrilateral $ABCD$ with diagonal $\overline{AC}$ shown below, segments GH and EF are drawn, $\overline{AE} \cong \overline{CG}$, $\overline{BE} \cong \overline{DG}$, $\overline{AH} \cong \overline{CF}$, and $\overline{AD} \cong \overline{CB}$. Prove: $\overline{EF} \cong \overline{GH}$ $\triangle AEG \cong \triangle CGH$



Statements	Reasons
① $\overline{AE} \cong \overline{CG}$	① given
② $\overline{BE} \cong \overline{DG}$	② given
③ $\overline{AB} \cong \overline{DC}$	③ Addition Property
④ $\overline{AH} \cong \overline{CF}$	④ given
⑤ $\overline{HF} \cong \overline{HF}$	⑤ Reflexive Property
⑥ $\overline{AF} \cong \overline{HC}$	⑥ Addition Property
⑦ $\overline{AD} \cong \overline{CB}$	⑦ given
⑧ ABCD is a parallelogram	⑧ A parallelogram has 2 pairs of opposite sides congruent
⑨ $\overline{AB} \parallel \overline{DC}$	⑨ A parallelogram has opposite sides parallel.
⑩ $\angle EAF \cong \angle HCG$	⑩ Parallel lines cut by a transversal create congruent alternate interior angles
⑪ $\triangle AEG \cong \triangle CGH$	⑪ SAS
⑫ $\overline{EF} \cong \overline{GH}$	⑫ CPCTC

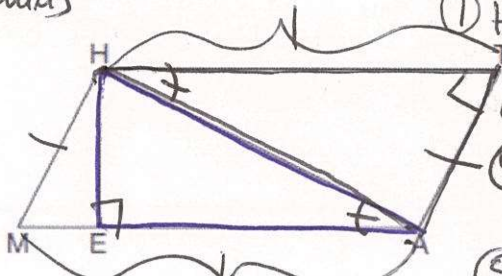
14. Given: Parallelogram $ABCD$, $\overline{BF} \perp \overline{AFD}$, and $\overline{DE} \perp \overline{BEC}$
Prove: $BEDF$ is a rectangle



Statements	Reasons
① Parallelogram ABCD	① given
② ∠BAF ≅ ∠BCD	② A P-gram has opposite ∡ congruent
③ $\overline{BA} \cong \overline{CD}$	③ A P-gram has opposite sides ≅
④ $\overline{BF} \perp \overline{AFD}$, $\overline{DE} \perp \overline{BEC}$	④ given
⑤ ∠BFA ≅ ∠CED	⑤ Perpendicular lines create ≅ right ∡
⑥ △BFA ≅ △DEC	⑥ AAS
⑦ $\overline{BC} \cong \overline{AD}$	⑦ A P-gram has opposite sides ≅
⑧ $\overline{EC} \cong \overline{AF}$	⑧ CPCTC
⑨ $\overline{BE} \cong \overline{FD}$	⑨ Subtraction Property
⑩ $\overline{BF} \cong \overline{ED}$	⑩ CPCTC
⑪ BEDF is a P-gram	⑪ A Parallelogram has 2 pairs of opposite sides congruent
⑫ ∠BFD is a right angle	⑫ Perpendicular lines form right angles
⑬ BEDF is a rectangle	⑬ A rectangle is a Parallelogram with a right angle

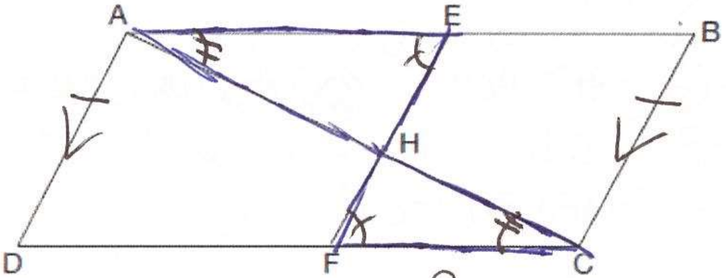
15. Given: Quadrilateral $MATH$, $\overline{HM} \cong \overline{AT}$, $\overline{HT} \cong \overline{AM}$, $\overline{HE} \perp \overline{MEA}$, and $\overline{HA} \perp \overline{AT}$.
Prove: $TA \bullet HA = HE \bullet TH$

work backwards



Statements	Reasons
① $\overline{HM} \cong \overline{AT}$, $\overline{HT} \cong \overline{AM}$	① given
② MATH is a P-gram	② A P-gram has 2 pairs of opp sides ≅
③ $\overline{HT} \parallel \overline{MA}$	③ A P-gram has opposite sides ∥
④ ∠THA ≅ ∠HAE	④ Parallel lines cut by a transversal create congruent alternate interior angles
⑤ $\overline{HE} \perp \overline{MEA}$, $\overline{HA} \perp \overline{AT}$	⑤ given
⑥ ∠HEA ≅ ∠HTA	⑥ Perpendicular lines form congruent right angles
⑦ △THA ~ △EAH	⑦ AA
⑧ $\frac{TA}{TH} = \frac{HE}{HA}$	⑧ CSSTP
⑨ $TA \cdot HA = HE \cdot TH$	⑨ cross products are equal

16. Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$. Prove: $(EH)(CH) = (FH)(AH)$ work backwards



Statements	Reasons
1) $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$	1) given
2) $\overline{AD} \parallel \overline{BC}$	2) Transitive Property
3) $\overline{AD} \cong BC$	3) given
4) ABCD is a parallelogram	4) A parallelogram has 1 pair of opposite sides congruent and parallel.
5) $\overline{AB} \parallel \overline{DC}$	5) A parallelogram has opposite sides parallel
6) $\angle AEH \cong \angle CFH$ $\angle EAH \cong \angle HCF$	6) Parallel lines cut by a transversal create congruent alternate interior angles
7) $\triangle EHA \sim \triangle FHC$	7) AA
8) $\dfrac{EH}{AH} = \dfrac{FH}{CH}$	8) CSSTP
9) $(EH)(CH) = (FH)(AH)$	9) cross products are equal

Coordinate Geometry Proofs

Distance (Length) = $\sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

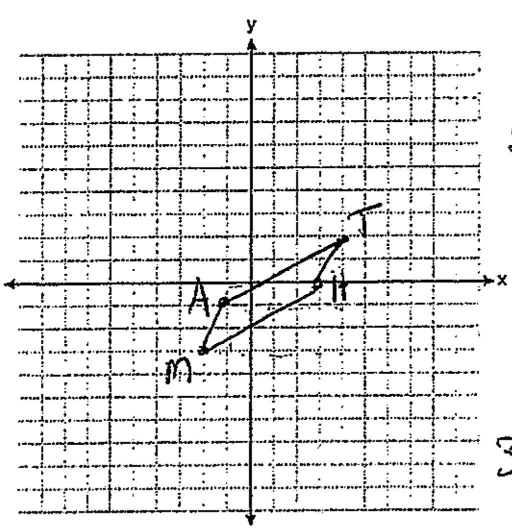
Slope = $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Midpoint = (*average x, average y*) = $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

How do you prove...?

...an **isosceles triangle**? (2 Distances)
Two Congruent Sides
. ... a **right triangle**? (3 Distances)
Show the sides fit into Pythagorean Theorem
... a **parallelogram**? (4 Distances)
Two Pairs of Opposite Sides Congruent
... a **rhombus**? (4 Distances)
All Sides Congruent
... a **rectangle**? (6 Distances)
1) Two Pairs of Opposite Sides Congruent
2) Diagonals Congruent
... a **square**? (6 Distances)
1) All Sides Congruent
2) Diagonals Congruent
...a **trapezoid**? (4 Slopes)
1) 1 pair of opposite sides parallel
2) 1 pair of opposite sides not parallel
...an **isosceles trapezoid**? (4 Slopes, 2 Distances)
1) 1 pair of opposite sides parallel
2) 1 pair of opposite sides not parallel
3) Congruent Legs

1. Graph the quadrilateral MATH: M(-2, -3) A(-1, -1) T(4, 2) H(3, 0). Prove that MATH **IS** a parallelogram but is **NOT** a rectangle.

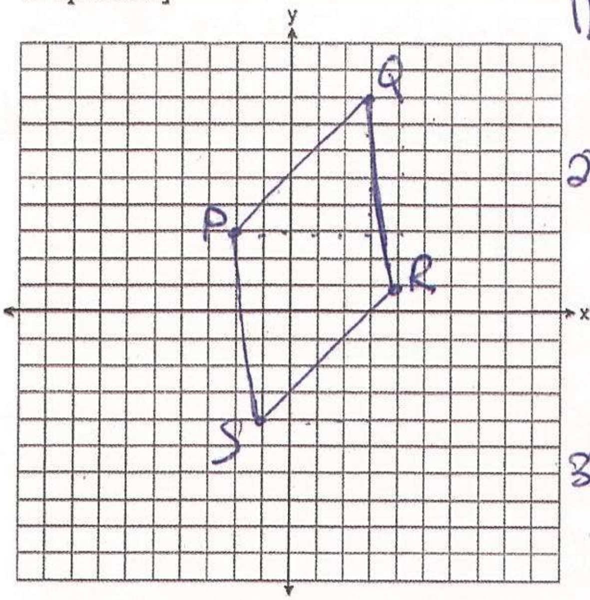


1) Math is a parallelogram because it has 2 pairs of opposite sides $\cong$
It is not a rectangle because diagonals are not $\cong$

2) $d\overline{MA} = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$
$d\overline{TH} = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$
$d\overline{AT} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$
$d\overline{MH} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$
$d\overline{AH} = \sqrt{4^2+1^2} = \sqrt{16+1} = \sqrt{17}$
$d\overline{MT} = \sqrt{6^2+5^2} = \sqrt{36+25} = \sqrt{61}$

3) $\overline{MA} \cong \overline{TH}$, $\overline{AT} \cong \overline{MH}$ because they have the same distance
$\overline{AH} \not\cong \overline{MT}$ because they don't have the same distance

2. Quadrilateral *PQRS* has vertices $P(-2,3)$, $Q(3,8)$, $R(4,1)$, and $S(-1,-4)$. Prove that *PQRS* is a rhombus. Prove that *PQRS* is *not* a square. [The use of the set of axes below is optional.]



1) Quadrilateral PQRS is a rhombes because all sides are congruent. It is not a square because diagonals are not congruent.

2) $d\overline{PQ} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$d\overline{QR} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$

$d\overline{RS} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$d\overline{SP} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$

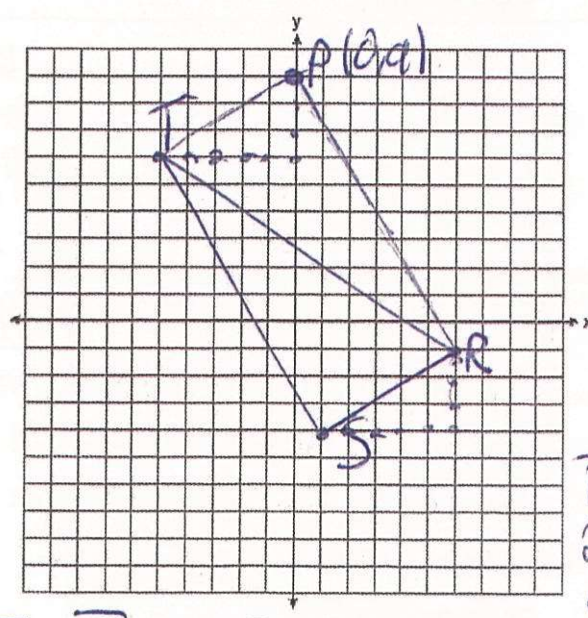
$d\overline{PR} = \sqrt{6^2+2^2} = \sqrt{36+4} = \sqrt{40}$

$d\overline{QS} = \sqrt{4^2+12^2} = \sqrt{16+144} = \sqrt{160}$

3) $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$ because they have the same distance

$\overline{PR} \ncong \overline{QS}$ because they don't have the same distance.

3. In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle. State the coordinates of point *P* such that quadrilateral *RSTP* is a rectangle. Prove that your quadrilateral *RSTP* is a rectangle. [The use of the set of axes below is optional.]



1) $\triangle RST$ is a right triangle because its sides fit into Pythagorean Theorem.

2) $d\overline{RS} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\overline{ST} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\overline{TR} = \sqrt{11^2+7^2} = \sqrt{121+49} = \sqrt{170}$

3) $a^2+b^2=c^2$

$\sqrt{34}^2+\sqrt{136}^2=\sqrt{170}^2$

$170=170$

1) RSTP is a rectangle because it has 2 pairs of opposite sides congruent and diagonals congruent

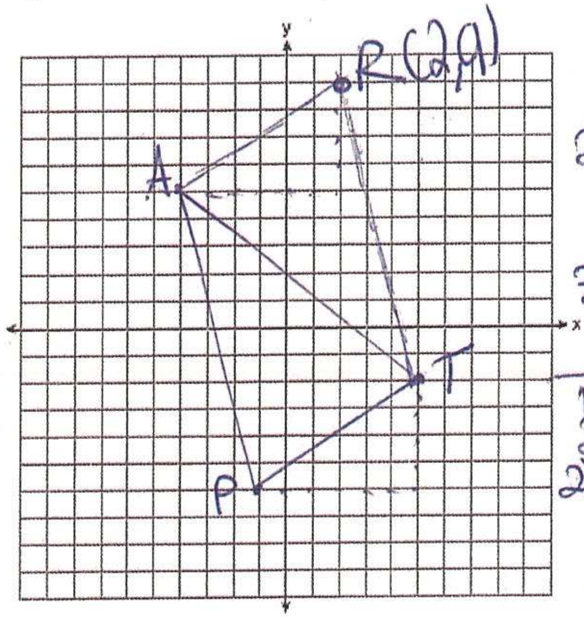
2) $d\overline{TP} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\overline{PR} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\overline{PS} = \sqrt{1^2+13^2} = \sqrt{1+169} = \sqrt{170}$

3) $\overline{RS} \cong \overline{TP}$, $\overline{ST} \cong \overline{PR}$, $\overline{TR} \cong \overline{PS}$ because they have the same distance

4. In the coordinate plane, the vertices of triangle PAT are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes below is optional.] State the coordinates of R so that quadrilateral $PART$ is a parallelogram. Prove that quadrilateral $PART$ is a parallelogram.



1) $\triangle PAT$ is an isosceles triangle because it has two congruent sides

2) $dPA = \sqrt{3^2+11^2} = \sqrt{9+121} = \sqrt{130}$

$dTA = \sqrt{9^2+7^2} = \sqrt{81+49} = \sqrt{130}$

3) $\overline{PA} \cong \overline{TA}$ because they have the same distance

1) PART is a parallelogram because it has 2 pairs of opposite sides congruent

2) $m\overline{AR} = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

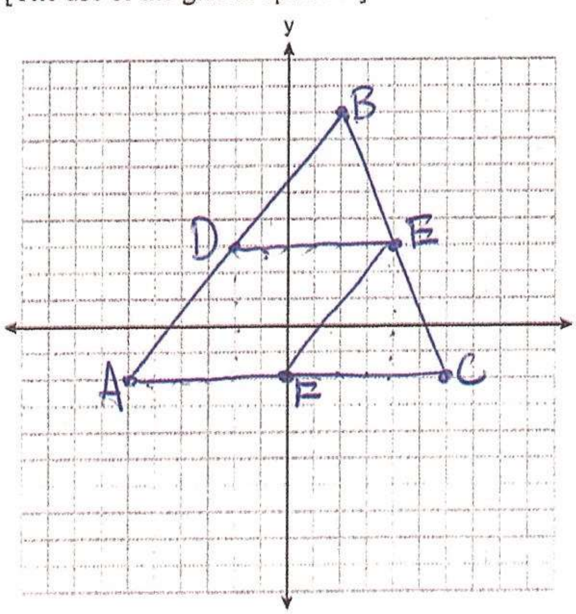
$mRT = \sqrt{3^2+11^2} = \sqrt{9+121} = \sqrt{130}$

$mPT = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

3) $\overline{AR} \cong \overline{PT}$, $\overline{PA} \cong \overline{RT}$ because they have the same distance

5. Given: $\triangle ABC$ with vertices $A(-6,-2)$, $B(2,8)$, and $C(6,-2)$. $\overline{AB}$ has midpoint D, $\overline{BC}$ has midpoint E, and $\overline{AC}$ has midpoint F.
Prove: $ADEF$ is a parallelogram
$ADEF$ is *not* a rhombus
[The use of the grid is optional.]

midpoint = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



F	E	D
midpoint AC	midpoint BC	midpoint AB
$\left(\frac{-6+6}{2}, \frac{-2+-2}{2}\right)$	$\left(\frac{2+6}{2}, \frac{8+-2}{2}\right)$	$\left(\frac{-6+2}{2}, \frac{-2+8}{2}\right)$
$\left(\frac{0}{2}, \frac{-4}{2}\right)$	$\left(\frac{8}{2}, \frac{6}{2}\right)$	$\left(\frac{-4}{2}, \frac{6}{2}\right)$
$(0,-2)$	$(4,3)$	$(-2,3)$

1) ADEF is a parallelogram because it has 2 pairs of opposite sides congruent. It is not a rhombus because not all sides are congruent.

2) $d\overline{AD} = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$

$d\overline{DE} = 6$

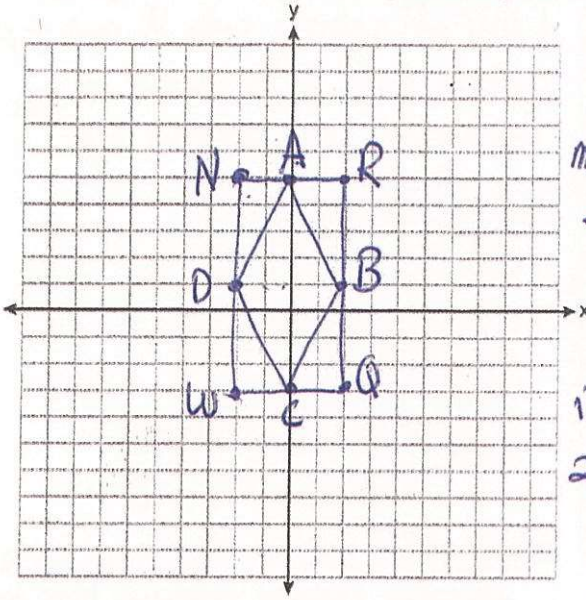
$d\overline{EF} = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$

$d\overline{FA} = 6$

3) $\overline{AD} \cong \overline{EF}$, $\overline{DE} \cong \overline{FA}$ because they have the same distance

$\overline{AD} \not\cong \overline{DE}$ because they don't have the same distance

6. The vertices of rectangle NRQW are N(-2,5), R(2,5), Q(2,-3), and W(-2,-3). If A is the midpoint $\overline{NR}$, B is the midpoint of $\overline{RQ}$, C is the midpoint of $\overline{QW}$, and D is the midpoint of $\overline{WN}$, prove that ABCD is a ~~parallelogram but *not*~~ a rhombus.



~~statements~~

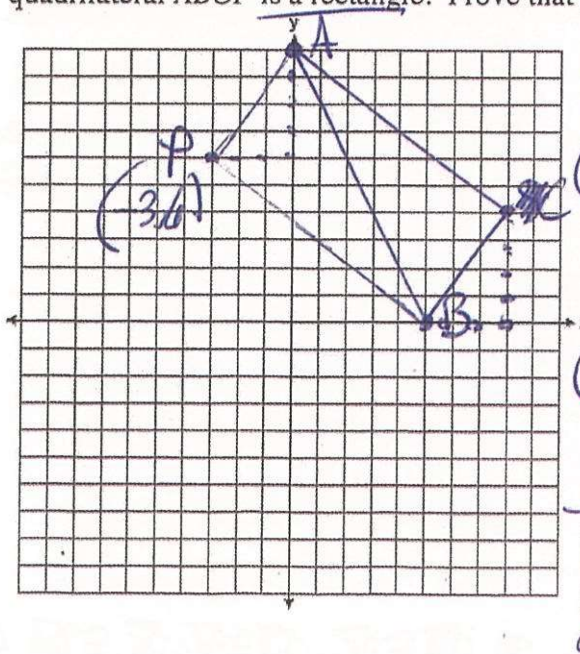
A	B	C	C
midpoint $\overline{NR}$	midpoint $\overline{RQ}$	midpoint $\overline{QW}$	midpoint
$\frac{-2+2}{2}, \frac{5+5}{2}$	$\frac{2+2}{2}, \frac{5+-3}{2}$	$\frac{2+-2}{2}, \frac{-3+-3}{2}$	$\frac{-2+-2}{2}, \frac{5}{}$
$\frac{0}{2}, \frac{10}{2}$	$\frac{4}{2}, \frac{2}{2}$	$\frac{0}{2}\ \frac{-6}{2}$	$\frac{-4}{2}, \frac{2}{2}$
(0,5)	(2,1)	(0,-3)	(-2,1)

1) ABCD is a rhombus because all sides are congruent

2) $d\overline{DA} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\overline{AB} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\overline{BC} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\overline{CD} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$

3) $\overline{DA} \cong \overline{AB} \cong \overline{BC} \cong \overline{CD}$ because they have the same distance

7. In the coordinate plane, the vertices of triangle ABC are A(0,10) B(5,0) and C(8,4). Prove that Triangle ABC is a right triangle. State the coordinates of point *P* such that quadrilateral *ABCP* is a rectangle. Prove that your quadrilateral *ABCP* is a rectangle.



① $\triangle ABC$ is a right triangle because its sides fit into Pythagorean Theorem

② $d\overline{BC} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$
$d\overline{CA} = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$
$d\overline{AB} = \sqrt{5^2+10^2} = \sqrt{25+100} = \sqrt{125}$

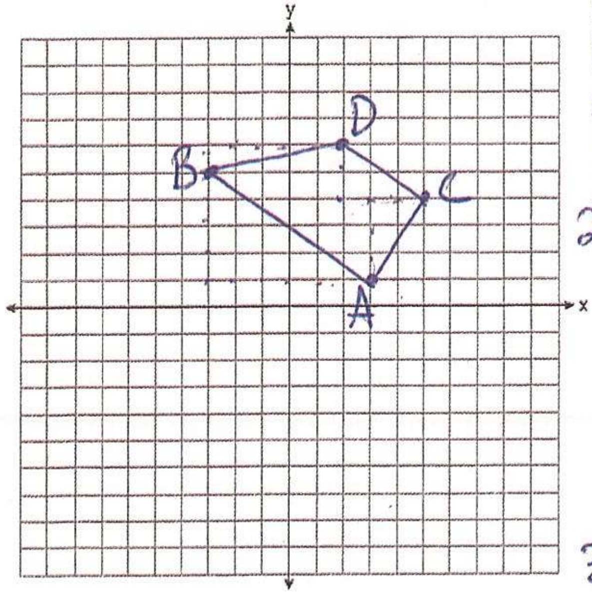
③ $a^2+b^2=c^2$
$\sqrt{25}^2+\sqrt{100}^2 = \sqrt{125}^2$
$125 = 125$ ✓

1) ABCP is a rectangle because it has 2 pairs of opposite sides congruent and diagonals congruent

2) $dPA = \sqrt{3^2+4^2} = \cancel{\sqrt{\quad}}\ \sqrt{9+16} = \sqrt{25}$
$d\overline{PB} = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$
$dPC = \sqrt{11^2+2^2} = \sqrt{121+4} = \sqrt{125}$

3) $\overline{PA} \cong \overline{BC}, \overline{PB} \cong \overline{AC}, \overline{PC} \cong \overline{AB}$ because they have the same distance

8. Quadrilateral ABCD has vertices A(3,1) B(-3,5) C(5,4) and D(2,6). Prove quadrilateral ABCD is a trapezoid but *not* an isosceles trapezoid.



1) ABCD is a trapezoid because it has 1 pair of opposite sides $\parallel$ and 1 pair of opposite sides $\nparallel$. It is not isoscoles because it does not have congruent legs

2) slope $\overline{DC} = \frac{-2}{3}$

slope $\overline{BA} = -\frac{4}{6} = \frac{-2}{3}$

slope $\overline{BD} = \frac{1}{5}$

slope $\overline{CA} = \frac{3}{2}$

$d\overline{BD} = \sqrt{5^2+1^2} = \sqrt{25+1} = \sqrt{26}$

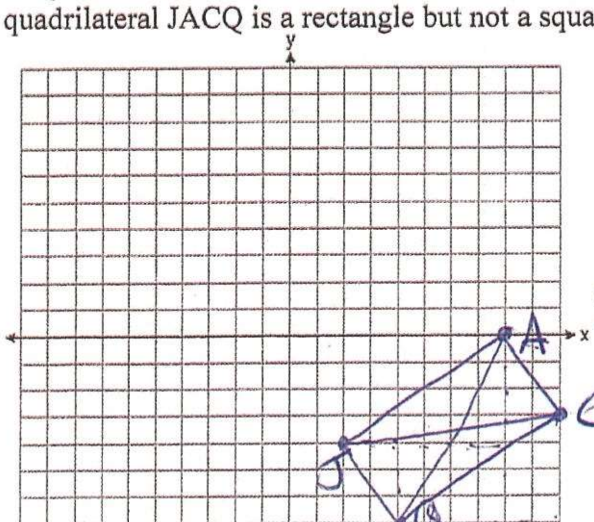
$d\overline{CA} = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$

3) $\overline{DC} \parallel \overline{BA}$ because they have the same slope

$\overline{BD} \nparallel \overline{CA}$ because they don't have the same slope

$\overline{BD} \ncong \overline{CA}$ because they don't have the same distance

9. Quadrilateral JACQ has vertices J(2,-4), A(8,0), C(10,-3), and Q(4,-7). Prove that quadrilateral JACQ is a rectangle but not a square.



1) JACQ is a rectangle because it has 2 pairs of opposite sides $\cong$ and diagonals congruent. It is not a square because all sides are not $\cong$.

2) $dJA = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

$dQC = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

$dJQ = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$

$dAC = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$

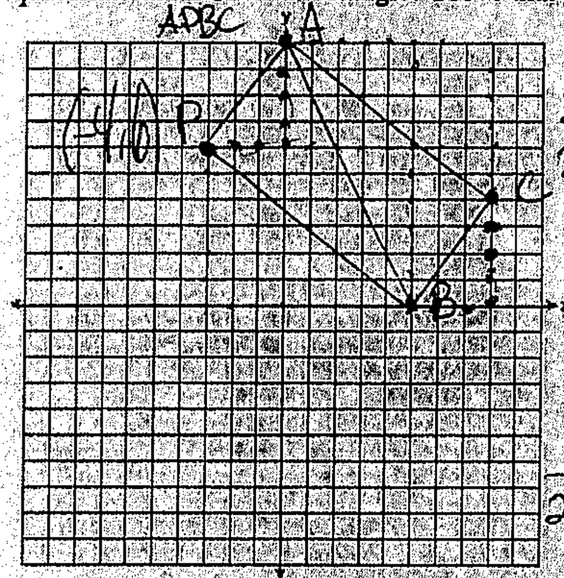
$dJC = \sqrt{8^2+1^2} = \sqrt{64+1} = \sqrt{65}$

$dQA = \sqrt{4^2+7^2} = \sqrt{16+49} = \sqrt{65}$

3) $\overline{JA} \cong \overline{QC}$, $\overline{JQ} \cong \overline{AC}$, $\overline{JC} \cong \overline{QA}$ because they have the same distance

$\overline{JA} \ncong \overline{AC}$ because they do not have the same distance

10. In the coordinate plane, the vertices of triangle ABC are A(0,10) B(5,0) and C(8,4). Prove that Triangle ABC is a right triangle. State the coordinates of point *P* such that quadrilateral ~~*ABCP*~~ APBC is a rectangle. Prove that your quadrilateral ~~*ABCP*~~ APBC is a rectangle.



1) ABC is a right triangle because its sides fit into Pythagorean Theorem.

2) $d\,\overline{AC} = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$

$d\,\overline{BC} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$

$d\,\overline{AB} = \sqrt{5^2+10^2} = \sqrt{25+100} = \sqrt{125}$

3) $a^2+b^2=c^2$

$\sqrt{100}^2+\sqrt{25}^2=\sqrt{125}^2$

$100+25=125$

$125=125$ ✓

1) APBC is a rectangle because it has 2 pairs of opposite sides ≅ and diagonals ≅.

2) $d\,\overline{PB} = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$

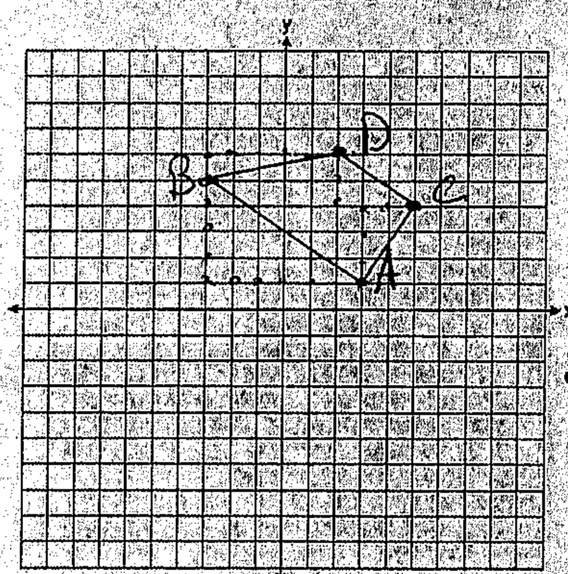
$d\,\overline{PA} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$

$d\,\overline{PC} = \sqrt{11^2+2^2} = \sqrt{121+4} = \sqrt{125}$

3) $\overline{AC} \cong \overline{PB}$, $\overline{AP} \cong \overline{CB}$, $\overline{PC} \cong \overline{AB}$ because they have the same distance.

11. Quadrilateral ABCD has vertices A(3,1) B(-3,5) C(5,4) and D(2,6). Prove quadrilateral ABCD is a trapezoid but *not* an isosceles trapezoid.

$m = \frac{\Delta y}{\Delta x}$



1) ABCD is a trapezoid because it has 1 pair of opposite sides ∥ and 1 pair of opposite sides ∦. It is not isosceles because it does not have congruent legs.

2) slope $\overline{DC} = -\frac{2}{3}$ slope $\overline{BD} = \frac{1}{5}$

slope $\overline{AB} = -\frac{4}{6} = -\frac{2}{3}$ slope $\overline{AC} = \frac{3}{2}$

$d\,\overline{BD} = \sqrt{5^2+1^2} = \sqrt{25+1} = \sqrt{26}$

$d\,\overline{AC} = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$

3) $\overline{DC} \parallel \overline{AB}$ because they have the same slope

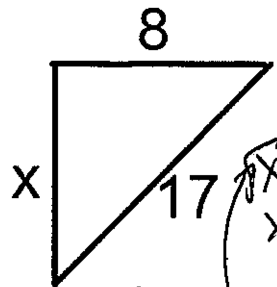
$\overline{BD} \nparallel \overline{AC}$ because they don't have the same slope

$\overline{BD} \ncong \overline{AC}$ because they don't have the same distance.

Hidden Right Triangles

Look out for hidden right triangles where you may need to use $a^2 + b^2 = c^2$

1.



$a^2+b^2=c^2$

$8^2+x^2=17^2$

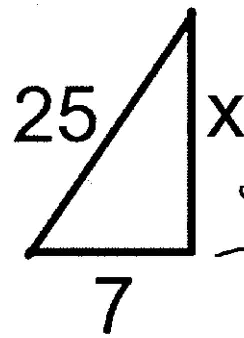
$64+x^2=289$

$-64 \quad -64$

$\sqrt{x^2}=\sqrt{225}$

$x=15$

2.



$a^2+b^2=c^2$

$7^2+x^2=25^2$

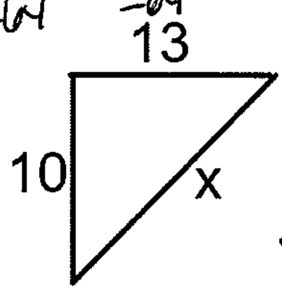
$49+x^2=625$

$-49 \quad -49$

$\sqrt{x^2}=\sqrt{576}$

$x=24$

3.



$a^2+b^2=c^2$

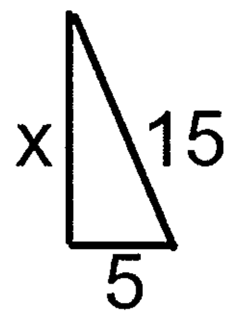
$10^2+13^2=x^2$

$100+169=x^2$

$\sqrt{269}=\sqrt{x^2}$

$16.4=x$

4.



$a^2+b^2=c^2$

$x^2+5^2=15^2$

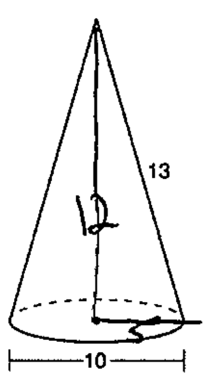
$x^2+\cancel{25}=225$

$-25 \quad -25$

$\sqrt{x^2}=\sqrt{200}$

$x=14.1$

5. In the diagram below, a right circular cone has a diameter of 10 and a slant height of 13. Determine and state the volume of the cone, in terms of π.



$a^2+b^2=c^2$

$5^2+b^2=13^2$

$25+b^2=169$

$-25 \quad -25$

$\sqrt{b^2}=\sqrt{144}$

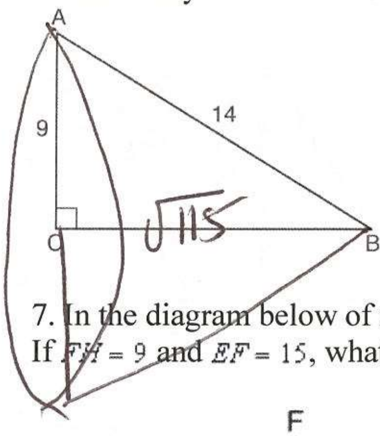
$b=12$

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi(5)^2(12)$

$V=100\pi$

6. In the diagram of right triangle *ABC* shown below, $AB = 14$ and $AC = 9$. What is the volume of the three dimensional object formed when the triangle is continuously rotated about side $\overline{BC}$ to the nearest tenth.



$9^2+b^2=c^2$

$9^2+b^2=14^2$

$81+b^2=196$

$-81 \quad -81$

$\sqrt{b^2}=\sqrt{115}$

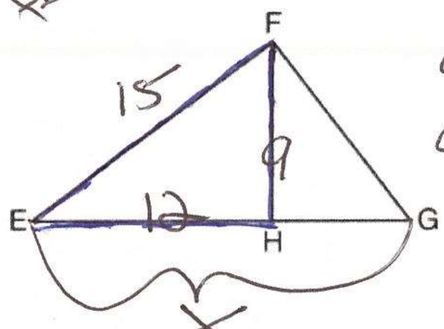
$b=\sqrt{115}$

$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi(9)^2(\sqrt{115})$

$V=909.6$

7. In the diagram below of right triangle *EFG*, altitude $\overline{FH}$ intersects hypotenuse $\overline{EG}$ at *H*. If $FH = 9$ and $EF = 15$, what is *EG*?



$a^2+b^2=c^2$

$a^2+9^2=15^2$

$a^2+81=225$

$-81 \quad -81$

$\sqrt{a^2}=\sqrt{144}$

$a=12$

$\frac{H}{L}=\frac{L}{S}$

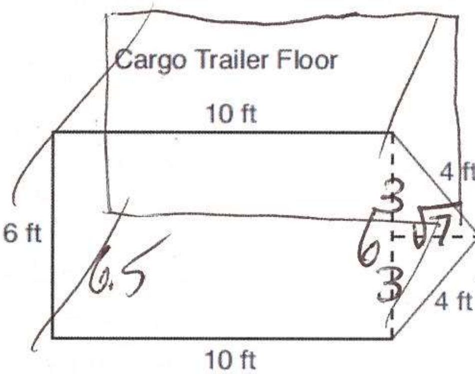
$\frac{x}{15}=\frac{15}{12}$

$\frac{12x}{12}=\frac{225}{12}$

$x=18.75$

8. A cargo trailer, pictured below, can be modeled by a rectangular prism and a triangular prism. Inside the trailer, the rectangular prism measures 6 feet wide and 10 feet long. The walls that form the triangular prism each measure 4 feet wide inside the trailer. The diagram below is of the floor, showing the inside measurements of the trailer.

Cargo Trailer



$9^2+b^2=c^2$

$3^2+b^2=4^2$

$9+b^2=16$

$-9 \quad -9$

$\sqrt{b^2}=\sqrt{7}$

$b=\sqrt{7}$

If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the *nearest cubic foot*?

rectangular prism

$V=lwh$

$V=10(6)(6.5)$

$V=390$

Triangular Prism:

$V=(\frac{1}{2}bh)H$

$V=\frac{1}{2}(6)(\sqrt{7})(6.5)$

$V=51..$

390

$+\ 51..$

442 ft^3

Common Core High School Math Reference Sheet
(Algebra I, Geometry, Algebra II)

CONVERSIONS

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilograms	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

FORMULAS

Triangle	$A = \frac{1}{2}bh$	Pythagorean Theorem	$a^2 + b^2 = c^2$
Parallelogram	$A = bh$	Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Circle	$A = \pi r^2$	Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Circle	$C = \pi d$ or $C = 2\pi r$	Geometric Sequence	$a_n = a_1 r^{n-1}$
General Prisms	$V = Bh$	Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1-r}$ where $r \neq 1$
Cylinder	$V = \pi r^2 h$	Radians	$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$
Sphere	$V = \frac{4}{3}\pi r^3$	Degrees	$1 \text{ degree} = \frac{\pi}{180} \text{ radians}$
Cone	$V = \frac{1}{3}\pi r^2 h$	Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$
Pyramid	$V = \frac{1}{3}Bh$		