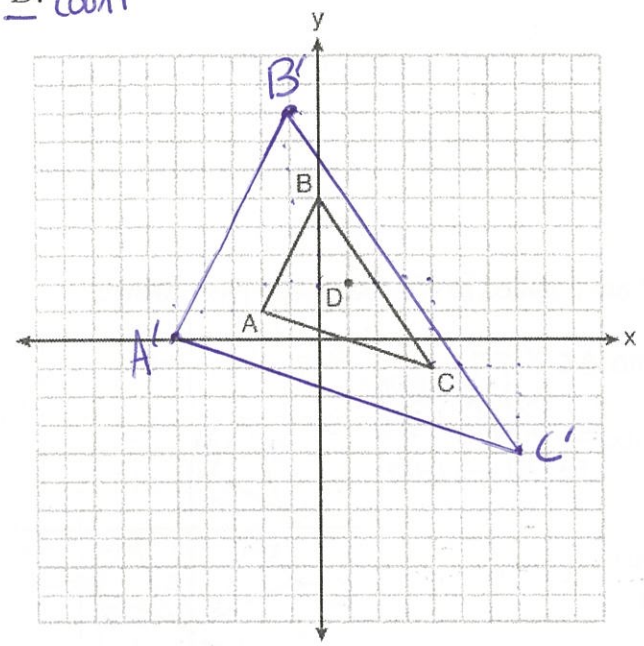
Name Schlansky
Mr. Schlansky

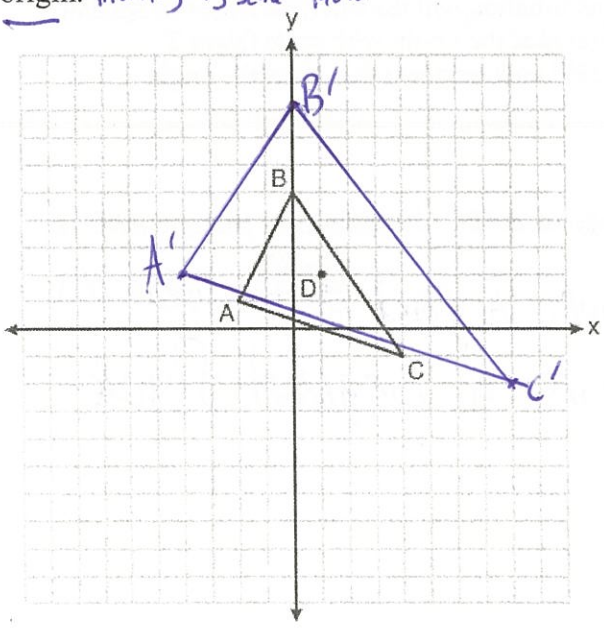
Date _____
Geometry

CCG Common Regents Test

1. Triangle ABC and point $D(1, 2)$ are graphed on the set of axes below.
Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point D. count

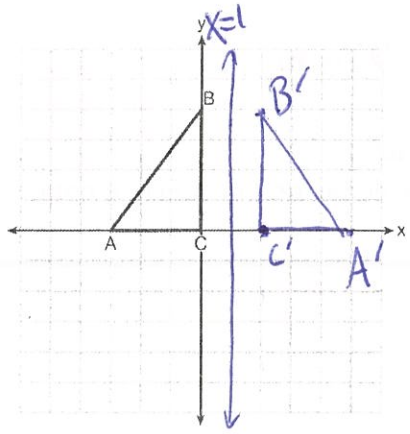


2. Triangle ABC and point $D(1, 2)$ are graphed on the set of axes below.
Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at the origin. multiply by scale factor



A(-2,1) x2 → (-4,2)
B(0,4) → (0,8)
C(4,-1) → (8,-2)

3. Triangle ABC is graphed on the set of axes below. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection over the line $x = 1$.

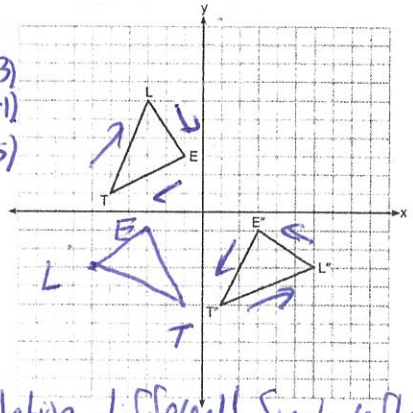


4. On the set of axes below, $\triangle LET$ and $\triangle L"E"T"$ are graphed in the coordinate plane where $\triangle LET \cong \triangle L"E"T"$.

Which sequence of rigid motions maps $\triangle LET$ onto $\triangle L"E"T"$?

1) a reflection over the *y*-axis followed by a reflection over the *x*-axis
2) a rotation of 180° about the origin
3) a rotation of 90° counterclockwise about the origin followed by a reflection over the *y*-axis
4) a reflection over the *x*-axis followed by a rotation of 90° clockwise about the origin

L(-3,6) → (-6,-3)
E(-1,3) → (-3,-1)
T(-5,1) → (-1,-5)



Orientation different! Single reflection

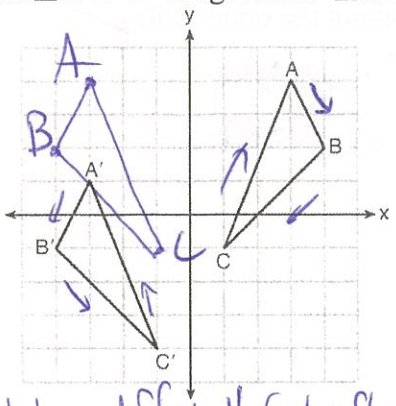
5. If $\triangle A'B'C'$ is the image of $\triangle ABC$, under which transformation will the triangles *not* be congruent?

1) reflection over the *x*-axis
2) translation to the left 5 and down 4
3) dilation centered at the origin with scale factor 2
4) rotation of 270° counterclockwise about the origin

6. After a reflection over a line, $\Delta A'B'C'$ is the image of ΔABC. Explain why triangle ABC is congruent to triangle $\Delta A'B'C'$.

A reflection is a rigid motion. A rigid motion preserves size and angle measure producing a congruent figure

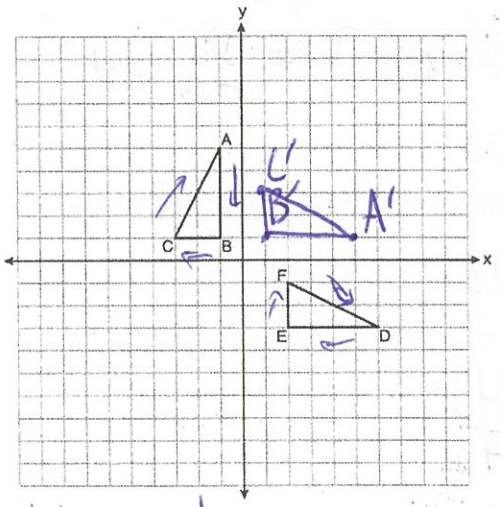
7. As graphed on the set of axes below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a sequence of transformations. Identify the sequences of transformations.
Is $\triangle A'B'C'$ congruent to $\triangle ABC$? Use the properties of rigid motion to explain your answer.



1) Reflect $\triangle ABC$ over the y-axis followed by a translation down 3 units.
2) Yes, a reflection and translation are rigid motions.
3) A rigid motion preserves size and angle measure producing a congruent figure.

orientation different! Single reflection

8. On the set of axes below, $\triangle ABC$ and $\triangle DEF$ are graphed. Describe a sequence of rigid motions that would map $\triangle ABC$ onto $\triangle DEF$.



1) rotate $\triangle ABC$ 270° counter-clockwise centered at the origin followed by a translation 1 unit right and 4 units down.

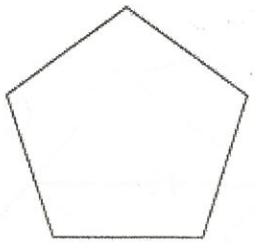
$A(-1,5) \xrightarrow{y,-x} (5,1)$
$B(-1,1) \rightarrow (1,1)$
$C(-3,1) \rightarrow (1,3)$

orientation same!
No reflection!

9. The regular polygon below is rotated about its center.
Which angle of rotation will carry the figure onto itself?

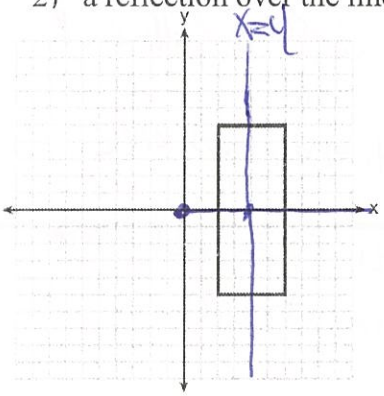
1) 60°
2) 108°
3) 216°
4) 540°

$\frac{360}{n}$ $\frac{360}{5} = 72$



10. As shown in the graph below, the quadrilateral is a rectangle.
Which transformation would *not* map the rectangle onto itself? not center of shape

1) a reflection over the x-axis ✓
2) a reflection over the line $x = 4$ ✓
3) a rotation of 180° about the origin ✗
4) a rotation of 180° about the point $(4, 0)$ ✓



line of reflection = line of symmetry
center of rotation = center of shape

11. After a dilation with center $(0, 0)$, the image of $\overline{DB}$ is $\overline{D'B'}$. If $DB = 4.5$ and $D'B' = 18$, the scale factor of this dilation is

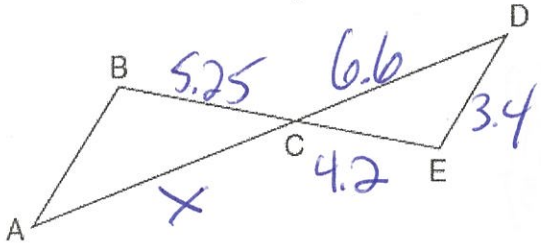
1) $\frac{1}{5}$
2) 5
3) $\frac{1}{4}$
4) 4 ✓

scale factor = $\frac{\text{image}}{\text{original}}$

scale factor = $\frac{18}{4.5} = 4$

12. In the diagram below, $\overline{AD}$ intersects $\overline{BE}$ at C, and $\overline{AB} \parallel \overline{DE}$.

If $CD = 6.6$ cm, $DE = 3.4$ cm, $CE = 4.2$ cm, and $BC = 5.25$ cm, what is the length of $\overline{AC}$, to the *nearest hundredth of a centimeter*?



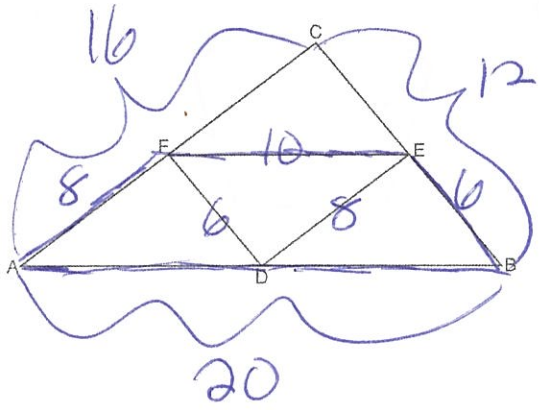
$\frac{X}{6.6} = \frac{5.25}{4.2}$

$\frac{4.2x}{4.2} = \frac{34.65}{4.2}$

$X = 8.25$

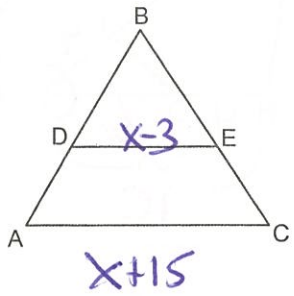
13. In the diagram of $\triangle ABC$ shown below, D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$, and F is the midpoint of $\overline{AC}$. If $AB = 20$, $BC = 12$, and $AC = 16$, what is the perimeter of trapezoid $ABEF$?

2(midsegment) = opposite side



20 + 6 + 8 + 6 = 44

14. D and E are midpoints of $\overline{AB}$ and $\overline{BC}$ respectively. If $\overline{AC} = x + 15$ and $\overline{DE} = x - 3$, find the measure of $\overline{DE}$.



2(midsegment) = opposite side

$2(x-3) = x+15$

$2x-6 = x+15$

$-x \quad -x$

$x-6 = 15$

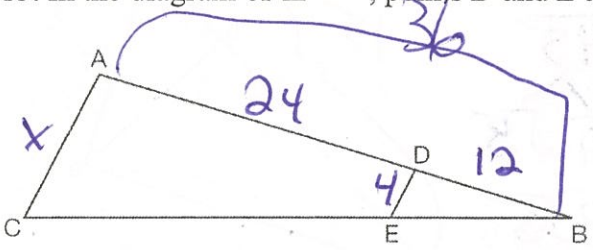
$+6 \quad +6$

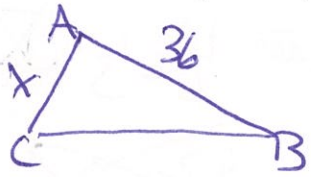
$x = 21$

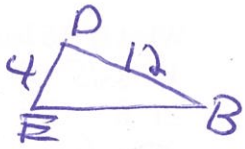
$DE = 21-3$

$DE = 18$

15. In the diagram of $\triangle ABC$, points D and E are on $\overline{AB}$ and $\overline{CB}$, respectively, such that $\overline{AC} \parallel \overline{DE}$.







If $AD = 24$, $DB = 12$, and $DE = 4$, what is the length of $\overline{AC}$?

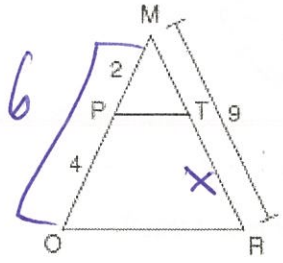
1) 8
2) 12
3) 16
4) 72

$\frac{x}{4} = \frac{36}{12}$

$\frac{12x}{12} = \frac{144}{12}$

$x = 12$

16. Given $\triangle MRO$ shown below, with trapezoid $PTRO$, $MR = 9$, $MP = 2$, and $PO = 4$.



$\frac{\text{to Hom}}{\text{to Hom}} = \frac{\text{side}}{\text{side}}$

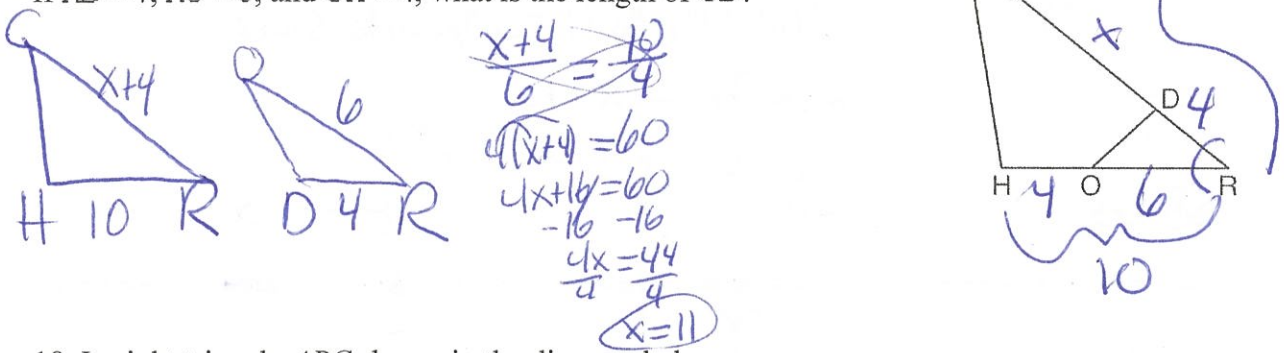
$\frac{4}{x} = \frac{6}{9}$ $\quad \frac{6x}{6} = \frac{36}{6}$

$x = 6$

What is the length of $\overline{TR}$?

1) 4.5
2) 5
3) 3
4) 6

17. In triangle CHR, O is on $\overline{HR}$, and D is on $\overline{CR}$ so that $\angle H \cong RDO$. If $RD = 4$, $RO = 6$, and $OH = 4$, what is the length of $\overline{CD}$?



$\frac{X+4}{6} = \frac{10}{4}$

$4(X+4) = 60$

$4X+16 = 60$

$-16 \quad -16$

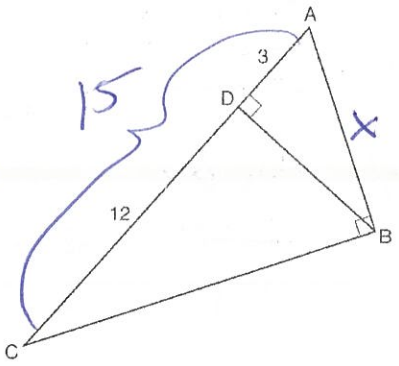
$\frac{4x}{4} = \frac{44}{4}$

$x = 11$

18. In right triangle ABC shown in the diagram below, altitude $\overline{BD}$ is drawn to hypotenuse $\overline{AC}$, $CD = 12$, and $AD = 3$.

What is the length of $\overline{AB}$?

1) $5\sqrt{3}$
2) 6
3) $3\sqrt{5}$
4) 9



$\frac{\#}{L} = \frac{L}{5}$ $\quad X^2 = \sqrt{45}$

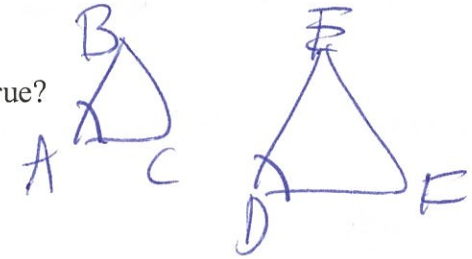
$\sqrt{9}\sqrt{5}$

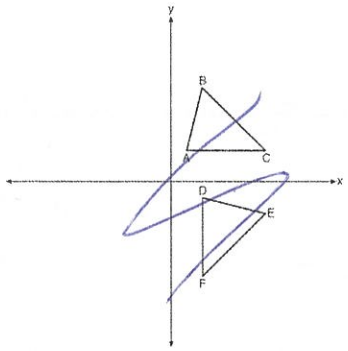
$\frac{15}{x} = \frac{x}{3}$ $\quad x = 3\sqrt{5}$

19. The image of $\triangle ABC$ after a rotation of 90° clockwise about the origin is $\triangle DEF$, as shown below.

Which statement is true?

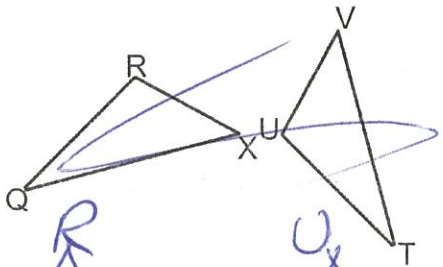
1) $\overline{BC} \cong \overline{DE}$
2) $\overline{AB} \cong \overline{DF}$
3) $\angle C \cong \angle E$
4) $\angle A \cong \angle D$

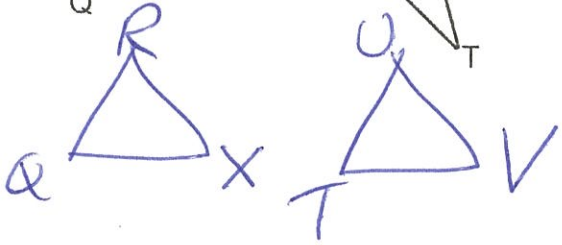




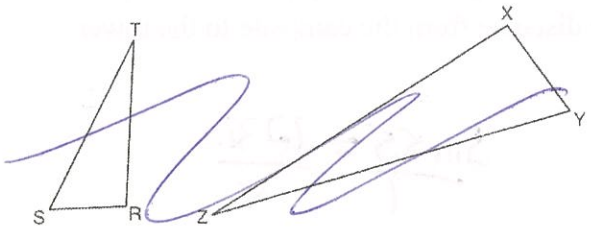
20. In the diagram below, $\Delta QRX \sim \Delta TUV$. Which of the following statements is *not* true?

1) $\frac{QR}{TU} = \frac{QX}{TV}$
2) $\frac{\angle X}{\angle V} = \frac{\angle Q}{\angle T}$
3) $\frac{RX}{UV} = \frac{VT}{XQ}$
4) $\frac{QX}{QR} = \frac{TV}{TU}$





21. Triangles RST and XYZ are drawn below. If $RS = 6$, $ST = 14$, $XY = 9$, $YZ = 21$, and $\angle S \cong \angle Y$, is $\triangle RST$ similar to $\triangle XYZ$? Justify your answer.

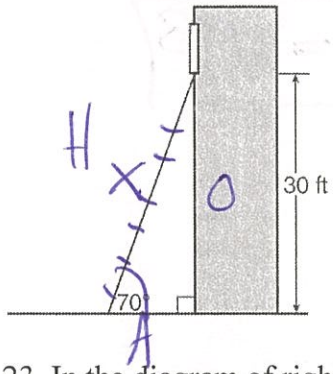


$\frac{6}{9} = \frac{14}{21}$

$\frac{2}{3} = \frac{2}{3}$ ✓

SAS

22. A carpenter leans an extension ladder against a house to reach the bottom of a window 30 feet above the ground. As shown in the diagram below, the ladder makes a 70° angle with the ground. To the *nearest foot*, determine and state the length of the ladder.



$\sin\theta = \frac{O}{H}$ $\quad$ SOH CAH TOA

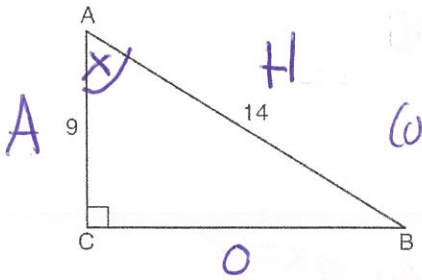
$\sin 70 = \frac{30}{x}$

$x = 32$

$x \sin 70 = 30$

$\frac{x \sin 70}{\sin 70} = \frac{30}{\sin 70}$

23. In the diagram of right triangle ABC shown below, $AB = 14$ and $AC = 9$. What is the measure of $\angle A$, to the *nearest degree*?



$\cos\theta = \frac{A}{H}$ $\quad x = \cos^{-1}\left(\frac{9}{14}\right)$

$\cos^{-1}$

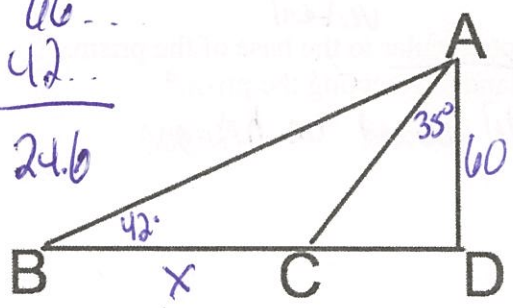
$\cos x = \frac{9}{14}$ $\quad x = 50°$

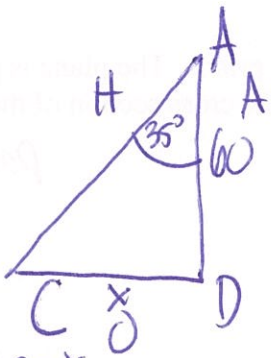
24. In the diagram below, $m\angle CAD = 35$, $m\angle ABD = 42$, and $m\overline{AD} = 60$. Find to the nearest tenth, $m\overline{BC}$.

66..

−42..

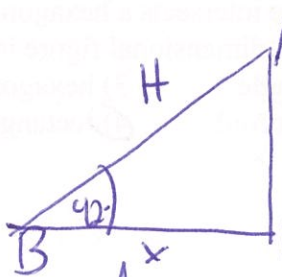
24.6





$\tan 35 = \frac{x}{60}$

$x = 60 \tan 35 = 42...$



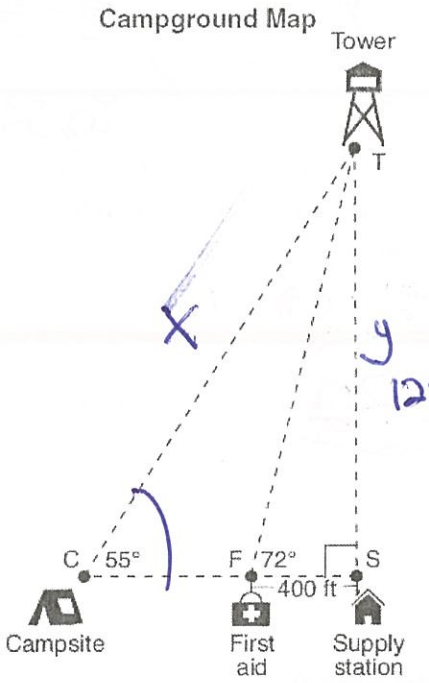
$\tan 42 = \frac{60}{x}$

$x \tan 42 = 60$

$\frac{x \tan 42}{\tan 42} = \frac{60}{\tan 42}$

$x = 66...$

25. The map of a campground is shown below. Campsite C, first aid station F, and supply station S lie along a straight path. The path from the supply station to the tower, T, is perpendicular to the path from the supply station to the campsite. The length of path $\overline{FS}$ is 400 feet. The angle formed by path $\overline{TF}$ and path $\overline{FS}$ is 72°. The angle formed by path $\overline{TC}$ and path $\overline{CS}$ is 55°. Determine and state, to the *nearest foot*, the distance from the campsite to the tower.



$\tan 72 = \frac{y}{400}$

$y = 400 \tan 72$

$y = 1231$

$\sin 55 = \frac{1231}{x}$

$x \sin 55 = 1231$

$x = 1503$

26. In right triangle ABC, $m\angle C = 90°$. If $\cos B = \frac{5}{13}$, which function also equals $\frac{5}{13}$?

1) $\tan A$
2) $\tan B$
3) $\sin A$
4) $\sin B$

$\sin A = \cos B$

27. If $\sin(2x + 7)° = \cos(4x - 7)°$, what is the value of x?

$A + B = 90$

$2x + 7 + 4x - 7 = 90$

$\frac{6x}{6} = \frac{90}{6}$

$x = 15$

1) 7
2) 15
3) 21
4) 30

28. A plane intersects a hexagonal prism. The plane is perpendicular to the base of the prism. Which two-dimensional figure is the cross section of the plane intersecting the prism?

1) triangle
2) trapezoid
3) hexagon
4) rectangle

vertical

parallel would be hexagon.

29. Find the volume of a sphere with a diameter of 8 in rounded to the nearest hundredth.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(4)^3$

$V = 268.08$

30. The volume of a cylinder is 12,566.4 cm^3. The height of the cylinder is 8 cm. Find the radius of the cylinder to the *nearest tenth of a centimeter*.

$V = \pi r^2 h$

$\frac{12,566.4}{8\pi} = \frac{\pi r^2(8)}{8\pi}$ → $\sqrt{500...} = \sqrt{r^2}$

$22.4 = r$

31. Town A has an area of 12 square miles. Town B has an area of 10 square miles. If town A has a population of 8,198 people and town B has a population of 7,384 people, which town has a greater population density? Justify your answer.

Population density = $\frac{\text{population}}{\text{area}}$

Town A: $\frac{8198}{12}$ = 683...

Town B: $\frac{7384}{10}$ = 738.4

Town B

32. A brick that weighs 1824 grams has dimensions that measure 4 cm by 3 cm by 8 cm. To the nearest tenth, what is the density of the brick?

density = $\frac{\text{mass}}{\text{volume}}$

density = $\frac{1824}{96} = 19 g/cm^3$

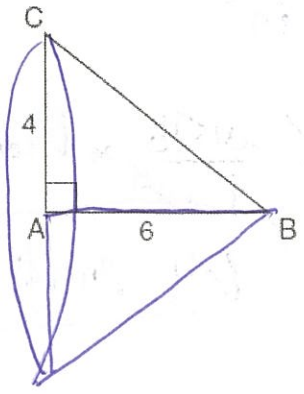
$V = lwh$

$V = 4(3)(8)$

$V = 96$

33. In the diagram below, right triangle ABC has legs whose lengths are 4 and 6. What is the volume of the three-dimensional object formed by continuously rotating the right triangle around $\overline{AB}$?

1) 32π
2) 48π
3) 96π
4) 144π

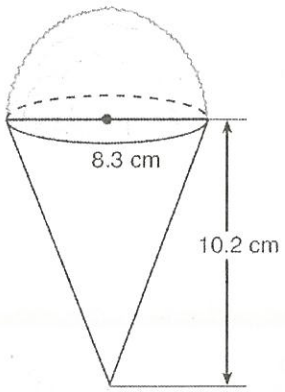


$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(4)^2(6)$

$V = 32\pi$

34. A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters. The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per kilogram, of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.



Cone

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(4.15)^2(10.2)$

$V = 183.$

hemisphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{1}{2}\left(\frac{4}{3}\pi(4.15)^3\right)$

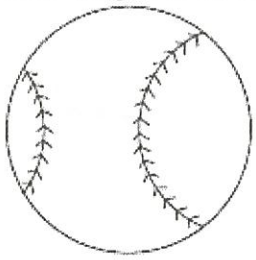
$V = 149..$

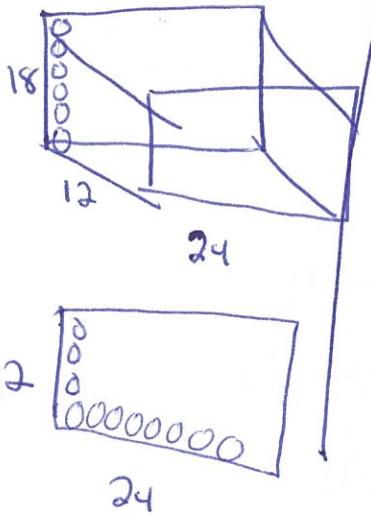
$183. + 149.. = 333..\ cm^3$

$333..\ cm^3 \cdot \frac{.697\ g}{1\ cm^3} \cdot \frac{1\ kg}{1000\ g} \cdot \frac{3.83\$}{1\ kg} \times 50$

\$44.53

35. A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches. Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs. The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the *nearest pound*, the total weight of all the baseballs in the fully packed box.





$2\ ft \cdot \frac{12\ in}{1\ ft} = 24\ in$

$1\ ft \cdot \frac{12\ in}{1\ ft} = 12\ in$

$\frac{24}{2.94} = 8.16$ → 8 $\quad \frac{12}{2.94} = 4.08$ → 4 $\quad \frac{18}{2.94} = 6.12$ → 6

# of balls = 8(4)(6) = 192

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(1.47)^3$

$V = 13.3\ in^3$

$13.3\ in^3 \cdot \frac{.025\ lb}{1\ in^3} \times 192$

64 pounds

$(-\frac{4}{2})^2 = 4 \quad (\frac{6}{2})^2 = 9$

36. What are the coordinates of the center and length of the radius of the circle whose equation is $x^2 + 6x + y^2 - 4y = 23$?

1) (3, −2) and 36
2) (3, −2) and 6
3) (−3, 2) and 36
4) (−3, 2) and 6

$x^2+6x+y^2-4y=23$

$x^2+6x+\boxed{9}+y^2-4y+\boxed{4}=23+\boxed{9}+\boxed{4}$

$(x+3)(x+3)+(y-2)(y-2)=36$

$(x+3)^2=(y-2)^2=36$

Center $=(-3,2)$ $r=6$

37. What is an equation of the image of the line $y = \frac{3}{2}x - 4$ after a dilation of a scale factor of $\frac{3}{4}$ centered at the origin?

1) $y = \frac{9}{8}x - 4$
2) $y = \frac{9}{8}x - 3$
3) $y = \frac{3}{2}x - 4$
4) $y = \frac{3}{2}x - 3$

multiply scale factor and b

same slope

$m = \frac{3}{2}$

$b = \frac{3}{4}(-4) = -3$

38. Line MN is dilated by a scale factor of 2 centered at the point (0, 6). If $\overleftrightarrow{MN}$ is represented by $y = -3x + 6$, which equation can represent $\overleftrightarrow{M'N'}$, the image of $\overleftrightarrow{MN}$?

same slope

same equation

1) $y = -3x + 12$
2) $y = -3x + 6$
3) $y = -6x + 12$
4) $y = -6x + 6$

39. The line $-3x + 4y = 8$ is transformed by a dilation centered at the origin. Which linear equation could represent its image?

1) $y = \frac{4}{3}x + 8$
2) $y = \frac{3}{4}x + 8$
3) $y = -\frac{3}{4}x - 8$
4) $y = -\frac{4}{3}x - 8$

same slope

$m = \frac{3}{4}$

$-3x+4y=8$

$+3x \quad +3x$

$\frac{4y}{4} = \frac{3x+8}{4}$

$y = \frac{3}{4}x + 2$ $\quad m = \frac{3}{4}$

40. Quadrilateral CAMI has a perimeter of 20 and an area of 15. What is the perimeter and area of quadrilateral CAMI after a dilation by a scale factor of 4?

new perimeter = scale factor · old perimeter

new area = (scale factor)^2 · old area

$P = 20(4) = 80$

$A = 15(4)^2 = 240$

negative reciprocal slopes

41. What is an equation of the line that contains the point $(3,-1)$ and is perpendicular to the line whose equation is $y=-3x+2$?

1) $y=-3x+8$

2) $y=-3x$

3) $y=\frac{1}{3}x$

4) $y=\frac{1}{3}x-2$

$m\perp=\frac{1}{3}$ $y-y_1=m(x-x_1)$

$x_1=3$ $y+1=\frac{1}{3}(x-3)$

$y_1=-1$ $y+1=\frac{1}{3}x-1$

$-1 \quad -1$

$y=\frac{1}{3}x-2$

Same slope

42. An equation of the line that passes through $(2,-1)$ and is parallel to the line $2y+3x=8$ is

$m\| = -\frac{3}{2}$

$x_1=2$

$y_1=-1$

1) $y+1=-\frac{3}{2}(x-2)$ $y-y_1=m(x-x_1)$ $y+1=-\frac{3}{2}(x-2)$

2) $y+1=\frac{2}{3}(x-2)$

3) $y-1=-\frac{3}{2}(x+2)$

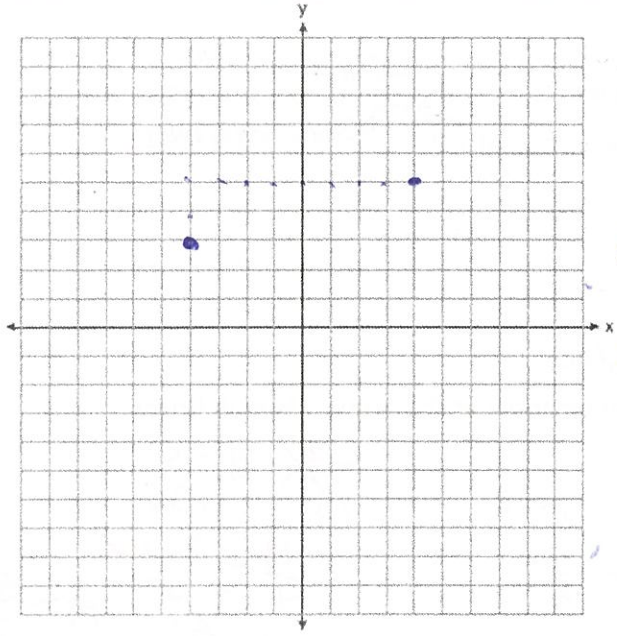
4) $y-1=\frac{2}{3}(x+2)$

$-3x \quad -3x$

$\frac{2y}{2}=\frac{-3x+8}{2}$

$y=-\frac{3}{2}x+4$

43. Write an equation of the perpendicular bisector of the line segment whose endpoints are (-4,3) and (4,5) in both point slope and slope intercept form.



$m=\frac{\Delta y}{\Delta x}$

$m=\frac{2}{8}=\frac{1}{4}$

$mp=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$mp=\left(\frac{-4+4}{2}, \frac{3+5}{2}\right)$

$mp=(0,4)$

$y-y_1=m(x-x_1)$

$y-4=-4(x-0)$

$m\perp=-4$

$x_1=0$

$y_1=4$

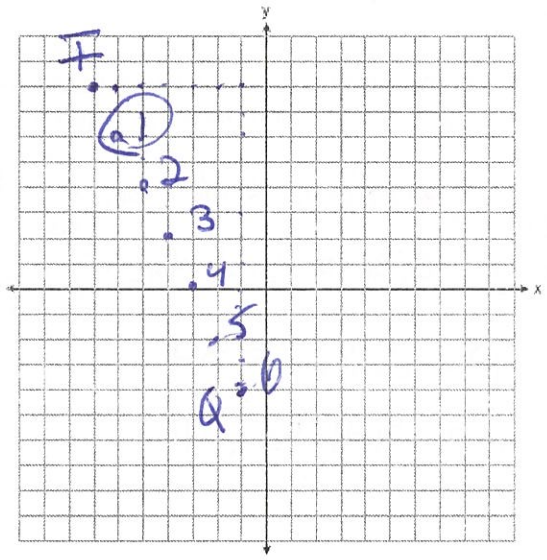
44. Directed line segment IQ has endpoints whose coordinates are $I(-7,8)$ and $Q(-1,-4)$. Determine the coordinates of point J that divides the segment in the ratio 1 to 5.

$P=6$

$\frac{\Delta x}{P}$ $\frac{\Delta y}{P}$ $(-6,6)$

$\frac{6}{6}$ $\frac{12}{6}$

1 2



45. On the set of axes below, the vertices of $\triangle PQR$ have coordinates $P(-6, 7)$, $Q(2, 1)$, and $R(-1, -3)$.

$A_{\square} = 10(8) = 80$

What is the area of $\triangle PQR$?

1) 10
2) 20
3) 25
4) 50

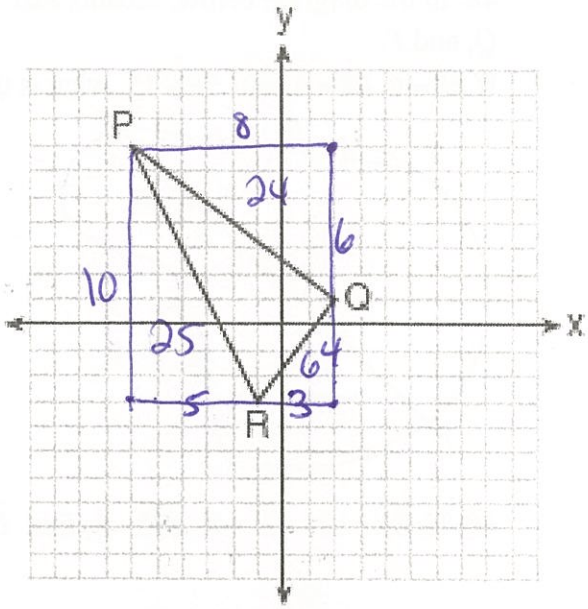
$A_{\triangle 1} = \frac{1}{2}(10)(5) = 25$

$A_{\triangle 2} = \frac{1}{2}(3)(4) = 6$

$A_{\triangle 3} = \frac{1}{2}(8)(6) = 24$

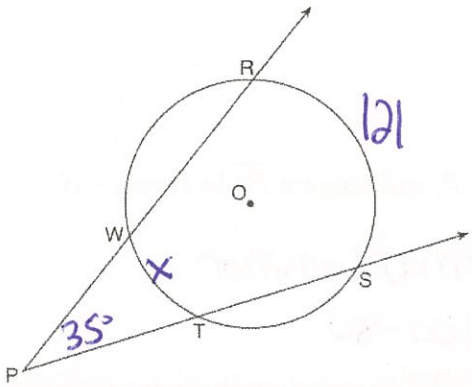
80
−25
55

25
+ 6
+24
55



46. As shown in the diagram below, secants $\overrightarrow{PWR}$ and $\overrightarrow{PTS}$ are drawn to circle O from external point P.

If $m\angle RPS = 35°$ and $m\overset{\frown}{RS} = 121°$, determine and state $m\overset{\frown}{WT}$.



2(EA) = major − minor

$2(35) = 121 - x$

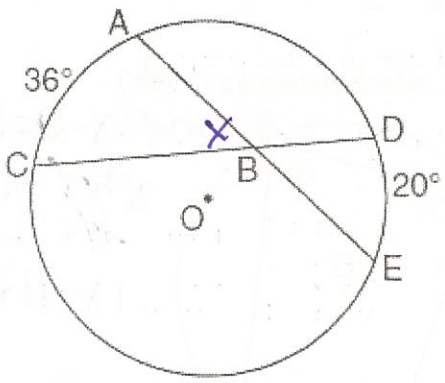
$70 = 121 - x$

$-121 \quad -121$

$\frac{-51}{-1} = \frac{-x}{-1}$

$51 = x$

47. In the diagram below of circle O, chords $\overline{AE}$ and $\overline{DC}$ intersect at point B, such that $m\overset{\frown}{AC} = 36$ and $m\overset{\frown}{DE} = 20$. What is $m\angle ABC$?



2(VA) = arc + arc

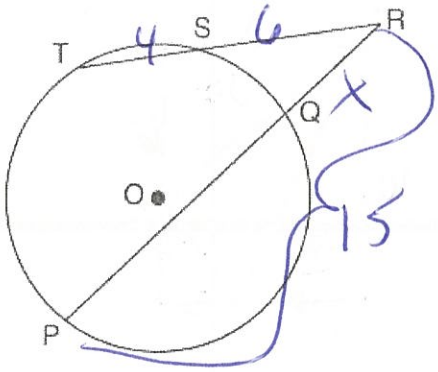
$2x = 36 + 20$

$\frac{2x}{2} = \frac{56}{2}$

$x = 28$

48. In the diagram below, secants $\overline{RST}$ and $\overline{RQP}$, drawn from point R, intersect circle O at S, T, Q, and P.

If $RS = 6$, $ST = 4$, and $RP = 15$, what is the length of $\overline{RQ}$?



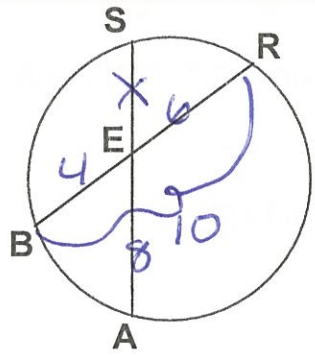
whole · exterior = whole · exterior

$10 \cdot 6 = 15 \cdot x$

$\frac{60}{15} = \frac{15x}{15}$

$4 = x$

49. If $\overline{BR} = 10$, $\overline{BE} = 4$, $\overline{AE} = 8$, find $\overline{ES}$



part · part = part · part

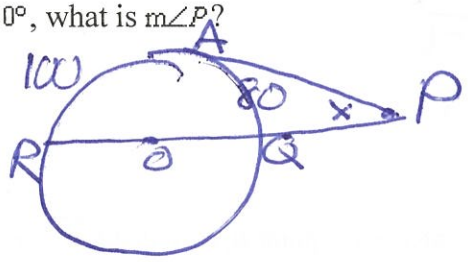
$4 \cdot 6 = 8 \cdot x$

$\frac{24}{8} = \frac{8x}{8}$

$3 = x$

50. Diameter $\overline{ROQ}$ of circle O is extended through Q to point P, and tangent $\overline{PA}$ is drawn. If $m\widehat{RA} = 100°$, what is $m\angle P$?

180
−100
80



2(EA) = major − minor

$2x = 100 - 80$

$\frac{2x}{2} = \frac{20}{2}$

$x = 10°$

51. Quadrilateral $ABCD$ is inscribed in circle O, as shown below.

If $m\angle A = 80°$, $m\angle B = 75°$, $m\angle C = (y + 30)°$, and $m\angle D = (x - 10)°$, which statement is true?

1) $x = 85$ and $y = 50$
2) $x = 90$ and $y = 45$
3) $x = 110$ and $y = 75$
4) $x = 115$ and $y = 70$

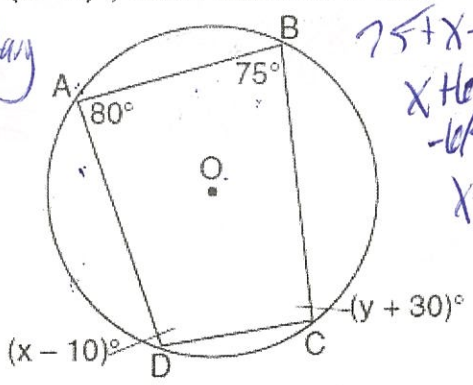
opposite angles are supplementary

$80 + y + 30 = 180$

$y + 110 = 180$

$-110 \quad -110$

$y = 70$



$75 + x - 10 = 180$

$x + 65 = 180$

$-65 \quad -65$

$x = 115$

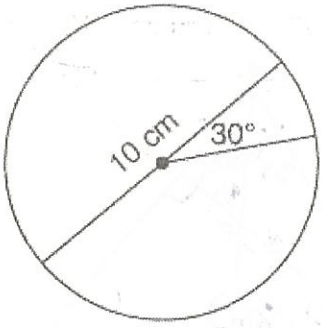
52. A circle with a diameter of 10 cm and a central angle of 30° is drawn below.
What is the area, to the *nearest tenth of a square centimeter*, of the sector formed by the 30° angle?

1) 5.2
2) 6.5
3) 13.1
4) 26.2

$A = \frac{\theta \pi r^2}{360}$ → type π in

$A = \frac{30\pi(5)^2}{360}$

$A = 6.5$



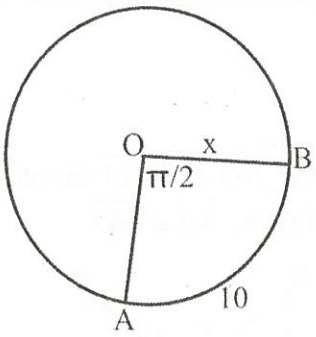
53. In circle O, the measure of central angle AOB is $\frac{\pi}{2}$ radians and the length of arc AB is 10 cm. What is the measure of radius $\overline{OB}$ to the *nearest tenth of a cm*?

$s = \theta r$

$2(10) = \left(\frac{\pi}{2}\right)x$

$\frac{20}{\pi} = \frac{\pi x}{\pi}$

$6.4 = x$



54. Given three distinct quadrilaterals, a square, a rectangle, and a rhombus, which quadrilaterals must have perpendicular diagonals?

1) the rhombus, only
2) the rectangle and the square
3) the rhombus and the square
4) the rectangle, the rhombus, and the square

55. In the diagram of parallelogram *FRED* shown below. $\overline{ED}$ is extended to *A*. and $\overline{AF}$ is drawn such that $\overline{AF} \cong \overline{DF}$.

If $m\angle R = 124°$, what is $m\angle AFD$?

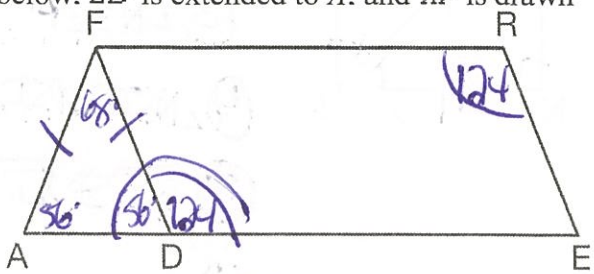
1) 124°
2) 112°
3) 68°
4) 56°

$56 + 56 + x = 180$

$112 + x = 180$

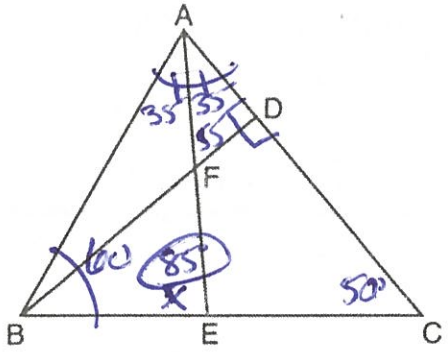
$-112 \quad -112$

$x = 68$



$180 - 124 = 56$

56. In the diagram of $\triangle ABC$ below, $\overline{AE}$ bisects angle BAC, and altitude $\overline{BD}$ is drawn. If $m\angle C = 50°$ and $m\angle ABC = 60°$, what is $m\angle FEB$?



$\triangle ABC$

$60+50+x=180$

$110+x=180$

$-110 \quad -110$

$x=70$

right ∠

$\triangle ADF$

$90+35+x=180$

$125+x=180$

$-125 \quad -125$

$x=55$

$\triangle AEB$

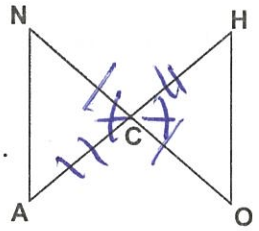
$60+35+x=180$

$95+x=180$

$-95 \quad -95$

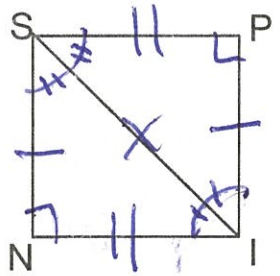
$x=85$

57. Given: $\overline{NO}$ and $\overline{HA}$ bisect each other
Prove: $\overline{NA} \cong \overline{HO}$



Statements	Reasons
(1) $\overline{NO}$ and $\overline{HA}$ bisect each other	(1) given
(2) $\overline{NC} \cong \overline{CO}$, $\overline{AC} \cong \overline{CH}$	(2) A line bisector creates 2 ≅ segments
(3) $\angle NCA \cong \angle HCO$	(3) vertical angles are ≅
(4) $\triangle NCA \cong \triangle OCH$	(4) SAS
(5) $\overline{NA} \cong \overline{HO}$	(5) CPCTC

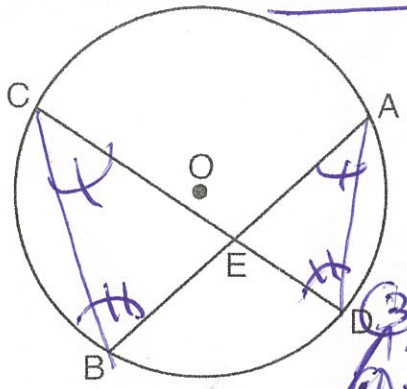
58. Given: SPIN is a square
Prove: $\triangle SNI \cong \triangle SPI$



Statements	Reasons
(1) SPIN is a square	(1) given
(2) $\overline{SN} \cong \overline{PI}$, $\overline{SP} \cong \overline{NI}$	(2) A square has all sides congruent
(3) $\angle N \cong \angle P$	(3) A square has congruent right angles
(4) $\angle NSI \cong \angle PSI$, $\angle NIS \cong \angle PIS$	(4) A square has diagonals that bisect the angles
(5) $\overline{SI} \cong \overline{SI}$	(5) Reflexive Property
(6) $\triangle SNI \cong \triangle SPI$	(6) SSS, SAS, ASA, AAS, HL

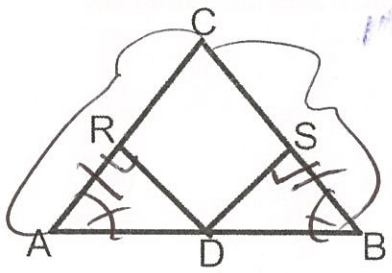
59. Given: Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E
Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. Prove this theorem by proving $AE \cdot EB = CE \cdot ED$.

→ work backwards



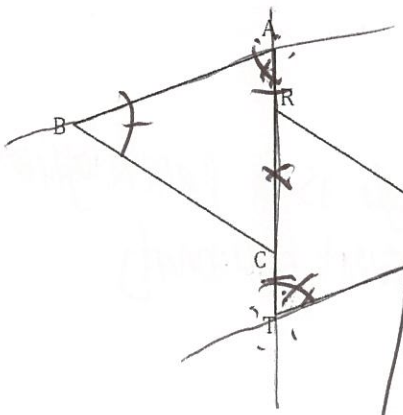
Statements	Reasons
① $\overline{CB}$, $\overline{AD}$	① Auxiliary lines can be drawn
② $\angle C \cong \angle A, \angle B \cong \angle D$	② Angles inscribed to the same arc are congruent
③ $\triangle BCE \sim \triangle DAE$	③ AA
④ $\frac{AE}{ED} = \frac{CE}{EB}$	④ CSSTIP
⑤ $AE \cdot EB = CE \cdot ED$	⑤ Cross products are equal

60. Given: In $\triangle ABC$, $\overline{CA} \cong \overline{CB}$, $\overline{AR} \cong \overline{BS}$, $\overline{DR} \perp \overline{AC}$, and $\overline{DS} \perp \overline{BC}$
Prove: $\overline{DR} \cong \overline{DS}$



Statements	Reasons
① $\overline{CA} \cong \overline{CB}$	① given
② $\angle A \cong \angle B$	② Isosceles Triangle Theorem
③ $\overline{AR} \cong \overline{BS}$	③ given
④ $\overline{DR} \perp \overline{AC}$, $\overline{DS} \perp \overline{BC}$	④ given
⑤ $\angle DRA \cong \angle DSB$	⑤ Perpendicular lines form congruent right angles
⑥ $\triangle DRA \cong \triangle DSB$	⑥ ASA
⑦ $\overline{DR} \cong \overline{DS}$	⑦ CPCTC

61. Given: $\angle B \cong \angle S$, $\overline{AB} \parallel \overline{ST}$, $\overline{AR} \cong \overline{TC}$
Prove: $\overline{BC} \cong \overline{SR}$



Statements	Reasons
① $\angle B \cong \angle S$	① given
② $\overline{AB} \parallel \overline{ST}$	② given
③ $\angle BAC \cong \angle STR$	③ Parallel lines cut by a transversal create congruent alternate interior $\angle$s
④ $\overline{AR} \cong \overline{TC}$	④ given
⑤ $\overline{RC} \cong \overline{RC}$	⑤ Reflexive Property
⑥ $\overline{AC} \cong \overline{RT}$	⑥ Addition Property
⑦ $\triangle BAC \cong \triangle STR$	⑦ AAS
⑧ $\overline{BC} \cong \overline{SR}$	⑧ CPCTC

62.

a) Name the two ways to prove a parallelogram is a rectangle.

Right angle
Congruent Diagonals

b) Name the three ways to prove a parallelogram is a rhombus.

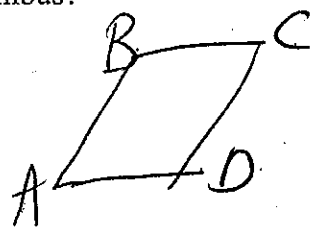
Consecutive sides congruent
diagonals perpendicular to each other
diagonals bisect the angles

63. A parallelogram must be a rectangle when its

1) diagonals are perpendicular
2) diagonals are congruent
3) opposite sides are parallel
4) opposite sides are congruent

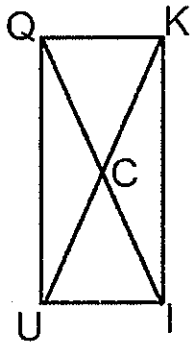
64. In parallelogram $ABCD$, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E. Which statement does *not* prove parallelogram $ABCD$ is a rhombus?

1) $\overline{AC} \cong \overline{DB}$ diagonals $\cong$
2) $\overline{AB} \cong \overline{BC}$ consecutive sides $\cong$
3) $\overline{AC} \perp \overline{DB}$ diagonals $\perp$
4) $\overline{AC}$ bisects $\angle DCB$ diagonals bisect angles



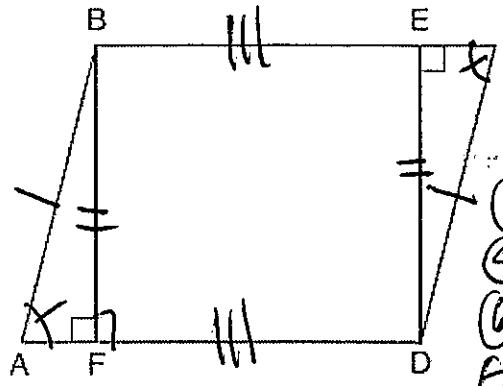
65. Given: QUIK is a parallelogram, $\overline{QI} \cong \overline{KU}$

Prove: QUIK is a rectangle



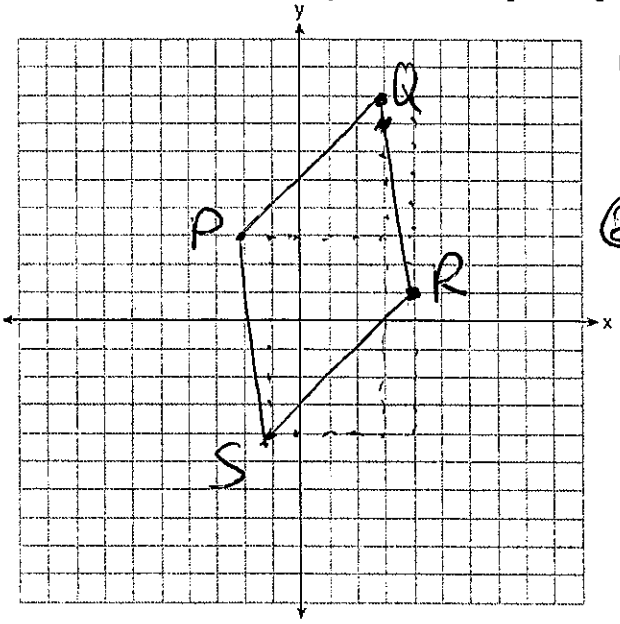
Statements	Reasons
1) QUIK is a parallelogram	1) given
2) $\overline{QI} \cong \overline{KU}$	2) given
3) QUIK is a rectangle	3) A rectangle is a parallelogram with congruent diagonals

66. Given: Parallelogram $ABCD$, $\overline{BF} \perp \overline{AFD}$, and $\overline{DE} \perp \overline{BEC}$
Prove: $BEDF$ is a rectangle



Statements	Reasons
① Parallelogram ABCD	① given
② $\overline{AB} \cong \overline{CD}$	② A p-gram has opposite sides ≅
③ $\angle BAF \cong \angle DCE$	③ A p-gram has opposite angles ≅
④ $\overline{BF} \perp \overline{AFD}$, $\overline{DE} \perp \overline{BEC}$	④ given
⑤ $\angle BFA \cong \angle DEC$	⑤ Perpendicular lines form ≅ right angles
⑥ $\triangle BFA \cong \triangle DEC$	⑥ AAS
⑦ $\overline{BF} \cong \overline{ED}$	⑦ CPCTC
⑧ $\overline{BC} \cong \overline{AD}$	⑧ A p-gram has opposite sides ≅
⑨ $\overline{BC} \cong \overline{AF}$	⑨ CPCTC
⑩ $\overline{BE} \cong \overline{FD}$	⑩ Subtraction Property
⑪ BEDF is a p-gram	⑪ A p-gram has 2 pairs of opposite sides congruent
⑫ $\angle BFD$ is a right angle	⑫ Perpendicular lines form right angles
⑬ BEDF is a rectangle	⑬ A rectangle is a p-gram with a right angle

67. Quadrilateral $PQRS$ has vertices $P(-2, 3)$, $Q(3, 8)$, $R(4, 1)$, and $S(-1, -4)$. Prove that $PQRS$ is a rhombus. Prove that $PQRS$ is *not* a square. [The use of the set of axes below is optional.]

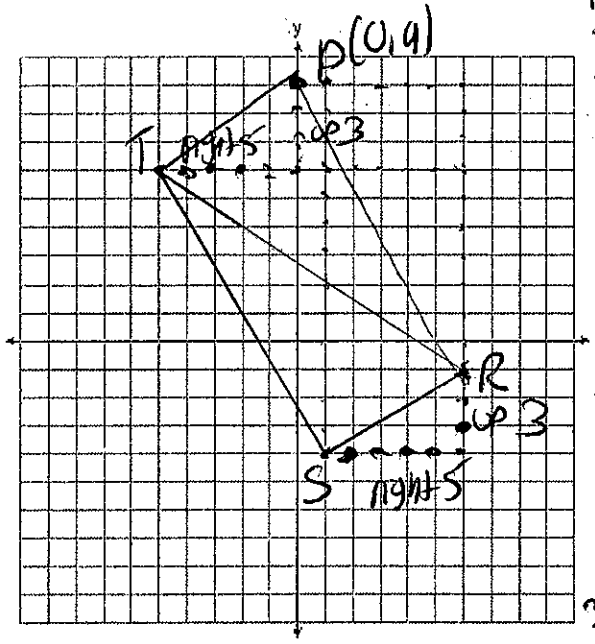


① PQRS is a rhombus because all sides are congruent. It is not a square because the diagonals are not congruent.

② $d_{\overline{PQ}} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$
$d_{\overline{QR}} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$
$d_{\overline{RS}} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$
$d_{\overline{SP}} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$
$d_{\overline{PR}} = \sqrt{6^2+2^2} = \sqrt{36+4} = \sqrt{40}$
$d_{\overline{QS}} = \sqrt{9^2+12^2} = \sqrt{16+144} = \sqrt{160}$

③ $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$ because they have the same distance.
$\overline{PR} \ncong \overline{QS}$ because they don't have the same distance.

68. In the coordinate plane, the vertices of ΔRST are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that ΔRST is a right triangle. State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle. Prove that your quadrilateral $RSTP$ is a rectangle. [The use of the set of axes below is optional.]



1) ΔRST is a right triangle because its sides fit into Pythagorean Theorem.

2) $dST = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$dTR = \sqrt{11^2+7^2} = \sqrt{121+49} = \sqrt{170}$

$dRS = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

3) $a^2+b^2=c^2$

$\sqrt{34}^2+\sqrt{136}^2 = \sqrt{170}^2$

$34+136=170$

$170=170$ ✓

1) RSTP is a rectangle because it has 2 pairs of opposite sides ≅ and diagonals congruent. 2

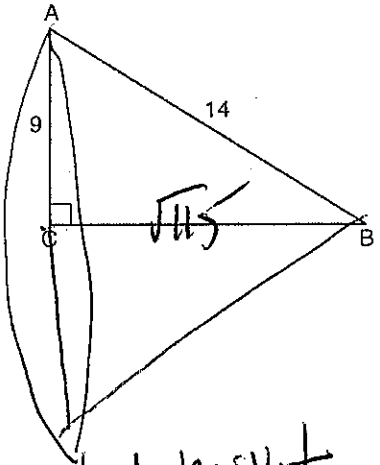
2) $dTP = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$dPR = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$dRS = \sqrt{1^2+13^2} = \sqrt{1+169} = \sqrt{170}$

3) $\overline{TP} \cong \overline{SR}$, $\overline{TS} \cong \overline{PR}$, $\overline{TR} \cong \overline{PS}$ because they have the same distance.

69. In the diagram of right triangle ABC shown below, $AB = 14$ and $AC = 9$.
What is the volume of the three dimensional object formed when the triangle is continuously rotated about side $\overline{BC}$ to the nearest tenth.



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(9^2)(\sqrt{115})$

$V = 904.6$

Need height

$a^2+b^2=c^2$

$9^2+b^2=14^2$

$81+b^2=196$

$-81 \quad -81$

$\sqrt{b^2} = \sqrt{115}$

$b = \sqrt{115}$