Name _____________ Date ______________
Mr. Schlansky Geometry

Common Core Geometry Key Understandings

USE A GRAPH!!!!

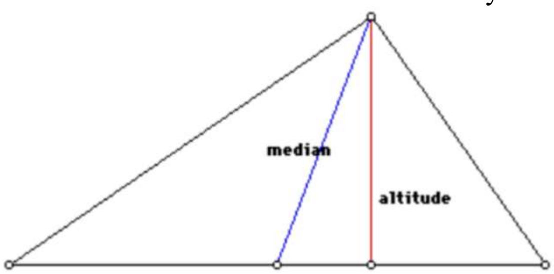
Triangles: (Look for angles of a triangle and linear pairs!)

Scalane triangles have 0 congruent sides/angles	Acute triangles have 3 acute angles
Isosceles triangles have 2 congruent sides/angles	Obtuse triangles have 1 obtuse angle
Equilateral triangles have 3 congruent sides/angles	Right triangles have 1 right angle

Complex Triangle Problems:

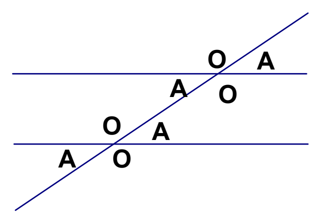
1) The three angles of a triangle add to equal 180º. Look for triangles.
2) Linear pairs add to 180º. Look for linear pairs.
3) Isosceles triangle has congruent angles opposite congruent sides (given congruent sides).
4) Equilateral triangle has angles 60, 60, 60 (given equilateral triangle).
5) An angle bisector cuts an angle into two congruent halves (given bisected angles).
6) Use parallel lines cut by a transversal (extend and follow the transversal, fill in 8 angles.)

A median is a line segment connecting a vertex to the midpoint of the opposite side.
An altitude is a line segment extending from a vertex and is perpendicular to the opposite side.
Medians are cut in a ratio of 2:1 when they intersect.



Parallel Lines:

<u>**Extend parallel lines and follow the transversal!!!!!!! Fill in all eight angles!**</u>



If lines are parallel:
-If the angles are the same (both acute or obtuse), set them equal to each other
-If the angles are different (one acute and one obtuse), add them to equal 180º.
If lines are *not* parallel:
- The same angles are *not* congruent and different angle are *not* supplementary
Names of angle pairs:
1 in, 1 out: Corresponding Angles are Congruent
2 in: Alternate Interior Angles are Congruent
2 in: Same Side Interior Angles are Supplementary
2 out: Alternate Exterior Angles are Congruent
2 out: Same Side Exterior Angles are Supplementary

Transformations:

Translations: Slide: Count on the graph.
Reflection: Flip: Count to what you are reflecting over.
y = # is horizontal line, x = # is vertical line
Rotations: Turn: Turn your paper to the left. Write down new points. Turn back. Plot.
Dilations (enlarge or shrink):

1) Count the distance from center of dilation to each point. Repeat that distance as many times as the scale factor.
2) If center of dilation is origin, multiply each coordinate by the scale factor.

Scale factor = $\frac{image}{original}$

They all preserve size/distance except dilation. They all preserve orientation except line reflection.

To prove triangles are congruent/similar using rigid motions/transformations

1) Identify the transformations (**Check for orientation to determine if reflection**) On the grid: reflect/rotate/dilate first Off the grid: translate first Translate ________ to ________ Reflect $\triangle$ ________ over Rotate $\triangle$ ________ about point __until it maps onto $\triangle$ ________ Dilation $\triangle$ ________ centered at point ____ by a scale factor of $\frac{image}{original}$	
Congruence	Similarity
2) A _______ and _______ are rigid motions. 3) **A rigid motion preserves size and angle measure producing a congruent figure.**	2) A dilation and ________ preserve angle measure producing a similar figure.

To map a shape onto itself:
Translation/Dilation: Never.
Reflection: **The line of reflection must be a line of symmetry** (cuts shape in half).
Rotation: **Center of rotation must be the center of the shape.** Common sense for degree measure.

To determine the minimum number of degrees a regular polygon must be rotated to be mapped onto itself:

1) The minimum rotation is $\frac{360}{n}$.
2) **Any multiple of that will also map the regular polygon onto itself!**

Solids:

Volume is the space inside a 3 dimensional shape. Surface Area is the area around the outside of a 3 dimensional shape.

Cavalieri's Principle: If the area of the bases are congruent, and the heights are congruent, then the volumes are congruent.

Volume = (<u>Area of the base</u>)(height), if it comes to a point, multiply by $\frac{1}{3}$.

Area of the base is USUALLY $A = lw$ (rectangle/square) or $A = \pi r^2$ (circle)

Most volume formulas are on the reference sheet. Be careful. B = area of the base

General Prism: $V = (area\ base)(height)$

Rectangular prism: $V = lwh$

Cylinder: $V = \pi r^2 h$

Pyramid: $V = \frac{1}{3}lwh$

Cone: $V = \frac{1}{3}\pi r^2 h$

Sphere: $V = \frac{4}{3}\pi r^3$

Weight = (weight)(volume)

Cost = (cost)(area/volume/mass)

Density = $\frac{mass}{volume}$, If given density, cross multiply to find mass.

Population Density = $\frac{population}{area}$

To convert units: Multiply to get units to cancel out. Example: $3\ in \bullet \frac{2.54\ cm}{1\ in}$

If given a percent: Convert to a decimal (divide by 100) and multiply

Surface Area (rectangular prism) = $2lw + 2hw + 2lh$

3 dimensional rotations ALMOST ALWAYS form a cylinder or cone

Reflect the shape in 2 dimensions and connect the images with curves

Cross Sections (2 dimensional slice of a 3 dimensional object):

The base of the shape is always one of its cross sections

Rectangular Prism: Rectangle, triangle

Cylinder: Circle, ellipse, rectangle

Cone: Circle, ellipse, triangle, "curved" rectangle

Pyramid: Rectangle, triangle

Sphere: Circle

Circumference is the distance around the outside of a circle

Euclidean Proofs:

If it is not specified, prove triangles are congruent
To prove triangles are congruent, prove 3 pairs of sides/angles are congruent
To prove segments or angles, use CPCTC
<u>*If you get stuck, make something up and keep on going!</u>

1) Do a mini proof with your givens

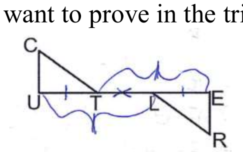
Altitude creates two congruent right angles
Median creates two congruent segments
Line bisector creates two congruent segments
Midpoint creates two congruent segments
Angle bisector creates two congruent angles
Perpendicular lines create two congruent right angles
Parallel lines cut by a transversal create
Congruent corresponding angles (1 in, 1 out) OR congruent alternate interior angles (2 out) OR congruent alternate exterior angles (2 out)
*Perpendicular bisector is perpendicular and line bisector (1 pair of congruent right angles, 1 pair of congruent segs)
*If segments bisect each other, they are both cut in half (2 pairs of congruent segments)

2) Use additional tools:

Vertical Angles are congruent (Look for an X)
Reflexive Property (A side/angle is in both triangles and is congruent to itself)
Isosceles Triangles (In a triangle, congruent angles are opposite congruent sides)
Addition and Subtraction Property (If you need more or less of a shared side)
*You must use three congruent statements to get one congruent statement for the triangles. The two that you are adding/subtracting and the one that you want to prove in the triangle.

7. Given: $\overline{UL} \cong \overline{TE}$
Prove: $\overline{UT} \cong \overline{LE}$

statements	Reasons
① $\overline{UL} \cong \overline{TE}$	① given
② $\overline{TL} \cong \overline{TL}$	② reflexive property
③ $\overline{UT} \cong \overline{LE}$	③ subtraction property



Parallelogram Theorems	Circle Theorems (Look for inscribed angles)
A parallelogram/rectangle/rhombus/square has: Two pairs of opposite sides congruent Two pairs of opposite sides parallel Diagonals that bisect each other Opposite angles congruent **A rectangle/square has:** Congruent right angles Congruent diagonals **A rhombus/square has:** All sides congruent Perpendicular diagonals Diagonals that bisect the angles	**Angles inscribed to the same arc are congruent** **An angle inscribed to a semicircle is a right angle** **A tangent and a radius/diameter form a right angles** All radii/diameters of a circle are congruent Congruent arcs have congruent chords have congruent central angles Parallel Lines intercept congruent arcs Tangents drawn from the same point are congruent

To prove triangles are SIMILAR, prove AA ≅ AA
If asked to prove a proportion/multiplication:

1) Prove triangles are similar
2) Corresponding Sides of Similar Triangle are In Proportion (CSSTIP)
3) Cross Products are Equal

Work Backwards!

③ $\triangle AED \sim \triangle CEB$ | ③ AA ≅ AA
④ $\frac{AE}{ED} = \frac{CE}{EB}$ | ④ CSSTIP
⑤ $AE \cdot EB = CE \cdot ED$ | ⑤ cross products are equal

To prove parallelograms: Always prove parallelogram first. You will probably have to use congruent triangles with CPCTC to get at least one of the properties.
A parallelogram has:
Two pairs of opposite sides congruent OR Two pairs of opposite sides parallel OR One pair of opposite sides congruent and parallel OR Diagonals that bisect each other OR Opposite angles congruent
A rectangle is a parallelogram with:
A right angle OR Congruent diagonals
A rhombus is a parallelogram with:
Consecutive sides congruent OR diagonals perpendicular to each other OR diagonals that bisect the angles
A square is a parallelogram with:
Consecutive sides congruent OR diagonals perpendicular to each other OR diagonals that bisect the angles
AND
A right angle OR Congruent diagonals
A trapezoid has one pair of opposite sides parallel and one pair of opposite sides not parallel
An isosceles trapezoid is a trapezoid with congruent legs

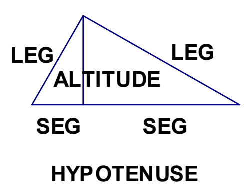
Similar Triangles:

Dilations create similar triangles
Corresponding ANGLES are congruent.
Corresponding SIDES are IN PROPORTION!

ROCS = Ratio of Perimeters $ROCS^2$ = Ratio of Areas
Ratio of Corresponding Angles = 1:1 (The corresponding angles are congruent)

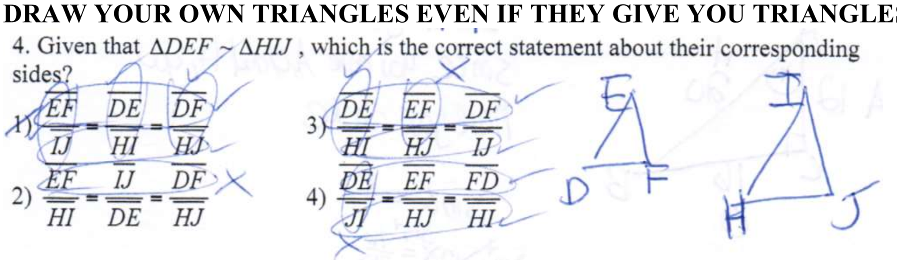
Candy Corn Problems:
If the bases are not involved: $\frac{top}{top} = \frac{bottom}{bottom} = \frac{side}{side}$
If bases are involved: separate your triangles!
If the midpoints are joined: 2(midsegment) = opposite parallel side

When an altitude is drawn to a right triangle:
HLLS and SAAS
$\frac{H}{L} = \frac{L}{S} \quad \frac{S}{A} = \frac{A}{S}$
If L is involved, use HLLS
If A is involved, use SAAS



To show triangles are similar:
1) AA (2 pairs of corresponding angles are congruent)
2) SAS (2 pairs of corresponding sides are in proportion and the corresponding angles between them are congruent)
3) SSS (3 pairs of corresponding sides are in proportion)
Show the sides are in proportion by creating a proportion

To determine whether sides or angles correspond, look at the picture OR the letters
For Example: If $\Delta ABC \cong \Delta DEF$, $\angle A \cong \angle D$ (because they are both written first) and $\overline{AC} \cong \overline{DF}$ (because they are both written first and third).
To determine if a proportion is correct, circle horizontally and vertically. One direction the sides should correspond, the other should be in the same triangle.
DRAW YOUR OWN TRIANGLES EVEN IF THEY GIVE YOU TRIANGLES


4. Given that $\Delta DEF \sim \Delta HIJ$, which is the correct statement about their corresponding sides?

1) $\frac{\overline{EF}}{\overline{IJ}} = \frac{\overline{DE}}{\overline{HI}} = \frac{\overline{DF}}{\overline{HJ}}$
2) $\frac{\overline{EF}}{\overline{HI}} = \frac{\overline{IJ}}{\overline{DE}} = \frac{\overline{DF}}{\overline{HJ}}$
3) $\frac{\overline{DE}}{\overline{HI}} = \frac{\overline{EF}}{\overline{HJ}} = \frac{\overline{DF}}{\overline{IJ}}$
4) $\frac{\overline{DE}}{\overline{JI}} = \frac{\overline{EF}}{\overline{HJ}} = \frac{\overline{FD}}{\overline{HI}}$

Right Triangles:

If only sides are involved, use Pythagorean theorem! ($a^2 + b^2 = c^2$ **)**
If an angle is involved, use SOHCAHTOA
1) Label each side with H, A, and O
2) Determine whether to use sine, cosine, or tangent (Which two are involved?)
3) Substitute into appropriate formula
*If finding a side, cross multiply and solve
*If finding an angle, use $\sin^{-1}, \cos^{-1}, or \tan^{-1}$

$\sin A = \cos B$ **: In a right triangle, the sine of one acute angle is equal to the cosine of the other acute angle**
$A + B = 90$ **: The two acute angles in a right triangle are complementary**

Equations of Circles and Lines:

Center and radius are key pieces of information for circles
To find center: Negate what is in the parenthesis. If there are no parentheses, the coordinate is 0.
Radius is the square root of the right hand side
$(x – a)^2 + (y – b)^2 = r^2$ where (a,b) is the center and r is the radius
To put into center-radius form: COMPLETE THE SQUARE TWICE
Completing the Square
1) Write the x's together, y's together, and move constant to the other side
$x^2 + bx + y^2 + by = c$
2) Add $\left(\frac{b}{2}\right)^2$ to both sides for each variable
3) Factor each trinomial (Both factors must be the same)
4) Rewrite the factors as a binomial squared

To write the equation of a line:
Slope-Intercept formula: $y = mx + b$ where m = slope and b = y intercept.
Point-Slope formula: $y - y_1 = m(x - x_1)$ where m = slope and (x_1, y_1) is any point on the line.
Parallel lines have the same slope.
Perpendicular lines have negative reciprocal slopes (flip it and negate it).

When asked for the equation of a line and given a point:
1) Find m by using parallel or perpendicular definitions
2) substitute into $y - y_1 = m(x - x_1)$ (Point-slope formula)
3) If necessary, solve for y to put it into $y = mx + b$ form (Slope-intercept form)

Line Dilations
The image is parallel. The slope always stays the same
Centered at origin: $b = kb$ (The new y-int is the scale factor times the original y int)
Centered on the line: $b = b$ (The image is the same as the original line)
Centered off the line: Count using the graph. Multiply the distance by the scale factor
If center of dilation is on the line:
The solution to a system of equations is the point of intersection of the two graphs

Coordinate Geometry

Distance (Length) = $\sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Slope = $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Midpoint = $(average\ x,\ average\ y) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Partitions (Directed Line Segment):

1) Find $\dfrac{\Delta x}{p}$ *and* $\dfrac{\Delta y}{p}$ where p is the number of partitions. For midpoint, it is always 2 (1:1).
2) Count those values out on the graph between the two endpoints
3) Circle and state the point that matches the given ratio. BE CAREFUL WHICH POINT YOU START FROM!

Use scrap graph paper if not given a graph!

Quadrilateral Properties:

<table>
<tr><td>A parallelogram/rectangle/rhombus/square has:
Two pairs of opposite sides congruent
Two pairs of opposite sides parallel
Diagonals that bisect each other
Opposite angles congruent</td><td rowspan="3">A rectangle is a parallelogram with:
A right angle OR Congruent diagonals
A rhombus is a parallelogram with:
Consecutive/all sides congruent OR diagonals perpendicular to each other OR diagonals that bisect the angles
A square is a parallelogram with:
Consecutive/all sides congruent OR diagonals perpendicular to each other OR diagonals that bisect the angles
AND
A right angle OR Congruent diagonals</td></tr>
<tr><td>A rectangle/square has:
Right angles
Congruent diagonals</td></tr>
<tr><td>A rhombus/square has:
All sides congruent
Perpendicular diagonals
Diagonals that bisect the angles</td></tr>
<tr><td>A trapezoid has 1 pair of opposite sides parallel and 1 pair of opposite sides not parallel</td><td></td></tr>
<tr><td>An isosceles trapezoid has:
Congruent legs
Congruent diagonals
2 pairs of congruent angles</td><td></td></tr>
</table>

Angles of a Parallelogram: (Combined with Complex Triangle Problems)

1) **Opposite angles of a parallelogram are congruent**
2) **Consecutive angles of a parallelogram are supplementary (add to 180)**
3) **The three angles of a triangle add to equal 180º. Look for triangles.**

***The four angles of a quadrilateral add to 360º.**

4) **Linear pairs add to 180º. Look for linear pairs.**
5) **Vertical angles are congruent. Look for an X (intersecting lines).**
6) **Isosceles triangle has congruent angles opposite congruent sides (given congruent sides).**
7) **Equilateral triangle has angles 60, 60, 60 (given equilateral triangle).**
8) **An angle bisector cuts an angle into two congruent halves (given bisected angles).**
9) **Use parallel lines cut by a transversal (follow the transversal and fill in all 8 angles)**

How do you prove…?

…an **isosceles triangle**? (2 Distances)
Two Congruent Sides

. … a **right triangle**? (3 Distances)
Show the sides fit into Pythagorean Theorem
… a **parallelogram**? (4 Distances)
Two Pairs of Opposite Sides Congruent

… a **rhombus**? (4 Distances)
All Sides Congruent

… a **rectangle**? (6 Distances)
1) Two Pairs of Opposite Sides Congruent
2) Diagonals Congruent

… a **square**? (6 Distances)
1) All Sides Congruent
2) Diagonals Congruent

…a **trapezoid**? (4 Slopes)
1) 1 pair of opposite sides parallel
2) 1 pair of opposite sides not parallel

…an **isosceles trapezoid**? (4 Slopes, 2 Distances)
1) 1 pair of opposite sides parallel
2) 1 pair of opposite sides not parallel
3) Congruent Legs

Circles:

Area and Perimeter/Arc Length:

Area and Area of a circle: $A = \pi r^2$
Circumference of a circle: $C = \pi d$

Area of a Sector

Degrees: $A = \frac{\theta}{360} \pi r^2$

Radians: $A = \frac{\theta}{2\pi} \pi r^2$

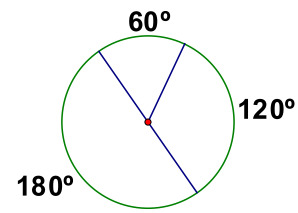
Arc Length

Degrees: $C = \frac{\theta}{360} \pi d$

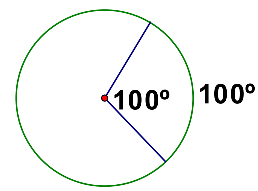
Radians: $C = \frac{\theta}{2\pi} \pi d$

Angle and Segment Rules:

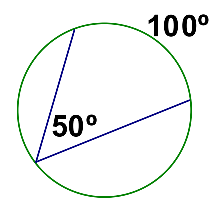
The arcs of a circle add to 360º
A diameter cuts a circle into 2 halves of 180º each



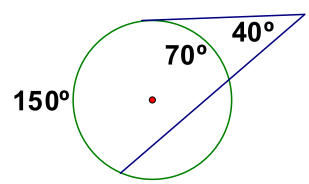
Central Angle: Has its vertex at the center of the circle
Central angle is equal to the measure of the intercepted arc

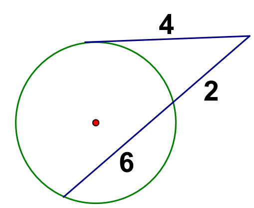


Inscribed Angle: Has its vertex on the circle
Inscribed angle is half of the measure of the intercepted arc

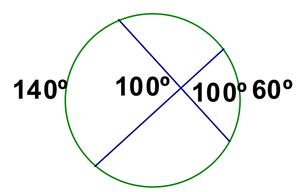


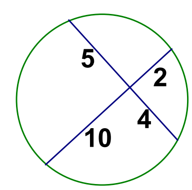
Exterior Angle:
Angles: 2(Exterior Angle) = (Major Arc – Minor Arc)
Segments: Whole ● Exterior = Whole ● Exterior



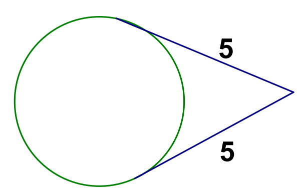


Intersecting Chords:
Angles: 2(Vertical Angle) = Arc + Arc
Segments: Part ● Part = Part ● Part

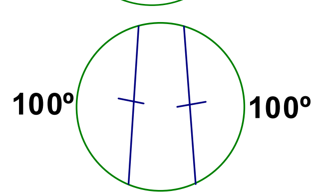




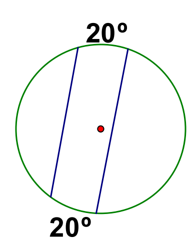
Two tangents drawn from the same point are congruent



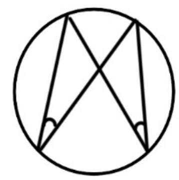
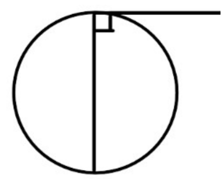
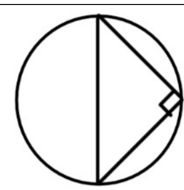
Congruent chords intercept congruent arcs



Parallel chords intercept congruent arcs



Special Angles in a Circle (Look for Inscribed Angles)

Angles inscribed to the same/congruent arcs are congruent.	
A tangent and radius/diameter intersect to form a right angle.	
An angle is inscribed to a semicircle/diameter is a right angle.	

Constructions:

Start by placing needle point on the key point(s)

Bisector Constructions:

<u>Perpendicular bisector (Jesus Fish):</u> Open compass more than half of the segment. Swing equal arcs above and below from each endpoint, connect points of intersection.

Angle bisector: Swing an arc from the vertex that hits each piece of the angle. Swing equal arcs from each new point. Connect the point of intersection to the vertex.

Using Perpendicular Bisector (Jesus Fish):

Median: Construct perpendicular bisector to find midpoint, connect vertex to midpoint.

Line of reflection: Connect any vertex with its image. Construct a perpendicular bisector of that line.

Center of rotation: Connect any two vertices with their images. Construct a perpendicular bisector of both lines. The center of rotation is the intersection of the two perpendicular bisectors.

Perpendicular lines through a given point (Smiley Face): Swing an arc from the point that intersects the line twice. Construct a perpendicular bisector using those two points.

Altitude (Smiley Face): Construct perpendicular line through a point using the vertex as the point and the opposite side as the line.

Triangles Constructions:

Equilateral triangle: Open compass exact amount of segment. Swing equal arcs from each endpoint. Connect the point of intersection to each endpoint.

Isosceles triangle: Open compass not the amount of segment. Swing equal arcs from each endpoint.

Scalene triangle: Open compass not the amount of segment. Swing *different* arcs from each endpoint.

Inscribed Circle Constructions:

Square inscribed inside a circle: Draw in an arbitrary diameter (through the center). Construct the perpendicular bisector. Connect the 4 points on the circle.

Equilateral triangle/Regular hexagon inscribed in a circle: Draw in an arbitrary radius. Open compass exact amount of radius and keep swinging arcs until you get completely around the circle. Connect all six points on the circle to construct hexagon, connect every other point to construct equilateral triangle.

Miscellaneous:

Algebra Skills:

Reducing Radicals

1) Separate into two radicals (perfect squares and non perfect squares). Find the largest perfect square that divides in
2) Take the square root of the perfect square. Bring the non-perfect square down

Solving Quadratic Equations

1) Bring everything to one side
2) Factor
3) Set each factor equal to zero

In Terms of x

1) Call the last thing x
2) Express everything else in terms of x

Look for hidden right triangles (Pythagorean Theorem)

Area with Coordinate Geometry

Box Method

1) Build a rectangle around the shape
2) Find the area of the rectangle $(A = lw)$
3) Find the area of the triangles outside of the shape $(A = \frac{1}{2}lw)$
4) Subtract the triangle areas from the rectangle areas