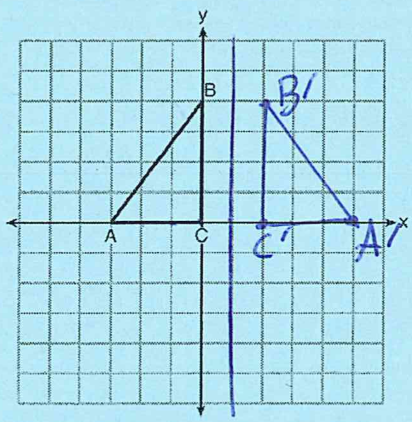
Name: Schlansky

Common Core Geometry *Not as Common* Regents Questions!

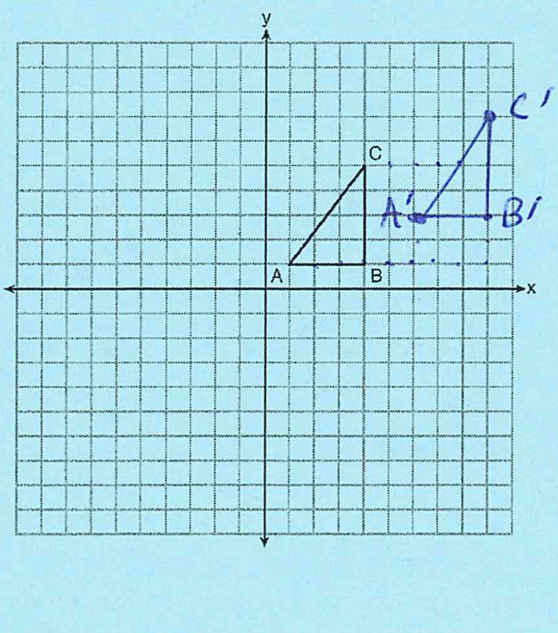
Mr. Schlansky

Rigid Motions
Translations: Slide on the graph
Rotations: Turn the graph counter-clockwise (left), write down coordinates, turn back and plot points
Reflections: Flip (Count to what you are reflecting over)
*Switch the coordinates for reflection over $y = x$
y = # is horizontal line, x = # is vertical line. You must graph these lines before you can reflect over them.

1. Triangle ABC is graphed on the set of axes below. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection over the line $x = 1$.



2. In the diagram below, $\triangle ABC$ has coordinates $A(1, 1)$, $B(4, 1)$, and $C(4, 5)$. Graph and the image of $\triangle ABC$ after the translation five units to the right and two units up.



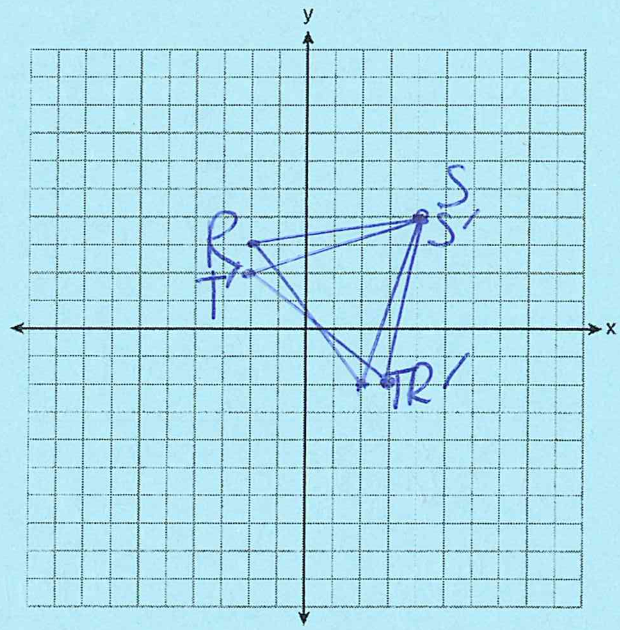
Switch the points

3. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a reflection in the line $y = x$.

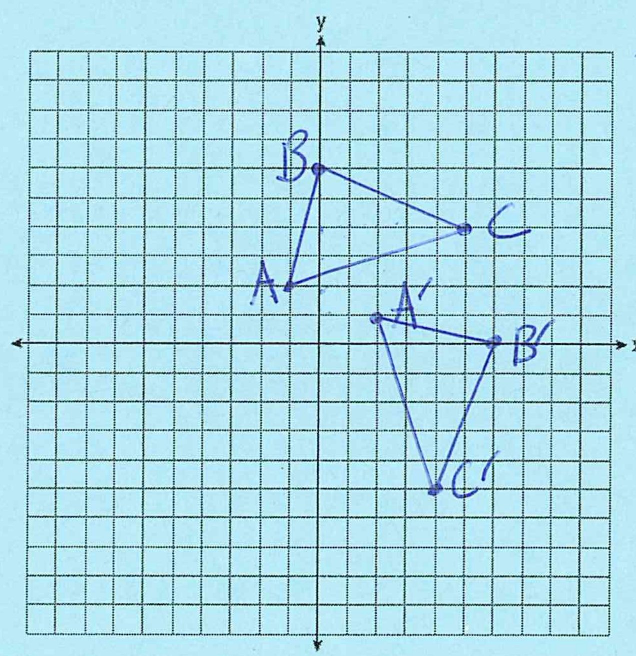
$R(-2,3) \xrightarrow{r_{y=x}} (3,-2)$

$S(4,4) \rightarrow (4,4)$

$T(2,-2) \rightarrow (-2,2)$



4. On the accompanying set of axes, graph $\triangle ABC$ with coordinates $A(-1,2)$, $B(0,6)$, and $C(5,4)$. Then graph $\triangle A'B'C'$, the image of $\triangle ABC$ after a rotation of 270 centered at the origin.

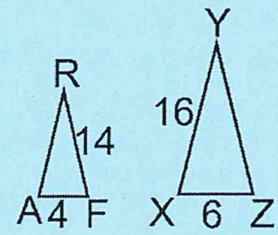


$A'(2,1)$ $B'(6,0)$ $C'(4,-5)$

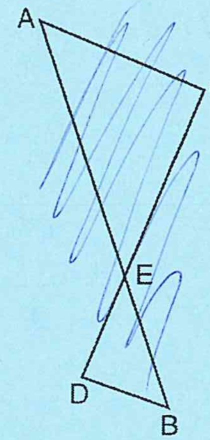
Scale factor = $\dfrac{\textit{image}}{\textit{original}}$

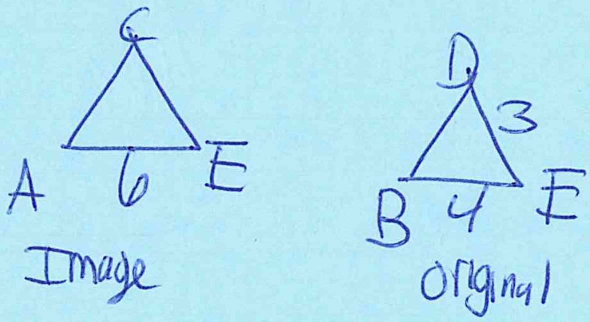
1. In the diagram below, ΔXYZ is the image of ΔARF after a dilation.

What is the scale factor of the dilation? $\frac{6}{4} = \frac{3}{2}$



2. In the diagram below, ΔACE is the image of ΔBDE after a sequence of transformations. If $\overline{AE} = 6$, $\overline{DE} = 3$, and $\overline{EB} = 4$, what is the scale factor?

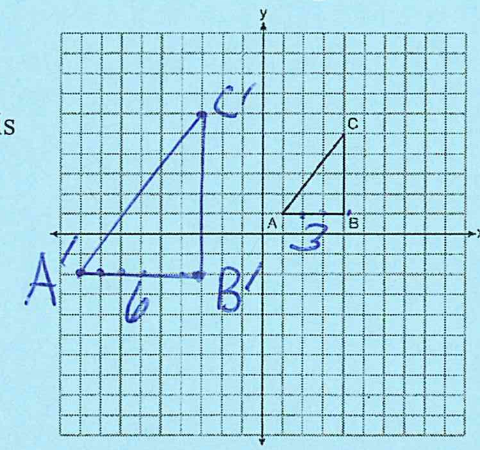




$\frac{6}{4} = 3$

3. In the diagram below, $\triangle ABC$ has coordinates $A(1,1)$, $B(4,1)$, and $C(4,5)$. The coordinates of its image after a sequence of transformations is $A'(-9,-2), B'(-3,-2),$ *and* $C'(-3,6)$. What is the scale factor?

$\frac{6}{3} = 2$



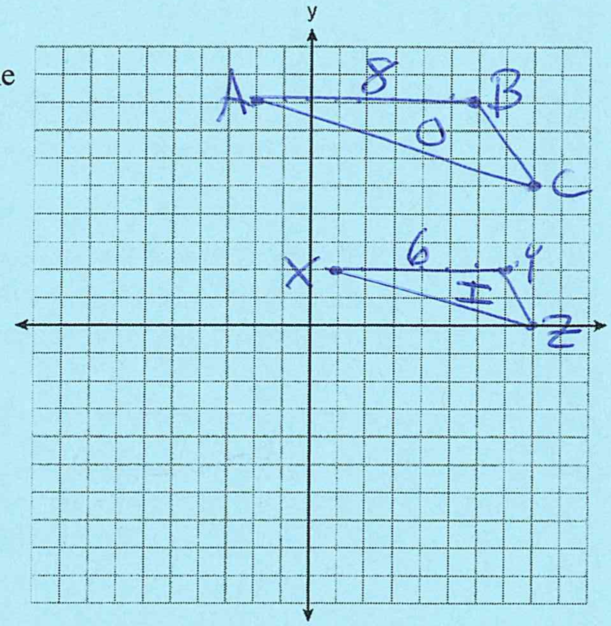
4. After a dilation with center $(0,0)$, the image of $\overline{DB}$ is $\overline{D'B'}$. If $DB = 4.5$ and $D'B' = 18$, the scale factor of this dilation is

1) $\frac{1}{5}$
2) 5
3) $\frac{1}{4}$
4) 4

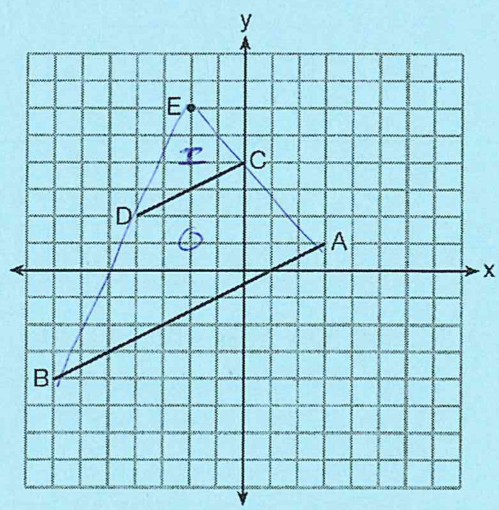
$\frac{18}{4.5} = 4$

5. $\triangle ABC$ has coordinates $A(-2,8), B(6,8),$ *and* $C(8,5)$. The coordinates of ΔXYZ, the image of ΔABC after a sequence of transformations is $X(1,2), Y(7,2),$ *and* $Z(8,0)$. What is the scale factor?

$6/8 = 3/4$



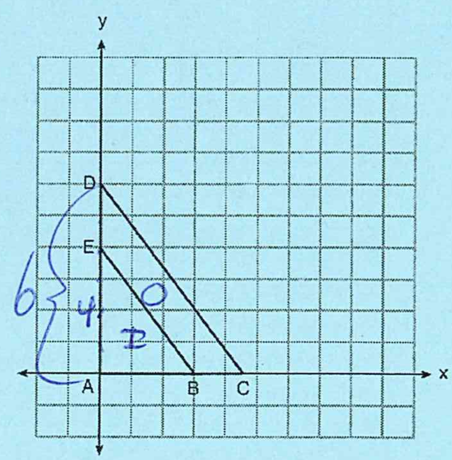
6. In the diagram below, $\overline{CD}$ is the image of $\overline{AB}$ after a dilation of scale factor k with center E.



Which ratio is equal to the scale factor k of the dilation?

1) $\frac{EC}{EA}$ $\frac{I}{O}$
2) $\frac{BA}{EA}$ $\frac{O}{O}$
3) $\frac{EA}{BA}$ $\frac{O}{O}$
4) $\frac{EA}{EC}$ $\frac{O}{I}$

7. In the diagram below, $\triangle ABE$ is the image of $\triangle ACD$ after a dilation centered at the origin. The coordinates of the vertices are $A(0,0)$, $B(3,0)$, $C(4.5,0)$, $D(0,6)$, and $E(0,4)$.



The scale factor of dilation is

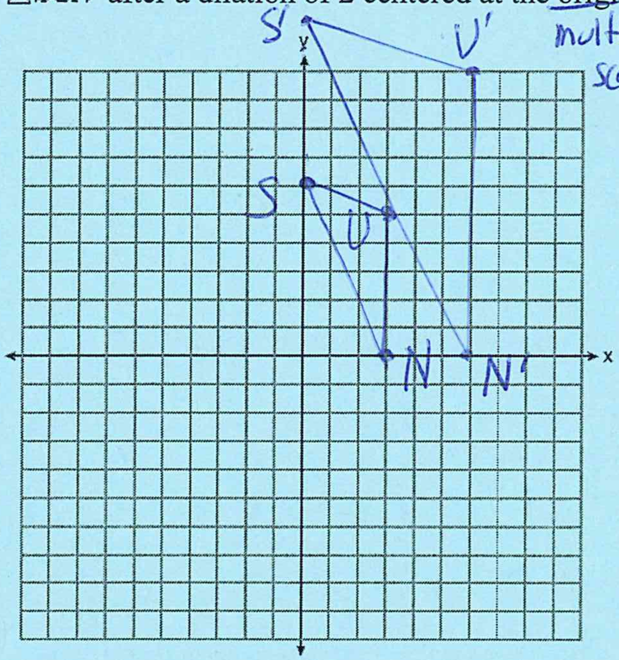
1) $\frac{2}{3}$
2) $\frac{3}{2}$
3) $\frac{3}{4}$
4) $\frac{4}{3}$

$4/6 = 2/3$

Dilations
If centered at the origin: multiply
If centered at a point: Count from the center to each point the number of times of the scale factor.

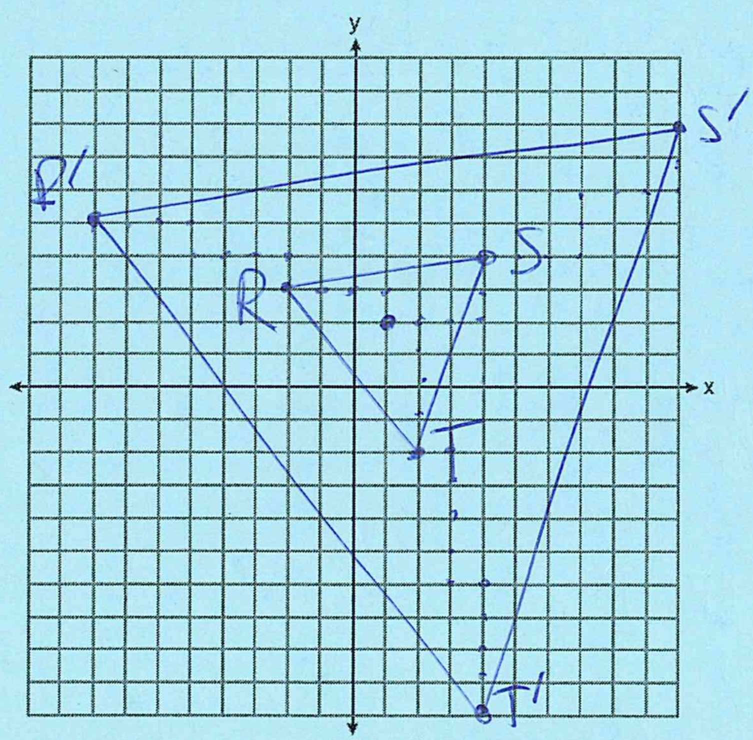
1. Triangle SUN has coordinates $S(0,6)$, $U(3,5)$, and $N(3,0)$. On the accompanying grid, draw and label $\triangle SUN$. Then, graph and state the coordinates of $\triangle S'U'N'$, the image of $\triangle SUN$ after a dilation of 2 centered at the origin.

multiply by scale factor

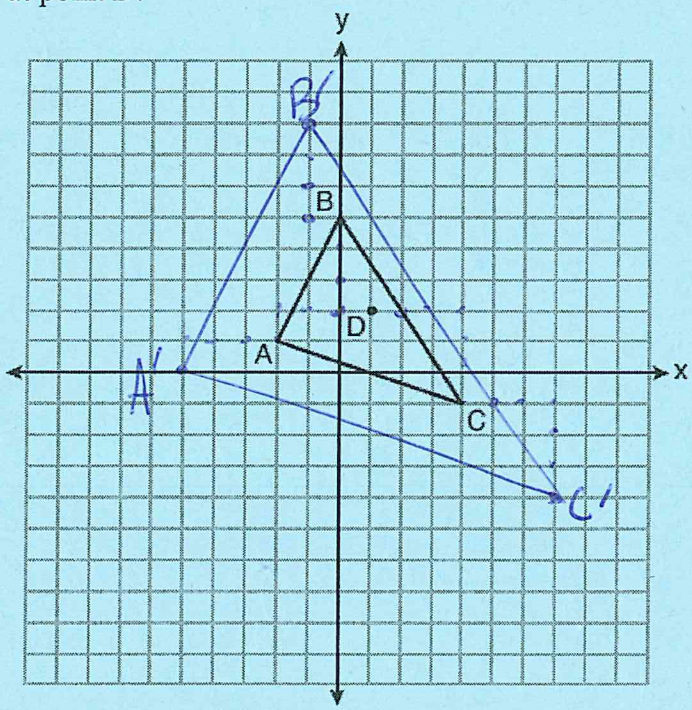


$S(0,6) \xrightarrow{D_2} (0,12)$
$U(3,5) \rightarrow (6,10)$
$N(3,0) \rightarrow (6,0)$

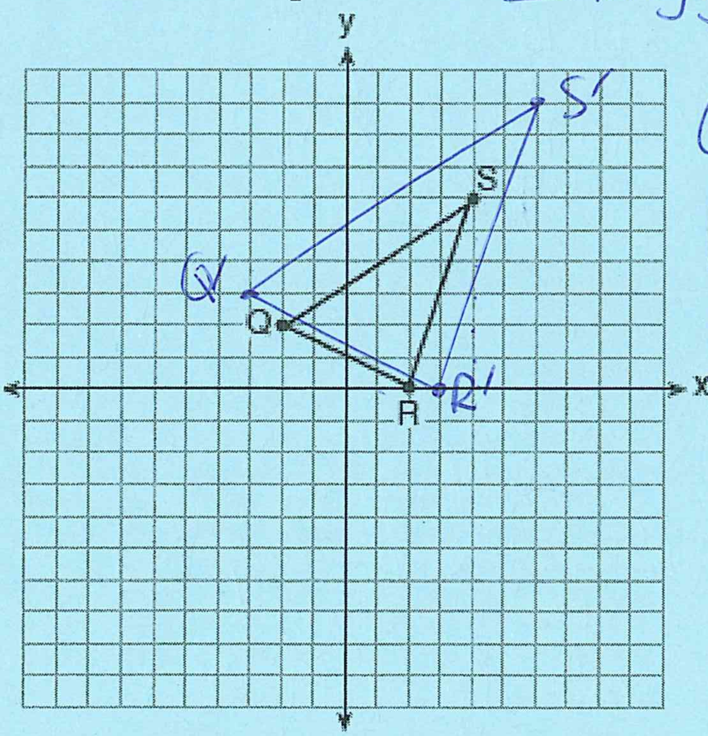
2. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$ and $\triangle R'S'T'$, the image of $\triangle RST$ after a dilation of 3 centered at (1,2).



3. Triangle ABC and point $D(1, 2)$ are graphed on the set of axes below.
Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point D.



4. Triangle QRS is graphed on the set of axes below.
On the same set of axes, graph and label $\triangle Q'R'S'$, the image of $\triangle QRS$ after a dilation with a scale factor of $\frac{3}{2}$ centered at the origin. multiply by the scale factor



$D_{3/2}$

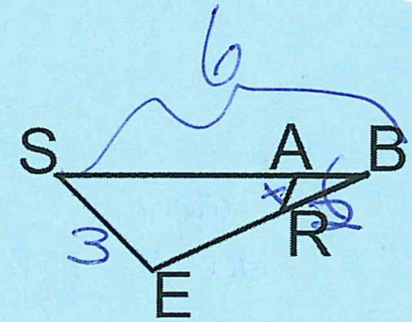
$Q(-2,2) \rightarrow (-3,3)$

$R(2,0) \rightarrow (3,0)$

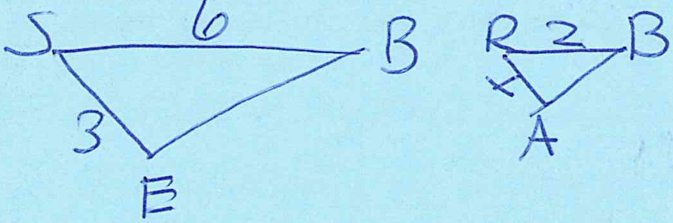
$S(4,6) \rightarrow (6,9)$

Overlapping Similar Triangles

1) Separate the triangles and draw them with the same orientation
2) Match up the corresponding letters (use reflexive property)
3) Create a proportion and solve



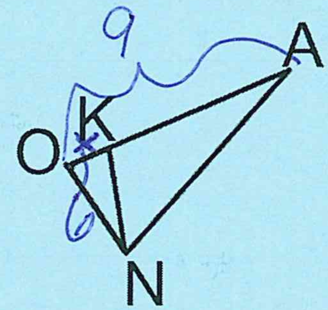
1. In triangle SEB, A is on $\overline{SB}$, and E is on $\overline{EB}$ so that $\angle E \cong \angle BAR$.
If $\overline{SB} = 6$, $\overline{RB} = 2$, and $\overline{SE} = 3$, find $\overline{RA}$.



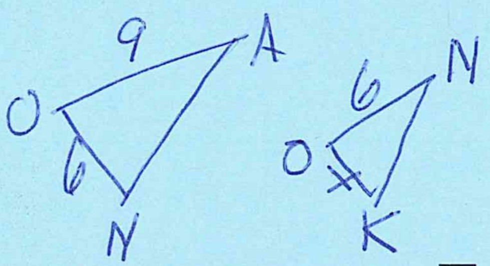
$\frac{6}{2} = \frac{3}{x}$

$\frac{6x}{6} = \frac{6}{6}$

$x = 1$



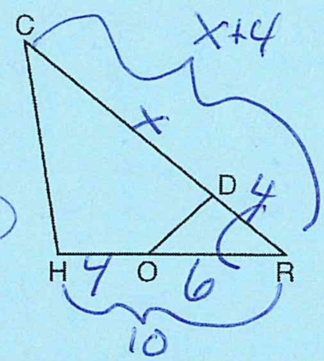
2. In triangle AON, K is on $\overline{AO}$ so that $\angle A \cong \angle ONK$.
If $\overline{ON} = 6$ and $\overline{OA} = 9$, find $\overline{OK}$.



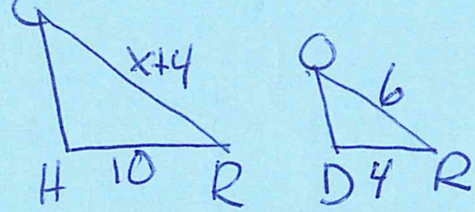
$\frac{9}{6} = \frac{6}{x}$

$\frac{9x}{9} = \frac{36}{9}$

$x = 4$



3. In triangle CHR, O is on $\overline{HR}$, and D is on $\overline{CR}$ so that $\angle H \cong RDO$.
If $RD = 4$, $RO = 6$, and $OH = 4$, what is the length of $\overline{CD}$?



$\frac{x+4}{6} = \frac{10}{4}$

$4(x+4) = 60$

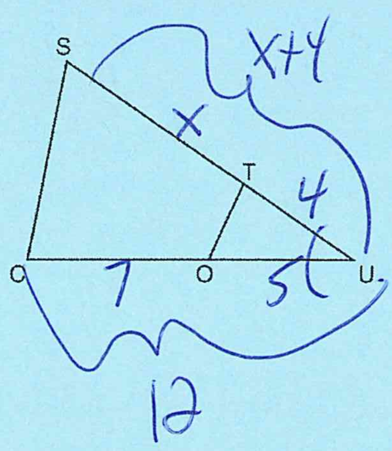
$4x + 16 = 60$

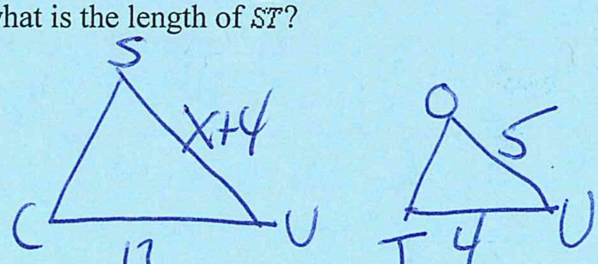
$-16 \quad -16$

$\frac{4x}{4} = \frac{44}{4}$

$x = 11$

4. In $\triangle SCU$ shown below, points T and O are on $\overline{SU}$ and $\overline{CU}$, respectively. Segment OT is drawn so that $\angle C \cong \angle OTU$.
If $TU = 4$, $OU = 5$, and $OC = 7$, what is the length of $\overline{ST}$?





$\frac{x+4}{5} = \frac{12}{4}$

$4(x+4) = 60$

$4x + 16 = 60$

$-16 \quad -16$

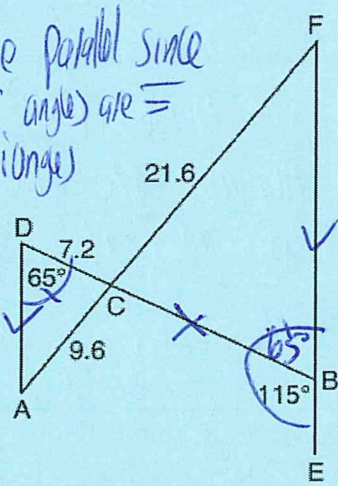
$\frac{4x}{4} = \frac{44}{4}$

$x = 11$

Similarity with Parallel Lines

1. If the lines are parallel, the triangles are similar and the sides are in proportion.
In the diagram below, $\overline{AF}$, and $\overline{DB}$ intersect at C, and $\overline{AD}$ and $\overline{FBE}$ are drawn such that $m\angle D = 65°$, $m\angle CBE = 115°$, $DC = 7.2$, $AC = 9.6$, and $FC = 21.6$. What is the length of $\overline{CB}$?

the lines are parallel since alternate interior angles are ≅ making the triangles similar



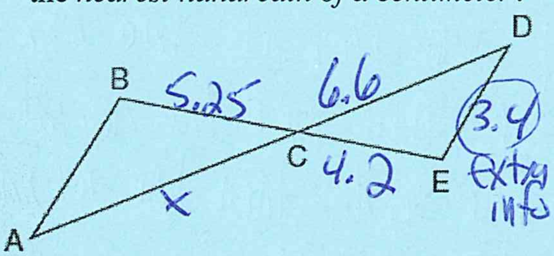
180
−115
65

$$\frac{7.2}{x} = \frac{9.6}{21.6}$$

$$\frac{9.6x}{9.6} = \frac{155.52}{9.6}$$

$$x = 16.2$$

2. In the diagram below, $\overline{AD}$ intersects $\overline{BE}$ at C, and $\overline{AB} \parallel \overline{DE}$.
If $CD = 6.6$ cm, $DE = 3.4$ cm, $CE = 4.2$ cm, and $BC = 5.25$ cm, what is the length of $\overline{AC}$, to the *nearest hundredth of a centimeter*?



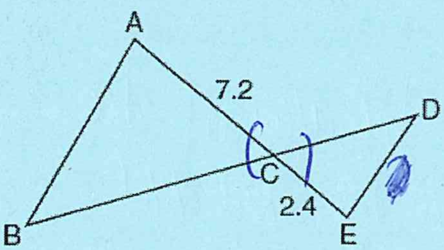
$$\frac{6.6}{x} = \frac{4.2}{5.25}$$

$$\frac{4.2x}{4.2} = \frac{34.65}{4.2}$$

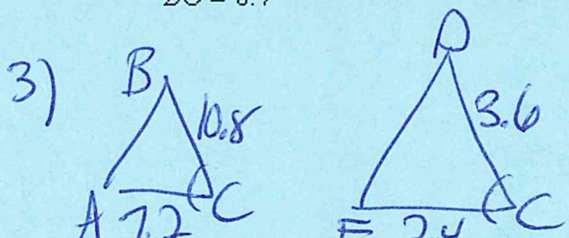
$$x = 8.25$$

3. In the diagram below, $AC = 7.2$ and $CE = 2.4$.
Which statement is *not* sufficient to prove $\triangle ABC \sim \triangle EDC$?

1) $\overline{AB} \parallel \overline{ED}$ ✓ If lines are ∥, △s are similar
2) $DE = 2.7$ and $AB = 8.1$
3) $CD = 3.6$ and $BC = 10.8$
4) $DE = 3.0$, $AB = 9.0$, $CD = 2.9$, and $BC = 8.7$



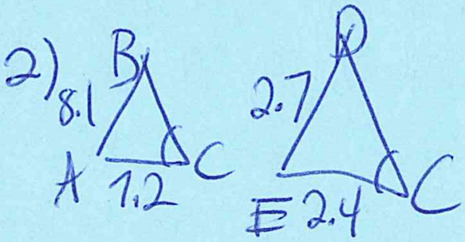
3)



$$\frac{10.8}{3} = \frac{7.2}{2.4}$$

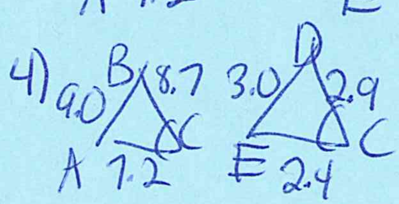
$21.6 = 25.92$ ✗
Not in proportion

2)



✗ Not AA, SAS, or SSS

4)



$$\frac{9}{3} = \frac{8.7}{2.9} = \frac{7.2}{2.4}$$

$3 = 3 = 3$ ✓

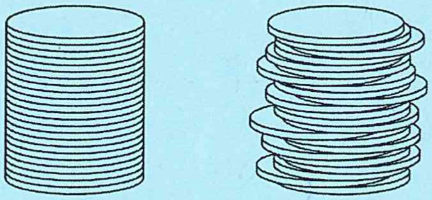
SSS

Cavalieri's Principle

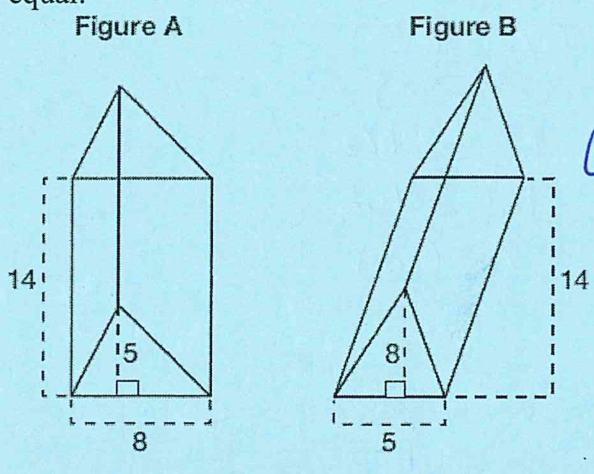
If the area of the bases are congruent, and the heights are congruent, then the volumes are congruent.

1. Two stacks of 23 quarters each are shown below. One stack forms a cylinder but the other stack does not form a cylinder.
Use Cavelieri's principle to explain why the volumes of these two stacks of quarters are equal.

If the area of the bases are the same and the heights are the same then the volumes are the same.



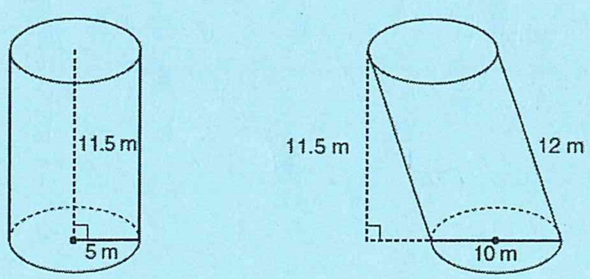
2. The diagram below shows two figures. Figure *A* is a right triangular prism and figure *B* is an oblique triangular prism. The base of figure *A* has a height of 5 and a length of 8 and the height of prism *A* is 14. The base of figure *B* has a height of 8 and a length of 5 and the height of prism *B* is 14.
Use Cavalieri's Principle to explain why the volumes of these two triangular prisms are equal.



If the area of the bases are the same and the heights are the same then the volumes are the same.

3. Sue believes that the two cylinders shown in the diagram below have equal volumes. Is Sue correct? Explain why.

Yes, if the area of the bases are the same and the heights are the same then the volumes are the same.



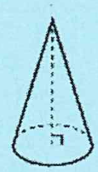
Cross Sections (2 dimensional slice of a 3 dimensional object):
The base of the shape is always one of its cross sections

Rectangular Prism: Rectangle, triangle
Cylinder: Circle, ellipse, rectangle
Cone: Circle, ellipse, triangle, "curved" rectangle
Pyramid: Rectangle, triangle
Sphere: Circle

1. Which type of shape can represent a two-dimensional cross-section of a sphere?
(1) circular (2) triangular (3) square (4) rectangular

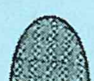
2. Which is *not* a possible two-dimensional cross section of a three-dimensional cylinder?
(1) circle (2) rectangle (3) ellipes (4) triangle

3.
William is drawing pictures of cross sections of the right circular cone below.

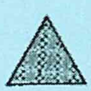


Which drawing can *not* be a cross section of a cone?

(1)



(3)



(2)

(4)

Vertical

4. A plane intersects a hexagonal prism. The plane is perpendicular to the base of the prism. Which two-dimensional figure is the cross section of the plane intersecting the prism?
1) triangle 3) hexagon
2) trapezoid 4) rectangle

5. The cross section of a regular pyramid contains the altitude of the pyramid. The shape of this cross section is a

Vertical

1) circle
2) square
3) triangle
4) rectangle

Area with Coordinate Geometry
Box Method

1) Build a rectangle around the shape
2) Find the area of the rectangle $(A = lw)$
3) Find the area of the triangles outside of the shape $(A = \frac{1}{2}lw)$
4) Subtract the triangle areas from the rectangle areas

1. Find the area of triangle ABC if A(0,2), B(2,3), and C(1,5).

$A_{Rectangle} = lw$
$= 2(3)$
$= 6$

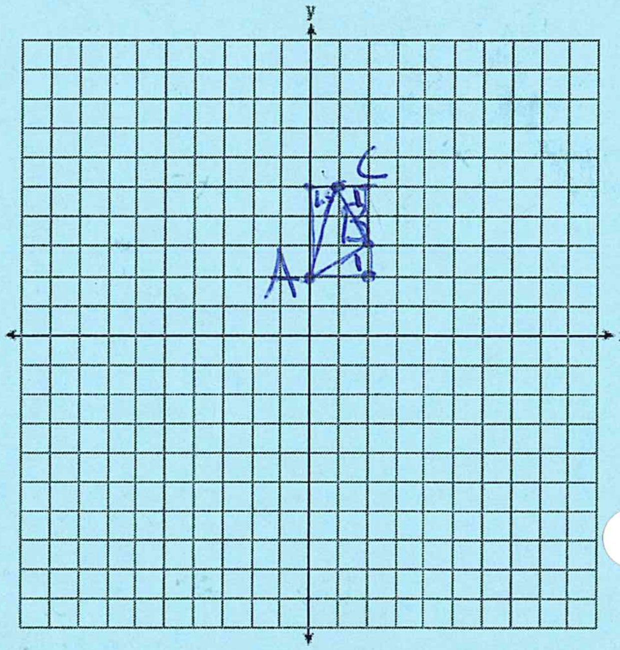
$A_{T1} = \frac{1}{2}bh$
$= \frac{1}{2}(2)(1)$
$= 1$

$A_{T2} = \frac{1}{2}bh$
$= \frac{1}{2}(1)(2)$
$= 1$

$A_{T3} = \frac{1}{2}bh$
$= \frac{1}{2}(1)(3)$
$= 1.5$

$1 + 1 + 1.5 = 3.5$

$A_{Rect} - A_{Tri's}$
$6 - 3.5 = 2.5$



2. Triangle USA has vertices U(4,-7), S(-3,-4), and A(7,0). Find the area of triangle USA.

$A_{rectangle} = lw$
$= 10(7)$
$= 70$

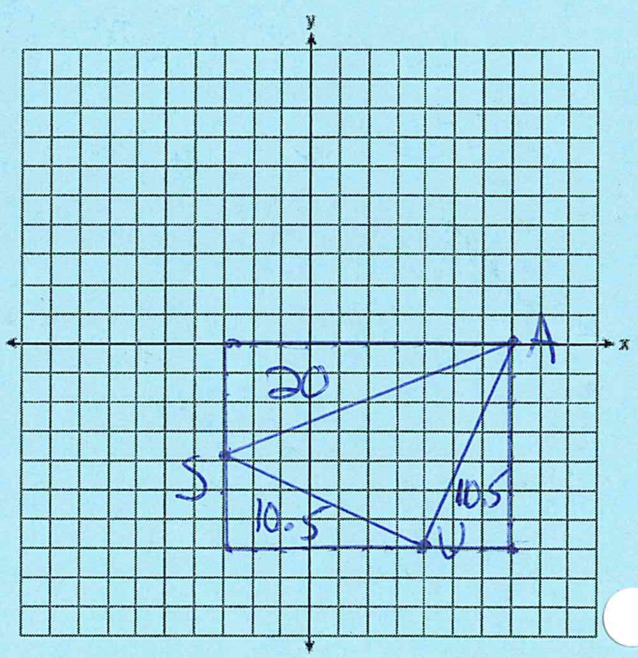
$A_{T1} = \frac{1}{2}bh$
$= \frac{1}{2}(7)(3)$
$= 10.5$

$A_{T2} = \frac{1}{2}bh$
$A_{T2} = \frac{1}{2}(3)(7)$
$= 10.5$

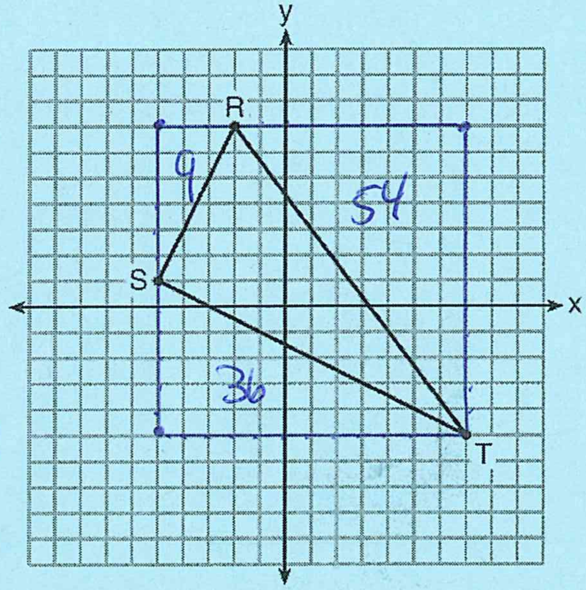
$A_{T3} = \frac{1}{2}bh$
$= \frac{1}{2}(10)(4)$
$= 20$

$10.5 + 10.5 + 20 = 41$

$A_{rect} - A_{Tri's}$
$70 - 41 = 29$



3. Triangle RST is graphed on the set of axes below.



How many square units are in the area of $\triangle RST$?

1) $9\sqrt{3}+15$
2) $9\sqrt{5}+15$
3) 45
4) 90

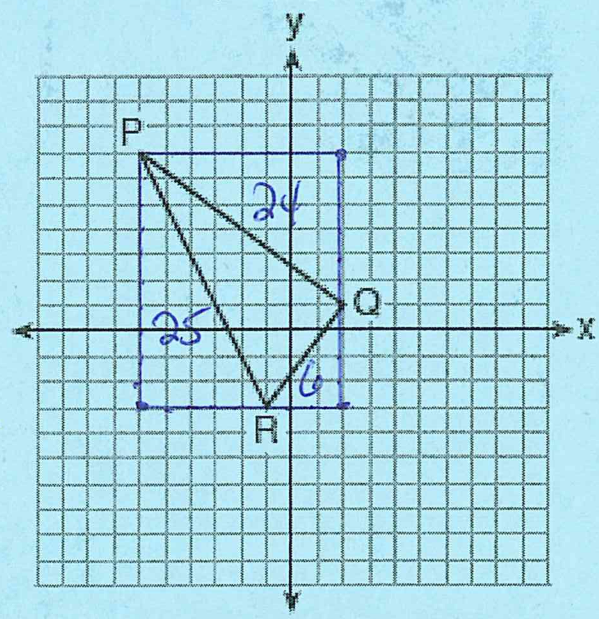
$A_{rect} = lw$
$A_{rect} = 12(12)$
$A_{rect} = 144$

$A_{T1} = \frac{1}{2}bh$
$= \frac{1}{2}(12)(6)$
$= 36$
$A_{T2} = \frac{1}{2}(6)(3)$
$= 9$
$A_{T3} = \frac{1}{2}(12)(9)$
$A_{T3} = 54$

$9+36+54 = 99$

$A_{rect} - A_{tris}$
$144 - 99 = 45$

4. On the set of axes below, the vertices of $\triangle PQR$ have coordinates $P(-6,7)$, $Q(2,1)$, and $R(-1,-3)$.



$A_{rect} = lw$
$= 8(10)$
$= 80$

$A_{T1} = \frac{1}{2}bh$
$= \frac{1}{2}(5)(10)$
$= 25$
$A_{T2} = \frac{1}{2}(3)(4)$
$= 6$
$A_{T3} = \frac{1}{2}(6)(8)$
$= 24$

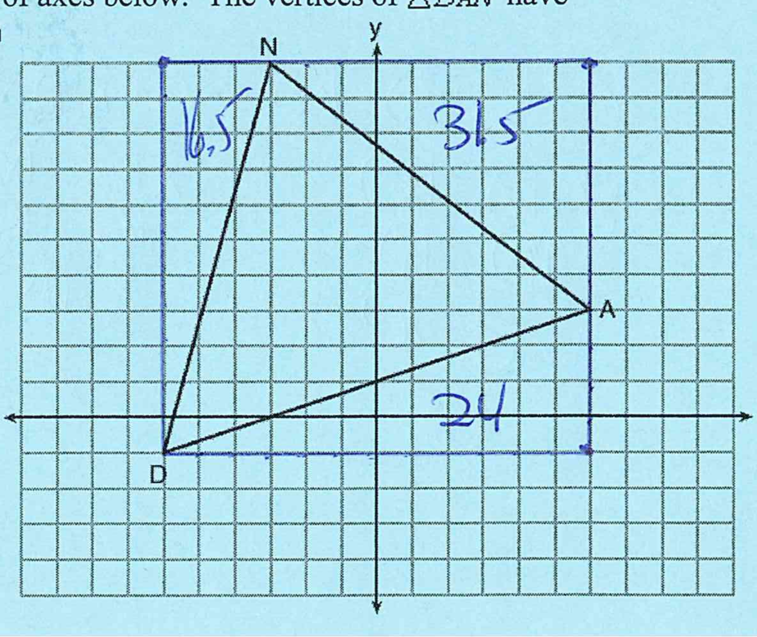
$25+24+6 = 55$

What is the area of $\triangle PQR$?

1) 10
2) 20
3) 25
4) 50

$A_{rect} - A_{tris}$
$80 - 55 = 25$

5. Triangle DAN is graphed on the set of axes below. The vertices of $\triangle DAN$ have coordinates $D(-6,-1)$, $A(6,3)$, and N



$A_{rect} = lw$
$= 12(11)$
$= 132$

What is the area of $\triangle DAN$?

1) 60
2) 120
3) $20\sqrt{13}$
4) $40\sqrt{13}$

$A_{T1} = \frac{1}{2}bh$
$= \frac{1}{2}(12)(4)$
$= 24$
$A_{T2} = \frac{1}{2}(9)(7)$
$= 31.5$
$A_{T3} = \frac{1}{2}(3)(11)$
$A_{T3} = 16.5$

24
+31.5
+16.5
72

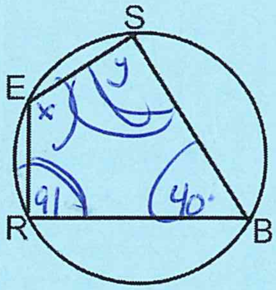
$A_{rect} - A_{tris}$
$132 - 72$
60

Quadrilateral Inscribed In a Circle
Opposite angles are supplementary (add to 180)

1. In the diagram below, quadrilateral $SBRE$ is inscribed in the circle. If $m\angle BRE = 91$ and $m\angle SBR = 40$, find $m\angle BSE$ and $m\angle SER$

y
89

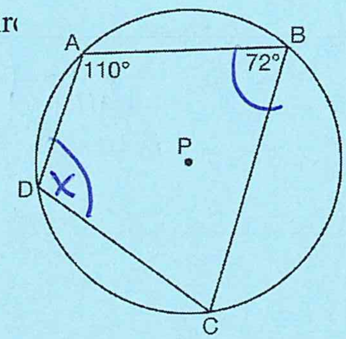
x
140



$x+40=180$
$-40 \quad -40$
$x=140$

$y+91=180$
$-91 \quad -91$
$y=89$

2. In the diagram below, quadrilateral $ABCD$ is inscribed in cir

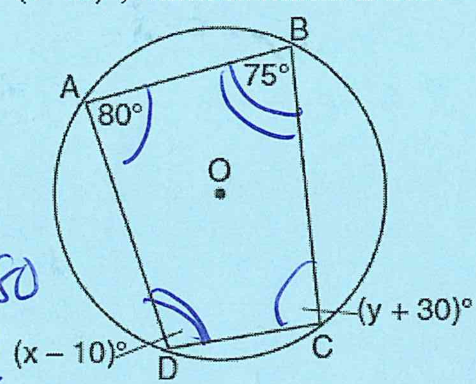


What is $m\angle ADC$?
1) 70°
2) 72°
3) 108°
4) 110°

$x+72=180$
$-72 \quad -72$
$x=108$

3. Quadrilateral $ABCD$ is inscribed in circle O, as shown below.

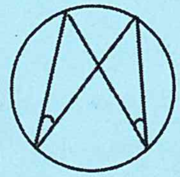
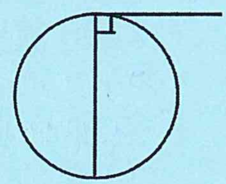
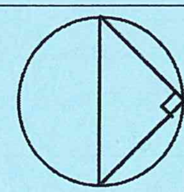
If $m\angle A = 80°$, $m\angle B = 75°$, $m\angle C = (y + 30)°$, and $m\angle D = (x - 10)°$, which statement is true?
1) $x = 85$ and $y = 50$
2) $x = 90$ and $y = 45$
3) $x = 110$ and $y = 75$
4) $x = 115$ and $y = 70$



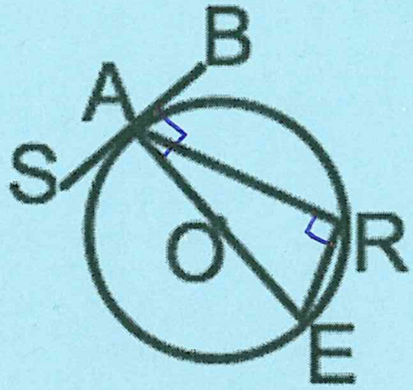
$80+y+30=180$
$y+110=180$
$-110 \quad -110$
$y=70$

$75+x-10=180$
$x+65=180$
$-65 \quad -65$
$x=115$

Special Angles in a Circle (Look for Inscribed Angles)

Angles inscribed to the same/congruent arcs are congruent.	
A tangent and radius/diameter intersect to form a right angle.	
An angle is inscribed to a semicircle/diameter is a right angle.	

1. In circle O shown below, $\overline{AE}$ is a diameter, $\overline{SB}$ is a tangent, and chord $\overline{AR}$ and $\overline{RE}$ are drawn.



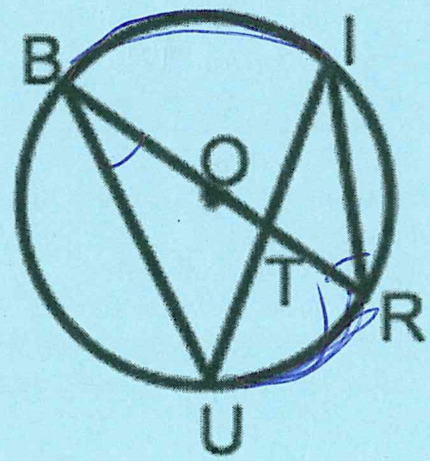
Which of the following statements is true?

1) $\angle EAR \cong \angle RAB$
2) $\angle REA \cong \angle SAE$
3) $\angle SAR \cong \angle BAE$
4) $\angle ERA \cong \angle BAE$

angle inscribed to semi-circle

tangent and diameter

2. In circle O shown below, $\overline{BR}$ is a diameter and chords $\overline{BU}$, $\overline{IU}$, and $\overline{IR}$ are drawn.



Which of the following statements is *not* true?

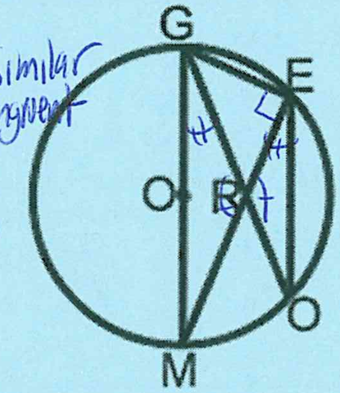
inscribed to same arc

1) $\angle BUI \cong \angle BRI$
2) $\angle ITR \cong \angle BTU$
3) $\angle UBT \cong \angle BRI$
4) $\angle RBU \cong \angle RIU$

vertical angles

inscribed to same arc

3. In circle O shown below, $\overline{GM}$ is a diameter and chords $\overline{EM}$, $\overline{OG}$, $\overline{EG}$ and $\overline{EO}$ are drawn.



inscribed to same arc

they are similar but not congruent

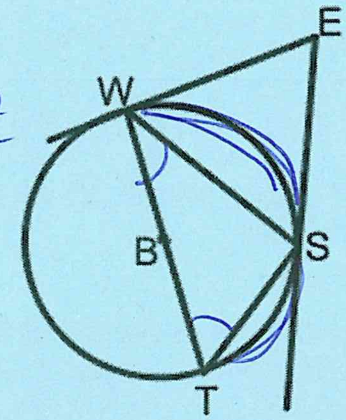
Which of the following statements is *not* true?

1) $\angle MEO \cong \angle OGM$ ✓
2) $\angle GRM \cong \angle ORE$ ✓
3) $\Delta MGR \cong \Delta EOR$ ✗
4) $\angle GEM$ is a right angle ✓

angle inscribed to a semicircle

vertical angles

4. In circle B shown below, $\overline{TW}$ is a diameter, tangents EW and ES are drawn and chords $\overline{WS}$ and $\overline{TS}$ are drawn.



inscribed to same arc

inscribed to same arc

Which of the following statements is *not* true?

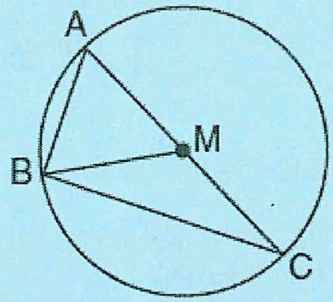
1) $\angle ESW \cong \angle WTS$ ✓
2) $\angle WST \cong \angle EWT$ ✓
3) $\angle EWS \cong \angle ESW$ ✓
4) $\angle TWS \cong \angle STW$ ✗

both right angles

angle inscribed to semicircle

tangent diameter

5. In circle M below, diameter $\overline{AC}$, chords $\overline{AB}$ and $\overline{BC}$, and radius $\overline{MB}$ are drawn.



Which statement is *not* true?

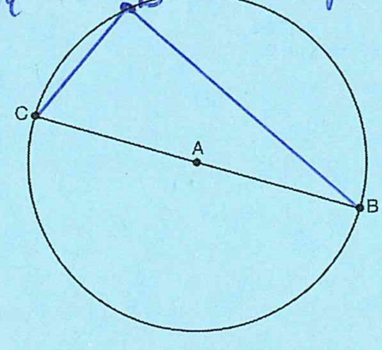
1) $\triangle ABC$ is a right triangle. ✓ angle inscribed to semicircle
2) $\triangle ABM$ is isosceles. ✓ $AM \cong BM$, All radii are $\cong$
3) $m\overset{\frown}{BC} = m\angle BMC$ ✓ central angle = intercepted arc
4) $m\overset{\frown}{AB} = \frac{1}{2} m\angle ACB$ ✗

$\frac{1}{2} m\overset{\frown}{AB} = m\angle ACB$

6. In the diagram below, $\overline{BC}$ is the diameter of circle A.

Point D, which is unique from points B and C, is plotted on circle A. Which statement must always be true?

1) $\triangle BCD$ is a right triangle. ✓ angle inscribed to semicircle
2) $\triangle BCD$ is an isosceles triangle. ✗
3) $\triangle BAD$ and $\triangle CBD$ are similar triangles. ✗
4) $\triangle BAD$ and $\triangle CAD$ are congruent triangles. ✗



put it somewhere random