Name _____________ Date _____________
Mr. Schlansky Geometry

Proofs Regents Review

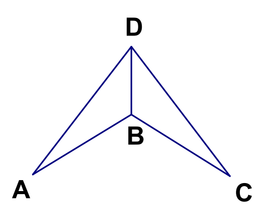
Euclidean Proofs (Basic)

If it is not specified, prove triangles are congruent
To prove triangles are congruent, prove 3 pairs of sides/angles are congruent
To prove segments or angles, use CPCTC
<u>*If you get stuck, make something up and keep on going!</u>

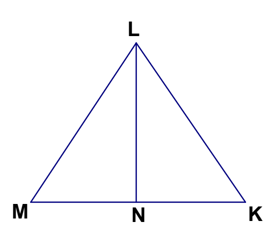
1) Do a mini proof with your givens
Altitude creates congruent right angles
Median creates congruent segments
Line bisector creates congruent segments
Midpoint creates congruent segments
Angle bisector creates congruent angles
Perpendicular lines create congruent right angles
When given parallel lines:
Corresponding angles are congruent OR Alternate interior angles are congruent OR Alternate exterior angles are congruent

2) Use additional tools:
Vertical Angles are congruent (Look for an X)
Reflexive Property (A side/angle is congruent to itself)

1. Given: $\overline{BD}$ bisects $\angle ADC$
$\overline{AD} \cong \overline{DC}$
Prove: $\overline{AB} \cong \overline{BC}$

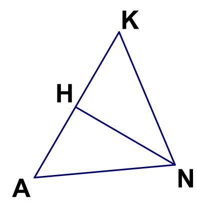


2. Given: $\overline{LN}$ bisects $\angle KLM$
$\angle LKM \cong \angle LMK$
Prove: $\overline{NM} \cong \overline{NK}$



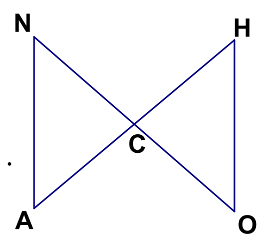
3. Given: $\overline{HN} \perp \overline{KA}$, $\overline{KN} \cong \overline{AN}$

Prove: $\angle HAN \cong \angle HKN$



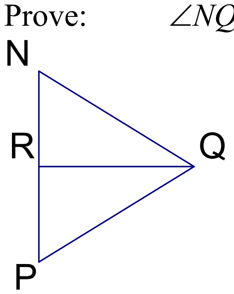
4. Given: $\overline{NO}$ and $\overline{HA}$ bisect each other

Prove: $\overline{NA} \cong \overline{HO}$



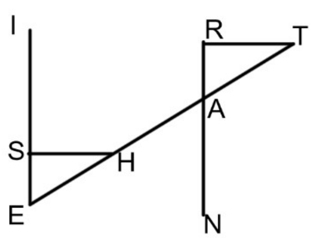
5. Given: $\overline{QR}$ is the perpendicular bisector of $\overline{NP}$

Prove: $\angle NQR \cong \angle PQR$



6. Given: $\overline{IE} \parallel \overline{RN}$, $\overline{TR} \perp \overline{RN}$, $\overline{HS} \perp \overline{IE}$, $\overline{EH} \cong \overline{AT}$

Prove: $\overline{SH} \cong \overline{RT}$



Euclidean Similar Triangle Proofs

To prove triangles are SIMILAR, prove AA $\cong$ AA

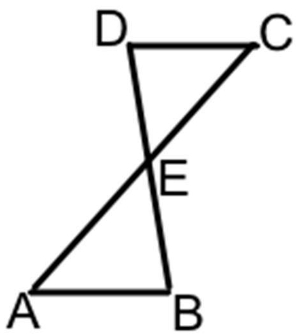
If asked to prove a proportion/multiplication:

1) **Prove triangles are similar**
2) **Corresponding Sides of Similar Triangle are In Proportion (CSSTIP)**
3) **Cross Products are Equal**

Work Backwards!

1. Given $\overline{AB} \parallel \overline{DC}$

   Prove: $\Delta ABE \sim \Delta CDE$



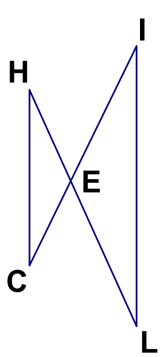
2. Given: $\overline{BC} \perp \overline{AC}$

   $\overline{DE} \perp \overline{AB}$

   Prove: $\Delta ABC \sim \Delta ADE$

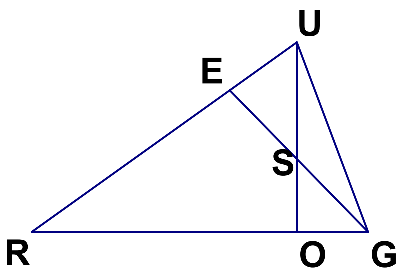
3. Given: $\angle HCE \cong \angle LIE$

   Prove: $\dfrac{\overline{CE}}{\overline{CH}} = \dfrac{\overline{EI}}{\overline{IL}}$



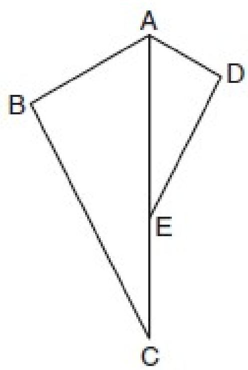
4. Given: $\overline{UO} \perp \overline{RG}$, $\overline{UR} \perp \overline{EG}$

Prove: $\frac{\overline{US}}{\overline{SO}} = \frac{\overline{EU}}{\overline{OG}}$



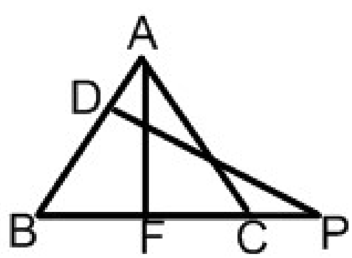
5. Given: $\overline{CA}$ bisects $\angle CAD$, $\angle ABC \cong \angle ADE$

Prove: $\overline{BC} \bullet \overline{AE} = \overline{DE} \bullet \overline{AC}$



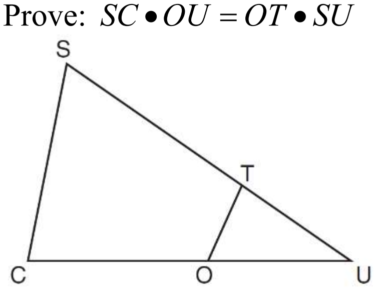
6. Given: $\overline{AB} \cong \overline{AC}$, $\overline{AF} \perp \overline{BC}$, $\overline{PD} \perp \overline{AB}$

Prove: $\overline{FC} \bullet \overline{PB} = \overline{DB} \bullet \overline{AC}$



7. In $\triangle SCU$ shown below, points T and O are on $\overline{SU}$ and $\overline{CU}$, respectively. Segment OT is drawn so that $\angle C \cong \angle OTU$.

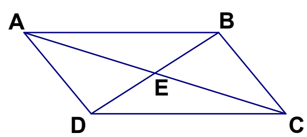
Prove: $\overline{SC} \bullet \overline{OU} = \overline{OT} \bullet \overline{SU}$



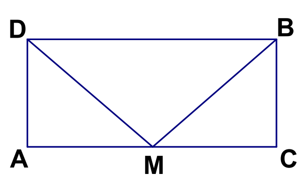
Euclidean Proofs with Parallelogram and Circle Theorems

<table>
<tr><th>Parallelogram Theorems</th><th>Circle Theorems</th></tr>
<tr><td>A parallelogram/rectangle/rhombus/square has:
Two pairs of opposite sides congruent
Two pairs of opposite sides parallel
Diagonals that bisect each other
Opposite angles congruent</td><td rowspan="3">All radii/diameters of a circle are congruent
Angles inscribed to the same arc are congruent
An angle inscribed to a semicircle is a right angle
A tangent and a radius/diameter form a right angles
Congruent arcs have congruent chords have congruent central angles
Parallel Lines intercept congruent arcs
Tangents drawn from the same point are congruent</td></tr>
<tr><td>A rectangle/square has:
A right angle
Congruent diagonals</td></tr>
<tr><td>A rhombus/square has:
All sides congruent
Perpendicular diagonals
Diagonals that bisect the angles</td></tr>
</table>

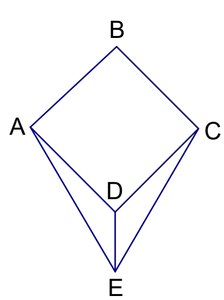
1. Given: ABCD is a parallelogram
Prove: $\Delta AED \cong \Delta CEB$



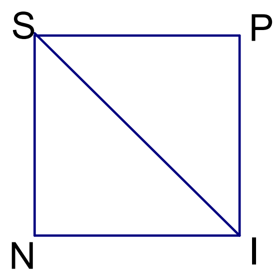
2. Given: ABCD is a rectangle, M is the midpoint of $\overline{AC}$
Prove: $\overline{DM} \cong \overline{BM}$



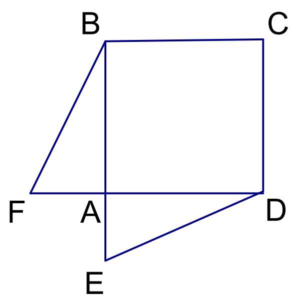
3. Given: ABCD is a rhombus, $\overline{AE} \cong \overline{CE}$
Prove: $\angle ADE \cong \angle CDE$



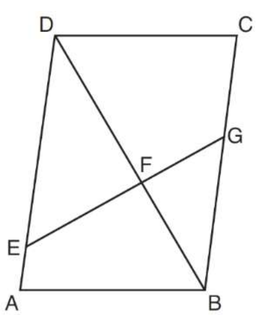
4. Given: SPIN is a square
Prove: $\Delta SNI \cong \Delta SPI$



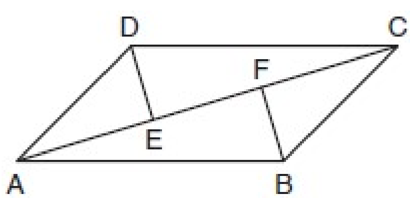
5. Given: ABCD is a square, $\overline{FA} \cong \overline{AE}$
Prove: $\overline{BF} \cong \overline{DE}$



6. Given: Parallelogram $ABCD$, $\overline{EFG}$, and diagonal $\overline{DFB}$
Prove: $\triangle DEF \sim \triangle BGF$

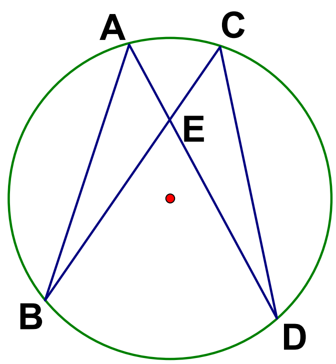


7. In parallelogram $ABCD$, $\overline{BF}$ and $\overline{DE}$ are perpendicular to diagonal $\overline{AC}$ at points F and E.
Prove: $\overline{AE} \cong \overline{CF}$



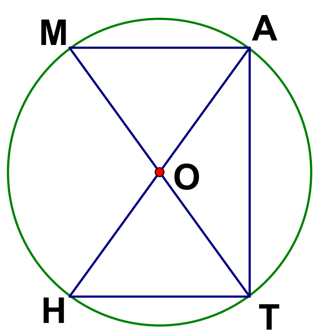
8. Given: Chords $\overline{AD}$ and $\overline{BC}$ of circle O intersect at E, $\overline{AB} \cong \overline{CD}$

Prove: $\overline{BC} \cong \overline{AD}$

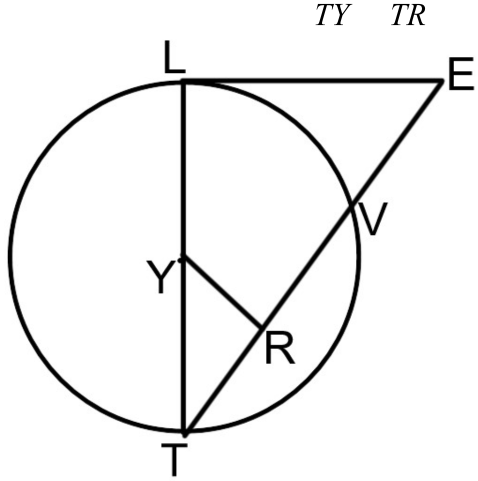


9. Given: Circle O with diameters $\overline{MOT}$ and $\overline{AOH}$.

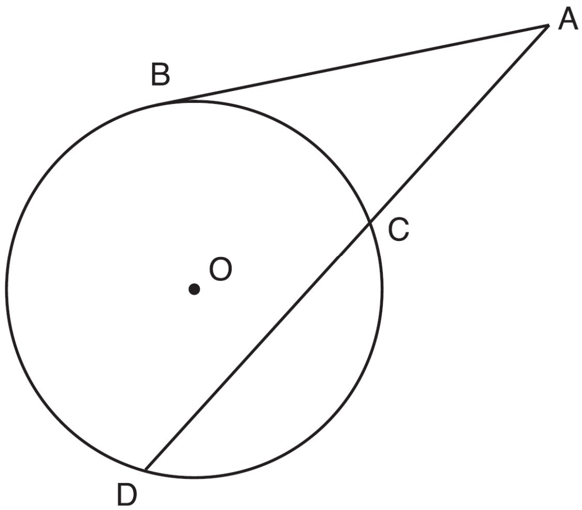
Prove: $\overline{MA} \cong \overline{HT}$



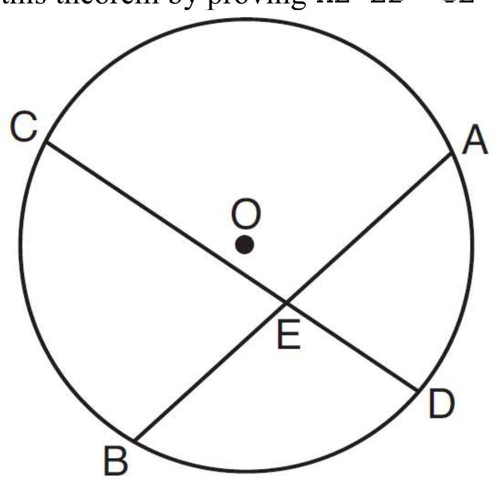
10. In circle Y, tangent $\overline{LE}$ is drawn to diameter $\overline{TYL}$ and $\overline{YR} \perp \overline{TE}$. Prove that $\frac{\overline{TE}}{\overline{TY}} = \frac{\overline{TL}}{\overline{TR}}$.



11. In the diagram below, secant $\overline{ACD}$ and tangent $\overline{AB}$ are drawn from external point A to circle O.
Prove the theorem: If a secant and a tangent are drawn to a circle from an external point, the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared. $(AC \cdot AD = AB^2)$



12. Given: Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E
Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. Prove this theorem by proving $AE \cdot EB = CE \cdot ED$.

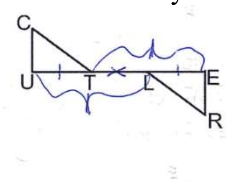


Euclidean Triangle Proofs with Additional Tools

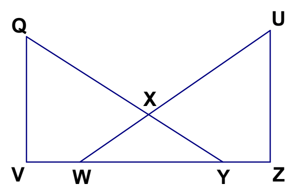
Vertical Angles are congruent (Look for an X)
Reflexive Property (A side/angle is congruent to itself)
Isosceles Triangles (In a triangle, congruent angles are opposite congruent sides)
Addition and Subtraction Property (If you need more or less of a shared side)
*You must use three congruent statements to get one congruent statement for the triangles. The two that you are adding/subtracting and the one that you want to prove in the triangle.

7. Given: $\overline{UL} \cong \overline{TE}$
Prove: $\overline{UT} \cong \overline{LE}$

statements	Reasons
① $\overline{UL} \cong \overline{TE}$	① given
② $\overline{TL} \cong \overline{TL}$	② reflexive property
③ $\overline{UT} \cong \overline{LE}$	③ subtraction property

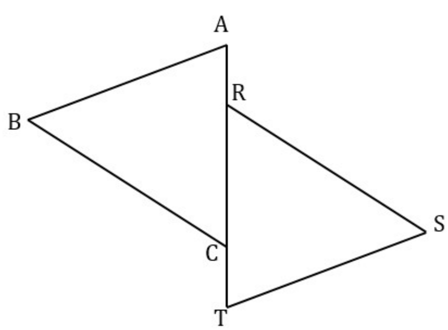


1. Given: $\overline{QV} \cong \overline{UZ}$
$\overline{VW} \cong \overline{YZ}$
$\overline{YQ} \cong \overline{WU}$
Prove: $\angle Q \cong \angle U$

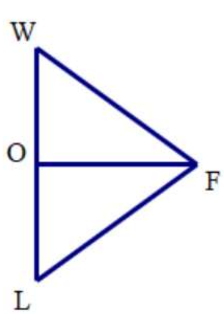


2. Given: $\angle B \cong \angle S$, $\overline{AB} \parallel \overline{ST}$, $\overline{AR} \cong \overline{TC}$

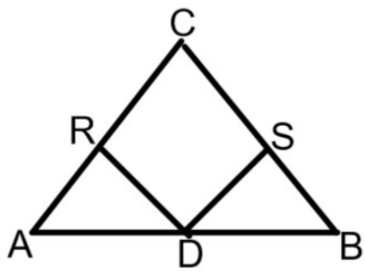
Prove: $\overline{BC} \cong \overline{SR}$



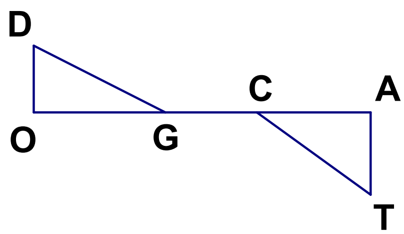
3. Given: $\overline{OF}$ is the perpendicular bisector of $\overline{WL}$
Prove: ΔWFL is isosceles



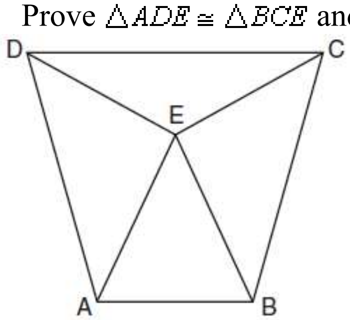
4. Given: In ΔABC, $\overline{CA} \cong \overline{CB}$, $\overline{AR} \cong \overline{BS}$, $\overline{DR} \perp \overline{AC}$,
and $\overline{DS} \perp \overline{BC}$
Prove: $\overline{DR} \cong \overline{DS}$



5. Given: $\overline{DO} \perp \overline{OA}$, $\overline{TA} \perp \overline{OA}$, $\overline{DO} \cong \overline{TA}$, $\overline{OC} \cong \overline{AG}$
Prove: $\overline{DG} \cong \overline{TC}$



6. Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.
Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.



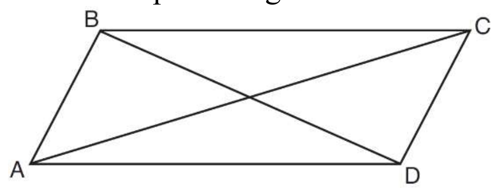
Euclidean Parallelogram Proofs/Parallelogram Properties

To prove parallelograms: Always prove parallelogram first. You will probably have to use congruent triangles with CPCTC to get at least one of the properties.

1. Quadrilateral *ABCD* with diagonals $\overline{AC}$ and $\overline{BD}$ is shown in the diagram below.

Which information is *not* enough to prove *ABCD* is a parallelogram?

1) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{DC}$
2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$
3) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$
4) $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$

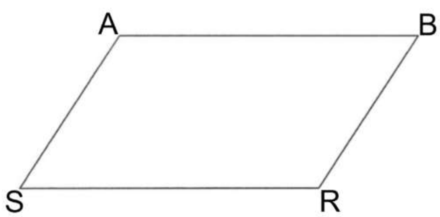


2. Quadrilateral *ABCD* has diagonals $\overline{AC}$ and $\overline{BD}$. Which information is *not* sufficient to prove *ABCD* is a parallelogram?

1) $\overline{AC}$ and $\overline{BD}$ bisect each other.
2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$
3) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$
4) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$

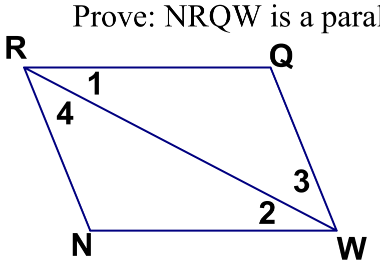
3. Given: $\overline{SA} \cong \overline{BR}$, $\overline{AB} \cong \overline{SR}$

Prove: SABR is a parallelogram

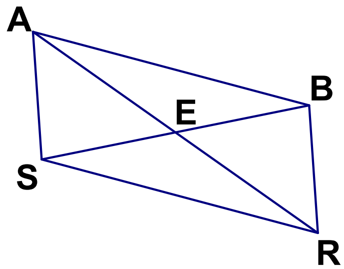


4. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

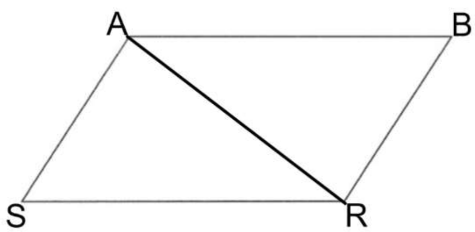
Prove: NRQW is a parallelogram



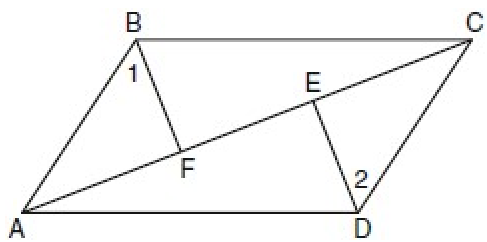
5. Given: E is the midpoint of $\overline{SB}$, $\overline{AE} \cong \overline{ER}$
Prove: SABR is a parallelogram



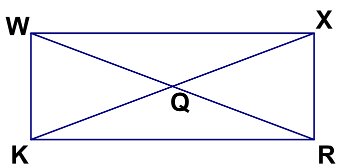
6. Given: $\Delta ASR \cong \Delta RBA$
Prove: SABR is a parallelogram



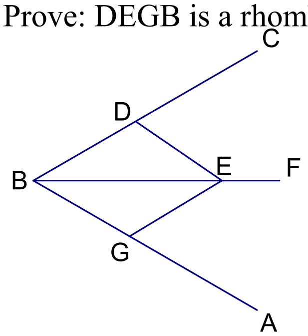
7. Given: Quadrilateral $ABCD$, diagonal $\overline{AFEC}$, $\overline{AE} \cong \overline{FC}$, $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$, $\angle 1 \cong \angle 2$
Prove: $ABCD$ is a parallelogram.



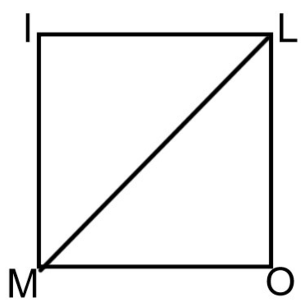
8. Given: WXRK is a parallelogram, $\overline{KW} \perp \overline{WX}$
Prove: WXRK is a rectangle



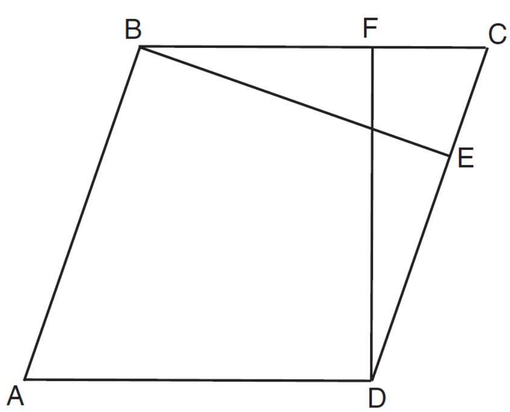
9. Given: BDEG is a parallelogram, $\overline{BF}$ bisects $\angle$CBA
Prove: DEGB is a rhombus



10. Given: MILO is a parallelogram, $\angle IML \cong \angle OML$, $\overline{MI} \perp \overline{IL}$
Prove: MILO is a square

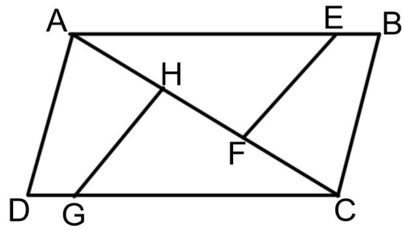


11. In the diagram of parallelogram *ABCD* below, $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$, $\overline{CE} \cong \overline{CF}$.
Prove *ABCD* is a rhombus.



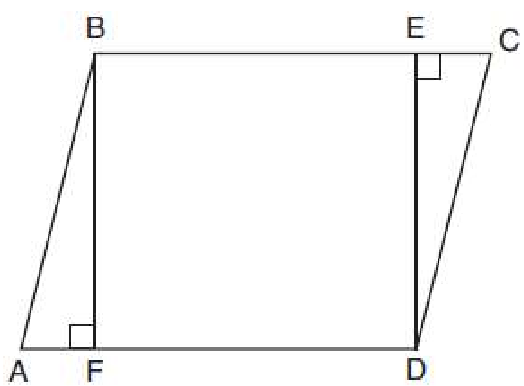
12. Given: $\overline{AE} \cong \overline{CG}$, $\overline{BE} \cong \overline{DG}$, $\overline{AH} \cong \overline{CF}$, $\overline{AD} \cong \overline{CB}$

Prove: $\overline{EF} \cong \overline{GH}$



13. Given: Parallelogram $ABCD$, $\overline{BF} \perp \overline{AFD}$, and $\overline{DE} \perp \overline{BEC}$

Prove: $BEDF$ is a rectangle



Coordinate Geometry Proofs

Distance (Length) = $\sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

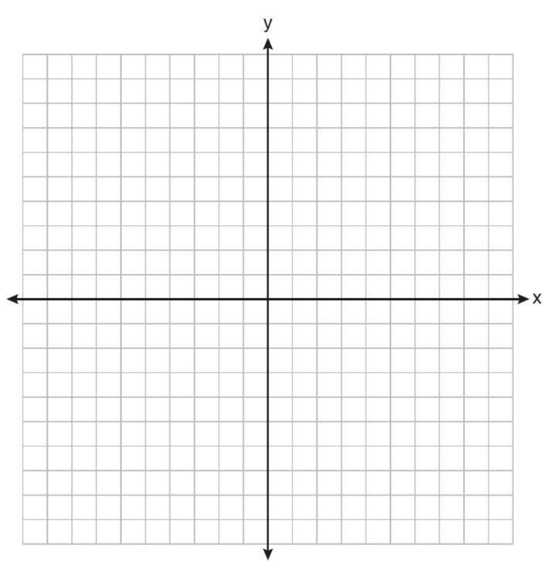
Slope = $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Midpoint = $(average\ x,\ average\ y) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

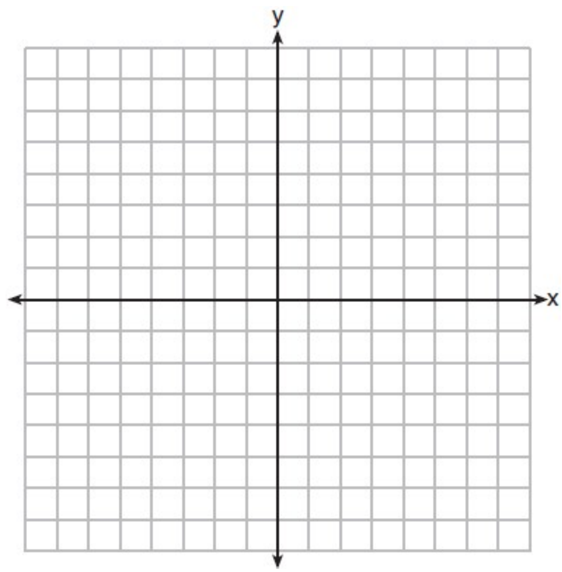
How do you prove…?

…an **isosceles triangle**? (2 Distances)
Two Congruent Sides
. … a **right triangle**? (3 Distances)
Show the sides fit into Pythagorean Theorem
… a **parallelogram**? (4 Distances)
Two Pairs of Opposite Sides Congruent
… a **rhombus**? (4 Distances)
All Sides Congruent
… a **rectangle**? (6 Distances)
1) Two Pairs of Opposite Sides Congruent
2) Diagonals Congruent
… a **square**? (6 Distances)
1) All Sides Congruent
2) Diagonals Congruent
…a **trapezoid**? (4 Slopes)
1) 1 pair of opposite sides parallel
2) 1 pair of opposite sides not parallel
…an **isosceles trapezoid**? (4 Slopes, 2 Distances)
1) 1 pair of opposite sides parallel
2) 1 pair of opposite sides not parallel
3) Congruent Legs

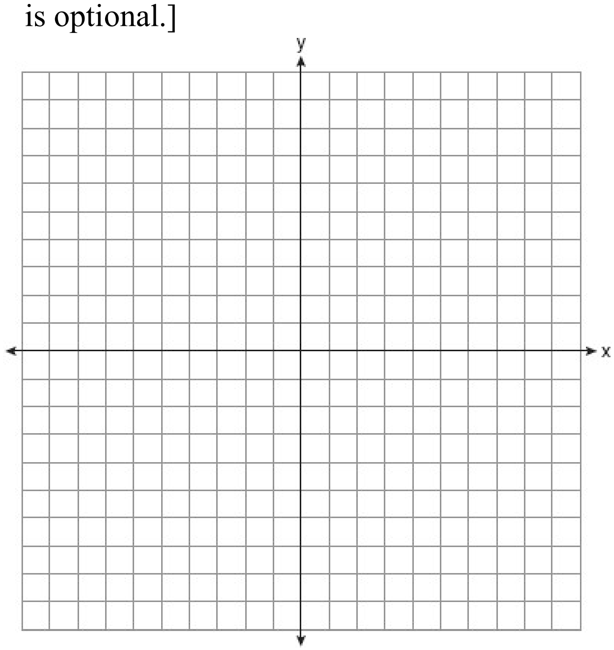
1. Graph the quadrilateral MATH: M(-2, -3) A(-1, -1) T(4, 2) H(3, 0). Prove that MATH **IS** a parallelogram but is **NOT** a rectangle.



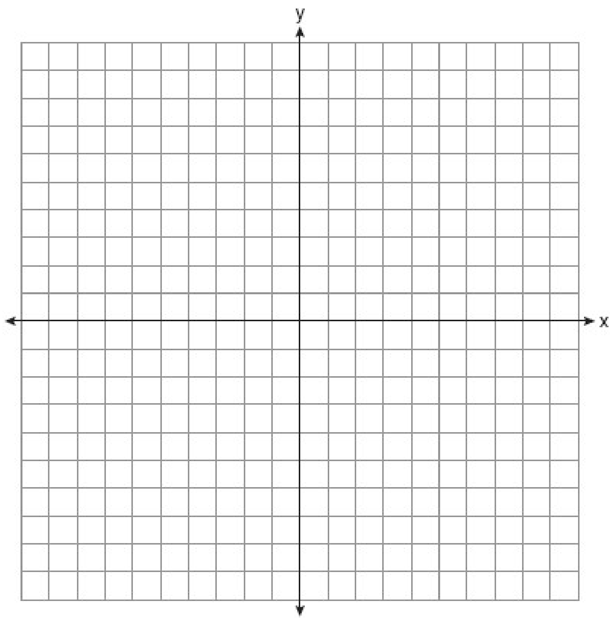
2. A triangle has vertices $A(-2,4)$, $B(6,2)$, and $C(1,-1)$. Prove that $\triangle ABC$ is an isosceles right triangle. [The use of the set of axes below is optional.]



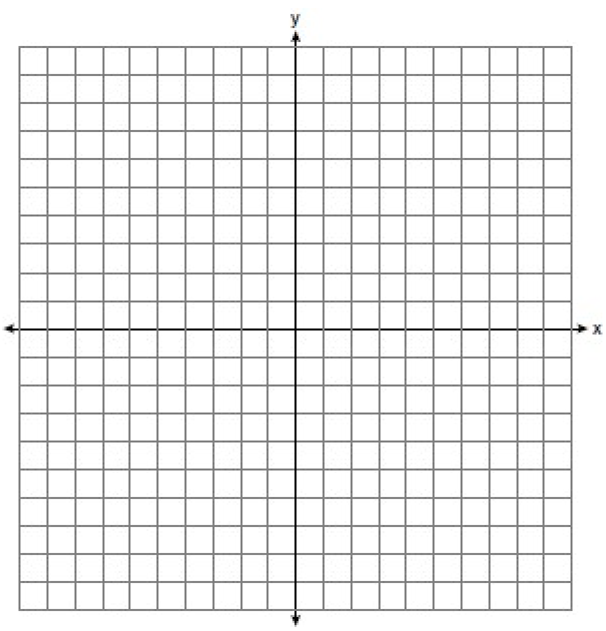
3. Quadrilateral *PQRS* has vertices $P(-2,3)$, $Q(3,8)$, $R(4,1)$, and $S(-1,-4)$. Prove that *PQRS* is a rhombus. Prove that *PQRS* is *not* a square. [The use of the set of axes below is optional.]



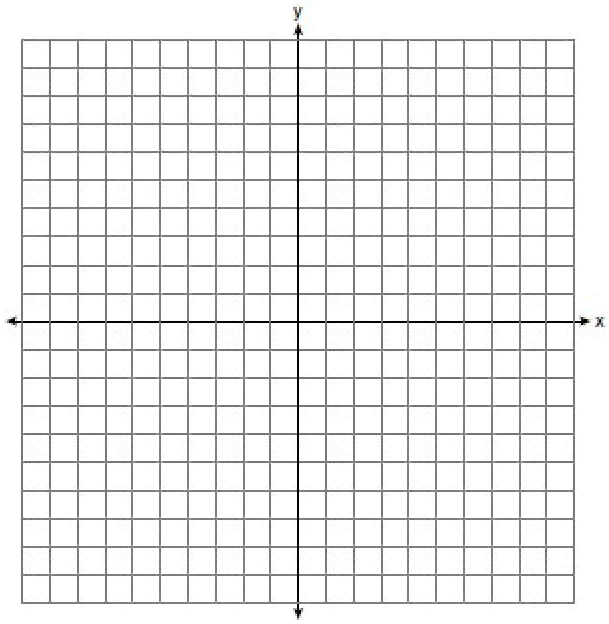
4. The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$. Prove that quadrilateral *MATH* is a parallelogram. Prove that quadrilateral *MATH* is a rectangle. [The use of the set of axes below is optional.]



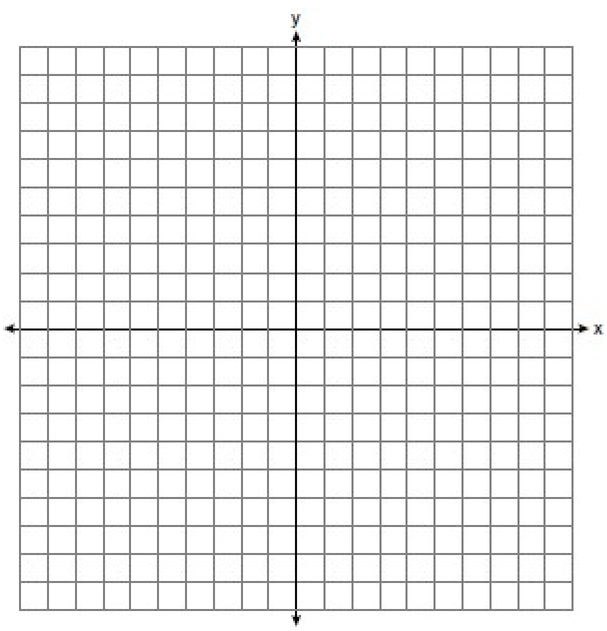
5. Triangle *ABC* has vertices with coordinates $A(-1,-1)$, $B(4,0)$, and $C(0,4)$. Prove that $\triangle ABC$ is an isosceles triangle but *not* an equilateral triangle. [The use of the set of axes below is optional.]



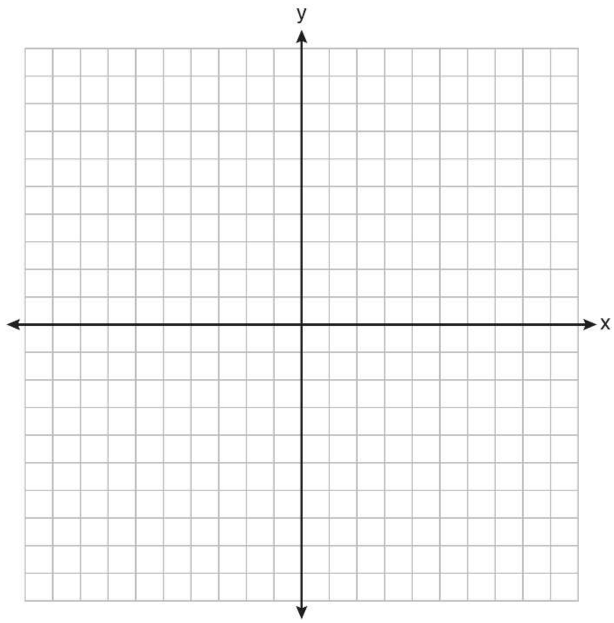
6. In the coordinate plane, the vertices of ΔRST are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that ΔRST is a right triangle. State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle. Prove that your quadrilateral $RSTP$ is a rectangle. [The use of the set of axes below is optional.]



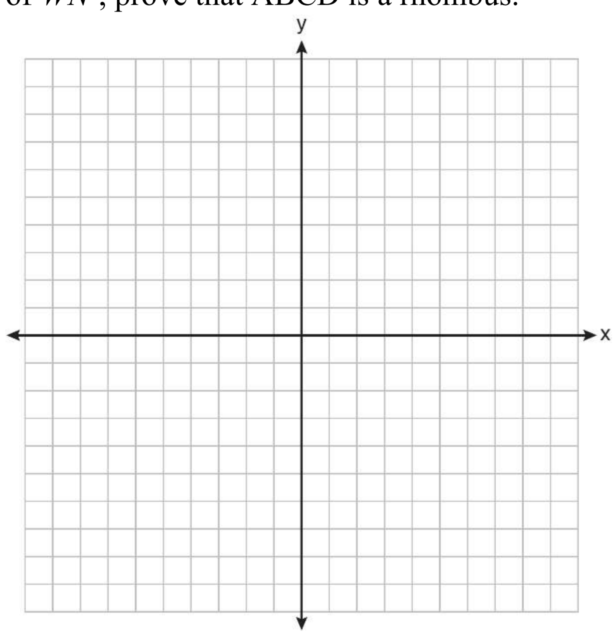
7. In the coordinate plane, the vertices of triangle PAT are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes below is optional.] State the coordinates of R so that quadrilateral $PART$ is a parallelogram. Prove that quadrilateral $PART$ is a parallelogram.



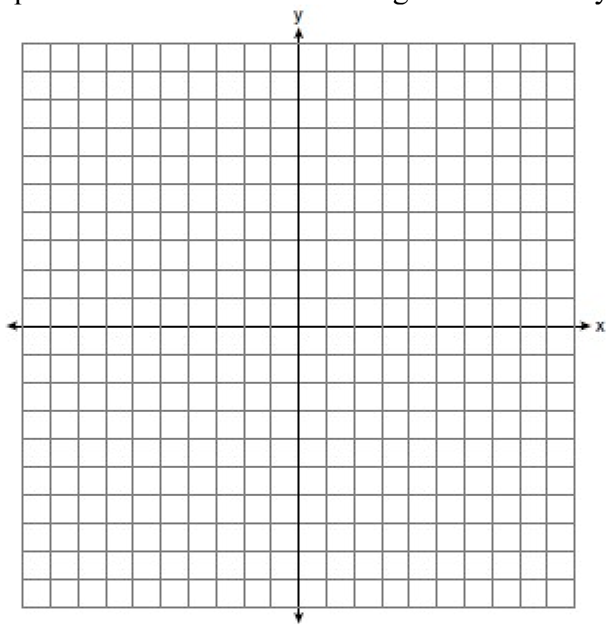
8. Given: $\triangle ABC$ with vertices $A(-6,-2)$, $B(2,8)$, and $C(6,-2)$. $\overline{AB}$ has midpoint D, $\overline{BC}$ has midpoint E, and $\overline{AC}$ has midpoint F.
Prove: *ADEF* is a parallelogram
ADEF is *not* a rhombus
[The use of the grid is optional.]



9. The vertices of rectangle NRQW are N(-2,5), R(2,5), Q(2,-3), and W(-2,-3). If A is the midpoint $\overline{NR}$, B is the midpoint of $\overline{RQ}$, C is the midpoint of $\overline{QW}$, and D is the midpoint of $\overline{WN}$, prove that ABCD is a rhombus.



10. In the coordinate plane, the vertices of triangle ABC are A(0,10) B(5,0) and C(8,4). Prove that Triangle ABC is a right triangle. State the coordinates of point *P* such that quadrilateral *ABCP* is a rectangle. Prove that your quadrilateral *ABCP* is a rectangle.



11. Quadrilateral ABCD has vertices A(3,1) B(-3,5) C(5,4) and D(2,6). Prove quadrilateral ABCD is a trapezoid but *not* an isosceles trapezoid.