Name Schlansky
Mr. Schlansky

Date ____________
Geometry

Proofs Regents Review

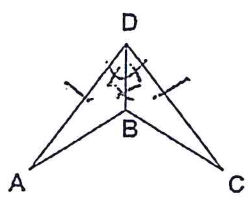
Euclidean Proofs (Basic)

If it is not specified, prove triangles are congruent
To prove triangles are congruent, prove 3 pairs of sides/angles are congruent
To prove segments or angles, use CPCTC
<u>*If you get stuck, make something up and keep on going!</u>

1) Do a mini proof with your givens
Altitude creates congruent right angles
Median creates congruent segments
Line bisector creates congruent segments
Midpoint creates congruent segments
Angle bisector creates congruent angles
Perpendicular lines create congruent right angles
When given parallel lines:
Corresponding angles are congruent OR Alternate interior angles are congruent OR Alternate exterior angles are congruent

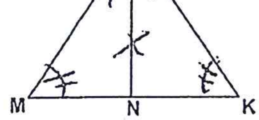
2) Use additional tools:
Vertical Angles are congruent (Look for an X)
Reflexive Property (A side/angle is congruent to itself)

1. Given: $\overline{BD}$ bisects $\angle ADC$
$\overline{AD} \cong \overline{DC}$
Prove: $\overline{AB} \cong \overline{BC}$



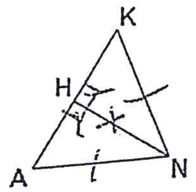
Statements	Reasons
① $\overline{BD}$ bisects $\angle ADC$	① given
② $\angle ADB \cong \angle CDB$	② An angle bisector creates 2 ≅ angles
③ $\overline{AD} \cong \overline{DC}$	③ given
④ $\overline{DB} \cong \overline{DB}$	④ Reflexive Property
⑤ $\triangle ADB \cong \triangle CDB$	⑤ SAS ≅ SAS
⑥ $\overline{AB} \cong \overline{BC}$	⑥ CPCTC

2. Given: $\overline{LN}$ bisects $\angle KLM$
$\angle LKM \cong \angle LMK$
Prove: $\overline{NM} \cong \overline{NK}$



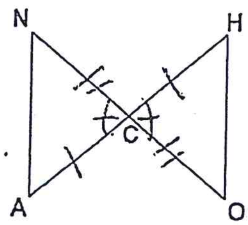
Statements	Reasons
① $\overline{LN}$ bisects $\angle KLM$	① given
② $\angle MLN \cong \angle KLN$	② An angle bisector creates 2 ≅ angles
③ $\angle LKM \cong \angle LMK$	③ given
④ $\overline{LN} \cong \overline{LN}$	④ Reflexive Property
⑤ $\triangle MLN \cong \triangle KLN$	⑤ AAS ≅ AAS
⑥ $\overline{NM} \cong \overline{NK}$	⑥ CPCTC

3. Given: $\overline{HN} \perp \overline{KA}$, $\overline{KN} \cong \overline{AN}$
Prove: $\angle HAN \cong \angle HKN$



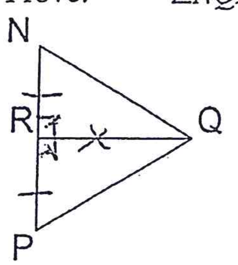
Statements	Reasons
① $\overline{HN} \perp \overline{KA}$	① given
② $\angle KHN \cong \angle AHN$	② Perpendicular lines create ≅ right angles
③ $\overline{KN} \cong \overline{AN}$	③ given
④ $\overline{HN} \cong \overline{HN}$	④ Reflexive Property
⑤ $\triangle AHN \cong \triangle KHN$	⑤ HL ≅ HL
⑥ $\angle HAN \cong \angle HKN$	⑥ CPCTC

4. Given: $\overline{NO}$ and $\overline{HA}$ bisect each other
Prove: $\overline{NA} \cong \overline{HO}$



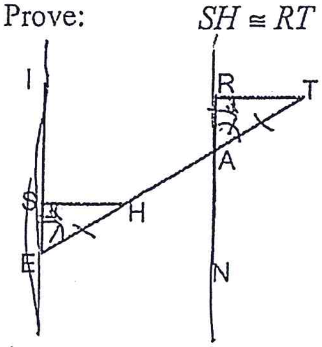
Statements	Reasons
① $\overline{NO}$ and $\overline{HA}$ bisect each other	① given
② $\overline{NC} \cong \overline{CO}$, $\overline{AC} \cong \overline{CH}$	② A line bisector creates 2 ≅ segments
③ $\angle NCA \cong \angle HCO$	③ vertical angles are congruent
④ $\triangle NCA \cong \triangle OCH$	④ SAS ≅ SAS
⑤ $\overline{NA} \cong \overline{HO}$	⑤ CPCTC

5. Given: $\overline{QR}$ is the perpendicular bisector of $\overline{NP}$
Prove: $\angle NQR \cong \angle PQR$



Statements	Reasons
① $\overline{QR}$ is the perpendicular bisector of $\overline{NP}$	① given
② $\angle NRQ \cong \angle PRQ$	② Perpendicular lines create ≅ right angles
③ $\overline{NR} \cong \overline{RP}$	③ A line bisector creates 2 ≅ segments
④ $\overline{RQ} \cong \overline{RQ}$	④ Reflexive Property
⑤ $\triangle NRQ \cong \triangle PRQ$	⑤ SAS ≅ SAS
⑥ $\angle NQR \cong \angle PQR$	⑥ CPCTC

6. Given: $\overline{IE} \parallel \overline{RN}$, $\overline{TR} \perp \overline{RN}$, $\overline{HS} \perp \overline{IE}$, $\overline{EH} \cong \overline{AT}$
Prove: $\overline{SH} \cong \overline{RT}$



Statements	Reasons
① $\overline{IE} \parallel \overline{RN}$	① given
② $\angle SEH \cong \angle RAT$	② Parallel lines cut by a transversal create congruent corresponding angles
③ $\overline{TR} \perp \overline{RN}$, $\overline{HS} \perp \overline{IE}$	③ given
④ $\angle HSE \cong \angle TRA$	④ Perpendicular lines create congruent right angles
⑤ $\overline{EH} \cong \overline{AT}$	⑤ given
⑥ $\triangle HSE \cong \triangle TRA$	⑥ AAS ≅ AAS
⑦ $\overline{SH} \cong \overline{RT}$	⑦ CPCTC

Euclidean Similar Triangle Proofs

To prove triangles are SIMILAR, prove AA ≅ AA

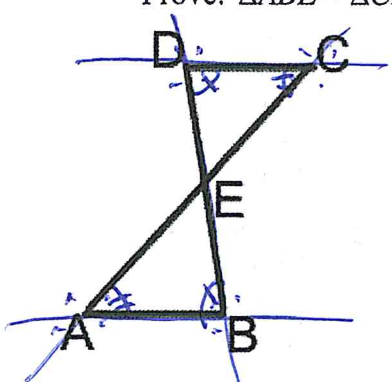
If asked to prove a proportion/multiplication:

1) **Prove triangles are similar**
2) **Corresponding Sides of Similar Triangle are In Proportion (CSSTIP)**
3) **Cross Products are Equal**

Work Backwards!

1. Given $\overline{AB} \parallel \overline{DC}$

   Prove: $\Delta ABE \sim \Delta CDE$



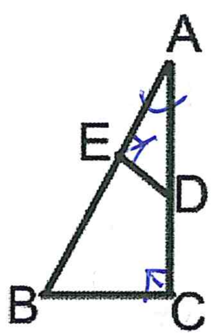
Statements	Reasons
① $\overline{AB} \parallel \overline{DC}$	① given
② $\angle CDE \cong \angle ABE$ $\angle DCE \cong \angle EAB$	② Parallel lines cut by a transversal create ≅ alternate interior angles
③ $\Delta ABE \sim \Delta CDE$	③ AA ≅ AA

*you could have also used vertical angles

2. Given: $\overline{BC} \perp \overline{AC}$

   $\overline{DE} \perp \overline{AB}$

   Prove: $\Delta ABC \sim \Delta ADE$

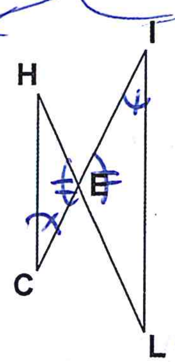


Statements	Reasons
① $\overline{BC} \perp \overline{AC}$, $\overline{DE} \perp \overline{AB}$	① given
② $\angle ABC \cong \angle AED$	② Perpendicular lines create ≅ right angles
③ $\angle EAD \cong \angle EAD$	③ Reflexive Property
④ $\Delta ABC \sim \Delta ADE$	④ AA ≅ AA

3. Given: $\angle HCE \cong \angle LIE$

   Prove: $\frac{CE}{CH} = \frac{EI}{IL}$

start backwards

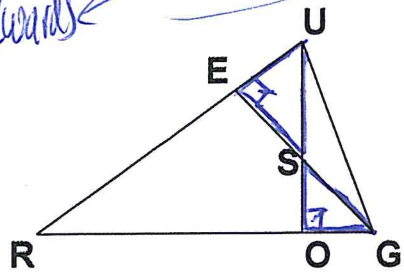


Statements	Reasons
① $\angle HCE \cong \angle LIE$	① given
② $\angle HEC \cong \angle IEL$	② vertical angles are congruent
③ $\Delta CEH \sim \Delta IEL$	③ AA ≅ AA
④ $\frac{CE}{CH} = \frac{EI}{IL}$	④ CSSTIP

4. Given: $\overline{UO} \perp \overline{RG}, \overline{UR} \perp \overline{EG}$

Prove: $\dfrac{\overline{US}}{\overline{SO}} = \dfrac{\overline{EU}}{\overline{OG}}$

work backwards

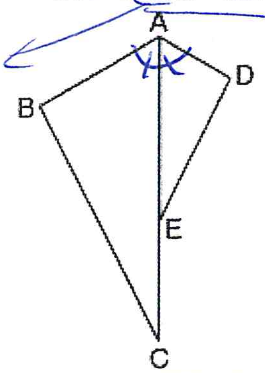


Statements	Reasons
① $\overline{UO}\perp\overline{RG}, \overline{UR}\perp\overline{EG}$	① given
② $\angle UES \cong \angle SOG$	② Perpendicular lines create congruent right angles
③ $\angle USE \cong \angle OSG$	③ vertical angles are congruent
④ $\triangle USE \sim \triangle GSO$	④ AA≅AA
⑤ $\dfrac{US}{SO} = \dfrac{EU}{OG}$	⑤ CSSTIP

5. Given: $\overline{CA}$ bisects $\angle CAD$, $\angle ABC \cong \angle ADE$

Prove: $\overline{BC} \bullet \overline{AE} = \overline{DE} \bullet \overline{AC}$

work backwards

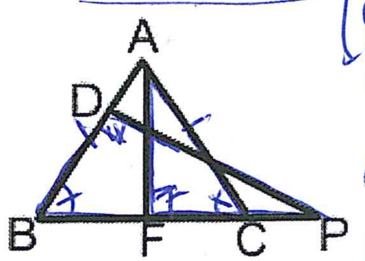


Statements	Reasons
① CA bisects $\angle CAD$	① given
② $\angle BAC \cong \angle DAE$	② An angle bisector creates 2 ≅ angles
③ $\angle ABC \cong \angle ADE$	③ given
④ $\triangle BCA \sim \triangle DEA$	④ AA≅AA
⑤ $\dfrac{BC}{AC} = \dfrac{DE}{AE}$	⑤ CSSTIP
⑥ $BC \cdot AE = DE \cdot AC$	⑥ cross products are equal

6. Given: $\overline{AB} \cong \overline{AC}$, $\overline{AF} \perp \overline{BC}$, $\overline{PD} \perp \overline{AB}$

Prove: $\overline{FC} \bullet \overline{PB} = \overline{DB} \bullet \overline{AC}$

work backwards

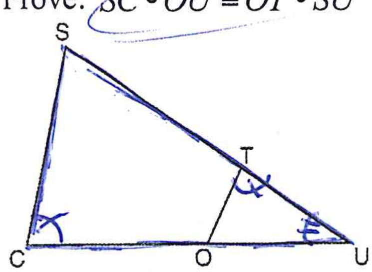


Statements	Reasons
① $\overline{AB} \cong \overline{AC}$	① given
② $\angle ABF \cong \angle ACF$	② Isosceles Triangle theorem
③ $\overline{AF}\perp\overline{BC}, \overline{PD}\perp\overline{AB}$	③ given
④ $\angle PDB \cong \angle AFC$	④ perpendicular lines create ≅ right angles
⑤ $\triangle AFC \sim \triangle DBP$	⑤ AA≅AA
⑥ $\dfrac{FC}{AC} = \dfrac{DB}{PB}$	⑥ CSSTIP
⑦ $FC \cdot PB = DB \cdot AC$	⑦ cross products are equal

7. In $\triangle SCU$ shown below, points T and O are on $\overline{SU}$ and $\overline{CU}$, respectively. Segment OT is drawn so that $\angle C \cong \angle OTU$.

Prove: $\overline{SC} \bullet \overline{OU} = \overline{OT} \bullet \overline{SU}$

work backwards



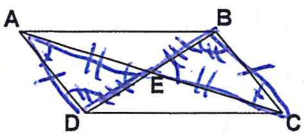
Statements	Reasons
① $\angle C \cong \angle OTU$	① given
② $\angle OUT \cong \angle OUT$	② Reflexive Property
③ $\triangle SCU \sim \triangle OTU$	③ AA≅AA
④ $\dfrac{SC}{SU} = \dfrac{OT}{OU}$	④ CSSTIP
⑤ $SC \cdot OU = OT \cdot SU$	⑤ cross products are equal

Euclidean Proofs with Parallelogram and Circle Theorems

Parallelogram Theorems	Circle Theorems
A parallelogram/rectangle/rhombus/square has: Two pairs of opposite sides congruent Two pairs of opposite sides parallel Diagonals that bisect each other Opposite angles congruent **A rectangle/square has:** A right angle Congruent diagonals **A rhombus/square has:** All sides congruent Perpendicular diagonals Diagonals that bisect the angles	**All radii/diameters of a circle are congruent** **Angles inscribed to the same arc are congruent** **An angle inscribed to a semicircle is a right angle** **A tangent and a radius/diameter form a right angles** Congruent arcs have congruent chords have congruent central angles Parallel Lines intercept congruent arcs Tangents drawn from the same point are congruent

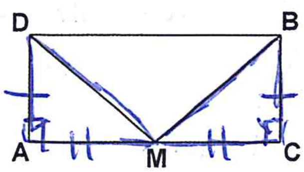
I'm giving you all 6 even though you only need 3

1. Given: ABCD is a parallelogram
Prove: $\Delta AED \cong \Delta CEB$



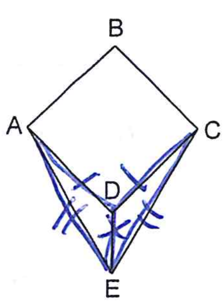
Statements	Reasons
① ABCD is a parallelogram	① given
② $\overline{AD} \cong \overline{BC}$	② A p-gram has opposite sides ≅
③ $\overline{AE} \cong \overline{EC}$, $\overline{BE} \cong \overline{ED}$	③ A p-gram has diagonals that bisect each other
④ $\angle EAD \cong \angle BCE$, $\angle ADE \cong \angle CBE$	④ A p-gram has ≅ alternate interior angles
⑤ $\angle AED \cong \angle BEC$	⑤ vertical angles are congruent
⑥ $\Delta AED \cong \Delta CEB$	⑥ SSS/SAS/ASA/AAS depending on which three you did

2. Given: ABCD is a rectangle, M is the midpoint of $\overline{AC}$
Prove: $\overline{DM} \cong \overline{BM}$



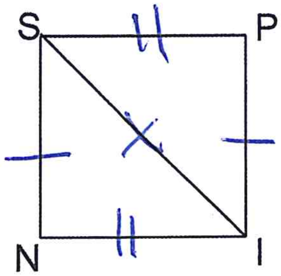
Statements	Reasons
① ABCD is a rectangle	① given
② $\overline{DA} \cong \overline{BC}$	② A rectangle has 2 pairs of opp sides ≅
③ $\angle DAM \cong \angle BCM$	③ A rectangle has ≅ right angles
④ M is midpoint of $\overline{AC}$	④ given
⑤ $\overline{AM} \cong \overline{MC}$	⑤ A midpoint creates 2 ≅ segments
⑥ $\Delta DAM \cong \Delta BCM$	⑥ SAS ≅ SAS
⑦ $\overline{DM} \cong \overline{BM}$	⑦ CPCTC

3. Given: ABCD is a rhombus, $\overline{AE} \cong \overline{CE}$
Prove: $\angle ADE \cong \angle CDE$



Statements	Reasons
① ABCD is a rhombus	① given
② $\overline{AD} \cong \overline{DC}$	② A rhombus has all sides ≅
③ $\overline{AE} \cong \overline{CE}$	③ given
④ $\overline{DE} \cong \overline{DE}$	④ Reflexive Property
⑤ $\Delta ADE \cong \Delta CDE$	⑤ SSS ≅ SSS
⑥ $\angle ADE \cong \angle CDE$	⑥ CPCTC

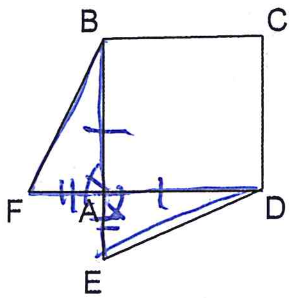
4. Given: SPIN is a square
Prove: $\Delta SNI \cong \Delta SPI$



Statements	Reasons
① SPIN is a square	① given
② $\overline{SN} \cong \overline{PI}$, $\overline{SP} \cong \overline{NI}$	② A square has ≅ ~~[illegible]~~ opposite sides
③ $\overline{SI} \cong \overline{SI}$	③ Reflexive Property
④ $\Delta SNI \cong \Delta SPI$	④ SSS ≅ SSS

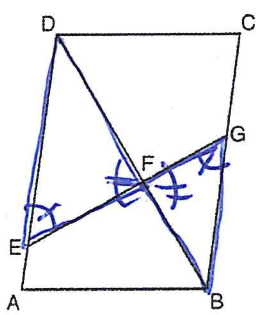
*Also right angles and alternate interior angles

5. Given: ABCD is a square, $\overline{FA} \cong \overline{AE}$
Prove: $\overline{BF} \cong \overline{DE}$



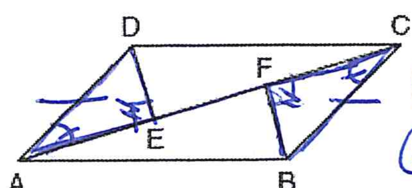
Statements	Reasons
① ABCD is a square	① given
② $\overline{BA} \cong \overline{AD}$	② A square has all sides ≅
③ $\overline{FA} \cong \overline{AE}$	③ given
④ $\angle FAB \cong \angle DAE$	④ Vertical angles are congruent
⑤ $\Delta BAF \cong \Delta DAE$	⑤ SAS ≅ SAS
⑥ $\overline{BF} \cong \overline{DE}$	⑥ CPCTC

6. Given: Parallelogram $ABCD$, $\overline{EFG}$, and diagonal $\overline{DFB}$
Prove: $\triangle DEF \sim \triangle BGF$



Statements	Reasons
① Parallelogram ABCD	① given
② $\angle DEF \cong \angle BGF$	② A parallelogram has ≅ alternate interior angles
③ $\angle DFE \cong \angle BFG$	③ Vertical angles are congruent
④ $\Delta DEF \sim \Delta BGF$	④ AA ≅ AA

7. In parallelogram $ABCD$, $\overline{BF}$ and $\overline{DE}$ are perpendicular to diagonal $\overline{AC}$ at points F and E.
Prove: $\overline{AE} \cong \overline{CF}$

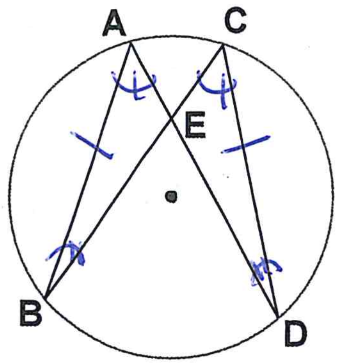


Statements	Reasons
① Parallelogram ABCD	① given
② $\overline{AD} \cong \overline{BC}$	② A parallelogram has opposite sides ≅
③ $\angle DAE \cong \angle BCF$	③ A parallelogram has congruent alternate interior angles
④ $\overline{BF}$ and $\overline{DE}$ are perpendicular to $\overline{AC}$	④ given
⑤ $\angle DEA \cong \angle CFB$	⑤ Perpendicular lines create ≅ right angles
⑥ $\Delta DEA \cong \Delta CFB$	⑥ AAS ≅ AAS
⑦ $\overline{AE} \cong \overline{CF}$	⑦ CPCTC

*Look for inscribed angles

8. Given: Chords $\overline{AD}$ and $\overline{BC}$ of circle O intersect at E, $\overline{AB} \cong \overline{CD}$

Prove: $\overline{BC} \cong \overline{AD}$

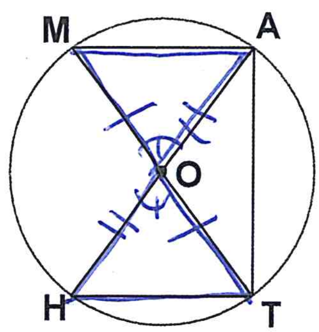


*you could have also used vertical angles

Statements	Reasons
① $\overline{AB} \cong \overline{CD}$	① given
② $\angle BAE \cong \angle DCE$, $\angle ABE \cong \angle CDE$	② Angles inscribed to the same arc are congruent
③ $\triangle BAE \cong \triangle DCE$	③ ASA≅ASA
④ $\overline{BC} \cong \overline{AD}$	④ CPCTC

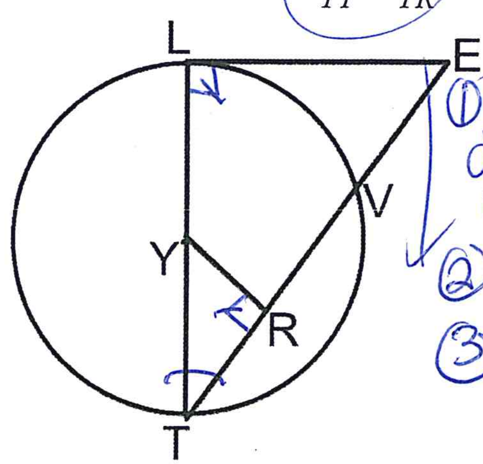
9. Given: Circle O with diameters $\overline{MOT}$ and $\overline{AOH}$.

Prove: $\overline{MA} \cong \overline{HT}$



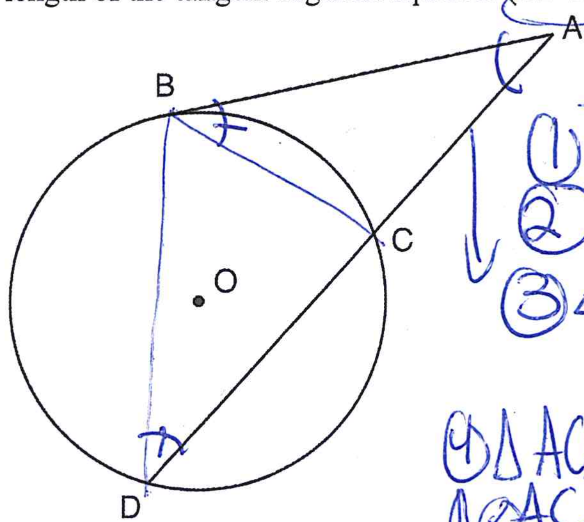
Statements	Reasons
① $\overline{MO} \cong \overline{OT}$, $\overline{AO} \cong \overline{OH}$	① All radii of a circle are ≅
② $\angle MOA \cong \angle HOT$	② vertical angles are ≅
③ $\triangle MOA \cong \triangle TOH$	③ SAS≅SAS
④ $\overline{MA} \cong \overline{HT}$	④ CPCTC

10. In circle Y, tangent $\overline{LE}$ is drawn to diameter $\overline{TYL}$ and $\overline{YR} \perp \overline{TE}$. Prove that $\frac{TE}{TY} = \frac{TL}{TR}$. work backwards



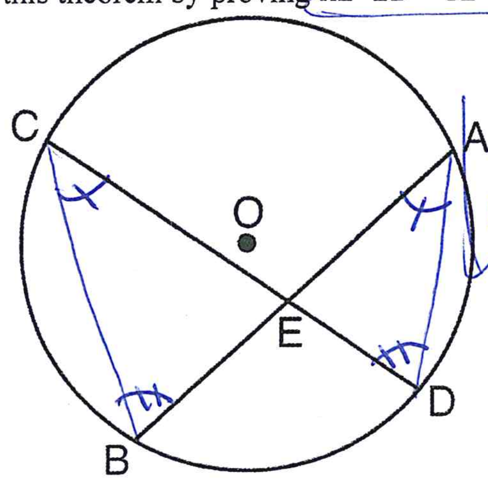
Statements	Reasons
① Tangent LE is drawn to diameter TYL, YR⊥TE	① given
② $\angle TLE \cong \angle TRY$	② An angle formed by a tangent and diameter and perpendicular lines form congruent right angles
③ $\angle RTY \cong \angle RTY$	③ Reflexive Property
④ $\triangle TLE \sim \triangle TRY$	④ AA≅AA
⑤ $\frac{TE}{TY} = \frac{TL}{TR}$	⑤ CSSTIP

11. In the diagram below, secant $\overline{ACD}$ and tangent $\overline{AB}$ are drawn from external point A to circle O.
Prove the theorem: If a secant and a tangent are drawn to a circle from an external point, the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared. ($AC \cdot AD = AB^2$) → work backwards



Statements	Reasons
① $\overline{BC}, \overline{BD}$	① Auxiliary lines can be drawn
② $\angle BAC \cong \angle BAC$	② Reflexive Property
③ $\angle BDC \cong \angle ABC$	③ Angles inscribed to the same arc are ≅
④ $\triangle ACB \sim \triangle ABD$	④ AA≅AA
⑤ $\frac{AC}{AB} = \frac{AB}{AD}$	⑤ CSSTP
⑥ $AC \cdot AD = AB^2$	⑥ Cross products are equal

12. Given: Circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E
Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. Prove this theorem by proving $AE \cdot EB = CE \cdot ED$. → work backwards



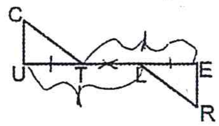
Statements	Reasons
① $\overline{CB}, \overline{AD}$	① Auxiliary lines can be drawn
② $\angle BCE \cong \angle DAE$, $\angle CBE \cong \angle ADE$	② Angles inscribed to the same arc are ≅
③ $\triangle AED \sim \triangle CEB$	③ AA≅AA
④ $\frac{AE}{ED} = \frac{CE}{EB}$	④ CSSTP
⑤ $AE \cdot EB = CE \cdot ED$	⑤ Cross products are equal

*you could have also used vertical angles

Euclidean Triangle Proofs with Additional Tools

Vertical Angles are congruent (Look for an X)
Reflexive Property (A side/angle is congruent to itself)
Isosceles Triangles (In a triangle, congruent angles are opposite congruent sides)
Addition and Subtraction Property (If you need more or less of a shared side)
*You must use three congruent statements to get one congruent statement for the triangles. The two that you are adding/subtracting and the one that you want to prove in the triangle.

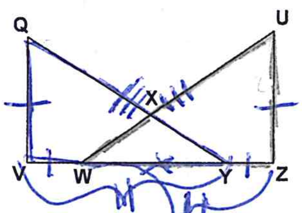
7. Given: $\overline{UL} \cong \overline{TE}$
Prove: $\overline{UT} \cong \overline{LE}$



statements	Reasons
① $\overline{UL} \cong \overline{TE}$	① given
② $\overline{TL} \cong \overline{TL}$	② reflexive property
③ $\overline{UT} \cong \overline{LE}$	③ subtraction property

1. Given: $\overline{QV} \cong \overline{UZ}$
$\overline{VW} \cong \overline{YZ}$
$\overline{YQ} \cong \overline{WU}$
Prove: $\angle Q \cong \angle U$

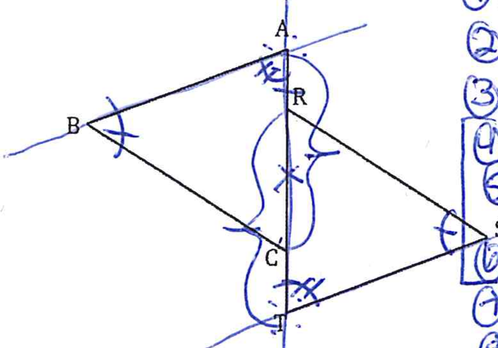
too little



Statements	Reasons
① $\overline{QV} \cong \overline{UZ}$	① given
② $\overline{VW} \cong \overline{YZ}$	② given
③ $\overline{WY} \cong \overline{WY}$	③ Reflexive Property
④ $\overline{VY} \cong \overline{WZ}$	④ Addition Property
⑤ $\overline{YQ} \cong \overline{WU}$	⑤ given
⑥ $\triangle QVY \cong \triangle UZW$	⑥ SSS≅SSS
⑦ $\angle Q \cong \angle U$	⑦ CPCTC

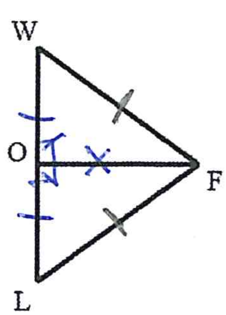
2. Given: $\angle B \cong \angle S$, $\overline{AB} \parallel \overline{ST}$, $\overline{AR} \cong \overline{TC}$

Prove: $\overline{BC} \cong \overline{SR}$



Statements	Reasons
① $\angle B \cong \angle S$	① given
② $\overline{AB} \parallel \overline{ST}$	② given
③ $\angle BAC \cong \angle STR$	③ Parallel lines cut by a transversal create congruent alternate interior angles
④ $\overline{AR} \cong \overline{TC}$	④ given
⑤ $\overline{RC} \cong \overline{RC}$	⑤ Reflexive Property
⑥ $\overline{AC} \cong \overline{RT}$	⑥ Addition Property
⑦ $\triangle BAC \cong \triangle STR$	⑦ AAS≅AAS
⑧ $\overline{BC} \cong \overline{SR}$	⑧ CPCTC

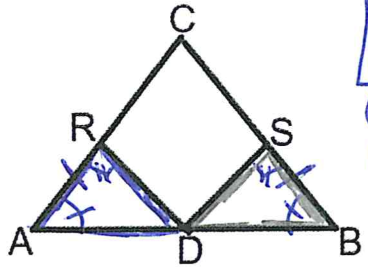
3. Given: $\overline{OF}$ is the perpendicular bisector of $\overline{WL}$
Prove: ΔWFL is isosceles



Statements	Reasons
① $\overline{OF}$ is the perpendicular bisector of $\overline{WL}$	① given
② $\overline{WO} \cong \overline{OL}$	② A line bisector creates two congruent segments
③ $\angle WOF \cong \angle LOF$	③ Perpendicular lines create congruent right angles
④ $\overline{OF} \cong \overline{OF}$	④ Reflexive Property
⑤ $\triangle WOF \cong \triangle LOF$	⑤ SAS≅SAS
⑥ $\overline{WF} \cong \overline{FL}$	⑥ CPCTC
⑦ $\triangle WFL$ is isosceles	⑦ Isosceles triangle theorem

4. Given: In $\triangle ABC$, $\overline{CA} \cong \overline{CB}$, $\overline{AR} \cong \overline{BS}$, $\overline{DR} \perp \overline{AC}$, and $\overline{DS} \perp \overline{BC}$

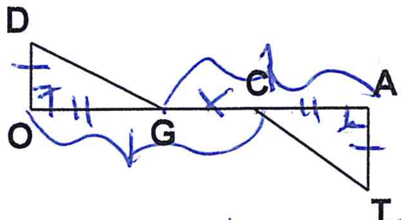
Prove: $\overline{DR} \cong \overline{DS}$



Statements	Reasons
① $\overline{CA} \cong \overline{CB}$	① given
② $\angle CAB \cong \angle CBA$	② Isosceles triangle theorem
③ $\overline{AR} \cong \overline{BS}$	③ given
④ $\overline{DR} \perp \overline{AC}$, $\overline{DS} \perp \overline{BC}$	④ given
⑤ $\angle DRA \cong \angle DSB$	⑤ Perpendicular lines create $\cong$ right angles
⑥ $\triangle DRA \cong \triangle DSB$	⑥ ASA≅ASA
⑦ $\overline{DR} \cong \overline{DS}$	⑦ CPCTC

5. Given: $\overline{DO} \perp \overline{OA}$, $\overline{TA} \perp \overline{OA}$, $\overline{DO} \cong \overline{TA}$, $\overline{OC} \cong \overline{AG}$

Prove: $\overline{DG} \cong \overline{TC}$

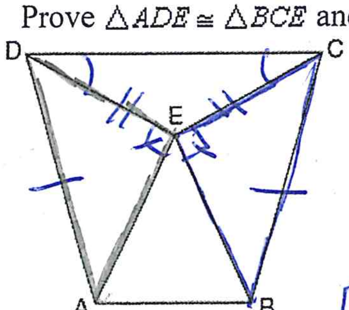


too much

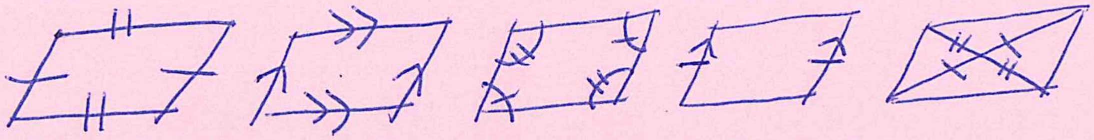
Statements	Reasons
① $\overline{DO} \perp \overline{OA}$, $\overline{TA} \perp \overline{OA}$	① given
② $\angle DOG \cong \angle CAT$	② Perpendicular lines create $\cong$ right angles
③ $\overline{DO} \cong \overline{TA}$	③ given
④ $\overline{OC} \cong \overline{AG}$	④ given
⑤ $\overline{GC} \cong \overline{GC}$	⑤ Reflexive Property
⑥ $\overline{OG} \cong \overline{CA}$	⑥ Subtraction property
⑦ $\triangle DOG \cong \triangle TAC$	⑦ SAS≅SAS
⑧ $\overline{DG} \cong \overline{TC}$	⑧ CPCTC

6. Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.

Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.



Statements	Reasons
① Isosceles trapezoid ABCD	① given
② $\overline{DA} \cong \overline{BC}$	② An isosceles trapezoid has congruent legs
③ $\angle CDE \cong \angle DCE$	③ given
④ $\overline{DE} \cong \overline{CE}$	④ Isosceles triangle theorem
⑤ $\overline{AE} \perp \overline{DE}$, $\overline{BE} \perp \overline{CE}$	⑤ given
⑥ $\angle DEA \cong \angle CEB$	⑥ Perpendicular lines create congruent right angles
⑦ $\triangle ADE \cong \triangle BCE$	⑦ HL≅HL
⑧ $\overline{AE} \cong \overline{EB}$	⑧ CPCTC
⑨ $\triangle AEB$ is isosceles	⑨ Isosceles Triangle Theorem



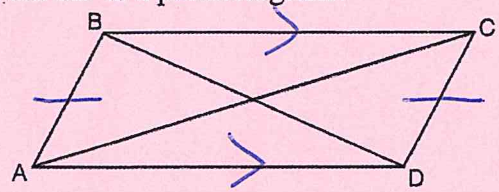
Euclidean Parallelogram Proofs/Parallelogram Properties

To prove parallelograms: Always prove parallelogram first. You will probably have to use congruent triangles with CPCTC to get at least one of the properties.

1. Quadrilateral *ABCD* with diagonals $\overline{AC}$ and $\overline{BD}$ is shown in the diagram below.

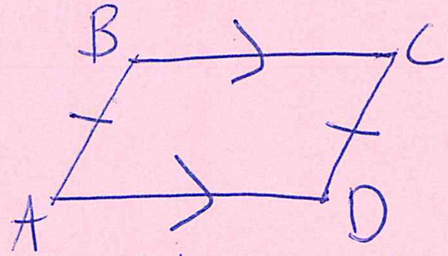
Which information is *not* enough to prove *ABCD* is a parallelogram?

1) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{DC}$
2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$
3) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$
4) $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$



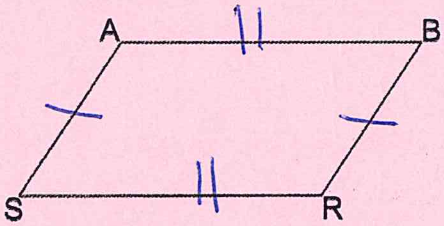
2. Quadrilateral *ABCD* has diagonals $\overline{AC}$ and $\overline{BD}$. Which information is *not* sufficient to prove *ABCD* is a parallelogram?

1) $\overline{AC}$ and $\overline{BD}$ bisect each other.
2) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$
3) $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$
4) $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$



3. Given: $\overline{SA} \cong \overline{BR}$, $\overline{AB} \cong \overline{SR}$

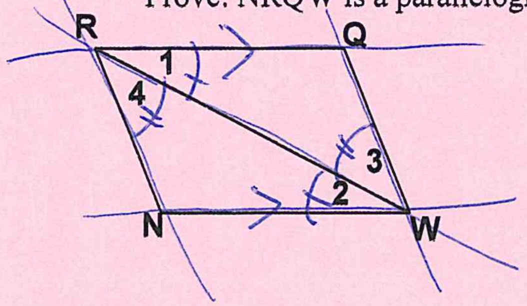
Prove: SABR is a parallelogram



Statements	Reasons
① $\overline{SA} \cong \overline{BR}$, $\overline{AB} \cong \overline{SR}$	① given
② SABR is a parallelogram	② A parallelogram has 2 pairs of opposite sides congruent

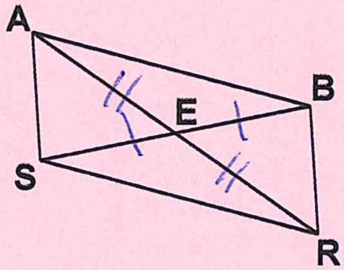
4. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove: NRQW is a parallelogram



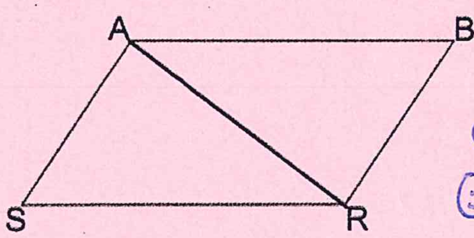
Statements	Reasons
① $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	① given
② $\overline{RQ} \parallel \overline{NW}$, $\overline{RN} \parallel \overline{QW}$	② Parallel lines cut by a transversal create congruent right angles
③ NRQW is a parallelogram	③ A Parallelogram has 2 pairs of opposite sides parallel.

5. Given: E is the midpoint of $\overline{SB}$, $\overline{AE} \cong \overline{ER}$
Prove: SABR is a parallelogram



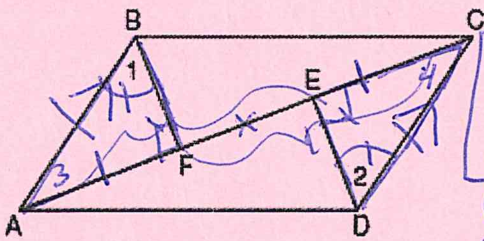
Statements	Reasons
① E is the midpoint of $\overline{SB}$	① given
② $\overline{SE} \cong \overline{EB}$	② A midpoint creates two congruent segments.
③ $\overline{AE} \cong \overline{ER}$	③ given
④ SABR is a parallelogram	④ A parallelogram has diagonals that bisect each other.

6. Given: $\Delta ASR \cong \Delta RBA$
Prove: SABR is a parallelogram



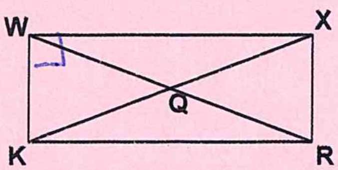
Statements	Reasons
① $\Delta ASR \cong \Delta RBA$	① given
② $\overline{AB} \cong \overline{SR}$, $\overline{SA} \cong \overline{BR}$	② CPCTC
③ SABR is a parallelogram	③ A parallelogram has 2 pairs of opposite sides congruent

7. Given: Quadrilateral *ABCD*, diagonal $\overline{AFEC}$, $\overline{AE} \cong \overline{FC}$, $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$, $\angle 1 \cong \angle 2$
Prove: *ABCD* is a parallelogram.



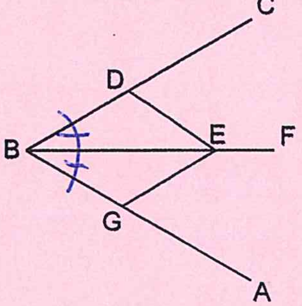
Statements	Reasons
① $\overline{AE} \cong \overline{FC}$	① given
② $\overline{EF} \cong \overline{EF}$	② Reflexive Property
③ $\overline{AF} \cong \overline{EC}$	③ Subtraction Property
④ $\angle 1 \cong \angle 2$	④ given
⑤ $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$	⑤ given
⑥ $\angle BFA \cong \angle DEC$	⑥ Perpendicular lines create ≅ right angles
⑦ $\Delta BFA \cong \Delta DEC$	⑦ AAS ≅ AAS
⑧ $\overline{AB} \cong \overline{CD}$, $\angle 3 \cong \angle 4$	⑧ CPCTC
⑨ $\overline{AB} \parallel \overline{CD}$	⑨ Parallel lines cut by a transversal form congruent alternate interior angles
⑩ ABCD is a parallelogram	⑪ A parallelogram has 1 pair of opposite sides ≅ and ∥.

8. Given: WXRK is a parallelogram, $\overline{KW} \perp \overline{WX}$
Prove: WXRK is a rectangle



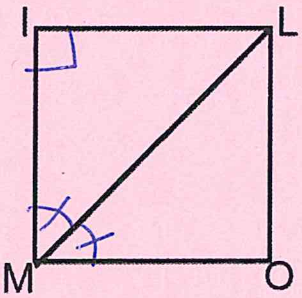
Statements	Reasons
① WXRK is a parallelogram	① given
② $\overline{KW} \perp \overline{WX}$	② given
③ $\angle KWX$ is a right angle	③ Perpendicular lines form right angles.
④ WXRK is a rectangle	④ A rectangle is a parallelogram with a right angle.

9. Given: BDEG is a parallelogram, $\overline{BF}$ bisects $\angle CBA$
Prove: DEGB is a rhombus



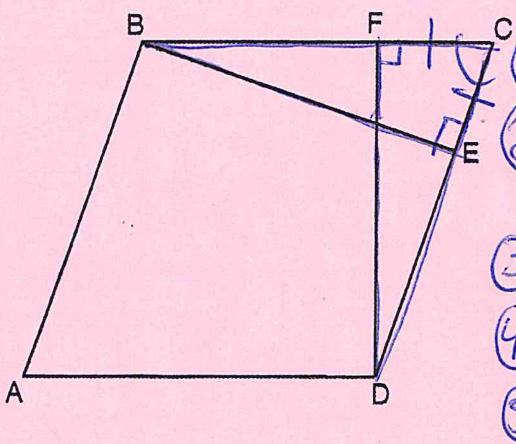
Statements	Reasons
① BDEG is a parallelogram	① given
② $\overline{BF}$ bisects ∠CBA	② given
③ BDEG is a rhombus.	③ A rhombus is a parallelogram with diagonals that bisect the angles.

10. Given: MILO is a parallelogram, $\angle IML \cong \angle OML$, $\overline{MI} \perp \overline{IL}$
Prove: MILO is a square



Statements	Reasons
① MILO is a parallelogram	① given
② $\angle IML \cong \angle OML$, $\overline{MI} \perp \overline{IL}$	② given
③ ∠MIL is a right angle	③ Perpendicular lines form right angles.
④ MILO is a square	④ A square is a parallelogram with diagonals that bisect the angles and right angles.

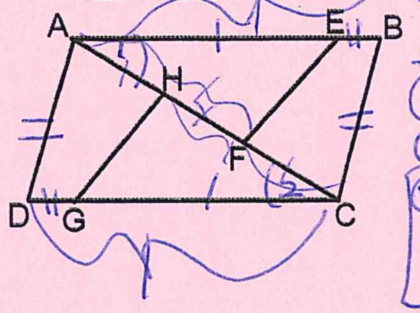
11. In the diagram of parallelogram *ABCD* below, $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$, $\overline{CE} \cong \overline{CF}$.
Prove *ABCD* is a rhombus.



Statements	Reasons
① Parallelogram ABCD	① given
② $\overline{BE} \perp \overline{CED}$, DF⊥BFC	② given
③ $\angle DFC \cong \angle BEC$	③ Perpendicular lines form ≅ right angles.
④ $\overline{CE} \cong \overline{CF}$	④ given
⑤ $\angle C \cong \angle C$	⑤ Reflexive Property
⑥ $\triangle BCE \cong \triangle DCF$	⑥ ASA ≅ ASA
⑦ $\overline{BC} \cong \overline{CD}$	⑦ CPCTC
⑧ ~~▱~~ ABCD is a rhombus	⑧ A rhombus is a parallelogram with consecutive sides congruent.

12. Given: $\overline{AE} \cong \overline{CG}$, $\overline{BE} \cong \overline{DG}$, $\overline{AH} \cong \overline{CF}$, $\overline{AD} \cong \overline{CB}$

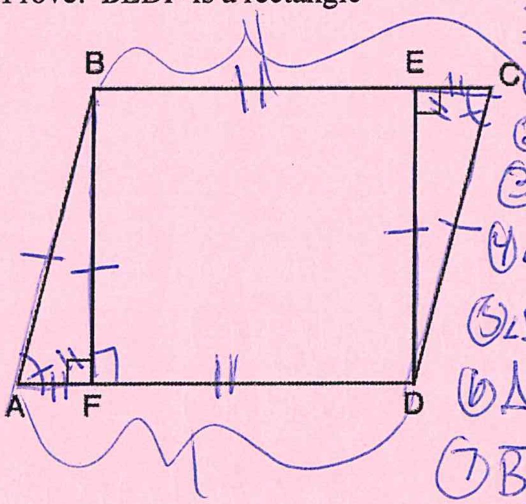
Prove: $\overline{EF} \cong \overline{GH}$



Statements	Reasons
① $\overline{AE} \cong \overline{GC}$	① given
② $\overline{BE} \cong \overline{DG}$	② given
③ $\overline{AH} \cong \overline{CF}$	③ given
④ $HF \cong HF$	④ Reflexive Property
⑤ $\overline{AF} \cong \overline{HC}$	⑤ Addition Property
⑥ $\overline{AB} \cong \overline{DC}$	⑥ Addition Property
⑦ $\overline{AD} \cong \overline{DC}$	⑦ given
⑧ ABCD is a Parallelogram	⑧ A Parallelogram has 2 pairs of opposite sides ≅
⑨ ∠1 ≅ ∠2	⑨ A parallelogram has parallel lines cut by a transversal forming ≅ alt int ∠
⑩ △EAF ≅ △GCH	⑩ SAS ≅ SAS
⑪ $\overline{EF} \cong \overline{GH}$	⑪ CPCTC

13. Given: Parallelogram $ABCD$, $\overline{BF} \perp \overline{AFD}$, and $\overline{DE} \perp \overline{BEC}$

Prove: $BEDF$ is a rectangle



Statements	Reasons
① Parallelogram ABCD	① given
② ∠A ≅ ∠C	② A parallelogram has opposite angles congruent.
③ $\overline{AB} \cong \overline{CD}$	③ A parallelogram has opposite sides congruent.
④ $\overline{BF} \perp \overline{AFD}$, $\overline{DE} \perp \overline{BEC}$	④ given
⑤ ∠BFA ≅ ∠DEC	⑤ Perpendicular lines create ≅ right angles
⑥ △BFA ≅ △DEC	⑥ AAS ≅ AAS
⑦ $\overline{BF} \cong \overline{ED}$, $\overline{AF} \cong \overline{CE}$	⑦ CPCTC
⑧ $\overline{BC} \cong \overline{AD}$	⑧ A parallelogram has 2 pairs of opposite sides ≅
⑨ BEDF is a parallelogram	⑨ A parallelogram has 2 pairs of opposite sides ≅.
⑩ ∠BFD is a right angle	⑩ Perpendicular lines form right angles
⑪ BEDF is a rectangle	⑪ A rectangle is a parallelogram with a right angle

Coordinate Geometry Proofs

Distance (Length) = $\sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Slope = $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

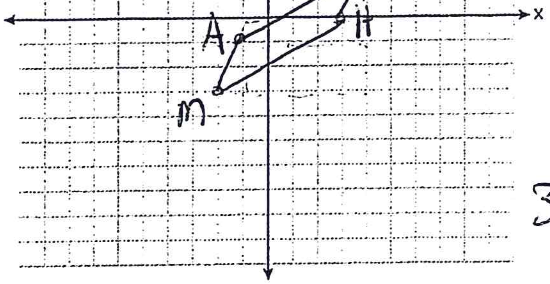
Midpoint = (*average x, average y*) = $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

How do you prove…?

…an **isosceles triangle**? (2 Distances)
Two Congruent Sides
…. a **right triangle**? (3 Distances)
Show the sides fit into Pythagorean Theorem
… a **parallelogram**? (4 Distances)
Two Pairs of Opposite Sides Congruent
… a **rhombus**? (4 Distances)
All Sides Congruent
… a **rectangle**? (6 Distances)
1) Two Pairs of Opposite Sides Congruent
2) Diagonals Congruent
… a **square**? (6 Distances)
1) All Sides Congruent
2) Diagonals Congruent
…a **trapezoid**? (4 Slopes)
1) 1 pair of opposite sides parallel
2) 1 pair of opposite sides not parallel
…an **isosceles trapezoid**? (4 Slopes, 2 Distances)
1) 1 pair of opposite sides parallel
2) 1 pair of opposite sides not parallel
3) Congruent Legs

1. Graph the quadrilateral MATH: M(-2, -3) A(-1, -1) T(4, 2) H(3, 0). Prove that MATH **IS** a parallelogram but is **NOT** a rectangle.

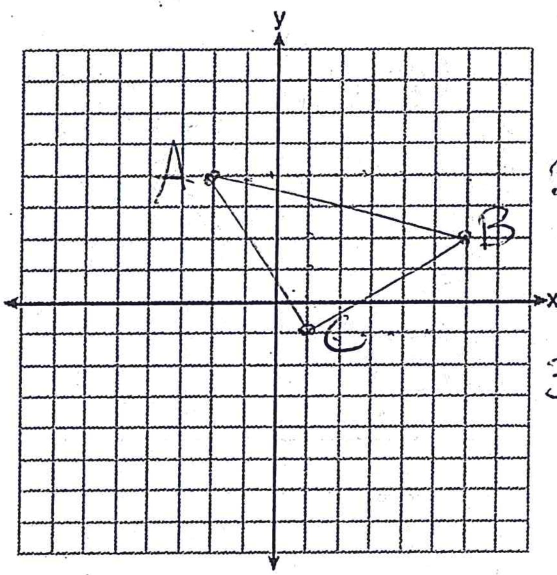
1) Math is a parallelogram because it has 2 pairs of opposite sides ≅
It is not a rectangle because diagonals are not ≅



2) $d\overline{MA} = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$
$d\overline{TH} = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$
$d\overline{AT} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$
$d\overline{MH} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$
$d\overline{AH} = \sqrt{4^2+1^2} = \sqrt{16+1} = \sqrt{17}$
$d\overline{MT} = \sqrt{6^2+5^2} = \sqrt{36+25} = \sqrt{61}$

3) $\overline{MA} \cong \overline{TH}$, $\overline{AT} \cong \overline{MH}$ because they have the Same distance
$\overline{AH} \not\cong \overline{MT}$ because they don't have the Same distang

2. A triangle has vertices $A(-2,4)$, $B(6,2)$, and $C(1,-1)$. Prove that $\triangle ABC$ is an isosceles right triangle. [The use of the set of axes below is optional.]



1) $\triangle ABC$ is an isosceles right triangle because it has two congruent sides and its sides fit into Pythagolean Theolem.

2) $d\,\overline{AC} = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$

$d\,\overline{BC} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\,\overline{AB} = \sqrt{8^2+2^2} = \sqrt{64+4} = \sqrt{68}$

3) $\overline{AC} \cong \overline{BC}$ because they have the same distance.

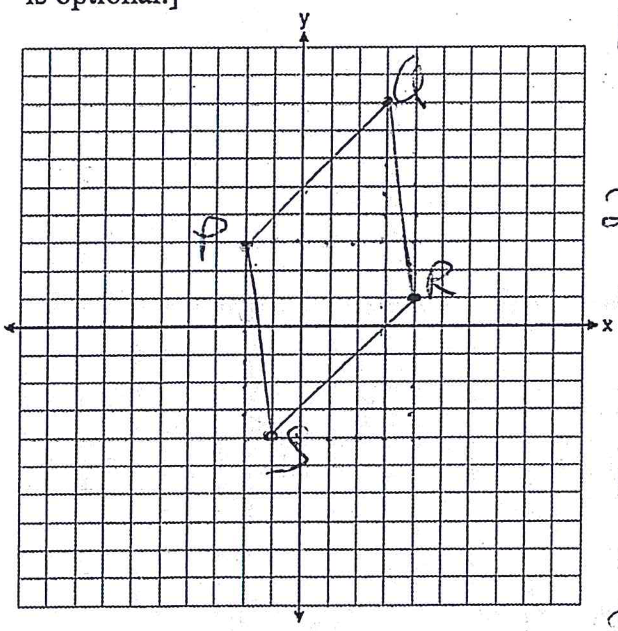
$a^2+b^2=c^2$

$\sqrt{34}^2+\sqrt{34}^2 = \sqrt{68}^2$

$34+34=68$

$68=68$ ✓

3. Quadrilateral *PQRS* has vertices $P(-2,3)$, $Q(3,8)$, $R(4,1)$, and $S(-1,-4)$. Prove that *PQRS* is a rhombus. Prove that *PQRS* is *not* a square. [The use of the set of axes below is optional.]



1) PQRS is a rhombus because all sides are congruent. It is not a square because diagonals are not congruent.

2) $d\,\overline{PQ} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$d\,\overline{QR} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$

$d\,\overline{RS} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$d\,\overline{SP} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$

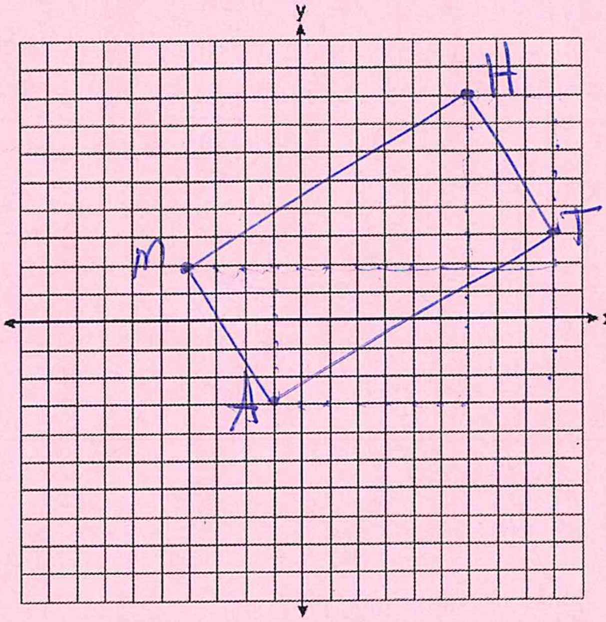
$d\,\overline{PR} = \sqrt{6^2+2^2} = \sqrt{36+4} = \sqrt{40}$

$d\,\overline{QS} = \sqrt{4^2+12^2} = \sqrt{16+144} = \sqrt{160}$

3) $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$ because they have the same distance.

$\overline{PR} \not\cong \overline{QS}$ because they don't have the same distance.

4. The vertices of quadrilateral *MATH* have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$. Prove that quadrilateral *MATH* is a parallelogram. Prove that quadrilateral *MATH* is a rectangle. [The use of the set of axes below is optional.]



1) MATH is a parallelogram because it has 2 pairs of opposite sides $\cong$. It is a rectangle because it is a parallelogram with congruent diagonals.

2) $d\overline{MA} = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$

$d\overline{HT} = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$

$d\overline{MH} = \sqrt{10^2+6^2} = \sqrt{100+36} = \sqrt{136}$

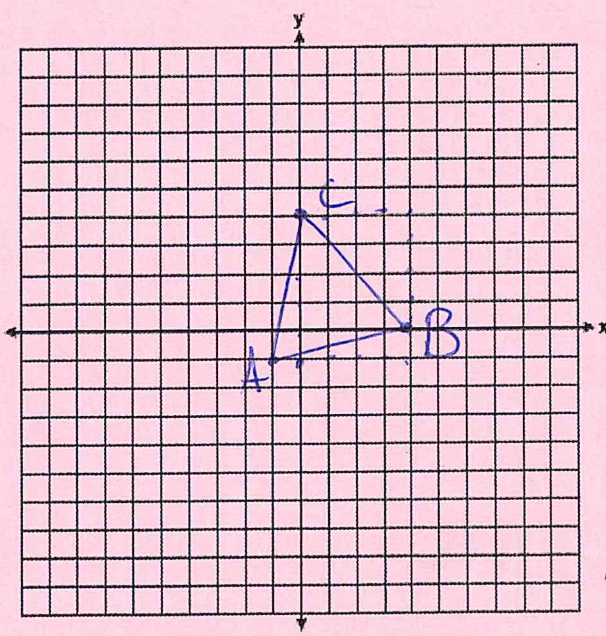
$d\overline{AT} = \sqrt{10^2+6^2} = \sqrt{100+36} = \sqrt{136}$

$d\overline{MT} = \sqrt{13^2+1^2} = \sqrt{169+1} = \sqrt{170}$

$d\overline{AH} = \sqrt{7^2+11^2} = \sqrt{49+121} = \sqrt{170}$

3) $\overline{MA} \cong \overline{HT}$, $\overline{MH} \cong \overline{AT}$, $\overline{MT} \cong \overline{AH}$ because they have the same distance

5. Triangle *ABC* has vertices with coordinates $A(-1,-1)$, $B(4,0)$, and $C(0,4)$. Prove that $\triangle ABC$ is an isosceles triangle but *not* an equilateral triangle. [The use of the set of axes below is optional.]



1) $\triangle ABC$ is isosceles because it has 2 $\cong$ sides. It is not equilateral because it does not have 3 $\cong$ sides.

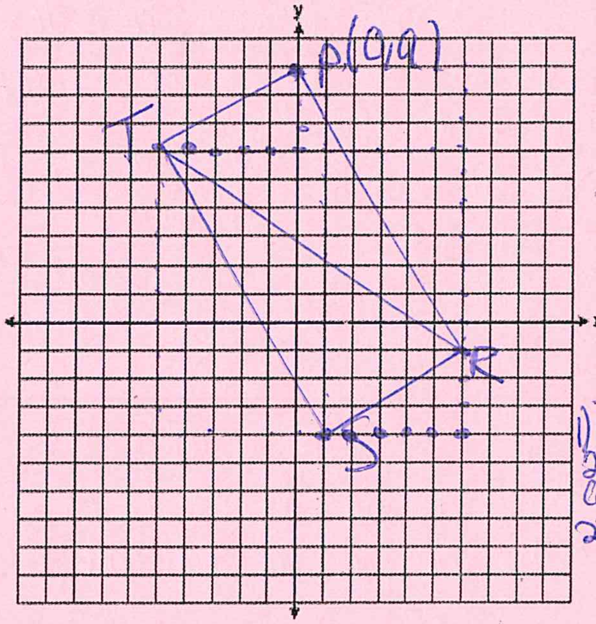
2) $d\overline{AB} = \sqrt{5^2+1^2} = \sqrt{25+1} = \sqrt{26}$

$d\overline{AC} = \sqrt{1^2+5^2} = \sqrt{1+25} = \sqrt{26}$

$d\overline{BC} = \sqrt{4^2+4^2} = \sqrt{16+16} = \sqrt{32}$

3) $\overline{AB} \cong \overline{AC}$ because they have the same distance. $\overline{AB} \ncong \overline{BC}$ because they don't have the same distance.

6. In the coordinate plane, the vertices of ΔRST are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that ΔRST is a right triangle. State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle. Prove that your quadrilateral $RSTP$ is a rectangle. [The use of the set of axes below is optional.]



1) ΔRST is a right triangle because its sides fit into Pythagorean theorem.

2) $d\,\overline{RS} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\,\overline{ST} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\,\overline{TR} = \sqrt{11^2+7^2} = \sqrt{121+49} = \sqrt{170}$

3) $a^2+b^2=c^2$

$\cancel{\sqrt{34}}^2 + \cancel{\sqrt{136}}^2 = \cancel{\sqrt{170}}^2$

$34+136=170$

$170=170$ ✓

1) RSTP is a rectangle because it has 2 pairs of opposite sides congruent and diagonals congruent.

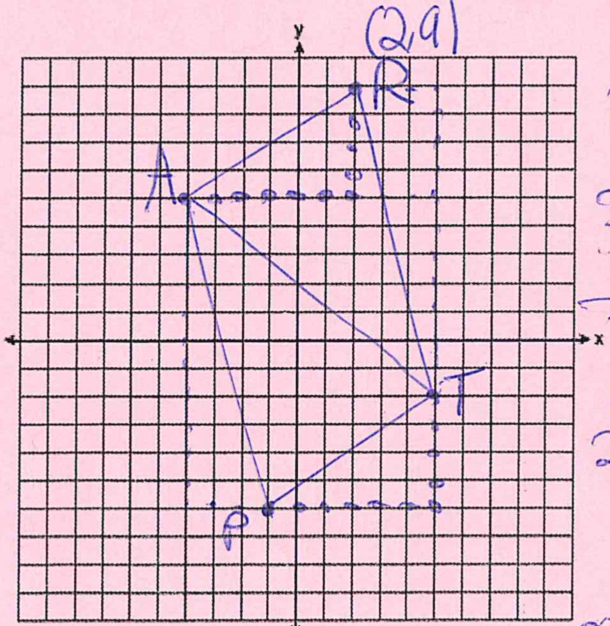
2) $d\,\overline{TP} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\,\overline{PR} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\,\overline{PS} = \sqrt{1^2+13^2} = \sqrt{1+169} = \sqrt{170}$

3) $\overline{TP} \cong \overline{SR}$, $\overline{TS} \cong \overline{PR}$, $\overline{TR} \cong \overline{PS}$ b/c they have the same distance.

7. In the coordinate plane, the vertices of triangle PAT are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes below is optional.] State the coordinates of R so that quadrilateral $PART$ is a parallelogram. Prove that quadrilateral $PART$ is a parallelogram.



1) ΔPAT is isosceles because it has two congruent sides.

2) $d\,\overline{PA} = \sqrt{3^2+11^2} = \sqrt{9+121} = \sqrt{130}$

$d\,\overline{AT} = \sqrt{9^2+7^2} = \sqrt{81+49} = \sqrt{130}$

3) $\overline{PA} \cong \overline{AT}$ because they have the same distance.

1) PART is a parallelogram because it has two pairs of opposite sides $\cong$

2) $d\,\overline{RT} = \sqrt{3^2+11^2} = \sqrt{9+121} = \sqrt{130}$

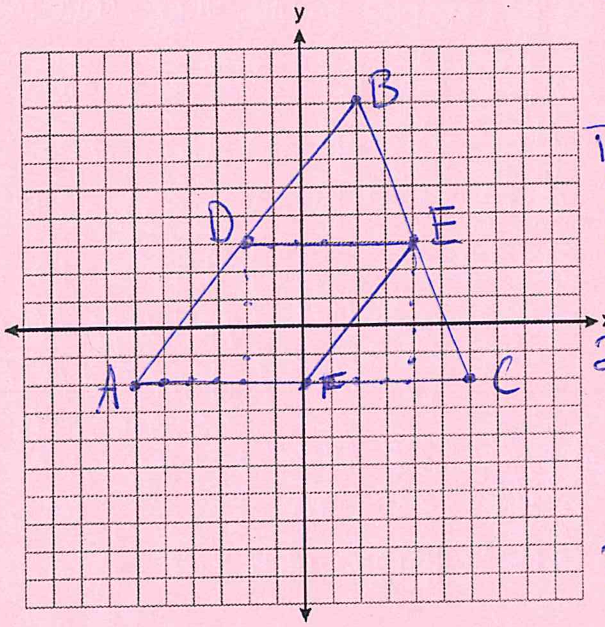
$d\,\overline{AR} = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

$d\,\overline{PT} = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

3) $\overline{AR} \cong \overline{PT}$, $\overline{PA} \cong \overline{RT}$ because they have the same distance

$mp = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

8. Given: $\triangle ABC$ with vertices $A(-6,-2)$, $B(2,8)$, and $C(6,-2)$. $\overline{AB}$ has midpoint D, $\overline{BC}$ has midpoint E, and $\overline{AC}$ has midpoint F.
Prove: *ADEF* is a parallelogram
ADEF is *not* a rhombus
[The use of the grid is optional.]



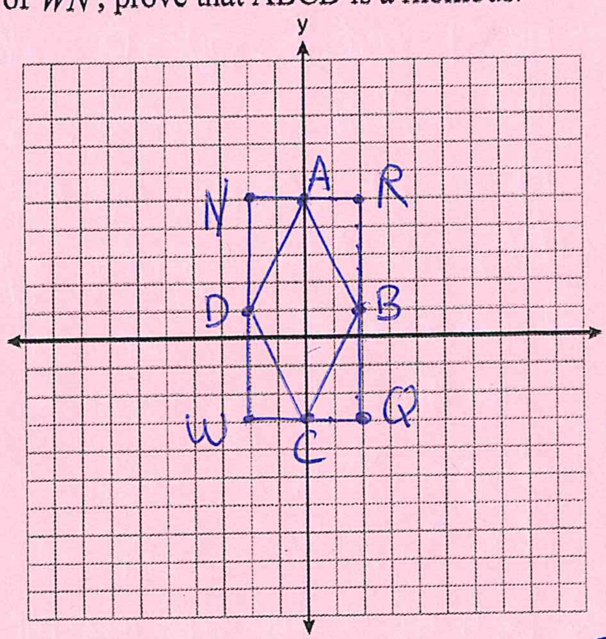
mp $\overline{AB}$ D	mp $\overline{BC}$ E	mp $\overline{AC}$ F
$\frac{-6+2}{2}, \frac{-2+8}{2}$	$\frac{2+6}{2}, \frac{8+-2}{2}$	$\frac{-6+6}{2}, \frac{-2+-2}{2}$
$(-2,3)$	$(4,3)$	$(0,-2)$

1) ADEF is a parallelogram because it has 2 pairs of opposite sides congruent. It is not a rhombus because not all sides are congruent.

2) $d\,\overline{AD} = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$
$d\,\overline{EF} = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$
$d\,\overline{DE} = 6$
$d\,\overline{AF} = 6$

3) $\overline{AD} \cong \overline{EF}$, $\overline{DE} \cong \overline{AF}$ b/c they have the same distance
$\overline{AD} \ncong \overline{DE}$ because they don't have the same distance.

9. The vertices of rectangle NRQW are N(-2,5), R(2,5), Q(2,-3), and W(-2,-3). If A is the midpoint $\overline{NR}$, B is the midpoint of $\overline{RQ}$, C is the midpoint of $\overline{QW}$, and D is the midpoint of $\overline{WN}$, prove that ABCD is a rhombus.



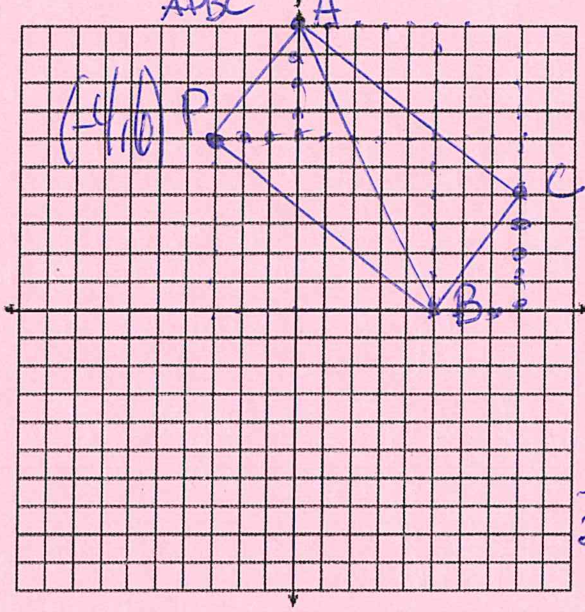
midpoint NR A	midpoint RQ B	midpoint QW C	midpoint WN D
$\frac{-2+2}{2}, \frac{5+5}{2}$	$\frac{2+2}{2}, \frac{5+-3}{2}$	$\frac{2+-2}{2}, \frac{-3+-3}{2}$	$\frac{-2+-2}{2}, \frac{5+-3}{2}$
$(0,5)$	$(2,1)$	$(0,-3)$	$(-2,1)$

1) ABCD is a rhombus because all sides are congruent.

2) $d\,\overline{AB} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\,\overline{BC} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\,\overline{CD} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\,\overline{DA} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$

3) $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$ because they have the same distance.

10. In the coordinate plane, the vertices of triangle ABC are A(0,10) B(5,0) and C(8,4). Prove that Triangle ABC is a right triangle. State the coordinates of point *P* such that quadrilateral ~~*ABCP*~~ APBC is a rectangle. Prove that your quadrilateral ~~*ABCP*~~ APBC is a rectangle.



1) ABC is a right triangle because its sides fit into Pythagorean Theorem.

2) $d\overline{AC} = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$

$d\overline{BC} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$

$d\overline{AB} = \sqrt{5^2+10^2} = \sqrt{25+100} = \sqrt{125}$

3) $a^2+b^2=c^2$

$\sqrt{100}^2+\sqrt{25}^2=\sqrt{125}^2$

$100+25=125$

$125=125$ ✓

1) APBC is a rectangle because it has 2 pairs of opposite sides ≅ and diagonals ≅.

2) $d\overline{PB} = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$

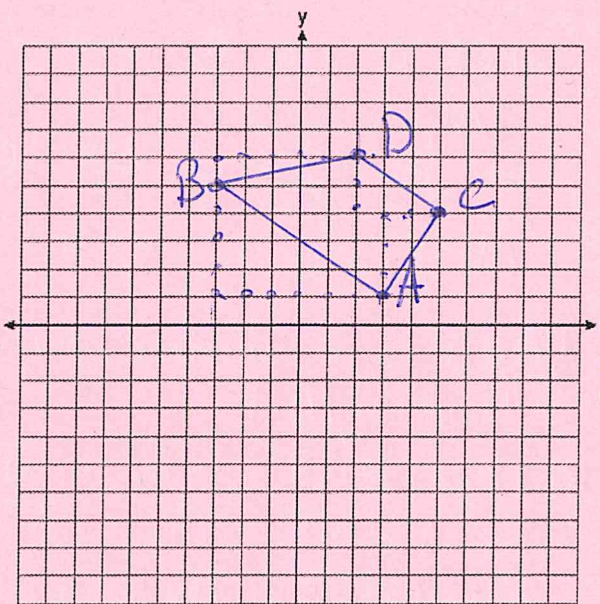
$d\overline{PA} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$

$d\overline{PC} = \sqrt{11^2+2^2} = \sqrt{121+4} = \sqrt{125}$

3) $\overline{AC} \cong \overline{PB}$, $\overline{AP} \cong \overline{CB}$, $\overline{PC} \cong \overline{AB}$ because they have the same distance.

11. Quadrilateral ABCD has vertices A(3,1) B(-3,5) C(5,4) and D(2,6). Prove quadrilateral ABCD is a trapezoid but *not* an isosceles trapezoid.

$m = \frac{\Delta y}{\Delta x}$



1) ABCD is a trapezoid because it has 1 pair of opposite sides ∥ and 1 pair of opposite sides ∦. It is not isosceles because it does not have congruent legs.

2) slope $\overline{DC} = -\frac{2}{3}$ slope $\overline{BD} = \frac{1}{5}$

slope $\overline{AB} = -\frac{4}{6} = -\frac{2}{3}$ slope $\overline{AC} = \frac{3}{2}$

$d\overline{BD} = \sqrt{5^2+1^2} = \sqrt{25+1} = \sqrt{26}$

$d\overline{AC} = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$

3) $\overline{DC} \parallel \overline{AB}$ because they have the same slope

$\overline{BD} \nparallel \overline{AC}$ because they don't have the same slope

$\overline{BD} \ncong \overline{AC}$ because they don't have the same distance.