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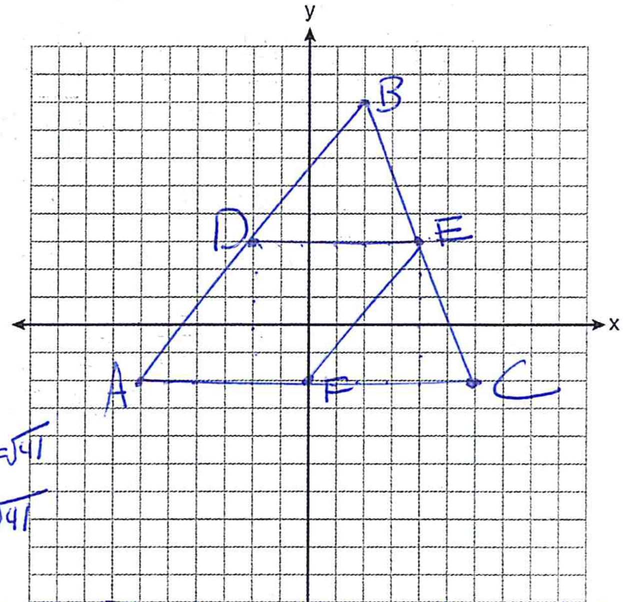
Date ____________
Geometry

Coordinate Geometry Proofs Applications

1. Given: $\triangle ABC$ with vertices $A(-6,-2)$, $B(2,8)$, and $C(6,-2)$. $\overline{AB}$ has midpoint D, $\overline{BC}$ has midpoint E, and $\overline{AC}$ has midpoint F.
Prove: *ADEF* is a parallelogram
ADEF is *not* a rhombus
[The use of the grid is optional.]



D
midpoint $\overline{AB}$
$\frac{-6+2}{2}, \frac{-2+8}{2}$
$\frac{-4}{2}, \frac{6}{2}$
$(-2,3)$

E
midpoint $\overline{BC}$
$\frac{2+6}{2}, \frac{8+-2}{2}$
$\frac{8}{2}, \frac{6}{2}$
$(4,3)$

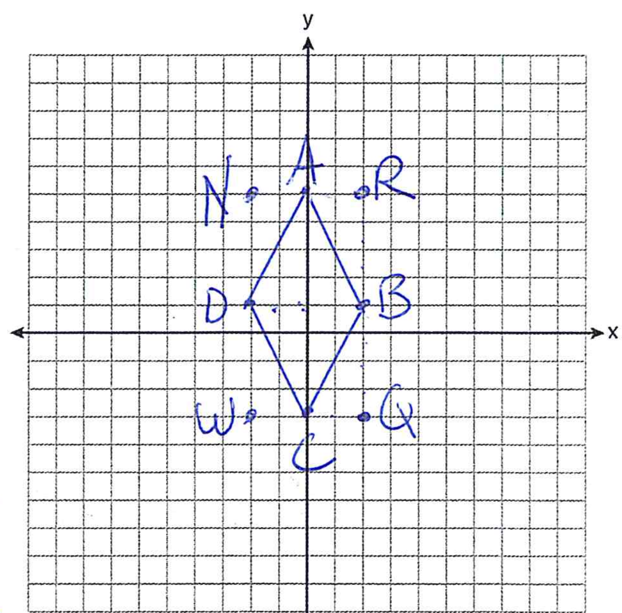
F
midpoint $\overline{AC}$
$\frac{-6+6}{2}, \frac{-2+-2}{2}$
$\frac{0}{2}, \frac{-4}{2}$
$(0,-2)$

1) ADEF is a parallelogram because it has 2 pairs of opposite sides congruent. It is not a rhombus because not all sides are congruent

2) $d\overline{AD} = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$
$d\overline{FE} = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$
$d\overline{DE} = 6$
$d\overline{AF} = 6$

3) $\overline{AD} \cong \overline{FE}$ and $\overline{DE} \cong \overline{AF}$ because they have the same distance
$\overline{AD} \not\cong \overline{DE}$ because they don't have the same distance.

2. The vertices of rectangle NRQW are N(-2,5), R(2,5), Q(2,-3), and W(-2,-3). If A is the midpoint $\overline{NR}$, B is the midpoint of $\overline{RQ}$, C is the midpoint of $\overline{QW}$, and D is the midpoint of $\overline{WN}$, prove that ABCD is a ~~parallelogram but *not* a~~ rhombus.



A
midpoint $\overline{NR}$
$\frac{-2+2}{2}, \frac{5+5}{2}$
$\frac{0}{2}, \frac{10}{2}$
$(0,5)$

B
midpoint $\overline{RQ}$
$\frac{2+2}{2}, \frac{5+-3}{2}$
$\frac{4}{2}, \frac{2}{2}$
$(2,1)$

C
midpoint $\overline{QW}$
$\frac{2+-2}{2}, \frac{-3+-3}{2}$
$\frac{0}{2}, \frac{-6}{2}$
$(0,-3)$

D
midpoint $\overline{WN}$
$\frac{-2+-2}{2}, \frac{5+-3}{2}$
$\frac{-4}{2}, \frac{2}{2}$
$(-2,1)$

1) ABCD is a rhombus because all sides are congruent

2) $d\overline{DA} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\overline{AB} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\overline{BC} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\overline{CD} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$

3) $\overline{DA} \cong \overline{AB} \cong \overline{BC} \cong \overline{CD}$ because they have the same distance

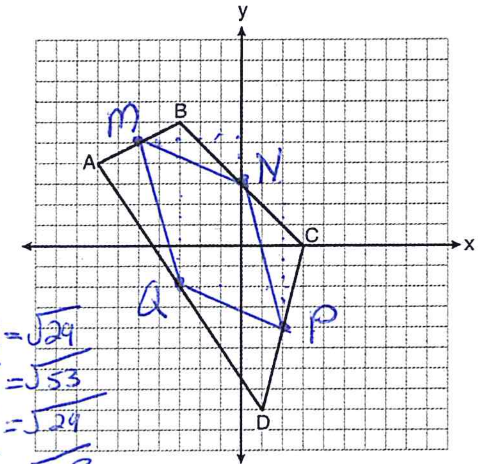
3. Quadrilateral *ABCD* with vertices $A(-7,4)$, $B(-3,6)$, $C(3,0)$, and $D(1,-8)$ is graphed on the set of axes below. Quadrilateral *MNPQ* is formed by joining *M*, *N*, *P*, and *Q*, the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$, respectively. Prove that quadrilateral *MNPQ* is a parallelogram. Prove that quadrilateral *MNPQ* is *not* a rhombus.

M midpoint AB: $\frac{-7+-3}{2}, \frac{4+6}{2}$; $-\frac{10}{2}, \frac{10}{2}$; $(-5,5)$

N midpoint BC: $\frac{-3+3}{2}, \frac{6+0}{2}$; $\frac{0}{2}, \frac{6}{2}$; $(0,3)$

P midpoint CD: $\frac{3+1}{2}, \frac{0+-8}{2}$; $\frac{4}{2}, -\frac{8}{2}$; $(2,-4)$

Q midpoint AD: $\frac{-7+1}{2}, \frac{4+-8}{2}$; $-\frac{6}{2}, -\frac{4}{2}$; $(-3,-2)$



1) MNPQ is a parallelogram because it has two pairs of opposite sides ≅. It is not a rhombus because not all sides are congruent.

2) $d\,\overline{MN} = \sqrt{5^2+2^2} = \sqrt{25+4} = \sqrt{29}$

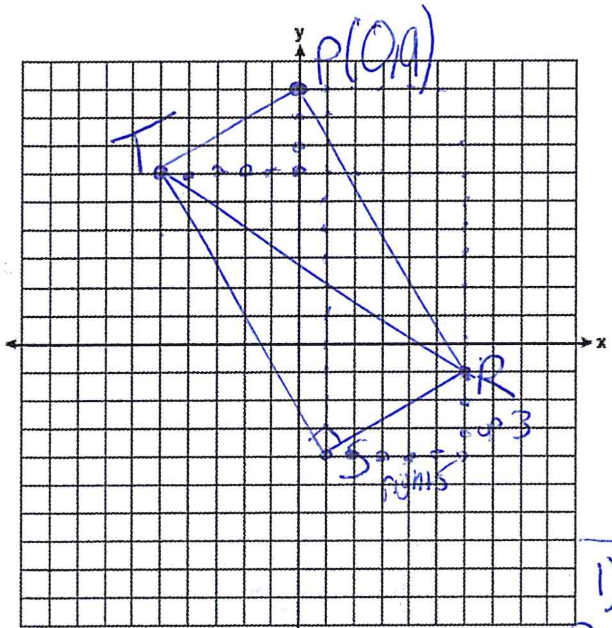
$d\,\overline{NP} = \sqrt{2^2+7^2} = \sqrt{4+49} = \sqrt{53}$

$d\,\overline{PQ} = \sqrt{5^2+2^2} = \sqrt{25+4} = \sqrt{29}$

$d\,\overline{QM} = \sqrt{2^2+7^2} = \sqrt{4+49} = \sqrt{53}$

3) $\overline{NM} \cong \overline{PQ}$ and $\overline{NP} \cong \overline{QM}$ because they have the same distance. $\overline{NM} \ncong \overline{NP}$ because they don't have the same distance.

4. In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle. State the coordinates of point *P* such that quadrilateral *RSTP* is a rectangle. Prove that your quadrilateral *RSTP* is a rectangle. [The use of the set of axes below is optional.]



1) $\triangle RST$ is a right triangle because its sides fit into Pythagorean Theorem.

2) $d\,\overline{RS} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\,\overline{ST} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\,\overline{TR} = \sqrt{11^2+7^2} = \sqrt{121+49} = \sqrt{170}$

3) $a^2+b^2=c^2$

$\sqrt{34}^2+\sqrt{136}^2=\sqrt{170}^2$

$34+136=170$

$170=170$ ✓

1) RSTP is a rectangle because it has 2 pairs of opposite sides congruent and diagonals congruent.

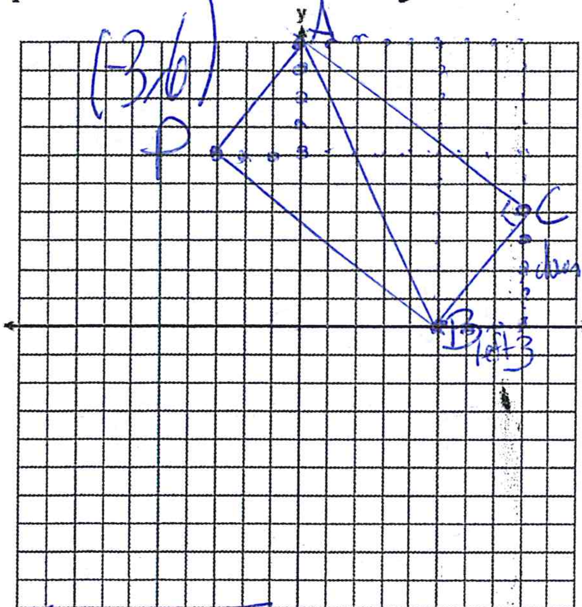
2) $d\,\overline{TP} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\,\overline{PR} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\,\overline{PS} = \sqrt{1^2+13^2} = \sqrt{1+169} = \sqrt{170}$

(3) ~~TR~~ $\overline{TP} \cong \overline{SR}$, $\overline{TS} \cong \overline{PR}$, $\overline{TR} \cong \overline{PS}$ because they have the same distance

5. In the coordinate plane, the vertices of Triangle ABC are A(0,10) B(5,0) and C(8,4). Prove that Triangle ABC is a right triangle. State the coordinates of point *P* such that quadrilateral *ABCP* is a rectangle. Prove that your quadrilateral *ABCP* is a rectangle.



1) ABC is a right triangle because its sides fit into Pythagorean Theorem.

2) $d\overline{BC} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$

$d\overline{CA} = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$

$d\overline{AB} = \sqrt{5^2+10^2} = \sqrt{25+100} = \sqrt{125}$

3) $a^2+b^2=c^2$

$\sqrt{25}^2+\sqrt{100}^2=\sqrt{125}^2$

$25+100=125$

$125=125$ ✓

1) ABCP is a rectangle because it has 2 pairs of opposite sides congruent and diagonals congruent.

2) $d\overline{PA} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$

$dPB = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$

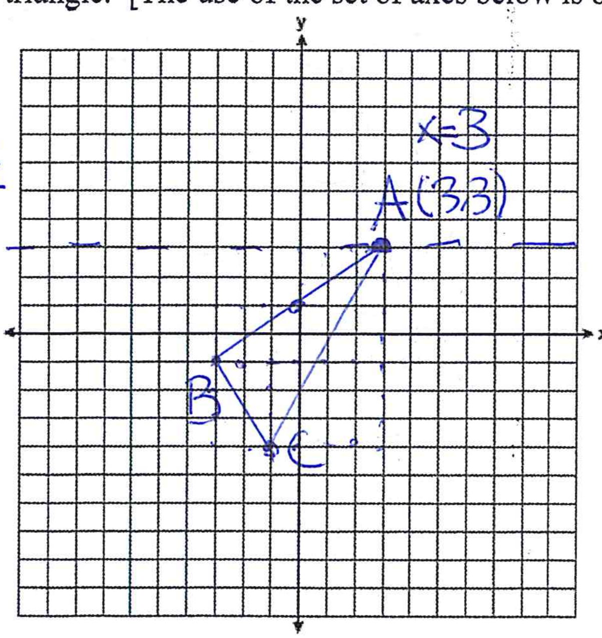
$dPC = \sqrt{11^2+2^2} = \sqrt{121+4} = \sqrt{125}$

3) ~~AB≅BC~~ $\overline{AP} \cong \overline{CB}$, $\overline{AC} \cong \overline{PB}$, and $\overline{AB} \cong \overline{PC}$ because they have the same distance.

6. Triangle *ABC* has vertices with $A(x,3)$, $B(-3,-1)$, and $C(-1,-4)$. Determine and state a value of *x* that would make triangle *ABC* a right triangle. Justify why $\triangle ABC$ is a right triangle. [The use of the set of axes below is optional.] Perpendicular lines have negative reciprocal slopes

$m\overline{BC} = -\frac{3}{2}$

$mBA = \frac{2}{3}$



1) ABC is a right triangle because its sides fit into Pythagorean Theorem.

2) $d\overline{AB} = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

$d\overline{BC} = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$

$d\overline{AC} = \sqrt{4^2+7^2} = \sqrt{16+49} = \sqrt{65}$

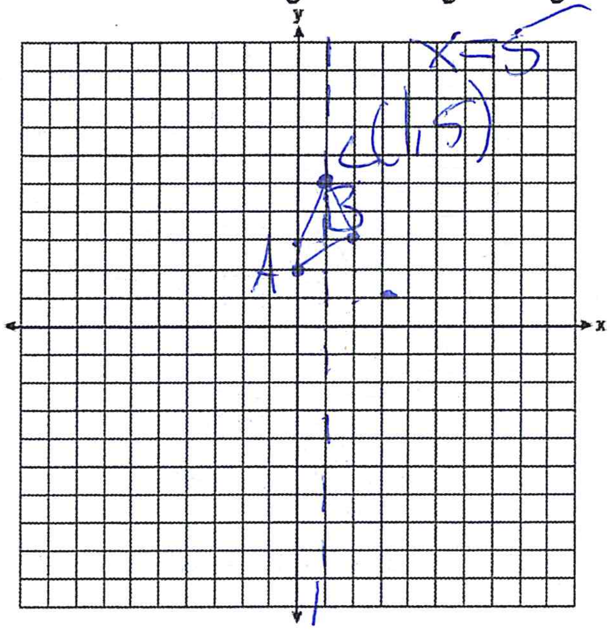
3) $a^2+b^2=c^2$

$\sqrt{52}^2+\sqrt{13}^2=\sqrt{65}^2$

$52+13=65$

$65=65$ ✓

7. Triangle ABC has vertices A(0,2), B(2,3), and C(1,x). Determine and state a value of x that would make triangle ABC a right triangle. Justify why $\triangle ABC$ is a right triangle.



$m\overline{AB} = \frac{1}{2}$ > negative reciprocal slopes

$m\overline{BC} = -\frac{2}{1}$

1) ~~AP~~ $\triangle ABC$ is a right triangle because its sides fit into Pythagorean Theorem.

2) $d\overline{AB} = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$

$d\overline{BC} = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$

$d\overline{AC} = \sqrt{1^2+3^2} = \sqrt{1+9} = \sqrt{10}$

3) $a^2+b^2=c^2$

$\cancel{\sqrt{}}5^2 + \cancel{\sqrt{}}5^2 = \cancel{\sqrt{}}10^2$

$5+5=10$

$10=10$ ✓