

$$\text{Margin of Error} = 2(\text{standard deviation})$$

$$\text{Confidence Interval} = \text{mean} \pm 2(\text{standard deviation})$$

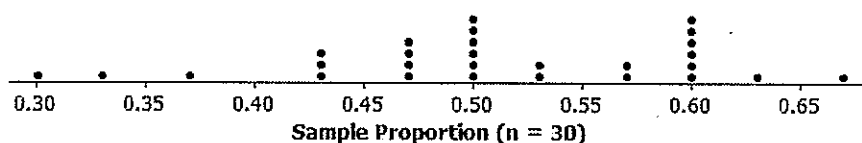
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Mr. Schlansky

As sample size increases, the mean relatively stays the same and the standard deviation decreases.

Date \_\_\_\_\_  
Algebra II

## Confidence Interval (Expected Values and Sample Size)

1. A group of eleventh graders wanted to estimate the population proportion of students in their high school who drink at least one soda per day. Each student selected a different random sample of 30 students and calculated the proportion that drink at least one soda per day. The dot plot below shows the sampling distribution. This distribution has a mean of 0.51 and a standard deviation of 0.09.



Find the margin of error and the confidence interval. Explain the meaning of the confidence interval in the context of the problem.

$$MOE = 2(SD)$$

$$MOE = 2(0.09)$$

$$MOE = .18$$

$$CI = \text{mean} \pm 2(SD)$$

$$= .51 + 2(0.09) = .69$$

$$.51 - 2(0.09) = .33$$

$$[.33, .69]$$

The range of expected values for the proportion of students who drink at least 1 soda per day is between .33 and .69.

What is your estimate for the proportion of *all* students who would report that they drink at least one soda per day?

Approximately .51. The sample mean approximates the population mean.

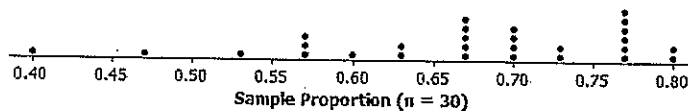
Is it reasonable to say that 40% of students drink at least one soda per day? Explain your answer.

Yes, .4 is inside the confidence interval.

If, instead of taking random samples of 30 students in the high school, the eleventh graders randomly selected samples of size 60, describe what will happen to the mean and standard deviation of the sampling distribution of the sample proportions.

The mean would relatively stay the same.  
The standard deviation decreases.

2. A class of 28 eleventh graders wanted to estimate the proportion of all juniors and seniors at their high school with part-time jobs after school. Each eleventh grader took a random sample of 30 juniors and seniors and then calculated the proportion with part-time jobs. A dot plot is created to represent the data. The mean is 0.67 and the standard deviation is 0.1.



Find the margin of error and the confidence interval. Explain what the confidence interval means in the context of the problem.

$$MOE = 26.11$$

$$MOE = .2$$

$$CI = .67 + 26.11 = .87$$

$$.67 - 26.11 = .47$$

$$[.47, .87]$$

The range of expected values for students who have part time jobs is between .47 and .87.

What is your estimate of the proportion of all juniors and seniors that have part time jobs after school? Explain.

Approximately .67. The sample mean approximates the population mean.

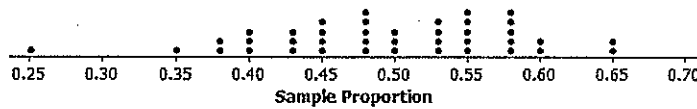
Do you think that the proportion of all juniors and seniors at the school with part-time jobs could be 0.7? Do you think it could be 0.5? Justify your answers.

Yes, both values are inside the confidence interval.

Suppose the eleventh graders had taken random samples of <sup>sample size increased</sup> size 60. How would the distribution of sample proportions based on samples of size 60 compare to the distribution for samples of size 30?

The mean would relatively stay the same.  
The standard deviation decreases.

3. The following is an example of a sampling distribution of sample proportions of heads in 40 flips of a coin. The mean is .4955 and the sample standard deviation is .0852.



Find the margin of error and the confidence interval. Explain the meaning of the confidence interval in the context of the problem.

$$\begin{aligned} \text{MOE} &= 2(.0852) \\ \text{MOE} &= .1704 \end{aligned} \quad \begin{aligned} \text{CI} &= .4955 + 2(.0852) = .5807 \\ &= .4955 - 2(.0852) = .4103 \\ &= [.4103, .5807] \end{aligned} \quad \begin{aligned} &\text{The range of expected} \\ &\text{values for the proportion} \\ &\text{of heads is between} \\ & .4103 \text{ and } .5807. \end{aligned}$$

What is the sample mean? What is population mean? How do they compare?

Sample mean = .4955  
Population mean = .5  
(.5)

They are relatively the same.

Fred flipped a coin 40 times and 65% of the flips came up heads. Is this an expected outcome? Explain your answer.

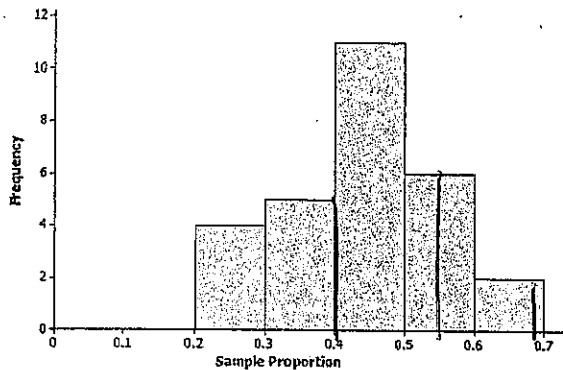
No, .65 is not inside the confidence interval.

If this experiment was performed where the coin was flipped 20 times, how would the data be affected?

Sample size decreased

The mean would relatively stay the same.  
The standard deviation would increase.

4. The nurse in your school district would like to study the proportion of all high school students in the district who usually get at least eight hours of sleep on school nights. Suppose each student in your class takes a random sample of 20 high school students in the district and each calculates their sample proportion of students who said that they usually get at least eight hours of sleep on school nights. Below is a histogram of the sampling distribution.



Do you think that the proportion of all high school students who usually get at least eight hours of sleep on school nights could have been 0.4? Do you think it could have been 0.55? Could it have been 0.68? Justify your answers based on the histogram.

- .4: yes, .4 is inside the confidence interval (the middle 95% of the data)
- .55: yes, .55 is inside the confidence interval
- .68: no, .68 is not inside the confidence interval

Suppose students had taken random samples of size 60. <sup>increased the sample size</sup> How would the distribution of sample proportions based on samples of size 60 differ from those of size 20?

- The mean would relatively stay the same.
- The standard deviation would decrease.

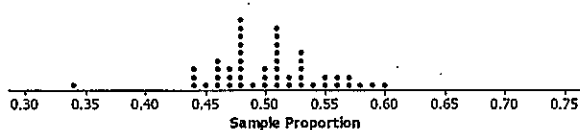
5. Suppose two sets of test scores have the same mean, but different standard deviations,  $\sigma_1$  and  $\sigma_2$ , with  $\sigma_2 > \sigma_1$ . Which statement best describes the variability of these data sets?

- 1) Data set one has the greater variability.
- 2) Data set two has the greater variability.
- 3) The variability will be the same for each data set.
- 4) No conclusion can be made regarding the variability of either set.

the greater the standard deviation, the greater the variability

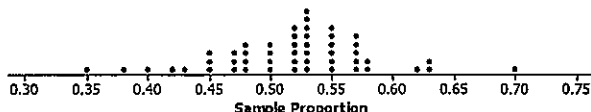
6. Below are three dot plots of the proportion of tails in 20, 60, or 120 simulated flips of a coin. The mean and standard deviation of the sample proportions are also shown for each of the three dot plots. Match each dot plot with the appropriate number of flips. Clearly explain how you matched the plots with the number of simulated flips.

Dot Plot 1:



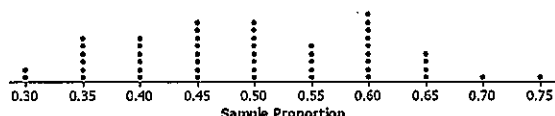
120 (least variability)

Dot Plot 2:



60 (middle variability)

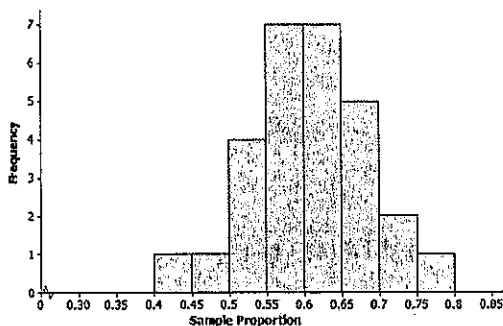
Dot Plot 3:



20 (greatest variability)

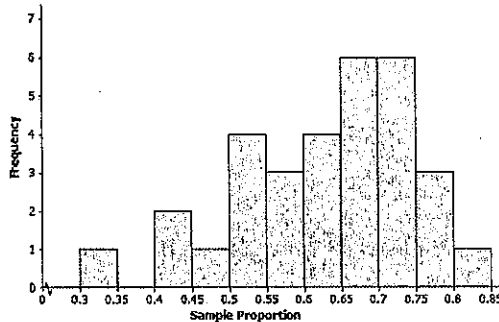
7. A group of eleventh graders wanted to estimate the proportion of all students at their high school who suffer from allergies. Each student in one group of eleventh graders took a random sample of 20 students, while each student in another group of eleventh graders each took a random sample of 40 students. Below are the two sampling distributions (shown as histograms) of the sample proportions of high school students who said that they suffer from allergies. Which histogram is based on random samples of size 40? Explain.

Histogram A



40

Histogram B



20

The lower sample size will have the greater variability.

$$6(60) = 360$$

8. Elizabeth waited for 6 minutes at the drive thru at her favorite fast-food restaurant the last time she visited. She was upset about having to wait that long and notified the manager. The manager assured her that her experience was very unusual and that it would not happen again. A study of customers commissioned by this restaurant found an approximately normal distribution of results. The mean wait time was 226 seconds and the standard deviation was 38 seconds. Given these data, and using a 95% level of confidence, was Elizabeth's wait time unusual? Justify your answer.

$$CI = \text{mean} \pm 2(SD)$$

$$CI = 226 + 2(38) = 302$$

$$226 - 2(38) = 150$$

$$[150, 302]$$

Yes, 360 is not

inside the

confidence interval.

confidence interval

9. Jessica got 20 math problems for homework and complained to her teacher that this was an unusual amount of homework. Her teacher told her to look at the number of questions in all of her past homework assignments from the school year and find the range of the expected number of math problems. She found that the mean was 11.2 and the standard deviation was 3. Was Jessica correct that 20 math problems was unusual? Justify your answer.

$$CI = 11.2 + 2(3) = 17.2$$

$$11.2 - 2(3) = 5.2$$

$$[5.2, 17.2]$$

Yes, 20 is unusual because it is not inside the confidence interval.

10. Fatima bought a chicken burrito for dinner and was unhappy with the amount of chicken that she received. She received 4.75 ounces of chicken and believed that this was less than normal. The manager conducted a study and found that the mean amount of chicken on their burritos was 5.1 ounces with a standard deviation of .25 ounces. Did Fatima's burrito have an expected amount of chicken? Justify your answer.

$$CI = 5.1 + 2(.25) = 5.6$$

$$5.1 - 2(.25) = \cancel{4.6} 4.6$$

$$[4.6, 5.6]$$

Yes, it was an expected amount because 4.75 is inside the confidence interval.