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Date _____
Geometry

Completing the Square with Circles

What is the center and radius of the circle with the following equations:

1. $x^2 + y^2 + 6x - 8y = 0$

$$x^2 + 6x + y^2 - 8y = 0$$

$$\left(\frac{6}{2}\right)^2 = 9$$

$$\left(-\frac{8}{2}\right)^2 = 16$$

Center: $(-3, 4)$

radius: 5

$$x^2 + 6x + \boxed{9} + y^2 - 8y + \boxed{16} = 0 + \boxed{9} + \boxed{16}$$

$$(x+3)(x+3) + (y-4)(y-4) = 25$$

$$(x+3)^2 + (y-4)^2 = 25$$

2. $x^2 + y^2 + 10x - 4y - 7 = 0$

$$x^2 + 10x + y^2 - 4y = 7$$

$$\left(\frac{10}{2}\right)^2 = 25 \quad \left(-\frac{4}{2}\right)^2 = 4$$

$$x^2 + 10x + \boxed{25} + y^2 - 4y + \boxed{4} = 7 + \boxed{25} + \boxed{4}$$

$$(x+5)(x+5) + (y-2)(y-2) = 36$$

$$(x+5)^2 + (y-2)^2 = 36$$

Center: $(-5, 2)$

radius: 6

3. $x^2 + y^2 + 16x + 6y + 1 = 0$

~~x^2~~

$-1 -1$

$$x^2 + 16x + y^2 + 6y = -1$$

$$x^2 + 16x + \boxed{64} + y^2 + 6y + \boxed{9} = -1 + \boxed{64} + \boxed{9}$$

$$(x+8)(x+8) + (y+3)(y+3) = 72$$

$$(x+8)^2 + (y+3)^2 = 72$$

Center: $(-8, -3)$

radius: $\sqrt{72}$

4. $x^2 + y^2 - 12x - 14y = 15$

$$x^2 - 12x + y^2 - 14y = 15$$

$$x^2 - 12x + \boxed{36} + y^2 - 14y + \boxed{49} = 15 + \boxed{36} + \boxed{49}$$

$$(x-6)(x-6) + (y-7)(y-7) = 100$$

$$(x-6)^2 + (y-7)^2 = 100$$

Center: $(6, 7)$

radius: 10

$$5. x^2 + y^2 - 4x + 8y + \frac{31}{4} = 0$$

$$\begin{aligned} x^2 - 4x \\ x^2 - 4x + \boxed{4} \\ (x-2)(x-2) \\ (x-2)^2 \end{aligned}$$

$$\begin{aligned} x^2 - 4x + y^2 + 8y &= -\frac{31}{4} \\ x^2 - 4x + \boxed{4} + y^2 + 8y + \boxed{16} &= -\frac{31}{4} + \boxed{4} + \boxed{16} \\ (x-2)(x-2) + (y+4)(y+4) &= \frac{49}{4} \\ (x-2)^2 + (y+4)^2 &= \frac{49}{4} \end{aligned}$$

Center: (2, -4)
radius: $\frac{7}{2}$

$$6. x^2 + 4x + y^2 - 2y = 3$$

$$\begin{aligned} x^2 + 4x + \boxed{4} + y^2 - 2y + \boxed{1} &= 3 + \boxed{4} + \boxed{1} \\ (x+2)(x+2) + (y-1)(y-1) &= 8 \\ (x+2)^2 + (y-1)^2 &= 8 \end{aligned}$$

Center: (-2, 1)
radius: $\sqrt{8}$

$$7. x^2 + y^2 + 6x - 2y = \frac{31}{9}$$

$$\begin{aligned} x^2 + 6x + \boxed{9} + y^2 - 2y + \boxed{1} &= \frac{31}{9} + \boxed{9} + \boxed{1} \\ (x+3)(x+3) + (y-1)(y-1) &= \frac{121}{9} \\ (x+3)^2 + (y-1)^2 &= \frac{121}{9} \end{aligned}$$

Center: (-3, 1)
radius: $\frac{11}{3} \sqrt{\frac{121}{9}}$

$$8. x^2 + y^2 + 6x - 10y + 4 = 0$$

$$\begin{aligned} x^2 + 6x + y^2 - 10y &= -4 \\ x^2 + 6x + \boxed{9} + y^2 - 10y + \boxed{25} &= -4 + \boxed{9} + \boxed{25} \\ (x+3)(x+3) + (y-5)(y-5) &= 30 \\ (x+3)^2 + (y-5)^2 &= 30 \end{aligned}$$

Center: (-3, 5)
radius: $\sqrt{30}$