Complex Triangle Problems
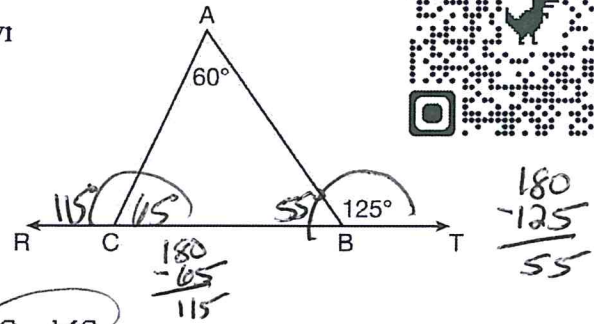
1. In the diagram below, $\overleftrightarrow{RCBT}$ and $\triangle ABC$ are shown
What is m∠ACR?
1) 125
2) 115
3) 65
4) 55

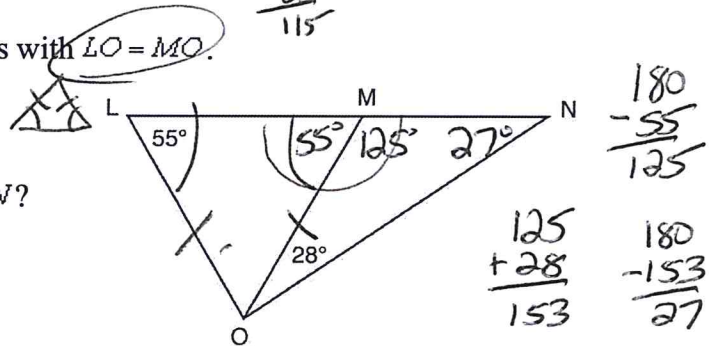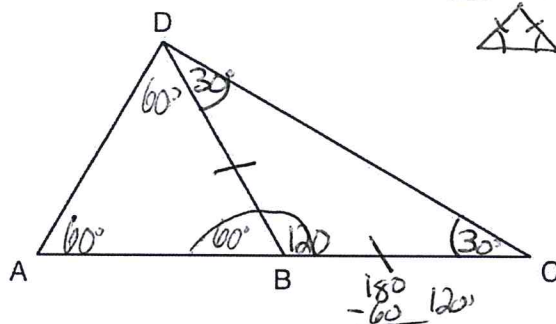$\begin{array}{r} 60 \\ +55 \\ \hline 115 \end{array}$ $\begin{array}{r} 180 \\ -115 \\ \hline 65 \end{array}$

60°

115° 65°   55° 125°

R   C   B   T

$\begin{array}{r} 180 \\ -65 \\ \hline 115 \end{array}$

$\begin{array}{r} 180 \\ -125 \\ \hline 55 \end{array}$

2. In the diagram below, $\triangle LMO$ is isosceles with $LO = MO$.

L   55°   M   55° 125° 27°   N

28°

O

If m∠L = 55 and m∠NOM = 28, what is m∠N?
1) 27
2) 28
3) 42
4) 70

$\begin{array}{r} 180 \\ -55 \\ \hline 125 \end{array}$

$\begin{array}{r} 125 \\ +28 \\ \hline 153 \end{array}$ $\begin{array}{r} 180 \\ -153 \\ \hline 27 \end{array}$
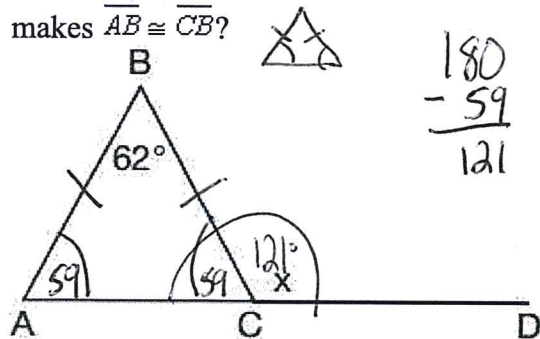
3. In the diagram below of $\triangle ACD$, B is a point on $\overline{AC}$ such that $\triangle ADB$ is an equilateral triangle, and $\triangle DBC$ is an isosceles triangle with $\overline{DB} \cong \overline{BC}$. Find m∠C.

60,60,60

D

60° 30°

60°   60° 120°   30°

A   B   C

$\begin{array}{r} 180 \\ -60 \\ \hline 120 \end{array}$ 120°

$\begin{array}{r} 180 \\ -60 \\ \hline 120 \end{array}$

$\begin{array}{r} 180 \\ -120 \\ \hline 60 \div 2 \\ 30 \end{array}$

∠C=30°

4. Given $\triangle ABC$ with m∠B = 62° and side $\overline{AC}$ extended to D, as shown below. Which value of x makes $\overline{AB} \cong \overline{CB}$?
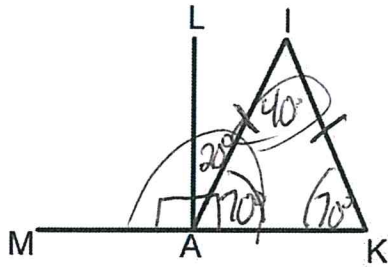
B

62°

59°   59° 121° x

A   C   D

$\begin{array}{r} 180 \\ -59 \\ \hline 121 \end{array}$

$\begin{array}{r} 180 \\ -62 \\ \hline 118 \end{array}$
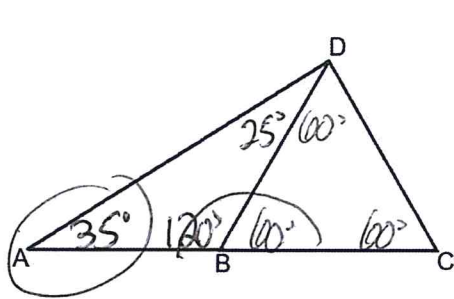
$\dfrac{118}{2} = 59$

5. In the diagram below, $m\angle MAL = 90$, $m\angle IAL = 20$, and $\overline{IA} \cong \overline{AK}$. Find $m\angle I$.
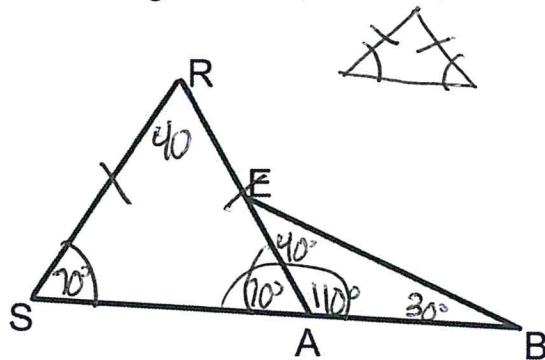

$$\begin{array}{cc} 90 & 180 \\ +20 & -110 \\ \hline 110 & 70° \end{array} \qquad \begin{array}{cc} 70 & 180 \\ +70 & -140 \\ \hline 140 & 40 \end{array}$$

6. In the diagram below, $\triangle DBC$ is an equilateral triangle and $m\angle ADB = 25$. Find $m\angle DAB$

60,60,60

$$\begin{array}{c} 180 \\ -60 \\ \hline 120 \end{array} \qquad \begin{array}{c} 120 \\ +25 \\ \hline 145 \end{array} \qquad \begin{array}{c} 180 \\ -145 \\ \hline 35° \end{array}$$
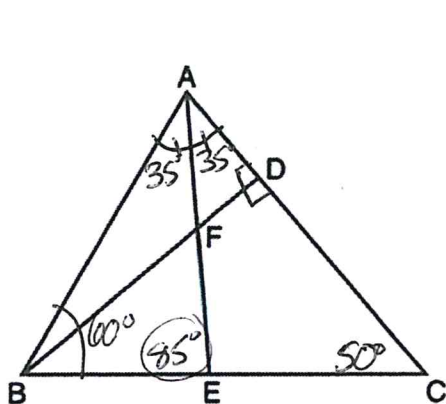
7. In the diagram below, $\overline{SR} \cong \overline{RA}$, $m\angle SRA = 40$, and $m\angle ABE = 30$. Find $m\angle BEA$.

$$\begin{array}{c} 180 \\ -40 \\ \hline 140 \end{array} \qquad \frac{140}{2}=70 \qquad \angle BEA = 40°$$

$$\begin{array}{c} 180 \\ -70 \\ \hline 110 \end{array} \qquad \begin{array}{cc} 110 & 180 \\ +30 & -140 \\ \hline 140 & 40 \end{array}$$

8. In the diagram of $\triangle ABC$ below, $\overline{AE}$ bisects angle BAC, and altitude $\overline{BD}$ is drawn. If $m\angle C = 50°$ and $m\angle ABC = 60°$, what is $m\angle FEB$?

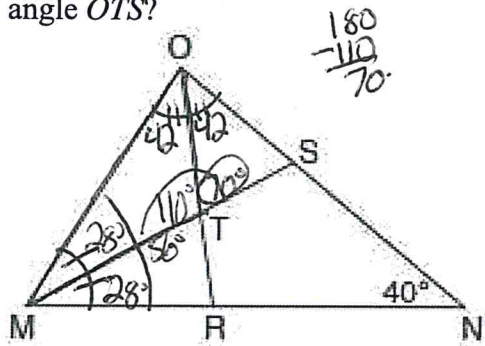$$\triangle ABC \qquad \frac{70}{2}=35° \qquad \triangle ABE$$

$$\begin{array}{cc} 60 & 180 \\ +50 & -110 \\ \hline 110 & 70 \end{array} \qquad\qquad \begin{array}{cc} 60 & 180 \\ +35 & -95 \\ \hline 95 & 85 \end{array}$$

$$\angle FEB = 85°$$

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9. In the diagram below of triangle *MNO*, ∠*M* and ∠*O* are <u>bisected</u> by $\overline{MS}$ and $\overline{OR}$, respectively. Segments *MS* and *OR* intersect at *T*, and m∠*N* = 40°. If m∠*TMR* = 28°, what is the measure of angle *OTS*?

Handwritten work:

$$\begin{array}{c}180\\-110\\\hline 70°\end{array}$$

$$\begin{array}{c}56\\+40\\\hline 96\end{array} \qquad \begin{array}{c}180\\-96\\\hline 84\end{array} \qquad \frac{84}{2}=42$$

$$\boxed{\angle OTS = 70°}$$

△MOT
$$\begin{array}{c}42\\+28\\\hline 70\end{array} \qquad \begin{array}{c}180\\-70\\\hline 110\end{array}$$

Diagram labels: O, S, T, 28°, 56°, 40°, M, R, N

10. In the diagram below, $\overline{EA}$ bisects ∠*SAR*, $\overline{RA} \cong \overline{AS}$ and m∠*SRA* = 55. Find and m∠*RAB* and m∠*REA*.
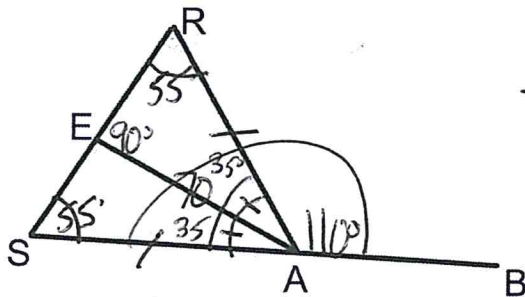
Handwritten work:

$$\begin{array}{c}55\\+55\\\hline 110\end{array} \qquad \begin{array}{c}180\\-110\\\hline 70\end{array} \qquad \frac{70}{2}=35°$$

$$\begin{array}{c}180\\-70\\\hline 110°\end{array}$$

∠RAB = 110°

∠REA = 90°

△REA
$$\begin{array}{c}55\\+35\\\hline 90\end{array} \qquad \begin{array}{c}180\\-90\\\hline 90\end{array}$$

Diagram labels: R, 55, E, 90°, 35, 55', 35, 110°, S, A, B

11. In the diagram below of isosceles triangle *ABC*, $\overline{AB} \cong \overline{CB}$ and angle bisectors $\overline{AD}$, $\overline{BF}$, and $\overline{CE}$ are drawn and intersect at *X*.
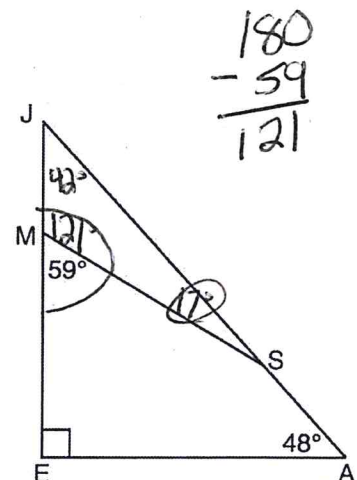If m∠*BAC* = 50°, find m∠*AXC*.

Handwritten work:

$$\begin{array}{c}25\\+25\\\hline 50\end{array} \qquad \begin{array}{c}180\\-50\\\hline 130\end{array}$$

Diagram labels: B, E, D, X, 130°, 25°, 25°, A, F, C
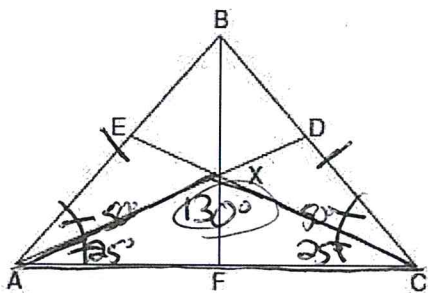
12. In the diagram of △*JEA* below, m∠*JEA* = 90 and m∠*EAJ* = 48.
Line segment *MS* connects points *M* and *S* on the triangle, such that m∠*EMS* = 59.
What is m∠*JSM*?

Handwritten work:

△JEA
$$\begin{array}{c}90\\+48\\\hline 138\end{array} \qquad \begin{array}{c}180\\-138\\\hline 42°\end{array}$$

$$\begin{array}{c}42\\+121\\\hline 163\end{array} \qquad \begin{array}{c}180\\-163\\\hline 17\end{array}$$

$$\begin{array}{c}180\\-59\\\hline 121\end{array}$$

Diagram labels: J, 42°, M, 121, 59°, 17°, S, 48°, E, A
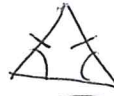
13. In the diagram below of △GJK, H is a point on $\overline{GJ}$, $\overline{HJ} \cong \overline{JK}$, m∠G = 28, and m∠GJK = 70.
Determine whether △GHK is an isosceles triangle and justify your answer.
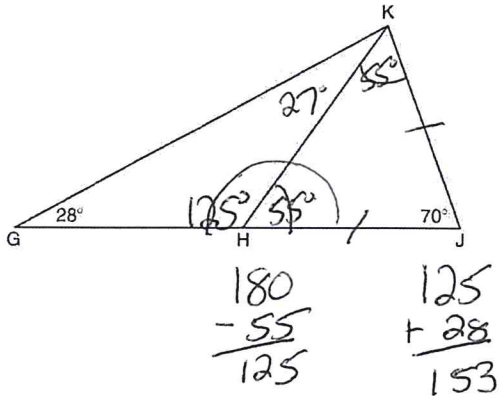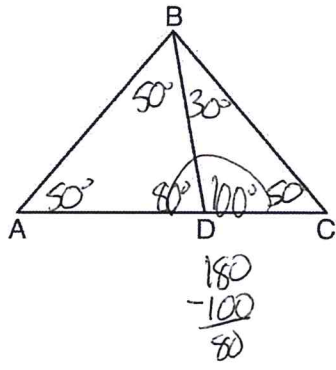
$$\begin{array}{r} 180 \\ -\ 70 \\ \hline 110 \end{array}$$

$$\frac{110}{2} = 55$$

No, it is not isosceles because all of the angles are different (scalene)

$$\begin{array}{r} 180 \\ -\ 55 \\ \hline 125 \end{array} \quad \begin{array}{r} 125 \\ +\ 28 \\ \hline 153 \end{array} \quad \begin{array}{r} 180 \\ -153 \\ \hline 27 \end{array}$$

14. In the diagram below, m∠BDC = 100°, m∠A = 50°, and m∠DBC = 30°.

Which statement is true?
1) △ABD is obtuse. ✗
2) △ABC is isosceles. ✓
3) m∠ABD = 80° ✗
4) △ABD is scalene. ✗

$$\begin{array}{r} 50 \\ +80 \\ \hline 130 \end{array} \quad \begin{array}{r} 180 \\ -130 \\ \hline 50 \end{array}$$

$$\begin{array}{r} 100 \\ +\ 30 \\ \hline 130 \end{array} \quad \begin{array}{r} 180 \\ -130 \\ \hline 50 \end{array}$$

$$\begin{array}{r} 180 \\ -100 \\ \hline 80 \end{array}$$

15. In the diagram of △BCD shown below, $\overline{BA}$ is drawn from vertex B to point A on $\overline{DC}$, such that $\overline{BC} \cong \overline{BA}$.
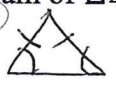
In △DAB, m∠D = x, m∠DAB = 5x − 30, and m∠DBA = 3x − 60. In △ABC, AB = 6y − 8 and BC = 4y − 2. [Only algebraic solutions can receive full credit.] Find m∠D. Find m∠BAC. Find the length of $\overline{BC}$. Find the length of $\overline{DC}$.
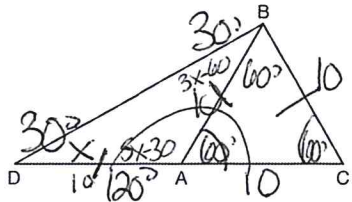
$$x + 3x - 60 + 5x - 30 = 180$$

$$9x - 90 = 180$$
$$+90 \qquad +90$$

$$\frac{9x}{9} = \frac{270}{9}$$

$$x = 30$$

$$\begin{array}{r} 60 \\ +60 \\ \hline 120 \end{array} \quad \begin{array}{r} 180 \\ -120 \\ \hline 60 \end{array}$$

4(3) − 2 = 10

$$\begin{array}{r} 180 \\ -120 \\ \hline 60 \end{array}$$

6y − 8 = 4y − 2
−4y   −4y
2y − 8 = −2
       +8 +8

$$\frac{2y}{2} = \frac{6}{2}$$
$$y = 3$$

∠D = 30°
∠BAC = 60°
BC = 10
DC = 20

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