

Coordinate Geometry Trapezoid Proofs

1. Given A(-4,6), B(2,9), C(8,6), D(-2,1). Prove ABCD is a trapezoid but *not* an isosceles trapezoid.

1) ABCD is a trapezoid because it has 1 pair of opposite sides \parallel and 1 pair of opposite sides \nparallel . It is not isosceles because it does not have congruent legs.

2) slope $\overline{AB} = \frac{3}{6} = \frac{1}{2}$

slope $\overline{DC} = \frac{5}{10} = \frac{1}{2}$

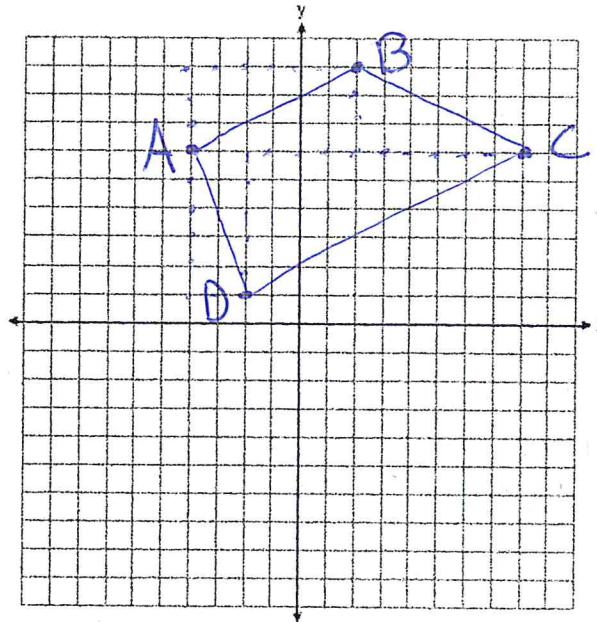
slope $\overline{AD} = \frac{-5}{2}$

slope $\overline{BC} = \frac{-3}{6} = -\frac{1}{2}$

$d_{AD} = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$

$d_{BC} = \sqrt{3^2 + 6^2} = \sqrt{9 + 36} = \sqrt{45}$

3) $\overline{AB} \parallel \overline{DC}$ because they have the same slope
 $\overline{AD} \nparallel \overline{BC}$ because they don't have the same slope
 $\overline{AD} \neq \overline{BC}$ because they don't have the same distance.



2. The coordinates of quadrilateral JKLM are J(1,-2), K(13,4), L(6,8), and M(-2,4). Prove that quadrilateral JKLM is a trapezoid but not an isosceles trapezoid.

1) JKLM is a trapezoid because it has 1 pair of opposite sides \parallel and 1 pair of opposite sides \nparallel . It is not isosceles because it does not have congruent legs.

2) slope $\overline{ML} = \frac{4}{8} = \frac{1}{2}$

slope $\overline{JK} = \frac{6}{12} = \frac{1}{2}$

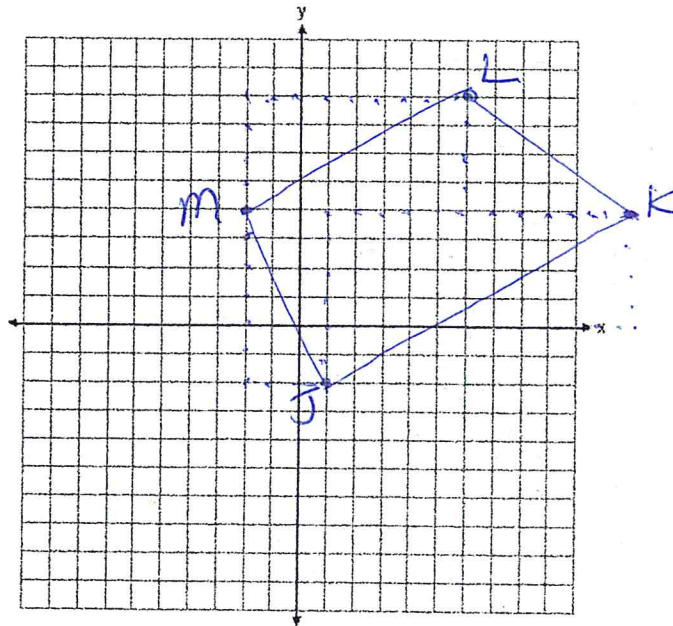
slope $\overline{MJ} = \frac{-6}{3} = -2$

slope $\overline{LK} = \frac{-4}{7}$

$d_{MJ} = \sqrt{3^2 + 6^2} = \sqrt{9 + 36} = \sqrt{45}$

$d_{LK} = \sqrt{4^2 + 7^2} = \sqrt{16 + 49} = \sqrt{65}$

3) $\overline{ML} \parallel \overline{JK}$ because they have the same slope
 $\overline{MJ} \nparallel \overline{LK}$ because they don't have the same slope
 $\overline{MJ} \neq \overline{LK}$ because they don't have the same distance.



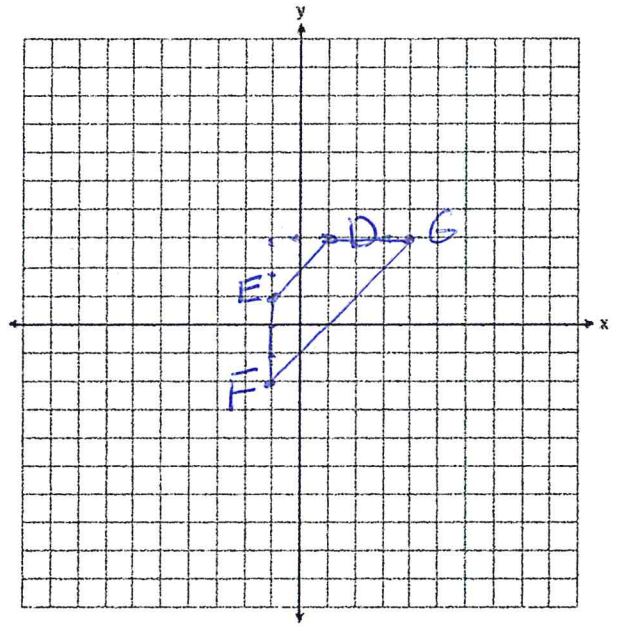
3. Quadrilateral DEFG has vertices D(1,3) E(-1,1) F(-1,-2) G(4,3). Prove that DEFG is an isosceles trapezoid.

1) DEFG is an isosceles trapezoid because it has 1 pair of opposite sides parallel, 1 pair of opposite sides not parallel, and congruent legs.

2) slope $\overline{ED} = \frac{2}{2} = 1$
 slope $\overline{FG} = \frac{4}{3} = 1$
 slope $\overline{EF} = \frac{3}{0} = \text{No slope}$
 slope $\overline{DG} = \frac{0}{3} = 0$

$d\overline{EF} = 3$ (don't have to do distance formula for straight lines)
 $d\overline{DG} = 3$

3) $\overline{ED} \parallel \overline{FG}$ because they have the same slope
 $\overline{EF} \perp \overline{DG}$ because they don't have the same slope, $\overline{EF} \cong \overline{DG}$ because they have the same distance.



4. Quadrilateral ABCD has vertices A(1,1), B(2,5), C(5,7) and D(7,5). Prove that quadrilateral ABCD is a trapezoid but NOT an isosceles trapezoid.

1) ABCD is a trapezoid because it has 1 pair of opposite sides \parallel and 1 pair of opposite sides \perp . It is not isosceles because it does not have congruent legs.

2) slope $\overline{BC} = \frac{2}{3}$
 slope $\overline{AD} = \frac{4}{6} = \frac{2}{3}$
 slope $\overline{AB} = \frac{4}{1} = 4$
 slope $\overline{CD} = \frac{-2}{2} = -1$

$d\overline{AB} = \sqrt{1^2 + 4^2} = \sqrt{1+16} = \sqrt{17}$

$d\overline{CD} = \sqrt{2^2 + 2^2} = \sqrt{4+4} = \sqrt{8}$

3) $\overline{BC} \parallel \overline{AD}$ because they have the same slope
 $\overline{AB} \perp \overline{CD}$ because they don't have the same slope.
 $\overline{AB} \not\cong \overline{CD}$ because they don't have the same distance.

