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Date _____
Algebra II

Equations and Systems Review Sheet

Solve the following equations

1. $3 = -x + \sqrt{x+5}$

Isolate square root, square both sides, solve, check!

$+x \quad +x$

$(x+3)^2 = (\sqrt{x+5})^2$

$(x+3)(x+3) = x+5$

$x^2+6x+9 = x+5$

$-x \quad -5 \quad -x \quad -5$

$x^2+5x+4=0$

	x	$+3$
x	x^2	$+3x$
$+3$	$+3x$	$+9$

x^2+6x+9

$(x+4)(x+1)=0$

$x=-4$ reject

$x=-1$ ✓

2. $x = 2 + \sqrt{x+4}$

$-2 \quad -2$

$(x-2)^2 = (\sqrt{x+4})^2$

$(x-2)(x-2) = x+4$

$x^2-4x+4 = x+4$

$-x \quad -4 \quad -x \quad -4$

$x^2-5x=0$

	x	-2
x	x^2	$-2x$
-2	$-2x$	$+4$

x^2-4x+4

$x(x-5)=0$

$x=0$ reject

$x=5$ ✓

3. $\left(\frac{x}{x-1}\right) = \left(\frac{2}{x}\right) + \left(\frac{1}{x-1}\right)$

multiply by LCD

$x(x-1)$

$x^2 = 2(x-1) + x$

$x^2 = 2x - 2 + x$

$x^2 = 3x - 2$

$-3x+2 \quad -3x+2$

$x^2 - 3x + 2 = 0$

F1: x
F2: $x-1$
LCD: $x(x-1)$

$(x-2)(x-1)=0$

$x=2$

$x=1$ reject (it makes denominator equal to 0)

4. $\frac{a}{a-2} - \frac{8}{a+3} = \frac{10}{a^2+a-6}$

$(a+3)(a-2)$

$(a+3)(a-2)$ $(a+3)(a-2)$ $(a+3)(a-2)$

$a(a+3) - 8(a-2) = 10$

$a^2+3a-8a+16=10$

$a^2-5a+16=10$

$-10 \quad -10$

$a^2-5a+6=0$

$(a-3)(a-2)=0$

F1: $a-2$
F2: $a+3$
LCD: $(a+3)(a-2)$

$a=3$

$a=2$ reject

5. Solve the following system of equations algebraically for all values of x, y, and z:

A $x + 2y - 3z = -2$
B $2x - 2y + z = 7$
C $x + y + 2z = -4$

A and B

$x+2y-3z=-2$
$+\ 2x-2y+z=7$

$3x-2z=5$

B and C

$1(2x-2y+z=7)$
$2(x+y+2z=-4)$

$2x-2y+z=7$
$+\ 2x+2y+4z=-8$

$4x+5z=-1$

D and E

$5(3x-2z=5)$
$2(4x+5z=-1)$

$15x-10z=25$
$+\ 8x+10z=-2$

$\frac{23x}{23} = \frac{23}{23}$

$x=1$

$4x+5z=-1$
$4(1)+5z=-1$
$4+5z=-1$
-4
$\frac{5z}{5} = \frac{-5}{5}$

$z=-1$

$2x-2y+z=7$
$2(1)-2y+-1=7$
$2-2y-1=7$
$-2y+1=7$
$-1 \quad -1$
$\frac{-2y}{-2} = \frac{6}{-2}$

$y=-3$

6. Which value is *not* contained in the solution of the system shown below?

$4x+2z=5y+130$
$3x+2y=7z-99$
$10x-6y-4z=112$

1) -8 3) 10
4) -12 4) 15

$4x-5y+2z=130$
$3x+2y-7z=-99$
$10x-6y-4z=112$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}B$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 & -5 & 2 \\ 3 & 2 & -7 \\ 10 & -6 & -4 \end{pmatrix}^{-1} \begin{pmatrix} 130 \\ -99 \\ 112 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ -12 \\ 15 \end{pmatrix}$

Spiral Review
Solving Polynomial Equations
1) Bring everything to one side
2) Factor
3) Set each factor equal to zero

Factoring by Grouping: (4 Terms or More)
1) Look for a pattern in the exponents to determine the groups. **You cannot have two terms with the same exponent in the same group.**
2) Factor out the GCF in each group
*You should be left with the same factor. If signs are reversed, factor out a negative
3) Combine coefficients and keep like term.
***Factor further if necessary**

7. $x^3+10x^2-9x-90=0$

$x^2(x+10)-9(x+10)=0$
$(x^2-9)(x+10)$
$(x+3)(x-3)(x+10)=0$
$x=-3 \quad x=3 \quad x=-10$

8. $27x^3+36x^2-12x=16$

$27x^3+36x^2-12x-16=0$
$9x^2(3x+4)-4(3x+4)=0$
$(9x^2-4)(3x+4)=0$
$(3x+2)(3x-2)(3x+4)=0$
$3x+2=0 \quad 3x-2=0 \quad 3x+4=0$
$3x=-2 \quad 3x=2 \quad 3x=-4$
$x=-\frac{2}{3} \quad x=\frac{2}{3} \quad x=-\frac{4}{3}$

Synthetic Division

1) List the coefficients of dividend (what you are dividing into)
2) Negate the divisor (what you are dividing by) and put it outside
3) Bring the first coefficient down
4) Multiply/add (repeat this step until you make it all the way through)
5) Take the new coefficients and decrease all the powers by 1. The last number is the remainder which goes over the divisor.

*Put 0 as a placeholder if necessary.

9. $\frac{2x^3-x-2}{x-4}$

$4 \mid 2 \;\; 0 \;\; -1 \;\; -2$

$\downarrow \;\; 8 \;\; 32 \;\; 124$

$2 \;\; 8 \;\; 31 \;\; 122$

$2x^2+8x+31+\frac{122}{x-4}$

10. $\frac{2x^3-3x^2+2x+5}{x-5}$

$5 \mid 2 \;\; -3 \;\; 2 \;\; 5$

$\downarrow \;\; 10 \;\; 35 \;\; 185$

$2 \;\; 7 \;\; 37 \;\; 190$

$2x^2+7x+37+\frac{190}{x-5}$

Quadratic Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

1) $ax^2+bx+c=0$
2) List a, b, and c values
3) Substitute values into quadratic formula
4) Type discriminant into the calculator (what is underneath the radical)
5) REDUCE THE RADICAL off to the side (If possible)
6) Reduce from all three terms (If possible)

*Separate into two fractions if there is an i involved.

11. $2x^2-6x=-5$

$+5 \quad +5$

$2x^2-6x+5=0$

$a=2 \quad b=-6 \quad c=5$

$x=\frac{6\pm\sqrt{(-6)^2-4(2)(5)}}{2(2)}$

$x=\frac{6\pm\sqrt{-4}}{4}$

$x=\frac{6\pm 2i}{4}$

$x=\frac{3}{2}\pm\frac{1}{2}i$

12. $3x^2=4x-2$

$-4x+2 \quad -4x+2$

$3x^2-4x+2=0$

$a=3 \quad b=-4 \quad c=2$

$x=\frac{4\pm\sqrt{(-4)^2-4(3)(2)}}{2(3)}$

$x=\frac{4\pm\sqrt{-8}}{6}$

$\sqrt{-8}$

$i\sqrt{8}$

$i\sqrt{4}\sqrt{2}$

$2i\sqrt{2}$

$x=\frac{4\pm 2i\sqrt{2}}{6}$

$x=\frac{2}{3}\pm\frac{1}{3}i\sqrt{2}$

To determine if $x-a$ is a factor:
Use remainder theorem and see if $p(a)=0$. If the remainder is 0, it is a factor. If the remainder is not 0, it is not a factor.

13. Which binomial is *not* a factor of the expression $x^3-4x^2-25x+28$?

1) $x+6$ $p(-6)=$
2) $x-7$ $p(7)=$
3) $x-1$ $p(1)=$
4) $x+4$ $p(-4)=$

14. Which binomial is a factor of the expression x^4+4x^2-32?

1) $x+8$ $p(-8)=$
2) $x-8$ $p(8)=$
3) $x-1$ $p(1)=$
4) $x+2$ $p(-2)=$

Definition of a Parabola: A parabola is the set of all points equidistant between a point (focus) and a line (directrix).
The vertex is directly in between the focus and the directrix. USE GRAPH PAPER AND COUNT!

$$\frac{(x-v)^2}{4p}=y-t$$

$(v,t)=vertex$

$p=dis\tan ce\ \ from\ \ vertex\ \ to\ \ focus\ /\ directrix$

15. Which equation represents the equation of the parabola with focus $(-3,3)$ and directrix $y=7$?

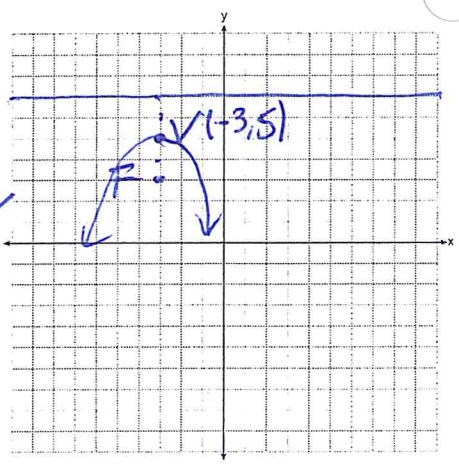
1) $y=\frac{1}{8}(x+3)^2-5$
2) $y=\frac{1}{8}(x-3)^2+5$
3) $y=-\frac{1}{8}(x+3)^2+5$
4) $y=-\frac{1}{8}(x-3)^2+5$

$V=-3$, $t=5$, $p=-2$

$\frac{(x-v)^2}{4p}=y-t$

$\frac{(x+3)^2}{4(-2)}=y-5$

$y=-\frac{1}{8}(x+3)^2+5$



16. Which equation represents a parabola with a focus of $(-2,5)$ and a directrix of $y=9$?

1) $(y-7)^2=8(x+2)$
2) $(y-7)^2=-8(x+2)$
3) $(x+2)^2=8(y-7)$
4) $(x+2)^2=-8(y-7)$

$y=9$

$V=-2$, $t=7$, $p=-2$

$\frac{(x-v)^2}{4p}=y-t$

$-8(y-7)=(x+2)^2$

$\frac{(x+2)^2}{4(-2)}=y-7$