

	30	45	60
SIN	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
CO	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
TAN	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

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Date _____
Pre Calculus

Evaluating Special Angles with Identities

Find the exact value of the following

$$\frac{\pi}{12} = 15^\circ$$

1. $\sin 75^\circ$

$$\begin{aligned} \sin(45+30) &= \sin A \cos B + \cos A \sin B \\ &= \sin 45 \cos 30 + \cos 45 \sin 30 \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

2. $\cos \frac{\pi}{12}$

$$\cos 15^\circ = \cos(45-30)$$

$$\begin{aligned} \cos(A-B) &= \cos A \cos B + \sin A \sin B \\ \cos(45-30) &= \cos 45 \cos 30 + \sin 45 \sin 30 \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

3. $\tan 15^\circ$

$$\begin{aligned} \tan(45-30) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\ \tan(45-30) &= \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} \\ &= \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}} \\ &= \frac{(3 - \sqrt{3})^2}{9 - 3} = \frac{9 - 6\sqrt{3} + 3}{6} = \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3} \end{aligned}$$

4. $\cos \frac{5\pi}{12}$

$$\cos(75^\circ) = \cos(45+30)$$

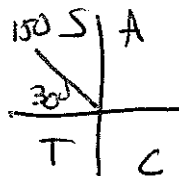
$$\begin{aligned} \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ \cos(45+30) &= \cos 45 \cos 30 - \sin 45 \sin 30 \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

5. $\cos 105^\circ$

$$\begin{aligned} \cos(60+45) &= \cos A \cos B - \sin A \sin B \\ \cos(60+45) &= \cos 60 \cos 45 - \sin 60 \sin 45 \\ &= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4} \end{aligned}$$

6. $\sin \frac{13\pi}{12}$

$$\sin 195^\circ = \sin(150+45)$$



$$\begin{aligned} \sin 150 &= \frac{1}{2} + \sin 30 = \frac{1}{2} \\ \cos 150 &= -\frac{1}{2} - \cos 30 = -\frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \sin(A+B) &= \sin A \cos B + \cos A \sin B \\ \sin(150+45) &= \sin 150 \cos 45 + \cos 150 \sin 45 \\ &= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4} \end{aligned}$$

7. $\tan 165^\circ = \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(135+30)$

$\frac{\tan 135 + \tan 30}{1 - \tan 135 \tan 30}$

$\tan(135+30) = \frac{\tan 135 + \tan 30}{1 - \tan 135 \tan 30}$

$= \frac{-1 + \frac{\sqrt{3}}{3}}{1 - (-1)(\frac{\sqrt{3}}{3})}$

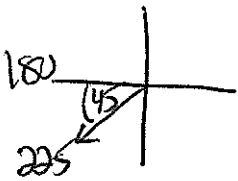
$= \frac{-1 + \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$

9. $\sin 285^\circ = \sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(225+60) = \sin 225 \cos 60 + \cos 225 \sin 60$

$= (-\frac{\sqrt{2}}{2})(\frac{1}{2}) + (-\frac{\sqrt{2}}{2})(\frac{\sqrt{3}}{2})$

$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$



$\sin 225 = \frac{y}{r} = \frac{-\sqrt{2}}{2}$

$\cos 225 = \frac{x}{r} = \frac{-\sqrt{2}}{2}$

8. $\cos \frac{11\pi}{12}$

$\frac{11(180)}{12} = 165^\circ$

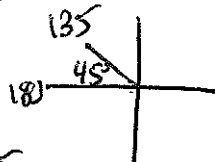
$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(135+30) = \cos 135 \cos 30 - \sin 135 \sin 30$

$= (-\frac{\sqrt{2}}{2})(\frac{\sqrt{3}}{2}) - (\frac{\sqrt{2}}{2})(\frac{1}{2})$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{-\sqrt{6} - \sqrt{2}}{4}$



$\cos 135 = \frac{x}{r} = \frac{-\sqrt{2}}{2}$
 $\sin 135 = \frac{y}{r} = \frac{\sqrt{2}}{2}$

10. $\cos \frac{17\pi}{12}$

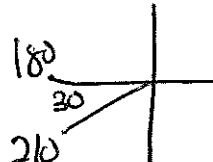
$\frac{17(180)}{12} = 255^\circ$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(210+45) = \cos 210 \cos 45 - \sin 210 \sin 45$

$= (-\frac{\sqrt{3}}{2})(\frac{\sqrt{2}}{2}) - (-\frac{1}{2})(\frac{\sqrt{2}}{2})$

$= \frac{-\sqrt{6} + \sqrt{2}}{4}$



$\sin 210 = \frac{y}{r} = \frac{-1}{2}$

$\cos 210 = \frac{x}{r} = \frac{-\sqrt{3}}{2}$

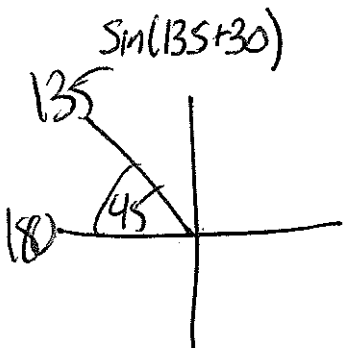
11. $\sin 525^\circ = \sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(135+30) = \sin 135 \cos 30 + \cos 135 \sin 30$

$= (\frac{\sqrt{2}}{2})(\frac{\sqrt{3}}{2}) + (-\frac{\sqrt{2}}{2})(\frac{1}{2})$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$



12. $\cos \frac{7\pi}{12} = \frac{-7(180)}{12}$

$\cos -105$

$\cos(255)$

*Same as #10

$\sin 135 = \frac{y}{r} = \frac{\sqrt{2}}{2}$

$\cos 135 = \frac{x}{r} = \frac{-\sqrt{2}}{2}$