Name Schlansky
Mr. Schlansky

Date ____________
Algebra II

Exponential Modeling Finding P and t

1. Melanie's car is currently worth $22,000. She bought it 3 years ago and the depreciate rate is compounded continuously at 7%. What was the initial value of the car?

$e \quad -.07$

$A = 22{,}000$
$P = P$
$r = -.07$
$t = 3$

$A = Pe^{rt}$

$$\frac{22000}{e^{-.07(3)}} = \frac{Pe^{-.07(3)}}{e^{-.07(3)}}$$

$27{,}140.92 = P$

2. Megan's savings account currently has $5,125 in it. If the account was opened 4 years ago and has an interest rate of 4.3% compounded weekly, how much money was initially put into the account?

$A = 5125$
$P = P$
$r = .043$
$n = 52$
$t = 4$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$5125 = P\left(1 + \frac{.043}{52}\right)^{52(4)}$

$$\frac{5125}{1.18\ldots} = \frac{P(1.18\ldots)}{1.18\ldots}$$

$\$4315.45 = P$

3. If the current balance of a bank account is $4,321 and the account was opened 3 years ago with an interest rate of 3% compounded continuously, what was the initial amount of the account?

$A = 4321$
$P = P$
$r = .03$
$t = 3$

$A = Pe^{rt}$

$$\frac{4321}{e^{.03(3)}} = \frac{Pe^{.03(3)}}{e^{.03(3)}}$$

$\$3949.10 = P$

4. Manny's savings account has a balance of \$6,391.52. He opened the account with \$5500.00 with a 5.2% interest rate that is compounded quarterly. How many years ago was the account opened?

$A = 6391.52$ $A = P(1+\frac{r}{n})^{nt}$

$P = 5500.00$

$r = .052$

$n = 4$

$t = t$

$\frac{6391.52}{5500} = \frac{5500(1+\frac{.052}{4})^{4t}}{5500}$ Isolate

$\log 1.16... = \log(1.013)^{4t}$

$\frac{\log 1.16...}{4\log 1.013} = \frac{4t\log 1.013}{4\log 1.013}$

$2.907... = t$

$3 = t$

5. If a bank account was opened with \$3000 and interest is compounded continuously at 5.2%, how much time has passed if there is now \$4000 in the account?

$A = 4000$ $A = Pe^{rt}$

$P = 3000$

$r = .052$

$t =$

$\frac{4000}{3000} = \frac{3000e^{.052t}}{3000}$

$\ln \frac{4}{3} = \ln e^{.052t}$

$\frac{\ln \frac{4}{3}}{.052\ln e} = \frac{.052t\ln e}{.052\ln e}$

$5.53 = t$

$6 \text{ years} = t$

6. Danielle currently has \$2125 in her savings account. If she opened the account with \$1700 and the account has an interest rate of 4.1% that is compounded continuously, how long has the account been open?

$A = 2125$ $A = Pe^{rt}$

$P = 1700$

$r = .041$

$t = t$

$\frac{2125}{1700} = \frac{1700e^{.041t}}{1700}$

$\ln 1.25 = \ln e^{.041t}$

$\frac{\ln 1.25}{.041\ln e} = \frac{.041t\ln e}{.041\ln e}$

$5.44. = t$

$6 = t$

$3\times$

7. Mike's bank account has tripled since he opened the account. If he opened the account with \$1000 and interest is compounded monthly at a rate of 8.1%, how much time, to the *nearest year*, has the account been open?

$A = 3(1000)$ 3000

$p = 1000$

$r = .081$

$n = 12$

$t = t$

$A = P(1+\frac{r}{n})^{nt}$

$\frac{3000}{1000} = \frac{1000}{1000}(1+\frac{.081}{12})^{12t}$

$\log 3 = \log 1.00675^{12t}$

$\frac{\log 3}{12\log 1.00675} = \frac{12t \log 1.00675}{12 \log 1.00675}$

$13.60 = t$

$14 = t$

$\times 1.25$

8. The amount of money in Jennifer's bank account has increased by 25% since she opened it. The initial investment was \$4800. If the interest is compounded continuously at a rate of 5.8%, how much time, to the *nearest tenth of a year*, has passed since the account has been opened?

$1.25(4800)$

$A = 6000$

$P = 4800$

$r = .058$

$t = t$

$A = Pe^{rt}$

$\frac{6000}{4800} = \frac{4800 e^{.058t}}{4800}$

$\ln 1.25 = \ln e^{.058t}$

$\frac{\ln 1.25}{.058 \ln e} = \frac{.058t \ln e}{.058 \ln e}$

$3.8 = t$

No compounding
Simple exponential

9. The value of a \$24000 car depreciates at a rate of 11% per year annually. After how many years will the car be worth 30% of its original value? Round your answer to the *nearest year*.

$.3(24000)$ $\times .3$

$A = 7200$

$P = 24000$

$r = .11$

$t = t$

$A = P(1-r)^t$

$\frac{7200}{24000} = \frac{24000}{24000}(1-.11)^t$

$\log .3 = \log(.89)^t$

$\frac{\log .3}{\log .89} = \frac{t \log .89}{\log .89}$

$t = 10$ years

10. Joe Manana just opened a bank account with a \$2000 initial balance. If the interest is compounded quarterly at a rate of 6.7%, how long would it take for his money to double?

$\times 2$

$A = 2(2000) = 4000$
$p = 2000$
$r = .067$
$n = 4$
$t = t$

$A = P(1+\frac{r}{n})^{nt}$

$\frac{4000}{2000} = \frac{2000(1+\frac{.067}{4})^{4t}}{2000}$

$\log 2 = \log(1.01675)^{4t}$

$\frac{\log 2}{4\log 1.01675} = \frac{4t \log 1.01675}{4\log 1.01675}$

$10 = t$

11. Jeff opened a bank account with a principal balance of \$2000. Interest is compounded monthly at a rate of 1.4%. After how many years, to the *nearest tenth of a year*, will it take for Jeff's account to increase by 50%? $\times 1.5$

$1.5(2000)$
$A = 3000$
$p = 2000$
$r = .014$
$n = 12$
$t = t$

$A = P(1+\frac{r}{n})^{nt}$

$\frac{3000}{2000} = \frac{2000(1+\frac{.014}{12})^{12t}}{2000}$

$\log 1.5 = \log(1.001...)^{12t}$

$\frac{\log 1.5}{12\log(1.001..)} = \frac{12t\log(1.001...)}{12\log(1.001..)}$

$28.97... = t$

$29.0 = t$

12. Seth's parents gave him \$5000 to invest for his 16th birthday. He is considering two investment options. Option *A* will pay him 4.5% interest compounded annually. Option *B* will pay him 4.6% compounded quarterly. Algebraically determine, to the *nearest tenth of a year*, how long it would take for option *B* to double Seth's initial investment.

$\times 2$

$2(5000)$
$A = 10000$
$p = 5000$
$r = .046$
$n = 4$
$t = t$

A

$A = P(1+r)^t$

$A = 5000(1+.045)^t$

$A = 5000(1.045)^t$

B

$A = P(1+\frac{r}{n})^{nt}$

$A = 5000(1+\frac{.046}{4})^{4t}$

$A = 5000(1.0115)^{4t}$

$\frac{10000}{5000} = \frac{5000(1.0115)^{4t}}{5000}$

$\log 2 = \log 1.0115^{4t}$

$\frac{\log 2}{4\log 1.0115} = \frac{4t\log 1.0115}{4\log 1.0115}$

$t = 15.2$

13. One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the *nearest day*, the amount of time needed before the amount of I–131 in the patient's body is approximately 7 milligrams.

$A=7$
$P=20$
$t=t$
$h=8.02$

$A=P\left(\frac{1}{2}\right)^{\frac{t}{h}}$

$\frac{7}{20}=\frac{20\left(\frac{1}{2}\right)^{\frac{t}{8.02}}}{20}$

$\log\frac{7}{20}=\log\left(\frac{1}{2}\right)^{\frac{t}{8.02}}$

$8.02\left(\log\left(\frac{7}{20}\right)=\frac{t}{8.02}\log\left(\frac{1}{2}\right)\right)8.02$

$\frac{8.02\log\left(\frac{7}{20}\right)}{\log\left(\frac{1}{2}\right)}=\frac{t\log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{2}\right)}$

$12=t$

14. The half-life of carbon-15 is 2.449 seconds. If Jackie has 17500 grams of carbon-15, write an equation that will represent the amount of grams of carbon-15 remaining after t seconds. After how much time will there be 500 grams of carbon-15 remaining? Round your answer to the *nearest tenth of a second.*

$A=500$
$P=17500$
$t=t$
$h=2.449$

$A=P\left(\frac{1}{2}\right)^{\frac{t}{h}}$

$\frac{500}{17500}=\frac{17500\left(\frac{1}{2}\right)^{\frac{t}{2.449}}}{17500}$

$\log\frac{1}{35}=\log\left(\frac{1}{2}\right)^{\frac{t}{2.449}}$

$2.449\left(\log\left(\frac{1}{35}\right)=\frac{t}{2.449}\log\left(\frac{1}{2}\right)\right)2.449$

$\frac{2.449\log\left(\frac{1}{35}\right)}{\log\left(\frac{1}{2}\right)}=\frac{t\log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{2}\right)}$

$12.6=t$

15. Jessica deposits \$2000 into a bank account where 4% interest is given every 2.4 years. To the *nearest tenth of a year*, how long will it take for Jessica's investment to reach \$5000?

$A=5000$
$P=2000$
$r=.04$
$t=t$
$h=2.4$

$A=P(1+r)^{\frac{t}{h}}$

$5000=2000(1+.04)^{\frac{t}{2.4}}$

$\frac{5000}{2000}=\frac{2000(1.04)^{\frac{t}{2.4}}}{2000}$

$\log 2.5=\log 1.04^{\frac{t}{2.4}}$

$2.4\left(\log 2.5=\frac{t}{2.4}\log 1.04\right)2.4$

$\frac{2.4\log 2.5}{\log 1.04}=\frac{t\log 1.04}{\log 1.04}$

$56.1=t$

16. The value of a stock doubles every 12 days. If the initial value of the stock was \$1500, how many full days will it take the stock to increase by 60%? $1+.6$ 1.6

$A=1.6(1500)$
$P=1500$
$t=t$
$h=12$

$A=P(2)^{\frac{t}{h}}$

$\frac{1.6(1500)}{1500}=\frac{1500(2)^{\frac{t}{12}}}{1500}$

$\log 1.6=\log 2^{\frac{t}{12}}$

$12\left(\log 1.6=\frac{t}{12}\log 2\right)12$

$\frac{12\log 1.6}{\log 2}=\frac{t\log 2}{\log 2}$

$8=t$

17. Christopher and Nolan are both preparing for the Nassau County Spelling Bee. There are a total of 5000 words that they are responsible for knowing how to spell. Currently, Christopher knows 1200 words and Nolan knows 1000 words. Every 4 days, Christopher will learn 20% of the remaining words. Every 6 days, Nolan will learn 25% of the remaining words. Create two functions to represent how many words Christopher and Nolan will be able to spell after *d* days. After how many days will they be able to spell the same number of words rounded to the nearest day.

irrelevant

$A=P(1+r)^{\frac{t}{h}}$

Christopher

$A=A$
$P=1200$
$r=.2$
$t=d$
$h=4$

$A=1200(1+.2)^{\frac{d}{4}}$
$A=1200(1.2)^{\frac{d}{4}}$

Nolan

$A=A$
$P=1000$
$r=.25$
$t=d$
$h=6$

$A=1000(1+.25)^{\frac{d}{6}}$
$A=1000(1.25)^{\frac{d}{6}}$

$\underbrace{1200(1.2)^{\frac{d}{4}}}_{Y_1}=\underbrace{1000(1.25)^{\frac{d}{6}}}_{Y_2}$ Intersect

No solution. Christopher will always know more

18. A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m. Write an equation in the form $A=A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ that models this situation, where *h* is the constant representing the number of hours in the half-life, A_0 is the initial mass, and *A* is the mass *t* hours after 3 p.m. Using this equation, solve for *h*, to the *nearest ten thousandth*. Determine when the mass of the radioactive substance will be 40 g. Round your answer to the *nearest tenth of an hour*.

$\frac{100}{140}=\frac{140}{140}\left(\frac{1}{2}\right)^{\frac{5}{h}}$

$\log\frac{5}{7}=\log\left(\frac{1}{2}\right)^{\frac{5}{h}}$

$h\left(\log\left(\frac{5}{7}\right)\right)=\left(\frac{5}{h}\log\left(\frac{1}{2}\right)\right)h$

$\frac{h\log\left(\frac{5}{7}\right)}{\log\left(\frac{5}{7}\right)}=\frac{5\log\left(\frac{1}{2}\right)}{\log\left(\frac{5}{7}\right)}$

$h=10.3002$

$A=40$
$A_0=140$
$t=t$
$h=10.3002$

$\frac{40}{140}=\frac{140}{140}\left(\frac{1}{2}\right)^{\frac{t}{10.3002}}$

$\log\frac{2}{7}=\log\left(\frac{1}{2}\right)^{\frac{t}{10.3002}}$

$10.3002\left(\log\left(\frac{2}{7}\right)\right)=\left(\frac{t}{10.3002}\log\left(\frac{1}{2}\right)\right)10.3002$

$\frac{10.3002\log\left(\frac{2}{7}\right)}{\log\left(\frac{1}{2}\right)}=\frac{t\log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{2}\right)}$

$18.6=t$