1) Isolate

2) reciprocal power (constant)

or

log of both sides (variable)

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Algebra II

Exponential Equations Word Problems

1. A population of wolves in a county is represented by the equation $P(t) = 80(0.98)^t$, where t is the number of years since 1998. Predict the number of wolves in the population in the year 2008. $t=10$

$P(10)=80(0.98)^{10}$

$P(10)=65$

2. After an oven is turned on, its temperature, T, is represented by the equation $T = 400 - 350(3.2)^{-0.1m}$, where m represents the number of minutes after the oven is turned on and T represents the temperature of the oven, in degrees Fahrenheit.
How many minutes does it take for the oven's temperature to reach 300°F? Round your answer to the *nearest minute*.

$300 = 400 - 350(3.2)^{-0.1m}$

$-400 \quad -400$

$\frac{-100}{-350} = \frac{-350(3.2)^{-0.1m}}{-350}$

$.2857... = 3.2^{-0.1m}$

$\log .2857... = \log 3.2^{-0.1m}$

$\frac{\log .2857...}{-0.1 \log 3.2} = \frac{-0.1m \log 3.2}{-0.1 \log 3.2}$

$10.77... = m$

$11 = m$

3. Meteorologists can determine how long a storm lasts by using the function $t(d) = 0.07d^{\frac{3}{2}}$, where d is the diameter of the storm, in miles, and t is the time, in hours. If the storm lasts 4.75 hours, find its diameter, to the nearest tenth of a mile. $t(d)$

$\frac{4.75}{.07} = \frac{.07d^{\frac{3}{2}}}{.07}$

$(67.857)^{\frac{2}{3}} = \left(d^{\frac{3}{2}}\right)^{\frac{2}{3}}$

$16.6 = d$

4. A population of rabbits doubles every 60 days according to the formula $P = 10(2)^{\frac{t}{60}}$, where P is the population of rabbits on day t. What is the value of t when the population is 320?

P

$\frac{320}{10} = \frac{10(2)^{\frac{t}{60}}}{10}$

$32 = 2^{\frac{t}{60}}$

$\log 32 = \log 2^{\frac{t}{60}}$

$60(\log 32) = \left(\frac{t}{60}\log 2\right)60$

$\frac{60\log 32}{\log 2} = \frac{t\log 2}{\log 2}$

$t = 300$

5. Growth of a certain strand of bacteria is modeled by the equation $G = A(2.7)^{0.584t}$, where G is the final number of bacteria, A is the initial amount of bacteria, and t is the time in hours. In approximately how many hours will 4 bacteria increase to 2500 bacteria? Round your answer to the nearest hour.

A G

$\frac{2500}{4} = \frac{4(2.7)^{0.584t}}{4}$

$625 = 2.7^{0.584t}$

$\log 625 = \log 2.7^{0.584t}$

$\frac{\log 625}{0.584\log 2.7} = \frac{0.584t\log 2.7}{0.584\log 2.7}$

$11.098 = t$

$11 = t$

6. The number of houses in Central Village, New York, grows every year according to the function $H(t) = 540(1.039)^t$, where H represents the number of houses, and t represents the number of years since January 1995. A civil engineering firm has suggested that a new, larger well must be built by the village to supply its water when the number of houses exceeds 1,000. During which year will this first happen?

$H(t)$

$\frac{1000}{540} = \frac{540(1.039)^t}{540}$

$1.85\ldots = 1.039^t$

$\log 1.85\ldots = \log 1.039^t$

$\frac{\log 1.85\ldots}{\log 1.039} = \frac{t\log 1.039}{\log 1.039}$

$16.105 = t$

$16 = t$

$1995 + 16 = 2011$

7. Drew's parents invested \$1,500 in an account such that the value of the investment doubles every seven years. The value of the investment, V, is determined by the equation $V = 1500(2)^{\frac{t}{7}}$, where t represents the number of years since the money was deposited. How many years, to the *nearest tenth of a year*, will it take the value of the investment to reach \$1,000,000?

V

$\frac{1{,}000{,}000}{1500} = \frac{1500(2)^{\frac{t}{7}}}{1500}$

$666.\overline{6} = 2^{\frac{t}{7}}$

$\log 666.\overline{6} = \log 2^{\frac{t}{7}}$

$7(\log 666.\overline{6}) = \left(\frac{t}{7}\log 2\right)7$

$\frac{7\log 666.\overline{6}}{\log 2} = \frac{t\log 2}{\log 2}$

$65.7 = t$

8. Juliette deposits $3000 into a bank account where the balance of the account $b(t)$ after t years can be represented by $b(t) = 3000(1.063)^t$. To the nearest tenth of a year:

a) how long will it take for Juliette's money to double?
b) how long will it take for Juliette's money to triple?
c) How long will it take for Juliette's money to increase by 50%?

a) $\frac{2(3000)}{3000} = \frac{3000(1.063)^t}{3000}$

$2 = 1.063^t$

$\log 2 = \log 1.063^t$

$\log 2 = t \log 1.063$

$\frac{\log 2}{\log 1.063} = \frac{t \log 1.063}{\log 1.063}$

$11.3 = t$

b) $\frac{3(3000)}{3000} = \frac{3000(1.063)^t}{3000}$

$3 = 1.063^t$

$\log 3 = \log 1.063^t$

$\log 3 = t \log 1.063$

$\frac{\log 3}{\log 1.063} = \frac{t \log 1.063}{\log 1.063}$

$18.0 = t$

c) $\frac{1.5(3000)}{3000} = \frac{3000(1.063)^t}{3000}$

$1.5 = 1.063^t$

$\frac{\log 1.5}{\log 1.063} = \frac{t \log 1.063}{\log 1.063}$

$6.6 = t$

9. 200 grams of a radioactive substance decays according to the formula $a(t) = 200(.094)^{2t}$ where $a(t)$ is the amount of the radioactive substance remaining after t years. To the nearest hundredth of a year:

a) How long will it take until there are 150 grams remaining?
b) How long will it take for the amount of the substance to decrease by 20%? $1 - .2 = .8$
c) How long will it take until there is 40% of the substance remaining?

a) $\frac{150}{200} = \frac{200(.094)^{2t}}{200}$

$.75 = .094^{2t}$

$\log .75 = \log .094^{2t}$

$\frac{\log .75}{2 \log .094} = \frac{2t \log .094}{2 \log .094}$

$.06 = t$

b) $\frac{.8(200)}{200} = \frac{200(.094)^{2t}}{200}$

$.8 = .094^{2t}$

$\log .8 = \log .094^{2t}$

$\frac{\log .8}{2 \log .094} = \frac{2t \log .094}{2 \log .094}$

$.05 = t$

c) $\frac{.4(200)}{200} = \frac{200(.094)^{2t}}{200}$

$.4 = .094^{2t}$

$\log .4 = \log .094^{2t}$

$\log .4 = 2t \log .094$

$\frac{\log .4}{2 \log .094} = \frac{2t \log .094}{2 \log .094}$

$.19 = t$

10. The number of subscribers to a website can be modeled by the equation $s(t) = s_0 t^{\frac{4}{3}}$ where $s(t)$ represents the number of subscribers, s_0 represents the initial number of subscribers, and t represents months. If there were initially 512 subscribers, to the nearest tenth, how long will it take the number of subscribers to increase by 60%?

$1.6(512)$

$$\frac{1.6(512)}{512} = \frac{512 t^{\frac{4}{3}}}{512}$$

$$(1.6)^{3/4} = \left(t^{\frac{4}{3}}\right)^{3/4}$$

$1.4 = t$

A_0 A $t=5$

11. A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m. Write an equation in the form $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ that models this situation, where h is the constant representing the number of hours in the half-life, A_0 is the initial mass, and A is the mass t hours after 3 p.m. Using this equation, solve for h, to the *nearest ten thousandth*. Determine when the mass of the radioactive substance will be 40 g. Round your answer to the *nearest tenth of an hour*.

A

$$\frac{100}{140} = \frac{140\left(\frac{1}{2}\right)^{\frac{5}{h}}}{140}$$

$$.714... = \left(\frac{1}{2}\right)^{\frac{5}{h}}$$

$$\log .714... = \log\left(\frac{1}{2}\right)^{\frac{5}{h}}$$

$$h\left(\log .714...\right) = \frac{5}{h}\log\left(\frac{1}{2}\right)h$$

$$\frac{h \log .714...}{\log .714...} = \frac{5\log\left(\frac{1}{2}\right)}{\log .714...}$$

$h = 10.3002$

$$\frac{40}{140} = \frac{140\left(\frac{1}{2}\right)^{\frac{t}{10.3002}}}{140}$$

$$.2857... = \frac{1}{2}^{\frac{t}{10.3002}}$$

$$10.3002 \log .2857... = \frac{t}{10.3002}\log\left(\frac{1}{2}\right) 10.3002$$

$$\frac{10.3002 \log .2857...}{\log\left(\frac{1}{2}\right)} = \frac{t \log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{2}\right)}$$

$t = 18.6$