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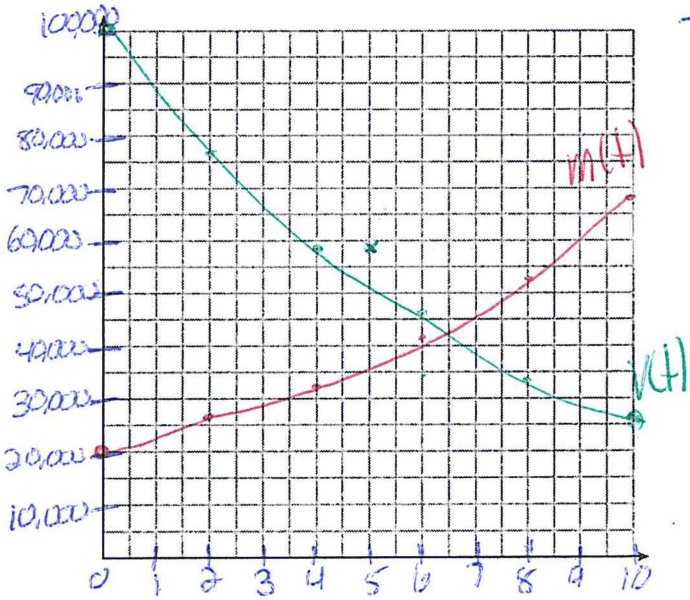
Date ___________
Algebra II

Exponential Graphs (Part IV)

1. The value of Tom's bank account is currently 100000 and is decreasing according to the equation $V(t)=100000(.876)^t$. The amount of money he has paid for his mortgage can be represented by the equation $M(t)=20000(1.1304)^t$. Graph and label $V(t)$ and $M(t)$ over the interval [0,10]. No arrows



V(t)

x	y
0	100,000
2	76738
4	58887
6	45188
8	34676
10	26610

m(t)

x	y
0	20,000
2	25556
4	32656
6	41728
8	53320
10	68132

V(t)=m(t) Intersect

After how many years will the value of Tom's bank account be equal to the amount of money he paid for his mortgage? Round your answer to the *nearest tenth of a year*. Tom will open a new bank account when the value of his account is \$20,000. After how many years, to the *nearest hundredth of a year*, will that happen?

V(t)

2nd Graph : Intersect
$t=6.3$ years.

$$\frac{20{,}000}{100{,}000}=\frac{100{,}000(.876)^t}{100{,}000}$$

$$\log .2=\log .876^t$$

$$\frac{\log .2}{\log .876}=\frac{t\log .876}{\log .876}$$

$$12.16=t$$

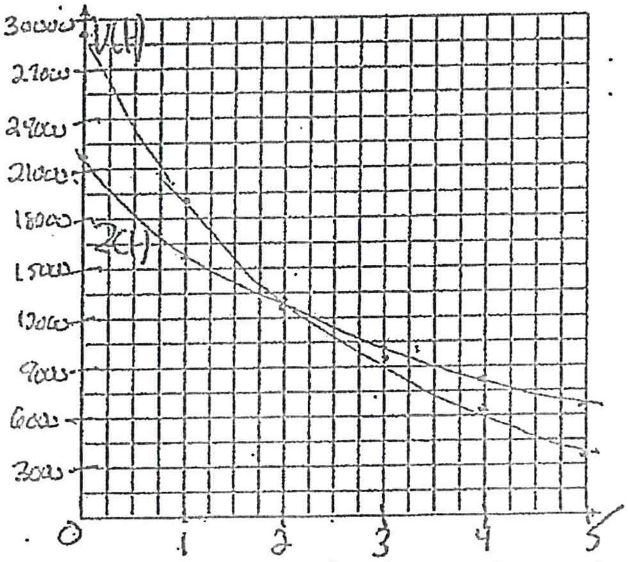
2 ~~1~~. The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time. Graph $V(t)$ and $Z(t)$ over the interval $0 \le t \le 5$, on the set of axes below.

V(t)

x	y
0	28483
1	19482
2	13326
3	9114.8
4	6234.6
5	4264.4

Z(t).

x	y
0	22151
1	17234
2	13408
3	10431
4	8115.6
5	6313.9



Scale

$x \geq \frac{5}{20}$ $\quad y \geq \frac{28483}{20}$

$x \geq .25$ $\quad y \geq 1424.15$

$y = 1500$

State when $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem. Zach takes out an insurance policy that requires him to pay a $3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the *nearest year*, how long will it take Zach to cancel this policy? Justify your answer. $V(t) = 3000$

$t = 1.95$

After 1.95 years, the value of the loans will be the same ($13569.24)

$V(t) = 28482.698(.684)^t$

$\frac{3000}{28482.698} = \frac{28482.698(.684)^t}{28482.698}$

$\log .105... = \log .684^t$

$\frac{\log .105}{\log .684} = \frac{t \log .684}{\log .684}$

$t = 6$

~~$Z(t) = 22151.327(0.778)^t$~~

$3000 = $ ~~22151.327~~$(0.778)^t$

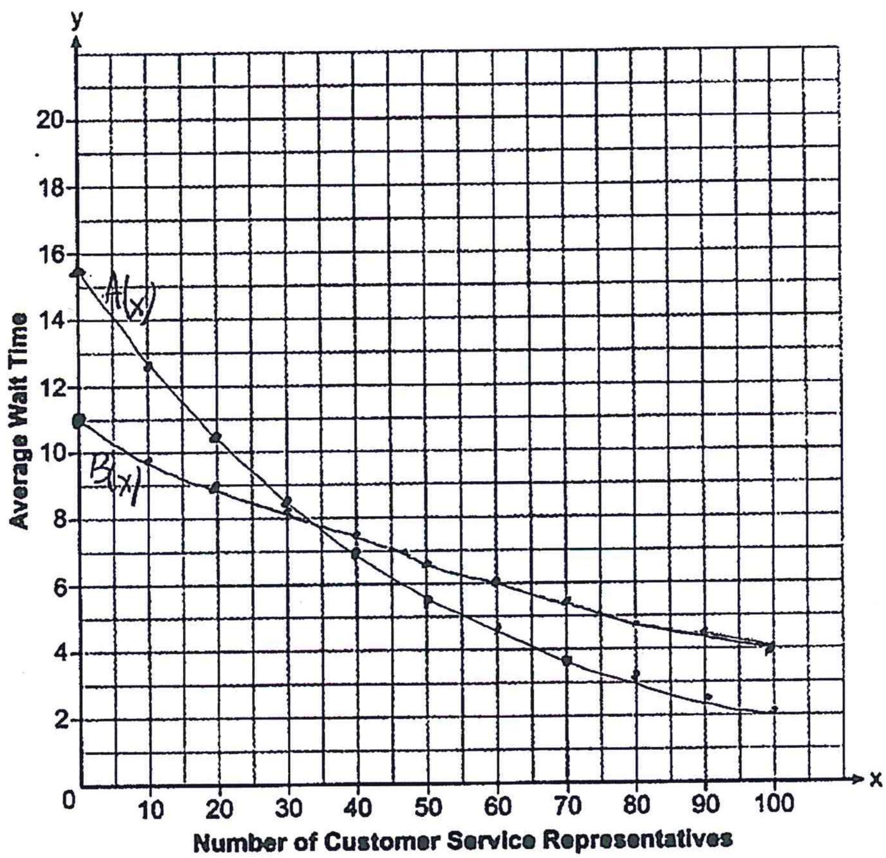
~~22151.32~~

13
36

3. A technology company is comparing two plans for speeding up its technical support time. Plan A can be modeled by the function $A(x) = 15.7(0.98)^x$ and plan B can be modeled by the function $B(x) = 11(0.99)^x$ where x is the number of customer service representatives employed by the company and $A(x)$ and $B(x)$ represent the average wait time, in minutes, of each customer. Graph $A(x)$ and $B(x)$ in the interval $0 \le x \le 100$ on the set of axes below.

A(x)

x	y
0	15.7
10	12.8
20	10.5
30	8.6
40	7.0
50	5.7
60	4.7
70	3.8
80	3.1
90	2.5
100	2.1



B(x)

x	y
0	11
10	9.9
20	9.0
30	8.1
40	7.4
50	6.7
60	6.0
70	5.4
80	4.9
90	4.5
100	4.0

To the *nearest integer*, solve the equation $A(x) = B(x)$. ~~Determine, to the *nearest minute*, $B(100) - A(100)$. Explain what this value represents in the given context.~~ How many customer service representatives would company B need in order for the average wait time to be 3 minutes? B(x)

41 42 intersect

$x = 35$

$B(100) - A(100)$

$4.0 - 2.1$

1.9

If they hire 100 customer service representatives, the average wait time time would be 1.9 minutes longer with Plan B.

$B(x) = 11(0.99)^x$

$\frac{3}{11} = \frac{11(0.99)^x}{11}$

$\log \frac{3}{11} = \log 0.99^x$

$\frac{\log \frac{3}{11}}{\log 0.99} = \frac{x \log 0.99}{\log 0.99}$

$129 = x$

37
15

4. Tony is evaluating his retirement savings. He currently has $318,000 in his account, which earns an interest rate of 7% compounded annually. He wants to determine how much he will have in the account in the future, even if he makes no additional contributions to the account. Write a function, $A(t)$, to represent the amount of money that will be in his account in t years. Graph $A(t)$ where $0 \le t \le 20$ on the set of axes below.

P

r

$A = P(1+r)^t$

$A = A(t)$

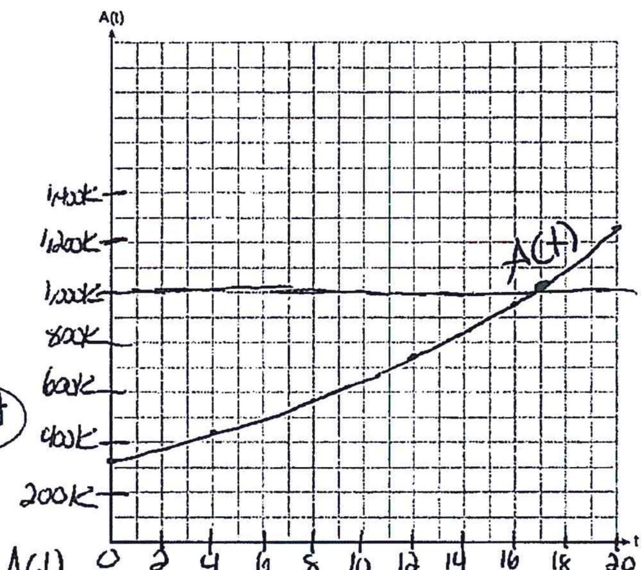
$P = 318{,}000$

$r = .07$

$t = t$

$A = 318{,}000(1+.07)^t$

$A(t) = 318{,}000(1.07)^t$



A(t)

x	y
0	318,000
4	416833
8	546383
12	716197
16	938788
20	1230000

Scale ≥ max / # of boxes

≥ 1230000 / 20

≥ 61500

Scale = 100,000

Tony's goal is to save $1,000,000. Determine algebraically, to the *nearest year*, how many years it will take for him to achieve his goal. Explain how your graph of $A(t)$ confirms your answer.

$$\frac{1{,}000{,}000}{318{,}000} = \frac{318{,}000(1.07)^t}{318{,}000}$$

$$\log \frac{500}{159} = \log 1.07^t$$

$$\frac{\log \frac{500}{159}}{\log 1.07} = \frac{t \log 1.07}{\log 1.07}$$

$$17 = t$$

17 is the first year the graph crosses 1,000,000.

5.

32. Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 N_0 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 N_0 milligrams of another drug with a decay rate of 0.231. Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.

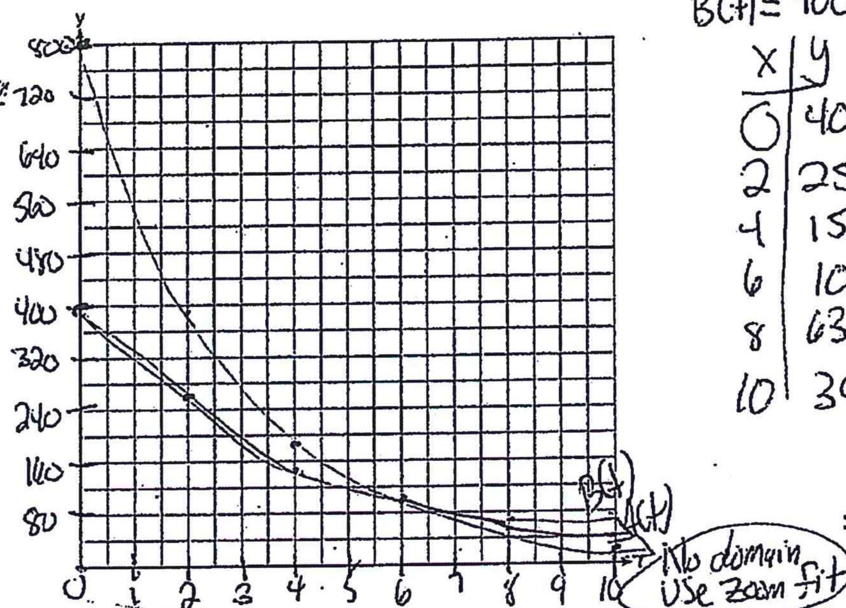
$A(t) = 800e^{-.347t}$

x	y
0	800
2	399.66
4	199.66
6	99.744
8	49.83
10	24.894

$B(t) = 400e^{-.231t}$

x	y
0	400
2	252.01
4	158.77
6	100.03
8	63.021
10	39.705

Scale: 120



x and y min = 0

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A? The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another ~~800 milligram~~ dose of the drug.

Same
Trace
intersect

(5.98, 100.10)
hours

Scale

$x \geq \frac{10}{20}$ $\quad y \geq \frac{800}{20}$

$x \geq .5$ $\quad y \geq 40$

$x = .5$ $\quad y = 40$

120 mg

$A(t) = .15(800)$ 120

$120 = .15(800) = 800e^{-.347t}$

y_1 $\quad y_2$ $\quad$ Find intersection

or

$\frac{120}{\frac{.15(800)}{800}} = \frac{800e^{-.347t}}{800}$

$\ln .15 = \ln e^{-.347t}$

$\frac{\ln .15}{-.347 \ln e} = \frac{-.347t \ln e}{-.347 \ln e}$

$5.5 = t$

39
155