Name:

Common Core Algebra II

Unit 5

Exponents and Logarithms

Mr. Schlansky

2025

Lesson 1: I can evaluate exponents using the exponents rules.

Multiplying: Add exponents $x^2 \bullet x^3 = x^{2+3} = x^5$

Dividing: Subtract exponents: $\dfrac{x^8}{x^5} = x^{8-5} = x^3$

When raising a power to a power, multiply exponents: $(x^2)^3 = x^{2\bullet 3} = x^6$

Anything to the zero power is equal to 1 $x^0 = 1$

Negative exponents are fractions! $x^{-2} = \dfrac{1}{x^2}$

If exponent is outside parenthesis, everything gets it $\left(\dfrac{xy}{z}\right)^3 = \dfrac{x^3 y^3}{z^3}$

Radicals are fractional exponents (Fractional exponent = $\dfrac{power}{root}$)

Get rid of parenthesis

Negative exponents are fractions (Move whatever is being raised to the negative power)

Clean it up (Multiply/Divide/Put back in radical)

Lesson 2: I can evaluate negative exponents by making a fraction and moving what's being raised to the negative power.

Same notes as Lesson 1.

Lesson 3: I can evaluate fractional exponents by understanding radicals are fractional exponents and putting $\dfrac{power}{root}$.

Same notes as Lesson 1.

Lesson 4: I can evaluate radicals by understanding radicals are fractional exponents and putting $\dfrac{power}{root}$.

If given a radical, re-write as a fractional exponent. Put everything inside the radical into parentheses and the fractional exponent on the outside of the parentheses.

Follow notes form lesson 1 from there.

Lesson 5: I can prepare for my exponents quiz by practicing and following my procedure.

Radicals are fractional exponents

Get rid of parenthesis

Negative exponents are fractions

Clean it up MDE

(Use notes from Lesson 1)

Lesson 6: I can evaluate logarithms by raising the base to a power to get the solution.
Evaluating Logs: The base to what power equals the answer. $\log_8 64 \rightarrow 8^x = 64$

Lesson 7: I can expand logarithms using the product, quotient, and power rules.
Product Rule: $\log a \bullet b = \log a + \log b$

Quotient Rule: $\log \frac{a}{b} = \log a - \log b$

Power Rule: $\log a^p = p \log a$
Apply product and quotient rules first
Apply power rule last
*If given radical, re-write as a fractional exponent first

Lesson 8: I can solve variable exponential equations by taking the log of both sides.
Variable Exponential Equations
Isolate the base
Take the log of both sides
Bring the exponent in front of the log
Solve the equation
*If the base is *e*, use *ln* instead of log (you don't have to but this is what is generally done in upper level courses)

Lesson 9: I can solve variable exponential equations multiple choice problems by storing each potential answer, typing in the left hand side, typing in the right hand side, and seeing if they match.
Same strategy as any multiple choice equation!
Store the expression given in each answer
Type the left hand side into the calculator
Type the right hand side into the calculator
If the left hand side matches the right hand side, that is the answer!

Lesson 10: I can solve word problems by solving exponential equations and taking the log of both sides.
Same notes as Lesson 8.
Read carefully to identifying what variable you are substituting in for.
If you're doubling, multiply initial value by 2, if tripling multiplying initial value by 3.

Lesson 11: I can solve Newton's Law of Heating/Cooling Problems by carefully substituting in for each variable, taking the problem one sentence at a time, and taking log of both sides if finding *k* or *t*.

The formula will be given to you. Write out what each variable represents and carefully substitute in. There may be multiple questions within the problem so make sure you read only one sentence at a time.

If solving for T, type the entire right hand side in.

If solving for *k* or *t*, solve the exponential equation by ISOLATING and taking the log/ln of both sides.

*Once you find *k*, you will need to use that *k* value to answer the next question. THE VALUES YOU USED IN THE FIRST QUESTION DO NOT APPLY TO THE SECOND QUESTION.

Lesson 12: I can graph exponential and logarithmic graphs by typing the equation into y =, plotting points from the table, and finding the asymptote using the horizontal shift or table tricks.

To graph an exponential function:

1) Type equation into y =, plot nice points from the table
2) Graph horizontal asymptote (The asymptote is the vertical shift OR the repeated value in the table. DO NOT plot the repeated value points, that value is the asymptote)
3) Draw a curve that gets closer and closer to the asymptote but never touches it

Domain: $(-\infty, \infty)$

Range: (HA, ∞) or $(-\infty, HA)$

Asymptote: y = vertical shift/repeated value in table

End Behavior:

One direction will be the asymptote value

The other will be $\pm\infty$

To graph a logarithmic function:

1) Type equation into y =, plot nice points from the table
2) Graph vertical asymptote (The asymptote is the horizontal shift OR the last error in the table before nice values start)
3) Draw a curve that gets closer and closer to the asymptote but never touches it

Domain: (VA, ∞) or $(-\infty, VA)$

Range: $(-\infty, \infty)$

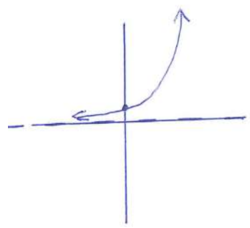
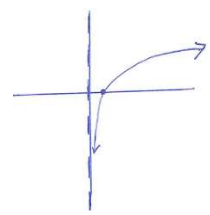
Asymptote: x = horizontal shift/last error in table

End Behavior:

As x approaches the asymptote, y will approach $\pm\infty$

As x approaches $\pm\infty$, y will approach $\pm\infty$

Lesson 13: I can answer exponential and logarithmic graph word problems by understanding their shape and that they are inverses of each other.

Exponential	**Logarithmic**
$y = 2^x$ 	$y = \log_2 x$ 
Horizontal Asymptote at $y = 0$	Vertical Asymptote at $x = 0$
Passes through $(0,1)$	Passes through $(1,0)$
Domain is all real numbers	Domain is all positive real numbers
Range is all positive real numbers	Range is all real numbers
Exponents and logarithms are inverses of each other!!!!!!!!!!!	

Lesson 14: I can graph exponential equations (Part IV) by typing the equation into y = and plotting points, finding the intersection using second trace, and finding t by solving exponential equations using logs.

1) Type equation(s) into y = and plot points. They will tell you what to graph between either in the problem or on the given graph.

*Create an appropriate scale that will fit all of the points. Each box must be worth the same value on each axis.

2) Plot the points carefully.

3) For $f(x) = g(x)$, 2nd Trace: Intersect and state the **x value**.

4) If given a value, substitute it into the appropriate values and solve the equation by taking log of both sides or Y1 Y2 Intersect strategy.

Lesson 15: I can prepare for my exponents/logarithms test by practicing!

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Exponents Rules

Express each of the following in simplest form

1. $(4x^3)(2x^5)$

2. $(3x^4)(7x^8)$

3. $(-2x^2y^3)(3x^5y)$

4. $\frac{12x^8}{4x^3}$

5. $\frac{24x^9}{3x^2}$

6. $\frac{48x^8y^9}{4xy^8}$

7. $(x^2)^3$

8. $(x^4)^6$

9. $(y^3)^6$

10. x^0

11. $(4x)^0$

12. $(12x^3y^2)^0$

13. 7^2

14. 2^4

15. 5^3

16. $(3x^4y)^2$

17. $\left(\dfrac{2x^2}{y^3}\right)^3$

18. $\left(\dfrac{x^2y}{2mn^3}\right)^4$

19. $(x^4y^3)^{\frac{1}{2}}$

20. $\left(\dfrac{x^2}{y^6z^9}\right)^{\frac{1}{3}}$

21. $\left(\dfrac{x^4y^8}{z^3}\right)^{\frac{3}{2}}$

22. $\dfrac{x^4y^5}{x^2y^8}$

23. $\dfrac{2x^7y^4}{4xy^6}$

24. $\dfrac{15x^8y^7}{10x^{10}y^2}$

25. $\dfrac{5x^6 \bullet 4x^3}{2x^5}$

26. $(2x^2y^3)^2(3xy)$

27. $\dfrac{(2x^3)^4}{4x^{15}}$

28. $\left(\dfrac{4x^0y^3}{z}\right)^3\left(\dfrac{2z}{y}\right)^2$

29. $\left(\dfrac{3x^4}{2z^2}\right)^3\left(\dfrac{x^2}{z}\right)^4$

30. $\left(\dfrac{x^2y}{z^8}\right)^{\frac{1}{2}}\left(xz^{10}\right)$

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Negative Exponents

Reduce each of the following and express with positive exponents

1. $\dfrac{14x^{-2}y^{3}}{-8x^{-5}y^{5}}$

2. $\dfrac{x^{2}y^{-3}}{x^{-3}y^{-2}}$

3. $(3y)^{2}(3zy^{4})^{-2}$

4. $\dfrac{(x^{2}y)^{-2}}{x^{2}y^{-3}}$

5. Which expression is equivalent to $x^{-1}\cdot y^{2}$?

1) xy^{2}

3) $\dfrac{x}{y^{2}}$

2) $\dfrac{y^{2}}{x}$

4) xy^{-2}

6. Which expression is equivalent to $\dfrac{x^{-1}y^{4}}{3x^{-5}y^{-1}}$?

1) $\dfrac{x^{4}y^{5}}{3}$

3) $3x^{4}y^{5}$

2) $\dfrac{x^{5}y^{4}}{3}$

4) $\dfrac{y^{4}}{3x^{5}}$

7. The expression $\dfrac{a^{2}b^{-3}}{a^{-4}b^{2}}$ is equivalent to

1) $\dfrac{a^{6}}{b^{5}}$

3) $\dfrac{a^{2}}{b}$

2) $\dfrac{b^{5}}{a^{6}}$

4) $a^{-2}b^{-1}$

Simplify the following expressions

8. $\dfrac{2x^{-2}y^{-2}}{4y^{-5}}$

9. $(5^{-2}a^{3}b^{-4})^{-1}$

10. $\dfrac{(3x^{-2}y^{2})^{2}}{9x^{-3}y^{-3}}$

11. $\dfrac{3x^{-4}y^{5}}{(2x^{3}y^{-7})^{-2}}$

12. $\dfrac{(4x^{-2})^{-2}}{(2x^{2})(2y)^{-3}}$

13. $\dfrac{(2x^{-3})^{-3}}{16(x^{2}y^{-1})^{-2}}$

14. $\dfrac{(2xy^{2})^{-2}}{(8x^{-2}y)^{-1}(2y^{2})^{-2}}$

15. $\dfrac{(3x^{2}y^{-2})^{2}}{(2x^{2}y^{-1})^{2}(3x^{-5})}$

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Fractional Exponents

Rewrite the following as radicals

1. $x^{\frac{2}{3}}$
2. $x^{\frac{3}{4}}$
3. $x^{\frac{5}{6}}$
4. $x^{\frac{1}{3}}$
5. $x^{\frac{3}{2}}$
6. $x^{\frac{1}{2}}$
7. $x^{\frac{4}{5}}$
8. $x^{\frac{1}{7}}$
9. $x^{\frac{5}{2}}$

Rewrite the following using fractional exponents

10. $\sqrt[3]{x^4}$
11. $\sqrt[5]{x^3}$
12. $\sqrt[4]{x^7}$
13. $\sqrt{x^3}$
14. $\sqrt[6]{x^5}$
15. $\sqrt{x}$
16. $\sqrt[8]{x^3}$
17. $\sqrt[5]{x^3}$
18. $\sqrt[3]{x}$

Evaluate each of the following:

19. $25^{\frac{1}{2}}$

20. $8^{\frac{1}{3}}$

21. $100^{\frac{1}{2}}$

22. $4^{\frac{3}{2}}$

23. $27^{\frac{2}{3}}$

24. $125^{\frac{5}{3}}$

25. $8^{\frac{5}{3}}$

26. $81^{\frac{3}{4}}$

27. $16^{\frac{3}{2}}$

28. $16^{\frac{5}{4}}$

29. $36^{\frac{3}{2}}$

30. $32^{\frac{2}{5}}$

31. Explain what a rational exponent, such as $\frac{5}{2}$ means. Use this explanation to evaluate $9^{\frac{5}{2}}$.

32. Explain how $125^{\frac{4}{3}}$ can be evaluated using properties of rational exponents to result in an integer answer.

33. Explain how $(-8)^{\frac{4}{3}}$ can be evaluated using properties of rational exponents to result in an integer answer.

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Given Radicals

Express the following without using radicals:

1. $\sqrt{x^{-2}y^{5}}$

2. $\sqrt[3]{27x^{6}y^{-8}}$

3. $\sqrt{25x^{3}y^{4}}$

4. $\sqrt[3]{64x^{-5}y^{-8}}$

5. The expression $\sqrt[4]{16x^{2}y^{7}}$ is equivalent to

1) $2x^{\frac{1}{2}}y^{\frac{7}{4}}$

2) $2x^{8}y^{28}$

3) $4x^{\frac{1}{2}}y^{\frac{7}{4}}$

4) $4x^{8}y^{28}$

6. The expression $\sqrt[4]{81x^{2}y^{5}}$ is equivalent to

1) $3x^{\frac{1}{2}}y^{\frac{5}{4}}$

2) $3x^{\frac{1}{2}}y^{\frac{4}{5}}$

3) $9xy^{\frac{5}{2}}$

4) $9xy^{\frac{2}{5}}$

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Fractional Exponents Regents Practice

For Multiple Choice, Use Multiple Choice Strategy!!!!!!

1. Given $y > 0$, the expression $\sqrt{3x^2y} \bullet \sqrt[3]{27x^3y^2}$ is equivalent to

1) $81x^5y^3$
2) $3^{1.5}x^2y$
3) $3^{\frac{5}{2}}x^2y^{\frac{5}{3}}$
4) $3^{\frac{3}{2}}x^2y^{\frac{7}{6}}$

2. The expression $\left(\dfrac{m^2}{m^{\frac{1}{3}}}\right)^{-\frac{1}{2}}$ is equivalent to

1) $-\sqrt[6]{m^5}$
2) $\dfrac{1}{\sqrt[6]{m^5}}$
3) $-m\sqrt[5]{m}$
4) $\dfrac{1}{m\sqrt[5]{m}}$

3. Which equation is equivalent to $P = 210x^{\frac{4}{3}}y^{\frac{7}{3}}$

1) $P = \sqrt[3]{210x^4y^7}$
2) $P = 70xy^2\sqrt[3]{xy}$
3) $P = 210xy^2\sqrt[3]{xy}$
4) $P = 210xy^2\sqrt[3]{x^3y^5}$

4. For $x \geq 0$, which equation is *false*?

1) $(x^{\frac{3}{2}})^2 = \sqrt[4]{x^3}$
2) $(x^3)^{\frac{1}{4}} = \sqrt[4]{x^3}$
3) $(x^{\frac{3}{2}})^{\frac{1}{2}} = \sqrt[4]{x^3}$
4) $(x^{\frac{2}{3}})^2 = \sqrt[3]{x^4}$

5. For $x \neq 0$, which expressions are equivalent to one divided by the sixth root of *x*?

I. $\dfrac{\sqrt[6]{x}}{\sqrt[3]{x}}$ II. $\dfrac{x^{\frac{1}{6}}}{x^{\frac{1}{3}}}$ III. $x^{\frac{-1}{6}}$

1) I and II, only
2) I and III, only
3) II and III, only
4) I, II, and III

Express the following in simplest form, with a rational exponent.

6. $a\sqrt[5]{a^4}$

7. $2xy^2\sqrt[3]{x^2y}$

8. $\dfrac{\sqrt[3]{x^2} \bullet \sqrt{x^5}}{\sqrt[6]{x}}$

9. $\dfrac{x\sqrt{x^3}}{\sqrt[3]{x^5}}$

Express the following in simplest radical form:

10. $\dfrac{x^{\frac{1}{5}}}{x^{\frac{1}{2}}}$

11. $\left(\dfrac{1}{x^{-2}}\right)^{-\frac{3}{4}}$

12. $\dfrac{2x^{\frac{3}{2}}}{\left(16x^4\right)^{\frac{1}{4}}}$

13. $\dfrac{(x^2y^4)^{\frac{1}{3}}}{xy}$

Determine the value of n in each of the following equations:

14. $\dfrac{\sqrt[3]{x^8}}{(x^4)^{\frac{1}{3}}} = x^n$

15. $\left(\dfrac{1}{\sqrt[3]{y^2}}\right) y^4 = y^n$

16. $\left(\dfrac{y^{\frac{17}{8}}}{y^{\frac{5}{4}}}\right)^{-4} = y^n$

17. Kenzie believes that for $x \geq 0$, the expression $\left(\sqrt[7]{x^2}\right)\left(\sqrt[5]{x^3}\right)$ is equivalent to $\sqrt[35]{x^6}$. Is she correct? Justify your response algebraically.

18. Justify why $\dfrac{\sqrt[3]{x^2y^5}}{\sqrt[4]{x^3y^4}}$ is equivalent to $x^{\frac{-1}{12}}y^{\frac{2}{3}}$ using properties of rational exponents, where $x \neq 0$ and $y \neq 0$.

19. For n and $p > 0$, is the expression $\left(p^2 n^{\frac{1}{2}}\right)^8 \sqrt{p^5 n^4}$ equivalent to $p^{18} n^6 \sqrt{p}$? Justify your answer.

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Evaluating Logarithms

Evaluate each of the following logarithms:

1. $\log_4 4$
2. $\log_{15} 1$
3. $\log_2 16$
4. $\log_3 27$
5. $\log_6 36$
6. $\log_4 64$
7. $\log_2 8$
8. $\log_5 125$
9. $\log 100000$
10. $\log_7 \frac{1}{49}$
11. $\log_2 \frac{1}{8}$
12. $\log 0.1$
13. $\log_6 216$
14. $\log_{11} \frac{1}{121}$
15. $\ln e$
16. $\log_{\frac{1}{2}} \frac{1}{64}$
17. $\log_9 27$
18. $\log_4 32$

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Logarithm Rules

Express as multiple logs

1. $\log xy$

2. $\ln \frac{7}{x}$

3. $\ln x^3$

4. $\log \frac{x^4 y^2}{z}$

5. $\ln x^3 y^2$

6. $\log \frac{a^2 b}{c^4}$

7. $\log \frac{a^5 b^3}{c^6}$

8. $\ln \frac{x^2 y^6}{c^3}$

9. $\ln \frac{\sqrt{x}}{y^3}$

10. $\log \frac{\sqrt[4]{x^3}\, y^2}{\sqrt[3]{z}}$

11. $\log \frac{x^5 y}{\sqrt[3]{z}}$

12. $\log \frac{m^3 \sqrt{n}}{k^2}$

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Variable Exponential Equations

Solve each of the following and round to the *nearest hundredth*.

1. $3^{2x} = 182$
2. $e^{2n} = 245$
3. $3(5)^{2x} = 60$
4. $20e^{4x} = 120$
5. $250(1.04)^{4x} = 500$
6. $48e^{.12x} = 60$
7. $1.2(4)^{2x} = 20$
8. $400(.987)^{2.5x} = 300$

9. $2(3)^{2x} + 8 = 18$

10. $4(2)^{3x} + 3 = 15$

11. $8 + 2e^{-5x} = 14$

12. $12 + 2(5)^{8x} = 2000$

13. $500e^{\frac{x}{2}} = 200$

14. $2000(2)^{\frac{x}{4.2}} = 1500$

15. $1.2(3)^{\frac{x}{4.1}} + 15 = 195$

16. $18 - 4(6)^{\frac{x}{3}} = 16$

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Exponential Equations Multiple Choice

1. Which is the solution to: $2(3)^{4x}+1=11$?

1) $\frac{\log 5}{4\log 3}$ 3) $\frac{\log 3}{4\log 5}$

2) $\frac{4\log 5}{\log 3}$ 4) $\frac{4\log 3}{\log 5}$

2. Which is the solution to: $256+4(2)^{6x}=2700$?

1) $\frac{\ln 4}{6\ln 2}$ 3) $\frac{\ln 611}{6\ln 2}$

2) $\frac{6\ln 423}{\ln 4}$ 4) $\frac{6\ln 2444}{\ln 4}$

3. Which is the solution to: $1-2(5)^{2x}=-5$?

1) $\frac{\ln 6}{2\ln 3}$ 3) $\frac{2\ln 4}{\ln 3}$

2) $\frac{2\ln 5}{\ln 1}$ 4) $\frac{\ln 3}{2\ln 5}$

4. Which is the solution to: $5(3)^{2x}=30$?

1) $\frac{\log 6}{3\log 2}$ 3) $\frac{2\log 6}{\log 3}$

2) $\frac{\log 6}{2\log 3}$ 4) $\frac{2\log 3}{\log 6}$

5. The solution to the equation $5e^{x+2} = 7$ is

1) $-2+\ln\left(\frac{7}{5}\right)$

2) $\left(\frac{\ln 7}{\ln 5}\right)-2$

3) $\frac{-3}{5}$

4) $-2+\ln(2)$

6. What is the solution of $2(3^{x+4}) = 56$?

1) $x = \log_3(28)-4$

2) $x = -1$

3) $x = \log(25)-4$

4) $x = \frac{\log(56)}{\log(6)}-4$

7. The solution to the equation $6\left(2^{x+4}\right) = 36$ is

1) -1

2) $\frac{\ln 36}{\ln 12}-4$

3) $\ln(3)-4$

4) $\frac{\ln 6}{\ln 2}-4$

8. Which expression is *not* a solution to the equation $2^t = \sqrt{10}$?

1) $\frac{1}{2}\log_2 10$

2) $\log_2\sqrt{10}$

3) $\log_4 10$

4) $\log_{10} 4$

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Exponential Equations Word Problems

1. A population of wolves in a county is represented by the equation $\mathrm{P}(t) = 80(0.98)^t$, where t is the number of years since 1998. After how many years will the population of wolves be 60 rounded to the *nearest year*?

2. Juliette deposits $3000 into a bank account where the balance of the account $b(t)$ after t years can be represented by $b(t) = 3000e^{.042t}$. To the nearest tenth of a year, how long will it take for Juliette's money to double?

3. 200 grams of a radioactive substance decays according to the formula $a(t) = 200(.094)^{2t}$ where $a(t)$ is the amount of the radioactive substance remaining after t years. To the nearest hundredth of a year, how long will it take until there are 150 grams remaining?

4. After an oven is turned on, its temperature, T, is represented by the equation $T = 400 - 350(3.2)^{-0.1m}$, where m represents the number of minutes after the oven is turned on and T represents the temperature of the oven, in degrees Fahrenheit.
How many minutes does it take for the oven's temperature to reach 300°F? Round your answer to the *nearest minute*.

5. Drew's parents invested $1,500 in an account such that the value of the investment doubles every seven years. The value of the investment, V, is determined by the equation $V = 1500(2)^{\frac{t}{7}}$, where t represents the number of years since the money was deposited. How many years, to the *nearest tenth of a year*, will it take the value of the investment to triple?

6. The number of houses in Central Village, New York, grows every year according to the function $\mathrm{H}(t) = 540(1.039)^{1.02t}$, where H represents the number of houses, and t represents the number of years since January 1995. A civil engineering firm has suggested that a new, larger well must be built by the village to supply its water when the number of houses exceeds 1,000. During which year will this first happen?

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Newton's Law of Heating and Cooling

1. The Fahrenheit temperature of a heated object can be modeled by the function below.

$$F(t) = F_S + (F_0 - F_S)e^{-kt}$$

$F(t)$ = the temperature of the object after t minutes
t = time in minutes
F_S = the surrounding temperature
F_0 = the initial temperature of the object
k = a constant
Hot chocolate at a temperature of 200°F is poured into a container. The room temperature is kept at a constant 68°F and $k = 0.05$.
After how much time, to the *nearest minute*, will the temperature of the hot chocolate be 150°F?

After how much time, to the *nearest tenth of a minute*, will the temperature of the hot chocolate be 120°F?

2. The Fahrenheit temperature, $F(t)$, of a heated object at time t, in minutes, can be modeled by the function below. F_s is the surrounding temperature, F_0 is the initial temperature of the object, and k is a constant.

$$F(t) = F_S + (F_0 - F_S)e^{-kt}$$

Coffee at a temperature of 195°F is poured into a container. The room temperature is kept at a constant 68°F and $k = 0.05$. Coffee is safe to drink when its temperature is, at most, 120°F. To the *nearest minute*, how long will it take until the coffee is safe to drink?

3. After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + \left(T_0 - T_a\right)e^{-kt}$$

T_a = the temperature surrounding the object

T_0 = the initial temperature of the object

t = the time in hours

T = the temperature of the object after t hours

k = decay constant

The turkey reaches the temperature of approximately 100° F after 2 hours. Find the value of k, to the *nearest thousandth*. Determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m.

4. Empanadas are taken out of an oven when they reached a temperature of 168°F and put on the kitchen table at room temperature (68°F). After 8 minutes, the temperature of the empanadas is 125°F. The temperature of a cooled object can be given by the formula below:

$T = T_a + \left(T_0 - T_a\right)e^{-kt}$

T = the temperature of the object after t minutes

t = time in minutes

T_a = the surrounding temperature

T_0 = the initial temperature of the object

k = decay constant

Find the value of k, rounded to the *nearest thousandth*. Using your value of k, to the *nearest minute*, how long will it take for the empanadas to reach 100°F?

5. Megan is performing an experiment in a lab where the air temperature is a constant 73ºF and the liquid is 237ºF. One and a half hours later, the temperature of the liquid is 112ºF. Newton's law of cooling states $T(t) = T_a + (T_0 - T_a)e^{-kt}$ where:

$T(t)$: temperature, °F, of the liquid at t hours

T_a: air temperature

T_0: initial temperature of the liquid

k: constant

Determine the value of k, to the *nearest thousandth*, for this liquid. Determine the temperature of the liquid using your value for k, to the *nearest degree*, after two and a half hours. Megan needs the temperature of the liquid to be 80ºF to perform the next step in her experiment. Use your value for k to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

6. Objects cool at different rates based on the formula below.

$T = (T_0 - T_R)e^{-rt} + T_R$

T_0: initial temperature

T_R: room temperature

r: rate of cooling of the object

t: time in minutes that the object cools to a temperature, T

Mark makes T-shirts using a hot press to transfer designs to the shirts. He removes a shirt from a press that heats the shirt to 400°F. The rate of cooling for the shirt is 0.0735 and the room temperature is 75°F. Find the temperature of the shirt, to the *nearest degree*, after five minutes. At the same time, Mark's friend Jeanine removes a hoodie from a press that heats the hoodie to 450°F. After eight minutes, the hoodie measured 270°F. The room temperature is still 75°F. Determine the rate of cooling of the hoodie, to the *nearest ten thousandth*. The T-shirt and hoodie were removed at the same time. Determine when the temperature will be the same, to the *nearest minute*.

Name ______________
Mr. Schlansky

Date _____________
Algebra II

Graphing Exponential and Logarithmic Functions

For the following equations, graph the equation and the asymptote. State the domain, range, equation of the asymptote, and end behavior.

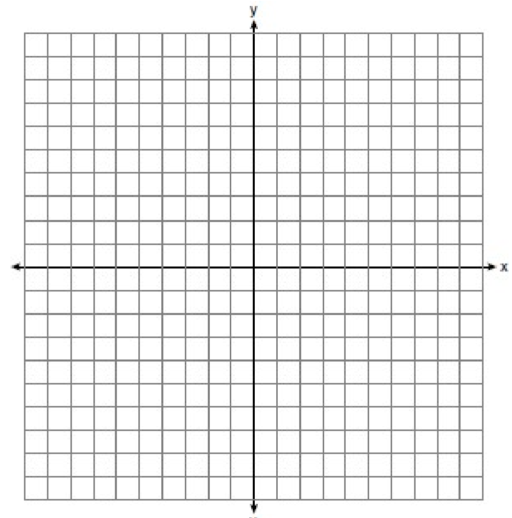
1. $y = 2^x - 3$

Domain:

Range:

Asymptote:

End Behavior:
$x \to -\infty,\ f(x) \to$
$x \to \infty,\ f(x) \to$



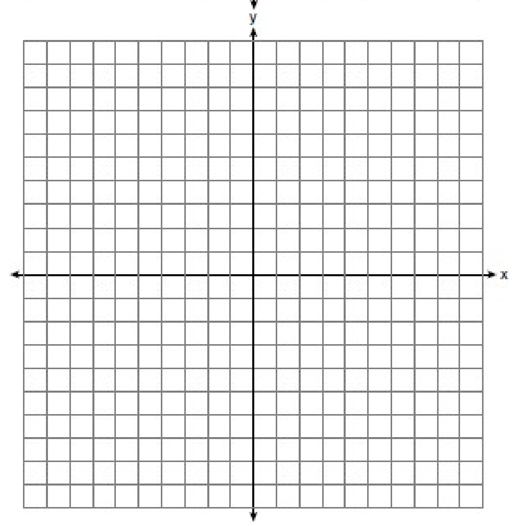
2. $y = \frac{1}{2}^{x-3} + 1$

Domain:

Range:

Asymptote:

End Behavior:
$x \to -\infty,\ f(x) \to$
$x \to \infty,\ f(x) \to$



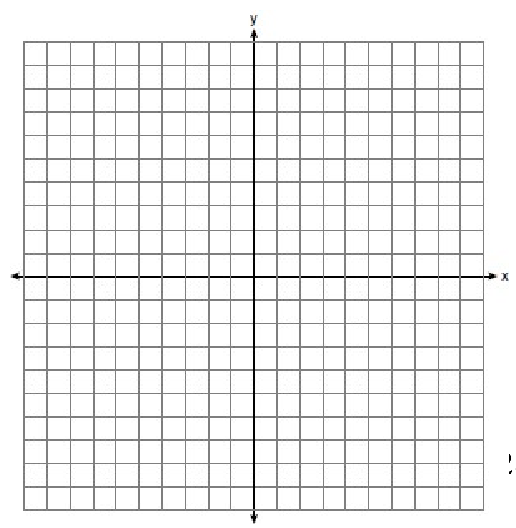
3. $y = -3^{x-2} + 4$

Domain:

Range:

Asymptote:

End Behavior:
$x \to -\infty,\ f(x) \to$
$x \to \infty,\ f(x) \to$

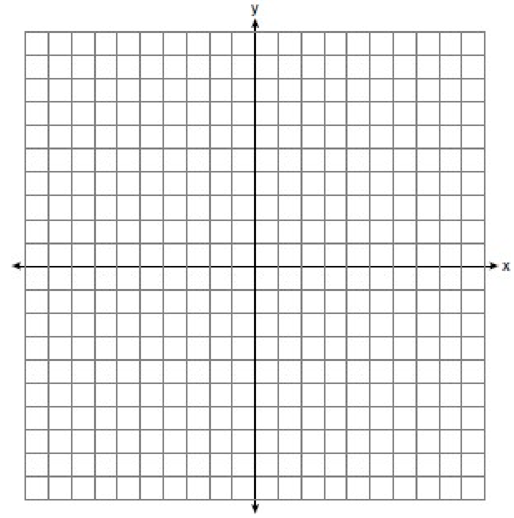


4. $y = 2(3)^{x+1} - 8$
Domain:

Range:

Asymptote:

End Behavior:
$x \to -\infty,\ f(x) \to$
$x \to \infty,\ f(x) \to$

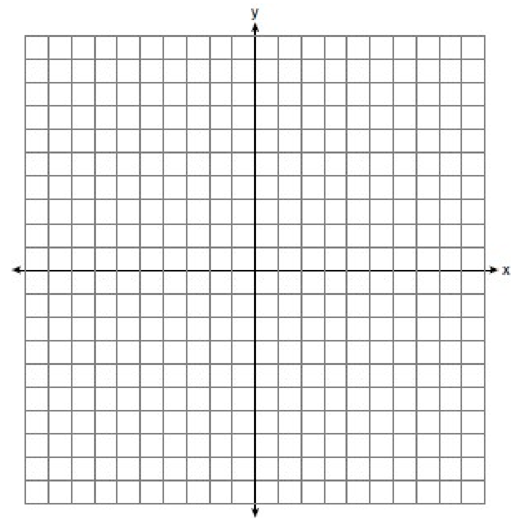


5. $y = -2\left(\frac{1}{3}\right)^{x-5} + 9$
Domain:

Range:

Asymptote:

End Behavior:
$x \to -\infty,\ f(x) \to$
$x \to \infty,\ f(x) \to$

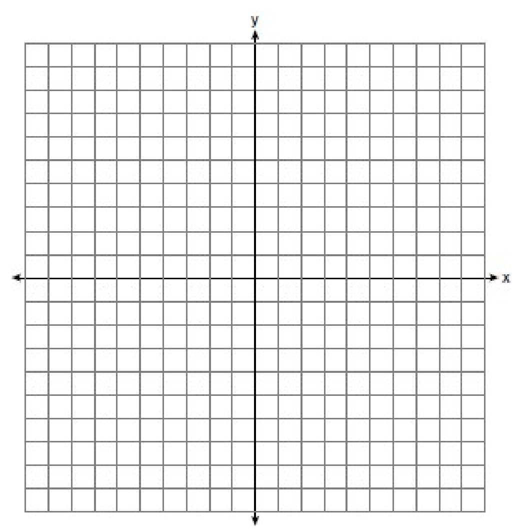


6. $y = 3\left(\frac{1}{2}\right)^{x+1} - 7$
Domain:

Range:

Asymptote:

End Behavior:
$x \to -\infty,\ f(x) \to$
$x \to \infty,\ f(x) \to$

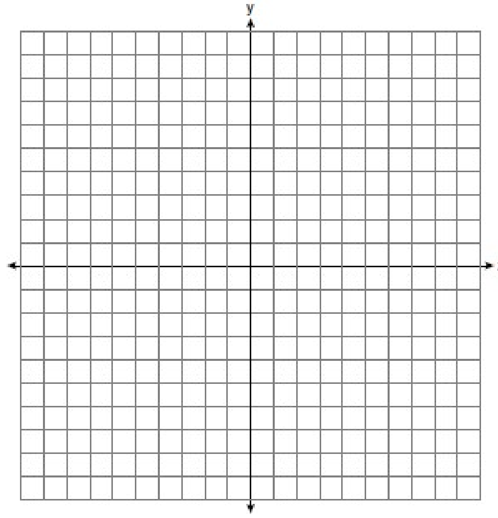


7. $y = \log_2(x) + 3$
Domain:

Range:

Asymptote:

End Behavior:
$x \to 0,\ f(x) \to$
$x \to \infty,\ f(x) \to$

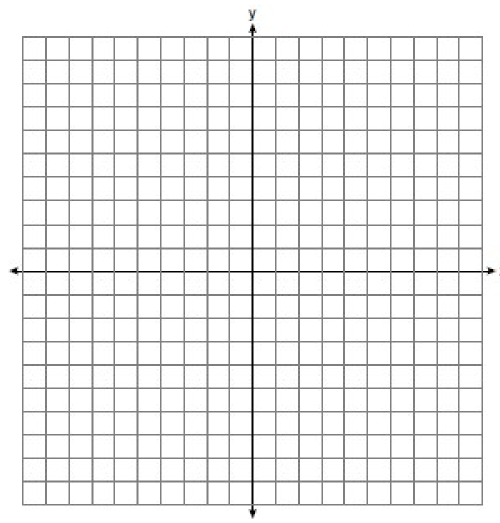


8. $y = \log_3(x+2) - 1$
Domain:

Range:

Asymptote:

End Behavior:
$x \to -2,\ f(x) \to$
$x \to \infty,\ f(x) \to$

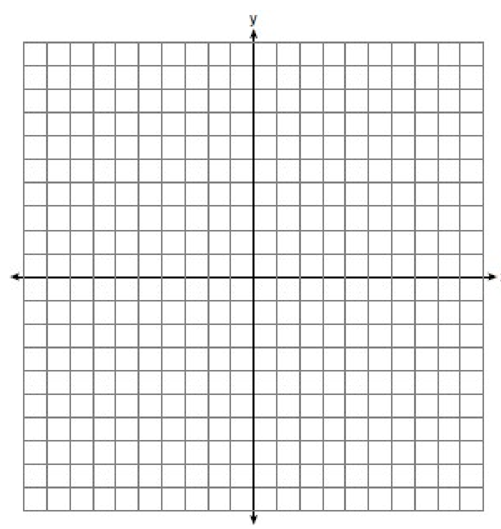


9. $y = -2\log_2(x+6) - 4$
Domain:

Range:

Asymptote:

End Behavior:
$x \to -6,\ f(x) \to$
$x \to \infty,\ f(x) \to$



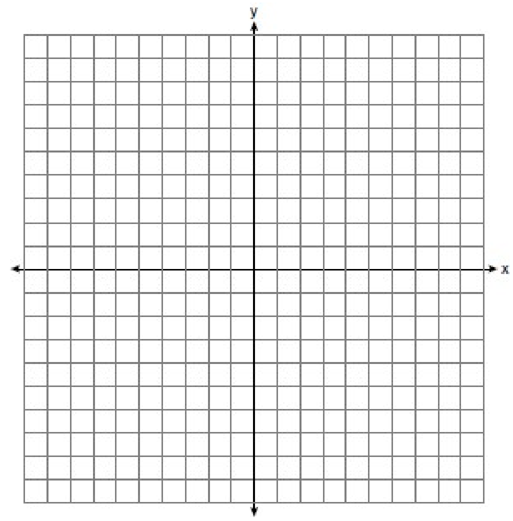
10. $y = 4\log_{\frac{1}{2}}(x-3)+1$

Domain:

Range:

Asymptote:

End Behavior:
$x \to 3,\ f(x) \to$
$x \to \infty,\ f(x) \to$



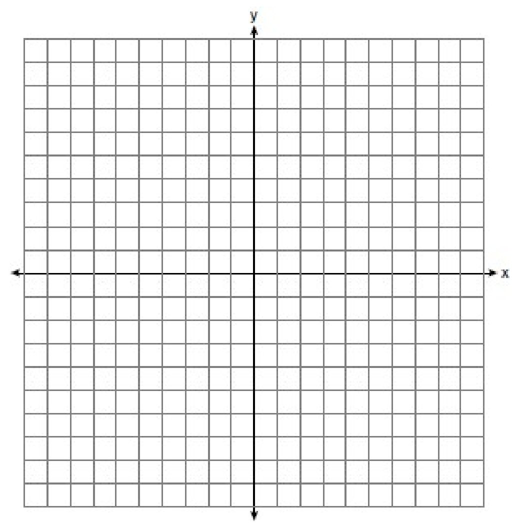
11. $y = 3\log_4(x+1)-8$

Domain:

Range:

Asymptote:

End Behavior:
$x \to -1,\ f(x) \to$
$x \to \infty,\ f(x) \to$



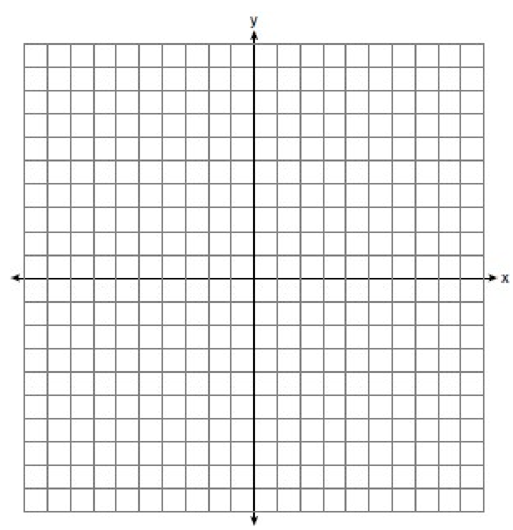
12. $y = -4\log_2(x+9)+4$

Domain:

Range:

Asymptote:

End Behavior:
$x \to -9,\ f(x) \to$
$x \to \infty,\ f(x) \to$



Name _____________ Date ____________
Mr. Schlansky Algebra II

Exponential and Logarithmic Graphs Multiple Choice

1. Which statement about the graph of $c(x) = \log_6 x$ is *false*?
 1) The asymptote has equation $y = 0$.
 2) The graph has no y-intercept.
 3) The domain is the set of positive reals.
 4) The range is the set of all real numbers.

2. Which statement about the graph of the equation $y = e^x$ is *not* true?
 1) It is asymptotic to the x-axis.
 2) The domain is the set of all real numbers.
 3) It lies in Quadrants I and II.
 4) It passes through the point $(e, 1)$.

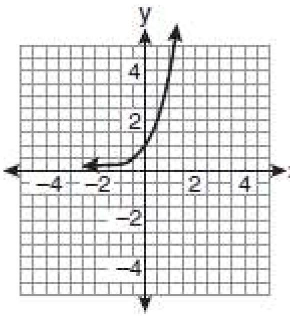
3. Which statement is true about the graph of $f(x) = \left(\frac{1}{8}\right)^x$?
 1) The graph is always increasing.
 2) The graph is always decreasing.
 3) The graph passes through $(1, 0)$.
 4) The graph has an asymptote, $x = 0$.

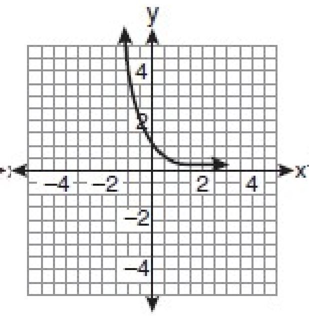
4. Which statement is *true* regarding the equation $f(x) = \log_7 x$?
1) It is always increasing
2) The graph passes through $(0,1)$
3) The domain is all real numbers
4) The equation of the asymptote is $y = 0$

5. Given the equation $f(x) = \pi^x$, which of the following statements is true?
1) The graph passes through $(\pi,1)$
2) The domain is $[0,\infty)$
3) The graph passes through $(0,1)$
4) The range is all real numbers

6. Which statement is *false* regarding the equation $f(x) = \ln x$?
1) It passes through $(1,0)$
2) It is always decreasing
3) The equation of the asymptote is $x = 0$
4) Its range is $(-\infty,\infty)$

7. If a function is defined by the equation $f(x) = 4^x$, which graph represents the inverse of this function?

1)


2)


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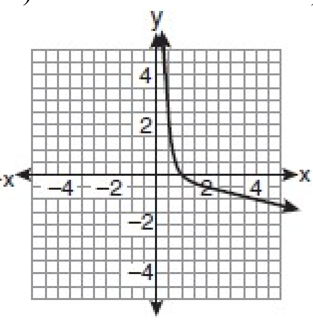

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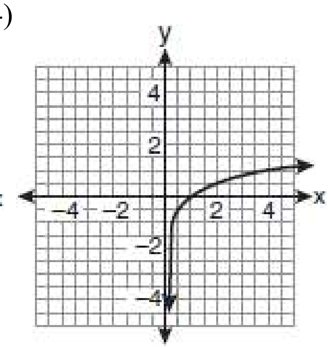
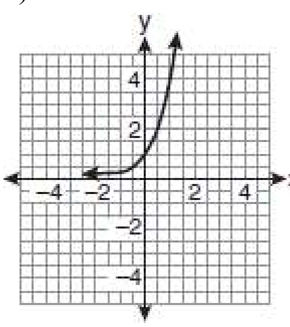


8. If a function is defined by the equation $f(x) = \log_4 x$, which graph represents the inverse of this function?

1)


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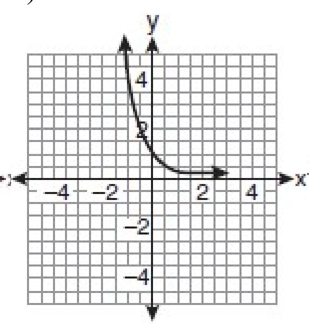

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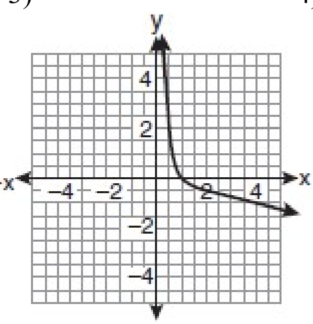

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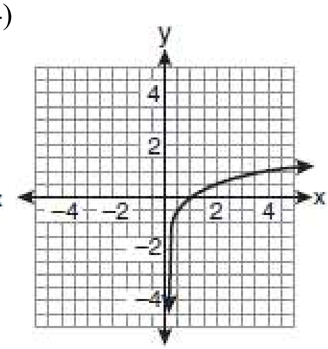
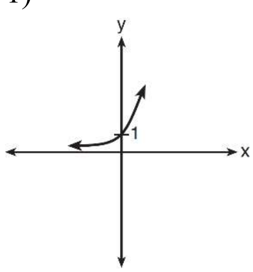


9. Which sketch shows the inverse of $y = a^x$, where $a > 1$?

1)


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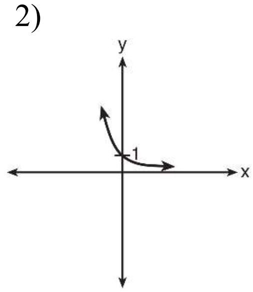

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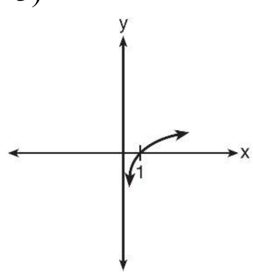
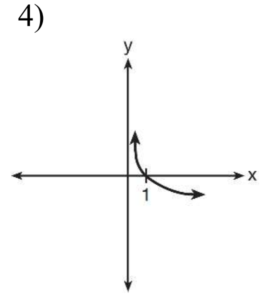


4)


10. What is the inverse of the function $y = \log_3 x$?

1) $y = x^3$ 2) $y = \log_x 3$ 3) $y = 3^x$ 4) $x = 3^y$

11. If $f(x) = a^x$ where $a > 1$, then the inverse of the function is

1) $f^{-1}(x) = \log_x a$

2) $f^{-1}(x) = a \log x$

3) $f^{-1}(x) = \log_a x$

4) $f^{-1}(x) = x \log a$

12. The asymptote of the graph of $f(x) = 5\log(x+4)$ is

1) $y = 6$
2) $x = -4$
3) $x = 4$
4) $y = 5$

13. The asymptote of the graph of $j(x) = 2e^{x-4} - 1$ is

1) $x = 4$
2) $x = -4$
3) $y = -1$
4) $y = 2$

14. The asymptote of the graph of $e(x) = \log_3(x-5) + 1$ is

1) $y = 1$
2) $x = 1$
3) $y = 5$
4) $x = 1$

15. The asymptote of the graph of $m(x) = -3(2)^{x+1} - 4$ is

1) $x = -1$
2) $x = 3$
3) $y = -4$
4) $y = -3$

16. For the equation $f(x) = 2^{x-3} + 1$, as $x \to -\infty$

1) $f(x) \to -\infty$
2) $f(x) \to 1$
3) $f(x) \to \infty$
4) $f(x) \to 3$

17. For the equation $f(x) = \log_2(x-4) + 3$, as $x \to 4$

1) $f(x) \to -\infty$
2) $f(x) \to 3$
3) $f(x) \to \infty$
4) $f(x) \to 4$

18. For the equation $f(x) = -\log_3(x+1) - 2$, as $x \to \infty$

1) $f(x) \to -\infty$
2) $f(x) \to -1$
3) $f(x) \to \infty$
4) $f(x) \to -2$

19. Given $f(x) = 3^{x-1} + 2$, as $x \to -\infty$

1) $f(x) \to -1$
2) $f(x) \to 0$
3) $f(x) \to 2$
4) $f(x) \to -\infty$

20. For the equation $f(x) = 3\ln(x-4) + 1$, $f(x) \to -\infty$ as

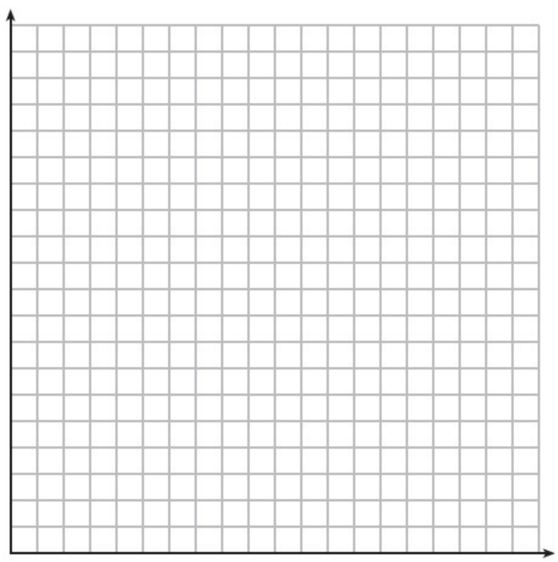
1) $x \to 4$
2) $x \to 1$
3) $x \to \infty$
4) $x \to -\infty$

Name _____________ Date ___________

Mr. Schlansky Algebra II

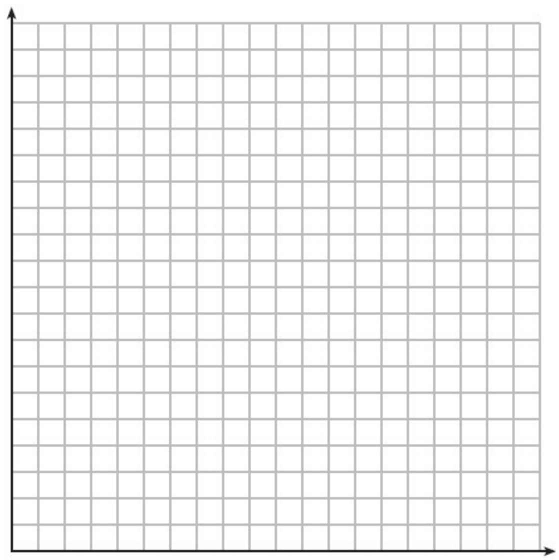
Exponential Graphs (Part IV)

1. The value of Tom's bank account is currently 100000 and is decreasing according to the equation $V(t)=100000(.876)^t$. The amount of money he has paid for his mortgage can be represented by the equation $M(t)=20000(1.1304)^t$. Graph and label $V(t)$ and $M(t)$ over the interval $[0,10]$.



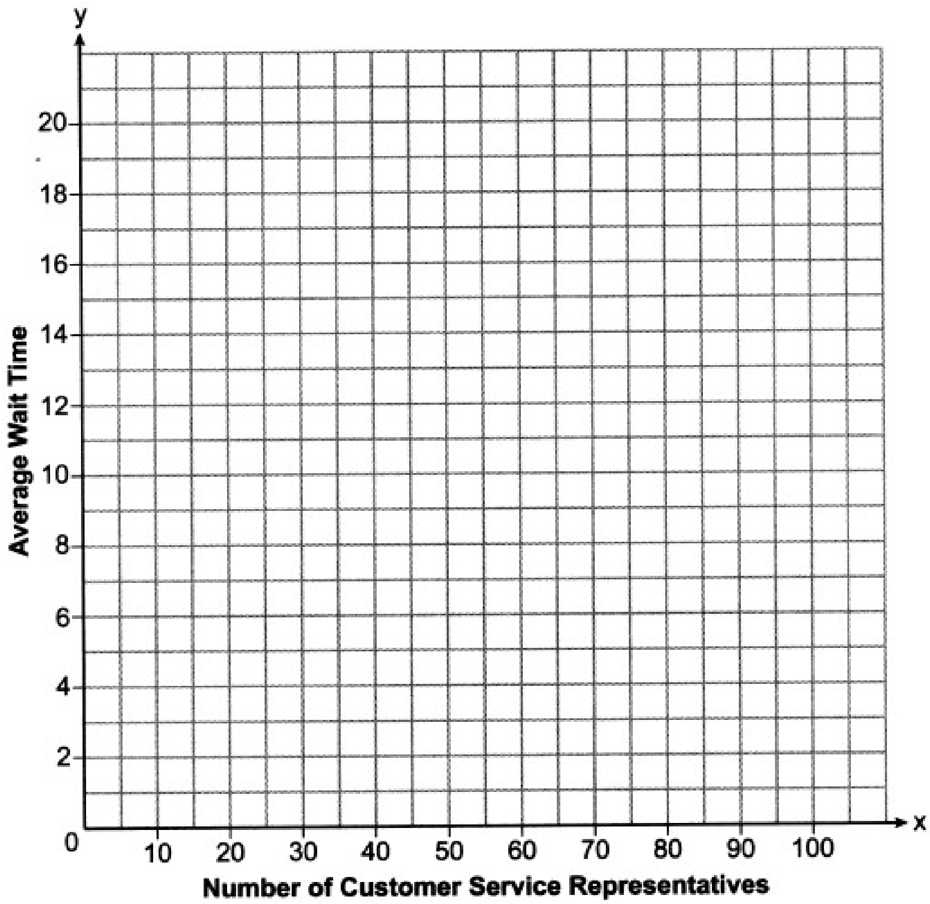
After how many years will the value of Tom's bank account be equal to the amount of money he paid for his mortgage? Round your answer to the *nearest tenth of a year*. Tom will open a new bank account when the value of his account is $20,000. After how many years, to the *nearest hundredth of a year*, will that happen?

2. The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time. Graph $V(t)$ and $Z(t)$ over the interval $0 \le t \le 5$, on the set of axes below.



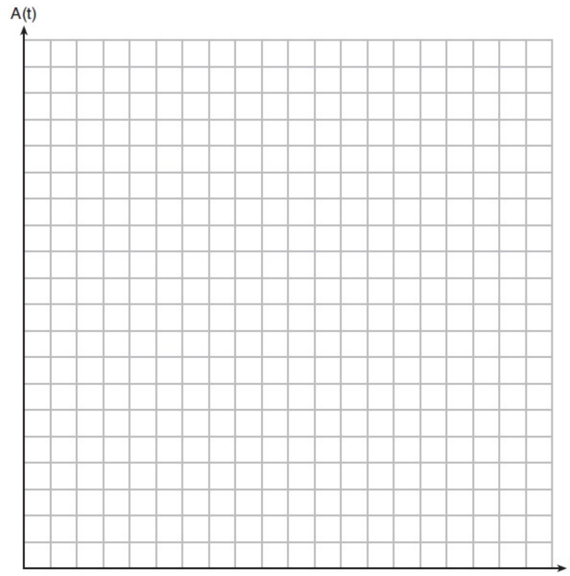
State when $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem. Zach will cancel the collision policy when the value of his car equals $3000. To the *nearest tenth of a year*, how long will it take Zach to cancel this policy? Justify your answer.

3. A technology company is comparing two plans for speeding up its technical support time. Plan A can be modeled by the function $A(x) = 15.7(0.98)^x$ and plan B can be modeled by the function $B(x) = 11(0.99)^x$ where x is the number of customer service representatives employed by the company and $A(x)$ and $B(x)$ represent the average wait time, in minutes, of each customer. Graph $A(x)$ and $B(x)$ in the interval $0 \leq x \leq 100$ on the set of axes below.



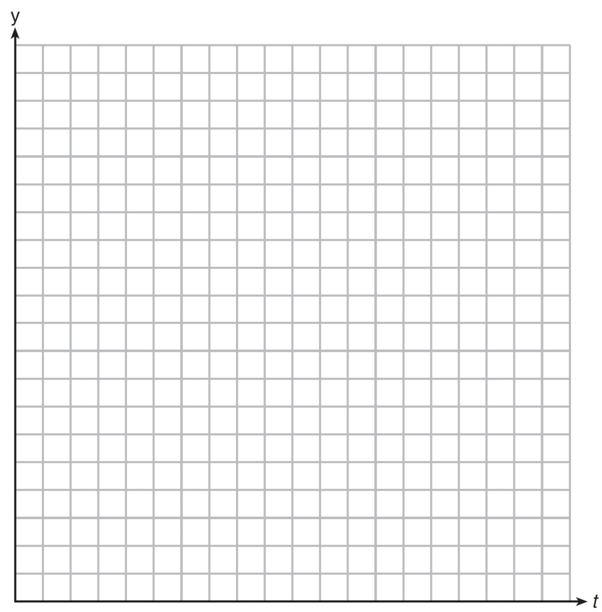
To the *nearest integer*, solve the equation $A(x) = B(x)$. How many Customer Service Representatives would the Company B need in order to the average wait time to be 3 minutes? Round to the *nearest representative*.

4. Tony is evaluating his retirement savings. The value of his account can be represented by $A(t) = 318000(1.07)^t$. Graph $A(t)$ where $0 \le t \le 20$ on the set of axes below.



Tony's goal is to save $1,000,000. Determine algebraically, to the *nearest year*, how many years it will take for him to achieve his goal. Explain how your graph of $A(t)$ confirms your answer.

5. Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231. Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.



To the *nearest tenth of an hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A? The doctor will allow patient A to take another dose of the drug once 120 milligrams is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another dose of the drug.

Name ______________ Date _____________
Mr. Schlansky Algebra II

Exponents/Logarithms/Functions Review Sheet

1. The expression $\left(\dfrac{m^2}{m^{\frac{1}{3}}}\right)^{-\frac{1}{2}}$ is equivalent to

1) $-\sqrt[6]{m^5}$
2) $\dfrac{1}{\sqrt[6]{m^5}}$
3) $-m\sqrt[5]{m}$
4) $\dfrac{1}{m\sqrt[5]{m}}$

2. The expression $\sqrt[4]{81x^2y^5}$ is equivalent to

1) $3x^{\frac{1}{2}}y^{\frac{5}{4}}$
2) $3x^{\frac{1}{2}}y^{\frac{4}{5}}$
3) $9xy^{\frac{5}{2}}$
4) $9xy^{\frac{2}{5}}$

3. The solution set of $\sqrt{3x+16}=x+2$ is

1) $\{-3,4\}$
2) $\{-4,3\}$
3) $\{3\}$
4) $\{-4\}$

4. The solution set of the equation $\sqrt{2x-4}=x-2$ is

1) $\{-2,-4\}$
2) $\{2,4\}$
3) $\{4\}$
4) $\{\ \}$

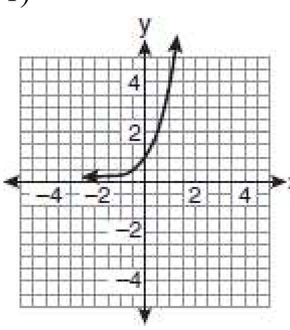
5. The solution to the equation $6\left(2^{x+4}\right) = 36$ is

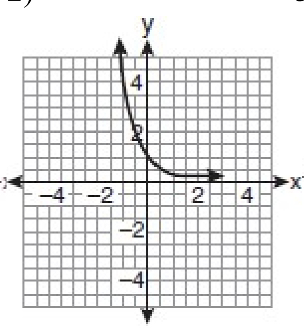
1) -1
2) $\frac{\ln 36}{\ln 12} - 4$
3) $\ln(3) - 4$
4) $\frac{\ln 6}{\ln 2} - 4$

6. Which is the solution to: $5(3)^{2x} = 30$?

1) $\frac{\log 6}{3\log 2}$
2) $\frac{\log 6}{2\log 3}$
3) $\frac{2\log 6}{\log 3}$
4) $\frac{2\log 3}{\log 6}$

7. If a function is defined by the equation $f(x) = \log_4 x$, which graph represents the inverse of this function?

1)


2)


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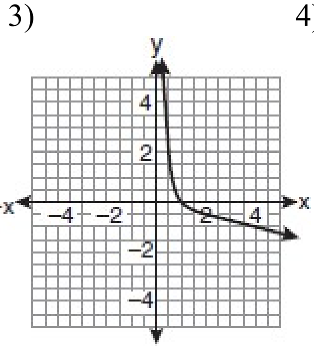

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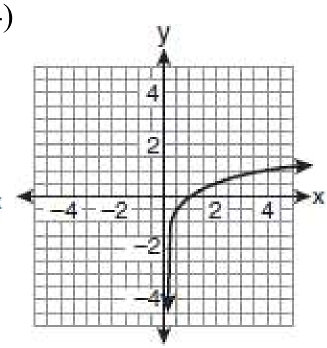
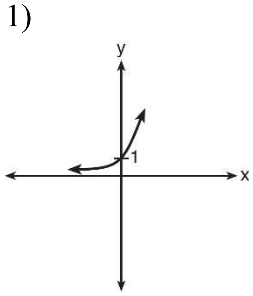
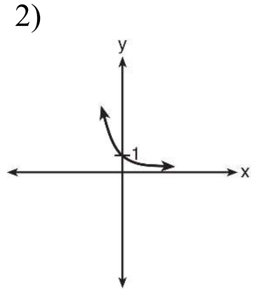
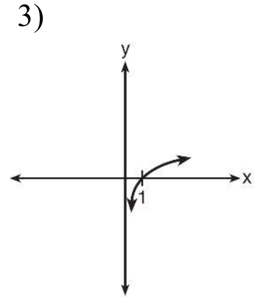
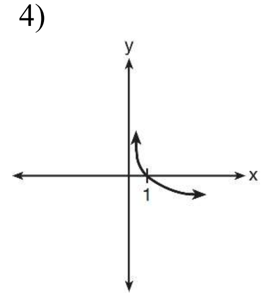


8. Which sketch shows the inverse of $y = a^x$, where $a > 1$?

1)


2)


3)


4)


9. Which statement about the graph of $f(x) = 2^x - 1$ is *true*?

1) It is always increasing with a y intercept of 0
2) It is always decreasing with a y intercept of 0
3) It is always increasing with a y intercept of -1
4) It is always decreasing with a y intercept of -1

10. Which statement about the graph of the equation $f(x) = \frac{1}{3}^x + 2$ is *true*?

1) It is always increasing with a y intercept of 2
2) It is always decreasing with a y intercept of 2
3) It is always increasing with a y intercept of 3
4) It is always decreasing with a y intercept of 3

11. Given $f(x) = 3^{x-1} + 2$, as $x \to -\infty$

1) $f(x) \to -1$
2) $f(x) \to 0$
3) $f(x) \to 2$
4) $f(x) \to -\infty$

12. For the equation $f(x) = -\log_3(x+1) - 2$, as $x \to \infty$

1) $f(x) \to -\infty$
2) $f(x) \to -1$
3) $f(x) \to \infty$
4) $f(x) \to -2$

13. $y = 2(3)^{x+1} - 8$

Domain:

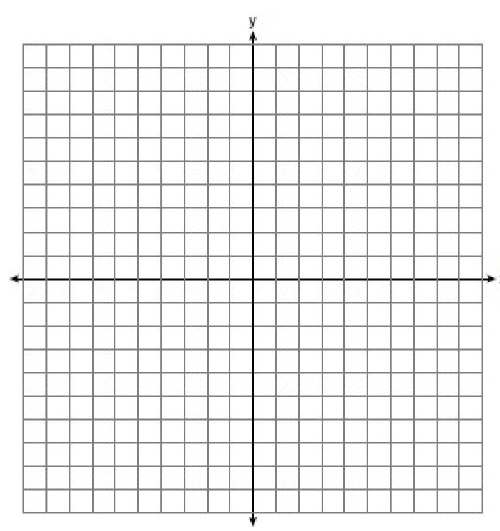
Range:

Asymptote:

End Behavior:

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$



14. $y = -2\left(\frac{1}{3}\right)^{x-5} + 9$

Domain:

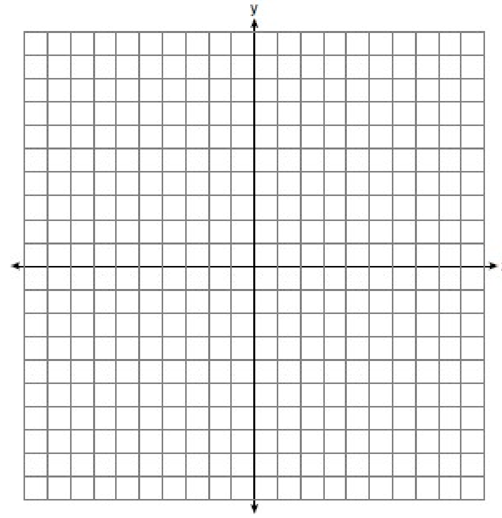
Range:

Asymptote:

End Behavior:

$x \to -\infty,\ f(x) \to$

$x \to \infty,\ f(x) \to$



15. $y = \log_3(x+2) - 1$

Domain:

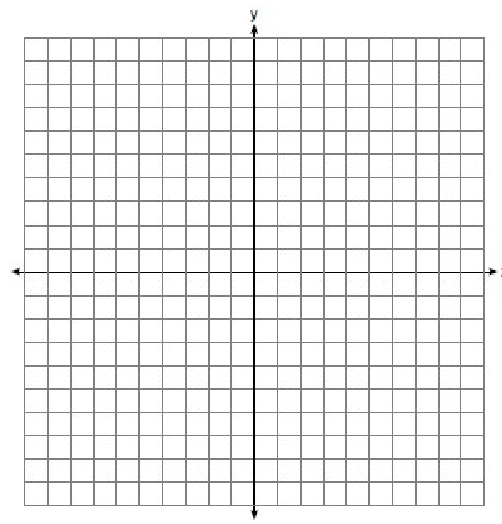
Range:

Asymptote:

End Behavior:

$x \to -2,\ f(x) \to$

$x \to \infty,\ f(x) \to$



16. $y = -2\log_2(x+6) - 4$

Domain:

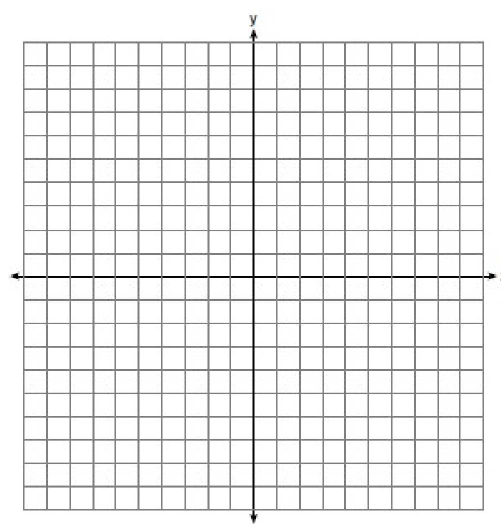
Range:

Asymptote:

End Behavior:

$x \to -6,\ f(x) \to$

$x \to \infty,\ f(x) \to$



Express the following in simplest form with a rational exponent

17. $\dfrac{\sqrt[3]{x^2} \bullet \sqrt{x^5}}{\sqrt[6]{x}}$

18. $\dfrac{x\sqrt{x^3}}{\sqrt[3]{x^5}}$

Express the following in radical form

19. $\dfrac{2x^{\frac{3}{2}}}{\left(16x^4\right)^{\frac{1}{4}}}$

20. $\dfrac{(x^2y^4)^{\frac{1}{3}}}{xy}$

21. 200 grams of a radioactive substance decays according to the formula $a(t) = 200(.094)^{2t}$ where $a(t)$ is the amount of the radioactive substance remaining after t years. To the nearest hundredth of a year, how long will it take until there are 50 grams remaining?

22. Juliette deposits $2500 into a bank account where the balance of the account $b(t)$ after t years can be represented by $b(t) = 2500(1.075)^t$. To the nearest tenth of a year, how long will it take for Juliette's money to reach $4000?

23. Empanadas are taken out of an oven when they reached a temperature of 168°F and put on the kitchen table at room temperature (68°F). After 8 minutes, the temperature of the empanadas is 125°F. The temperature of a cooled object can be given by the formula below:

$$T = T_a + \left(T_0 - T_a\right)e^{-kt}$$

T = the temperature of the object after t minutes
t = time in minutes
T_a = the surrounding temperature
T_0 = the initial temperature of the object
k = decay constant

Algebraically determine the value of k, rounded to the *nearest thousandth*. Using your value of k, to the *nearest minute*, algebraically determine how long will it take for the empanadas to reach 100°F?

24. After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + \left(T_0 - T_a\right)e^{-kt}$$

T_a = the temperature surrounding the object

T_0 = the initial temperature of the object

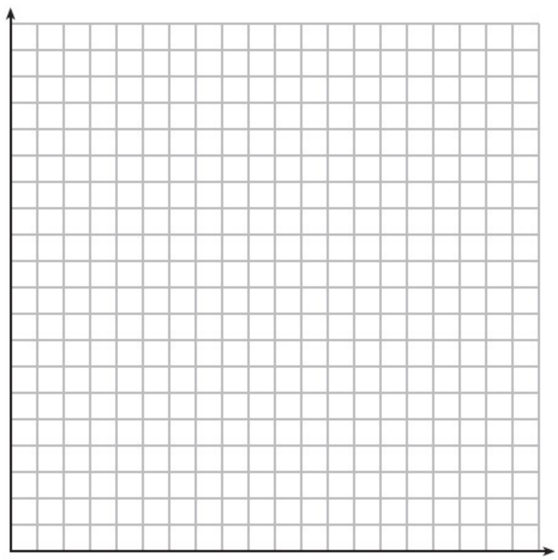
t = the time in hours

T = the temperature of the object after t hours

k = decay constant

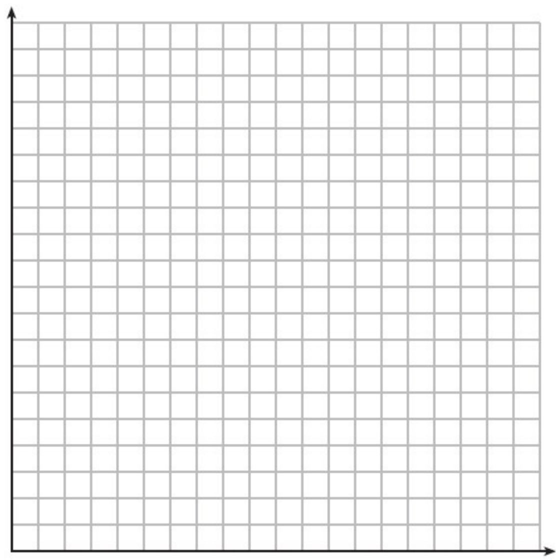
The turkey reaches the temperature of approximately 100° F after 2 hours. Algebraically determine the value of *k*, to the *nearest thousandth*. Using your value of *k,* algebraically determine how many hours after 8 a.m. the turkey will be 160° to the *nearest tenth of a year*?

25. The value of Tom's bank account is currently 100000 and is decreasing according to the equation $V(t) = 90000(.887)^t$. The amount of money he has paid for his mortgage can be represented by the equation $M(t) = 18000(1.152)^t$. Graph and label $V(t)$ and $M(t)$ over the interval $[0,10]$.



After how many years will the value of Tom's bank account be equal to the amount of money he paid for his mortgage? Round your answer to the *nearest tenth of a year*. Tom will open a new bank account when the value of his account is \$25,000. Algebraically, determine after how many years, to the *nearest hundredth of a year*, will that happen?

26. The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time. Graph $V(t)$ and $Z(t)$ over the interval $0 \le t \le 5$, on the set of axes below.



State when $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem. Zach will cancel the collision policy when the value of his car equals \$3000. To the *nearest tenth of a year*, how long will it take Zach to cancel this policy? Justify your answer.