

## Fractional Equations with Factoring

Solve the following fractional equations and list the solutions as well as the extraneous solutions

1.  $\frac{1}{x-2} + \frac{4}{x+5} = \frac{7}{x^2+3x-10}$   
 FI:  $x-2$   
 F2:  $x+5$   
 LCD:  $(x-2)(x+5)$   
 $\frac{1}{\cancel{x-2}} + \frac{4}{\cancel{x+5}} = \frac{7}{(x-2)(x+5)}$   
 $1(x+5) + 4(x-2) = 7$   
 $x+5+4x-8=7$   
 $5x-3=7$   
 $5x=10$   
 $x=2$  reject  
**No solution**  
 2 is an extraneous solution

2.  $\frac{x}{x+2} + \frac{1}{x^2-4} = \frac{4}{x-2}$   
 FI:  $x+2$   
 F2:  $x-2$   
 LCD:  $(x+2)(x-2)$   
 $\frac{x}{\cancel{x+2}} + \frac{1}{\cancel{(x+2)(x-2)}} = \frac{4}{\cancel{x-2}}$   
 $x(x-2) + 1 = 4(x+2)$   
 $x^2-2x+1 = 4x+8$   
 $-4x-8$   
 $x^2-6x-7=0$   
 $(x-7)(x+1)=0$   
 $x=7$  |  $x=-1$   
**Σ -1, 7**  
 No extraneous solutions

3.  $\frac{1}{b-3} - \frac{3}{2b+6} = \frac{b}{b^2-9}$   
 FI: 2  
 F2:  $b+3$   
 F3:  $b-3$   
 LCD:  $2(b+3)(b-3)$   
 $\frac{1}{\cancel{b-3}} - \frac{3}{2\cancel{b+3}} = \frac{b}{(b-3)(b+3)}$   
 $2(b+3) - 3(b-3) = 2b$   
 $2b+6-3b+9=2b$   
 $-b+15=2b$   
 $15=3b$   
 $5=3$   
**Σ 3**  
 No extraneous solutions

4.  $\frac{a-7}{a+2} - \frac{a+3}{a^2+a-6} = \frac{10}{a^2+a-6}$   
 FI:  $a+3$   
 F2:  $a-2$   
 LCD:  $(a+3)(a-2)$   
 $\frac{(a-7)(a+3)}{\cancel{(a+3)(a-2)}} - \frac{(a+3)}{\cancel{(a+3)(a-2)}} = \frac{10}{\cancel{(a+3)(a-2)}}$   
 $a(a+3) - 8(a-2) = 10$   
 $a^2+3a-8a+16=10$   
 $a^2-5a+6=0$   
 $(a-3)(a-2)=0$   
 $a=3$  |  $a=2$  reject  
**Σ 3**  
 2 is an extraneous solution

5.  $\frac{1}{y} + \frac{6}{y^2+2y} = \frac{5}{y+2}$   
 FI:  $y$   
 F2:  $y+2$   
 LCD:  $y(y+2)$   
 $\frac{1}{\cancel{y}} + \frac{6}{\cancel{y}(y+2)} = \frac{5}{\cancel{y+2}}$   
 $1(y+2) + 6 = 5y$   
 $y+2+6=5y$   
 $y+8=5y$   
 $8=4y$   
 $2=y$   
**No extraneous solutions**

6.  $\frac{8}{x^2-121} = \frac{x+11}{x-11} - \frac{2}{x+11}$   
 FI:  $x+11$   
 F2:  $x-11$   
 LCD:  $(x+11)(x-11)$   
 $\frac{8}{\cancel{(x+11)(x-11)}} = \frac{(x+11)(x-11)}{\cancel{(x+11)(x-11)}} - \frac{2}{\cancel{x+11}}$   
 $8 = x(x-11) - 2(x+11)$   
 $8 = x^2 - 11x - 2x - 22$   
 $8 = x^2 - 13x - 22$   
 $0 = x^2 - 13x - 30$   
 $0 = (x-15)(x+2)$   
 $x=15$  |  $x=-2$   
**Σ -2, 15**  
 No extraneous solutions



$$\begin{array}{|c|c|c|} \hline & x & +2 \\ \hline x & x^2 & 4x \\ \hline -2 & 2x & -4 \\ \hline \end{array} \quad x^2=4$$

$$7. \frac{1}{x-2} + \frac{x+2}{x+5} = \frac{3}{x^2+3x-10}$$

F1:  $x+5$   
 F2:  $x-2$   
 LCD:  $(x+5)(x-2)$

$$\frac{1}{x-2} + \frac{x+2}{x+5} = \frac{3}{(x+5)(x-2)}$$

$$1(x+5) + (x+2)(x-2) = 3 \quad (x+2)(x-1) = 0$$

$$x+5 + x^2 - 4 = 3$$

$$x^2 + x + 1 = 3$$

$$x^2 + x - 2 = 0$$

$x = -2, 1$   
 No extraneous solutions

$$\begin{array}{|c|c|c|} \hline & x & +1 \\ \hline x & x^2 & 4x \\ \hline 3 & 3x & +3 \\ \hline \end{array} \quad x^2+4x+3$$

$$8. \frac{x+1}{x+5} + \frac{18}{x^2+8x+15} = \frac{9}{x+3}$$

F1:  $x+5$   
 F2:  $x+3$   
 LCD:  $(x+5)(x+3)$

$$\frac{x+1}{x+5} + \frac{18}{(x+5)(x+3)} = \frac{9}{x+3}$$

$$(x+1)(x+3) + 18 = 9(x+5)$$

$$x^2 + 4x + 21 = 9x + 45$$

$$x^2 - 5x - 24 = 0$$

$(x-8)(x+3) = 0$   
 $x = 8, x = -3$  reject  
 ESS  
 -3 is an extraneous solution

$$9. \frac{2}{x+3} - \frac{3}{4-x} = \frac{2x-2}{x^2-x-12}$$

F1:  $x+3$   
 F2:  $(x-4)$   
 LCD:  $(x+3)(x-4)$

$$\frac{2}{x+3} - \frac{3}{4-x} = \frac{2x-2}{(x+3)(x-4)}$$

$$2(x-4) - 3(x+3) = 2x-2$$

$$2x - 8 + 3 + 9 = 2x - 2$$

$$5x + 1 = 2x - 2$$

$3x + 4 = -2$  ESS  
 $3x = -6$   
 $x = -2$   
 No extraneous solutions

$$10. \frac{1}{x+3} - \frac{4}{3-x} = \frac{14}{x^2-9}$$

F1:  $x+3$   
 F2:  $x-3$

$$\frac{1}{x+3} - \frac{4}{3-x} = \frac{14}{(x+3)(x-3)}$$

$$1(x-3) - 4(x+3) = 14$$

$$x - 3 + 4x + 12 = 14$$

$$5x + 9 = 14$$

$5x = 5$   
 $x = 1$   
 No extraneous solutions

$$11. \text{Solve for } x \text{ in simplest radical form: } \frac{6}{x} + \frac{x}{x-7} = \frac{12}{x^2-7x}$$

F1:  $x$   
 F2:  $x-7$   
 LCD:  $x(x-7)$

$$\sqrt{252} = \sqrt{36 \cdot 7} = 6\sqrt{7}$$

$$\frac{6}{x} + \frac{x}{x-7} = \frac{12}{x(x-7)}$$

$$6(x-7) + x^2 = 12$$

$$6x - 42 + x^2 = 12$$

$$x^2 + 6x - 54 = 0$$

$a=1$   
 $b=6$   
 $c=-54$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-54)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{252}}{2}$$

$$x = \frac{-6 \pm 6\sqrt{7}}{2}$$

$$x = -3 \pm 3\sqrt{7}$$

$$12. \text{Solve for } x \text{ in simplest radical form: } \frac{x}{x-5} - \frac{4}{x} = \frac{28}{x^2-5x}$$

F1:  $x$   
 F2:  $x-5$   
 LCD:  $x(x-5)$

$$\sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$$

$$\frac{x}{x-5} - \frac{4}{x} = \frac{28}{x(x-5)}$$

$$x^2 - 4(x-5) = 28$$

$$x^2 - 4x + 20 = 28$$

$$x^2 - 4x - 8 = 0$$

$a=1$   
 $b=-4$   
 $c=-8$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{48}}{2}$$

$$x = \frac{4 \pm 4\sqrt{3}}{2}$$

$$x = 2 \pm 2\sqrt{3}$$





13. Which of the following is true based on the equation  $\frac{x}{x+3} + \frac{2}{x+1} = \frac{6}{x^2+4x+3}$ ?

F1:  $x+3$   
 F2:  $x+1$   
 LCD:  $(x+3)(x+1)$

- 1) -3 is an extraneous solution
- 2) -1 is an extraneous solution
- 3) -3 and -1 are extraneous solutions
- 4) -3 and 0 are extraneous solutions

$$\frac{x}{x+3} + \frac{2}{x+1} = \frac{6}{(x+3)(x+1)}$$

$$x(x+1) + 2(x+3) = 6$$

$$x^2 + x + 2x + 6 = 6$$

$$x^2 + 3x + 6 = 6$$

$$x^2 + 3x = 0$$

$$x(x+3) = 0$$

$$x=0 \quad x=-3 \text{ reject}$$

-3 is extraneous solution

14. To solve  $\frac{2x}{x-2} - \frac{11}{x} = \frac{8}{x^2-2x}$ , Ren multiplied both sides by the least common denominator.

Which statement is true?

- 1) 2 is an extraneous solution.
- 2)  $\frac{7}{2}$  is an extraneous solution.
- 3) 0 and 2 are extraneous solutions.
- 4) This equation does not contain any extraneous solutions.

F1:  $x$   
 F2:  $x-2$   
 LCD:  $x(x-2)$

$$\frac{2x}{x-2} - \frac{11}{x} = \frac{8}{x(x-2)}$$

$$2x^2 - 11(x-2) = 8$$

$$2x^2 - 11x + 22 = 8$$

$$2x^2 - 11x + 14 = 0$$

$$x^2 - 11x + 28 = 0$$

$$(x-7)(x-4) = 0$$

$$(2x-7)(x-2) = 0$$

$$2x-7=0 \quad x=2 \text{ reject}$$

$$\frac{2x}{2} = \frac{7}{2}$$

$$x = \frac{7}{2}$$

