Fractional Equations: MULTIPLY BY THE LCD

To find a common denominator:

1) Factor (if necessary)

2) Put all of your factors together

LCD: $3x$

1. $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$3 - x = -1$

$-3 \qquad -3$

$\frac{-x}{-1} = \frac{-4}{-1}$

$x = 4$

LCD: $4x$

2. $\frac{5x}{2} = \frac{1}{x} + \frac{x}{4}$

$2x(5x) = 4 + x^2$

$10x^2 = 4 + x^2$

$-x^2 - 4 \quad -4 - x^2$

DOTS $\quad 9x^2 - 4 = 0$

$(3x+2)(3x-2) = 0$

$3x+2=0 \qquad 3x-2=0$

$-2 \quad -2 \qquad +2 \quad +2$

$\frac{3x}{3} = \frac{-2}{3} \qquad \frac{3x}{3} = \frac{2}{3}$

$x = \frac{-2}{3} \qquad x = \frac{2}{3}$

LCD: $3x$

3. $\frac{x+2}{x} + \frac{x}{3} = \frac{2x^2+6}{3x}$

$3(x+2) + x^2 = 2x^2 + 6$

$3x + 6 + x^2 = 2x^2 + 6$

$-3x - 6 \ -x^2 \quad -x^2 - 3x - 6$

GCF $\quad 0 = x^2 - 3x$

$0 = x(x-3)$

$\cancel{x=0}$ reject $\qquad x - 3 = 0$

$+3 \quad +3$

$x = 3$

LCD: $6x$

4. $\frac{1}{2x} - \frac{5}{6} = \frac{3}{x}$

$3 - 5x = 18$

$-3 \qquad -3$

$\frac{-5x}{-5} = \frac{15}{-5}$

$x = -3$

LCD: $x(x+2)$

5. $\frac{3}{x} + \frac{x}{x+2} = -\frac{2}{x+2}$

$3(x+2) + x^2 = -2x$

$3x + 6 + x^2 = -2x$

$+2x \qquad +2x$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x+3=0 \qquad x+2=0$

$-3 \quad -3 \qquad -2 \quad -2$

$x = -3 \qquad \cancel{x=-2}$ reject

LCD: $x(x-1)$

6. $\frac{x}{x-1} = \frac{2}{x} + \frac{1}{x-1}$

$x^2 = 2(x-1) + x$

$x^2 = 2x - 2 + x$

$x^2 = 3x - 2$

$-3x+2 \quad -3x+2$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x-2=0 \qquad x-1=0$

$+2 \quad +2 \qquad +1 \quad +1$

$x = 2 \qquad \cancel{x=1}$ reject

LCD: $x(x+7)$

7. $\frac{3x+25}{x+7} - 5 = \frac{3}{x}$

$x(3x+25) - 5x(x+7) = 3(x+7)$

$3x^2+25x-5x^2-35x = 3x+21$

$-2x^2-10x = 3x+21$

$+2x^2+10x \quad +2x^2+10x$

$0 = 2x^2+13x+21$

$x^2+13x+42 \rightarrow \frac{(x+7)(x+6)}{2 \quad 2}$

$0 = (2x+7)(x+3)$

$2x+7=0 \qquad x+3=0$

$-7 \;\; -7 \qquad -3 \;\; -3$

$\frac{2x}{2} = \frac{-7}{2} \qquad x=-3$

$x = -\frac{7}{2}$

LCD: $x(x+5)$

8. $\frac{8}{x+5} - \frac{3}{x} = 5$

$8x - 3(x+5) = 5x(x+5)$

$8x-3x-15 = 5x^2+25x$

$5x-15 = 5x^2+25x$

$-5x+15 \quad -5x+15$

$\frac{0}{5} = \frac{5x^2+20x+15}{5}$

$0 = x^2+4x+3$

$0 = (x+3)(x+1)$

$x+3=0 \qquad x+1=0$

$-3 \;\; -3 \qquad -1 \;\; -1$

$x=-3 \qquad x=-1$

LCD: $4x(x+1)$

9. $\frac{7}{2x} - \frac{2}{x+1} = \frac{1}{4}$

$14(x+1) - 8x = x(x+1)$

$14x+14-8x = x^2+1x$

$6x+14 = x^2+1x$

$-6x-14 \quad -6x-14$

$0 = x^2-5x-14$

$0 = (x-7)(x+2)$

$x-7=0 \qquad x+2=0$

$+7 \;\; +7 \qquad -2 \;\; -2$

$x=7 \qquad x=-2$

	x	$+3$
x	x^2	$+3x$
$+2$	$+2x$	$+6$

10. $\frac{2}{n^2} + \frac{3}{n} = \frac{4}{n^2}$

$2+3n=4$

$-2 \quad -2$

$\frac{3n}{3} = \frac{2}{3}$

$n = \frac{2}{3}$

	x	$+3$
x	x^2	$+3x$
-3	$-3x$	-9

LCD: $(x-5)(x+2)$

11. $\frac{x+3}{x-5} + \frac{6}{x+2} = \frac{6+10x}{(x-5)(x+2)}$

$(x+3)(x+2) + 6(x-5) = 6+10x$

$x^2+5x+6+6x-30 = 6+10x$

$x^2+11x-24 = 6+10x$

$-10x-6 \quad -6-10x$

$x^2+x-30=0$

$(x+6)(x-5)=0$

$x+6=0 \qquad x-5=0$

$-6 \;\; -6 \qquad +5 \;\; +5$

$x=-6 \qquad \cancel{x=5}$ reject

LCD: $(x+3)(x-3)$

12. $\frac{30}{x^2-9} + 1 = \frac{5}{x-3}$

$30 + (x+3)(x-3) = 5(x+3)$

$30+x^2-9 = 5x+15$

$x^2+21 = 5x+15$

$-5x-15 \quad -5x-15$

$x^2-5x+6=0$

$(x-3)(x-2)=0$

$x-3=0 \qquad x-2=0$

$+3 \;\; +3 \qquad +2 \;\; +2$

$\cancel{x=3}$ reject $\qquad x=2$

LCD: $(x+5)(x-2)$

LCD: $(x-4)(x+3)$

13. $\frac{1}{x-2}+\frac{4}{x+5}=\frac{7}{x^2+3x-10}$

$(x+5)(x-2)\left(\frac{1}{x-2}\right)+(x+5)(x-2)\left(\frac{4}{x+5}\right)=(x+5)(x-2)\left(\frac{7}{(x+5)(x-2)}\right)$

$1(x+5)+4(x-2)=7$

$1x+5+4x-8=7$

$5x-3=7$

$+3 \quad +3$

$5x=10$

$\frac{5x}{5}=\frac{10}{5}$

$\cancel{x=2}$ reject

No solution

14. $\frac{x}{x+3}+\frac{2}{x-4}=\frac{2x+27}{x^2-x-12}$

$(x-4)(x+3)\left(\frac{x}{x+3}\right)+(x-4)(x+3)\left(\frac{2}{x-4}\right)=(x-4)(x+3)\left(\frac{2x+27}{(x-4)(x+3)}\right)$

$x(x-4)+2(x+3)=2x+27$

$x^2-4x+2x+6=2x+27$

$x^2-2x+6=2x+27$

$-2x-27 \quad -2x-27$

$x^2-4x-21=0$

$(x-7)(x+3)=0$

$x-7=0$ | $x+3=0$

$+7 \; +7$ | $-3 \; -3$

$x=7$ | $\cancel{x=-3}$ reject

LCD $(x-6)(x-2)$

15. $\frac{1}{x-6}+\frac{x}{x-2}=\frac{4}{x^2-8x+12}$

$(x-6)(x-2)\left(\frac{1}{x-6}\right)+(x-6)(x-2)\left(\frac{x}{x-2}\right)=(x-6)(x-2)\left(\frac{4}{(x-6)(x-2)}\right)$

$1(x-2)+x(x-6)=4$

$1x-2+x^2-6x=4$

$x^2-5x-2=4$

$-4 \quad -4$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$x-6=0$ | $x+1=0$

$+6 \; +6$ | $-1 \; -1$

$\cancel{x=6}$ reject | $x=-1$

16. $\frac{4}{k^2-8k+12}=\frac{k}{k-2}+\frac{1}{k-6}$

$\frac{4}{(k-6)(k-2)}=\frac{k}{k-2}+\frac{1}{k-6}$

It's the same question!

LCD $(x+2)(x-2)$

17. $\frac{x}{x+2}+\frac{1}{x^2-4}=\frac{4}{x-2}$

$(x+2)(x-2)\left(\frac{x}{x+2}\right)+(x+2)(x-2)\left(\frac{1}{(x+2)(x-2)}\right)=(x+2)(x-2)\left(\frac{4}{x-2}\right)$

$x(x-2)+1=4(x+2)$

$x^2-2x+1=4x+8$

$-4x-8 \quad -4x-8$

$x^2-6x-7=0$

$(x-7)(x+1)=0$

$x-7=0$ | $x+1=0$

$+7 \; +7$ | $-1 \; -1$

$x=7$ | $x=-1$

LCD $(x+3)(x-4)$

18. $\frac{2}{x+3}-\frac{3}{4-x}=\frac{2x-2}{x^2-x-12}$

(-1)

$(x+3)(x-4)\frac{2}{x+3}-(x+3)(x-4)\frac{3}{4-x}=(x+3)(x-4)\frac{2x-2}{(x-4)(x+3)}$

$2(x-4)+3(x+3)=2x-2$

$2x-8+3x+9=2x-2$

$5x+1=2x-2$

$-2x \quad -2x$

$3x+1=-2$

$-1 \quad -1$

$\frac{3x}{3}=\frac{-3}{3}$

$x=-1$

19. Markus is a long-distance walker. In one race, he walked 55 miles in t hours and in another race walked 65 miles in $t+3$ hours. His rates are shown in the equations below.

$$r = \frac{55}{t} \quad r = \frac{65}{t+3}$$

Markus walked at an equivalent rate, r, for each race. Determine the number of hours that each of the two races took.

LCD: $t(t+3)$

$\cancel{t}\cancel{(t+3)}\left(\frac{65}{\cancel{t+3}}\right) = \left(\frac{55}{\cancel{t}}\right)\cancel{t}(t+3)$

$65t = 55(t+3)$

$65t = 55t + 165$

$-55t \quad -55t$

$\frac{10t}{10} = \frac{165}{10}$

$t = 16.5$

$t = 16.5$

$t+3 = 16.5+3 = 19.5$

16.5 and 19.5

20. Sarah is fighting a sinus infection. Her doctor prescribed a nasal spray and an antibiotic to fight the infection. The active ingredients, in milligrams, remaining in the bloodstream from the nasal spray, $n(t)$, and the antibiotic, $a(t)$, are modeled in the functions below, where t is the time in hours since the medications were taken.

$$n(t) = \frac{t+1}{t+5} + \frac{18}{t^2 + 8t + 15}$$

$$a(t) = \frac{9}{t+3}$$

$t=0$

Determine which drug is made with a greater initial amount of active ingredient. Justify your answer. Sarah's doctor told her to take both drugs at the same time. Determine algebraically the number of hours after taking the medications when both medications will have the same amount of active ingredient remaining in her bloodstream.

set them equal

$n(0) = \frac{0+1}{0+5} + \frac{18}{(0)^2+8(0)+15} = \frac{7}{5}$

$a(0) = \frac{9}{0+3} = 3$

antibiotic has greater initial amount of active ingredient

	t	$+3$
t	t^2	$+3t$
$+1$	$+1t$	$+3$

t^2+4t+3

$\frac{t+1}{t+5} + \frac{18}{t^2+8t+15} = \frac{9}{t+3}$ LCD $(t+5)(t+3)$

$\cancel{(t+5)}(t+3)\left(\frac{t+1}{\cancel{t+5}}\right) + \cancel{(t+5)(t+3)}\left(\frac{18}{\cancel{(t+5)(t+3)}}\right) = \left(\frac{9}{\cancel{t+3}}\right)(t+5)\cancel{(t+3)}$

$(t+3)(t+1) + 18 = 9(t+5)$

$t^2+4t+3+18 = 9t+45$

$t^2+4t+21 = 9t+45$

$-9t-45 \quad -9t-45$

$t^2-5t-24=0$

$(t-8)(t+3)=0$

$t-8=0 \qquad t+3=0$

$+8 \; +8 \qquad -3 \; -3$

$t=8 \qquad \cancel{t=-3}$ reject

8 hours

extraneous solution means it's a reject!

LCD. $(x+7)(x-7)$

$(x+7)(x-7)\quad (x+7)(x-7)\quad (x+7)(x-7)$

21. To solve the equation $\frac{7}{x+7}+\frac{4x}{x-7}=\frac{3x+7}{x-7}$, Joan's first step is to multiply both sides by the least common denominator. Which statement is true?

1) -14 is an extraneous solution.
2) 7 and -7 are extraneous solutions.
3) 7 is an extraneous solution.
4) There are no extraneous solutions.

$7(x-7)+4x(x+7)=(3x+7)(x+7)$ → at this point, you can use y1, y2, intersect

$7x-49+4x^2+28x=3x^2+28x+49$

$4x^2+35x-49=3x^2+28x+49$

$-3x^2-28x-49 \quad -3x^2-28x-49$

$x^2+7x-98=0$

$(x+14)(x-7)=0$

$x+14=0 \quad x-7=0$

$-14 \; -14 \quad +7 \; +7$

$x=-14$ ✓ $\quad$ ~~$x=7$~~ reject

	$3x$	$+7$
x	$3x^2$	$+7x$
$+7$	$+21x$	$+49$

$3x^2+28x+49$

LCD $x(x-2)$

22. To solve $\frac{2x}{x-2}-\frac{11}{x}=\frac{8}{x^2-2x}$, Ren multiplied both sides by the least common denominator. Which statement is true?

1) 2 is an extraneous solution.
2) $\frac{7}{2}$ is an extraneous solution.
3) 0 and 2 are extraneous solutions.
4) This equation does not contain any extraneous solutions.

$\left(\frac{2x}{x-2}\right)x(x-2)-\left(\frac{11}{x}\right)x(x-2)=\left(\frac{8}{x(x-2)}\right)x(x-2)$

$2x^2-11(x-2)=8$

$2x^2-11x+22=8$

$-8 \quad -8$

$2x^2-11x+14=0$

$x^2-11x+28$

$\left(x-\frac{7}{2}\right)\left(x-\frac{4}{2}\right)$

$(2x-7)(x-2)=0$

$2x-7=0 \quad x-2=0$

$+7 \; +7 \quad +2 \; +2$

$\frac{2x}{2}=\frac{7}{2}$ $\quad$ ~~$x=2$~~ reject

$x=\frac{7}{2}$ ✓

23. Jin solved the equation $\left(\sqrt{4-x}\right)^2=(x+8)^2$ by squaring both sides. What extraneous solution did he find?

1) -5
2) -12
3) 3
4) 4

$4-x=(x+8)(x+8)$

$4-x=x^2+16x+64$

$-4+x \qquad +x-4$

$0=x^2+17x+60$

$0=(x+12)(x+5)$

$x+12=0 \quad x+5=0$

$-12 \; -12 \quad -5 \; -5$

~~$x=-12$~~ reject $\quad x=-5$ ✓

	x	$+8$
x	x^2	$+8x$
$+8$	$+8x$	$+64$

$x^2+16x+64$