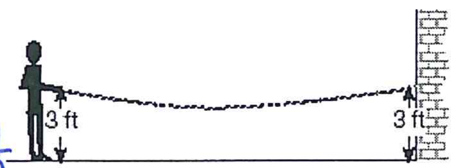
Name Schlansky
Mr. Schlansky

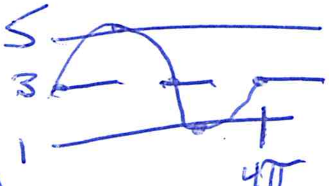
Date ____________
Algebra I

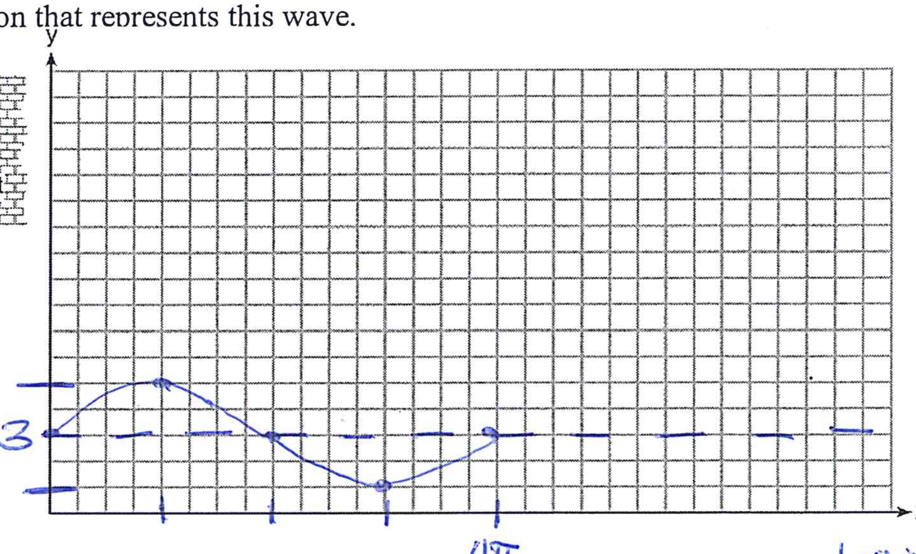
Graphing Sinusoidal Models

1. A student attaches one end of a rope to a wall at a fixed point 3 feet above the ground, as shown in the accompanying diagram, and moves the other end of the rope up and down, producing a wave described by the equation $y = a \sin bx + c$. The range of the rope's height above the ground is between 1 and 5 feet. The period of the wave is 4π. Graph one full wavelength and write the equation that represents this wave.

amp = 2
+sin
$f = \frac{2\pi}{p} = \frac{2\pi}{4\pi} = \frac{1}{2}$
shift = 3



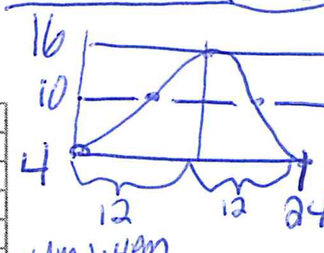




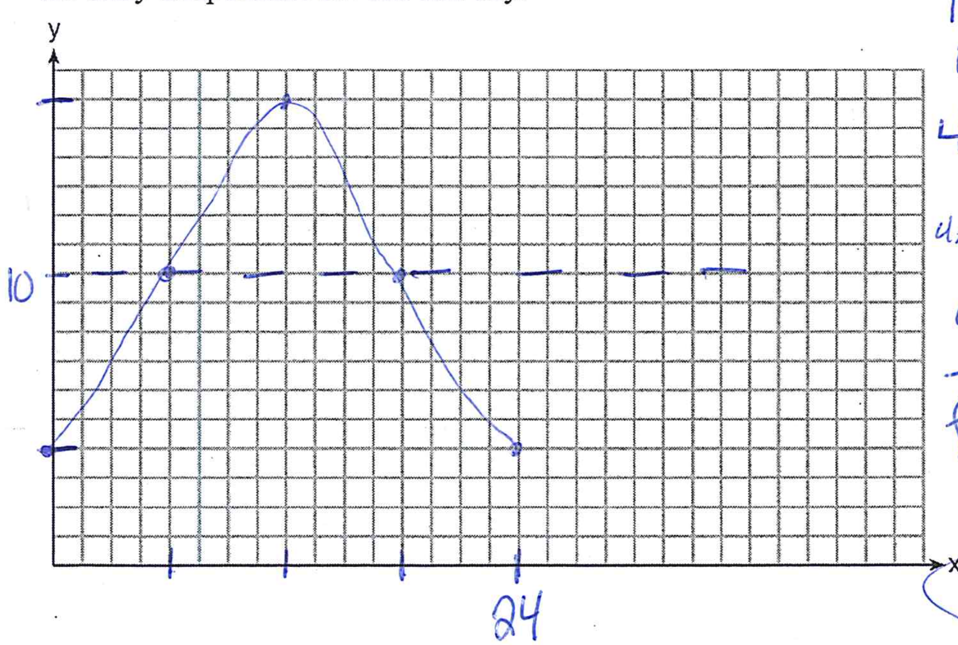
$y = 2\sin\frac{1}{2}x + 3$

2. In Johannesburg in June, the daily low temperature is usually around 4 degrees Celcius at 4AM, and the daily high temperature is around 16 degrees Celcius at 4PM. Write the equation of a sinusoidal function that models the temperature T in Johannesburg t hours after 4AM. Graph the daily temperature for one full day.

t=0 is 4Am
4Am is min
−cos



$mid = \frac{min+max}{2}$
$mid = \frac{4+16}{2}$
mid = 10

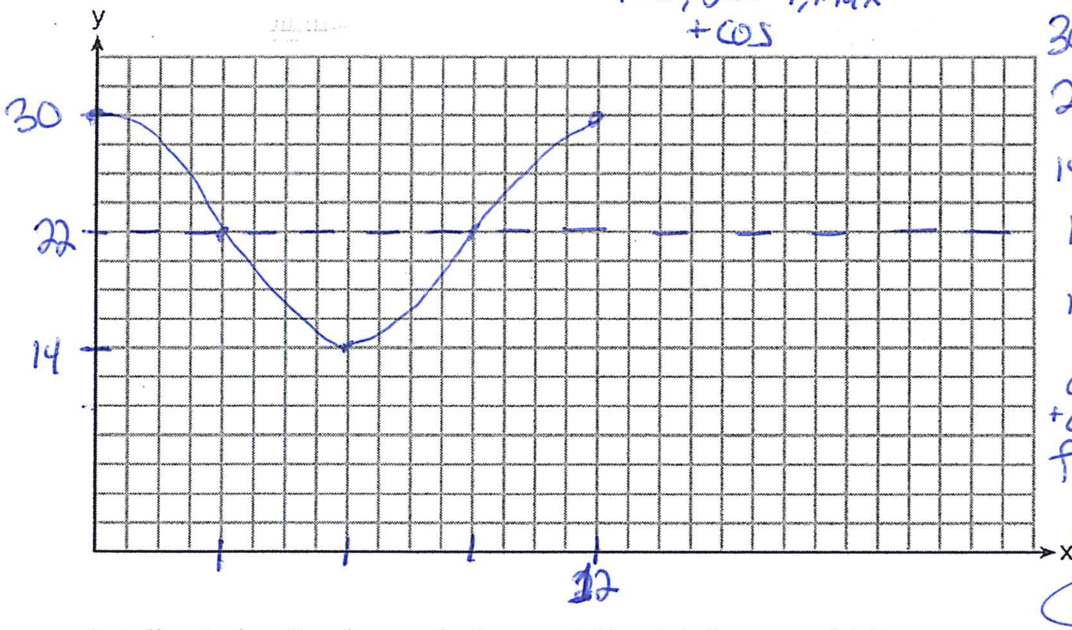


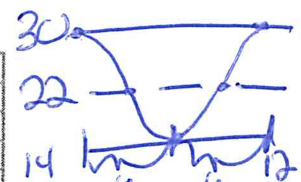
amp = 6
−cos
$freq = \frac{2\pi}{P} = \frac{2\pi}{24} = \frac{\pi}{12}$
shift = 10

$y = -6\cos\frac{\pi}{12}t + 10$

3. The hottest day of the year in Santiago, Chile, on average is January 7 when the average high temperature is 30 degrees Celcius. Six months later, the coolest day of the year has an average high temperature of 14 degrees Celcius. Use a trigonometric function to model the temperature in Santiago, Chile, where t represents months since January 7. Graph your trigonometric function on the grid below.

t=0, Jan 7, max
+cos





$mid = \frac{min + max}{2}$

$mid = \frac{14+30}{2} = \frac{44}{2} = 22$

amp = 8

+cos

$freq = \frac{2\pi}{P} = \frac{2\pi}{12} = \frac{\pi}{6}$

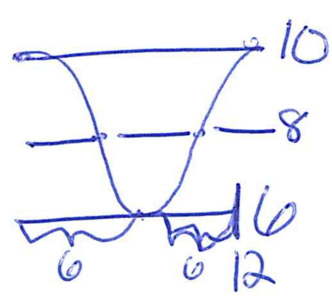
shift = 22

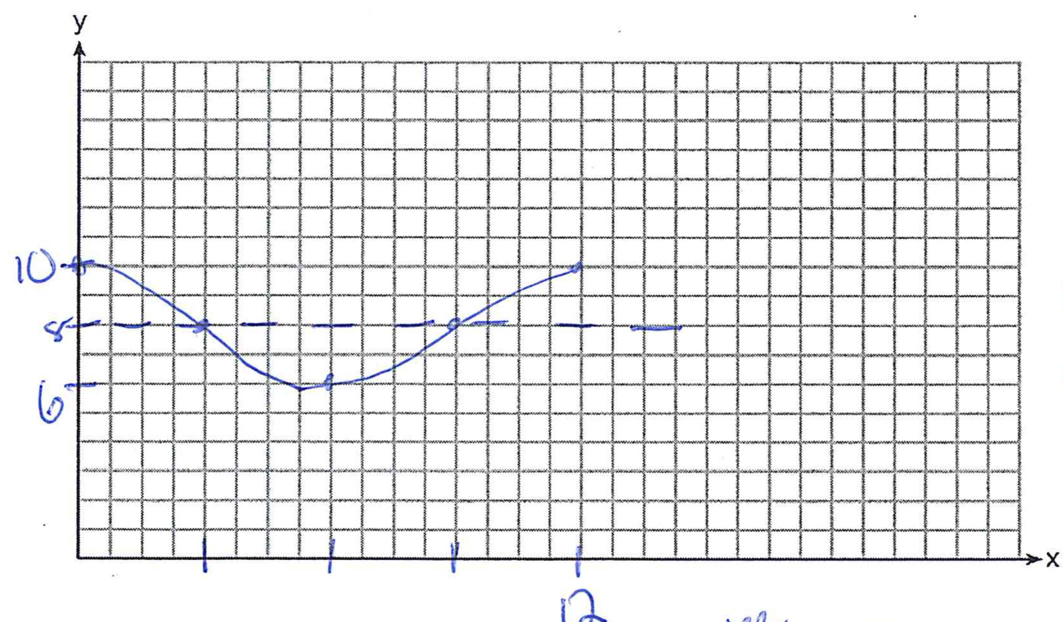
$y = 8\cos\frac{\pi}{6}t + 22$

4. Aditya's dog Sparky routinely eats Aditya's leftovers, which vary seasonally. As a result, his weight fluctuates throughout the year.

The dog's weight $W(t)$ as a function of time t (in months) over the course of a year can be modeled by a sinusoidal expression of the form $a\cos(bt)+d$. At $t=0$ months, the start of the year, he is at his maximum weight of 10 kg. Six months later, when $t=6$ months, he is at his minimum weight of 6.0 kg. Write an equation for $W(t)$, the weight of Sparky after t months. Sketch the graph of Sparky's weight over one full year.

t=0 max
+cos





$mid = \frac{min + max}{2}$

$mid = \frac{6+10}{2} = 8$

amp = 2

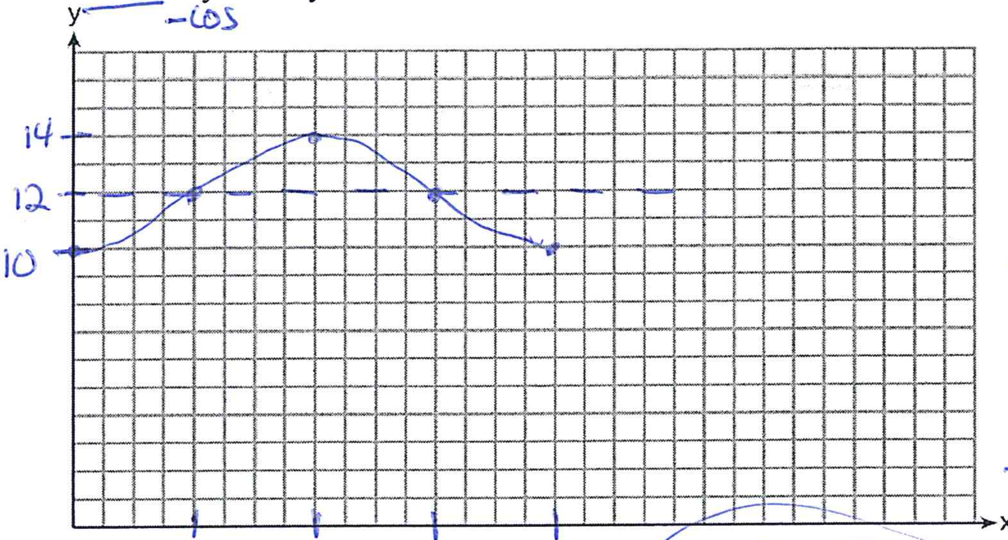
+cos

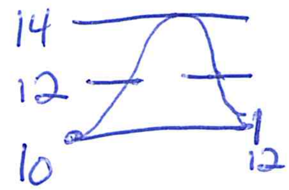
$freq = \frac{2\pi}{P} = \frac{2\pi}{12} = \frac{\pi}{6}$

shift = 8

$W(t) = 2\cos\frac{\pi}{6}t + 8$

5. The shortest day of the year in Town A is 10 hours and the longest day of the year is 14 hours. Create a cosine function to represent this situation. Graph the cosine function on the grid below to represent the length of the days in Town A for one full year. Let t represents months since the shortest day of the year.





mid = (min + max)/2

mid = (10 + 14)/2 = 12

amp = 2

−cos

freq = 2π/p = 2π/12 = π/6

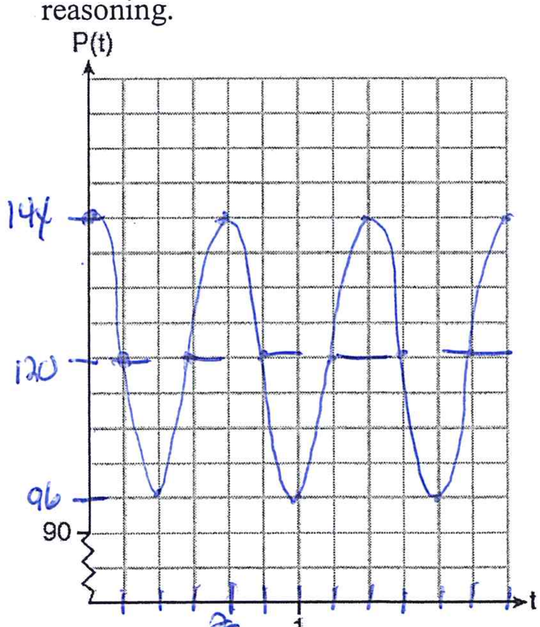
shift = 12

$y = -2\cos\frac{\pi}{6}t + 12$

6. The resting blood pressure of an adult patient can be modeled by the function P below, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \le t \le 2$.
Determine the period of P. Explain what this value represents in the given context. Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.



P(t) = 24 cos(3πt) + 120

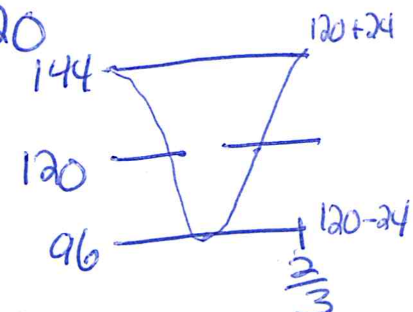
amp = 24

+cos

freq = 3π

shift = 120

~~f~~ p = 2π/f = 2π/3π = 2/3



Period = 2/3. It takes 2/3 of a second for the patient's blood pressure to cycle from its max to its min back to its max.

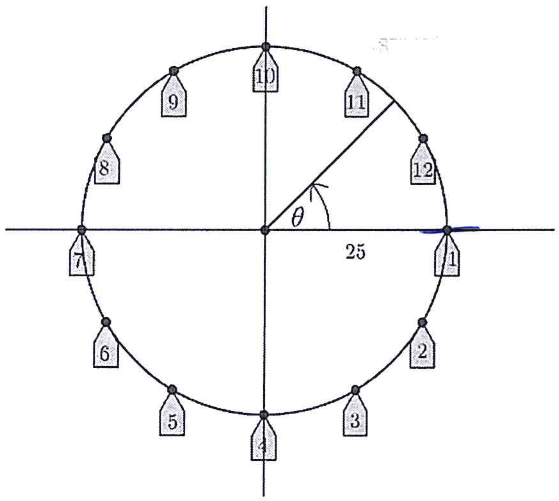
High blood pressure. The max and min are above 140 and 90.

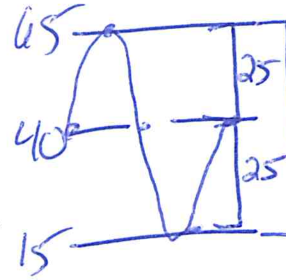
*I made my y scale 6 since my min/mid/max are multiples of 6

*x scale is 1/6
2/3 = 4/6

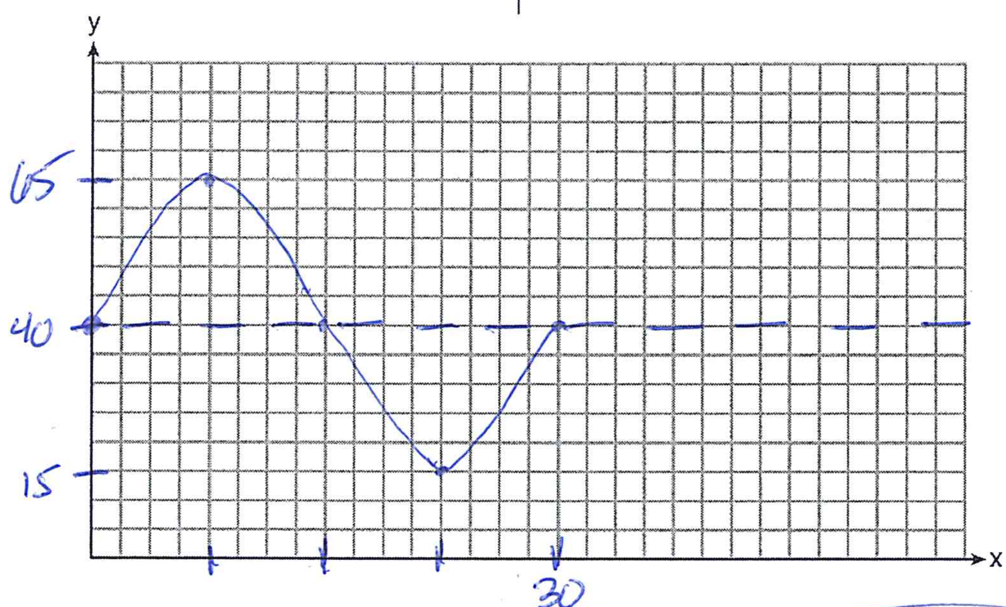
7. A carnival has a Ferris wheel that is 50 feet in diameter with 12 passenger cars. When viewed from the side where passengers board, the Ferris wheel rotates counterclockwise and makes two full turns each minute. Riders board the Ferris wheel from a platform that is 40 feet above the ground. Create a graph of the height of car 1 as a function of time in seconds. Write an equation for your graph.

They board at the midline = sin.
Counterclockwise goes up first = +sin





2 full turns in 60 seconds
1 full turn in 30 seconds
period



amp=25
+sin
$freq = \frac{2\pi}{p} = \frac{2\pi}{30} = \frac{\pi}{15}$
shift = 40

$y = 25\sin\frac{\pi}{15}x + 40$

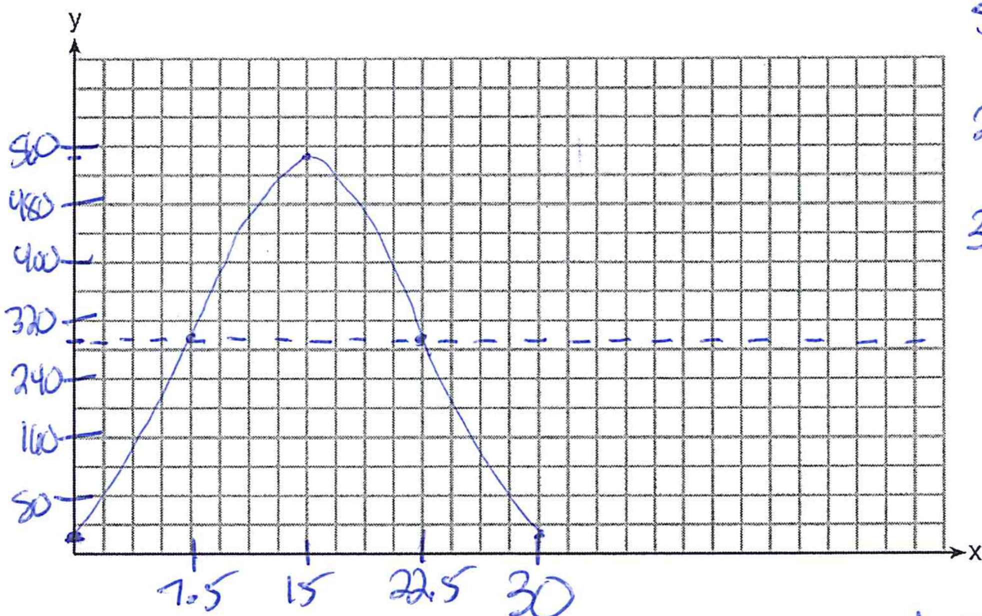
8. The High Roller, a Ferris wheel in Las Vegas, Nevada, opened in March 2014. The 550 ft. tall wheel has a diameter of 520 feet. A ride on one of its 28 passenger cars lasts 30 minutes, the time it takes the wheel to complete one full rotation. Riders board the passenger cars at the bottom of the wheel. Assume that once the wheel is in motion, it maintains a constant speed for the 30-minute ride and is rotating in a counterclockwise direction.

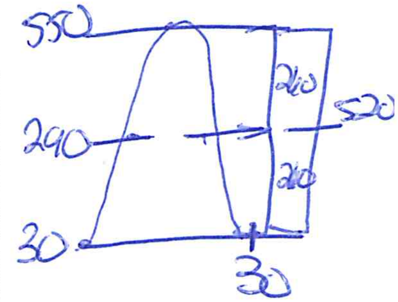
max

period

$-\cos$

a) Sketch a graph of the height of a passenger car on the High Roller as a function of the time the ride began.
b) Write a sinusoidal function H that represents the height of a passenger car t minutes after the ride begins.
c) Identify the amplitude, midline, frequency, and period and explain how they relate to the situation.
d) If you were on this ride, how high would you be above the ground after 22.5 minutes?





Scale ≥ $\frac{\text{max}}{\text{\#of boxes}}$

Scale ≥ $\frac{550}{17}$

Scale ≥ 32

Scale = 40

amp = 260

$-\cos$

freq $= \frac{2\pi}{P} = \frac{2\pi}{30} = \frac{\pi}{15}$

Shift = 290

$H = -260\cos\frac{\pi}{15}t + 290$

period = 30

scale $= \frac{P}{4}$

scale $= \frac{30}{4}$

Scale = 7.5

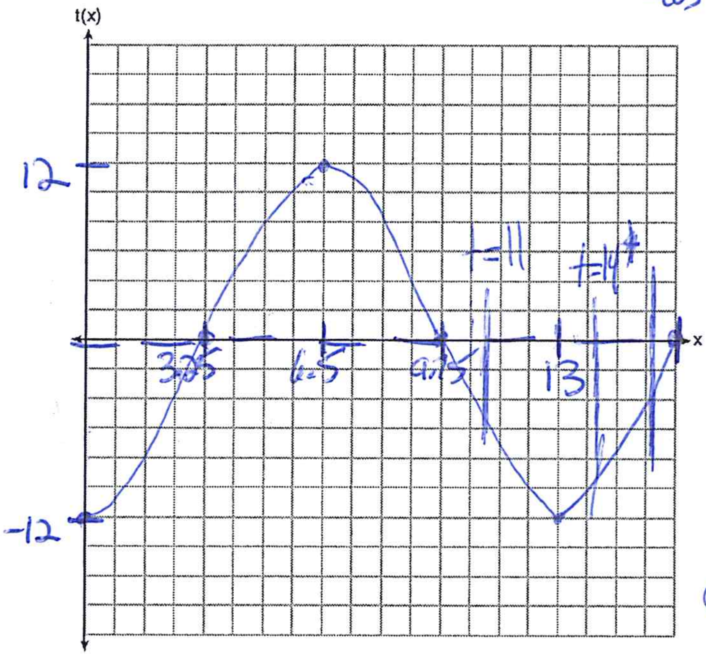
$H(22.5) = 290$

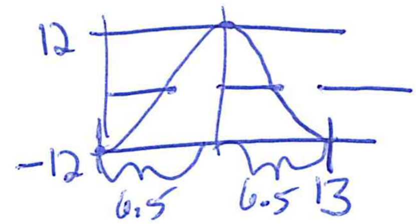
290 feet

9. The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above the average local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively. Write a cosine function of the form $f(t) = A\cos(Bt)$, where A and B are real numbers, that models the water level, $f(t)$, in inches above or below the average Carter Beach sea level, as a function of the time measured in t hours since 8:30 a.m. On the grid below, graph one cycle of this function.

6. Show

min
−cos





$mid = \frac{min + max}{2}$

$mid = \frac{-12+12}{2} = 0$

$scale = \frac{p}{4} = \frac{13}{4} = 3.25$

amp = 12

−cos

$freq = \frac{2\pi}{p} = \frac{2\pi}{13}$

Shift = 0

$f(t) = -12\cos\frac{2\pi}{13}t$

People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.

t = 11
decreasing

t = 14
increasing

10:30 PM because it is increasing then.