1) Isolate/Combine logs (put coefficient to exponent)
2) Convert to exponential form (cycle)
3) Solve equation
4) Check (You can't take the log of a negative or 0)

Name Schlansky
Mr. Schlansky

Date
Algebra II

Logarithmic Equations

Solve the following logarithmic equations

1. $\log_5 x = 4$

$5^4 = x$

$625 = x$

2. $\log_b 27 = 3$

$(b^3)^{\frac{1}{3}} = (27)^{\frac{1}{3}}$

$b = 3$

3. $\log_y 64 = 2$

$(y^2)^{\frac{1}{2}} = (64)^{\frac{1}{2}}$

$y = 8$

4. $\log_{(x+1)} 64 = 3$

$((x+1)^3)^{\frac{1}{3}} = (64)^{\frac{1}{3}}$

$x+1 = 4$

$-1 \quad -1$

$x = 3$

5. $\log_{(5x-1)} 4 = \frac{1}{3}$

$((5x-1)^{\frac{1}{3}})^3 = (4)^3$

$5x-1 = 64$

$+1 \quad +1$

$5x = 65$

$\frac{5x}{5} = \frac{65}{5}$

$x = 13$

6. $\log_{(x+4)}(17x-4) = 2$

$(x+4)^2 = 17x-4$

$x^2+8x+16 = 17x-4$

$-17x+4 \quad -17x+4$

$x^2-9x+20 = 0$

$(x-5)(x-4) = 0$

$x=5 \quad x=4$

	x	$+4$
x	x^2	$+4x$
$+4$	$+4x$	$+16$

$x^2+8x+16$

7. $\log_{27}(2x-1) = \frac{4}{3}$

$27^{\frac{4}{3}} = 2x-1$

$81 = 2x-1$

$+1 \quad +1$

$\frac{82}{2} = \frac{2x}{2}$

$41 = x$

8. $2\log_4(5x) = 3$

$\log_4(5x)^2 = 3$

$4^3 = (5x)^2$

$\frac{64}{25} = \frac{25x^2}{25}$

$\sqrt{\frac{64}{25}} = \sqrt{x^2}$

$\frac{8}{5} = x$

9. $2\log_4(2x) = 1$

$\log_4(2x)^2 = 1$

$4^1 = 4x^2$

$\frac{4}{4} = \frac{4x^2}{4}$

$\sqrt{1} = \sqrt{x^2}$

$1 = x$

10. $3\log_2(2x) = 5$

$\log_2(2x)^3 = 5$

$2^5 = (2x)^3$

$\frac{32}{8} = \frac{8x^3}{8}$

$(4)^{\frac{1}{3}} = (x^3)^{\frac{1}{3}}$

$x = 4^{\frac{1}{3}}$

11. $\log 45x - \log 3 = 1$

Combine into a single log using log rules

$\log_{10} \frac{45x}{3} = 1$

$10^1 = \frac{45x}{3}$

$\frac{10}{15} = \frac{15x}{15}$

$\frac{2}{3} = x$

12. $\log_3 x + \log_3(x-8) = 2$

$\log_3 x(x-8) = 2$

$3^2 = x(x-8)$

$9 = x^2 - 8x$

$-9 \quad -9$

$0 = x^2 - 8x - 9$

$(x-9)(x+1)$

$x = 9$ | ~~$x = -1$~~

the answer can't make the log negative or zero.

13. $\log x + \log(x-3) = 1$

$\log_{10} x(x-3) = 1$

$10^1 = x(x-3)$

$10 = x^2 - 3x$

$-10 \quad -10$

$0 = x^2 - 3x - 10$

$0 = (x-5)(x+2)$

$x = 5$ | ~~$x = -2$~~

log can't be negative

14. $\log x + \log(x+9) = 1$

$\log_{10} x(x+9) = 1$

$10^1 = x(x+9)$

$10 = x^2 + 9x$

$-10 \quad -10$

$0 = x^2 + 9x - 10$

$(x+10)(x-1)$

~~$x = -10$~~ | $x = 1$

log can't be negative

15. $2\log_4 x - \log_4(x-1) = 1$

$\log_4 x^2 - \log_4 x - 1 = 1$

$\log_4 \frac{x^2}{x-1} = 1$

$4^1 = \frac{x^2}{x-1}$

Multiply by the LCD to get rid of fractions

$x-1\,(4) = \left(\frac{x^2}{x-1}\right) x-1$

$4(x-1) = x^2$

$4x - 4 = x^2$

$-4x+4 \quad -4x+4$

$0 = x^2 - 4x + 4$

$0 = (x-2)(x-2)$

$x = 2$ | $x = 2$

16. $\log\left(x + \frac{3}{10}\right) + \log x + 1 = 0$

$-1 \quad -1$

$\log\left(x + \frac{3}{10}\right) + \log x = -1$

$\log_{10} x\left(x + \frac{3}{10}\right) = -1$

$10^{-1} = x\left(x + \frac{3}{10}\right)$

$10\left(\frac{1}{10}\right)10 = (x^2) + \left(\frac{3x}{10}\right)10$

$1 = 10x^2 + 3x$

$-1 \quad -1 \quad -10$

$0 = 10x^2 + 3x - 1$

Tricky Trinomial

$0 = 10x^2 + 5x - 2x - 1$

$5x(2x+1) - 1(2x+1)$

$0 = (5x-1)(2x+1)$

$5x - 1 = 0$ | $2x + 1 = 0$

$+1 \quad +1$ | $-1 \quad -1$

$\frac{5x}{5} = \frac{1}{5}$ | $\frac{2x}{2} = \frac{-1}{2}$

$x = \frac{1}{5}$ | ~~$x = -\frac{1}{2}$~~

log can't be negative

17. $\log_{x+3} \dfrac{x^3 + x - 2}{x} = 2$

	x	$+3$
x	x^2	$+3x$
$+3$	$+3x$	$+9$

x^2+6x+9

$(x+3)^2 = \dfrac{x^3+x-2}{x}$

$x(x^2+6x+9) = \left(\dfrac{x^3+x-2}{x}\right)x$

$x^3+6x^2+9x = x^3+x-2$

$-x^3 \quad -x+2 \quad -x^3-x+2$

$6x^2+8x+2=0$

$\dfrac{6x^2}{2}+\dfrac{8x}{2}+\dfrac{2}{2}=\dfrac{0}{2}$

$3x^2+4x+1=0$

$3x^2+3x+1x+1=0$

$3x(x+1)+1(x+1)=0$

$(3x+1)(x+1)=0$

$3x+1=0 \quad x+1=0$

$-1 \;\; -1 \quad -1 \;\; -1$

$\dfrac{3x}{3}=\dfrac{-1}{3}$ $\quad x=-1$

$x=-\dfrac{1}{3}$

even though they're negative, they don't make the log negative

18. $\log_{16}(p^2 - p + 4) - \log_{16}(2p + 11) = \dfrac{3}{4}$

$\log_{16} \dfrac{p^2-p+4}{2p+11} = \dfrac{3}{4}$

$16^{\frac{3}{4}} = \dfrac{p^2-p+4}{2p+11}$

$2p+11\,(8) = \left(\dfrac{p^2-p+4}{2p+11}\right)2p+11$

$8(2p+11) = p^2-p+4$

$16p+88 = p^2-p+4$

$-16p-88 \quad -16p-88$

$p^2-17p-84=0$

$(p-21)(p+4)=0$

$p=21 \quad p=-4$

even though it's negative, it doesn't make the log negative

19. $\log_{(x+3)}(2x + 3) + \log_{(x+3)}(x + 5) = 2$

$\log_{x+3}(2x+3)(x+5) = 2$

	$2x$	$+3$
x	$2x^2$	$+3x$
$+5$	$+10x$	$+15$

$2x^2+13x+15$

$(x+3)^2 = (2x+3)(x+5)$

	x	$+3$
x	x^2	$+3x$
$+3$	$+3x$	$+9$

x^2+6x+9

$x^2+6x+9 = 2x^2+13x+15$

$-x^2-6x-9 \quad -x^2-6x-9$

$0 = x^2+7x+6$

$(x+6)(x+1)$

~~$x=-6$~~ $\quad x=-1$

makes the log negative $\quad$ doesn't make the log negative

20. $\log_2(x^2 - 7x + 12) - \log_2(2x - 10) = 3$ (nearest tenth)

$\log_2 \dfrac{x^2-7x+12}{2x-10} = 3$

$2^3 = \dfrac{x^2-7x+12}{2x-10}$

$2x-10\,(8) = \left(\dfrac{x^2-7x+12}{2x-10}\right)2x-10$

$8(2x-10) = x^2-7x+12$

$a=1$
$b=-23$
$c=92$

$16x-80 = x^2-7x+12$

$-16x+80 \quad -16x+80$

$0 = x^2-23x+92$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{23 \pm \sqrt{(-23)^2-4(1)(92)}}{2(1)}$

$x = \dfrac{23+\sqrt{161}}{2} = 17.8$

$x = \dfrac{23-\sqrt{161}}{2} = 5.2$