

Modeling Exponential Functions Review Sheet

If t represents years, find the yearly rate of increase/decrease for the following functions. Round to the nearest tenth of a percent.

1. $A = 38,000(0.987)^{12t}$

$A = 38,000(0.987^{12})^t$
 $A = 38,000(0.85468...)^t$
 $1 - 0.85468... = 0.145315... (100)$
 14.5% decrease

2. $A = 16,000(0.887)^{8.4t}$

$A = 16,000(0.887^{8.4})^t$
 $A = 16,000(0.3652...)^t$
 $1 - 0.3652... = 0.63477... (100)$
 63.5% decrease

3. $A = 9,200(1.985)^{\frac{t}{2}}$

$A = 9,200(1.985^{\frac{1}{2}})^t$
 $A = 9,200(1.4089...)^t$
 $1.4089... - 1 = 0.4089... (100)$
 40.9% increase

4. $A = 9,324(1.562)^{\frac{t}{5}}$

$A = 9,324(1.562^{\frac{1}{5}})^t$
 $A = 9,324(1.093...)^t$
 $1.093... - 1 = 0.093... (100) = 9.3%$
 increase

5. A study of black bears in the Adirondacks reveals that their population can be represented by the function $P(t) = 3500(1.025)^t$, where t is the number of years since the study began. Which function is correctly rewritten to reveal the monthly growth rate of the black bear population?

1) $P(t) = 3500(1.00206)^{12t}$

2) $P(t) = 3500(1.00206)^{\frac{t}{12}}$

Monthly rate 12 times per year

3) $P(t) = 3500(1.34489)^{12t}$

4) $P(t) = 3500(1.34489)^{\frac{t}{12}}$

$1.025^{\frac{1}{12}}$
 1.00206

6. Driven by conservation efforts in Asia, the global population of tigers in the wild has shown a significant increase in the past few years. In 2010 there were estimated to be 3,200 tigers in the wild and that number has grown by approximately 3.3% per year since. Which formula can be used to determine, T , the number of wild tigers, d days since 2010?

1) $T(t) = 3,200(1.00009)^d$

2) $T(t) = 3,200(1.00009)^{365d}$

3) $T(t) = 3,200(1.033)^{\frac{365}{d}}$

4) $T(t) = 3,200(1.033)^d$

daily rate one time per day
initial (1 + r)^t

$1.033 = 1.033^+$
 $1.033^{\frac{1}{365}} = 1.00009$

7. The function $A = 3,600(1.025)^t$ represents the value of a bank account after t years. Which of the following statements is false?

1) The initial investment of the bank account was \$3,600. ✓

2) The annual interest rate of the bank account is 2.5%. ✓

3) The value of the account after 5 years is \$4073.07. ✓

4) It will take 12 years for the value of the account to double. ✗

$3600(1.025)^5 = 4073.07$ ✓
 $3600(1.025)^{12} = 4841...$ ✗

8. The function $v(t) = 40,000(0.887)^t$ represents the value of a 2020 Subaru Ascent after t years.

Which of the following statements is *false*?

- 1) The initial value of the car was \$40,000. ✓
- 2) The value of the car is decreasing by 11.3% each year. ✓
- 3) The car is worth \$15,324.18 after 5 years. $40,000(0.887)^5 = 21962.31$ ✗
- 4) The decreased \$3,556.20 from years 2 to 3. $40,000(0.887)^2 - 40,000(0.887)^3 = 3556.20$ ✓

9. A bank account opened up 3 years ago with an initial balance of \$12000 now has a balance of \$12824. Find the annual growth rate, to the nearest tenth of a percent.

$A = 12824$
 $P = 12000$
 $r = r$
 $t = 3$

$A = P(1+r)^t$
 $12824 = 12000(1+r)^3$
 $\frac{12824}{12000} = \frac{12000}{12000}(1+r)^3$
 $\sqrt[3]{1.0686} = (1+r)^3$

$1.0223 = 1+r$
 -1
 $.0223 = r$
 $.0223(100) = 2.2\%$

10. The principal value of a loan is \$424,100. If there is \$110,000 remaining on the loan after 19 years, what was the annual rate of decrease to the nearest tenth of a percent?

$A = 110,000$
 $P = 424,100$
 $r = r$
 $t = 19$

$A = P(1+r)^t$
 $110,000 = 424,100(1-r)^{19}$
 $\frac{110,000}{424,100} = \frac{424,100}{424,100}(1-r)^{19}$
 $\sqrt[19]{\frac{110}{424.1}} = (1-r)^{19}$

$.9314 = 1-r$
 -1
 $-.06856 = -r$
 $.06856 = r$
 $.06856(100) = 6.9\%$

11. Joe Manana just opened a bank account with a \$5000 initial balance with interest compounded quarterly at a rate of 2.8%. Write an equation to represent $b(t)$, the balance of his account after t years. Using your equation, how long will it take for his money to double? Round your answer to the nearest tenth of a year.

$A = b(t)$
 $P = 5000$
 $r = .028$
 $n = 4$
 $t = t$

$A = P(1 + \frac{r}{n})^{nt}$
 $b(t) = 5000(1 + \frac{.028}{4})^{4t}$
 $b(t) = 5000(1.007)^{4t}$

$b(t) = 2(5000) = 10,000$
 $\frac{10,000}{5000} = \frac{5000}{5000}(1.007)^{4t}$
 $2 = 1.007^{4t}$
 $\log 2 = 4t \log 1.007$
 $\frac{\log 2}{4 \log 1.007} = \frac{4t \log 1.007}{4 \log 1.007}$
 $24.8 = t$

12. The half-life of substance X is 12.4 minutes. Write an equation for $p(t)$, the amount of a 300 mg sample remaining after t minutes. Using your equation, how much of substance X remain after 30 minutes to the nearest milligram?

$A = P(t)$
 $P = 300$
 $t = t$
 $h = 12.4$

$A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$
 $p(t) = 300\left(\frac{1}{2}\right)^{\frac{t}{12.4}}$

$p(30) = 300\left(\frac{1}{2}\right)^{\frac{30}{12.4}}$
 $p(30) = 56$

13. A bank account is opened with \$2500, and is compounded continuously with an interest rate of 5.16%. Write an equation for $A(t)$, the amount in the account after t years. Using your equation, how much money will be in his account after 6.5 years rounded to the nearest cent?

$A = A(t)$
 $P = 2500$
 $r = .0516$
 $t = t$

$A = Pe^{rt}$
 $A(t) = 2500e^{-.0516t}$

$A(6.5) = 2500e^{.0516(6.5)}$
 $A(6.5) = 3496.25$

14. Jay borrowed \$15,000 from Aaron and they came to an agreement regarding how the interest will be paid. Every five days, the loan will accumulate 2.5% interest. Write an equation for $f(t)$, the amount owed after t days. Using your equation, to the nearest day, after how many days will Jay owe \$25,000?

$A = f(t)$
 $P = 15,000$
 $r = .025$
 $t = t$
 $h = 5$

$A = P(1+r)^{\frac{t}{h}}$
 $f(t) = 15,000(1+.025)^{\frac{t}{5}}$
 $f(t) = 15,000(1.025)^{\frac{t}{5}}$

$25,000 = 15,000(1.025)^{\frac{t}{5}}$
 $\frac{25,000}{15,000} = 1.025^{\frac{t}{5}}$
 $\log \frac{5}{3} = \log 1.025^{\frac{t}{5}}$
 $5(\log \frac{5}{3}) = \frac{t}{5}(\log 1.025)$
 $5 \log \frac{5}{3} = \frac{t \log 1.025}{5}$
 $10 \log \frac{5}{3} = t \log 1.025$
 $103 = t$

15. Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*. The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest hundredth of an hour*, that the meat can be kept at room temperature safely.

| Hours (x) | Average Number of Spores (y) |
|-----------|------------------------------|
| 0 | 4 |
| 0.5 | 10 |
| 1 | 15 |
| 2 | 60 |
| 3 | 260 |
| 4 | 1130 |
| 6 | 16,380 |

Exp Reg

$$y = a(b)^x$$

$$a = 4.168$$

$$b = 3.981$$

$$y = 4.168(3.981)^x$$

$$100 = 4.168(3.981)^x$$

$$\frac{100}{4.168} = \frac{4.168}{4.168}(3.981)^x$$

$$23... = 3.981^x$$

$$\frac{\log 23...}{\log 3.981} = \frac{x \log 3.981}{\log 3.981}$$

$$2.30 = x$$

16. The table below shows the average yearly balance in a savings account where interest is compounded annually. No money is deposited or withdrawn after the initial amount is deposited.

Write an exponential regression equation to represent this situation. Round all coefficients to the *nearest ten-thousandth*. Use your equation to determine to the *nearest tenth of a year*, how long it will take for the balance to reach \$1,000,000.

| Year | Balance, in Dollars |
|------|---------------------|
| 0 | 380.00 |
| 10 | 562.49 |
| 20 | 832.63 |
| 30 | 1232.49 |
| 40 | 1824.39 |
| 50 | 2700.54 |

$$20.8 = x$$

Exp Reg

$$y = a(b)^x$$

$$a = 379.9996$$

$$b = 1.0400$$

$$y = 379.9996(1.0400)^x$$

$$1,000,000 = 379.9996(1.0400)^x$$

$$\frac{1,000,000}{379.9996} = \frac{379.9996}{379.9996}(1.0400)^x$$

$$\log 2631... = \frac{x \log 1.0400}{\log 1.0400}$$