

Exponential Modeling Finding t

1. Megan opens a savings account with \$5,000 in it. If interest is compounded weekly at a rate of 4.3%, write an equation for $b(t)$, the balance of her account after t years. Using your equation, how long will it take for Megan's money to reach \$8,000?

n formula n=52

$A = b(t)$
 $P = 5,000$
 $r = .043$
 $n = 52$
 $t = t$

$A = P(1 + \frac{r}{n})^{nt}$
 $b(t) = 5,000(1 + \frac{.043}{52})^{52t}$
 $b(t) = 5,000(1.000826923)^{52t}$

$8000 = 5000(1.000826923)^{52t}$
 $\frac{8000}{5000} = \frac{5000}{5000}(1.000826923)^{52t}$
 $1.6 = 1.000826923^{52t}$
 $\log 1.6 = 52t \log 1.000826923$
 $52 \log 1.000826923 = 52 \log 1.000826923$
 $11 = t$

2. One of the medical uses of Iodine-131 (I-131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I-131 is approximately 8.02 days. A patient is injected with 20 milligrams of I-131. Create an equation for $a(t)$, the amount of Iodine-131 remaining after t days. Determine, to the nearest day, the amount of time needed before the amount of I-131 in the patient's body is approximately 7 milligrams.

half life

$A = a(t)$
 $P = 20$
 $t = t$
 $n = 8.02$

$A = P(\frac{1}{2})^{\frac{t}{n}}$
 $a(t) = 20(\frac{1}{2})^{\frac{t}{8.02}}$
 $7 = 20(\frac{1}{2})^{\frac{t}{8.02}}$
 $\frac{7}{20} = \frac{20}{20}(\frac{1}{2})^{\frac{t}{8.02}}$
 $\log \frac{7}{20} = \log \frac{1}{2}$
 $8.02(\log .35) = \frac{t}{8.02} \log(\frac{1}{2})$
 $8.02 \log .35 = \frac{t \log(\frac{1}{2})}{8.02}$
 $\frac{8.02 \log .35}{\log(\frac{1}{2})} = \frac{t \log(\frac{1}{2})}{\log(\frac{1}{2})}$

$12 = t$

3. Tyler opens a bank account with \$5,450 with an annual interest rate of 5.3% compounded continuously. Write an equation for $b(t)$, the balance of Tyler's account after t years. Using your equation, to the nearest hundredth of a year, how long will it take for Tyler's account to triple?

Pert

$A = b(t)$
 $P = 5450$
 $r = .053$
 $t = t$

$A = Pert$
 $b(t) = 5450e^{.053t}$
 $16350 = 5450e^{.053t}$
 $\frac{16350}{5450} = \frac{5450}{5450}e^{.053t}$
 $\ln 3 = \ln e^{.053t}$
 $\ln 3 = .053t$
 $\frac{\ln 3}{.053} = \frac{.053t}{.053}$
 $20.73 = t$
 $b(t) = 3(5450)$
 $b(t) = 16350$

irregular time

h

4. Jessica deposits \$2000 into a bank account where 4% interest is given every 2.4 years. Write an equation for $v(t)$, the value of Jessica's account after t years. Using your equation, to the nearest tenth of a year, how long will it take for Jessica's investment to reach \$5000?

$$\begin{aligned}
 A &= v(t) \\
 P &= 2000 \\
 r &= .04 \\
 t &= t \\
 h &= 2.4
 \end{aligned}$$

$$\begin{aligned}
 A &= P(1+r)^{\frac{t}{h}} \\
 v(t) &= 2000(1+.04)^{\frac{t}{2.4}} \\
 v(t) &= 2000(1.04)^{\frac{t}{2.4}}
 \end{aligned}$$

$$\begin{aligned}
 5000 &= 2000(1.04)^{\frac{t}{2.4}} \\
 \frac{5000}{2000} &= \frac{2000}{2000}(1.04)^{\frac{t}{2.4}} \\
 \log 2.5 &= \log 1.04^{\frac{t}{2.4}} \\
 2.4(\log 2.5) &= \frac{t}{2.4} \log 1.04 \\
 2.4 \log 2.5 &= \frac{t \log 1.04}{\log 1.04} \\
 &= \frac{t}{\log 1.04}
 \end{aligned}$$

$t = 56.1$

n formula

5. Manny opens a savings account with \$6,400.00 with a 5.2% interest rate that is compounded quarterly. Write an equation for $b(t)$, the balance of the account after t years. Using your equation, to the nearest tenth of a year, how long will it take for Manny's balance to double?

$$\begin{aligned}
 A &= b(t) \\
 P &= 6400 \\
 r &= .052 \\
 n &= 4 \\
 t &= t
 \end{aligned}$$

$$\begin{aligned}
 A &= P(1+r)^{nt} \\
 b(t) &= 6400(1+\frac{.052}{4})^{4t} \\
 b(t) &= 6400(1.013)^{4t}
 \end{aligned}$$

$$\begin{aligned}
 12800 &= 6400(1.013)^{4t} \\
 \frac{12800}{6400} &= \frac{6400}{6400}(1.013)^{4t} \\
 \log 2 &= 4t \log 1.013 \\
 \frac{\log 2}{4 \log 1.013} &= \frac{4t \log 1.013}{4 \log 1.013} \\
 &= \frac{t}{\log 1.013}
 \end{aligned}$$

$b(t) = 2(6400) = 12800$

irregular time

$13.4 = t$

6. Christopher is preparing for the Nassau County Spelling Bee. Currently, Christopher knows 1200 words and will learn 20% more words every 4 days. Write an equation, $A(t)$, to represent how many words Christopher will be able to spell after t days. After how many days, to the nearest day, will Christopher be able to spell 5000 words?

$$\begin{aligned}
 A &= A(t) \\
 P &= 1200 \\
 r &= .2 \\
 t &= t \\
 h &= 4
 \end{aligned}$$

$$\begin{aligned}
 A &= P(1+r)^{\frac{t}{h}} \\
 A(t) &= 1200(1+.2)^{\frac{t}{4}} \\
 A(t) &= 1200(1.2)^{\frac{t}{4}}
 \end{aligned}$$

$$\begin{aligned}
 5000 &= 1200(1.2)^{\frac{t}{4}} \\
 \frac{5000}{1200} &= \frac{1200}{1200}(1.2)^{\frac{t}{4}} \\
 \log 2.5 &= \log 1.2^{\frac{t}{4}} \\
 4 \log 2.5 &= \frac{t \log 1.2}{\log 1.2} \\
 &= \frac{t}{\log 1.2}
 \end{aligned}$$

$31 = t$

7. If a bank account was opened with \$3000 and interest is compounded continuously at 5.2%. Write an equation for $v(t)$, the value of the account after t years. To the nearest hundredth of a year, how long will it take for the value of the account to reach \$4000?

$A = v(t)$
 $P = 3000$
 $r = .052$
 $t = t$

$A = Pe^{rt}$
 $v(t) = 3000e^{.052t}$

$\frac{4000}{3000} = \frac{3000e^{.052t}}{3000}$
 $\ln \frac{4}{3} = \ln e^{.052t}$
 $\frac{\ln \frac{4}{3}}{.052} = \frac{.052t}{.052}$
 $6.53 = t$

nothing below

8. Danielle bought a basketball card for \$2125 its value is increasing by 4.1% each year. Create an equation for $v(t)$, the value of the basketball card after t years. Using your equation, how long will it take for the value of the basketball card to reach \$10000?

$A = v(t)$
 $P = 2125$
 $r = .041$
 $t = t$

$A = P(1+r)^t$
 $v(t) = 2125(1+.041)^t$
 $v(t) = 2125(1.041)^t$

$\frac{10000}{2125} = \frac{2125(1.041)^t}{2125}$
 $\log 4.7 = \log 1.041^t$
 $\frac{\log 4.7}{\log 1.041} = \frac{t \log 1.041}{\log 1.041}$
 $39 = t$

9. Miguel opened a bank account with \$1000 and interest is compounded monthly at a rate of 8.1%. Write an equation to represent $b(t)$, the balance of Miguel's account after t years. Using your equation, how much time, to the nearest year, will it take for Miguel's money to triple?

$A = b(t)$
 $P = 1000$
 $r = .081$
 $n = 12$
 $t = t$

$A = P(1 + \frac{r}{n})^{nt}$
 $b(t) = 1000(1 + \frac{.081}{12})^{12t}$
 $b(t) = 1000(1.00675)^{12t}$

$\frac{3000}{1000} = \frac{1000(1.00675)^{12t}}{1000}$
 $\log 3 = \log 1.00675^{12t}$
 $\frac{\log 3}{12 \log 1.00675} = \frac{12t \log 1.00675}{12 \log 1.00675}$
 $14 = t$

nothing below

10. Melanie bought a car for $\$52,000$ and the car depreciates at a rate of 10% each year. Write an equation to represent the value of the car, $v(t)$, after t years. Using your equation, to the nearest tenth of a year, how long will it take until the value of her car reaches $\$22,000$?

$A = v(t)$
 $P = 52000$
 $r = .10$
 $t = t$
 $A = P(1 \pm r)^t$
 $v(t) = 52000(1 - .10)^t$
 $v(t) = 52000(.9)^t$

$\frac{22000}{52000} = \frac{52000(.9)^t}{52000}$
 $\log .42 = \log .9^t$
 $\frac{\log .42}{\log .9} = \frac{t \log .9}{\log .9}$

$8.2 = t$

11. Jennifer initially invested $\$4800$ in a bank account compounded continuously at a rate of 5.8% . Write an equation for $C(t)$, the value of her account after t years. After how much time, to the nearest tenth of a year, will it take for Jennifer's money to double?

$A = C(t)$
 $P = 4800$
 $r = .058$
 $t = t$

$A = Pe^{rt}$
 $C(t) = 4800e^{-.058t}$

$C(t) = 2(4800) = 9600$
 $\frac{9600}{4800} = \frac{4800e^{-.058t}}{4800}$
 $1.2 = e^{-.058t}$
 $\frac{\ln 2}{-.058} = \frac{-.058t + \ln 2}{-.058}$

$11.915 = t$

half life h

12. The half-life of carbon-15 is 2.449 seconds. If Jackie has 17500 grams of carbon-15, write an equation for $j(t)$, the amount of grams of carbon-15 remaining after t seconds. After how much time will there be 500 grams of carbon-15 remaining? Round your answer to the nearest tenth of a second.

$A = j(t)$
 $P = 17500$
 $t = t$
 $h = 2.449$

$A = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$
 $j(t) = 17500\left(\frac{1}{2}\right)^{\frac{t}{2.449}}$

$\frac{500}{17500} = \frac{17500\left(\frac{1}{2}\right)^{\frac{t}{2.449}}}{17500}$
 $\frac{1}{35} = \left(\frac{1}{2}\right)^{\frac{t}{2.449}}$

$2.449 \left(\log \frac{1}{35} \right) = \frac{t}{2.449} \left(\log \frac{1}{2} \right)$

$\frac{2.449 \log \frac{1}{35}}{\log \frac{1}{2}} = \frac{t \log \frac{1}{2}}{\log \frac{1}{2}}$

$12.6 = t$