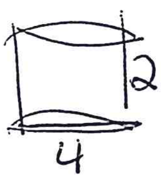
1) Calculate Volume
2) Perform unit analysis
*convert first if necessary

Name Schlansky
Mr. Schlansky

Date ____________
Geometry

Modeling Volume

1. Cylindrical bricks are needed to fill a hole in a homeowner's backyard. Each brick is to have a diameter of 4 cm and a height of 2 cm. The weight of the concrete that the brick is going to be made from is 2.1 ounces per cubic centimeter. If the concrete costs $.14 per ounce, how much would it cost to purchase four bricks? Round your answer to the *nearest cent.*



$25\ldots \cancel{cm^3} \times \frac{2.1\ \cancel{oz}}{1\ \cancel{cm^3}} \times \frac{.14\ \$}{1\ \cancel{oz}} \times 4$

$V=\pi r^2 h$
$V=\pi(2)^2(2)$
$V=25\ldots cm^3$

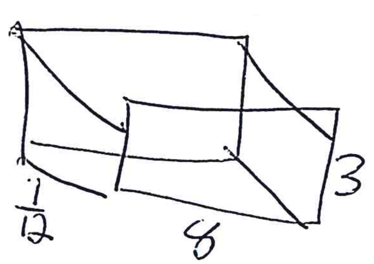
$29.56

2. A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. If the maple costs $12.59 per pound, how much will it cost to make 5 tabletops?

convert to feet

$1\ \cancel{in} \times \frac{1\ ft}{12\ \cancel{in}} = \frac{1}{12} ft$

$2\ \cancel{ft^3} \times \frac{43\ \cancel{pounds}}{1\ \cancel{ft^3}} \times \frac{12.59\ \$}{1\ \cancel{pounds}} \times 5$

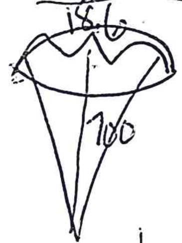


$5413.70

$V=lwh$
$V=\frac{1}{12}(8)(3)$
$V=2ft^3$

$.7\,\text{km} \cdot \frac{1000\,\text{m}}{1\,\text{km}} = 700\text{m}$

3. A town in upstate New York keeps sand in a silo that is in the shape of a cone. They use this sand to help de-ice the roads after a snowstorm. The silo has a diameter of 18.6 meters and a height of .7 kilometers. The weight of the sand is 1.2 ounces per cubic meter. If the sand costs $.12 per ounce, how much will it cost the town to fill 80% of the silo?



$63400\ldots\,\text{m}^3 \cdot \frac{1.2\,\text{oz}}{1\,\text{m}^3} \cdot \frac{.12\,\$}{1\,\text{oz}} \times .8 = \$7303.74$

$V = \frac{1}{3}\pi r^2 h$

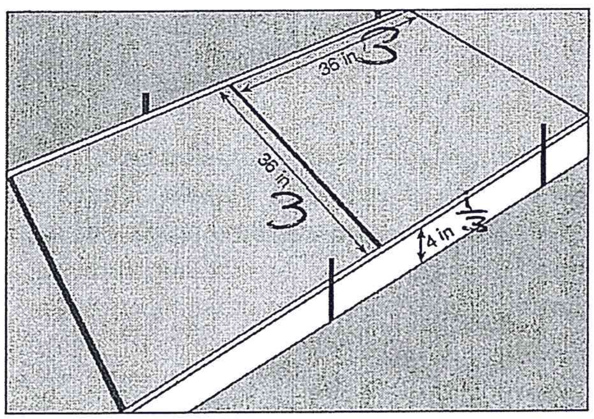
$V = \frac{1}{3}\pi (9.3)^2 (700)$

$V = 63400\ldots\,\text{m}^3$

4. Ian needs to replace two concrete sections in his sidewalk, as modeled below. Each section is 36 inches by 36 inches and 4 inches deep. He can mix his own concrete for $3.25 per cubic foot.

$36\,\text{in} \cdot \frac{1\,\text{ft}}{12\,\text{in}} = 3\text{ft}$

$4\,\text{in} \cdot \frac{1\,\text{ft}}{12\,\text{in}} = \frac{1}{3}\text{ft}$



How much money will it cost Ian to replace the two concrete sections?

$V = lwh$

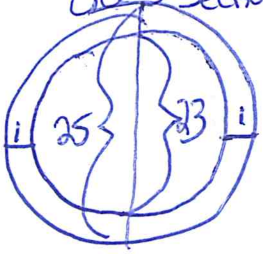
$V = 3(3)(\frac{1}{3})$

$V = 3\text{ft}^3$

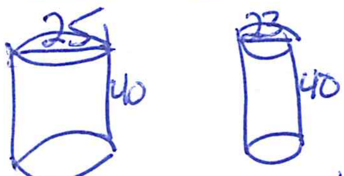
$3\,\text{ft}^3 \cdot \frac{3.25\,\$}{1\,\text{ft}^3} \times 2 = \$19.50$

5. A cylindrical casing is to be put around a garbage can in a busy street in Manhattan. The diameter is 25 inches. The height of the case will be 40 inches and the casing will be 1 inch thick. The density of the metal is .841 grams per cubic inch. What will be the mass of the casing?

Cross Section



Whole Inside



$3015..in^3 \cdot \frac{.841\ g}{1\ in^3} = 2536g$

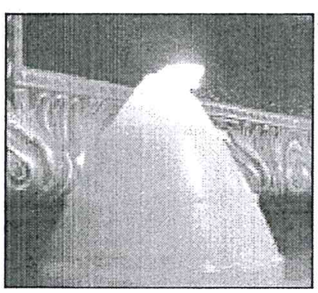
$V=\pi r^2h$ $V=\pi r^2h$

$V=\pi(12.5)^2(40)$ $V=\pi(11.5)^2(40)$

$V=19634...in^3$ $V=16619..in^3$

$19634..-16619..=3015.in^3$

6. A candle maker uses a mold to make candles like the one shown below.
The height of the candle is 13 cm and the circumference of the candle at its widest measure is 31.416 cm. Use modeling to approximate how much wax, to the *nearest cubic centimeter*, is needed to make this candle. Justify your answer.



$V=\frac{1}{3}\pi r^2h$

$V=\frac{1}{3}\pi(5)^2(13)$

$V=340\ cm^3$

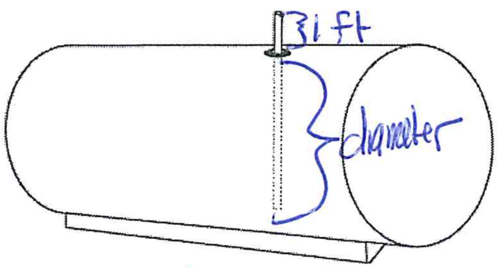
$C=\pi d$

$\frac{31.416}{\pi}=\frac{\pi d}{\pi}$

$10...=d$

volume, convert

7. A gas station has a cylindrical fueling tank that holds the gasoline for its pumps, as modeled below. The tank holds a maximum of 20,000 gallons of gasoline and has a length of 34.5 feet. A metal pole is used to measure how much gas is in the tank. To the *nearest tenth of a foot*, how long does the pole need to be in order to reach the bottom of the tank and still extend one foot outside the tank? Justify your answer. [1 ft^3=7.48 gallons]



$20000 \text{ gal} \cdot \frac{1 \text{ ft}^3}{7.48 \text{ gal}} = 2673... \text{ft}^3$

$V=\pi r^2 h$

$\frac{2673...}{34.5\pi} = \frac{\pi r^2(34.5)}{34.5\pi}$

$\sqrt{24..} = \sqrt{r^2}$

$4... = r$

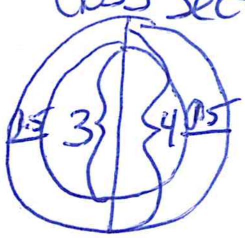
$d=2(4..)$

$d=9.. + 1 = 10.93366959$

10.9 ft

8. A bakery sells hollow chocolate spheres. The larger diameter of each sphere is 4 cm. The thickness of the chocolate of each sphere is 0.5 cm. Determine and state, to the *nearest tenth of a cubic centimeter*, the amount of chocolate in each hollow sphere. The bakery packages 8 of them into a box. If the density of the chocolate is 1.308 g/cm^3, determine and state, to the *nearest gram*, the total mass of the chocolate in the box.

cross section



whole

$V=\frac{4}{3}\pi r^3$

$V=\frac{4}{3}\pi(2)^3$

inside

$V=\frac{4}{3}\pi r^3$

$V=\frac{4}{3}\pi(1.5)^3$

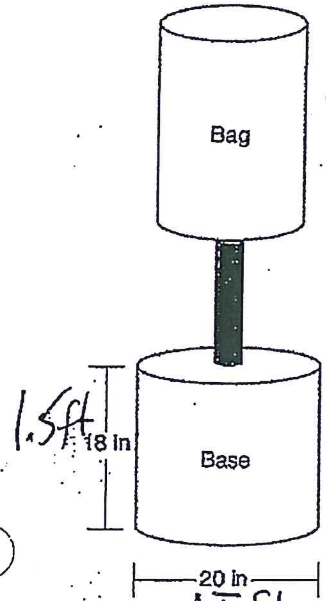
$V=33... - V= 14... = 19..\text{cm}^3$

$19... \text{cm}^3 \cdot \frac{1.308 \text{ g}}{\text{cm}^3} \times 8 = 203 \text{ grams}$

$18\text{ in}\cdot\frac{1\text{ ft}}{12\text{ in}}=1.5\text{ft}$ $20\text{ in}\cdot\frac{1\text{ ft}}{12\text{ in}}=1.6\text{ft}$

9. Shae has recently begun kickboxing and purchased training equipment as modeled in the diagram below. The total weight of the bag, pole, and unfilled base is 270 pounds. The cylindrical base is 18 inches tall with a diameter of 20 inches. The dry sand used to fill the base weighs 95.46 lbs per cubic foot.
To the *nearest pound*, determine and state the total weight of the training equipment if the base is filled to 85% of its capacity.

weight of equipment = weight of unit + weight of sand
weight of unit = 270 pounds
weight of sand = 265... pounds
270 + 265... = 536 pounds



$V=\pi r^2h$
$V=\pi(.83)^2(1.5)$
$V=3...\text{ft}^3$

$3...\text{ft}^3\cdot\frac{95.46\text{ lb}}{1\text{ ft}^3}\cdot .85=265...\text{lbs}$

10. Theresa has a rectangular pool 30 ft long, 15 ft wide, and ~~4~~ 3.5 ft deep. Theresa fills her pool using city water at a rate of \$3.95 per 100 gallons of water. Nancy has a circular pool with a diameter of 24 ft and a depth of ~~4~~ 3.5 ft. Nancy fills her pool with a water delivery service at a rate of \$200 per 6000 gallons. If Theresa and Nancy both fill their pools 6 inches from the top of the pool, determine and state who paid more to fill her pool. [1ft^3 water = 7.48 gallons]

$6\text{ in}\cdot\frac{1\text{ ft}}{12\text{ in}}=.5\text{ft}$
height = 4 − .5 = 3.5 ft

Nancy
$V=\pi r^2h$
$V=\pi(12)^2(3.5)$
$V=1583...\text{ft}^3$

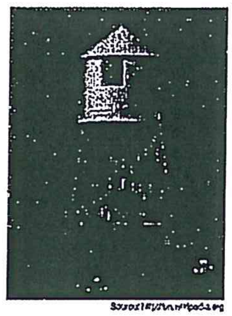
$1583...\text{ft}^3\cdot\frac{7.48\text{ gal}}{1\text{ ft}^3}\cdot\frac{200\ \$}{6000\text{ gal}}=\$394...$

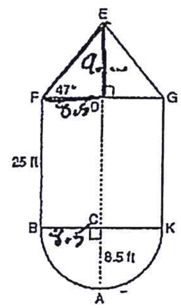
Theresa
$V=lwh$
$V=30(15)(3.5)$
$V=1575\text{ft}^3$

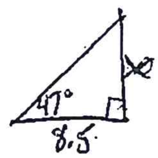
$1575\text{ft}^3\cdot\frac{7.48\text{ gal}}{1\text{ ft}^3}\cdot\frac{3.95\ \$}{100\text{ gal}}=\$465...$

Theresa paid more

11. The water tower in the picture below is modeled by the two-dimensional figure beside it. The water tower is composed of a hemisphere, a cylinder, and a cone. Let C be the center of the hemisphere and let D be the center of the base of the cone.







$\tan 47 = \frac{x}{8.5}$

$\frac{1.0724}{1} = \frac{x}{8.5}$

$x = 9.1...$

If $AC = 8.5$ feet, $BF = 25$ feet, and $m\angle EFD = 47°$, determine and state, to the *nearest cubic foot*, the volume of the water tower. The water tower was constructed to hold a maximum of 400,000 pounds of water. If water weighs 62.4 pounds per cubic foot, can the water tower be filled to 85% of its volume and *not* exceed the weight limit? Justify your answer.

don't care

Cone	Cylinder	Hemisphere
$V = \frac{1}{3}\pi r^2 h$	$V = \pi r^2 h$	$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$
$V = \frac{1}{3}\pi (8.5)^2 (9.1...)$	$V = \pi (8.5)^2 (25)$	$V = \frac{1}{2}\left(\frac{4}{3}\pi (8.5)^3\right)$
$V = 689...$ +	$V = 5674...$ +	$V = 1286...$ $= 7650\ ft^3$

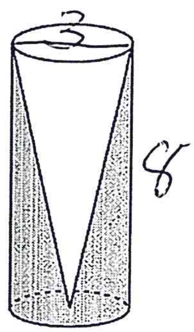
$7650\ ft^3 \cdot \frac{62.4\ \text{pounds}}{1\ ft^3} \times .85 = 405756$ pounds

$405756 > 400000$

No!

12. Walter wants to make 100 candles in the shape of a cone for his new candle business. The mold shown below will be used to make the candles. Each mold will have a height of 8 inches and a diameter of 3 inches. To the *nearest cubic inch*, what will be the total volume of 100 candles?

Walter goes to a hobby store to buy the wax for his candles. The wax costs \$0.10 per ounce. If the weight of the wax is 0.52 ounce per cubic inch, how much will it cost Walter to buy the wax for 100 candles? If Walter spent a total of \$37.83 for the molds and charges \$1.95 for each candle, what is Walter's profit after selling 100 candles?



$$1885\,\cancel{in^3} \cdot \frac{.52\,\cancel{oz}}{1\,\cancel{in^3}} \cdot \frac{.10\,\$}{1\,\cancel{oz}} = 98.02$$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (1.5)^2 (8)$

$V = 18..$

$18..(100) = 1885\ in^3$

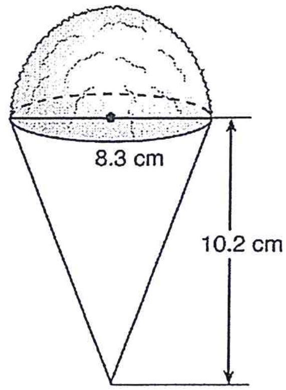
Profit = amount made − amount spent

amount made = $1.95(100) = 195$

amount spent = $98.02 + 37.83 = \cancel{138}$ 135.85

$195 - 135.85 = \$59.15$

13. ~~14~~. A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters.



The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per kilogram, of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.

cone — hemisphere

$V=\frac{1}{3}\pi r^2 h \qquad V=\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V=\frac{1}{3}\pi(4.15)^2(10.2) \qquad V=\frac{1}{2}\left(\frac{4}{3}\pi(4.15)^3\right)$

$V=183\ldots \qquad V=149\ldots$

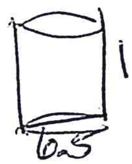
$183\ldots + 149\ldots = 333\ldots \text{ cm}^3$

$333\ldots \cancel{\text{cm}^3} \cdot \frac{.697 \cancel{g}}{1 \cancel{\text{cm}^3}} \cdot \frac{1 \cancel{kg}}{1000 \cancel{g}} \cdot \frac{3.83\ \$}{1 \cancel{kg}} \times 50 = \$44.53$

$12\,\cancel{in} \times \frac{1\,ft}{12\,\cancel{in}} = \frac{12}{12} = 1\,ft$

convert to feet

14. A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of $6\frac{1}{2}$ feet and a height of 12 inches. The pool is filled with water to $\frac{2}{3}$ of its height. Determine and state the volume of the water in the pool, to the *nearest cubic foot*. One cubic foot equals 7.48 gallons of water. Determine and state, to the *nearest gallon*, the number of gallons of water in the pool.



$22\,\cancel{ft^3} \times \frac{7.48\,gal}{1\,\cancel{ft^3}} = 165$ gallons

$V = \frac{2}{3}(\pi r^2 h)$

$V = \frac{2}{3}\pi(3.25)^2(1)$

$V = 22.\ ft^3$

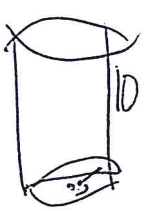
$V = 22\ ft^3$

15. Trees that are cut down and stripped of their branches for timber are approximately cylindrical. A timber company specializes in a certain type of tree that has a typical diameter of 50 cm and a typical height of about 10 meters. The density of the wood is 380 kilograms per cubic meter, and the wood can be sold by mass at a rate of \$4.75 per kilogram. Determine and state the minimum number of whole trees that must be sold to raise at least \$50,000.

convert to m

$50\,\cancel{cm} \times \frac{1\,m}{100\,\cancel{cm}}$

$.5\,m$



$V = \pi r^2 h$

$V = \pi(.25)^2(10)$

$V = 1.96.\ m^3$

$1.96\,\cancel{m^3} \times \frac{380\,kg}{1\,\cancel{m^3}} \times \frac{4.75\,\$}{1\,kg} = \$354.$ for 1 tree

$\frac{\$50{,}000}{\$354..} = 141..$

15 trees

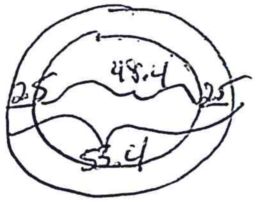
must convert $7.5\,m \cdot \frac{100\,cm}{1\,m} = 750\,cm$

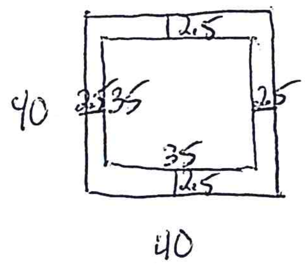
ollow
ake
cross
ction

15. New streetlights will be installed along a section of the highway. The posts for the streetlights will be 7.5 m tall and made of aluminum. The city can choose to buy the posts shaped like cylinders or the posts shaped like rectangular prisms. The cylindrical posts have a hollow core, with aluminum 2.5 cm thick, and an outer diameter of 53.4 cm. The rectangular-prism posts have a hollow core, with aluminum 2.5 cm thick, and a square base that measures 40 cm on each side. The density of aluminum is 2.7 g/cm^3, and the cost of aluminum is \$0.38 per kilogram. If all posts must be the same shape, which post design will cost the town less? How much money will be saved per streetlight post with the less expensive design?

$53.4 - 2(2.5) = 48.4$

$40 - 2(2.5) = 35$





Whole

$V = \pi r^2 h$

$= \pi (26.7)^2 (750)$

$= 1679707.49$

inside

$V = \pi r^2 h$

$V = \pi (24.2)^2 (750)$

$V = 1379881.741$

$1679707.49 - 1379881.741$

$V = 299825.749\,cm^3$

whole

$V = lwh$

$V = 40(40)(750)$

$V = 1200000$

inside

$V = lwh$

$V = 35(35)(750)$

$V = 918750$

$1200000 - 918750$

$V = 281250\,cm^3$

$299825.749 - 281250$

$18575.749\,cm^3 \rightarrow$ Difference in volume for 1 post

$18575.749\,cm^3 \cdot \frac{2.7\,g}{1\,cm^3} \cdot \frac{1\,kg}{1000\,g} \cdot \frac{.38\,\$}{1\,kg} = \$19.06$

Rectangular posts