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Next Gen Algebra I Everything You Need to Know Packet!

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2024

Multiple Choice Strategy with Variables

If variables in the problems and answers:

10 STO → X, 15 STO → Y   See which matches up.

Type in original problem, ~~2^{nd} Math (Test), =~~, type in each solution. ~~1 is equivalent, 0 is~~ not ~~equivalent~~. Make sure to try all four choices.

1. Which expression represents $\frac{(2x^3)(8x^5)}{4x^6}$ 400 in simplest form?

1) x^2 100
2) x^9 1,000,000,000
3) $4x^2$ 400
4) $4x^9$ 4,000,000,000

−4025

2. Factored, the expression $16x^2 - 25y^2$ is equivalent to

1) $(4x-5y)(4x+5y)$ −4025
2) $(4x-5y)(4x-5y)$ 1225
3) $(8x-5y)(8x+5y)$ 775
4) $(8x-5y)(8x-5y)$ 25

288

3. Factored completely, the expression $2x^2+10x-12$ is equivalent to

1) $2(x-6)(x+1)$ 88
2) $2(x+6)(x-1)$ 288
3) $2(x+2)(x+3)$ 312
4) $2(x-2)(x-3)$ 112

800

4. The expression $9x^2-100$ is equivalent to

1) $(9x-10)(x+10)$ 1600
2) $(3x-10)(3x+10)$ 800
3) $(3x-100)(3x-1)$ −2030
4) $(9x-100)(x+1)$ −110

$\frac{4}{75}$

5. What is $\frac{6}{5x}-\frac{2}{3x}$ in simplest form?

1) $\frac{8}{15x^2}$ $\frac{2}{375}$
2) $\frac{8}{15x}$ $\frac{4}{75}$
3) $\frac{4}{15x}$ $\frac{2}{75}$
4) $\frac{4}{2x}$ $\frac{1}{5}$

6. Which expression represents $\frac{12x^3-6x^2+2x}{2x}$ in simplest form? 571

1) $6x^2-3x$ 570
2) $10x^2-4x$ 960
3) $6x^2-3x+1$ 571
4) $10x^2-4x+1$ 961

7. The sum of $4x^3+6x^2+2x-3$ and $3x^3+3x^2-5x-5$ is 7862

1) $7x^3+3x^2-3x-8$ 7262
2) $7x^3+3x^2+7x+2$ 7372
3) $7x^3+9x^2-3x-8$ 7862
4) $7x^6+9x^4-3x^2-8$ 7089692

8. What is the sum of $\frac{-x+7}{2x+4}$ + and $\frac{2x+5}{2x+4}$? $\frac{11}{12}$

1) $\frac{x+12}{2x+4}$ $\frac{11}{12}$
2) $\frac{3x+12}{2x+4}$ $\frac{7}{4}$
3) $\frac{x+12}{4x+8}$ $\frac{11}{24}$
4) $\frac{3x+12}{4x+8}$ $\frac{7}{8}$

9. Factored completely, the expression $3x^2 - 3x - 18$ is equivalent to 252

1) $3(x^2 - x - 6)$ 252
2) $3(x-3)(x+2)$ 252
The only choice factored completely
3) $(3x-9)(x+2)$ 252
4) $(3x+6)(x-3)$ 252

10. Four expressions are shown below.

I $2(2x^2 - 2x - 60)$ 240
II $4(x^2 - x - 30)$ 240
III $4(x+6)(x-5)$ 320
IV $4x(x-1) - 120$ 240

The expression $4x^2 - 4x - 120$ 240 is equivalent to

1) I and II, only
2) II and IV, only
3) I, II, and IV
4) II, III, and IV

11. Which trinomial is equivalent to $3(x-2)^2 - 2(x-1)$? 174

1) $3x^2 - 2x - 10$ 270
2) $3x^2 - 2x - 14$ 266
3) $3x^2 - 14x + 10$ 170
4) $3x^2 - 14x + 14$ 174

12. When factored completely, $x^3 - 13x^2 - 30x$ is -600

1) $x(x+3)(x-10)$ 0
2) $x(x-3)(x-10)$ 0
3) $x(x+2)(x-15)$ -600
4) $x(x-2)(x+15)$ 2000

13. The expression $x^4 - 16$ 9984 is equivalent to

1) $(x^2+8)(x^2-8)$ 9936
2) $(x^2-8)(x^2-8)$ 8464
3) $(x^2+4)(x^2-4)$ 9984
4) $(x^2-4)(x^2-4)$ 9216

14. The expression $3(x^2 - 1) - (x^2 - 7x + 10)$ is equivalent to 257

1) $2x^2 - 7x + 7$ 137
2) $2x^2 + 7x - 13$ 257
3) $2x^2 - 7x + 9$ 139
4) $2x^2 + 7x - 11$ 259

Multiple Choice Strategy with Equations
Store each potential answer (_____ STO → X)
Type in left hand side
Type in right hand side
See if the match up
*Check all potential answers

1. What is the value of x in the equation $2(x-4) = 4(2x+1)$?

1) -2 $-12 = -12$
2) 2 $-4 \neq 20$
3) $-\frac{1}{2}$ $-9 \neq 0$
4) $\frac{1}{2}$ $-7 \neq 8$

2. Solve for x: $15x - 3(3x + 4) = 6$

1) 1 $-6 \neq 6$ 3) 3 $6 = 6$
2) $-\frac{1}{2}$ 4) $\frac{1}{3}$ $-10 \neq 6$
$-15 \neq 6$

3. Which value of x is a solution of $\frac{5}{x} = \frac{x+13}{6}$?

1) -2 $-\frac{5}{2} \neq \frac{11}{6}$ 3) -10 $-\frac{1}{2} \neq \frac{1}{2}$
2) -3 $-\frac{5}{3} \neq \frac{5}{3}$ 4) -15 $-\frac{1}{3} = -\frac{1}{3}$

4. What is the solution of $\frac{k+4}{2} = \frac{k+9}{3}$?

1) 1 $\frac{5}{2} \neq \frac{10}{3}$ 3) 6 $5 = 5$
2) 5 $\frac{9}{2} \neq \frac{14}{3}$ 4) 14 $9 \neq \frac{23}{3}$

5. What is the value of x in the equation $\frac{2}{x} - 3 = \frac{26}{x}$?

1) -8 $-\frac{13}{4} = -\frac{13}{4}$ 3) $\frac{1}{8}$ $13 \neq 208$
2) $-\frac{1}{8}$ $-19 \neq -208$ 4) 8 $-\frac{11}{4} \neq \frac{13}{4}$

6. Which value of x is the solution of the equation $\frac{2x}{3} + \frac{x}{6} = 5$?

1) 6 $5 = 5$ 3) 15 $\frac{25}{2} \neq 5$
2) 10 $\frac{25}{3} \neq 5$ 4) 30 $25 \neq 5$

7. Solve for x: $\frac{3}{5}(x+2) = x-4$

1) 8 $6 \neq 4$
2) 13 $9 = 9$
3) 15 $\frac{51}{5} \neq 11$
4) 23 $15 \neq 19$

8. Which value of x is the solution of $\frac{x}{3} + \frac{x+1}{2} = x$?

1) 1 $\frac{4}{3} \neq 1$
2) -1 $-\frac{1}{3} \neq -1$
3) 3 $3 = 3$
4) -3 $-2 \neq 3$

9. Which value of x is the solution of $\frac{2x-3}{x-4} = \frac{2}{3}$?

1) $-\frac{1}{4}$ $\frac{14}{17} \neq \frac{2}{3}$
2) $\frac{1}{4}$ $\frac{2}{3} = \frac{2}{3}$
3) -4 $\frac{11}{8} \neq \frac{2}{3}$
4) 4 ERROR

10. Which value of x satisfies the equation $\frac{7}{3}\left(x + \frac{9}{28}\right) = 20$?

1) 8.25 $20 = 20$
2) 8.89 $21.49\overline{3} \neq 20$
3) 19.25 $45.6 \neq 20$
4) 44.92 $105.56\overline{3} \neq 20$

11. Which value of x satisfies the equation $\frac{5}{6}\left(\frac{3}{8} - x\right) = 16$?

1) -19.575 $16.625 \neq 16$
2) -18.825 $16 = 16$
3) -16.3125 $13.90625 \neq 16$
4) -15.6875 $13.38541\overline{6} \neq 16$

12. What is the solution to the equation $\frac{3}{5}\left(x + \frac{4}{3}\right) = 1.04$?

1) $3.0\overline{6}$ $2.64 \neq 1.04$
2) 0.4 $1.04 = 1.04$
3) $-0.4\overline{8}$ $.50\overline{6} \neq 1.04$
4) $-0.709\overline{3}$ $.3744 \neq 1.04$

Equivalent Expressions

Use multiple choice strategy! See if the two expressions are equal.

6.591... E12

1. A computer application generates a sequence of musical notes using the function $f(n) = 6(16)^n$, where n is the number of the note in the sequence and $f(n)$ is the note frequency in hertz. Which function will generate the same note sequence as $f(n)$?

1) $g(n) = 12(2)^{4n}$ 1.31...E13
3) $p(n) = 12(4)^{2n}$ 1.31...E13
2) $h(n) = 6(2)^{4n}$ 6.591.. E12
4) $k(n) = 6(8)^{2n}$ 6.917.. E18

431

2. The function $f(x) = 3x^2 + 12x + 11$ can be written in vertex form as

1) $f(x) = (3x+6)^2 - 25$ 1271
3) $f(x) = 3(x+2)^2 - 1$ 431
2) $f(x) = 3(x+6)^2 - 25$ 743
4) $f(x) = 3(x+2)^2 + 7$ 439

3. Mario's \$15,000 car depreciates in value at a rate of 19% per year. The value, V, after t years can be modeled by the function $V = 15,000(0.81)^t$. Which function is equivalent to the original function? 1823...

1) $V = 15,000(0.9)^{9t}$ 1.14...
3) $V = 15,000(0.9)^{\frac{t}{9}}$ 13342..
2) $V = 15,000(0.9)^{2t}$ 1823...
4) $V = 15,000(0.9)^{\frac{t}{2}}$ 8857...

7857...

4. Nora inherited a savings account that was started by her grandmother 25 years ago. This scenario is modeled by the function $A(t) = 5000(1.013)^{t+25}$, where $A(t)$ represents the value of the account, in dollars, t years after the inheritance. Which function below is equivalent to $A(t)$?

1) $A(t) = 5000[(1.013^t)]^{25}$ 126279...
3) $A(t) = (5000)^t(1.013)^{25}$ 1.34.. E37
2) $A(t) = 5000[(1.013)^t + (1.013)^{25}]$ 12595...
4) $A(t) = 5000(1.013)^t(1.013)^{25}$ 7857...

3.2.. E14

5. The number of bacteria grown in a lab can be modeled by $P(t) = 300 \bullet 2^{4t}$, where t is the number of hours. Which expression is equivalent to $P(t)$?

1) $300 \bullet 8^t$ 3.2... E11
3) $300^t \bullet 2^4$ 9.44..E25
2) $300 \bullet 16^t$ 3.2...E14
4) $300^{2t} \bullet 2^{2t}$ 3.6..E55

9543...

6. The growth of a certain organism can be modeled by $C(t) = 10(1.029)^{24t}$, where $C(t)$ is the total number of cells after t hours. Which function is approximately equivalent to $C(t)$?

1) $C(t) = 240(.083)^{24t}$ 0
3) $C(t) = 10(1.986)^t$ 9545...
2) $C(t) = 10(.083)^t$ 1.55..E10
4) $C(t) = 240(1.986)^{\frac{t}{24}}$ 319...

Evaluating Functions
Algebraically: Substitute the value in parenthesis in for x.
Graphically: Find the y value for the x value that is in parenthesis.

1. If $f(x) = 6x - 3$, $g(x) = -x^2 + x$, $h(x) = 2\sqrt{x^2 + 2}$ find:

a) f(3)
$f(3) = 6(3) - 3$
$f(3) = 15$

b) f(0)
$f(0) = 6(0) - 3$
$f(0) = -3$

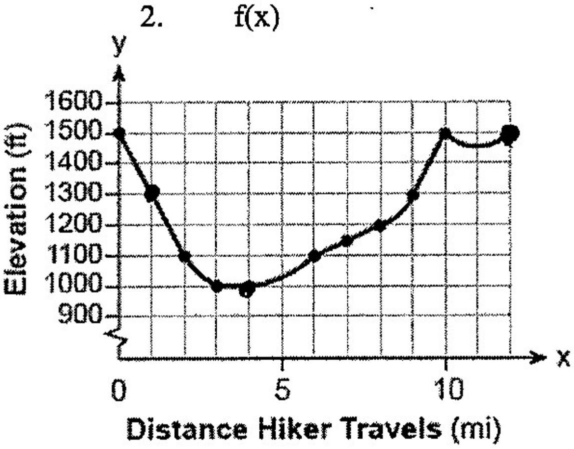
d) g(0)
$g(0) = -(0)^2 + 0$
$g(0) = 0$

e) g(4.2)
$g(4.2) = -(4.2)^2 + 4.2$
$g(4.2) = -13.44$

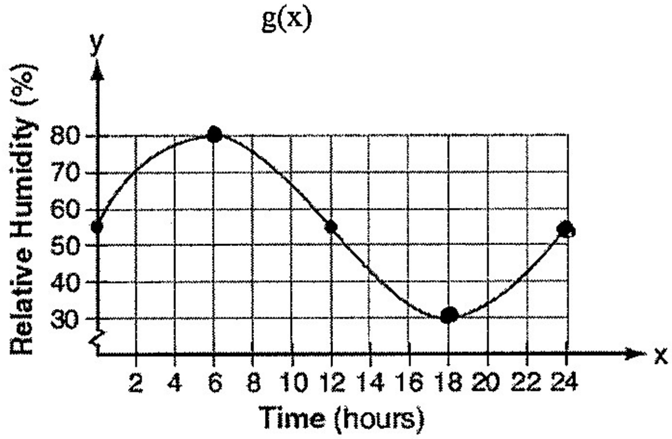
f) h(1)
$h(1) = 2\sqrt{(1)^2 + 2}$
$h(1) = 2\sqrt{3}$

g) h(7)
$h(7) = 2\sqrt{(7)^2 + 2}$
$h(7) = 2\sqrt{51}$

2. f(x)



g(x)



a) Evaluate f(4)
$f(4) = 1000$

b) Evaluate f(12)
$f(12) = 1500$

c) Evaluate f(1)
$f(1) = 1300$

d) Evaluate g(6)
$g(6) = 80$

e) Evaluate g(18)
$g(18) = 30$

f) Evaluate g(24)
$g(24) = 55$

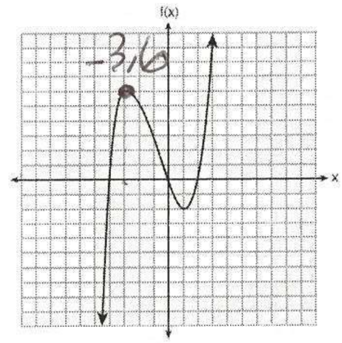
3. If $f(x) = \frac{1}{2}x^2 - \left(\frac{1}{4}x + 3\right)$, what is the value of $f(8)$? $f(8) = \frac{1}{2}(8)^2 - \left(\frac{1}{4}(8) + 3\right) = 27$

1) 11
2) 17
3) 27
4) 33

$k(9) = 2(9)^2 - 3\sqrt{9} = 153$

4. If $k(x) = 2x^2 - 3\sqrt{x}$, then $k(9)$ is
1) 315
2) 307
3) 159
4) 153

5. The graph of $f(x)$ is shown below. What is the value of $f(-3)$?
1) 6
2) 2
3) -2
4) -4



6. Lynn, Jude, and Anne were given the function $f(x) = -2x^2 + 32$, and they were asked to find $f(3)$. Lynn's answer was 14, Jude's answer was 4, and Anne's answer was ±4. Who is correct?
1) Lynn, only
2) Jude, only
3) Anne, only
4) Both Lynn and Jude

$f(3) = -2(3)^2 + 32$
$f(3) = 14$

5. Faith wants to use the formula $C(f) = \frac{5}{9}(f - 32)$ to convert degrees Fahrenheit, f, to degrees Celsius, $C(f)$. If Faith calculated $C(68)$, what would her result be?
1) 20° Celsius
2) 20° Fahrenheit
3) 154° Celsius
4) 154° Fahrenheit

$C(68) = \frac{5}{9}(68 - 32)$
$C(68) = 20$

6. If $f(n) = (n - 1)^2 + 3n$, which statement is true?
1) $f(3) = -2$ $f(3) = (3-1)^2 + 3(3) = 13$
2) $f(-2) = 3$ $f(-2) = (-2-1)^2 + 3(-2) = 3$
3) $f(-2) = -15$
4) $f(-15) = -2$ $f(-15) = (-15-1)^2 + 3(-15) = 211$

7. Which value of x results in equal outputs for $j(x) = 3x - 2$ and $b(x) = |x + 2|$?
1) -2 $-8 \neq 0$
2) 2 $4 = 4$
3) $\frac{2}{3}$ $1 \neq 2\frac{2}{3}$
4) 4 $10 \neq 12$

7. Which value of x results in equal outputs for $j(x) = 3x - 2$ and $b(x) = |x + 2|$?

1) -2 $j(-2) = 3(-2) - 2 = -8$ $b(-2) = |-2+2| = 0$

2) 2 $j(2) = 3(2) - 2 = 4$ $b(2) = |2+2| = 4$

3) $\frac{2}{3}$ $j(\frac{2}{3}) = 3(\frac{2}{3}) - 2 = 0$ $b(\frac{2}{3}) = |\frac{2}{3}+2| = \frac{8}{3}$

4) 4 $j(4) = 3(4) - 2 = 10$ $b(4) = |4+2| = 6$

8. As x increases beyond 25, which function will have the largest value?

1) $f(x) = 1.5^x$ $1.5^{26} = 37876...$

2) $g(x) = 1.5x + 3$ $1.5(26) + 3 = 42$

3) $h(x) = 1.5x^2$ $1.5(26)^2 = 1014$

4) $k(x) = 1.5x^3 + 1.5x^2$ $1.5(26)^3 + 1.5(26)^2 = 27378$

9. What is the largest integer, x, for which the value of $f(x) = 5x^4 + 30x^2 + 9$ will be greater than the value of $g(x) = 3^x$?

1) 7 $5(7)^4 + 30(7)^2 + 9 = 13484$ $3^7 = 2187$ ✓

2) 8 $5(8)^4 + 30(8)^2 + 9 = 22409$ $3^8 = 6561$ ✓

3) 9 $5(9)^4 + 30(9)^2 + 9 = 35244$ $3^9 = 19683$ ✓

4) 10 $5(10)^4 + 30(10)^2 + 9 = 53009$ $3^{10} = 59049$ X

10. The function $g(x)$ is defined as $g(x) = -2x^2 + 3x$. The value of $g(-3)$ is

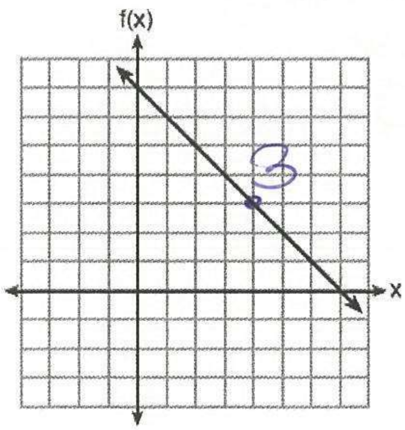
1) -27 $g(-3) = -2(-3)^2 + 3(-3) = -27$

2) -9

3) 27

4) 45

11. The functions $f(x)$, $q(x)$, and $p(x)$ are shown below.



$q(x) = (x - 1)^2 - 6$

~~$q(4) = (3-1)^2 - 6 = -2$~~

$q(4) = (4-1)^2 - 6 = 3$

x	p(x)
2	5
3	4
4	3
5	4
6	5

3

When the input is 4, which functions have the same output value?

1) $f(x)$ and $q(x)$, only

2) $f(x)$ and $p(x)$, only

3) $q(x)$ and $p(x)$, only

4) $f(x)$, $q(x)$, and $p(x)$

Number Properties

$2(4+3) = 2 \bullet 4 + 2 \bullet 3$	Distributive Property
$4 \bullet 6 = 6 \bullet 4$	Commutative Property of Multiplication
$2 + 7 = 7 + 2$	Commutative Property of Addition
$5 + (2+3) = (5+2) + 3$	Associative Property of Addition
$5 \bullet (4 \bullet 3) = (5 \bullet 4) \bullet 3$	Associative Property of Multiplication
$4 + 0 = 4$	Additive Identity
$7 \bullet 1 = 7$	Multiplicative Identity
$5 + -5 = 0$	Additive Inverse
$4 \bullet \frac{1}{4} = 1$	Multiplicative Inverse
$3 \bullet 0 = 0$	Multiplication Property of Zero
$4(3x^2+2) - 9 = 8x^2 + 7 \rightarrow 4(3x^2+2) = 8x^2 + 16$	Addition Property of Equality
$2x + 8 = 4x + 4 \rightarrow 8 = 2x + 4$	Subtraction Property of Equality
$2x^2 = 8x + 10 \rightarrow x^2 = 4x + 5$	Division Property of Equality
$\frac{2x+5}{3} = 5 \rightarrow 2x + 5 = 15$	Multiplication Property of Equality

ASSociative property has two sets of parenthesis
Commutative property has numbers commute (move)
Identity is where you start and end with the same thing
Inverse is when you end up with the identity element (Addition 0, Multiplication 1)

1. Which property is illustrated by the equation $ax + ay = a(x+y)$?
1) associative
2) commutative
3) distributive
4) identity

start and end with same thing

2. The statement $2 + 0 = 2$ is an example of the use of which property of real numbers?
1) associative
2) additive identity
3) additive inverse
4) distributive

3. Which equation illustrates the associative property?
1) $x + y + z = x + y + z$
2) $x(y+z) = xy + xz$
3) $x + y + z = z + y + x$
4) $(x+y) + z = x + (y+z)$

two sets of parenthesis grouped differently

4. If M and A represent integers, $M + A = A + M$ is an example of which property?
(1) commutative
(2) associative
(3) distributive
(4) closure

moving property

5. Which equation illustrates the multiplicative inverse property?
1) $a \cdot 1 = a$
2) $a \cdot 0 = 0$
3) $a\left(\frac{1}{a}\right) = 1$
4) $(-a)(-a) = a^2$

end with identity element

Commutative they moved distributive

6. Britney is solving a quadratic equation. Her first step is shown below.

Problem: $3x^2 - 8 - 10x = 3(2x + 3)$

Step 1: $3x^2 - 10x - 8 = 6x + 9$

Which two properties did Britney use to get to step 1?

I. addition property of equality
II. commutative property of addition
III. multiplication property of equality
IV. distributive property of multiplication over addition

1) I and III
2) I and IV
3) II and III
4) II and IV

distributive

7. A part of Jennifer's work to solve the equation $2(6x^2 - 3) = 11x^2 - x$ is shown below.

Given: $2(6x^2 - 3) = 11x^2 - x$

Step 1: $12x^2 - 6 = 11x^2 - x$

Which property justifies her first step?

1) identity property of multiplication
2) multiplication property of equality
3) commutative property of multiplication
4) distributive property of multiplication over subtraction

8. In the process of solving the equation $10x^2 - 12x - 16x = 6$, George wrote $2(5x^2 - 14x) = 2(3)$, followed by $5x^2 - 14x = 3$. Which properties justify George's process?

$\frac{2(5x^2 - 14x)}{2} = \frac{2(3)}{2}$

A. addition property of equality
B. division property of equality
C. commutative property of addition
D. distributive property

1) A and C
2) A and B
3) D and C
4) D and B

9. When solving $p^2 + 5 = 8p - 7$, Kate wrote $p^2 + 12 = 8p$. The property she used is

+7 +7

1) the associative property
2) the commutative property
3) the distributive property
4) the addition property of equality

10. John was given the equation $4(2a + 3) = -3(a - 1) + 31 - 11a$ to solve. Some of the steps and their reasons have already been completed. State a property of numbers for each missing reason.

$4(2a + 3) = -3(a - 1) + 31 - 11a$	Given
$8a + 12 = -3a + 3 + 31 - 11a$	distributive property
$8a + 12 = 34 - 14a$ +14a +14a	Combining like terms
$22a + 12 = 34$	addition property of equality

11. A student is in the process of solving an equation. The original equation and the first step are shown below.

Original: $3a + 6 = 2 - 5a + 7$ moved them around.

Step one: $3a + 6 = 2 + 7 - 5a$

Which property did the student use for the first step? Explain why this property is correct.

Commutative property because the order changed.

Rational vs. Irrational

Rational	Irrational
Ends of continues with a pattern	Never ends with no pattern
Fraction	π
Perfect Square Radicals	Non Perfect Square Radicals

Addition/Subtraction: If at least one number is irrational, the result is irrational.
Multiplication/Division: If one number is irrational, the result is irrational. If both are irrational, the result can either be rational or irrational.
***If an irrational number is involved, the result is almost always irrational.**

1. Which statement is *not* always true?
1) The product of two irrational numbers is irrational. $I \cdot I = R \text{ or } I$ X
2) The product of two rational numbers is rational. $R \cdot R = R$ ✓
3) The sum of two rational numbers is rational. $R + R = R$ ✓
4) The sum of a rational number and an irrational number is irrational. $R + I = I$ ✓

2. Which statement is *not* always true?
1) The sum of two rational numbers is rational. $R + R = R$ ✓
2) The product of two irrational numbers is rational. $I \cdot I = R \text{ or } I$ X
3) The sum of a rational number and an irrational number is irrational. $R + I = I$ ✓
4) The product of a nonzero rational number and an irrational number is irrational. $R \cdot I = I$ ✓

3. Which expression results in a rational number?
1) $\sqrt{121} - \sqrt{21}$ PS NPS $R - I = I$
2) $\sqrt{25} \bullet \sqrt{50}$ PS NPS $R \cdot I = I$
3) $\sqrt{36} \div \sqrt{225}$ PS PS $R \div R = R$
4) $3\sqrt{5} + 2\sqrt{5}$ NPS NPS $I + I = I$

$R \cdot I = I$

4. The product of $\sqrt{576}$ and $\sqrt{684}$ is
1) irrational because both factors are irrational
2) rational because both factors are rational
3) irrational because one factor is irrational
4) rational because one factor is rational

5. Given the following expressions:

I. $-\frac{5}{8} + \frac{3}{5}$ $R + R = R$

III. $(\sqrt{5}) \cdot (\sqrt{5})$ I I $I \cdot I = R \text{ or } I$ $\sqrt{5} \cdot \sqrt{5} = 5$ R

II. $\frac{1}{2} + \sqrt{2}$ NPS $R + I = I$

IV. $3 \cdot (\sqrt{49})$ PS $R \cdot R = R$

Which expression(s) result in an irrational number?
1) II, only
2) III, only
3) I, III, IV
4) II, III, IV

6. Which expression results in a rational number? $L = \sqrt{2}$ I NPS

1) $L + M$ ~~3)~~ $N + P$ $M = 3\sqrt{3}$ I NPS

2) $M + N$ 4) $P + L$

$N = \sqrt{16}$ R PS

$P = \sqrt{9}$ R PS

7. State whether $7 - \sqrt{2}$ (NPS) is rational or irrational. Explain your answer.

R - I = I A rational minus an irrational is irrational.

8. Determine if the product of $3\sqrt{2}$ (I) and $8\sqrt{18}$ (I = R or I) is rational or irrational. Explain your answer.

$3\sqrt{2} \cdot 8\sqrt{18} = 24(6) = R$

Rational. the product of two irrational numbers could be rational or irrational. In this case, it was rational.

9. Jakob is working on his math homework. He decides that the sum of the expression $\frac{1}{3} + \frac{6\sqrt{5}}{7}$ must be rational because it is a fraction. Is Jakob correct? Explain.

R + I = I No, a rational plus an irrational is irrational

10. Is the sum of $3\sqrt{2}$ and $4\sqrt{2}$ rational or irrational? Explain your answer.

I + I = I

Irrational. Irrational plus irrational is irrational.

11. Ms. Fox asked her class "Is the sum of 4.2 (R) and $\sqrt{2}$ (I NPS) rational or irrational?" Patrick answered that the sum would be irrational. State whether Patrick is correct or incorrect. Justify your reasoning.

R + I = I

Yes, rational plus irrational is irrational.

12. A teacher wrote the following set of numbers on the board:
Explain why $a + b$ is irrational, but $b + c$ is rational.

$a = \sqrt{20}$ (NPS, I) $b = 2.5$ (R) $c = \sqrt{225}$ (PS, R)

a+b: I+R = I Irrational + rational is irrational.

b+C: R+R = R Rational + rational is rational.

$\sqrt{225}$ is rational b/c it is a perfect square.

13. Is the product of $\sqrt{16}$ (PS, R) and $\frac{4}{7}$ (R) rational or irrational? Explain your reasoning.

R·R = R

rational times a rational is rational.

Polynomial Standard Form

A polynomial in standard form has the term with the highest exponent first and is followed by terms in decreasing exponential order.

The number in front of the term with the highest exponent is called the leading coefficient

Write the following polynomials in standard form, state their degree, leading coefficient, and constant term.

1. $9x^3 - x^4 + 1 + 2x^6$

$2x^6 - x^4 + 9x^3 + 1$

degree: 6

Leading coefficient: 2

Constant term: 1

2. $5y + 4 - 3y^5$

$-3y^5 + 5y + 4$

degree: 5

leading coefficient: -3

constant term: 4

3. An expression of the fifth degree is written with a leading coefficient of seven and a constant of six. Which expression is correctly written for these conditions?

1) $6x^5 + x^4 + 7$
2) $7x^6 - 6x^4 + 5$
3) $6x^7 - x^5 + 5$
4) $7x^5 + 2x^2 + 6$

4. Mrs. Allard asked her students to identify which of the polynomials below are in standard form and explain why.

I. $15x^4 - 6x + 3x^2 - 1$ ✗

II. $12x^3 + 8x + 4$ ✓

III. $2x^5 + 8x^2 + 10x$ ✓

Which student's response is correct?

1) Tyler said I and II because the coefficients are decreasing.
2) Susan said only II because all the numbers are decreasing.
3) Fred said II and III because the exponents are decreasing.
4) Alyssa said II and III because they each have three terms.

5. When multiplying polynomials for a math assignment, Pat found the product to be $-4x + 8x^2 - 2x^3 + 5$. He then had to state the leading coefficient of this polynomial. Pat wrote down -4. Do you agree with Pat's answer? Explain your reasoning.

No, the coefficient of the term with the highest exponent is -2.

6. Mrs. Allard asked her students to identify which of the polynomials below are in standard form and explain why.

I. $15x^4 - 6x + 3x^2 - 1$

II. $12x^3 + 8x + 4$ ✓

III. $2x^5 + 8x^2 + 10x$ ✓

Which student's response is correct?

1) Tyler said I and II because the coefficients are decreasing.
2) Susan said only II because all the numbers are decreasing.
3) Fred said II and III because the exponents are decreasing.
4) Alyssa said II and III because they each have three terms.

7. Students were asked to write $6x^5 + 8x - 3x^3 + 7x^7$ in standard form. Shown below are four student responses.

Anne: $7x^7 + 6x^5 - 3x^3 + 8x$ highest exponent first and then decreasing order

Bob: $-3x^3 + 6x^5 + 7x^7 + 8x$

Carrie: $8x + 7x^7 + 6x^5 - 3x^3$

Dylan: $8x - 3x^3 + 6x^5 + 7x^7$

Which student is correct?

1) Anne
2) Bob
3) Carrie
4) Dylan

8. Which polynomial has a leading coefficient of 4 and a degree of 3?

1) $3x^4 - 2x^2 + 4x - 7$
2) $4 + x - 4x^2 + 5x^3$
3) $4x^4 - 3x^3 + 2x^2$
4) $2x + x^2 + 4x^3$

9. Students were asked to write an expression which had a leading coefficient of 3 and a constant term of -4. Which response is correct?

1) $3 - 2x^3 - 4x$
2) $7x^3 - 3x^5 - 4$
3) $4 - 7x + 3x^3$
4) $-4x^2 + 3x^4 - 4$ → constant term

leading coefficient

10. An example of a sixth-degree polynomial with a leading coefficient of seven and a constant term of four is

1) $6x^7 - x^5 + 2x + 4$
2) $4 + x + 7x^6 - 3x^2$
3) $7x^4 + 6 + x^2$
4) $5x + 4x^6 + 7$

11. What is the constant term of the polynomial $4d + 6 + 3d^2$?

1) 6
2) 2
3) 3
4) 4

Operations with Polynomials
Adding: Combine like terms
Subtracting: Distribute the negative and then combine like terms (Keep Change Change)
***Subtracting from: from comes first!**
Multiplying: Box Method (Multiply to determine what's in the boxes, add to combine like terms)
Dividing: Divide every term in the numerator by the denominator
*Use Multiple Choice Strategy if Multiple Choice

1. If $A = 3x^2 + 5x - 6$ and $B = -2x^2 - 6x + 7$, then $A - B$ equals
1) $-5x^2 - 11x + 13$
2) $5x^2 + 11x - 13$
3) $-5x^2 - x + 1$
4) $5x^2 - x + 1$

$(3x^2+5x-6)-(-2x^2-6x+7)$

$3x^2+5x-6$
$+\ 2x^2+6x-7$
$5x^2+11x-13$

2. What is the result when $6x^2 - 13x + 12$ is subtracted from $-3x^2 + 6x + 7$?
1) $3x^2 - 7x + 19$
2) $9x^2 - 19x + 5$
3) $9x^2 - 7x + 19$
4) $-9x^2 + 19x - 5$

$(-3x^2+6x+7)-(6x^2-13x+12)$

$-3x^2+6x+7$
$+\ -6x^2+13x-12$
$-9x^2+19x-5$

3. What is the result when $4x^2 - 17x + 36$ is subtracted from $2x^2 - 5x + 25$?
1) $6x^2 - 22x + 61$
2) $2x^2 - 12x + 11$
3) $-2x^2 - 22x + 61$
4) $-2x^2 + 12x - 11$

$(2x^2-5x+25)-(4x^2-17x+36)$

$2x^2-5x+25$
$+\ -4x^2+17x-36$
$-2x^2+12x-11$

4. Which expression is equivalent to $2(3g - 4) - 1(8g + 3)$?
1) $-2g - 1$
2) $-2g - 5$
3) $-2g - 7$
4) $-2g - 11$

$6g-8-8g-3$ $\quad -2g-11$

5. What is the product of $2x + 3$ and $4x^2 - 5x + 6$?
1) $8x^3 - 2x^2 + 3x + 18$
2) $8x^3 - 2x^2 - 3x + 18$
3) $8x^3 + 2x^2 - 3x + 18$
4) $8x^3 + 2x^2 + 3x + 18$

$(2x+3)(4x^2-5x+6)$

	$4x^2$	$-5x$	$+6$
$2x$	$8x^3$	$-10x^2$	$+12x$
$+3$	$+12x^2$	$-15x$	$+18$

$8x^3+2x^2-3x+18$

6. The expression $3(x^2 - 1) - 1(x^2 - 7x + 10)$ is equivalent to
1) $2x^2 - 7x + 7$
2) $2x^2 + 7x - 13$
3) $2x^2 - 7x + 9$
4) $2x^2 + 7x - 11$

$3x^2-3-x^2+7x-10$

$2x^2+7x-13$

7. Express in simplest form: $(3x^2 + 4x - 8) - (-2x^2 + 4x + 2)$

$3x^2+4x-8$
$+2x^2-4x-2$

$5x^2-10$

8. Express the product of $2x^2 + 7x - 10$ and $x + 5$ in standard form.

	$2x^2$	$+7x$	-10
x	$2x^3$	$+7x^2$	$-10x$
$+5$	$+10x^2$	$+35x$	-50

$2x^3+17x^2+25x-50$

9. Multiply $(2x^2 + 3x - 2)(x - 2)$

	$2x^2$	$+3x$	-2
x	$2x^3$	$+3x^2$	$-2x$
-2	$-4x^2$	$-6x$	$+4$

$2x^3-x^2-8x+4$

10. Write the expression $5x + 4x^2(2x + 7) - 6x^2 - 9x$ as a polynomial in standard form.

$5x+8x^3+28x^2-6x^2-9x$

$8x^3+22x^2-4x$

11. Given that $f(x) = 2x + 1$, find $g(x)$ if $g(x) = 2[f(x)]^2 - 1$.

	$2x$	$+1$
$2x$	$4x^2$	$+2x$
$+1$	$+2x$	$+1$

$4x^2+4x+1$

$2(2x+1)^2-1$

$2(4x^2+4x+1)-1$

$8x^2+8x+2-1$

$8x^2+8x+1$

Solving Linear Equations and Inequalities

1) Get rid of fractions (Multiply by the LCD)
2) Get rid of parenthesis (Distribute)
3) Combine like terms on each side
4) Bring all variables to one side
5) Isolate variable (add/subtract first, divide last)

*When dividing/multiplying by a negative in an inequality, switch the inequality!

*Be careful which direction the inequality sign is facing when you write your solution

Solve the following equations

1. $-2(1-4x) = 3x+8$ is

$-2+8x = 3x+8$

$-3x \quad -3x$

$-2+5x = 8$

$+2 \quad +2$

$\frac{5x}{5} = \frac{10}{5}$

$x = 2$

2. $4(x-7) = 0.3(x+2)+2.11$

$4x-28 = .3x+.6+2.11$

$4x-28 = .3x+2.71$

$-.3x \quad -.3x$

$3.7x-28 = 2.71$

$+28 \quad +28$

$\frac{3.7x}{3.7} = \frac{30.71}{3.7}$

$x = 8.3$

3. $\left(\frac{x-2}{3}\right)+\left(\frac{1}{6}\right)=\left(\frac{5}{6}\right)$

$2(x-2)+1=5$

$2x-4+1=5$

$2x-3=5$

$+3 \quad +3$

$\frac{2x}{2}=\frac{8}{2}$

$x=4$

4. $\left(\frac{1}{7}\right)+\left(\frac{2x}{3}\right)=\left(\frac{15x-3}{21}\right)$

$3+14x=15x-3$

$-14x \quad -14x$

$3=x-3$

$+3 \quad +3$

$6=x$

5. $\left(\frac{2x}{3}\right)+\left(\frac{x}{6}\right)=(5)$

$4x+x=30$

$\frac{5x}{5}=\frac{30}{5}$

$\boxed{x=6}$

6. $\left(\frac{2x}{5}\right)+\left(\frac{1}{3}\right)=\left(\frac{7x-2}{15}\right)$

$6x+5=7x-2$

$-6x \quad -6x$

$5=x-2$

$+2 \quad +2$

$7=x$

7. $\left(\frac{2}{3}x\right)+\left(\frac{1}{2}\right)=\left(\frac{5}{6}\right)$

$$4x+3=5$$
$$-3 \quad -3$$
$$\frac{4x}{4}=\frac{2}{4}$$
$$x=\frac{1}{2}$$

8. $\left(\frac{m}{5}\right)+\left(\frac{3(m-1)}{2}\right)=\left(2(m-3)\right)$

$$2m+15(m-1)=20(m-3)$$
$$2m+15m-15=20m-60$$
$$17m-15=20m-60$$
$$-17m \quad -17m$$
$$-15=3m-60$$
$$+60 \quad +60$$
$$\frac{45}{3}=\frac{3m}{3}$$
$$15=m$$

9. $\left(-\frac{2}{3}(x+12)\right)+\left(\frac{2}{3}x\right)=\left(-\frac{5}{4}x\right)+\left(2\right)$

$$-8(x+12)+8x=-15x+24$$
$$-8x-96+8x=-15x+24$$
$$-96=-15x+24$$
$$-24 \quad -24$$
$$\frac{-120}{-15}=\frac{-15x}{-15}$$
$$8=x$$

10. $\left(6\right)-\left(\frac{2}{3}(x+5)\right)=\left(4x\right)$

$$18-2(x+5)=12x$$
$$18-2x-10=12x$$
$$-2x+8=12x$$
$$+2x \quad +2x$$
$$\frac{8}{14}=\frac{14x}{14}$$
$$\frac{4}{7}=x$$

11. When $3x+2 \leq 5(x-4)$ is solved for x, the solution is

1) $x \leq 3$
2) $x \geq 3$
3) $x \leq -11$
4) $x \geq 11$

$$3x+2 \leq 5x-20$$
$$-3x \quad -3x$$
$$2 \leq 2x-20$$
$$+20 \quad +20$$
$$\frac{22}{2} \leq \frac{2x}{2}$$
$$11 \leq x$$
$$x \geq 11$$

12. What is the solution to $2h+8>3h-6$

1) $h<14$
2) $h<\frac{14}{5}$
3) $h>14$
4) $h>\frac{14}{5}$

$$-2h \quad -2h$$
$$8>h-6$$
$$+6 \quad +6$$
$$14>h$$
$$h<14$$

13. The solution to $4p+2<2(p+5)$ is

1) $p>-6$
2) $p<-6$
3) $p>4$
4) $p<4$

$$4p+2<2p+10$$
$$-2p \quad -2p$$
$$2p+2<10$$
$$-2 \quad -2$$
$$\frac{2p}{2}<\frac{8}{2}$$
$$p<4$$

14. What is the solution to the inequality $2+\frac{4}{9}x \ge 4+x$?

1) $x \le -\frac{18}{5}$
2) $x \ge -\frac{18}{5}$
3) $x \le \frac{54}{5}$
4) $x \ge \frac{54}{5}$

$18+4x \ge 36+9x$

$-4x \quad -4x$

$18 \ge 36+5x$

$-36 \quad -36$

$\frac{-18}{5} \ge \frac{5x}{5}$

$x \le -\frac{18}{5}$

15. The inequality $7-\frac{2}{3}x < x-8$ is equivalent to

1) $x > 9$
2) $x > -\frac{3}{5}$
3) $x < 9$
4) $x < -\frac{3}{5}$

$21-2x < 3x-24$

$+2x \quad +2x$

$21 < 5x-24$

$+24 \quad +24$

$\frac{45}{5} < \frac{5x}{5}$

$9 < x$

$x > 9$

16. Which value would be a solution for x in the inequality $47 - 4x < 7$?

1) -13
2) -10
3) 10
4) 11

$-47 \quad -47$

$\frac{-4x}{-4} < \frac{-40}{-4}$

$x > 10$

Switch the inequality when dividing by a negative

11 is the only value greater than 10

17. Which value of x is not contained in the solution set of $7x - 3(4x - 8) \le 6x + 12 - 9x$?

1) 4
2) 6
3) 8
4) 10

the only value not ≥ 6

$7x-12x+24 \le 6x+12-9x$

$-5x+24 \le -3x+12$

$+5x \quad +5x$

$24 \le 2x+12$

$-12 \quad -12$

$\frac{12}{2} \le \frac{2x}{2}$

$6 \le x$

$x \ge 6$

18. Solve the inequality below to determine and state the smallest possible value for x in the solution set.

$3(x+3) \le 5x-3$

$3x+9 \le 5x-3$

$-3x \quad -3x$

$9 \le 2x-3$

$+3 \quad +3$

$\frac{12}{2} \le \frac{2x}{2}$

$6 \le x$

$x \ge 6$

$\boxed{6}$

19. Determine the smallest integer that makes $-3x + 7 - 5x < 15$ true.

$-8x+7 < 15$

$-7 \quad -7$

$\frac{-8x}{-8} < \frac{8}{-8}$

$x > -1$

-2 -1 0

$\boxed{0}$

Literal Equations
Follow same steps as equation solving. Don't combine unlike terms.
When isolating, add or subtract first, divide last

1. If $abx - 5 = 0$, what is x in terms of a and b?

1) $x = \frac{5}{ab}$
2) $x = -\frac{5}{ab}$
3) $x = 5 - ab$
4) $x = ab - 5$

$abx - 5 = 0$
$+5 \quad +5$
$\frac{abx}{ab} = \frac{5}{ab}$
$x = \frac{5}{ab}$

2. The formula for electrical power, P, is $P = I^2R$, where I is current and R is resistance. The formula for I in terms of P and R is

1) $I = \left(\frac{P}{R}\right)^2$
2) $I = \sqrt{\frac{P}{R}}$
3) $I = (P - R)^2$
4) $I = \sqrt{P - R}$

$\frac{P}{R} = \frac{I^2R}{R}$
$\sqrt{\frac{P}{R}} = \sqrt{I^2}$
$\sqrt{\frac{P}{R}} = I$

3. Boyle's Law involves the pressure and volume of gas in a container. It can be represented by the formula $P_1V_1 = P_2V_2$. When the formula is solved for P_2, the result is

1) $P_1V_1V_2$
2) $\frac{V_2}{P_1V_1}$
3) $\frac{P_1V_1}{V_2}$
4) $\frac{P_1V_2}{V_1}$

$\frac{P_1V_1}{V_2} = \frac{P_2V_2}{V_2}$
$\frac{P_1V_1}{V_2} = P_2$

4. The formula for the sum of the degree measures of the interior angles of a polygon is $S = 180(n - 2)$. Solve for n, the number of sides of the polygon, in terms of S.

$S = 180(n-2)$
$S = 180n - 360$
$+360 \quad +360$
$\frac{S+360}{180} = \frac{180n}{180}$
$\frac{S+360}{180} = n$

5. The equation for the volume of a cylinder is $V = \pi r^2 h$. The positive value of r, in terms of h and V, is

1) $r = \sqrt{\frac{V}{\pi h}}$
2) $r = \sqrt{V\pi h}$
3) $r = 2V\pi h$
4) $r = \frac{V}{2\pi}$

$\frac{V}{\pi h} = \frac{\pi r^2 h}{\pi h}$
$\sqrt{\frac{V}{\pi h}} = \sqrt{r^2}$
$\sqrt{\frac{V}{\pi h}} = r$

6. The amount of energy, Q, in joules, needed to raise the temperature of m grams of a substance is given by the formula $Q = mC(T_f - T_i)$, where C is the specific heat capacity of the substance. If its initial temperature is T_i, an equation to find its final temperature, T_f, is

1) $T_f = \frac{Q}{mC} - T_i$

2) $T_f = \frac{Q}{mC} + T_i$

3) $T_f = \frac{T_i + Q}{mC}$

4) $T_f = \frac{Q - mC}{T_i}$

$Q = mC(T_f - T_i)$

$Q = mCT_f - mCT_i$

$+mCT_i \quad +mCT_i$

$\frac{Q + mCT_i = mCT_f}{mC}$

$\frac{Q + mCT_i}{mC} = T_f$

$\frac{Q}{mC} + T_i = T_f$

7. The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. Express b_1 in terms of A, h, and b_2.

$2A = h(b_1 + b_2)$

$2A = hb_1 + hb_2$

$-hb_2 \quad -hb_2$

$\frac{2A - hb_2}{h} = \frac{hb_1}{h}$

$\frac{2A - hb_2}{h} = b_1$

8. The formula for converting degrees Fahrenheit (F) to degrees Kelvin (K) is

$K = \frac{5}{9}(F + 459.67)$

Solve for F, in terms of K.

$9K = 5(F + 459.67)$

$9K = 5F + 2298.35$

$-2298.35 \quad -2298.35$

$\frac{9K - 2298.35}{5} = \frac{5F}{5}$

$\frac{9K - 2298.35}{5} = F$

9. The volume of a trapezoidal prism can be found using the formula $V = \frac{1}{2}a(b + c)h$.

Which equation is correctly solved for b?

1) $b = \frac{V}{2ah} + c$

2) $b = \frac{V}{2ah} - c$

3) $b = \frac{2V}{ah} + c$

4) $b = \frac{2V}{ah} - c$

$2V = ah(b + c)$

$2V = abh + ach$

$-ach \quad -ach$

$\frac{2V - ach}{ah} = \frac{abh}{ah}$

$\frac{2V - ach}{ah} = b$

$\frac{2V}{ah} - c = b$

10. The formula $F_g = \frac{GM_1M_2}{r^2}$ calculates the gravitational force between two objects where G is the gravitational constant, M_1 is the mass of one object, M_2 is the mass of the other object, and r is the distance between them. Solve for the positive value of r in terms of F_g, G, M_1, and M_2.

$\frac{r^2 F_g}{F_g} = \frac{GM_1M_2}{F_g}$

$\sqrt{r^2} = \sqrt{\frac{GM_1M_2}{F_g}}$

$r = \sqrt{\frac{GM_1M_2}{F_g}}$

In Terms of x

1) List the things you are comparing
2) Call the last thing x
3) Express everything else in terms of x
4) Create an equation to represent the situation and solve
5) Substitute x into your original expressions and answer the question

1. Jeff and Danielle collect basketball cards. Jeff has 4 less than twice as many basketball cards than Danielle has. If they have a total of 44 basketball cards, which equation could be used to determine the number of basketball cards, d, that Danielle has?

J: 2d-4
D: d

1) $d+(d-4)=44$
2) $(2d-4)+d=44$
3) $2d-4=44$
4) $d+(4-2d)=44$

2. The Yankees won 6 less than twice as many games as they lost in a season. If there are 162 games in a season, which equation can be used to determine how many games the Yankees lost, x?

w: 2x-6
l: x

1) $2x-6=162$
2) $x+(2x-6)=162$
3) $(6+2x)+x=162$
4) $(6-2x)+x=162$

3. Nicci's sister is 7 years less than twice Nicci's age, *a*. The sum of Nicci's age and her sister's age is 41. Which equation represents this relationship?

S: 2a-7
N: a

1) $a+(7-2a)=41$
2) $a+(2a-7)=41$
3) $2a-7=41$
4) $a=2a-7$

4. Nadia has some change at the bottom of her purse. She has nine more dimes than quarters. If the value of her change is \$3.35, which equation can be used to determine the number of quarters, x, she has in her purse?

D: x+9
Q: x

1) $.25x+.10x=3.35$
2) $x+x+9=3.35$
3) $.10x+.25(x+9)=3.35$
4) $.10(x+9)+.25x=3.35$

5. John has four more nickels than dimes in his pocket, for a total of \$1.25. Which equation could be used to determine the number of dimes, *x*, in his pocket?

N: x+4
D: x

1) $0.10(x+4)+0.05(x)=\$1.25$
2) $0.05(x+4)+0.10(x)=\$1.25$
3) $0.10(4x)+0.05(x)=\$1.25$
4) $0.05(4x)+0.10(x)=\$1.25$

6. Joe has dimes and nickels in his piggy bank totaling \$1.45. The number of nickels he has is 5 more than twice the number of dimes, *d*. Which equation could be used to find the number of dimes he has?

N: 2d+5
D: d

1) $0.10d+0.05(2d+5)=1.45$
2) $0.10(2d+5)+0.05d=1.45$
3) $d+(2d+5)=1.45$
4) $(d-5)+2d=1.45$

7. Jamie is 5 years older than her sister Amy. If the sum of their ages is 19, how old is Jamie?

J: $x+5$ $\quad 7+5=\boxed{12}$

A: x

$x+x+5=19$

$2x+\cancel{5}=19$
$-\cancel{5} \quad -5$

$\frac{2x}{2}=\frac{14}{2}$

$x=7$

8. Ben has four more than twice as many CDs as Jake. If they have a total of 31 CDs, how many CDs does Jake have?

B: $2x+4$

J: $x=\boxed{9}$

$x+2x+4=31$

$3x+\cancel{4}=31$
$-\cancel{4} \quad -4$

$\frac{3x}{3}=\frac{27}{3}$

$x=9$

9. A varsity basketball team has 3 less than four times as many senior as sophomores and 5 more juniors than sophomores. If there are 20 players on the team, how many are sophomores?

Se: $4x-3$

So: $x=\boxed{3}$

Ju: $x+5$

$4x-3+x+x+5=20$

$6x+\cancel{2}=20$
$-\cancel{2} \quad -2$

$\frac{6x}{6}=\frac{18}{6}$

$x=3$

10. At midtown high school, the sophomore class has 30 more students than the freshman class. The junior class has 20 less students than the freshman class, and the senior class has 50 less than twice the students in the freshman class. If there are 180 students in the schools, how many students are in the sophomore class?

So: $x+30$ $\quad 44+30=\boxed{74}$

F: x

J: $x-20$

Se: $2x-50$

$x+30+x+x-20+2x-50=180$

$5x-\cancel{40}=180$
$+\cancel{40} \quad +40$

$\frac{5x}{5}=\frac{220}{5}$

$x=44$

11. A store sells grapes for \$1.99 per pound, strawberries for \$2.50 per pound, and pineapples for \$2.99 each. Jonathan has \$25 to buy fruit. He plans to buy 2 more pounds of strawberries than grapes. He also plans to buy 2 pineapples. If x represents the number of pounds of grapes, write an inequality in one variable that models this scenario. Determine algebraically the maximum number of whole pounds of grapes he can buy.

S: x+2
g: $x = \boxed{3}$
P: 2

$1.99x + 2.50(x+2) + 2(2.99) \leq 25$
$1.99x + 2.50x + 5 + 5.98 \leq 25$
$4.49x + 10.98 \leq 25$
$-10.98 \quad -10.98$

$\frac{4.49x}{4.49} \leq \frac{14.02}{4.49}$
$x \leq 3.12..$
$x = 3$

12. Hannah went to the school store to buy supplies and spent \$16. She bought four more pencils than pens and two fewer erasers than pens. Pens cost \$1.25 each, pencils cost \$0.55 each, and erasers cost \$0.75 each. If x represents the number of pens Hannah bought, write an equation in terms of x that can be used to find how many of each item she bought. Use your equation to determine algebraically how many pens Hannah bought.

Pencil: x+4
pen: $x = \boxed{6}$
eraser: x-2

$.55(x+4) + 1.25x + .75(x-2) = 16$
$.55x + 2.2 + 1.25x + .75x - 1.5 = 16$
$2.55x + .7 = 16$
$-.7 \quad -.7$

$\frac{2.55x}{2.55} = \frac{15.3}{2.55}$
$x = 6$

13. Franklin has a jar full of nickels and dimes in a jar that he is bringing to the bank. He has one less than twice as many nickels are dimes. If his coins have a value of \$5.95, how many nickels does he have?

N: 2x-1   2(30)-1=59
D: x
30
59 Nickels

$.05(2x-1) + .10x = 5.95$
$.1x - .05 + .1x = 5.95$
$.2x - .05 = 5.95$
$+.05 \quad +.05$

$\frac{.2x}{.2} = \frac{6}{.2}$
$x = 30$

14. Dave has four more quarters than dimes. He has six less nickels than dimes. If he has a total of \$4.30 in his pocket, how many dimes does he have?

Q: x+4   9+4=13
D: x   9
N: x-6
9-6=3

$.05(x-6) + .10x + .25(x+4) = 4.30$
$.05x - .3 + .1x + .25x + 1 = 4.30$
$.4x + .7 = 4.30$
$-.7 \quad -.7$
$.4x = 3.6$
$\frac{.4x}{.4} = \frac{3.6}{.4}$
$x = 9$

9 Dimes

Modeling Linear Functions
y = Per/Eachx + one time fee
y = slopex + y-intercept
Slope = per/each, y intercept = one time fee/starting amount
Per or each (slope) goes in front of x, the one time fee or one time starting amount (y-intercept) goes at the end. x represents what the per is for. For example, if something costs $5 per hour, x is hours.
Same: Set the two equations equal to each other
Context for Linear Functions: The slope is the y unit per x unit. The y intercept is the starting amount y units or the amount when x is 0.

1. The cost of airing a commercial on television is modeled by the function
perx + ITF
$C(n) = 110n + 900$, where n is the number of times the commercial is aired. Based on this model, which statement is true?
1) The commercial costs $0 to produce and $110 per airing up to $900.
2) The commercial costs $110 to produce and $900 each time it is aired.
3) The commercial costs $900 to produce and $110 each time it is aired.
4) The commercial costs $1010 to produce and can air an unlimited number of times.

2. The cost of airing a commercial on television is modeled by the function
$C(n) = 110n + 900$, where n is the number of times the commercial is aired. Based on this model, which statement is true?
1) The commercial costs $0 to produce and $110 per airing up to $900.
2) The commercial costs $110 to produce and $900 each time it is aired.
3) The commercial costs $900 to produce and $110 each time it is aired.
4) The commercial costs $1010 to produce and can air an unlimited number of times.

3. A company that manufactures radios first pays a start-up cost, and then spends a certain amount of money to manufacture each radio. If the cost of manufacturing r radios is given by the function $c(r) = 5.25r + 125$, then the value 5.25 best represents
perx + ITF
1) the start-up cost
2) the profit earned from the sale of one radio
3) the amount spent to manufacture each radio
4) the average number of radios manufactured

4. A satellite television company charges a one-time installation fee and a monthly service charge. The total cost is modeled by the function $y = 40 + 90x$. Which statement represents the meaning of each part of the function? ITF Perx
1) y is the total cost, x is the number of months of service, $90 is the installation fee, and $40 is the service charge per month.
2) y is the total cost, x is the number of months of service, $40 is the installation fee, and $90 is the service charge per month.
3) x is the total cost, y is the number of months of service, $40 is the installation fee, and $90 is the service charge per month.
4) x is the total cost, y is the number of months of service, $90 is the installation fee, and $40 is the service charge per month.

2.40x

5. Each day, a local dog shelter spends an average of \$2.40 on food per dog. The manager estimates the shelter's daily expenses, assuming there is at least one dog in the shelter, using the function $E(x) = 30 + 2.40x$. Which statements regarding the function $E(x)$ are correct?

I. x represents the number of dogs at the shelter per day. ✓
II. x represents the number of volunteers at the shelter per day. X
III. 30 represents the shelter's total expenses per day. X
IV. 30 represents the shelter's nonfood expenses per day. ✓ one time fee

1) I and III
2) I and IV
3) II and III
4) II and IV

6. A plumber has a set fee for a house call and charges by the hour for repairs. The total cost of her services can be modeled by $c(t) = 125t + 95$. Which statements about this function are true?

per x + ITF

I. A house call fee costs \$95. one time fee ✓
II. The plumber charges \$125 per hour. ✓
III. The number of hours the job takes is represented by t. ✓ per hour

1) I and II, only
2) I and III, only
3) II and III, only
4) I, II, and III

per x + ITF

7. The amount Mike gets paid weekly can be represented by the expression $2.50a + 290$, where a is the number of cell phone accessories he sells that week. What is the constant term in this expression and what does it represent?

1) $2.50a$, the amount he is guaranteed to be paid each week
2) $2.50a$, the amount he earns when he sells a accessories
3) 290, the amount he is guaranteed to be paid each week
4) 290, the amount he earns when he sells a accessories

8. A car leaves Albany, NY, and travels west toward Buffalo, NY. The equation $D = 280 - 59t$ can be used to represent the distance, D, from Buffalo after t hours. In this equation, the 59 represents the

per

1) car's distance from Albany
2) speed of the car
3) distance between Buffalo and Albany
4) number of hours driving

miles per hour

9. The table below shows the height in feet, $h(t)$, of a hot-air balloon and the number of minutes, t, the balloon is in the air.

Time (min)	2	5	7	10	12
Height (ft)	64	168	222	318	369

per x + ITF

The function $h(t) = 30.5t + 8.7$ can be used to model this data table. Explain the meaning of the slope in the context of the problem. Explain the meaning of the y-intercept in the context of the problem.

slope: The height of the ballon increased 30.5ft per minute

y-intercept: The initial height of the ballon was 8.7 ft.

Per x + ITF

10. The cost of belonging to a gym can be modeled by $C(m) = 50m + 79.50$, where $C(m)$ is the total cost for m months of membership. State the meaning of the slope and y-intercept of this function with respect to the costs associated with the gym membership.

slope: The gym costs $50 per month
y-int: The one time registration fee is $79.50.

11. Omar has a piece of rope. He ties a knot in the rope and measures the new length of the rope. He then repeats this process several times. Some of the data collected are listed in the table below.

Number of Knots	4	5	6	7	8
Length of Rope (cm)	64	58	49	39	31

Per x + ITF

The equation $y = -8.5x + 99.2$ represents this situation. Explain what the y-intercept means in the context of the problem. Explain what the slope means in the context of the problem.

slope: The length of the rope decreases by 8.5cm per knot.
y-int: The initial length of the rope is 99.2cm.

12. Tanya is making homemade greeting cards. The data table below represents the amount she spends in dollars, $f(x)$, in terms of the number of cards she makes, x. The data can be represented by the equation $f(x) = .75x + 4.5$. Explain what the slope and y-intercept of $f(x)$ mean in the given context. Per x + ITF

x	f(x)
4	7.50
6	9
9	11.25
10	12

Slope: The cost is $.75 per greeting card
y-int: The initial cost for supplies is $4.50.

13. During a recent snowstorm in Red Hook, NY, Jaime noted that there were 4 inches of snow on the ground at 3:00 p.m., and there were 6 inches of snow on the ground at 7:00 p.m. The situation can be represented by the equation $f(t) = \frac{1}{2}t$ where $f(t)$ represents the amount of snow after t hours. What does the slope of the line represent in the context of this problem?

slope: The amount of snow on the ground increased by $\frac{1}{2}$ inch per hour.

$.05x$

14. A cell phone company charges \$60.00 a month for up to 1 gigabyte of data. The cost of additional data is \$0.05 per megabyte. If d represents the number of additional megabytes used and c represents the total charges at the end of the month, which linear equation can be used to determine a user's monthly bill?

1) $c = 60 - 0.05d$
2) $c = 60.05d$
3) $c = 60d - 0.05$
4) $c = 60 + 0.05d$

one time fee

15. At Benny's Cafe, a mixed-greens salad costs \$5.75. Additional toppings can be added for \$0.75 each. Which function could be used to determine the cost, $c(s)$, in dollars, of a salad with s additional toppings? $.75x$

1) $c(s) = 5.75s + 0.75$
2) $c(s) = 0.75s + 5.75$
3) $c(s) = 5.00s + 0.75$
4) $c(s) = 0.75s + 5.00$

16. A gardener is planting two types of trees:

Type A is 36 inches tall and grows at a rate of 15 inches per year. $15x$

Type B is 48 inches tall and grows at a rate of 10 inches per year. $10x$

Algebraically determine exactly how many years it will take for these trees to be the same height.

equal

$A(x) = 15x + 36$
$B(x) = 10x + 48$

$15x + 36 = 10x + 48$
$-10x \quad -10x$
$5x + 36 = 48$
$-36 \quad -36$

$\frac{5x}{5} = \frac{12}{5}$
$x = 2.4$

17. A local business was looking to hire a landscaper to work on their property. They narrowed their choices to two companies. Flourish Landscaping Company charges a flat rate of \$120 per hour. Green Thumb Landscapers charges \$70 per hour plus a \$1600 equipment fee. Write a system of equations representing how much each company charges. Determine and state the number of hours that must be worked for the cost of each company to be the same. Justify your answer. $120x$

equal

$70x$

$g(x) = 70x + 1600$
$f(x) = 120x$

$70x + 1600 = 120x$
$-70x \quad -70x$
$\frac{1600}{50} = \frac{50x}{50}$

$x = 32$

$60x$

18. Ian is borrowing \$1000 from his parents to buy a notebook computer. He plans to pay them back at the rate of \$60 per month. Ken is borrowing \$600 from his parents to purchase a snowboard. He plans to pay his parents back at the rate of \$20 per month. Write an equation that can be used to determine after how many months the boys will owe the same amount. Determine algebraically and state in how many months the two boys will owe the same amount. State the amount they will owe at this time. $20x$ equal

$I(x) = 1000 - 60x$
$K(x) = 600 - 20x$

$1000 - 60x = 600 - 20x$
$+60x \quad +60x$
$1000 = 600 + 40x$
$-600 \quad -600$
$\frac{400}{40} = \frac{40x}{40}$

$10 = x$

10 weeks

$600 - 20(10)$
$600 - 200$
\$ 400

*the amount they owe is declasing as the pay their patents back

19. Next weekend Marnie wants to attend either carnival A or carnival B. Carnival A charges \$6 for admission and an additional \$1.50 per ride. Carnival B charges \$2.50 for admission and an additional \$2 per ride. $2x$ $1.50x$

a) In function notation, write $A(x)$ to represent the total cost of attending carnival A and going on x rides. In function notation, write $B(x)$ to represent the total cost of attending carnival B and going on x rides.

b) Determine the number of rides Marnie can go on such that the total cost of attending each carnival is the same. equal

$A(x) = 1.50x + 6$

$B(x) = 2x + 2.50$

$1.50x + 6 = 2x + 2.50$

$-1.50x \quad -1.50x$

$6 = .50x + 2.50$

$-2.50 \quad -2.50$

$\frac{3.50}{.50} = \frac{.50x}{.50}$

$7 = x$

7 rides

20. Central High School had five members on their swim team in 2010. Over the next several years, the team increased by an average of 10 members per year. The same school had 35 members in their chorus in 2010. The chorus saw an increase of 5 members per year. Write a system of equations to model this situation, where x represents the number of years since 2010. After how many weeks will the swim team and the chorus have the same number of members? $10x$ $5x$

equal

$S(x) = 10x + 5$

$C(x) = 5x + 35$

$10x + 5 = 5x + 35$

$-5x \quad -5x$

$5x + 5 = 35$

$-5 \quad -5$

$\frac{5x}{5} = \frac{30}{5}$

$x = 6$

6 weeks

21. Aidan and his sister Ella are having a race. Aidan runs at a rate of 10 feet per second. Ella runs at a rate of 6 feet per second. Since Ella is younger, Aidan is letting her begin 30 feet ahead of the starting line. Let y represent the distance from the starting line and x represent the time elapsed, in seconds. Write an equation to model the distance Aidan traveled. Write an equation to model the distance Ella traveled. Exactly how many seconds does it take Aidan to catch up to Ella? Justify your answer. $10x$ $6x$

equal

$A(x) = 10x$

$E(x) = 6x + 30$

$10x = 6x + 30$

$-6x \quad -6x$

$\frac{4x}{4} = \frac{30}{4}$

$x = 7.5$ seconds

Systems of Equations with Elimination

1) Choose a variable to cancel and multiply each equation by the other's coefficient

*multiply by negative if they are the same sign

2) Add equations together

3) Solve equation for one variable

4) Substitute answer in to either equation to find the second variable

*You can find an equivalent system by multiplying either equation by any constant

*For word problems, the first equation is just $x + y =$ for an amount. The second equation is usually a money equation.

***For money, the second equation is** $.01p$, $.05n$, $.10d$, *or* $.25q$

1. $9(c - 2d = 14)$
   $2(3c + 9d = 27)$

   $9c - 18d = 126$
   $+\ 6c + 18d = 54$
   $15c = 180$; $\frac{15c}{15} = \frac{180}{15}$
   $c = 12$
   $3c + 9d = 27$
   $3(12) + 9d = 27$
   $36 + 9d = 27$
   $-36 \quad -36$
   $\frac{9d}{9} = \frac{-9}{9}$
   $d = -1$

2. $3(3a - b = 3)$
   $1(a + 3b = 11)$

   $9a - 3b = 9$
   $+\ a + 3b = 11$
   $\frac{10a}{10} = \frac{20}{10}$
   $a = 2$
   $a + 3b = 11$
   $2 + 3b = 11$
   $-2 \quad -2$
   $\frac{3b}{3} = \frac{9}{3}$
   $b = 3$

3. $1(2x + y = 3)$
   $2(-x + 3y = -12)$

   $2x + y = 3$
   $+\ -2x + 6y = -24$
   $\frac{7y}{7} = \frac{-21}{7}$
   $y = -3$
   $2x + y = 3$
   $2x - 3 = 3$
   $+3 \quad +3$
   $\frac{2x}{2} = \frac{6}{2}$
   $x = 3$

4. $1(2x + 3y = 12)$
   $3(5x - y = 13)$

   $2x + 3y = 12$
   $+\ 15x - 3y = 39$
   $\frac{17x}{17} = \frac{51}{17}$
   $x = 3$
   $2x + 3y = 12$
   $2(3) + 3y = 12$
   $6 + 3y = 12$
   $-6 \quad -6$
   $\frac{3y}{3} = \frac{6}{3}$
   $y = 2$

5. $2(-3x + 4y = 12)$
   $3(2x + y = -8)$

   $-6x + 8y = 24$
   $+\ 6x + 3y = -24$
   $\frac{11y}{11} = \frac{0}{11}$
   $y = 0$
   $2x + y = -8$
   $2x + 0 = -8$
   $\frac{2x}{2} = \frac{-8}{2}$
   $x = -4$

6. $3(2x + 4y = -4)$
   $-2(3x + 5y = -3)$

   $6x + 12y = -12$
   $+\ -6x - 10y = 6$
   $\frac{2y}{2} = \frac{-6}{2}$
   $y = -3$
   $2x + 4y = -4$
   $2x + 4(-3) = -4$
   $2x - 12 = -4$
   $+12 \quad +12$
   $\frac{2x}{2} = \frac{8}{2}$
   $x = 4$

7. Which system of equations will yield the same solution as the system below?

$$2(x - y = 3)$$
$$2x - 3y = -1$$

1) $-2x - 2y = -6$	3) $2x - 2y = 6$
$2x - 3y = -1$	$2x - 3y = -1$
2) $-2x + 2y = 3$	4) $3x + 3y = 9$
$2x - 3y = -1$	$2x - 3y = -1$

7 12. Lizzy has 30 coins that total \$4.80. All of her coins are dimes, D, and quarters, Q. Which system of equations models this situation?

~~1) $D+Q=4.80$~~

$.10D+.25Q=30$

2) $D+Q=30$

$.10D+.25Q=4.80$

3) $D+Q=30$

$.25D+.10Q=4.80$

~~4) $D+Q=4.80$~~

$.25D+.10Q=30$

8 13. Alicia purchased H half-gallons of ice cream for \$3.50 each and P packages of ice cream cones for \$2.50 each. She purchased 14 items and spent \$43. Which system of equations could be used to determine how many of each item Alicia purchased?

1) $3.50H+2.50P=43$

$H+P=14$

2) $3.50P+2.50H=43$

$P+H=14$

3) $3.50H+2.50P=14$

~~$H+P=43$~~

4) $3.50P+2.50H=14$

~~$P+H=43$~~

9 14. The Celluloid Cinema sold 150 tickets to a movie. Some of these were child tickets and the rest were adult tickets. A child ticket cost \$7.75 and an adult ticket cost \$10.25. If the cinema sold \$1470 worth of tickets, which system of equations could be used to determine how many adult tickets, a, and how many child tickets, c, were sold?

1) $a+c=150$

$10.25a+7.75c=1470$

2) ~~$a+c=1470$~~

$10.25a+7.75c=150$

3) $a+c=150$

$7.75a+10.25c=1470$

4) $a+c=1470$

$7.75a+10.25c=150$

10 15. During its first week of business, a market sold a total of 108 apples and oranges. The second week, five times the number of apples and three times the number of oranges were sold. A total of 452 apples and oranges were sold during the second week. Determine how many apples and how many oranges were sold the first week.

$-5(a+o=108)$

$1(5a+3o=452)$

$-\cancel{5a}-5o=-540$

$+\ \cancel{5a}+3o=452$

$-2o=-88$

$\frac{-2}{-2}\quad\frac{}{-2}$

$o=44$

$a+o=108$

$a+44=108$

$-44\quad -44$

$a=64$

16. Dylan has a bank that sorts coins as they are dropped into it. A panel on the front displays the total number of coins inside as well as the total value of these coins. The panel shows 90 coins with a value of $17.55 inside of the bank. If Dylan only collects dimes and quarters, write a system of equations in two variables or an equation in one variable that could be used to model this situation. Using your equation or system of equations, algebraically determine the number of quarters Dylan has in his bank.

$-.10(d+q=90)$
$1(.10d+.25q=17.55)$

$-.10d-.10q=-9$
$+\ .10d+.25q=17.55$
$.15q=8.55$
$\frac{.15q}{.15}=\frac{8.55}{.15}$
$q=57$

17. Mo's farm stand sold a total of 165 pounds of apples and peaches. She sold apples for $1.75 per pound and peaches for $2.50 per pound. If she made $337.50, how many pounds of peaches did she sell?

$-1.75(a+p=165)$
$1(1.75a+2.50p=337.50)$
$-1.75a-1.75p=-288.75$
$+\ 1.75a+2.50p=337.50$
$.75p=48.75$
$\frac{.75p}{.75}=\frac{48.75}{.75}$
$p=65$

$a+p=165$
$a+65=165$
$-65\ -65$
$a=100$

18. Last week, a candle store received $355.60 for selling 20 candles. Small candles sell for $10.98 and large candles sell for $27.98. How many large candles did the store sell?

$-10.98(s+l=20)$
$1(10.98s+27.98l=355.60)$
$-10.98s-10.98l=-219.6$
$+\ 10.98s+27.98l=355.60$
$17l=136$
$\frac{17l}{17}=\frac{136}{17}$
$l=8$

$s+l=20$
$s+8=20$
$-8\ -8$
$s=12$

14. An animal shelter spends \$2.35 per day to care for each cat and \$5.50 per day to care for each dog. Pat noticed that the shelter spent \$89.50 caring for cats and dogs on Wednesday. Write an equation to represent the possible numbers of cats and dogs that could have been at the shelter on Wednesday. Pat said that there might have been 8 cats and 14 dogs at the shelter on Wednesday. Are Pat's numbers possible? Use your equation to justify your answer. Later, Pat found a record showing that there were a total of 22 cats and dogs at the shelter on Wednesday. How many cats were at the shelter on Wednesday?

$2.35c + 5.50d = 89.50$

$2.35(8) + 5.50(14) = 89.50$

$95.8 \neq 89.50$

No

cancel d

$-5.5(c + d = 22)$

$1(2.35c + 5.50d = 89.50)$

$-5.5c - 5.5d = -121$

$2.35c + 5.5d = 89.50$

$\frac{-3.15c}{-3.15} = \frac{-31.5}{-3.15}$

$c = 10$

15. For a class picnic, two teachers went to the same store to purchase drinks. One teacher purchased 18 juice boxes and 32 bottles of water, and spent \$19.92. The other teacher purchased 14 juice boxes and 26 bottles of water, and spent \$15.76. Write a system of equations to represent the costs of a juice box, *j*, and a bottle of water, *w*. Kara said that the juice boxes might have cost 52 cents each and that the bottles of water might have cost 33 cents each. Use your system of equations to justify that Kara's prices are *not* possible. Solve your system of equations to determine the actual cost, in dollars, of each juice box and each bottle of water.

$-14(18j + 32w = 19.92)$

$18(14j + 26w = 15.76)$

$-252j - 448w = -278.8$

$252j + 468w = 283.68$

$\frac{20w}{20} = \frac{4.8}{20}$

$w = .24$

$18(.52) + 32(.33) = 19.92$

$19.92 = 19.92$ ✓

$14(.52) + 26(.33) \neq 15.76$

$15.86 \neq 15.76$ ✗

No!

$18j + 32(.24) = 19.92$

$18j + 7.68 = 19.92$

$-7.68 \quad -7.68$

$\frac{18j}{18} = \frac{12.24}{18}$

$j = .68$

16. Two friends went to a restaurant and ordered one plain pizza and two sodas. Their bill totaled \$15.95. Later that day, five friends went to the same restaurant. They ordered three plain pizzas and each person had one soda. Their bill totaled \$45.90. Write and solve a system of equations to determine the price of one plain pizza. [Only an algebraic solution can receive full credit.]

$$-3(1p+2s=15.95)$$
$$1(3p+5s=45.90)$$
$$-3p-6s=-47.85$$
$$+3p+5s=45.90$$
$$-1s=-1.95$$

$s=1.95$

$p+2s=15.95$

$p+2(1.95)=15.95$

$p+3.9=15.95$

$-3.9 \quad -3.9$

$p=12.05$

17. A recreation center ordered a total of 15 tricycles and bicycles from a sporting goods store. The number of wheels for all the tricycles and bicycles totaled 38. Write a linear system of equations that models this scenario, where *t* represents the number of tricycles and *b* represents the number of bicycles ordered. Based on your graph of this scenario, could the recreation center have ordered 10 tricycles? Explain your reasoning.

$$-2(b+t=15)$$
$$1(2b+3t=38)$$
$$-\cancel{2b}-2t=-30$$
$$+\cancel{2b}+3t=38$$
$$t=8$$

No, they ordered 8 tricycles

18. Franco and Caryl went to a bakery to buy desserts. Franco bought 3 packages of cupcakes and 2 packages of brownies for \$19. Caryl bought 2 packages of cupcakes and 4 packages of brownies for \$24. Let *x* equal the price of one package of cupcakes and *y* equal the price of one package of brownies. Write a system of equations that describes the given situation. Determine the exact cost of one package of cupcakes and the exact cost of one package of brownies in dollars and cents. Justify your solution.

$$-2(3c+2b=19)$$
$$3(2c+4b=24)$$
$$-\cancel{6c}-4b=-38$$
$$+\cancel{6c}+12b=72$$
$$\frac{8b}{8}=\frac{34}{8}$$
$$b=4.25$$

$3c+2b=19$

$3c+2(4.25)=19$

$3c+8.5=19$

$-8.5 \quad -8.5$

$\frac{3c}{3}=\frac{10.5}{3}$

$c=3.5$

brownies: \$4.25

cupcakes: \$3.50

Writing the Equation of a Line

Find slope and y intercept!

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute the slope and the coordinates of a point into $y - y_1 = m(x - x_1)$

Distribute and isolate y in order to put it into $y = mx + b$ form if necessary

Write the equation of the line that passes through the given point and has the given slope in point slope form.

1. $(2,7), m = -4$

$y-7=-4(x-2)$
$y-7=-4x+8$
$+7 \quad +7$
$y=4x+15$

2. $(12,5), m = -3$

$y-5=-3(x-12)$
$y-5=-3x+36$
$+5 \quad +5$
$y=-3x+41$

3. $(4,-5), m = 6$

$y+5=6(x-4)$
$y+5=6x-24$
$-5 \quad -5$
$y=6x-29$

4. $(-6,-2), m = 3$

$y+2=3(x+6)$
$y+2=3x+18$
$-2 \quad -2$
$y=3x+16$

5. $(7,-6), m = \frac{1}{2}$

$y+6=\frac{1}{2}(x-7)$
$y+6=\frac{1}{2}x-3.5$
$-6 \quad -6$
$y=\frac{1}{2}x-9.5$

6. $(-8,2), m = -\frac{3}{4}$

$y-2=-\frac{3}{4}(x+8)$
$y-2=-\frac{3}{4}x-6$
$+2 \quad +2$
$y=-\frac{3}{4}x-4$

Write the equation of the line passing through the given points in point slope and slope intercept form

7. (4,1) and (2,5)

$m=\frac{5-1}{2-4}$
$m=-2$

$y-1=-2(x-4)$
$y-1=-2x+8$
$+1 \quad +1$
$y=-2x+9$

8. (9,1) and (7,-5)

$m=\frac{-5-1}{7-9}=3$

$y-1=3(x-9)$
$y-1=3x-27$
$+1 \quad +1$
$y=3x-26$

9. (3,10) and (-1,8)

$m=\frac{8-10}{-1-3}$
$m=\frac{1}{2}$

$y-10=\frac{1}{2}(x-3)$
$y-10=\frac{1}{2}x-1.5$
$+10 \quad +10$
$y=\frac{1}{2}x+8.5$

10. (3,1) and (5,-7)

$m=\frac{-7-1}{5-3}=-4$

$y-1=-4(x-3)$
$y-1=-4x+12$
$+1 \quad +1$
$y=-4x+13$

x y x y
11. (-2,1) and (-4, -1) $m = \frac{-1-1}{-4-2} = 1$

$y - 1 = 1(x+2)$

$y - 1 = x + 2$
$+1 \quad +1$

$y = x + 3$

x y x y
12. (10,-1) and (0, 4) $m = \frac{4 - -1}{0-10} = -\frac{1}{2}$

$y + 1 = -\frac{1}{2}(x - 10)$

$y + 1 = -\frac{1}{2}x + 5$
$-1 \quad -1$

$y = -\frac{1}{2}x + 4$

$x_1 y_1$ $x_2 y_2$
13. (8,2) and (6,4) $m = \frac{4-2}{6-8} = -1$

$y - 2 = -1(x - 8)$

$y - 2 = -1x + 8$
$+2 \quad +2$

$y = -1x + 10$

x y x y
14. (2,-2) and (-3, 8) $m = \frac{8 - -2}{-3-2} = -2$

$y - 8 = -2(x + 3)$

$y - 8 = -2x - 6$
$+8 \quad +8$

$y = -2x + 2$

x y x y
15. (-6,2) and (-3,3) $m = \frac{3-2}{-3--6} = \frac{1}{3}$

1) $y - 2 = \frac{1}{3}(x+6)$

2) $y + 2 = \frac{1}{3}(x-6)$

3) $y - 2 = 3(x+6)$

4) $y + 2 = 3(x-6)$

$y - 2 = \frac{1}{3}(x+6)$

x y x y
16. (0,4) and (-1, 6) $m = \frac{6-4}{-1-0} = -2$

1) $y + 6 = -\frac{1}{2}(x-1)$

2) $y + 6 = -2(x-1)$

3) $y - 6 = -2(x+1)$

4) $y - 6 = -\frac{1}{2}(x+1)$

$y - 6 = -2(x+1)$

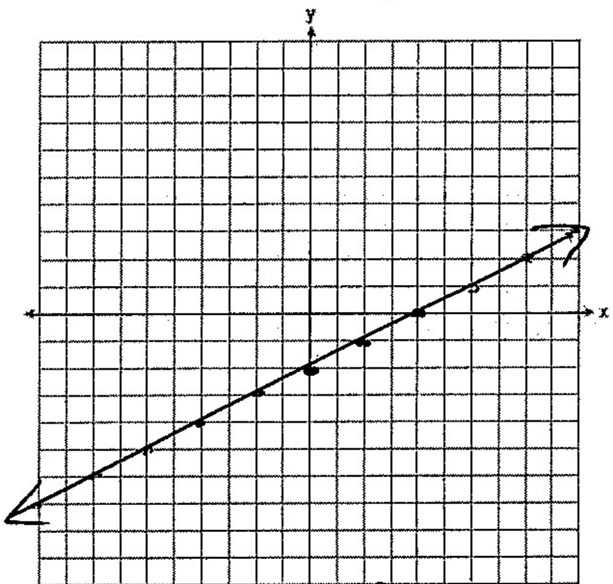
Graphing Lines

1) Put equation into $y = mx + b$ form.

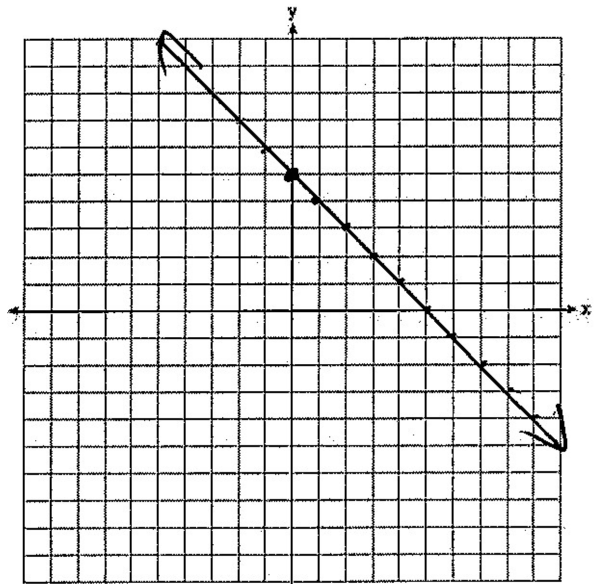
2) Begin with b. Plot the y intercept.

3) Apply slope $\left(\frac{rise}{run}\right)$

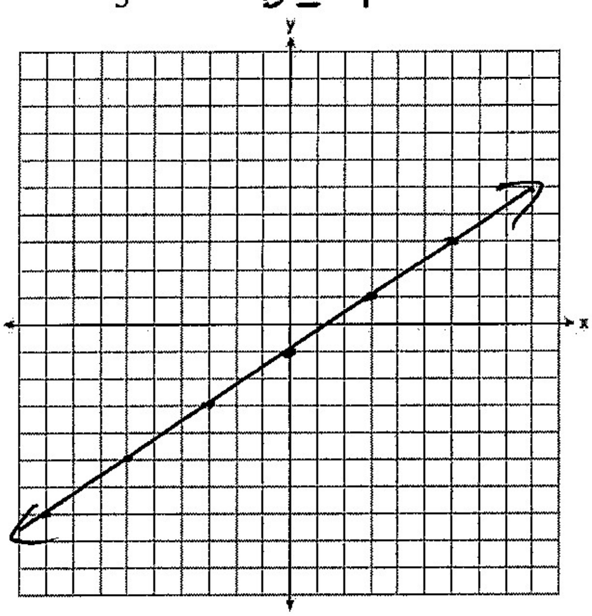
1. $y = \frac{1}{2}x - 2$ $m = \frac{1}{2}$ $b = -2$



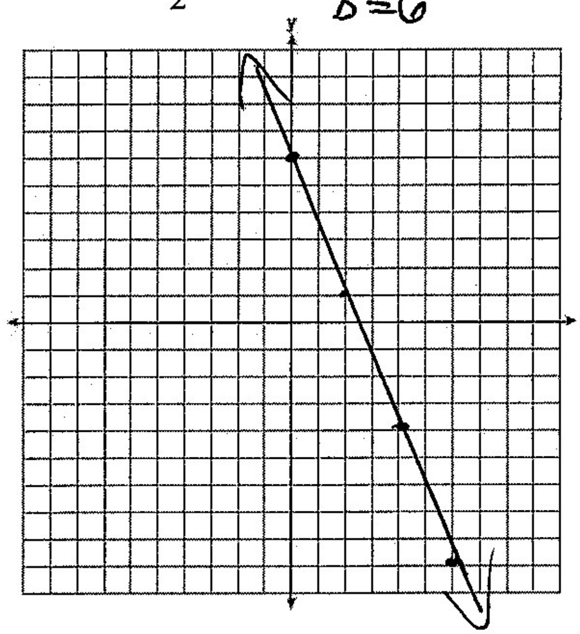
2. $y = -x + 5$ $m = -1$ $-\frac{1}{1}$ $b = 5$



3. $y = \frac{2}{3}x - 1$ $m = \frac{2}{3}$ $b = -1$



4. $y = -\frac{5}{2}x + 6$ $m = -\frac{5}{2}$ $b = 6$

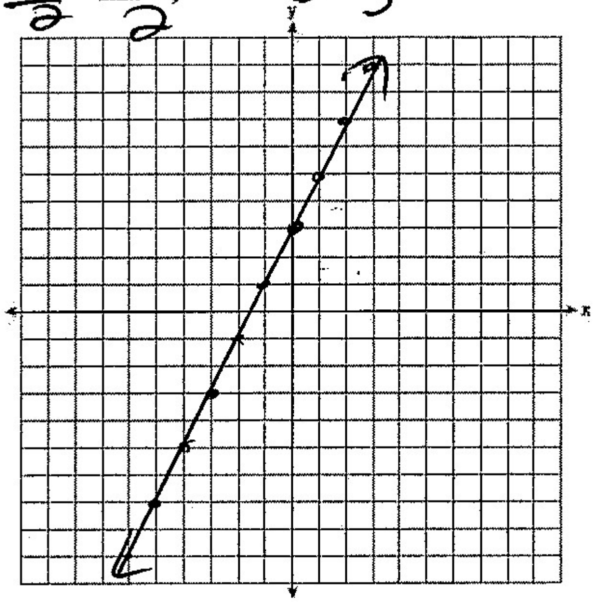


5. $\frac{2y}{2} = \frac{4x + 6}{2}$

$y = 2x + 3$

$m = \frac{2}{1}$

$b = 3$

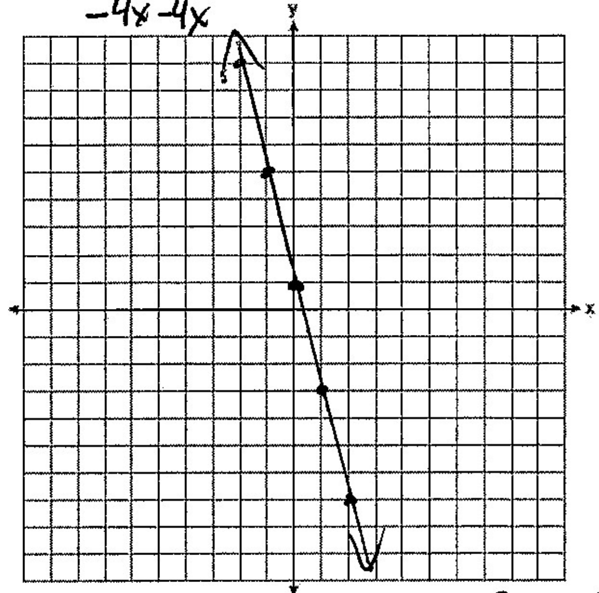


6. $y + 4x = 1$

$-4x \quad -4x$

$y = -4x + 1$ $\quad m = \frac{-4}{1}$

$b = 1$

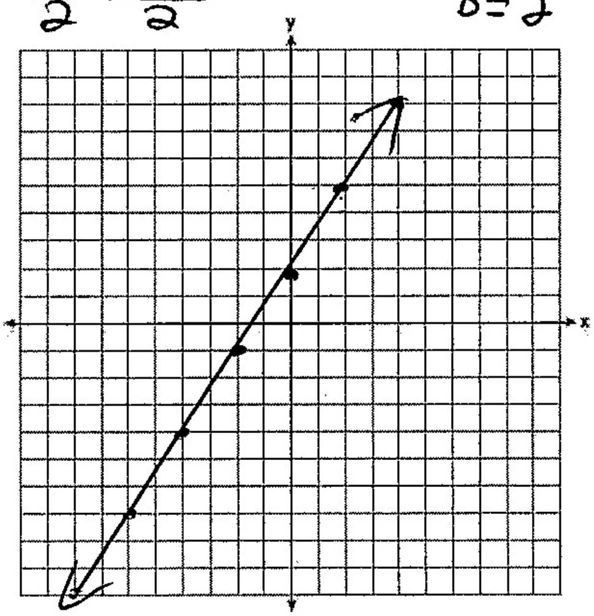


7. $\frac{2y}{2} = \frac{3x + 4}{2}$

$y = \frac{3}{2}x + 2$

$m = \frac{3}{2}$

$b = 2$



8. $2y + 2x = 6$

$2y + \cancel{2x} = 6$

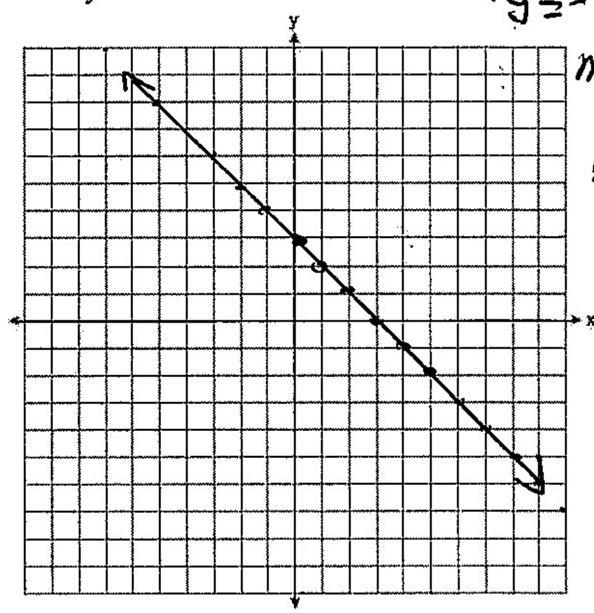
$-\cancel{2x} \quad -2x$

$\frac{2y}{2} = \frac{-2x + 6}{2}$

$y = -x + 3$

$m = \frac{-1}{1}$

$b = 3$



Systems of Inequalities

<: shade below dashed line ≤: shade below solid line

>: shade above dashed line ≥: shade above solid line

The solution set is the region that both graphs are shaded. Mark with an S.

*For word problems, the first inequality is just $x+y$ for an amount. The second inequality is usually a money inequality.

1. Graph the following systems of inequalities on the set of axes below:
Based upon your graph, explain why $(6,1)$ is a solution to this system and why $(-6,7)$ is *not* a solution to this system.

$2y \geq 3x - 16$

$y + 2x > -5$

$\frac{2y}{2} \geq \frac{3x-16}{2}$

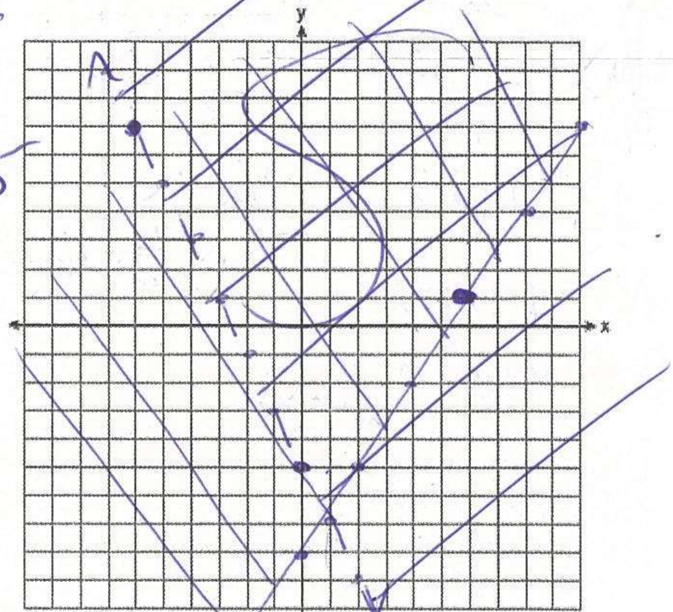
$y \geq \frac{3}{2}x - 8$

$y + 2x > -5$
$-2x \quad -2x$

$y > -2x - 5$

(6,1) is in the solution set. On the line is included if it is a solid line.

(-6,7) is not in the solution set. On the line is not included if it's a dashed line.



2. On the set of axes below, graph the following system of inequalities:
Determine if the point $(1,2)$ is in the solution set. Explain your answer.

$2y + 3x \leq 14$

$4x - y < 2$

$4x - y < 2$
$-4x \quad -4x$

$\frac{-y}{-1} < \frac{-4x+2}{-1}$

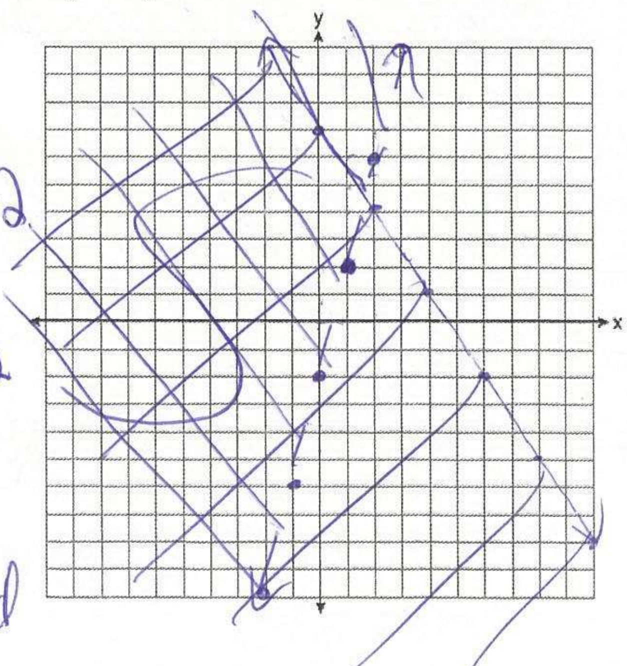
$y > 4x - 2$

$2y + 3x \leq 14$
$-3x \quad -3x$

$\frac{2y}{2} \leq \frac{-3x+14}{2}$

$y \leq -\frac{3}{2}x + 7$

(1,2) is not in the solution set. The dashed line is not included.

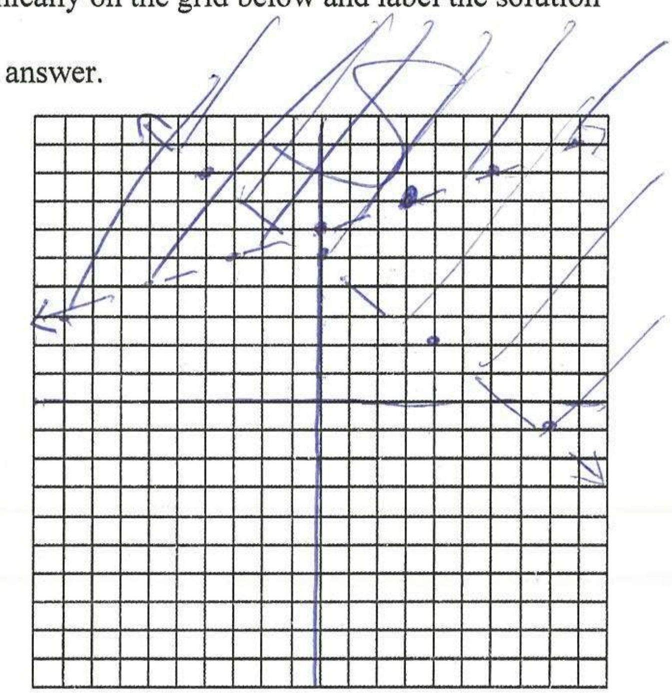


3. Solve the following system of inequalities graphically on the grid below and label the solution *S*.

Is the point (3, 7) in the solution set? Explain your answer.

$3x + 4y > 20$

$x < 3y - 18$

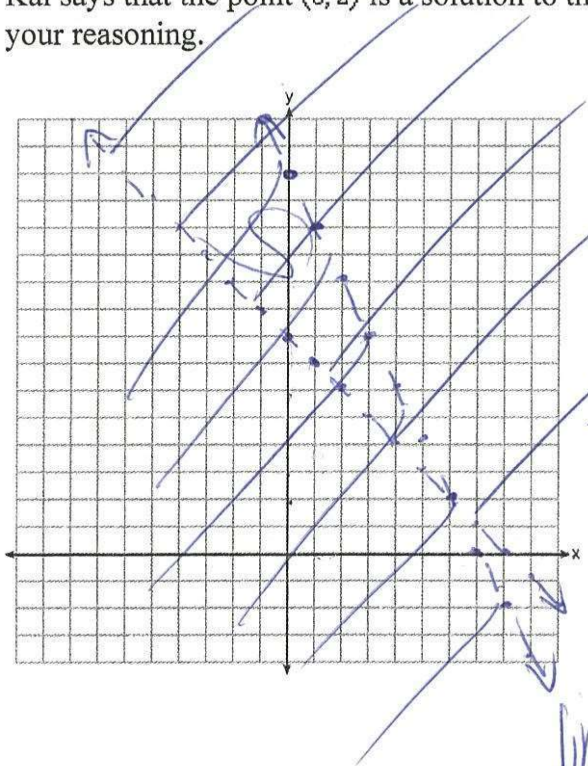


$3x + 4y > 20$
$-3x \quad -3x$
$\frac{4y}{4} > \frac{-3x+20}{4}$
$y > -\frac{3}{4}x + 5$

$x < 3y - 18$
$+18 \quad +18$
$\frac{x+18}{3} < \frac{3y}{3}$
$\frac{1}{3}x + 6 < y$
$y > \frac{1}{3}x + 6$

No, the dashed line is not included in the solution set

4. The sum of two numbers, *x* and *y*, is more than 8. When you double *x* and add it to *y*, the sum is less than 14. Graph the inequalities that represent this scenario on the set of axes below. Kai says that the point (6, 2) is a solution to this system. Determine if he is correct and explain your reasoning.



$x + y > 8$

$2x + y < 14$

$x + y > 8$
$-x \quad -x$
$y > -x + 8$

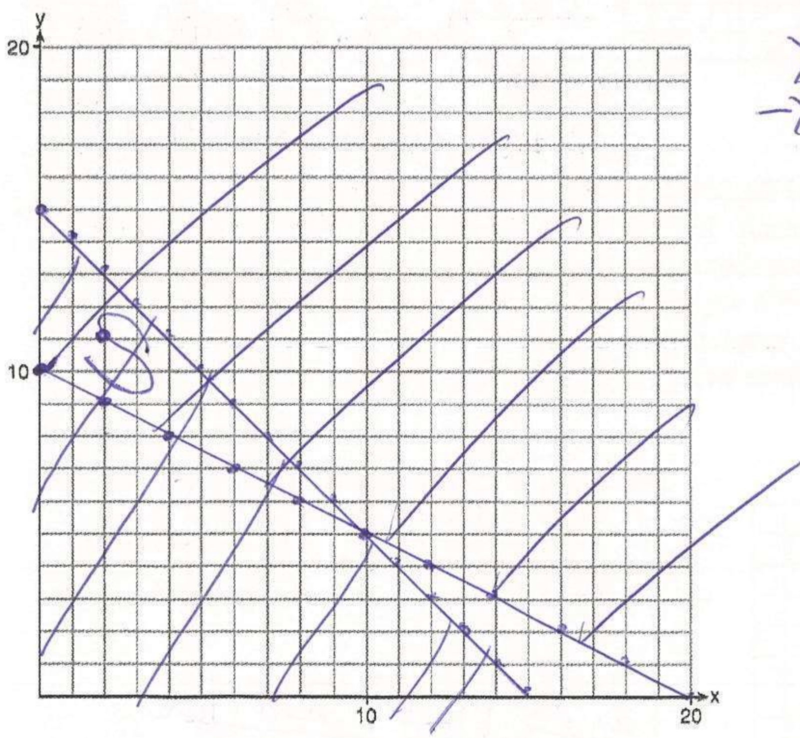
$2x + y < 14$
$-2x \quad -2x$
$y < -2x + 14$

No, the dashed line is not included.

5. Jordan works for a landscape company during his summer vacation. He is paid \$12 per hour for mowing lawns and \$14 per hour for planting gardens. He can work a maximum of 40 hours per week, and would like to earn at least \$250 this week. If m represents the number of hours mowing lawns and g represents the number of hours planting gardens, which system of inequalities could be used to represent the given conditions?

1) $m + g \leq 40$
   $12m + 14g \geq 250$
2) $m + g \geq 40$
   $12m + 14g \leq 250$
3) $m + g \leq 40$
   $12m + 14g \leq 250$
4) $m + g \geq 40$
   $12m + 14g \geq 250$

6. Edith babysits for x hours a week after school at a job that pays \$4 an hour. She has accepted a job that pays \$8 an hour as a library assistant working y hours a week. She will work both jobs. She is able to work no more than 15 hours a week, due to school commitments. Edith wants to earn at least \$80 a week, working a combination of both jobs. Write a system of inequalities that can be used to represent the situation. Graph these inequalities on the set of axes below.



$x + y \leq 15$
$-x \quad -x$
$y \leq -x + 15$

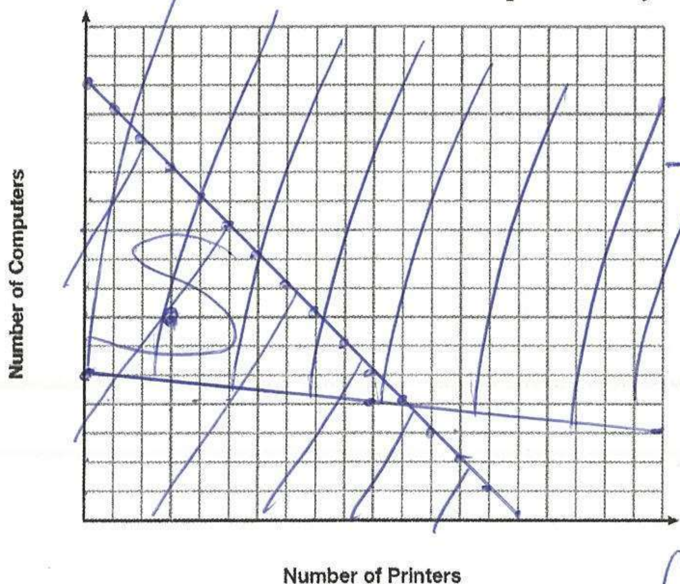
$4x + 8y \geq 80$
$-4x \quad -4x$
$\frac{8y}{8} \geq \frac{-4x + 80}{8}$
$y \geq -\frac{1}{2}x + 10$

Determine and state one combination of hours that will allow Edith to earn *at least* \$80 per week while working *no more than* 15 hours.

(2, 11)

2 hours babysitting and 11 hours as a library assistant.

7. An on-line electronics store must sell at least \$2500 worth of printers and computers per day. Each printer costs \$50 and each computer costs \$500. The store can ship a maximum of 15 items per day. On the set of axes below, graph a system of inequalities that models these constraints. Determine a combination of printers and computers that would allow the electronics store to meet all of the constraints. Explain how you obtained your answer.



$x+y \le 15$ $\quad 50x+500y \ge 2500$

$-x \quad -50x \quad -50x$

$y \le -x+15$ $\quad \frac{500y}{500} \ge \frac{-50x+2500}{500}$

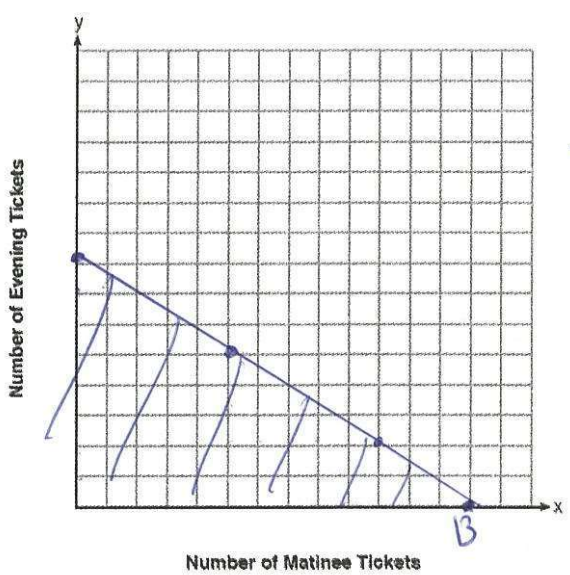
$y \ge -\frac{1}{10}x+5$

(3,7)

3 printers and 7 computers

(3,7) is in the solution set.

8. Myranda received a movie gift card for \$100 to her local theater. Matinee tickets cost \$7.50 each and evening tickets cost \$12.50 each. If x represents the number of matinee tickets she could purchase, and y represents the number of evening tickets she could purchase, write an inequality that represents all the possible ways Myranda could spend her gift card on movies at the theater. On the set of axes below, graph this inequality.
What is the maximum number of matinee tickets Myranda could purchase with her gift card? Explain your answer.



$7.50x+12.50y \le 100$

$-7.50x \quad -7.50x$

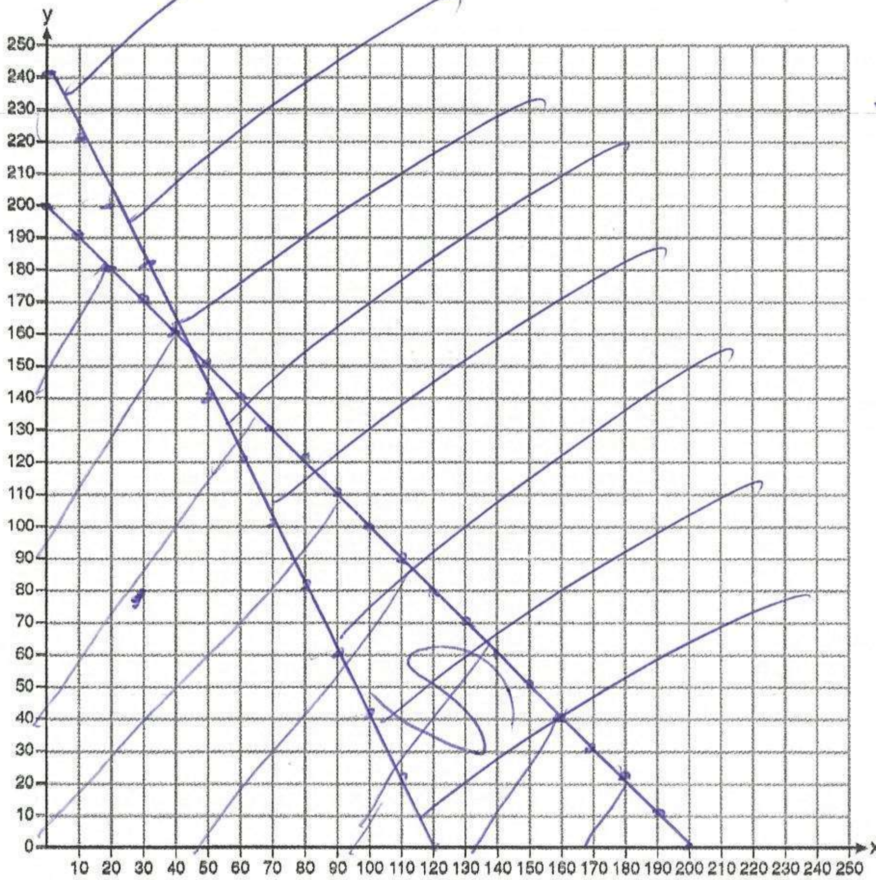
$\frac{12.50y}{12.50} \le \frac{-7.50x+100}{12.50}$

$y \le -\frac{3}{5}x+8$

13 matinee tickets.

13 is the largest x value in the solution set.

9. The Reel Good Cinema is conducting a mathematical study. In its theater, there are 200 seats. Adult tickets cost $12.50 and child tickets cost $6.25. The cinema's goal is to sell at least $1500 worth of tickets for the theater. Write a system of linear inequalities that can be used to find the possible combinations of adult tickets, x, and child tickets, y, that would satisfy the cinema's goal. Graph the solution to this system of inequalities on the set of axes below. Label the solution with an S. Marta claims that selling 30 adult tickets and 80 child tickets will result in meeting the cinema's goal. Explain whether she is correct or incorrect, based on the graph drawn.



$x+y \le 200$ $\quad$ $12.50x+6.25y \ge 1[500]$

$-x \quad -x$ $\quad$ $-12.50x \quad -125$

$y \le -x+200$ $\quad$ $\frac{6.25y}{6.25} \ge \frac{-12.50x+1500}{6.25}$

$y \ge -2x+240$

No, (30,80) is not in the solution set.

Modeling Exponential Functions

Exponential/ Interest/Depreciation Problems: $A = P(1 \pm r)^t$, where A is the current amount, P is the initial amount (y-intercept), r is the rate as a decimal (divide by 100), and t is time.
Given an exponential function: What is in front of the parenthesis is the INITIAL amount, what is inside the parenthesis is 1 + the rate or 1 – the rate.

Example: $A = 500(1.2)^t$: 500 is initial amount, rate is .2 or 20% growth (1 + .2)

$A = 500(0.8)^t$: 500 is initial amount, rate is .2 or 20% decay (1 - .2)

1. Anne invested $1000 in an account with a 1.3% annual interest rate. She made no deposits or withdrawals on the account for 2 years. If interest was compounded annually, which equation represents the balance in the account after the 2 years?

1) $A = 1000(1 - 0.013)^2$ 3) $A = 1000(1 - 1.3)^2$

2) $A = 1000(1 + 0.013)^2$ 4) $A = 1000(1 + 1.3)^2$

2. Dylan invested $600 in a savings account at a 1.6% annual interest rate. He made no deposits or withdrawals on the account for 2 years. The interest was compounded annually. Find, to the *nearest cent*, the balance in the account after 2 years.

$A = A$ $A = P(1+r)^t$ $A = 619.35$
$P = 600$ $A = 600(1+.016)^2$
$r = .016$ $A = 600(1.016)^2$
$t = 2$

3. A car worth $41,235 depreciates at a rate of 11.5% each year. Find the value of the car after 7 years to the nearest cent?

$A = A$ $A = P(1-r)^t$
$P = 41,235$ $A = 41,235(1-.115)^7$
$r = .115$ $A = 41,235(.885)^7$
$t = 7$ $A = 17533.51$

4. Sheba opened a retirement account with $36,500. Her account grew at a rate of 7% per year compounded annually. She made no deposits or withdrawals on the account. At the end of 20 years, what was the account worth, to the *nearest dollar*?

$A = A$ $A = P(1+r)^t$ $A = 141243$
$P = 36,500$ $A = 36,500(1+.07)^{20}$
$r = .07$ $A = 36,500(1.07)^{20}$
$t = 20$

5. A certain car depreciates at a rate of 15% each year. If the car was initially worth $8125, what is the value of the car, rounded to the nearest cent, 11 years later?

$A = A$ $A = P(1-r)^t$
$P = 8125$ $A = 8125(1-.15)^{11}$
$r = .15$ $A = 8125(.85)^{11}$
$t = 11$ $A = \$1359.66$

6. Marilyn collects old dolls. She purchases a doll for $450. Research shows this doll's value will increase by 2.5% each year. Write an equation that determines the value, V, of the doll t years after purchase. Assuming the doll's rate of appreciation remains the same, will the doll's value be doubled in 20 years? Justify your reasoning.

$A = V$
$P = 450$
$r = .025$
$t = t$

$A = P(1+r)^t$

$V = 450(1+.025)^t$

$V = 450(1+.025)^{20}$
$V = 737.38$

$450(2) = 900$
No!
$737 < 900$

7. The function $V(t) = 1350(1.017)^t$ represents the value $V(t)$, in dollars, of a comic book t years after its purchase. The yearly rate of appreciation of the comic book is
1) 17%
2) 1.7%
3) 1.017%
4) 0.017%

1.017
-1
$.017(100) = 1.7\%$

$1 + r = 1.017$

8. Milton has his money invested in a stock portfolio. The value, $v(x)$, of his portfolio can be modeled with the function $v(x) = 30,000(0.78)^x$, where x is the number of years since he made his investment. Which statement describes the rate of change of the value of his portfolio?
1) It decreases 78% per year.
2) It decreases 22% per year.
3) It increases 78% per year.
4) It increases 22% per year.

$\cancel{1} - r = .78$
$-1 \quad -1$
$\frac{-r}{-1} = \frac{-.22}{-1}$

$r = .22(100)$
$r = 22\%$

9. The equation $A = 1300(1.02)^7$ is being used to calculate the amount of money in a savings account. What does 1.02 represent in this equation?
1) 0.02% decay
2) 0.02% growth
3) 2% decay
4) 2% growth

$1 + r = 1.02$
$-1 \quad -1$
$r = .02(100)$
$r = 2\%$

10. A car's depreciated value can be represented by the function $v(t) = 25500(.83)^t$. What was the initial value of the car and what is the depreciation rate?

Initial value: 25500

$\cancel{1} - r = .83$
$-1 \quad -1$
$-r = -.17$
$\frac{-r}{-1} = \frac{-.17}{-1}$
$r = .17(100)$
$r = 17\%$

11. The value, $v(t)$, of a car depreciates according to the function $v(t) = P(.85)^t$, where P is the purchase price of the car and t is the time, in years, since the car was purchased. State the percent that the value of the car *decreases* by each year. Justify your answer.

$1 - r = .85$
$-1 \quad -1$
$-r = -.15$
$\div -1 \quad \div -1$
$r = .15(100)$
$r = 15\%$

12. Some banks charge a fee on savings accounts that are left inactive for an extended period of time. The equation $y = 5000(0.98)^x$ represents the value, y, of one account that was left inactive for a period of x years. What is the y-intercept of this equation and what does it represent?
1) 0.98, the percent of money in the account initially
2) 0.98, the percent of money in the account after x years
3) 5000, the amount of money in the account initially
4) 5000, the amount of money in the account after x years

initial value

13. The number of carbon atoms in a fossil is given by the function $y = 5100(0.95)^x$, where x represents the number of years since being discovered. What is the percent of change each year? Explain how you arrived at your answer.

1−r is what is inside the parenthesis

$1 - r = .95$
$-1 \quad -1$
$-r = -.05$
$\div -1 \quad \div -1$
$r = .05(100)$
$r = 5\%$

14. A population of rabbits in a lab, $p(x)$, can be modeled by the function $p(x) = 20(1.014)^x$, where x represents the number of days since the population was first counted. Explain what 20 and 1.014 represent in the context of the problem.

20 is the initial number of rabbits
1.4% is the growth rate.

$1.014 = 1 + r$
$-1 \quad -1$
$100(.014) = r$
$r = 1.4\%$

15. The breakdown of a sample of a chemical compound is represented by the function $p(t) = 300(0.5)^t$, where $p(t)$ represents the number of milligrams of the substance and t represents the time, in years. In the function $p(t)$, explain what 0.5 and 300 represent.

300 is the initial number of milligrams of the substance.

50% is the rate of decrease.

$1 - r = .5$
$-1 \quad -1$
$-r = -.5$
$\div -1 \quad \div -1$
$r = .5(100)$
$r = 50\%$

Linear vs. Exponential

Linear	Exponential
Add/Subtract Constant Amount	**Multiply/Divide Constant Amount** Add/subtract increasing/decreasing amount
Per x + 1TF	AP1RT
	Percent/Rate

Linear increases/decreases by a constant amount.
Exponential increases/decreases by a constant **percent**!

1. One characteristic of all linear functions is that they change by
1) equal factors over equal intervals
2) unequal factors over equal intervals
3) equal differences over equal intervals
4) unequal differences over equal intervals

2. Which statement below is true about linear functions?
1) Linear functions grow by equal factors over equal intervals.
2) Linear functions grow by equal differences over equal intervals.
3) Linear functions grow by equal differences over unequal intervals.
4) Linear functions grow by unequal factors over equal intervals.

3. The table below shows the average yearly balance in a savings account where interest is compounded annually. No money is deposited or withdrawn after the initial amount is deposited.

Year	Balance, in Dollars
0	380.00 +≈180
10	562.49
20	832.63
30	1232.49
40	1824.39 +≈900
50	2700.54

Which type of function best models the given data?
1) linear function with a negative rate of change
2) linear function with a positive rate. of change
3) exponential decay function
4) exponential growth function

4. The number of people who attended a school's last six basketball games increased as the team neared the state sectional games. The table below shows the data. State the type of function that best fits the given data. Justify your choice of a function type.

Game	13	14	15	16	17	18
Attendance	348	435	522	609	696	783

+87 +87 +87 +87 +87

Linear, it is increasing by a constant amount.

5. The function, $t(x)$, is shown in the table below. Determine whether $t(x)$ is linear or exponential. Explain your answer.

Linear, it is decreasing by a Constant amount.

x	t(x)	
−3	10	−2.5
−1	7.5	−2.5
1	5	−2.5
3	2.5	−2.5
5	0	−2.5

6. Caleb claims that the ordered pairs shown in the table below are from a nonlinear function. State if Caleb is correct. Explain your reasoning.

x	f(x)	
0	2	+2
1	4	+4
2	8	+8
3	16	

Yes, it is not increasing by a Constant amount.

7. The tables below show the values of four different functions for given values of x. Which table represents a linear function?

1) ~~$f(x)$~~
2) $g(x)$
3) $h(x)$
4) $k(x)$

x	f(x)	
1	12	+7
2	19	+7
3	26	+7
4	33	

x	g(x)	
1	−1	+2
2	1	+4
3	5	+8
4	13	

x	h(x)	
1	9	+3
2	12	+5
3	17	+7
4	24	

x	k(x)	
1	−2	+6
2	4	+10
3	14	+14
4	28	

8. Which table of values represents a linear relationship?

1)

x	f(x)	
−1	−3	+1
0	−2	+3
1	1	+5
2	6	+7
3	13	

2)

x	f(x)	
−1	$\frac{1}{2}$	$+\frac{1}{2}$
0	1	+1
1	2	+2
2	4	+4
3	8	

~~3)~~

x	f(x)	
−1	−3	+2
0	−1	+2
1	1	+2
2	3	+2
3	5	

4)

x	f(x)	
−1	−1	+1
0	0	+1
1	1	+7
2	8	+19
3	27	

9. During physical education class, Andrew recorded the exercise times in minutes and heart rates in beats per minute (bpm) of four of his classmates. Which table best represents a linear model of exercise time and heart rate?

1) Student 1

Exercise Time (in minutes)	Heart Rate (bpm)
0	60
1	65
2	70
3	75
4	80

+5 +5 +5 +5

2) Student 2

Exercise Time (in minutes)	Heart Rate (bpm)
0	62
1	70
2	83
3	88
4	90

+8 +13 +5 +2

3) Student 3

Exercise Time (in minutes)	Heart Rate (bpm)
0	58
1	65
2	70
3	75
4	79

+7 +5 +5 +4

4) Student 4

Exercise Time (in minutes)	Heart Rate (bpm)
0	62
1	65
2	66
3	73
4	75

+3 +1 +7 +2

10. Determine and state whether the sequence 1, 3, 9, 27, . . . displays exponential behavior. Explain how you arrived at your decision.

·3 ·3 ·3

Yes, it is increasing by a constant factor.

11. Ian is saving up to buy a new baseball glove. Every month he puts \$10 into a jar. Which type of function best models the total amount of money in the jar after a given number of months?

+10

1) linear
2) exponential
3) quadratic
4) square root

12. The highest possible grade for a book report is 100. The teacher deducts 10 points for each day the report is late. Which kind of function describes this situation?

1) linear
2) quadratic
3) exponential growth
4) exponential decay

−10

13. Which situation is *not* a linear function?

1) A gym charges a membership fee of \$10.00 down and \$10.00 per month.
2) A cab company charges \$2.50 initially and \$3.00 per mile.
3) A restaurant employee earns \$12.50 per hour.
4) A \$12,000 car depreciates 15% per year.

exponential percents

14. Which scenario represents exponential growth? multiplication/percents → exponential
1) A water tank is filled at a rate of 2 gallons/minute.
2) A vine grows 6 inches every week.
3) A species of fly doubles its population every month during the summer.
4) A car increases its distance from a garage as it travels at a constant speed of 25 miles per hour.

15. Which situation could be modeled by using a linear function?
1) a bank account balance that grows at a rate of 5% per year, compounded annually exponential
2) a population of bacteria that doubles every 4.5 hours → exponential
3) the cost of cell phone service that charges a base amount plus 20 cents per minute
4) the concentration of medicine in a person's body that decays by a factor of one-third every hour

exponential

16. Which of the three situations given below is best modeled by an exponential function?
I. A bacteria culture doubles in size every day. → exponential
II. A plant grows by 1 inch every 4 days. → +1, linear
III. The population of a town declines by 5% every 3 years. → exponential
1) I, only
2) II, only
3) I and II
4) I and III

percent → exponential

17. Which situation could be modeled with an exponential function?
1) the amount of money in a savings account where $150 is deducted every month.
2) the amount of money in Suzy's piggy bank which she adds $10 to each week.
3) the amount of money in a certificate of deposit that gets 4% interest each year
4) the amount of money in Jaclyn's wallet which increases and decreases by a different amount each week.

18. Which situation could be modeled with a linear function?
1) the height of a ball that is thrown in the air quadratic
2) the price of a car that depreciates 20% per year exponential
3) the amount of money Jonathan pays for a certain number of gallons of gas at $3.85 per gallon
4) a bacteria colony which doubles in number every 4 hours

→ exponential

19. Which situation could be modeled with an exponential function?
1) the amount of money in a savings account where $150 is deducted every month.
2) the amount of money in Suzy's piggy bank which she adds $10 to each week.
3) the amount of money in a certificate of deposit that gets 4% interest each year
4) the amount of money in Jaclyn's wallet which increases and decreases by a different amount each week.

exponential

Factoring
Greatest Common Factor: GCF(   )
Difference of Two Squares: $(\sqrt{\ }_1 + \sqrt{\ }_2)(\sqrt{\ }_1 - \sqrt{\ }_2)$
Trinomials: (x   )(x   )
1) First sign comes down
2) The two signs must multiply for the last sign
3) Find two numbers that multiply to the last number and add/subtract to the middle number
Bridge Method: (Trinomial with a leading coefficient bigger than 1)
1) Build a bridge between the first and last numbers (Multiply)
2) Factor Trinomial Normally
3) Pay the toll (Divide by the leading coefficient)
*If possible, reduce the fraction
If they divide nicely, divide them
If not, put the denominator in front of the variable inside the parenthesis
***Factor further if necessary**

Factor each expression

1. $4x + 8$
   $4(x+2)$

2. $12x + 18$
   $6(2x+3)$

3. $x^2 - 7x$
   $x(x-7)$

4. $2x^2 - 4xy$
   $2x(x-2y)$

5. $5x^2y - 20x$
   $5x(xy-4)$

6. $x^2 - 64$
   $(x+8)(x-8)$

7. $y^2 - 36$
   $(y+6)(y-6)$

8. $4t^2 - 25$
   $(2t+5)(2t-5)$

9. $9x^2 - 16y^4$
   $(3x+4y^2)(3x-4y^2)$

10. $36 - 25x^2$
    $(6+5x)(6-5x)$

11. $\sqrt{100y^4}-\sqrt{49t^6}$

$(10y^2+7t^3)(10y^2-7t^3)$

12. $\sqrt{1}-\sqrt{9x^8y^4}$

$(1+3x^4y^2)(1-3x^4y^2)$

13. $x^2+4x-12$ 1,12 2,6 3,4

$(x+6)(x-2)$

14. y^2+3y+2 1,2

$(y+2)(y+1)$

15. $m^2-8m+15$ 1,15 3,5

$(m-5)(m-3)$

16. $x^2-8x-20$ 1,20 2,10 4,5

$(x-10)(x+2)$

17. $y^2+5y-14$ 1,14 2,7

$(y+7)(y-2)$

18. x^2+x-12 1,12 2,6 3,4

$(x+4)(x-3)$

19. $x^2-3x-10$ 1,10 2,5

$(x-5)(x+2)$

20. $x^2-7x+12$ 1,12 2,6 3,4

$(x-4)(x-3)$

21. $x^2-9x-36$ 1,36 2,18 3,12 4,9 6,6

$(x-12)(x+3)$

22. $y^2-21y+110$ 1,110 2,55 5,22 10,11

$(y-11)(y-10)$

23. x^4+4x^2-12 1,12 2,6 3,4

$(x^2+6)(x^2-2)$

24. x^6-6x^3+9 1,9 3,3

$(x^3-3)(x^3-3)$

1,9
3,3

25. $x^4 - 8x^2 - 9$

$(x^2-9)(x^2+1)$

$(x+3)(x-3)(x^2+1)$

26. $x^4 + x^2 - 2$

$(x^2+2)(x^2-1)$

$(x^2+2)(x+1)(x-1)$

27. $\frac{2x^2}{2} - \frac{50}{2}$

$2(x^2-25)$

$2(x+5)(x-5)$

28. $\frac{2x^2}{2} - \frac{8x}{2} - \frac{10}{2}$ 1,5

$2(x^2-4x-5)$

$2(x-5)(x+1)$

29. $\frac{3x^2}{3} + \frac{9x}{3} - \frac{12}{3}$

$3(x^2+3x-4)$ 1,4 / 2,2

$3(x+4)(x-1)$

30. $\frac{6x^2}{6} - \frac{54}{6}$

$6(x^2-9)$

$6(x+3)(x-3)$

31. $\frac{2x^2}{2} + \frac{14x}{2} + \frac{24}{2}$

$2(x^2+7x+12)$

$2(x+4)(x+3)$

32. $\frac{5x^2}{5} - \frac{500}{5}$

$5(x^2-100)$

$5(x+10)(x-10)$

33. $\frac{ax^2}{a} - \frac{2ax}{a} - \frac{8a}{a}$

1,8
2,4

$a(x^2-2x-8)$

$a(x-4)(x+2)$

34. $\frac{yx^2}{y} - \frac{64y}{y}$

$y(x^2-64)$

$y(x+8)(x-8)$

35. $\frac{12x^2}{3} - \frac{75}{3}$

$3(4x^2-25)$

$3(2x+5)(2x-5)$

36. $x^4 - 81$

$(x^2-9)(x^2+9)$

$(x+3)(x-3)(x^2+9)$

Solving Quadratic Equations by Factoring
Quadratic Equations

Mr. x^2 wants to party. Before he can party, all of his friends have to come over. Once all of his friends come over, they party! At the party, they want to blow bubbles(factor).

*Divide out an integer GCF if possible

1) Bring all terms to the side with x^2 (x^2 should be positive)
2) Factor (Follow the steps of factoring)
3) Set each factor equal to zero (T-chart) and solve

1. $y^2-5y-6=0$

1,6
2,3

$(y-6)(y+1)=0$

$y-6=0$ | $y+1=0$
$+6\ +6$ | $-1\ -1$
$y=6$ | $y=-1$

2. $x^2+4x=0$

$x(x+4)=0$

$x=0$ | $x+4=0$
$-4\ -4$
$x=-4$

3. $a^2-8a=20$
$-20\ -20$

1,20
2,10
4,5

$a^2-8a-20=0$
$(a-10)(a+2)=0$

$a-10=0$ | $a+2=0$
$+10\ +10$ | $-2\ -2$
$a=10$ | $a=-2$

4. $3x^2=48$
$-48\ -48$

$\frac{3x^2}{3}-\frac{48}{3}=\frac{0}{3}$

$x^2-16=0$
$(x+4)(x-4)=0$

$x+4=0$ | $x-4=0$
$-4\ -4$ | $+4\ +4$
$x=-4$ | $x=4$

5. $x^2-5x=7x-20$
$-7x+20\ -7x+20$

$x^2-12x+20=0$
$(x-10)(x-2)=0$

$x-10=0$ | $x-2=0$
$+10\ +10$ | $+2\ +2$
$x=10$ | $x=2$

6. $x(x-6)=7$

$x^2-6x=7$
$-7\ -7$

$x^2-6x-7=0$
$(x-7)(x+1)=0$

$x-7=0$ | $x+1=0$
$+7\ +7$ | $-1\ -1$
$x=7$ | $x=-1$

7. $x^2+3x=8x-4$

$-8x+4 \quad -8x+4$

$x^2-5x+4=0$ 1,4 / 2,2

$(x-4)(x-1)=0$

$x-4=0 \quad x-1=0$

$+4 \ +4 \quad +1 \ +1$

$x=4 \quad x=1$

8. $x^2+6x+2=3x+20$

$-3x-20 \quad -3x-20$

$x^2+3x-18=0$

$(x+6)(x-3)=0$

$x+6=0 \quad x-3=0$

$-6 \ -6 \quad +3 \ +3$

$x=-6 \quad x=3$

9. $x^2+8x+4=4x+9$ 1,5

$-4x-9 \quad -4x-9$

$x^2+4x-5=0$

$(x+5)(x-1)=0$

$x+5=0 \quad x-1=0$

$-5 \ -5 \quad +1 \ +1$

$x=-5 \quad x=1$

10. $x^2+x-4=2x^2+x-29$

$-x^2-x+4 \quad -x^2-x+4$

$0=x^2-25$

$0=(x+5)(x-5)$

$x+5=0 \quad x-5=0$

$-5 \ -5 \quad +5 \ +5$

$x=-5 \quad x=5$

11. $x(x-1)+x=9$

$x^2-\cancel{1x}+\cancel{1x}=9$

$-9 \quad -9$

$x^2-9=0$

$(x+3)(x-3)=0$

$x+3=0 \quad x-3=0$

$-3 \ -3 \quad +3 \ +3$

$x=-3 \quad x=3$

12. $x^2+5(x+2)-4=2x+6$

$x^2+5x+10-4=2x+6$

$x^2+5x+6=2x+6$

$-2x-6 \quad -2x-6$

$x^2+3x=0$

$x(x+3)=0$

$x=0 \quad x+3=0$

$-3 \ -3$

$x=-3$

Solving Linear-Quadratic Systems of Equations Algebraically

1) Get y by itself for both equations
2) Substitute one equation into the other (set them equal to each other)
3) Solve the quadratic equations
   - -bring everything to 1 side
   - -factor
   - -set each factor equal to zero
4) Substitute each x value into one of the original equations to find the corresponding y values

Solve each of the following systems of equations for all values of x and y

1. $y = x^2 - 3x + 1$
   $y = -5x + 9$

$x^2 - 3x + 1 = -5x + 9$
$+5x - 9 \quad +5x - 9$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x+4=0 \quad x-2=0$
$-4 \; -4 \quad +2 \; +2$
$x=-4 \quad x=2$

$x=-4 \quad x=2$
$y=-5x+9 \quad y=-5x+9$
$y=-5(-4)+9 \quad y=-5(2)+9$
$y=29 \quad y=-1$
$(-4,29) \quad (2,-1)$

2. $y = x^2 - 5x + 7$
   $y = -2x + 17$

$x^2 - 5x + 7 = -2x + 17$
$+2x - 17 \quad +2x - 17$
$x^2 - 3x - 10 = 0$
$(x-5)(x+2) = 0$
$x-5=0 \quad x+2=0$
$+5 \; +5 \quad -2 \; -2$
$x=5 \quad x=-2$
$y=-2x+17 \quad y=-2x+17$
$y=-2(5)+17 \quad y=-2(-2)+17$
$y=7 \quad y=21$
$(5,7) \quad (-2,21)$

3. $y = x^2 - 5x + 1$
   $y = -5x + 10$

$x^2 - \cancel{5x} + 1 = -\cancel{5x} + 10$
$+5x - 10 \quad +5x - 10$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x+3=0 \quad x-3=0$
$-3 \; -3 \quad +3 \; +3$

$x=-3 \quad x=3$
$y=-5x+10 \quad y=-5x+10$
$y=-5(-3)+10 \quad y=-5(3)+10$
$y=25 \quad y=-5$
$(-3,25) \quad (3,-5)$

4. $y = x^2 - 7x + 1$
   $y = 2x + 1$

$x^2 - 7x + \cancel{1} = 2x + \cancel{1}$
$-2x - 1 \quad -2x - 1$

$x^2 - 9x = 0$

$x(x-9) = 0$

$x=0 \quad x-9=0$
$+9 \; +9$

$x=9$

$x=0 \quad x=9$
$y=2x+1 \quad y=2x+1$
$y=2(0)+1 \quad y=2(9)+1$
$y=1 \quad y=19$

$(0,1) \quad (9,19)$

5. $y = x^2 - 4x + 3$
$y + 1 = x$

$-1 \quad -1$

$y = x - 1$

$x^2 - 4x + 3 = x - 1$

$-x + 1 \quad -x + 1$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x - 4 = 0 \quad x - 1 = 0$

$+4 \; +4 \quad +1 \; +1$

$x = 4 \quad x = 1$

$x = 4 \quad x = 1$

$y = x - 1 \quad y = x - 1$

$y = 4 - 1 \quad y = 1 - 1$

$y = 3 \quad y = 0$

$(4, 3) \quad (1, 0)$

6. $y - 5 = x^2 - 2x$
$y + 7 = 5x$

$+5 \quad +5$

$y = x^2 - 2x + 5$

$-7 \quad -7$

$y = 5x - 7$

$x^2 - 2x + 5 = 5x - 7$

$-5x + 7 \quad -5x + 7$

$x^2 - 7x + 12 = 0$

$(x-4)(x-3) = 0$

$x - 4 = 0 \quad x - 3 = 0$

$+4 \; +4 \quad +3 \; +3$

$x = 4 \quad x = 3$

$y = 5x - 7 \quad y = 5x - 7$

$y = 5(4) - 7 \quad y = 5(3) - 7$

$y = 13 \quad y = 8$

$(4, 13) \quad (3, 8)$

7. $y = 3x^2 + 6x - 1$
$y + 2x = x^2 + 23$

$-2x \quad -2x$

$y = x^2 - 2x + 23$

$3x^2 + 6x - 1 = x^2 - 2x + 23$

$-x^2 + 2x - 23 \quad -x^2 + 2x - 23$

$\frac{2x^2 + 8x - 24 = 0}{2 \qquad 2}$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$x + 6 = 0 \quad x - 2 = 0$

$-6 \; -6 \quad +2 \; +2$

$x = -6 \quad x = 2$

$y = (-6)^2 - 2(-6) + 23 = 71$

$y = (2)^2 - 2(2) + 23 = 23$

$(-6, 71) \quad (2, 23)$

8. $y = 2x^2 - 7x + 1$
$y + 7x = 9$

$-7x \quad -7x$

$y = -7x + 9$

$2x^2 - 7x + 1 = -7x + 9$

$+7x - 9 \quad +7x - 9$

$\frac{2x^2 - 8}{2} = \frac{0}{2}$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x + 2 = 0 \quad x - 2 = 0$

$-2 \; -2 \quad +2 \; +2$

$x = -2 \quad x = 2$

$y = -7x + 9 \quad y = -7x + 9$

$y = -7(-2) + 9 \quad y = -7(2) + 9$

$y = 23 \quad y = -5$

$(-2, 23) \quad (2, -5)$

9. $x^2 - y = 5$
$y = 3x - 1$

$x^2 - y = 5$

$-x^2 \quad -x^2$

$\frac{-y}{-1} = \frac{-x^2 + 5}{-1}$

$y = x^2 - 5$

$x^2 - 5 = 3x - 1$

$-3x + 1 \quad -3x + 1$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x - 4 = 0 \quad x + 1 = 0$

$+4 \; +4 \quad -1 \; -1$

$x = 4 \quad x = -1$

$y = 3x - 1 \quad y = 3x - 1$

$y = 3(4) - 1 \quad y = 3(-1) - 1$

$y = 11 \quad y = -4$

$(4, 11) \quad (-1, -4)$

10. $y = -2x + 1$
$y = -2x^2 + 3x + 1$

$-2x + 1 = -2x^2 + 3x + 1$

$+2x - 1 \quad +2x - 1$

$0 = -2x^2 + 5x$

$0 = x(-2x + 5)$

$x = 0 \quad -2x + 5 = 0$

$-5 \; -5$

$\frac{-2x}{-2} = \frac{-5}{-2}$

$x = 2.5$

$x = 0 \quad x = 2.5$

$y = -2x + 1 \quad y = -2x + 1$

$y = -2(0) + 1 \quad y = -2(2.5) + 1$

$y = 1 \quad y = -4$

$(0, 1) \quad (2.5, -4)$

Reducing Radicals
Find the largest perfect square that divides into the radicand
Separate into two radicals, perfect squares and non perfect squares
Take the square root of the perfect square
Keep the non perfect square in the radical

PS
1
4
9
16
25
36
49
64
81
100

Express the following in simplest radical form

1. $\sqrt{12}$

$\sqrt{4}\sqrt{3}$

$2\sqrt{3}$

2. $\sqrt{50}$

$\sqrt{25}\sqrt{2}$

$5\sqrt{2}$

3. $3\sqrt{162}$

$3\sqrt{81}\sqrt{2}$

$3(9)\sqrt{2}$

$27\sqrt{2}$

4. $2\sqrt{32}$

$2\sqrt{16}\sqrt{2}$

$2(4)\sqrt{2}$

$8\sqrt{2}$

5. $4\sqrt{45}$

$4\sqrt{9}\sqrt{5}$

$4(3)\sqrt{5}$

$12\sqrt{5}$

6. $2\sqrt{75}$

$2\sqrt{25}\sqrt{3}$

$2(5)\sqrt{3}$

$10\sqrt{3}$

7. $3\sqrt{20}$

$3\sqrt{4}\sqrt{5}$

$3(2)\sqrt{5}$

$6\sqrt{5}$

8. $5\sqrt{54}$

$5\sqrt{9}\sqrt{6}$

$5(3)\sqrt{6}$

$15\sqrt{6}$

Operations with Radicals

Multiplication: Multiply first, reduce last

Multiply your outsides and keep it outside, multiply your insides and keep it inside

Reduce the radical

Addition/Subtraction: Reduce first, add/subtract last

Reduce first so they have the same radicand

Add/subtract coefficients, keep radicand

PS
1
4
9
16
25
36
49
64
81
100

1. $\sqrt{10} \bullet \sqrt{5}$

$\sqrt{50}$

$\sqrt{25}\sqrt{2}$

$5\sqrt{2}$

2. $\sqrt{5} \bullet \sqrt{8}$

$\sqrt{40}$

$\sqrt{4}\sqrt{10}$

$2\sqrt{10}$

3. $5\sqrt{12} \bullet 2\sqrt{6}$

$10\sqrt{72}$

$10\sqrt{36}\sqrt{2}$

$10(6)\sqrt{2}$

$60\sqrt{2}$

4. $6\sqrt{2} \bullet 2\sqrt{14}$

$12\sqrt{28}$

$12\sqrt{4}\sqrt{7}$

$12(2)\sqrt{7}$

$24\sqrt{7}$

5. $-2\sqrt{5} \bullet 2\sqrt{18}$

$-4\sqrt{90}$

$-4\sqrt{9}\sqrt{10}$

$-4(3)\sqrt{10}$

$-12\sqrt{10}$

6. $3\sqrt{10} \bullet -2\sqrt{2}$

$-6\sqrt{20}$

$-6\sqrt{4}\sqrt{5}$

$-6(2)\sqrt{5}$

$-12\sqrt{5}$

7. $\sqrt{63}+\sqrt{7}$

$\sqrt{9}\ 7$

$3\sqrt{7}+1\sqrt{7}$

$4\sqrt{7}$

8. $\sqrt{45}-\sqrt{125}$

$\sqrt{9}\sqrt{5}\ \sqrt{25}\sqrt{5}$

$3\sqrt{5}-5\sqrt{5}$

$-2\sqrt{5}$

9. $4\sqrt{18}+2\sqrt{72}$

$4\sqrt{9}\ \sqrt{2}+2\sqrt{36}\ \sqrt{2}$

$4(3)\sqrt{2}+2(6)\sqrt{2}$

$12\sqrt{2}+12\sqrt{2}$

$24\sqrt{2}$

10. $5\sqrt{27}+2\sqrt{75}$

$5\sqrt{9}\ \sqrt{3}+2\sqrt{25}\ \sqrt{3}$

$5(3)\sqrt{3}+2(5)\sqrt{3}$

$15\sqrt{3}+10\sqrt{3}$

$25\sqrt{3}$

11. $k\sqrt{200}-2k\sqrt{18}$

$K\sqrt{100}\ \sqrt{2}-2K\sqrt{9}\ \sqrt{2}$

$K(10)\sqrt{2}-2K(3)\sqrt{2}$

$10K\sqrt{2}-6K\sqrt{2}$

$4K\sqrt{2}$

12. $2y\sqrt{12}+3y\sqrt{27}$

$2y\sqrt{4}\ \sqrt{3}+3y\sqrt{9}\ \sqrt{3}$

$2y(2)\sqrt{3}+3y(3)\sqrt{3}$

$4y\sqrt{3}+9y\sqrt{3}$

$13y\sqrt{3}$

13. $2\sqrt{50}+3\sqrt{75}-\sqrt{8}$

$2\sqrt{25}\ \sqrt{2}+3\sqrt{25}\ \sqrt{3}-\sqrt{4}\ \sqrt{2}$

$2(5)\sqrt{2}+3(5)\sqrt{3}-2\sqrt{2}$

$\underline{10\sqrt{2}}+15\sqrt{3}\underline{-2\sqrt{2}}$

$8\sqrt{2}+15\sqrt{3}$

14. $5\sqrt{294}-3\sqrt{216}-4\sqrt{180}$

$\sqrt{49}\ \sqrt{6}\ \sqrt{36}\ \sqrt{6}\ \sqrt{36}\ \sqrt{5}$

$5(7)\sqrt{6}-3(6)\sqrt{6}-4(6)\sqrt{5}$

$\underline{35\sqrt{6}}\underline{-18\sqrt{6}}-24\sqrt{5}$

$17\sqrt{6}-24\sqrt{5}$

Rationalizing the Denominator

Rationalizing the Denominator

Multiply top and bottom by the radical
*When multiplying a radical by itself, the radical cancels
Reduce the fraction if possible

1. $\frac{2}{\sqrt{5}} \frac{\sqrt{5}}{\sqrt{5}}$

$\frac{2\sqrt{5}}{5}$

2. $\frac{-7}{\sqrt{11}} \frac{\sqrt{11}}{\sqrt{11}}$ $\quad \frac{-7\sqrt{11}}{11}$

3. $\frac{3}{\sqrt{6}} \frac{\sqrt{6}}{\sqrt{6}}$

$\frac{\overset{\div 2}{3\sqrt{6}}}{6 \div 2}$ $\quad \frac{1\sqrt{6}}{2}$

4. $\frac{15}{\sqrt{10}} \frac{\sqrt{10}}{\sqrt{10}}$ $\quad \frac{\overset{\div 5}{15\sqrt{10}}}{10 \div 5}$ $\quad \frac{3\sqrt{10}}{2}$

5. $\frac{3}{2\sqrt{5}} \frac{\sqrt{5}}{\sqrt{5}}$ $\quad \frac{3\sqrt{5}}{2(5)}$ $\quad \frac{3\sqrt{5}}{10}$

6. $\frac{6}{5\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}}$ $\quad \frac{6\sqrt{2}}{5(2)} = \frac{\overset{\div 2}{6\sqrt{2}}}{10 \div 2}$ $\quad \frac{3\sqrt{2}}{5}$

7. $\frac{8}{2\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}}$

$\frac{8\sqrt{3}}{2(3)}$ $\quad \frac{\overset{\div 2}{8\sqrt{3}}}{6 \div 2}$

$\frac{4\sqrt{3}}{3}$

8. $\frac{-6}{3\sqrt{5}} \frac{\sqrt{5}}{\sqrt{5}}$ $\quad \frac{-6\sqrt{5}}{3(5)} = \frac{\overset{\div 3}{-6\sqrt{5}}}{15 \div 3}$

$\frac{-2\sqrt{5}}{5}$

Solving Quadratic Equations Using the Quadratic Formula
Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) $ax^2 + bx + c = 0$
2) List a, b, and c values
3) Substitute values into quadratic formula
4) Type into calculator, one with plus, one with minus.
5) Round to the given value.

Solve the following equations rounding all answers to the *nearest* tenth.

1. $3x^2 - 5x - 7 = 0$

$a=3$
$b=-5$
$c=-7$

$x = \frac{5 \pm \sqrt{(-5)^2-4(3)(-7)}}{2(3)} = 2.6$

$x = \frac{5 - \sqrt{(-5)^2-4(3)(-7)}}{2(3)} = -.9$

2. $4w^2 + 12w - 44 = 0$

$a=4$
$b=12$
$c=-44$

$x = \frac{-12+\sqrt{(12)^2-4(4)(-44)}}{2(4)} = 2.1$

$x = \frac{-12-\sqrt{(12)^2-4(4)(-44)}}{2(4)} = -5.1$

3. $x^2 + x - 5 = 0$

$a=1$
$b=1$
$c=-5$

$x = \frac{-1+\sqrt{(1)^2-4(1)(-5)}}{2(1)} = 1.8$

$x = \frac{-1-\sqrt{(1)^2-4(1)(-5)}}{2(1)} = -2.8$

4. $2x^2 + 4x = 1$

$-1 \quad -1$

$2x^2+4x-1=0$

$a=2$
$b=4$
$c=-1$

$x = \frac{-4+\sqrt{(4)^2-4(2)(-1)}}{2(2)} = .2$

$x = \frac{-4-\sqrt{(4)^2-4(2)(-1)}}{2(2)} = -2.2$

5. $6x^2 + 5x - 6 = 0$

$a=6$
$b=5$
$c=-6$

$x = \frac{-5+\sqrt{(5)^2-4(6)(-6)}}{2(6)} = \frac{2}{3}$

$x = \frac{-5-\sqrt{(5)^2-4(6)(-6)}}{2(6)} = \frac{-3}{2}$

6. $7x^2 + 2x - 2 = 0$

$a=7$
$b=2$
$c=-2$

$x = \frac{-2+\sqrt{(2)^2-4(7)(-2)}}{2(7)} = .4$

$x = \frac{-2-\sqrt{(2)^2-4(7)(-2)}}{2(7)} = -.7$

7. $3x^2 + 2x = 8$

$-8 \quad -8$

$3x^2+2x-8=0$

$a=3$

$b=2$

$c=-8$

$x=\frac{-2+\sqrt{(2)^2-4(3)(-8)}}{2(3)}=\frac{4}{3}$

$x=\frac{-2-\sqrt{(2)^2-4(3)(-8)}}{2(3)}=-2$

8. $4x^2 + 3x = -2$

$+2 \quad +2$

$4x^2+3x+2=0$

$a=4$

$b=3$

$c=2$

$x=\frac{-3+\sqrt{(3)^2-4(4)(2)}}{2(4)}$

$x=\frac{-3-\sqrt{(3)^2-4(4)(2)}}{2(4)}$

No Solution

9. $3x^2 + 2x = 3$

$-3 \quad -3$

$3x^2+2x-3=0$

$a=3$

$b=2$

$c=-3$

$x=\frac{-2+\sqrt{(2)^2-4(3)(-3)}}{2(3)}=.7$

$x=\frac{-2-\sqrt{(2)^2-4(3)(-3)}}{2(3)}=-1.4$

10. $5x^2 - 2x = 3$

$-3 \quad -3$

$5x^2-2x-3=0$

$a=5$

$b=-2$

$c=-3$

$x=\frac{2+\sqrt{(-2)^2-4(5)(-3)}}{2(5)}=1$

$x=\frac{2-\sqrt{(-2)^2-4(5)(-3)}}{2(5)}=-\frac{3}{5}$

11. $5x^2 + 2x = -3x + 2$

$+3x-2 \quad +3x-2$

$5x^2+5x-2=0$

$a=5$

$b=5$

$c=2$

$x=\frac{-5+\sqrt{(5)^2-4(5)(-2)}}{2(5)}=.3$

$x=\frac{-5-\sqrt{(5)^2-4(5)(-2)}}{2(5)}=-1.3$

12. $6x^2 + x = 4x + 5$

$-4x-5 \quad -4x-5$

$6x^2-3x-5=0$

$a=6$

$b=-3$

$c=-5$

$x=\frac{3+\sqrt{(-3)^2-4(6)(-5)}}{2(6)}=1.2$

$x=\frac{3-\sqrt{(-3)^2-4(6)(-5)}}{2(6)}=-.7$

Solving Quadratic Equations Using Completing the Square

Completing the Square: $(x-a)^2$

*Divide out a GCF if possible

1) Bring both variable terms to one side and the constant term to the other side
2) Add $\left(\frac{b}{2}\right)^2$ to both sides
3) Factor the trinomial (Both factors must be the same)
4) Rewrite the factors as a binomial squared
5) Take the square root of both sides (The right hand side should have a $\pm$)
6) Add or subtract to isolate x

$\left(\frac{-6}{2}\right)^2=9$

1. Which equation has the same solution as $x^2 - 6x - \cancel{12} = 0$?

1) $(x+3)^2 = 21$
2) $(x-3)^2 = 21$
3) $(x+3)^2 = 3$
4) $(x-3)^2 = 3$

$+12 \quad +12$

$x^2-6x=12$

$x^2-6x+\boxed{9}=12+\boxed{9}$

$(x-3)(x-3)=21$

$(x-3)^2=21$

$\left(\frac{-8}{2}\right)^2=16$

2. Which equation has the same solutions as $x^2 - 8x + \cancel{3} = 0$?

(1) $(x-8)^2 = 16$ (3) $(x-4)^2 = 13$

(2) $(x-8)^2 = 13$ (4) $(x-4)^2 = 61$

$-3 \quad -3$

$x^2-8x=-3$

$x^2-8x+\boxed{16}=-3+\boxed{16}$

$(x-4)(x-4)=13$

$(x-4)^2=13$

3. When solving the equation $x^2 - 8x - \cancel{7} = 0$ by completing the square, which equation is a step in the process?

1) $(x-4)^2 = 9$
2) $(x-4)^2 = 23$
3) $(x-8)^2 = 9$
4) $(x-8)^2 = 23$

$+7 \quad +7$

$\left(\frac{-8}{2}\right)^2=16$

$x^2-8x=7$

$x^2-8x+\boxed{16}=7+\boxed{16}$

$(x-4)(x-4)=23$

$(x-4)^2=23$

$\left(\frac{6}{2}\right)^2=9$

4. Which equation has the same solutions as $x^2 + 6x - \cancel{7} = 0$?

1) $(x+3)^2 = 2$
2) $(x-3)^2 = 2$
3) $(x-3)^2 = 16$
4) $(x+3)^2 = 16$

$+7 \quad +7$

$x^2+6x=7$

$x^2+6x+\boxed{9}=7+\boxed{9}$

$(x+3)(x+3)=16$

$(x+3)^2=16$

5. The method of completing the square was used to solve the equation $\frac{2x^2 - 12x + 6 = 0}{2 \quad\quad 2}$. Which equation is a correct step when using this method?

$\left(\frac{-6}{2}\right)^2 = 9$

1) $(x-3)^2 = 6$
2) $(x-3)^2 = -6$
3) $(x-3)^2 = 3$
4) $(x-3)^2 = -3$

$x^2 - 6x + \cancel{3} = 0$
$\quad\; -3 \quad -3$
$x^2 - 6x = -3$
$x^2 - 6x + \boxed{9} = -3 + \boxed{9}$
$(x-3)(x-3) = 6$
$(x-3)^2 = 6$

6. The quadratic equation $x^2 - 6x = 12$ is rewritten in the form $(x+p)^2 = q$, where q is a constant. What is the value of p?

1) -12
2) -9
3) -3
4) 9

$\left(\frac{-6}{2}\right)^2 = 9$
$x^2 - 6x + \boxed{9} = 12 + \boxed{9}$
$(x-3)(x-3) = 21$
$(x-3)^2 = 21$
$p = -3$

7. What are the solutions to the equation $x^2 - 8x = 10$?

1) $4 \pm \sqrt{10}$
2) $4 \pm \sqrt{26}$
3) $-4 \pm \sqrt{10}$
4) $-4 \pm \sqrt{26}$

$\left(\frac{-8}{2}\right)^2 = 16$
$x^2 - 8x + \boxed{16} = 10 + \boxed{16}$
$(x-4)(x-4) = 26$
$\sqrt{(x-4)^2} = \pm\sqrt{26}$
$x - \cancel{4} = \pm\sqrt{26}$
$\quad +4 \quad +4$
$x = 4 \pm \sqrt{26}$

$\left(\frac{4}{2}\right)^2$
4

8. What are the roots of the equation $x^2 + 4x - \cancel{16} = 0$?

1) $2 \pm 2\sqrt{5}$
2) $-2 \pm 2\sqrt{5}$
3) $2 \pm 4\sqrt{5}$
4) $-2 \pm 4\sqrt{5}$

$+16 \quad +16$
$x^2 + 4x = 16$
$x^2 + 4x + \boxed{4} = 16 + \boxed{4}$
$(x+2)(x+2) = 20$
$\sqrt{(x+2)^2} = \pm\sqrt{20}$
$x + \cancel{2} = \pm\sqrt{20}$
$\quad -2 \quad -2$
$x = -2 \pm \sqrt{20}$
$x = -2 \pm \sqrt{20}$
$\sqrt{4} \; \sqrt{5}$
$x = -2 \pm 2\sqrt{5}$

$\left(\frac{-6}{2}\right)^2$
9

9. Solve the equation $x^2 - 6x - \cancel{19} = 0$ using the completing the square method.

$+19 \quad +19$
$x^2 - 6x = 19$
$x^2 - 6x + \boxed{9} = 19 + \boxed{9}$
$(x-3)(x-3) = 28$
$\sqrt{(x-3)^2} = \pm\sqrt{28}$
$x - \cancel{3} = \pm\sqrt{28}$
$\quad +3 \quad +3$
$x = 3 \pm \sqrt{28}$
$x = 3 \pm \sqrt{28}$
$\sqrt{4} \; \sqrt{7}$
$x = 3 \pm 2\sqrt{7}$

$\left(\frac{-6}{2}\right)^2$
9

10. Solve the equation $x^2 - 6x = 15$ by completing the square.

$x^2 - 6x + \boxed{9} = 15 + \boxed{9}$
$(x-3)(x-3) = 24$
$\sqrt{(x-3)^2} = \pm\sqrt{24}$
$x - \cancel{3} = \pm\sqrt{24}$
$\quad +3 \quad +3$
$x = 3 \pm \sqrt{24}$
$\sqrt{4} \; \sqrt{6}$
$x = 3 \pm 2\sqrt{6}$

$\left(\frac{4}{2}\right)^2$
4

11. Solve the following equation by completing the square: $x^2 + 4x = 2$

$x^2 + 4x + \boxed{4} = 2 + \boxed{4}$
$(x+2)(x+2) = 6$
$\sqrt{(x+2)^2} = \pm\sqrt{6}$
$x + \cancel{2} = \pm\sqrt{6}$
$\quad -2 \quad -2$
$x = -2 \pm \sqrt{6}$

In Terms of x with Quadratics

1) List the things you are comparing
2) Call the last thing x
3) Express everything else in terms of x
4) Create an equation to represent the situation
5) Solve the quadratic equation
    a. Everything to one side
    b. Factor
    c. Set each factor equal to zero
6) Substitute x into your original expressions and answer the question

1. Javon's homework is to determine the dimensions of his rectangular backyard. He knows that the length is 10 feet more than the width, and the total area is 144 square feet. Write an equation that Javon could use to solve this problem. Then find the dimensions, in feet, of his backyard.

$8+10=18$ $l=x+10$ $A=lw$ $0=x^2+10x-144$ length=18

8 $w=x$ $144=x(x+10)$ $0=(x+18)(x-8)$ width=8

$A=144$ $144=x^2+10x$ $x+18=0$ | $x-8=0$

$-144 \quad -144$ $-18\ -18$ $+8\ +8$

$\cancel{x=-18}$ $x=8$

2. The length of a rectangle is 3 inches more than its width. The area of the rectangle is 40 square inches. What is the length, in inches, of the rectangle?

$5+3=8$ $l=x+3$ $A=lw$ $0=x^2+3x-40$ length=8

5 $w=x$ $40=x(x+3)$ $0=(x+8)(x-5)$

$A=40$ $40=x^2+3x$ $x+8=0$ | $x-5=0$

$-40 \quad -40$ $-8\ -8$ $+5\ +5$

$\cancel{x=-8}$ $x=5$

3. A rectangle has an area of 24 square units. The width is 5 units less than the length. What is the length, in units, of the rectangle?

8 $l=x$ $A=lw$ $0=x^2-5x-24$ length= 8

$8-5=3$ $w=x-5$ $24=x(x-5)$ $0=(x-8)(x+3)$

$A=24$ $24=x^2-5x$ $x-8=0$ | $x+3=0$

$-24 \quad -24$ $+8\ +8$ $-3\ -3$

$x=8$ $\cancel{x=-3}$

4. A landscaper is creating a rectangular flower bed such that the width is half of the length. The area of the flower bed is 34 square feet. Write and solve an equation to determine the width of the flower bed, to the *nearest tenth of a foot*.

$w=\frac{1}{2}x$ $A=lw$

$l=x$ $34=\frac{1}{2}x(x)$

$A=34$ $2(34)=(\frac{1}{2}x^2)2$

$\sqrt{68}=\sqrt{x^2}$

$8.2=x$

5. The length of a rectangular sign is 6 inches more than half its width. The area of this sign is 432 square inches. Write an equation in one variable that could be used to find the number of inches in the dimensions of this sign. Solve this equation algebraically to determine the dimensions of this sign, in inches.

$\frac{1}{2}(24)+6$
18
24

$l = \frac{1}{2}x+6$
$w = x$
$A = 432$

$A = lw$
$432 = x(\frac{1}{2}x+6)$
$432 = \frac{1}{2}x^2+6x$
$-432 \quad -432$

$2(0 = \frac{1}{2}x^2+6x-432)2$
$0 = x^2+12x-864$
$0 = (x+36)(x-24)$
$x+36=0 \quad x-24=0$
$-36 \; -36 \quad +24 \; +24$
$\cancel{x=-36} \quad x=24$

length = 18
width = 24

6. When 36 is subtracted from the square of a number, the result is five times the number. What is the positive solution?

$x^2-36=5x$
$-5x \quad -5x$
$x^2-5x-36=0$
$(x-9)(x+4)=0$

$x-9=0 \quad x+4=0$
$+9 \; +9 \quad -4 \; -4$
$x=9 \quad \cancel{x=-4}$

7. Jordan and Aaron are brothers. Jordan's age is four more than Aaron's age. If the product of their ages is 32, how old is Jordan?

4+4=8
4

$J=x+4$
$A=x$

$x(x+4)=32$
$x^2+4x=32$
$-32 \; -32$

$x^2+4x-32=0$
$(x+8)(x-4)=0$
$x+8=0 \quad x-4=0$
$-8 \; -8 \quad +4 \; +4$
$\cancel{x=-8} \quad x=4$

Jordan is 8.

8. Tamara has two sisters. One of the sisters is 7 years older than Tamara. The other sister is 3 years younger than Tamara. The product of Tamara's sisters' ages is 24. How old is Tamara?

5+7=12
S1: $x+7$
5 T: x
5-3=2 S2: $x-3$

	x	$+7$
x	x^2	$+7x$
-3	$-3x$	-21

$x^2+4x-21$

$(x+7)(x-3)=24$
$x^2+4x-21=24$
$-24 \; -24$
$x^2+4x-45=0$
$(x+9)(x-5)=0$
$x+9=0 \quad x-5=0$
$-9 \; -9 \quad +5 \; +5$
$\cancel{x=-9} \quad x=5$

Tamara is 5

Zeros, Vertex, Axis of Symmetry
The zeros (roots) hit the x axis. Graph the equation in the calculator and look at the graph.
The vertex (maximum/minimum) is the turning point.
The axis of symmetry (AOS) is the vertical line that cuts the graph in half. $x = \#$

1. The zeros of the function $f(x) = (x+2)^2 - 25$ are
1) -2 and 5
2) -3 and 7
3) -5 and 2
4) -7 and 3

2. The zeros of the function $f(x) = 2x^2 - 4x - 6$ are
1) 3 and -1
2) 3 and 1
3) -3 and 1
4) -3 and -1

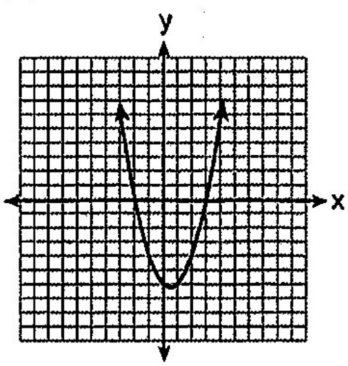
3. The zeros of the function $p(x) = x^2 - 2x - 24$ are
1) -8 and 3
2) -6 and 4
3) -4 and 6
4) -3 and 8

4. For which function defined by a polynomial are the zeros of the polynomial -4 and -6?
1) $y = x^2 - 10x - 24$
2) $y = x^2 + 10x + 24$
3) $y = x^2 + 10x - 24$
4) $y = x^2 - 10x + 24$

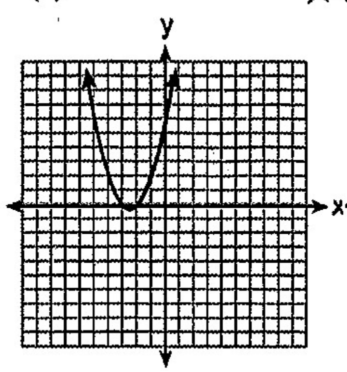
5. If $4x^2 - 100 = 0$, the roots of the equation are
1) -25 and 25
2) -25, only
3) -5 and 5
4) -5, only

6. The graphs below represent functions defined by polynomials. For which function are the zeros of the polynomials 2 and -3?

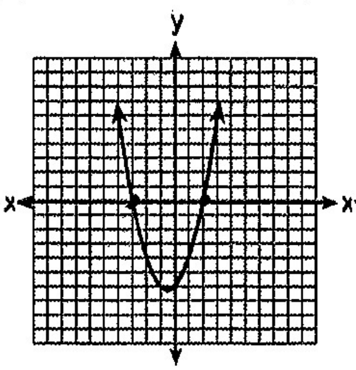
(1)



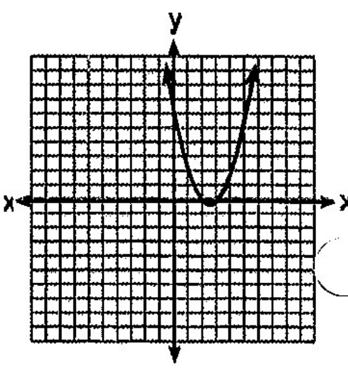
(2)



(3)



(4)

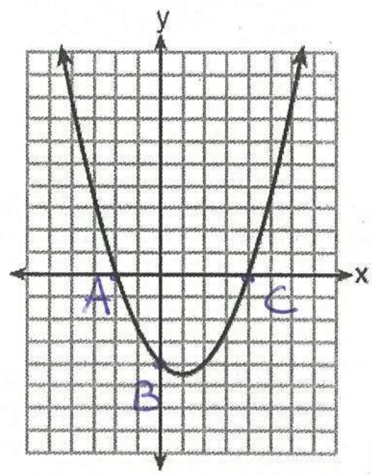


7. Which polynomial function has zeros at -3, 0, and 4?

1) $f(x) = (x+3)(x^2+4)$
2) $f(x) = (x^2-3)(x-4)$
3) $f(x) = x(x+3)(x-4)$
4) $f(x) = x(x-3)(x+4)$

8. The graph of $y = \frac{1}{2}x^2 - x - 4$ is shown below. The points $A(-2,0)$, $B(0,-4)$, and $C(4,0)$ lie on this graph.

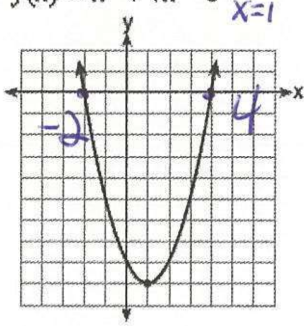
Which of these points can determine the zeros of the equation $y = \frac{1}{2}x^2 - x - 4$?

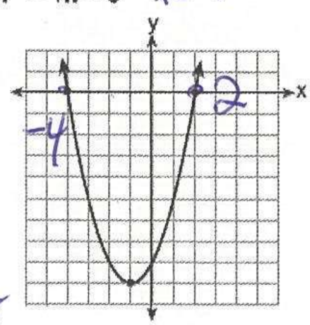


1) A, only
2) B, only
3) A and C, only
4) A, B, and C

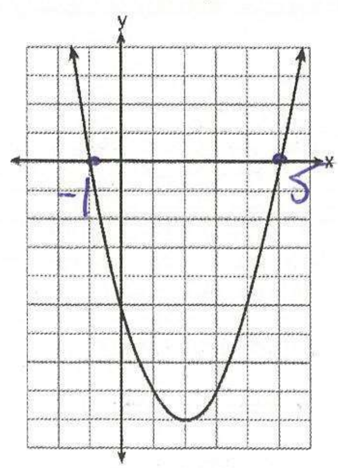
9. Which function has zeros of -4 and 2?

1) $f(x) = x^2 + 7x - 8$ $x=-8$ $x=1$
3) $g(x) = x^2 - 7x - 8$ $x=8$ $x=-1$

2)


4)


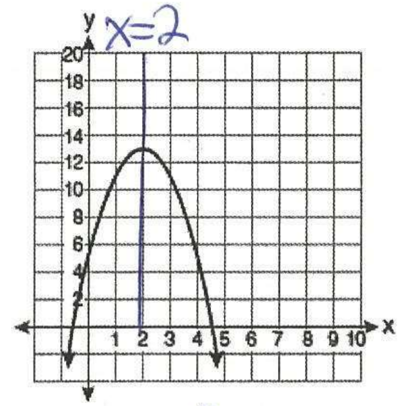
10. The graph of $f(x)$ is shown below.



Based on this graph, what are the roots of the equation $f(x) = 0$?

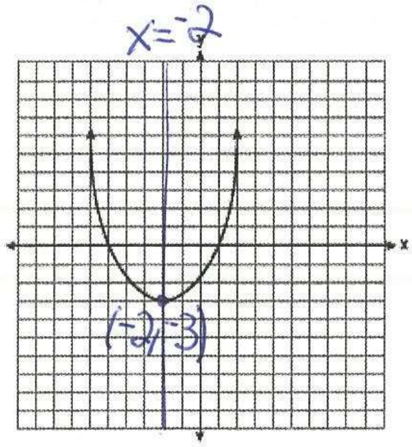
1) 1 and −5
2) −1 and 5
3) 2 and −9
4) −1 and −5 and 5

11. What is the equation of the axis of symmetry of the parabola shown in the diagram below?



1) $x = -0.5$
2) $x = 2$
3) $x = 4.5$
4) $x = 13$

12. What are the vertex and the axis of symmetry of the parabola shown in the diagram below?

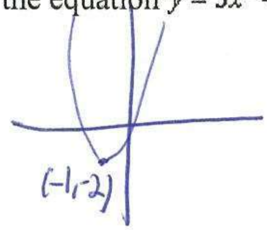


1) The vertex is $(-2,-3)$, and the axis of symmetry is $x = -2$.
2) The vertex is $(-2,-3)$, and the axis of symmetry is $y = -2$.
3) The vertex is $(-3,-2)$, and the axis of symmetry is $y = -2$.
4) The vertex is $(-3,-2)$, and the axis of symmetry is $x = -2$.

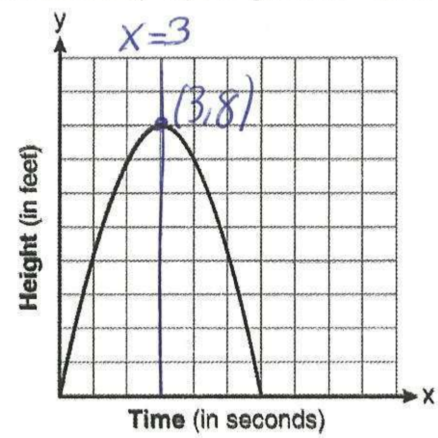
type in calc

13. What is the vertex of the graph of the equation $y = 3x^2 + 6x + 1$?

1) $(-1,-2)$
2) $(-1,10)$
3) $(1,-2)$
4) $(1,10)$



14. The graph below represents the parabolic path of a ball kicked by a young child. What are the vertex and the axis of symmetry for the parabola?



1) vertex: $(3,8)$; axis of symmetry: $x = 3$
2) vertex: $(3,8)$; axis of symmetry: $y = 3$
3) vertex: $(8,3)$; axis of symmetry: $x = 3$
4) vertex: $(8,3)$; axis of symmetry: $y = 3$

Vertex Form of a Parabola
Axis of Symmetry Method

1) Find the axis of symmetry graphically or using $x = -\frac{b}{2a}$
2) Find the y coordinate of the vertex by substituting axis of symmetry into equation
3) Substitute vertex is into $f(x) = a(x - v)^2 + t$ where (v, t) is the vertex

Completing the Square Method

1) $f(x) + c = a(x^2 + bx)$
2) Add the *distributed value* of $\left(\frac{b}{2}\right)^2$ to both sides
3) Factor the trinomial
4) Re-write as a binomial squared
5) Isolate $f(x)$

Rewrite the following equations in vertex form and state the vertex

1. $f(x) = x^2 + 6x + 2$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{-6}{2(1)}$

$x = -3$

$y = (-3)^2 + 6(-3) + 2$

$y = -7 \quad (-3, -7)$

$y = (x+3)^2 - 7$

CTS method

$f(x) - 2 = x^2 + 6x$

$9 + f(x) - 2 = x^2 + 6x + 9$

$f(x) + 7 = (x+3)^2$
$\quad -7 \qquad -7$

$f(x) = (x+3)^2 - 7$

$(-3, -7)$

2. $f(x) = x^2 - 8x + 3$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{8}{2(1)}$

$x = 4$

$y = (4)^2 - 8(4) + 3$

$y = -13 \quad (4, -13)$

$y = (x-4)^2 - 13$

CTS Method

$f(x) - 3 = x^2 - 8x$

$16 + f(x) - 3 = x^2 - 8x + 16$

$f(x) + 13 = (x-4)^2$
$\quad -13 \qquad -13$

$f(x) = (x-4)^2 - 13$

$(4, -13)$

3. $f(x) = 2x^2 + 12x - 6$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{-12}{2(2)}$

$x = -3$

$f(-3) = 2(-3)^2 + 12(-3) - 6$

$f(-3) = -24$

$(-3, -24)$

$f(x) = 2(x+3)^2 - 24$

CTS Method

$f(x) + 6 = 2(x^2 + 6x)$ 18

$18 + f(x) + 6 = 2(x^2 + 6x + 9)$

$f(x) + 24 = 2(x+3)^2$
$\quad -24 \qquad -24$

$f(x) = 2(x+3)^2 - 24$

$(-3, -24)$

4. $y = 4x^2 + 8x - 6$

AOS method

$x = \frac{-b}{2a}$

$x = \frac{-8}{2(4)}$

$x = -1$

$y = 4(-1)^2 + 8(-1) - 6$

$y = -10$

$(-1, -10)$

$y = 4(x+1)^2 - 10$

CTS Method

$y + 6 = 4(x^2 + 2x)$

$4 + y + 6 = 4(x^2 + 2x + 1)$

$y + 10 = 4(x+1)^2$
$\quad -10 \qquad -10$

$y = 4(x+1)^2 - 10$

$(-1, -10)$

CTS ~~AOS~~ Method

5. $f(x) = -x^2 + 4x + 16$

$f(x) - 16 = -1(x^2 - 4x)$ -4

$-4 + f(x) - 16 = -1(x^2 - 4x + 4)$

$f(x) - 20 = -1(x-2)^2$

$+20 \quad +20$

$f(x) = -1(x-2)^2 + 20$

$(2, 20)$

AOS Method

$x = -\frac{b}{2a}$

$x = \frac{-4}{2(-1)}$

$x = 2$

$f(2) = -(2)^2 + 4(2) + 16$

$f(2) = 20$

$(2, 20)$

$f(x) = -1(x-2)^2 + 20$

6. $f(x) = x^2 + 12x + 2$

CTS Method

$f(x) - 2 = x^2 + 12x$

$36 + f(x) - 2 = x^2 + 12x + 36$

$f(x) + 34 = (x+6)^2$

$-34 \quad -34$

$f(x) = (x+6)^2 - 34$

$(-6, -34)$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{-12}{2(1)}$

$x = -6$

$f(-6) = (-6)^2 + 12(-6) + 2$

$f(-6) = -34$ $(-6, -34)$

$f(x) = (x+6)^2 - 34$

7. $f(x) = -x^2 + 14x + 20$

CTS method

$f(x) - 20 = -1(x^2 - 14x)$ -49

$-49 + f(x) - 20 = -1(x^2 - 14x + 49)$

$f(x) - 69 = -1(x-7)^2$

$+69 \quad +69$

$f(x) = -1(x-7)^2 + 69$

$(7, 69)$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{-14}{2(-1)}$

$x = 7$

$f(7) = -(7)^2 + 14(7) + 20$

$f(7) = 69$

$(7, 69)$

8. $f(x) = 4x^2 + 12x - 28$

CTS Method

$f(x) + 28 = 4(x^2 + 3x)$

$9 + f(x) + 28 = 4(x^2 + 3x + \frac{9}{4})$ 9

$f(x) + 37 = 4(x + \frac{3}{2})^2$

$-37 \quad -37$

$f(x) = 4(x + \frac{3}{2})^2 - 37$

$(-\frac{3}{2}, -37)$

AOS Method

$x = \frac{-b}{2a}$

$x = \frac{-12}{2(4)}$

$x = -\frac{3}{2}$

$f(-\frac{3}{2}) = 4(-\frac{3}{2})^2 + 12(-\frac{3}{2}) - 28$

$f(-\frac{3}{2}) = -37$

$(-\frac{3}{2}, -37)$

8. Identify the turning point of the function $f(x) = x^2 - 2x + 8$ by writing its equation in vertex form.

$-8 \quad -8$

$1 + f(x) - 8 = x^2 - 2x + 1$

$f(x) - 7 = (x-1)^2$

$+7 \quad +7$

$f(x) = (x-1)^2 + 7$

$(1, 7)$

9. Given the function $f(x) = -x^2 + 8x + 9$, state whether the vertex represents a maximum or minimum point for the function. Explain your answer.
Rewrite $f(x)$ in vertex form by completing the square.

maximum because it opens down.

$f(x) - 9 = -1(x^2 - 8x)$ -16

$-16 + f(x) - 9 = -1(x^2 - 8x + 16)$

$f(x) - 25 = -1(x-4)^2$

$+25 \quad +25$

$f(x) = -1(x-4)^2 + 25$

Modeling Parabolas

The initial height is the y-intercept/constant term.
The domain is from [0,second zero].
The vertex is the turning point. 2^{nd} Trace (Calc): Maximum/Minimum
The object hits the ground at the second zero
To find the zeros algebraically, set the equation equal to zero, factor, set each factor equal to 0.

1. The expression $-4.9t^2 + 50t + 2$ represents the height, in meters, of a toy rocket t seconds after launch. The initial height of the rocket, in meters, is

y-intercept

1) 0
2) 2
3) 4.9
4) 50

2. The height of a ball Doreen tossed into the air can be modeled by the function $h(x) = -4.9x^2 + 6x + 5$, where x is the time elapsed in seconds, and $h(x)$ is the height in meters. The number 5 in the function represents

1) the initial height of the ball

y-intercept

2) the time at which the ball reaches the ground
3) the time at which the ball was at its highest point
4) the maximum height the ball attained when thrown in the air

3. A toy rocket is launched from the ground straight upward. The height of the rocket above the ground, in feet, is given by the equation $h(t) = -16t^2 + 64t$, where t is the time in seconds. Determine the domain for this function in the given context. Explain your reasoning.

[0,4]
The rocket hits the ground at $t=4$.

4. The function $h(t) = -16t^2 + 144$ represents the height, $h(t)$, in feet, of an object from the ground at t seconds after it is dropped. A realistic domain for this function is

1) $-3 \le t \le 3$ [0,3] the zero is 3
2) $0 \le t \le 3$
3) $0 \le h(t) \le 144$
4) all real numbers

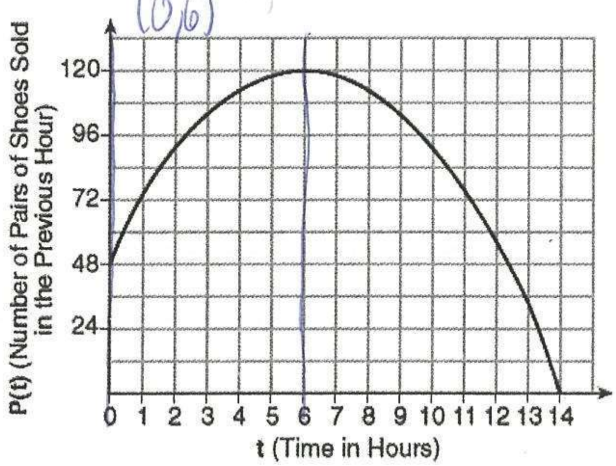
5. Morgan throws a ball up into the air. The height of the ball above the ground, in feet, is modeled by the function $h(t) = -16t^2 + 24t$, where t represents the time, in seconds, since the ball was thrown. What is the appropriate domain for this situation?

1) $0 \le t \le 1.5$ 1.5 is a zero
2) $0 \le t \le 9$
3) $0 \le h(t) \le 1.5$
4) $0 \le h(t) \le 9$

domain is x values

6. A manager wanted to analyze the online shoe sales for his business. He collected data for the number of pairs of shoes sold each hour over a 14-hour time period. He created a graph to model the data, as shown below.
The manager believes the set of integers would be the most appropriate domain for this model. Explain why he is *incorrect*. State the entire interval for which the number of pairs of shoes sold is increasing. Determine the average rate of change between the sixth and fourteenth hours, and explain what it means in the context of the problem.

increasing
(0,6)



Shoes can be sold at fractional hours. Also, they can't be sold at negative hours.

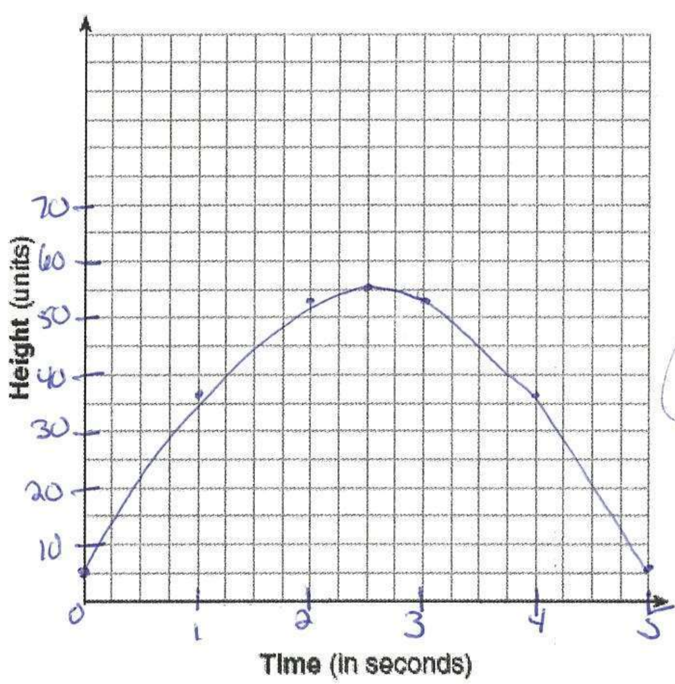
$\frac{f(b)-f(a)}{b-a}$

x	y
6	120
14	0

$\frac{0-120}{14-6} = -15$

On average, between 6 and 14 hours, the number of pairs of shoes sold decreased by 15 pairs per hour.

7. Alex launched a ball into the air. The height of the ball can be represented by the equation $h = -8t^2 + 40t + 5$, where h is the height, in units, and t is the time, in seconds, after the ball was launched. Graph the equation from $t = 0$ to $t = 5$ seconds.
State the coordinates of the vertex and explain its meaning in the context of the problem.



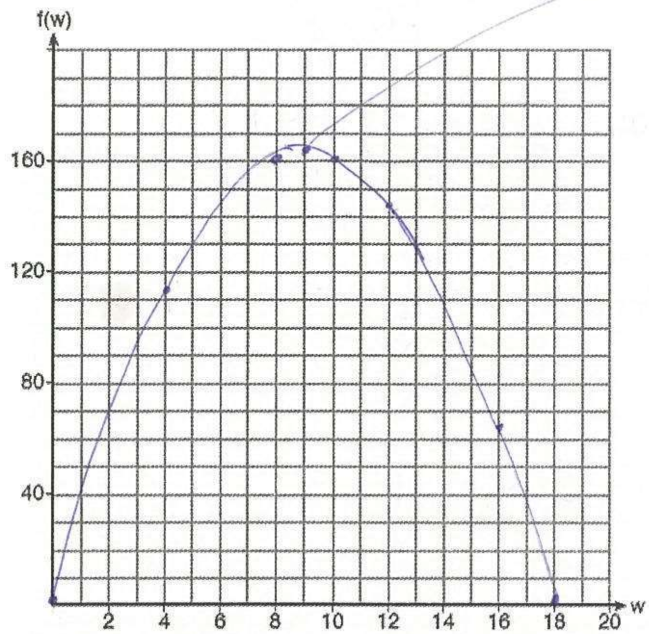
x	y
0	5
1	37
2	53
3	53
4	37
5	5

(2.5, 55)
2nd Trace, Maximum

At 2.5 seconds, the maximum height of the ball is 55 units

8. Paul plans to have a rectangular garden adjacent to his garage. He will use 36 feet of fence to enclose three sides of the garden. The area of the garden, in square feet, can be modeled by $f(w) = w(36 - 2w)$, where w is the width in feet. On the set of axes below, sketch the graph of $f(w)$. Explain the meaning of the vertex in the context of the problem.

2nd Trace, Maximum
(9,162)



x	y
0	0
4	112
8	160
~~12~~	~~144~~
~~16~~	~~64~~
9	162
10	160
12	144
16	64
18	0

At a width of 9 feet, the maximum area of the garden will be 162 square feet.

9. An Air Force pilot is flying at a cruising altitude of 9000 feet and is forced to eject from her aircraft. The function $h(t) = -16t^2 + 128t + 9000$ models the height, in feet, of the pilot above the ground, where t is the time, in seconds, after she is ejected from the aircraft. Determine and state the vertex of $h(t)$. Explain what the second coordinate of the vertex represents in the context of the problem. After the pilot was ejected, what is the maximum number of feet she was above the aircraft's cruising altitude? Justify your answer.

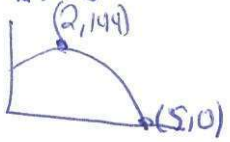
2nd Trace, maximum (4,9256) The maximum height of the pilot was 9,256 feet.

$$\begin{array}{r} 9256 \\ -9000 \\ \hline 256 \text{ feet} \end{array}$$

10. Let $h(t) = -16t^2 + 64t + 80$ represent the height of an object above the ground after t seconds. Determine the number of seconds it takes to achieve its maximum height. Justify your answer. State the time interval, in seconds, during which the height of the object *decreases*. Explain your reasoning.

2^{nd} Trace, maximum
(2, 144)
2 Seconds

*Adjust window



2^{nd} Trace, Zero
(5, 0)

decreases (2, 5)
It is decreasing from its maximum until it hits the ground.

11. When an apple is dropped from a tower 256 feet high, the function $h(t) = -16t^2 + 256$ models the height of the apple, in feet, after t seconds. Determine, algebraically, the number of seconds it takes the apple to hit the ground.

Factor

$f(x) = 0$

$\frac{0}{-16} = \frac{-16x^2 + 256}{-16}$

$0 = x^2 - 16$

$0 = (x+4)(x-4)$

$\cancel{x = -4}$ | $x = 4$

Can't have negative time

4 Seconds

12. The height, H, in feet, of an object dropped from the top of a building after t seconds is given by $H(t) = -16t^2 + 144$. How many feet did the object fall between one and two seconds after it was dropped? Determine, algebraically, how many seconds it will take for the object to reach the ground.

$H(1) = -16(1)^2 + 144 = 128$

$H(2) = -16(2)^2 + 144 = 80$

$128 - 80 = 48$ feet

$f(x) = 0$

$\frac{0}{-16} = \frac{-16t^2 + 144}{-16}$

$0 = t^2 - 9$

$0 = (t+3)(t-3)$

$\cancel{t = -3}$ | $t = 3$

3 Seconds

13. The height, H, in feet, of an object dropped from the top of a building after t seconds is given by $H(t) = -16t^2 + 144$. How many feet did the object fall between one and two seconds after it was dropped? Determine, algebraically, how many seconds it will take for the object to reach the ground.

Same as 12

Functions

A function is when each x value corresponds ("talks") to only one y value (x does not repeat)
A graph is a function if it passes the vertical line test (no vertical line can touch twice)

1. Which relation is *not* a function?
1) $\{(2,4),(1,2),(0,0),(-1,2),(-2,4)\}$
2) $\{(2,4),(1,1),(0,0),(-1,1),(-2,4)\}$
3) $\{(2,2),(1,1),(0,0),(-1,1),(-2,2)\}$
4) $\{(2,2),(1,1),(0,0),(1,-1),(2,-2)\}$ x values repeat

2. Which set is a function?
1) $\{(3,4),(3,5),(3,6),(3,7)\}$ ✗
2) $\{(1,2),(3,4),(4,3),(2,1)\}$ ✓
3) $\{(6,7),(7,8),(8,9),(6,5)\}$ ✗
4) $\{(0,2),(3,4),(0,8),(5,6)\}$ ✗

3. Which set of ordered pairs does *not* represent a function?
1) {(3,-2), (-2,3), (4,-1), (-1,4)}
2) {(3,-2), (3,-4), (4,-1), (4,-3)} x values repeat
3) {(3,-2), (4,-3), (5,-4), (6,-5)}
4) {(3,-2), (5,-2), (4,-2), (-1,-2)}

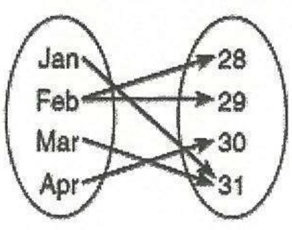
4. A function is defined as $\{(0,1),(2,3),(5,8),(7,2)\}$. Isaac is asked to create one more ordered pair for the function. Which ordered pair can he add to the set to keep it a function?
1) $(0,2)$ 0 would repeat
2) $(5,3)$ 5 would repeat
3) $(7,0)$ 7 would repeat
4) $(1,3)$ 1 would not repeat

5. A function is shown in the table below.
If included in the table, which ordered pair, $(-4,1)$ or $(1,-4)$, would result in a relation that is no longer a function? Explain your answer.

(-4,1) would make -4 repeat. There would be two outputs for one input.

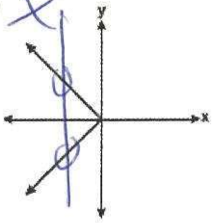
x	f(x)
−4	2
−1	−4
0	−2
3	16

6. A mapping is shown in the diagram below.
This mapping is
1) a function, because Feb has two outputs, 28 and 29
2) a function, because two inputs, Jan and Mar, result in the output 31
3) not a function, because Feb has two outputs, 28 and 29
4) not a function, because two inputs, Jan and Mar, result in the output 31

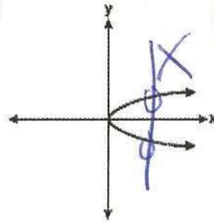


7. Which graph represents a function?

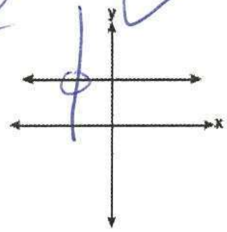
1)



2)



3)



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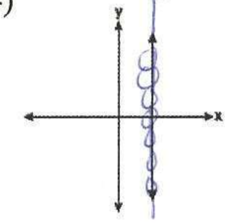
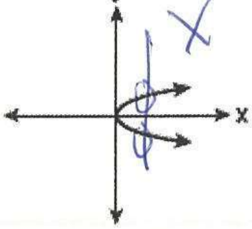
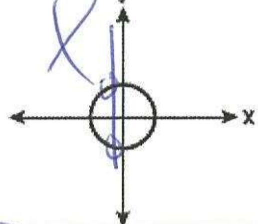


8. Which graph represents a function?

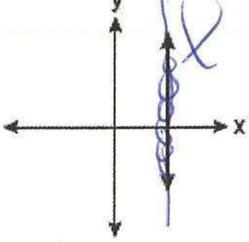
1)



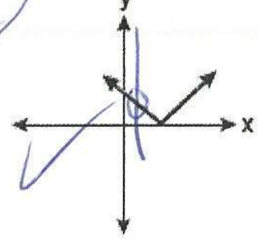
3)



2)

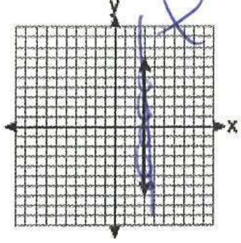


4)

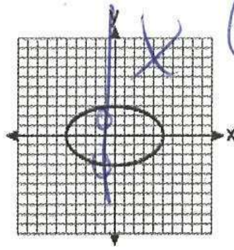


9. Which graph represents a function?

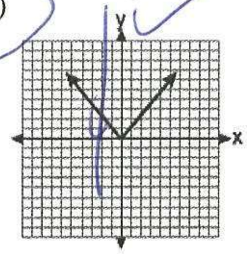
1)



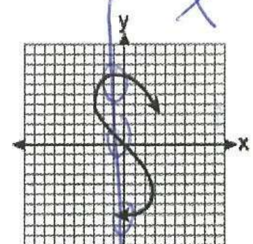
2)



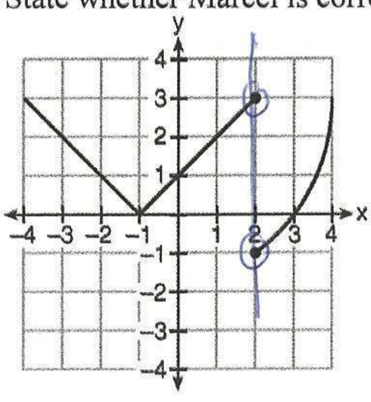
3)



4)

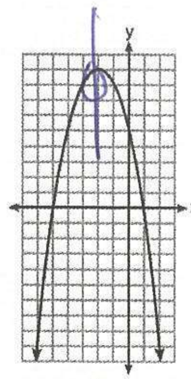


10. Marcel claims that the graph below represents a function.
State whether Marcel is correct. Justify your answer.



No, it does not pass the vertical line test. 2 has 2 outputs.

11. A relation is graphed on the set of axes below. Based on this graph, the relation is



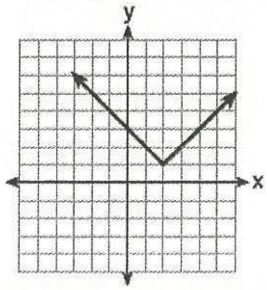
1) a function because it passes the horizontal line test
2) a function because it passes the vertical line test
3) not a function because it fails the horizontal line test
4) not a function because it fails the vertical line test

12. Which relation does *not* represent a function?

1)

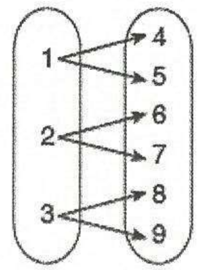
x	1	2	3	4	5	6
y	3.2	4	5.1	6	7.4	8.8

2)



3) $y = 3\sqrt{x+1} - 2$

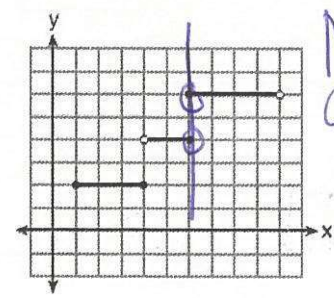
4)



Each input has multiple outputs

13. Four relations are shown below. State which relation(s) are functions. Explain why the other relation(s) are *not* functions.

III and IV are functions.



I

Not a function because it does not pass the vertical line test.

$\{(1, 2), (2, 5), (3, 8), (2, -5), (1, -2)\}$

II

Not a function because 2 has multiple outputs

x	y
−4	1
0	3
4	5
6	6

III

$y = x^2$

IV

Domain and Range
Domain is all possible x values
Range is all possible y values
If given an equation, use your calculator
If given a graph, using your pencil, for domain travel vertically along the x axis and for range travel horizontally along the y axis to see where the values begin and end.
Modeling Domain: The answer is almost always:
Non-Negative Integers {0,1,2,3,4,...}: If 0 is possible
OR
Positive Integers {1,2,3,4,...}: If 0 is not possible (workers needed to complete a job)

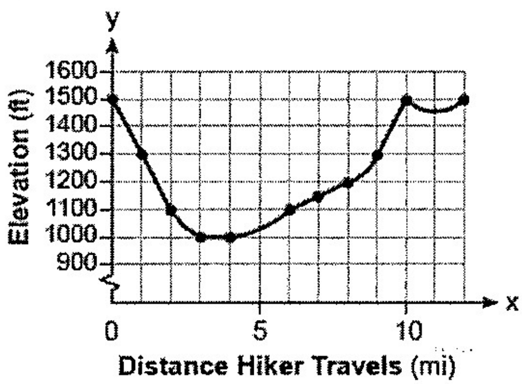
1. What is the domain of the relation shown below?

$\{(4,2),(1,1),(0,0),(1,-1),(4,-2)\}$

1) $\{0,1,4\}$
2) $\{-2,-1,0,1,2\}$
3) $\{-2,-1,0,1,2,4\}$
4) $\{-2,-1,0,0,1,1,1,2,4,4\}$

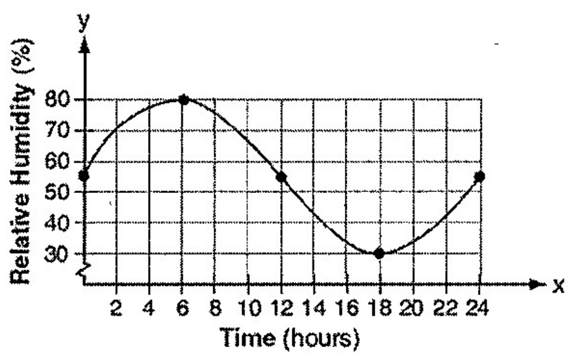
Determine the domain and range of the following graphs in both interval and set builder notation

2.



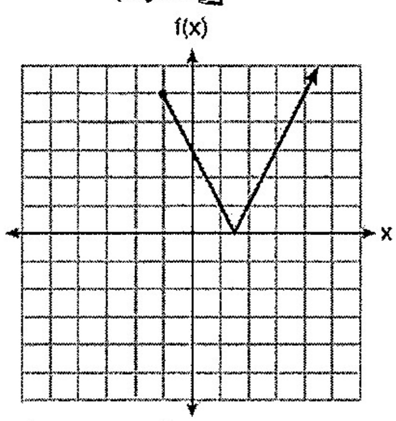
D: [0,12]
R: [1000,1500]

3.



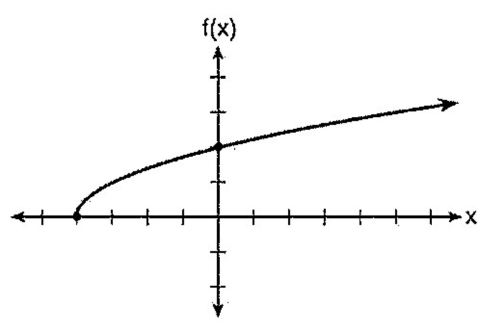
D: [0,24]
R: [30,80]

4.



D: [-1,∞)
R: [0,∞)

5.



D: [-4,∞)
R: [0,∞)

6. If the domain of the function $f(x) = 2x^2 - 8$ is $\{-2, 3, 5\}$, then the range is

1) $\{-16, 4, 92\}$
2) $\{-16, 10, 42\}$
3) $\{0, 10, 42\}$
4) $\{0, 4, 92\}$

x	y
-2	0
3	10
5	42

7. The function $f(x) = 2x^2 + 6x - 12$ has a domain consisting of the integers from -2 to 1, inclusive. Which set represents the corresponding range values for $f(x)$?

1) $\{-32, -20, -12, -4\}$
2) $\{-16, -12, -4\}$
3) $\{-32, -4\}$
4) $\{-16, -4\}$

x	y
-2	-16
-1	-16
0	-12
1	-4

8. If the function $f(x) = x^2$ has the domain $\{0, 1, 4, 9\}$, what is its range?

1) $\{0, 1, 2, 3\}$
2) $\{0, 1, 16, 81\}$
3) $\{0, -1, 1, -2, 2, -3, 3\}$
4) $\{0, -1, 1, -16, 16, -81, 81\}$

x	y
0	0
1	1
4	16
9	81

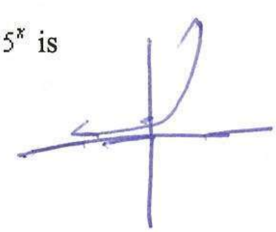
9. Let f be a function such that $f(x) = 2x - 4$ is defined on the domain $2 \le x \le 6$. The range of this function is

1) $0 \le y \le 8$
2) $0 \le y < \infty$
3) $2 \le y \le 6$
4) $-\infty < y < \infty$

x	y
2	0
6	8

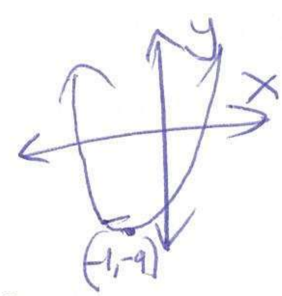
10. The range of the function defined as $y = 5^x$ is

1) $y < 0$
2) $y > 0$
3) $y \le 0$
4) $y \ge 0$



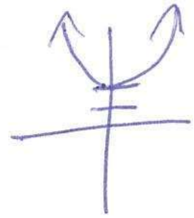
11. The range of the function $f(x) = x^2 + 2x - 8$ is all real numbers

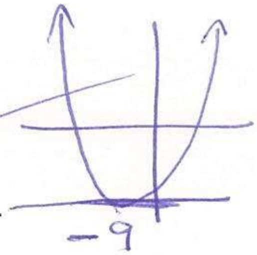
1) less than or equal to -9
2) greater than or equal to -9
3) less than or equal to -1
4) greater than or equal to -1



12. If $f(x) = x^2 + 2$, which interval describes the range of this function?

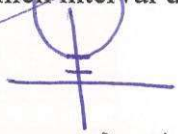
1) $(-\infty, \infty)$
2) $[0, \infty)$
3) $[2, \infty)$
4) $(-\infty, 2]$





11. The range of the function $f(x) = x^2 + 2x - 8$ is all real numbers
1) less than or equal to -9
2) greater than or equal to -9
3) less than or equal to -1
4) greater than or equal to -1

12. If $f(x) = x^2 + 2$, which interval describes the range of this function?



1) $(-\infty, \infty)$
2) $[0, \infty)$
3) $[2, \infty)$
4) $(-\infty, 2]$

13. Officials in a town use a function, C, to analyze traffic patterns. $C(n)$ represents the rate of traffic through an intersection where n is the number of observed vehicles in a specified time interval. What would be the most appropriate domain for the function?
1) $\{\ldots -2, -1, 0, 1, 2, 3, \ldots\}$
2) $\{-2, -1, 0, 1, 2, 3\}$
3) $\{0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}\}$
4) $\{0, 1, 2, 3, \ldots\}$

14. Which domain would be the most appropriate set to use for a function that predicts the number of household online-devices in terms of the number of people in the household?
1) integers
2) whole numbers
3) irrational numbers
4) rational numbers

15. A store sells self-serve frozen yogurt sundaes. The function $C(w)$ represents the cost, in dollars, of a sundae weighing w ounces. An appropriate domain for the function is
1) integers
2) rational numbers
3) nonnegative integers
4) nonnegative rational numbers

16. If the function h(x) represents the number of full hours that it takes a person to assemble x sets of tires in a factory, which would be an appropriate domain for the function?
(1) the set of real numbers
(2) the set of negative integers
(3) the set of integers
(4) the set of non-negative integers

Real Life Domain
$\{0,1,2,3,\ldots\}$
or
$\{1,2,3,4,5,\ldots\}$

17. An online company lets you download songs for \$0.99 each after you have paid a \$5 membership fee. Which domain would be most appropriate to calculate the cost to download songs?
1) rational numbers greater than zero
2) whole numbers greater than or equal to one
3) integers less than or equal to zero
4) whole numbers less than or equal to one

18. At an ice cream shop, the profit, $P(c)$, is modeled by the function $P(c) = 0.87c$, where c represents the number of ice cream cones sold. An appropriate domain for this function is
1) an integer ≤ 0
2) an integer ≥ 0
3) a rational number ≤ 0
4) a rational number ≥ 0

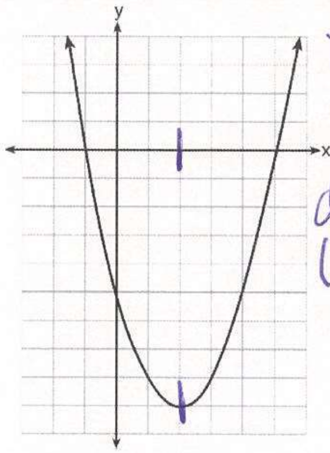
19. The daily cost of production in a factory is calculated using $c(x) = 200 + 16x$, where x is the number of complete products manufactured. Which set of numbers best defines the domain of $c(x)$?
1) integers
2) positive real numbers
3) positive rational numbers
4) whole numbers

Increasing/Decreasing

The interval where a function is increasing/decreasing is the x values where the interval starts and stops. From left to right, uphill is increasing and downhill is decreasing.

State the intervals where the following graphs are increasing and decreasing.

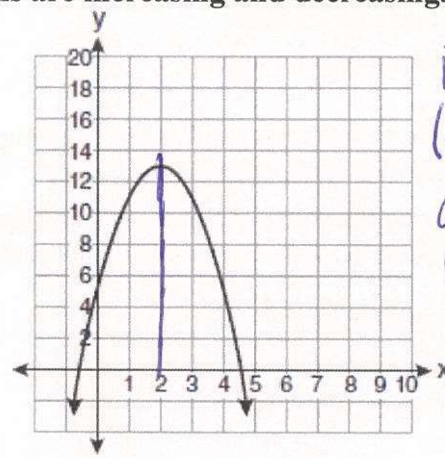
1.



increasing: $(2, \infty)$

decreasing: $(-\infty, 2)$

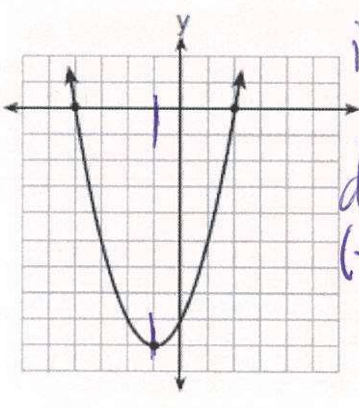
2.



increasing: $(-\infty, 2)$

decreasing $(2, \infty)$

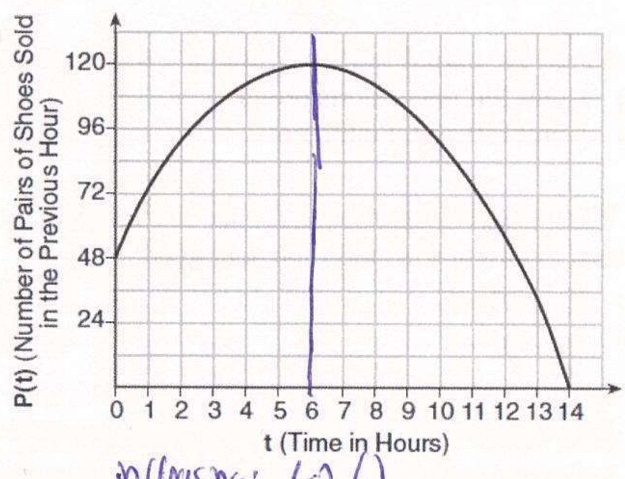
3.



increasing: $(-1, \infty)$

decreasing: $(-\infty, -1)$

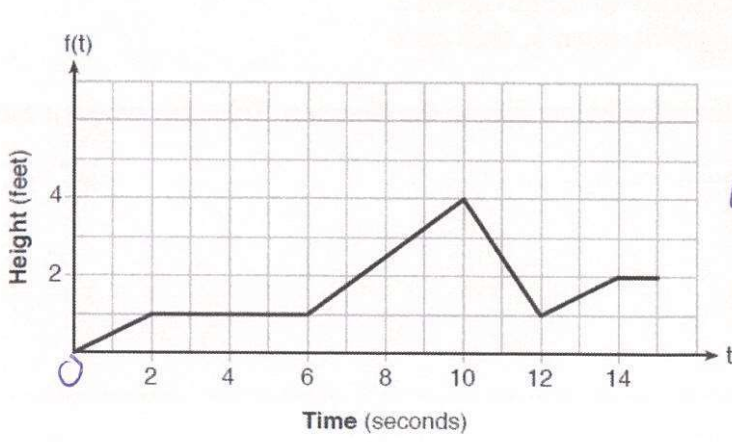
4.



increasing: $(0, 6)$

decreasing: $(6, 14)$

5.



increasing: $(0,2)$ $(6,10)$ $(12,14)$

decreasing $(10,12)$

constant: $(2,6)$ $(14,15)$

Transforming Functions

If adding to f(x), the graph moves up or down
If adding to x, the graph moves left or right (the opposite direction in which you would think)

$y = f(x) + a$ moves UP a units

$y = f(x) - a$ moves DOWN a units

$y = f(x + a)$ moves LEFT a units

$y = f(x - a)$ moves RIGHT a units

If the x is negated, the graph is reflected over the y axis
If the f(x) (aka y) is negated, the graph is reflected over the x axis

$y = f(-x)$ reflect over y axis

$y = -f(x)$ reflect over x axis

If a is positive, the vertex is a minimum and the graph opens upward
If a is negative, the vertex is a maximum and the graph opens downward

$y = af(x)$ Vertical Dilation

If $|a| > 1$, vertical stretch, narrower

If $|a| < 1$, vertical shrink, wider

$y = f(ax)$ Horizontal Dilation

If $|a| > 1$, Horizontal shrink

If $|a| < 1$, Horizontal stretch

right 2 up 3

1. Compared to the graph of $f(x) = x^2$, the graph of $g(x) = (x - 2)^2 + 3$ is the result of translating $f(x)$

1) 2 units up and 3 units right
2) 2 units down and 3 units up
3) 2 units right and 3 units up
4) 2 units left and 3 units right

right 3 down 2

2. Given the parent function $f(x) = x^3$, the function $g(x) = (x - 1)^3 - 2$ is the result of a shift of $f(x)$

1) 1 unit left and 2 units down
2) 1 unit left and 2 units up
3) 1 unit right and 2 units down
4) 1 unit right and 2 units up

(x+3)

3. If the original function $f(x) = 2x^2 - 1$ is shifted to the left 3 units to make the function $g(x)$, which expression would represent $g(x)$?

1) $2(x - 3)^2 - 1$
2) $2(x + 3)^2 - 1$
3) $2x^2 + 2$
4) $2x^2 - 4$

4. Joey's math class is studying the basic quadratic function, $f(x) = x^2$. Each student is supposed to make two new functions by adding or subtracting a constant to the function. Joey chooses the functions $g(x) = x^2 - 5$ and $h(x) = x^2 + 2$. What transformations would map $f(x)$ to $g(x)$ and $f(x)$ to $h(x)$? down 5 up 2

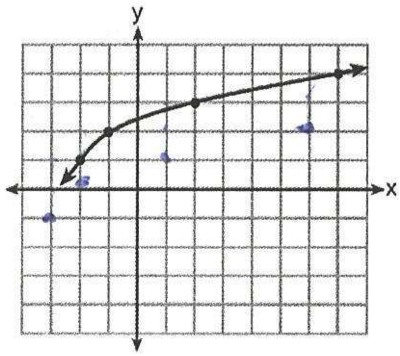
(1) shift left 5, shift right 2
(2) shift right 5, shift left 2
(3) shift up 5, shift down 2
(4) shift down 5, shift up 2

5. Describe the effect that each transformation below has on the function $f(x) = |x|$, where $a > 0$.

$g(x) = |x - a|$ right a units

$h(x) = |x| - a$ down a units

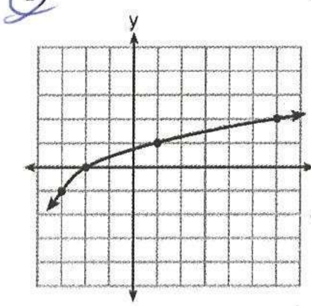
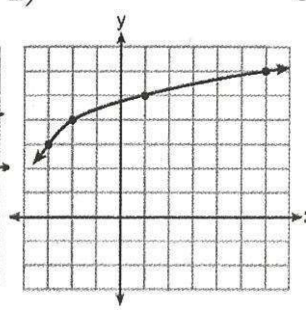
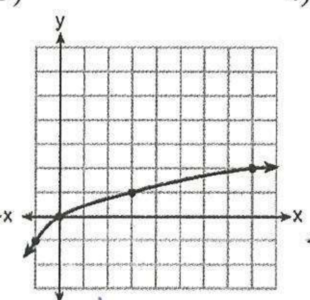
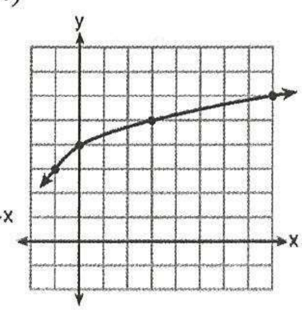
6. The graph of $y = f(x)$ is shown below.



left 1 down 2

What is the graph of $y = f(x + 1) - 2$?

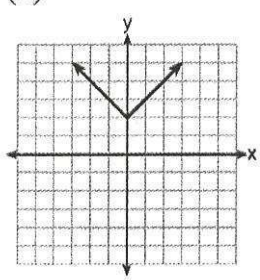
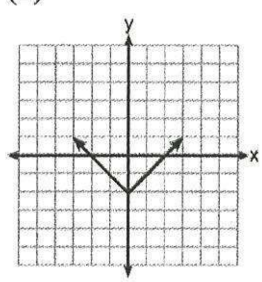
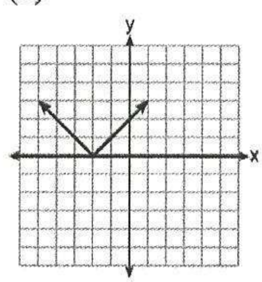
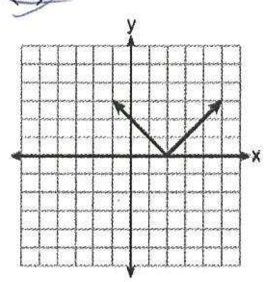
1) 2) 3) 4)

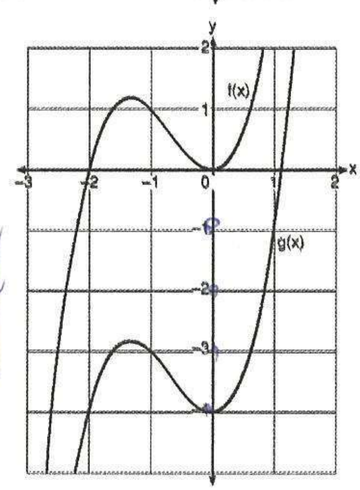
right 2

7. Which graph represents the equation $y = |x - 2|$?

(1) (2) (3) (4)

8. In the diagram below, $f(x) = x^3 + 2x^2$ is graphed. Also graphed is $g(x)$, the result of a translation of $f(x)$. Determine an equation of $g(x)$. Explain your reasoning.



$g(x) = x^3 + 2x^2 - 4$

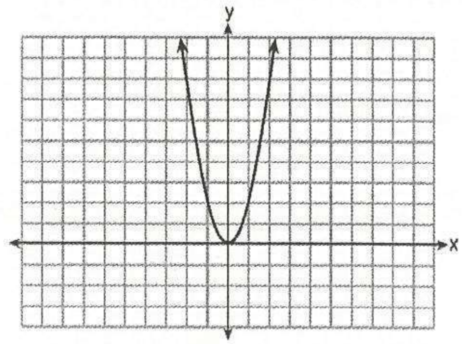
The graph was translated down 4.

down 4

narrower right 2 up 1

9. How does the graph of $f(x) = 3(x-2)^2 + 1$ compare to the graph of $g(x) = x^2$?
1) The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.
2) The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.
3) The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.
4) The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.

10. The graph of the equation $y = ax^2$ is shown below.



opens downward wider

If a is multiplied by $-\frac{1}{2}$, the graph of the new equation is
1) wider and opens downward
2) wider and opens upward
3) narrower and opens downward
4) narrower and opens upward

11. When the function $f(x) = x^2$ is multiplied by the value a, where $a > 1$, the graph of the new function, $g(x) = ax^2$

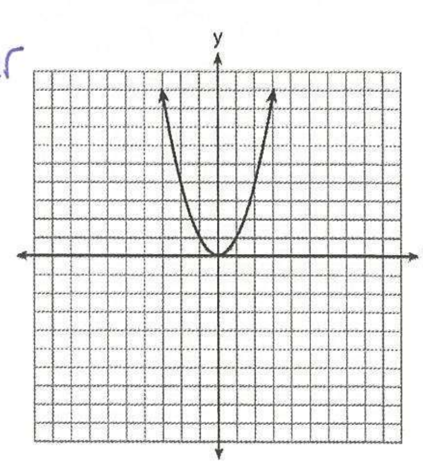
upward narrower

1) opens upward and is wider
2) opens upward and is narrower
3) opens downward and is wider
4) opens downward and is narrower

$\frac{1}{2}$ wider $\frac{1}{2}$ wider

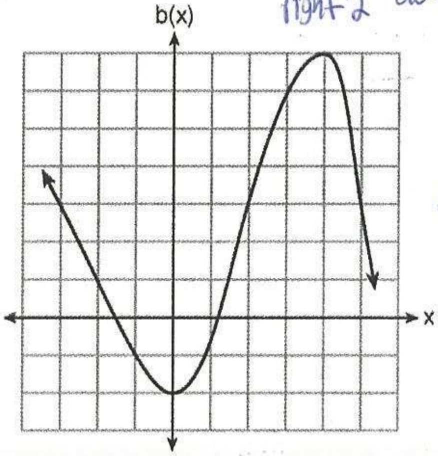
12. In the functions $f(x) = \cancel{k}x^2$ and $g(x) = |\cancel{k}x|$, k is a positive integer. If k is replaced by $\frac{1}{2}$, which statement about these new functions is true?
1) The graphs of both $f(x)$ and $g(x)$ become wider.
2) The graph of $f(x)$ becomes narrower and the graph of $g(x)$ shifts left.
3) The graphs of both $f(x)$ and $g(x)$ shift vertically.
4) The graph of $f(x)$ shifts left and the graph of $g(x)$ becomes wider.

13. The graph of the equation $y = x^2$ is shown below. Which statement best describes the change in this graph when the coefficient of x^2 is multiplied by 4? narrower
1) The parabola becomes wider.
2) The parabola becomes narrower.
3) The parabola will shift up four units.
4) The parabola will shift right four units.



14. Richard is asked to transform the graph of $b(x)$ below. The graph of $b(x)$ is transformed using the equation $h(x) = b(x-2) - 3$. Describe how the graph of $b(x)$ changed to form the graph of $h(x)$.

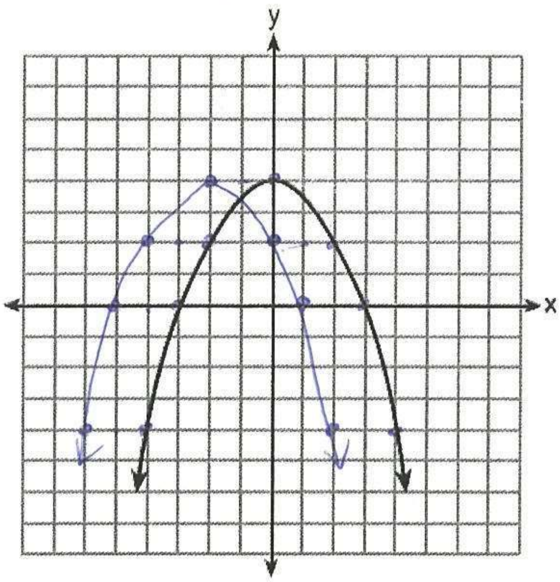
right 2 down 3



It is shifted right 2 and down 3.

15. The graph of the function $p(x)$ is represented below. On the same set of axes, sketch the function $p(x+2)$.

left 2



Average rate of change: $\frac{f(b)-f(a)}{b-a}$

Use a table to organize your values. If given an equation, type it into $y=$. If given a graph, pull the values from the graph. $f(b)$ and $f(a)$ are y values. b and a are x values.

If asked which interval has the greatest rate of change, find the average rate of change for each interval.

"On average, between a and b, the function is increasing/decreasing x units per unit of time."

1. Joey enlarged a 3-inch by 5-inch photograph on a copy machine. He enlarged it four times. The table below shows the area of the photograph after each enlargement.

Enlargement	0	1	2	3	4
Area (square inches)	15	18.8	23.4	29.3	36.6

What is the average rate of change of the area from the original photograph to the fourth enlargement, to the *nearest tenth*?

1) 4.3
2) 4.5
3) 5.4
4) 6.0

$\frac{f(b)-f(a)}{b-a}$ $\frac{36.6-15}{4-0}=5.4$

2. A family is traveling from their home to a vacation resort hotel. The table below shows their distance from home as a function of time.

Determine the average rate of change between hour 2 and hour 7, including units.

Time (hrs)	0	2	5	7
Distance (mi)	0	140	375	480

$\frac{f(b)-f(a)}{b-a}$ $\frac{480-140}{7-2}=68$ miles per hour.

3. The table below shows the average diameter of a pupil in a person's eye as he or she grows older.

What is the average rate of change, in millimeters per year, of a person's pupil diameter from age 20 to age 80?

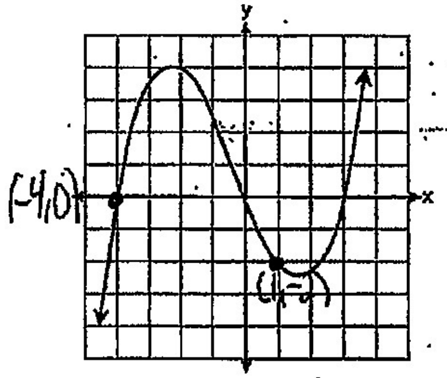
1) 2.4
2) 0.04
3) −2.4
4) −0.04

$\frac{f(b)-f(a)}{b-a}$ $\frac{2.3-4.7}{80-20}=-.04$

Age (years)	**Average Pupil Diameter** (mm)
20	4.7
30	4.3
40	3.9
50	3.5
60	3.1
70	2.7
80	2.3

Pull the y value from the graph

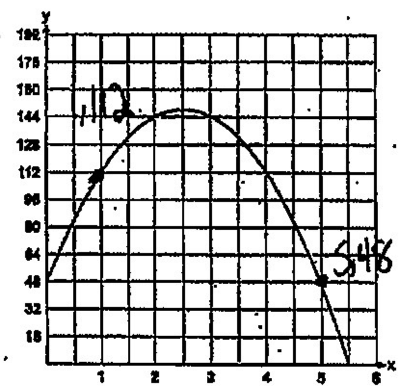
4. The graph of $p(x)$ is shown below. What is the average rate of change over the interval $-4 \le x \le 1$?



x	y
-4	0
1	-2

$\frac{-2-0}{1--4}$ $\frac{-2}{5}$

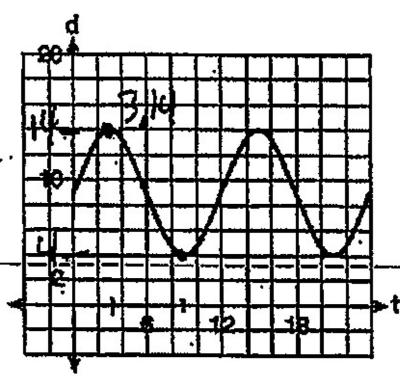
5. A ball is thrown into the air from the edge of a 48-foot-high cliff so that it eventually lands on the ground. The graph below shows the height, y, of the ball from the ground after x seconds. What is the average rate of change of the ball between 1 and 5 seconds?



x	y
1	112
5	48

$\frac{48-112}{5-1} = -16$

6. The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below. If the depth, d, is measured in feet and time, t, is measured in hours since midnight, what is the average rate of change of the depth of the water between 3AM and 9AM?



x	y
3	14
9	4

$\frac{4-14}{9-3} = -\frac{5}{3}$

7. For the function $f(x) = 3^x$, find the average rate of change over the interval 0 to 5.

x	y
0	1
5	243

$\frac{243-1}{5-0} = 48.4$

8. An astronaut drops a rock off the edge of a cliff on the Moon. The distance, $d(t)$, in meters, the rock travels after t seconds can be modeled by the function $d(t) = 0.8t^2$. What is the average speed, in meters per second, of the rock between 5 and 10 seconds after it was dropped?

1) 12
2) 20
3) 60
4) 80

x	y
5	20
10	80

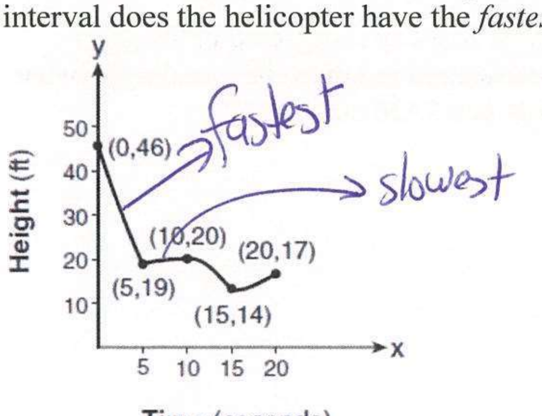
$\frac{80-20}{10-5} = 12$

9. A population of rabbits in a lab, $p(x)$, can be modeled by the function $p(x) = 20(1.014)^x$, where x represents the number of days since the population was first counted. Determine, to the *nearest tenth*, the average rate of change from day 50 to day 100.

x	y
50	40.08
100	80.32

$\frac{80.32-40.08}{100-50} = .8$

10. The graph below models the height of a remote-control helicopter over 20 seconds during flight.
Over which interval does the helicopter have the *slowest* average rate of change? Over which interval does the helicopter have the *fastest* average rate of change?



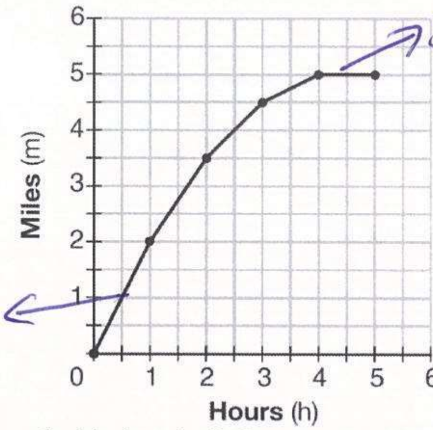
fastest: (0,5)
slowest: (5,10)

11. The graph below shows the distance in miles, m, hiked from a camp in h hours. Which hourly interval had the greatest rate of change? Which hourly interval had the least average rate of change?

1) hour 0 to hour 1 fastest
2) hour 1 to hour 2
3) hour 2 to hour 3
4) hour 3 to hour 4

hour 4 to hours 5 slowest

slowest



fastest

12. The table below shows the year and the number of households in a building that had high-speed broadband internet access.
For which interval of time was the average rate of change the *smallest*?

1) 2002 - 2004
2) 2003 - 2005
3) 2004 - 2006
4) 2005 - 2007

Number of Households	11	16	23	33	42	47
Year	2002	2003	2004	2005	2006	2007

1)

x	y
2002	11
2004	23

$\frac{23-11}{2004-2002}$ 6

2)

x	y
2003	16
2005	33

$\frac{33-16}{2005-2003}$ 8.5

3)

x	y
2004	23
2006	42

$\frac{42-23}{2006-2004}$ 9.5

4)

x	y
2005	33
2007	47

$\frac{47-33}{2007-2005}$ 7

13. The table below shows the cost of mailing a postcard in different years. During which time interval did the cost increase at the greatest average rate?

1) 1898-1971
2) 1971-1985
3) 1985-2006
4) 2006-2012

Year	1898	1971	1985	2006	2012
Cost (¢)	1	6	14	24	35

1)

x	y
1898	1
1971	6

$\frac{6-1}{1971-1898}$ .06849..

2)

x	y
1971	6
1985	14

$\frac{14-6}{1985-1971}$ .5714..

3)

x	y
1985	14
2006	24

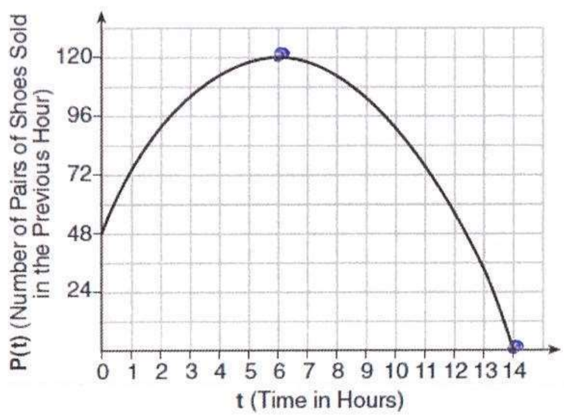
$\frac{24-14}{2006-1985}$ .47619...

4)

x	y
2006	24
2012	35

$\frac{35-24}{2012-2006}$ $1.8\overline{3}$

14. A manager wanted to analyze the online shoe sales for his business. He collected data for the number of pairs of shoes sold each hour over a 14-hour time period. He created a graph to model the data, as shown below. Determine the average rate of change between the sixth and fourteenth hours, and explain what it means in the context of the problem.



x	y
6	120
14	0

$$\frac{0-120}{14-6}$$

$$-15$$

On average, from hour 6 to hour 14, the number of pairs of shoes sold decreased by 15 pairs per hour.

15. The distance needed to stop a car after applying the brakes varies directly with the square of the car's speed. The table below shows stopping distances for various speeds.
Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph. Explain what this rate of change means as it relates to braking distance.

Speed (mph)	10	20	30	40	50	60	70
Distance (ft)	6.25	25	56.25	100	156.25	225	306.25

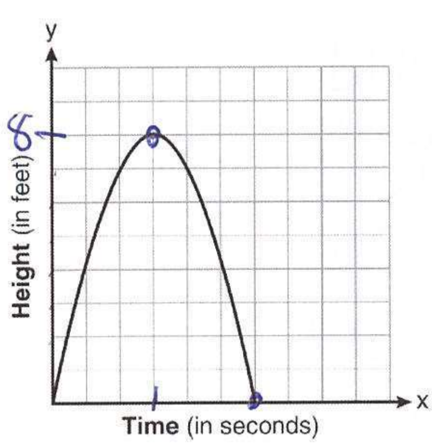
x	y
50	156.25
70	306.25

$$\frac{306.25-156.25}{70-50}$$

$$7.5$$

On average, from 50mph to 70mph, the braking distance increases by 7.5 ft per mph.

16. The graph below represents the parabolic path of a ball kicked by a young child. Find the average rate of change from 3 to 6 seconds. Explain its meaning in the context of the problem.



x	y
3	8
6	0

$$\frac{0-8}{6-3}$$

$$-\frac{8}{3}$$

On average, from 3 to 6 seconds, the height of the ball decreases by $\frac{8}{3}$ ft per second

17. The population, $P(t)$, of a town increased according to the function $P(t) = 12,000(1.03)^t$, where t is the number of years since 2000. Find the average rate of change from $t = 10$ to $t = 20$ rounding to the nearest integer. Explain its meaning in the context of the problem.

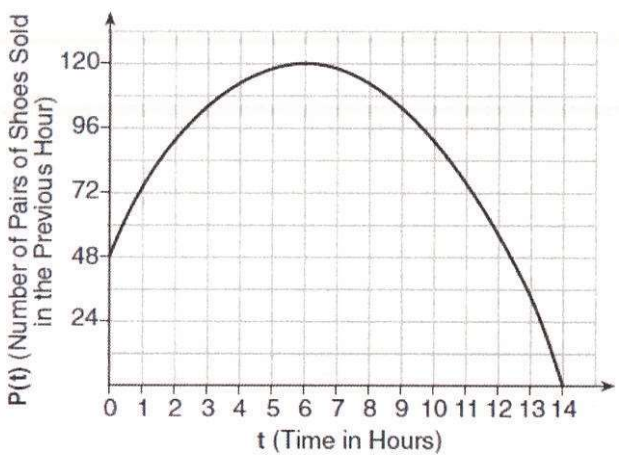
x	y
10	16127
20	21673

$$\frac{21673 - 16127}{20 - 10}$$

555

On average, from 2010 to 2020, the population of the town increased by 555 people per year

18. A manager wanted to analyze the online shoe sales for his business. He created a graph to model the data, as shown below. Determine the average rate of change between the sixth and fourteenth hours, and explain what it means in the context of the problem.



Same as 14

19. The table below shows the number of hours of daylight on the first day of each month in Rochester, NY. Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st? Interpret what this means in the context of the problem.

Month	Hours of Daylight
Jan. 1	9.4
Feb.	10.6
March	11.9
April 4	13.9
May	14.7
June	15.4
July	15.1
Aug.	13.9
Sept.	12.5
Oct.	11.1
Nov.	9.7
Dec.	9.0

$$\frac{13.9 - 9.4}{4 - 1}$$

1.5

On average, from January to April, the number of hours of daylight increased by 1.5 hours per month.

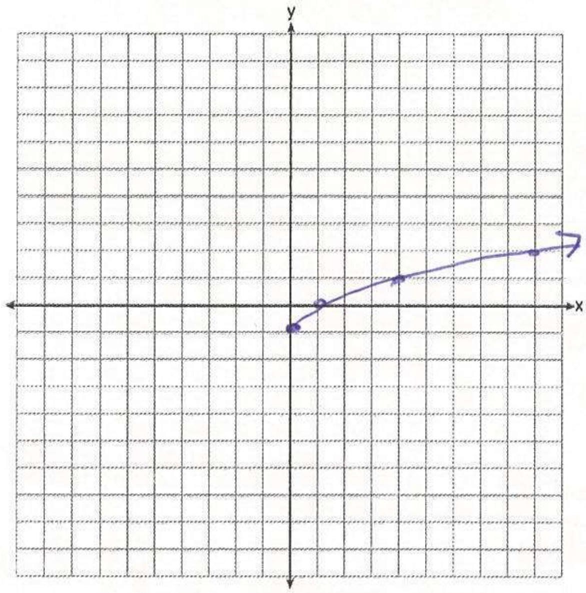
Graphing Non-Linear Functions

1) Get y by itself
2) Type into y =
3) If there is an interval/domain, plot only nice points between those values: no arrows
   If not, plot all "nice" points that fit on the graph (usually -10 to 10) and use arrows

* $a < x \leq b$ means all numbers between a and b

*For quadratic and absolute value, find a mirror image

1. Draw the graph of $y = \sqrt{x} - 1$ on the set of axes below.

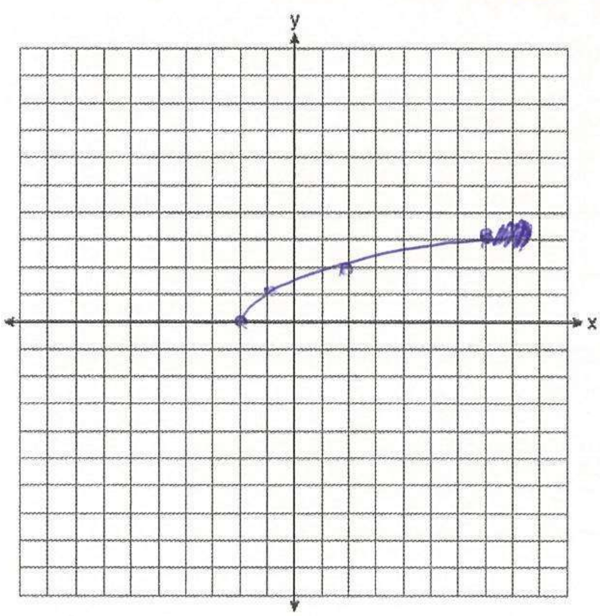


x	y
0	-1
1	0
4	1
9	2

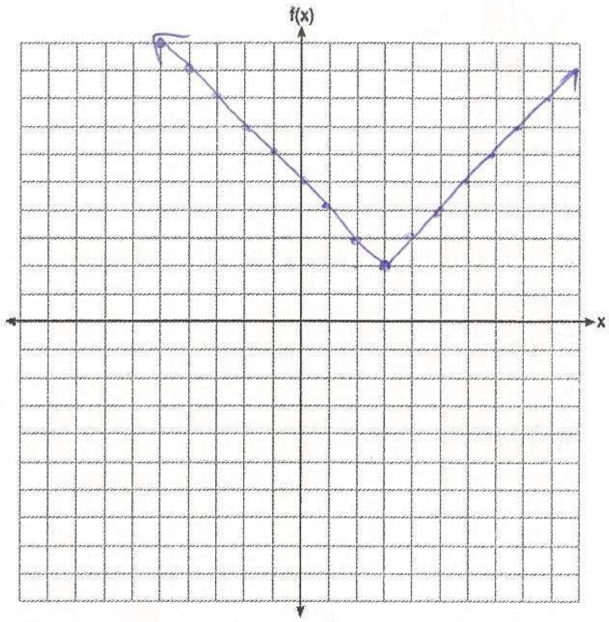
no arrows

2. Graph $f(x) = \sqrt{x+2}$ over the domain $-2 \leq x \leq 7$.

x	y
-2	0
-1	1
2	2
7	3

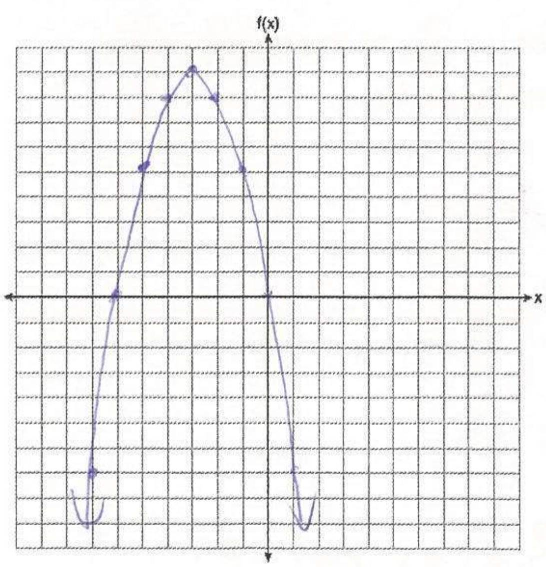


3. On the set of axes below, graph $f(x) = |x - 3| + 2$.



X	y
-5	10
-4	9
-3	8
-2	7
-1	6
0	5
1	4
2	3
3	2
4	3
5	4
6	5
7	6
8	7
9	8
10	9

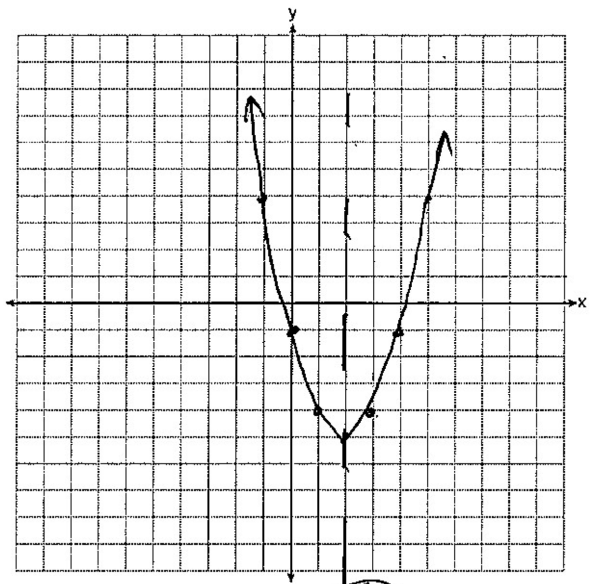
4. Graph the function $f(x) = -x^2 - 6x$ on the set of axes below. State the coordinates of the vertex of the graph.



X	y
-7	-7
-6	0
-5	5
-4	8
-3	9
-2	8
-1	5
0	0
1	-7

vertex (-3,9)

5. On the set of axes below, draw the graph of $y = x^2 - 4x - 1$. State the equation of the axis of symmetry.



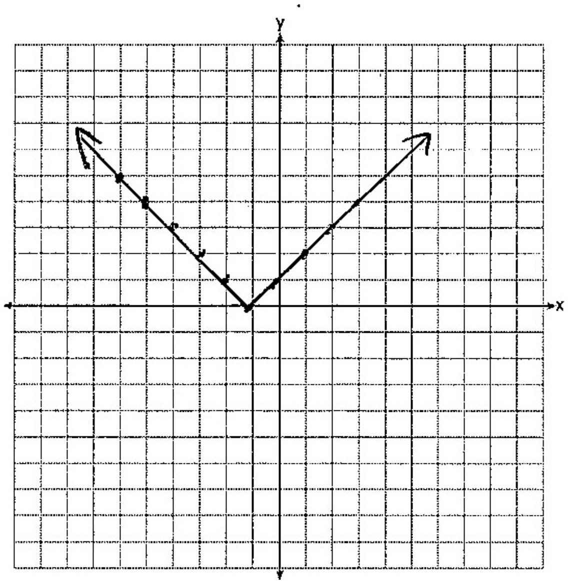
x	y
-1	4
0	-1
1	-4
2	-5
3	-4
4	-1
5	4

mirror image

$x=2$

axis of symmetry

6. On the set of axes below, graph the function $y = |x + 1|$.

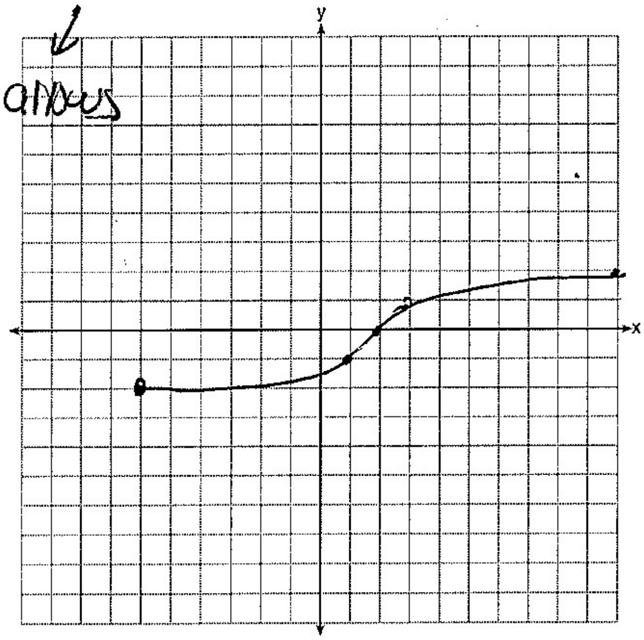


x	y
-6	5
-5	4
-4	3
-3	2
-2	1
-1	0
0	1
1	2
2	3
3	4
4	5

mirror image

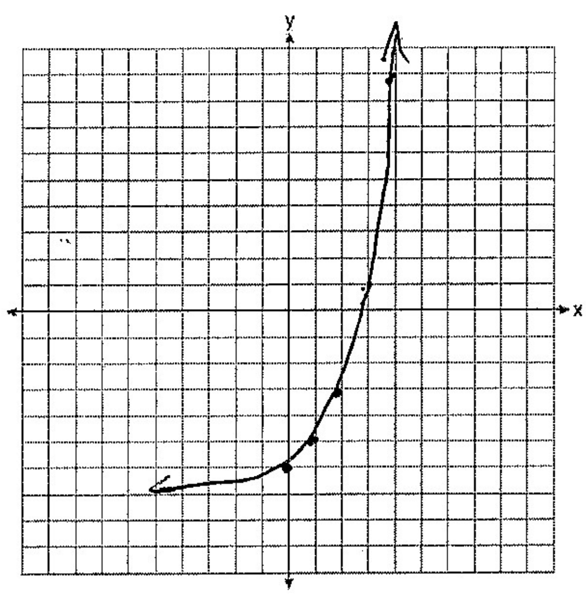
7. On the set of axes below, graph the function represented by $y = \sqrt[3]{x-2}$ for the domain $-6 \le x \le 10$.

no arrows



x	y
-6	-2
1	-1
2	0
3	1
10	2

8. Graph the function $f(x) = 2^x - 7$ on the set of axes below.



x	y
0	-6
1	-5
2	-3
3	1
4	9

9. Graph $f(x)$ and $g(x)$ on the set of axes below. Based on your graph, state *one* value of x that satisfies $f(x) = g(x)$. Explain your reasoning.

$f(x) = x^2 - 4x + 3$

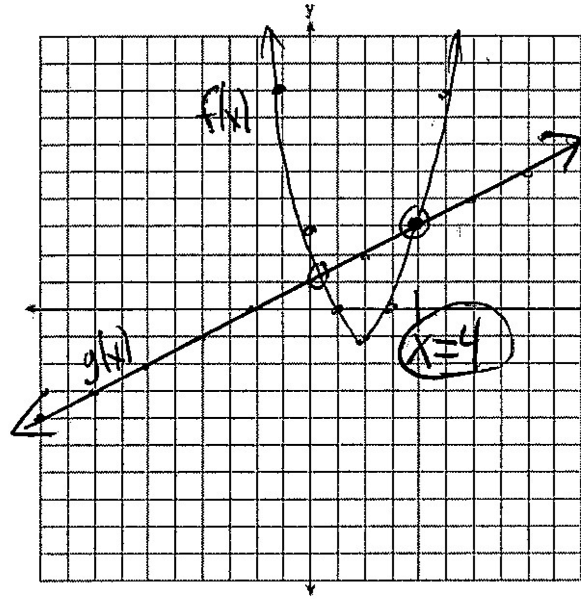
$g(x) = \frac{1}{2}x + 1$

f(x)

X	y
-1	8
0	3
1	0
2	-1
3	0
4	3
5	8

g(x)

x	y
-10	-4
-8	-3
-6	-2
-4	-1
-2	0
0	1
2	2
4	3
6	4
8	5
10	6



10. Graph $y = f(x)$ and $y = g(x)$ on the set of axes below. Determine and state all values of x for which $f(x) = g(x)$.

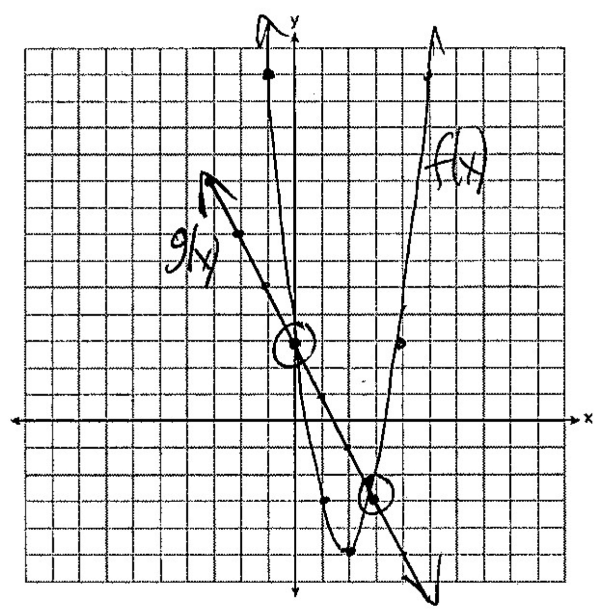
$f(x) = 2x^2 - 8x + 3$
$g(x) = -2x + 3$

f(x)

X	y
-1	13
0	3
1	-3
2	-5
3	-3
4	3
5	13

g(x)

X	y
-3	9
-2	7
-1	5
0	3
1	1
2	-1
3	-3
4	-5
5	-7
6	-9



X=0

X=3

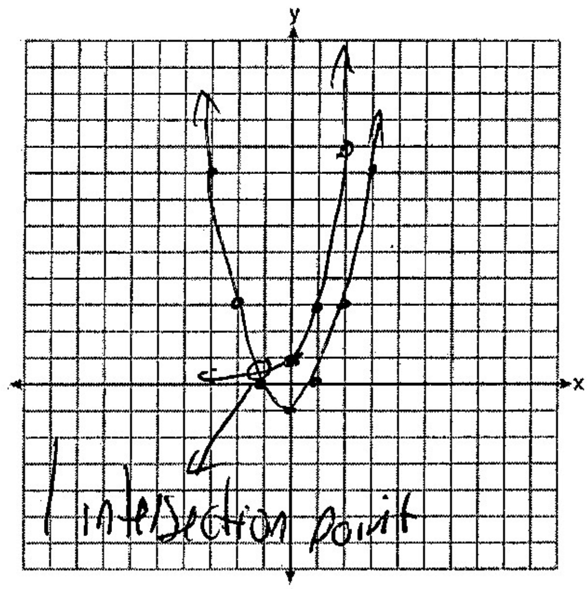
11. On the set of axes below, graph $f(x) = x^2 - 1$ and $g(x) = 3^x$. Based on your graph, for how many values of x does $f(x) = g(x)$? Explain your reasoning.

f(x)

x	y
-3	8
-2	3
-1	0
0	-1
1	0
2	3
3	8

g(x)

x	y
0	1
1	3
2	9



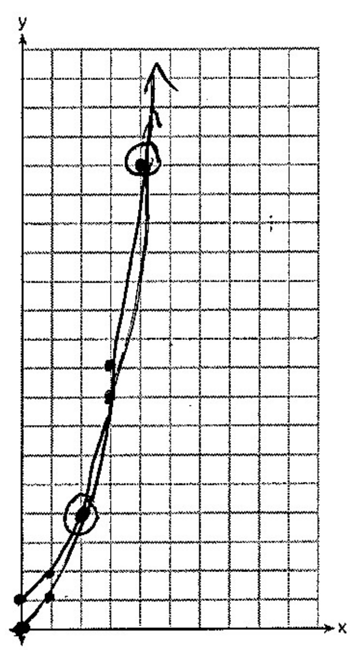
12. Graph $f(x) = x^2$ and $g(x) = 2^x$ for $x \geq 0$ on the set of axes below. Determine and state all values of x for which $f(x) = g(x)$.

f(x)

x	y
0	0
1	1
2	4
3	9
4	16

g(x)

x	y
0	1
1	2
2	4
3	8
4	16



$x=2$

$x=4$

Systems of Equations Word Problems with Graphing

1) Create a system of equations
2) Get y by itself and graph each
3) The point of intersection is the solution to the system. The x coordinate is the value for whatever x represents and the y coordinate is the value for whatever y represents.

1. Two families went to Rollercoaster World. The Brown family paid $170 for 3 children and 2 adults. The Peckham family paid $360 for 4 children and 6 adults. If *x* is the price of a child's ticket in dollars and *y* is the price of an adult's ticket in dollars, write a system of equations that models this situation. Graph your system of equations on the set of axes below.

Brown	Peckham
$3x+2y=170$	$4x+6y=360$
$-3x \quad -3x$	$-4x \quad -4x$
$\frac{2y}{2}=\frac{-3x+170}{2}$	$\frac{6y}{6}=\frac{-4x+360}{6}$
$y=-\frac{3}{2}x+85$	$y=-\frac{2}{3}x+60$

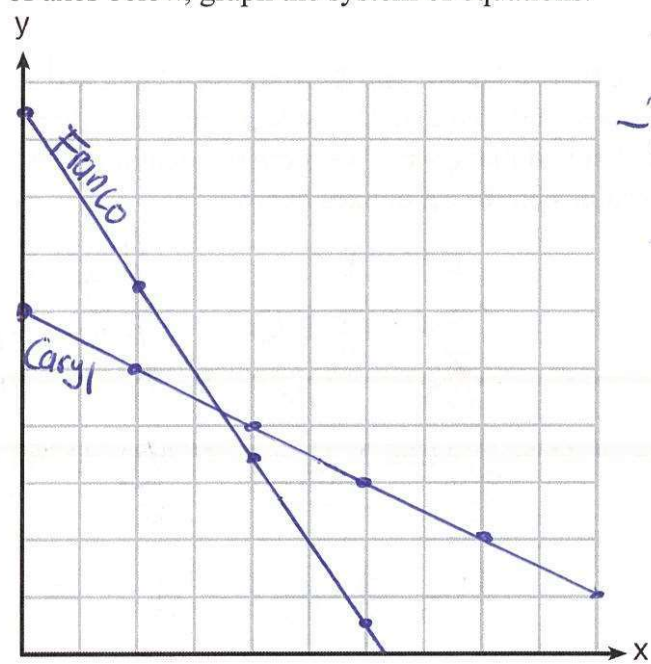
State the coordinates of the point of intersection. Explain what each coordinate of the point of intersection means in the context of the problem.

(30,40)

The cost of a child ticket is $30

The cost of an adult ticket is $40.

2. Franco and Caryl went to a bakery to buy desserts. Franco bought 3 packages of cupcakes and 2 packages of brownies for \$19. Caryl bought 2 packages of cupcakes and 4 packages of brownies for \$24. Let x equal the price of one package of cupcakes and y equal the price of one package of brownies. Write a system of equations that describes the given situation. On the set of axes below, graph the system of equations.



Franco

$3x + 2y = 19$

$-3x \qquad -3x$

$\frac{2y}{2} = \frac{-3x + 19}{2}$

$y = -\frac{3}{2}x + 9.5$

Caryl

$2x + 4y = 24$

$-2x \qquad -2x$

$\frac{4y}{4} = \frac{-2x + 24}{4}$

$y = -\frac{1}{2}x + 6$

Determine the exact cost of one package of cupcakes and the exact cost of one package of brownies in dollars and cents. Justify your solution

$-2(3x + 2y = 19)$

$3(2x + 4y = 24)$

$-6x - 4y = -38$

$6x + 12y = 72$

$\frac{8y}{8} = \frac{34}{8}$

$y = 4.25$

$2x + 4y = 24$

$2x + 4(4.25) = 24$

$2x + 17 = 24$

$-17 \quad -17$

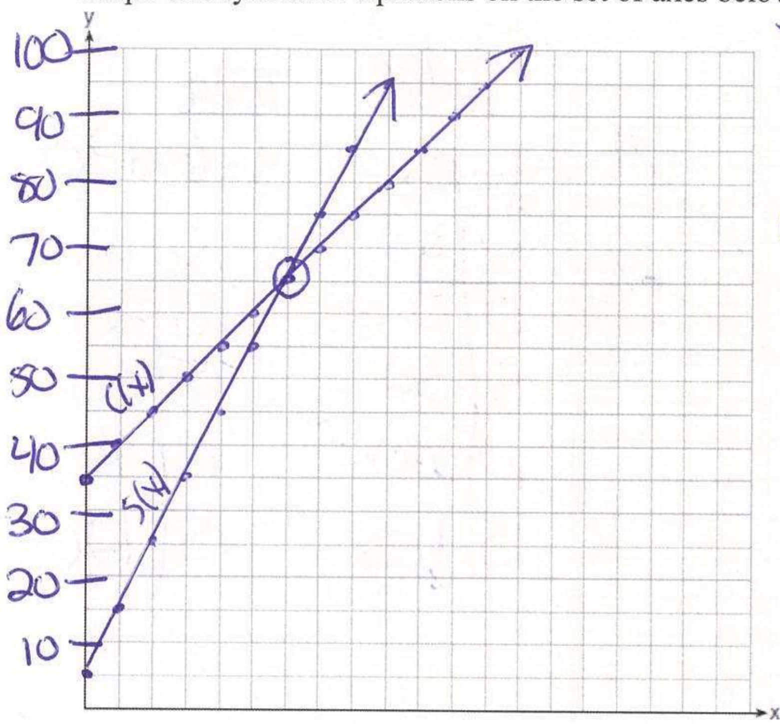
$\frac{2x}{2} = \frac{7}{2}$

$x = 3.5$

Cupcakes: \$3.50

brownies: \$4.25

3. Central High School had five members on their swim team in 2010. Over the next several years, the team increased by an average of 10 members per year. The same school had 35 members in their chorus in 2010. The chorus saw an increase of 5 members per year. Write a system of equations to model this situation, where x represents the number of years since 2010. Graph this system of equations on the set of axes below.



$S(x) = 10x + 5$

$C(x) = 5x + 35$

S(x)

x	y
0	5
1	15
2	25
3	35
4	45
5	55
6	65
7	75
8	85
9	95
10	105

C(x)

x	y
0	35
1	40
2	45
3	50
4	55
5	60
6	65
7	70
8	75
9	80
10	85

*Since scale isn't the same, much easier to use table

Explain in detail what each coordinate of the point of intersection of these equations means in the context of this problem.

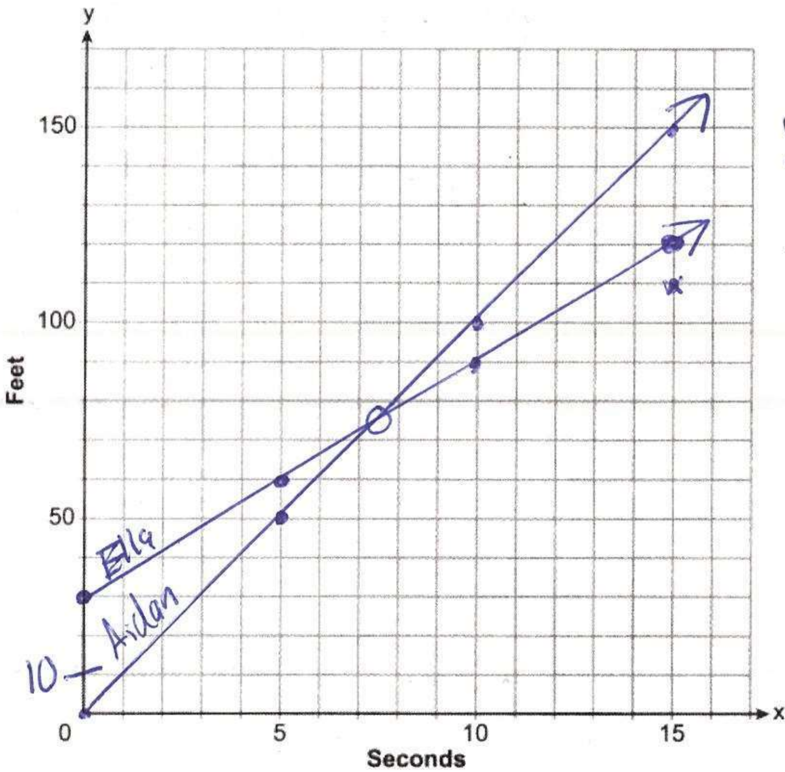
(6,65)

After 6 years, both clubs will have 65 members

4. Aidan and his sister Ella are having a race. Aidan runs at a rate of 10 feet per second. Ella runs at a rate of 6 feet per second. Since Ella is younger, Aidan is letting her begin 30 feet ahead of the starting line. Let *y* represent the distance from the starting line and *x* represent the time elapsed, in seconds. Write an equation to model the distance Aidan traveled. Write an equation to model the distance Ella traveled. On the set of axes below, graph your equations.

10x

6x



Aidan

$y=10x$

x	y
0	0
5	50
10	100
15	150

Ella

$y=6x+30$

x	y
0	30
5	60
10	90
15	120

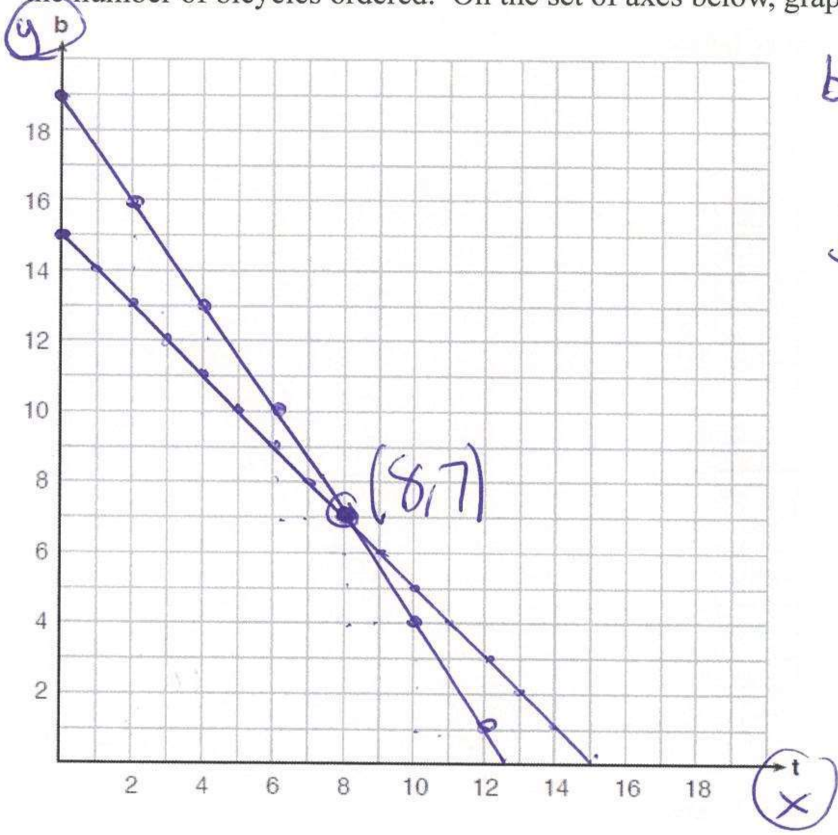
Exactly how many seconds does it take Aidan to catch up to Ella? Justify your answer.

$10x=6x+30$
$-6x\quad -6x$

$4x=30$
$\frac{4x}{4}=\frac{30}{4}$

$x=7.5$ Seconds

5. A recreation center ordered a total of 15 tricycles and bicycles from a sporting goods store. The number of wheels for all the tricycles and bicycles totaled 38. Write a linear system of equations that models this scenario, where *t* represents the number of tricycles and *b* represents the number of bicycles ordered. On the set of axes below, graph this system of equations.



$b+\cancel{t}=15$ $\quad$ $2b+3\cancel{t}=38$

$\quad -\cancel{t}\ \ -t$ $\quad\quad -3\cancel{t}\ \ -3t$

$b=-t+15$ $\quad$ $\frac{\cancel{2}b}{\cancel{2}}=\frac{-3t+38}{2}$

$b=-\frac{3}{2}t+19$

*same scale so I used slope and y-intercept

Based on your graph of this scenario, could the recreation center have ordered 10 tricycles? Explain your reasoning.

No, they ordered 8 tricycles and 7 bicycles

Graphing Piecewise Functions

Create a table of values for each piece

Graph all points

*Make sure the border value without the equals gets an open circle.

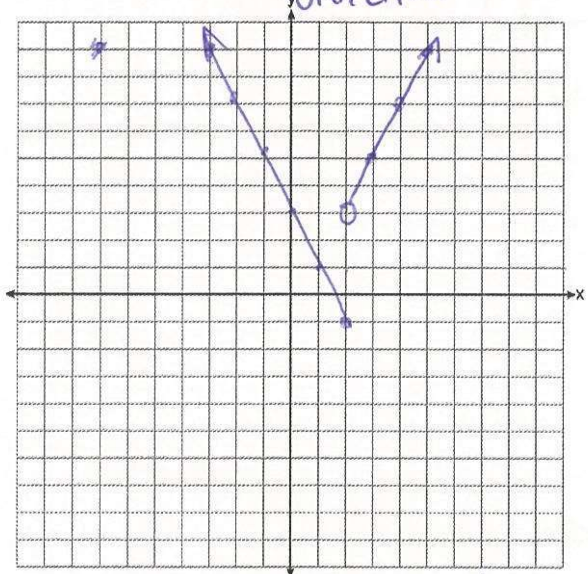
closed circle

1. Graph the following on the set of axes below:

$$f(x) = \begin{cases} 3 - 2x, & x \le 2 \\ 2x - 1, & x > 2 \end{cases}$$

This value must be in both tables

open circle



3-2x

x	y
-3	9
-2	7
-1	5
0	3
1	1
2	-1

2x-1

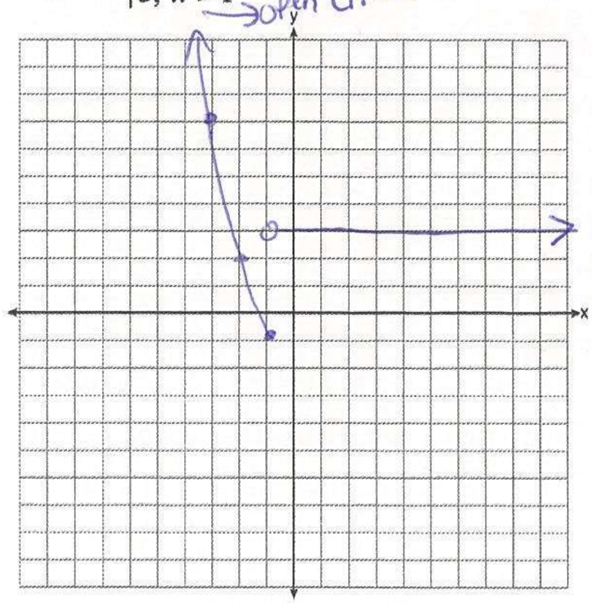
x	y
2	3
3	5
4	7
5	9

closed circle

2. Graph the following on the set of axes below:

$$f(x) = \begin{cases} x^2 - 2, & x \le -1 \\ 3, & x > -1 \end{cases}$$

open circle



$x^2 - 2$

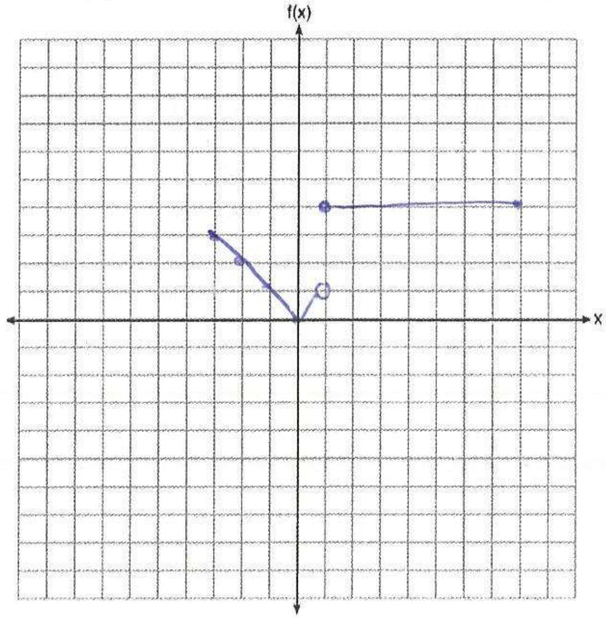
x	y
-3	7
-2	2
-1	-1

3

x	y
-1	3
0	3
1	3
2	3
↓	3

3. Graph the following function on the set of axes below.

$$f(x) = \begin{cases} |x|, & -3 \le x < 1 \text{ open circle} \\ 4, & 1 \le x \le 8 \text{ closed circle} \end{cases}$$



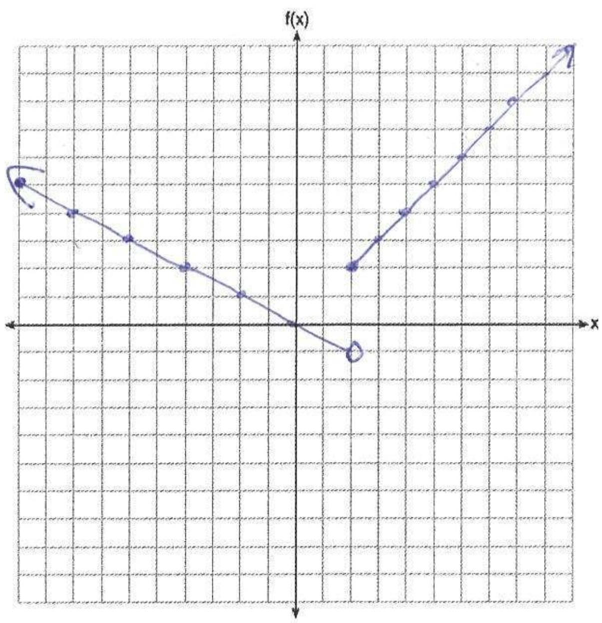
$|x|$

x	y
-3	3
-2	2
-1	1
0	0
1	1

4

x	y
1	4
2	4
3	4
4	4
5	4
6	4
7	4
8	4

4. On the set of axes below, graph the piecewise function: $f(x) = \begin{cases} -\frac{1}{2}x, & x < 2 \text{ open circle} \\ x, & x \ge 2 \text{ closed circle} \end{cases}$



$-\frac{1}{2}x$

x	y
-10	5
-8	4
-6	3
-4	2
-2	1
0	0
2	-1

x

x	y
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10

5. The function g is defined as

$$g(x) = \begin{cases} |x+3|, & x < -2 \\ x^2 + 1, & -2 \le x \le 2 \end{cases}$$

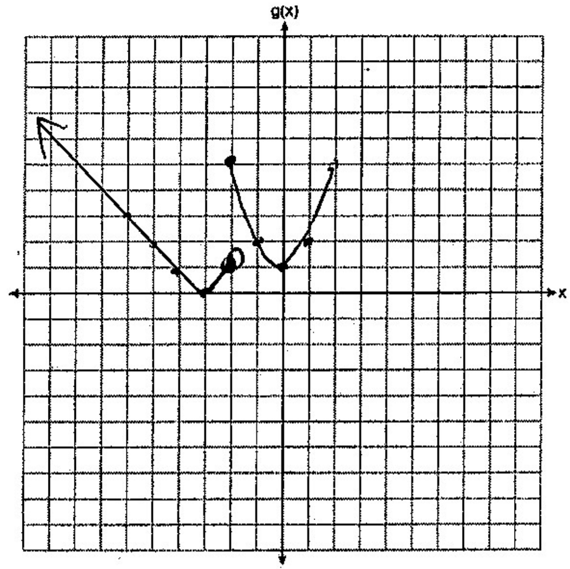
On the set of axes below, graph $g(x)$.

$|x+3|$

x	y
-2	1
-3	0
-4	1
-5	2
-6	3

x^2+1

x	y
-2	5
-1	2
0	1
1	2
2	5



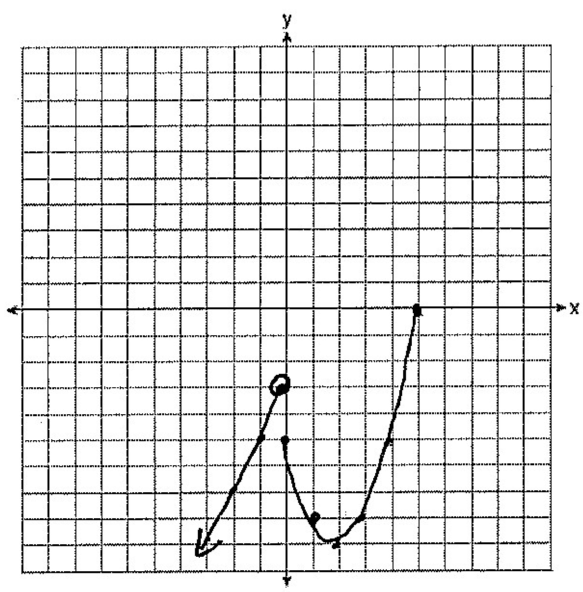
6. Graph the function: $h(x) = \begin{cases} 2x - 3, & x < 0 \\ x^2 - 4x - 5, & 0 \le x \le 5 \end{cases}$

$2x-3$

x	y
0	-3
-1	-5
-2	-7
-3	-9

x^2-4x-5

x	y
0	$\cancel{-3}$ -5
1	-8
2	-9
3	-8
4	-5
5	0

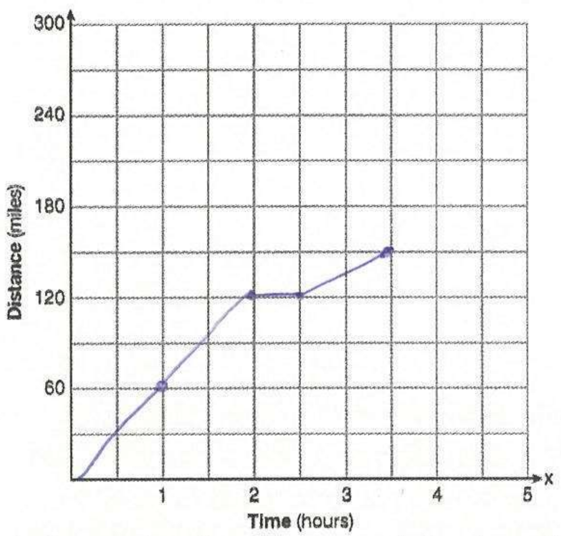


Irregular Graphs

If the y axis is distance, 0 slope means there is no movement. The greater the slope, the faster the movement.

If the y axis is rate, 0 slope means the rate is staying the same. The greater the slope, the greater the rate is increasing.

1. A driver leaves home for a business trip and drives at a constant speed of 60 miles per hour for 2 hours. Her car gets a flat tire, and she spends 30 minutes changing the tire. She resumes driving and drives at 30 miles per hour for the remaining one hour until she reaches her destination. On the set of axes below, draw a graph that models the driver's distance from home.



2. One spring day, Elroy noted the time of day and the temperature, in degrees Fahrenheit. His findings are stated below.

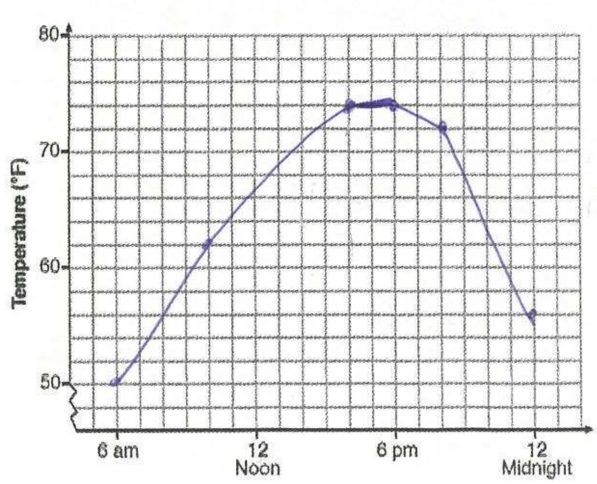
10 Am $50+4(3)=62$

At 6 a.m., the temperature was 50°F. For the next 4 hours, the temperature rose 3° per hour. The next 6 hours, it rose 2° per hour. The temperature then stayed steady until 6 p.m. For the next 2 hours, the temperature dropped 1° per hour. The temperature then dropped steadily until the temperature was 56°F at midnight.

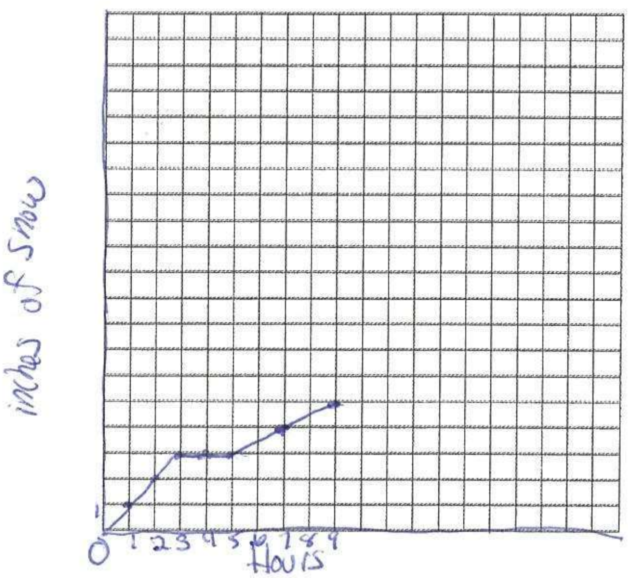
4pm $62+6(2)=74$

On the set of axes below, graph Elroy's data.

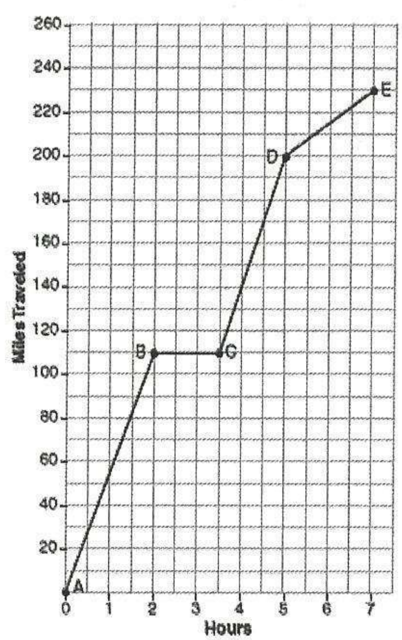
8Pm $74-2(1)=72$



3. During a snowstorm, a meteorologist tracks the amount of accumulating snow. For the first three hours of the storm, the snow fell at a constant rate of one inch per hour. The storm then stopped for two hours and then started again at a constant rate of one-half inch per hour for the next four hours. On the grid below, draw and label a graph that models the accumulation of snow over time using the data the meteorologist collected.



4. The graph below models Craig's trip to visit his friend in another state. In the course of his travels, he encountered both highway and city driving. Based on the graph, during which interval did Craig most likely drive in the city? Explain your reasoning. Explain what might have happened in the interval between B and C. Determine Craig's average speed, to the *nearest tenth of a mile per hour*, for his entire trip.



He drove in the city from D to E because it is the slowest rate where the car was in motion.

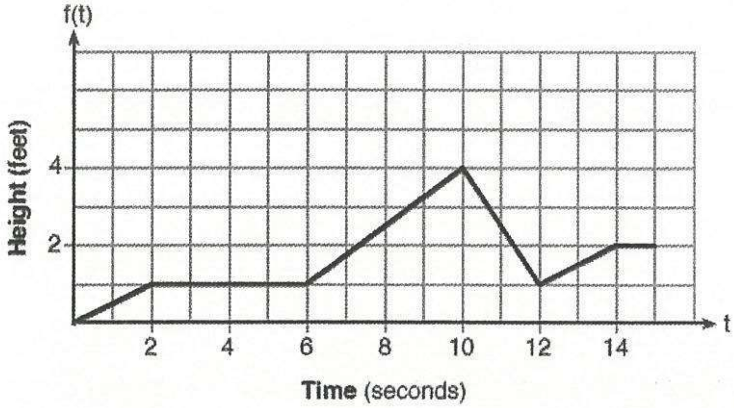
He may have stopped for lunch between B and C because he did not move although time was passing.

Average rate of change

x	y
0	0
7	230

$\frac{f(b)-f(a)}{b-a}$ $\frac{230-0}{7-0} = 3.3$ mph

5. The graph of $f(t)$ models the height, in feet, that a bee is flying above the ground with respect to the time it traveled in t seconds. State all time intervals when the bee's rate of change is zero feet per second. Explain your reasoning.



(2,6)
(14,15)

The height did not change as time passed.

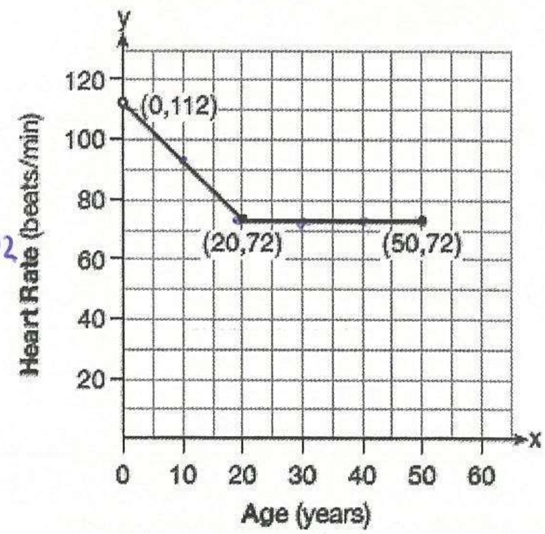
6. A graph of average resting heart rates is shown below. The average resting heart rate for adults is 72 beats per minute, but doctors consider resting rates from 60-100 beats per minute within normal range.

Which statement about average resting heart rates is *not* supported by the graph?

1) A 10-year-old has the same average resting heart rate as a 20-year-old. 92 72 X
2) A 20-year-old has the same average resting heart rate as a 30-year-old. 72
3) A 40-year-old may have the same average resting heart rate for ten years. 72 40 and 50 are both 72
4) The average resting heart rate for teenagers steadily decreases.

Yes, decreases from 13 to 20.

Average Resting Heart Rate by Age

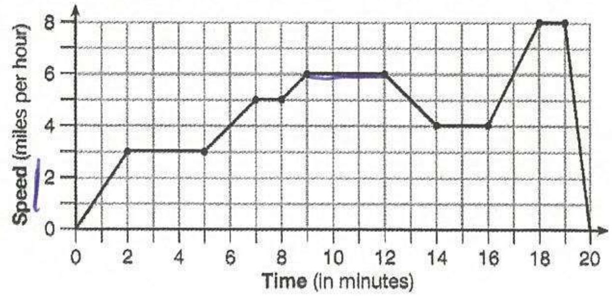


7. The graph below represents a jogger's speed during her 20-minute jog around her neighborhood.

Which statement best describes what the jogger was doing during the 9 – 12 minute interval of her jog?

1) She was standing still.
2) She was increasing her speed.
3) She was decreasing her speed
4) She was jogging at a constant rate.

The speed did not change.



Identifying Functions

If asked which equation represents a table or graph, type the equation into the calculator and see if it matches the table or graph.

Look Carefully!

1. The table below represents the function F. The equation that represents this function is

x	3	4	6	7	8
$F(x)$	9	17	65	129	257

1) $F(x) = 3^x$
2) $F(x) = 3x$
3) $F(x) = 2^x + 1$
4) $F(x) = 2x + 3$

2. A laboratory technician studied the population growth of a colony of bacteria. He recorded the number of bacteria every other day, as shown in the partial table below.
Which function would accurately model the technician's data?

t (time, in days)	0	2	4
f(t) (bacteria)	25	15,625	9,765,625

1) $f(t) = 25^t$
2) $f(t) = 25^{t+1}$
3) $f(t) = 25t$
4) $f(t) = 25(t+1)$

3. Which function is shown in the table below?

x	f(x)
−2	$\frac{1}{9}$
−1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

1) $f(x) = 3x$
2) $f(x) = x + 3$
3) $f(x) = -x^3$
4) $f(x) = 3^x$

4. Marc bought a new laptop for $1250. He kept track of the value of the laptop over the next three years, as shown in the table below.

Years After Purchase	Value in Dollars
1	1000
2	800
3	640

Which function can be used to determine the value of the laptop for x years after the purchase?

1) $f(x) = 1000(1.2)^x$
2) $f(x) = 1000(0.8)^x$
3) $f(x) = 1250(1.2)^x$
4) $f(x) = 1250(0.8)^x$

5. Which chart could represent the function $f(x) = -2x + 6$?

1)

x	f(x)
0	6
2	10
4	14
6	18

2)

x	f(x)
0	4
2	6
4	8
6	10

3)

x	f(x)
0	8
2	10
4	12
6	14

4)

x	f(x)
0	6
2	2
4	–2
6	–6

6. The table below shows the temperature, $T(m)$, of a cup of hot chocolate that is allowed to chill over several minutes, m.

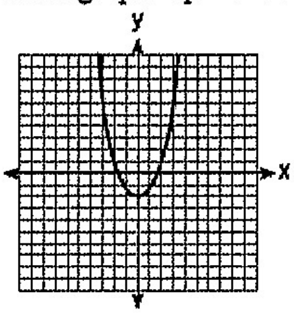
Which expression best fits the data for $T(m)$?

1) $150(0.85)^m$
2) $150(1.15)^m$
3) $150(0.85)^{m-1}$
4) $150(1.15)^{m-1}$

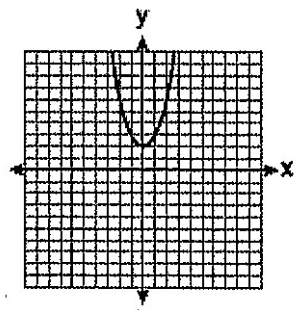
Time, m (minutes)	0	2	4	6	8
Temperature, T(m) (°F)	150	108	78	56	41

7. Which graph represents $y = \sqrt{x-2}$?

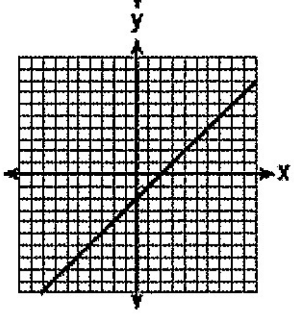
1)



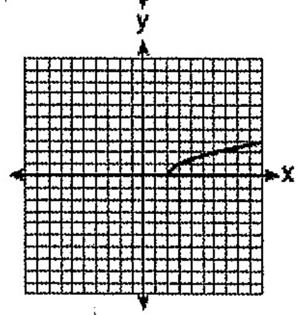
3)



2)

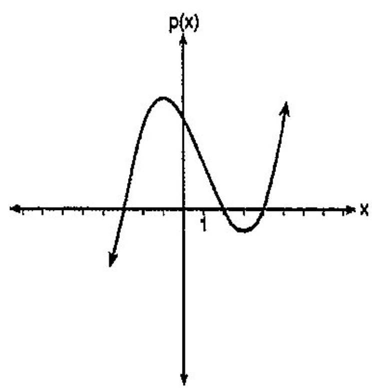


4)

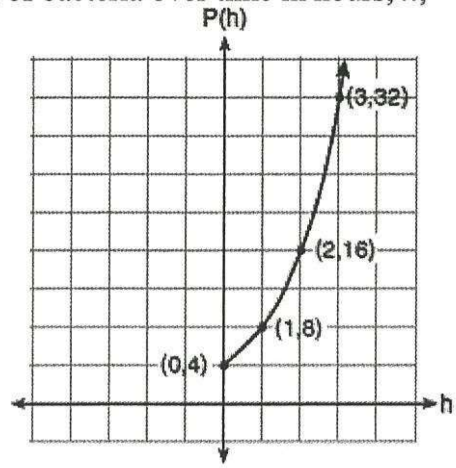


8. Based on the graph below, which expression is a possible factorization of $p(x)$?

1) $(x+3)(x-2)(x-4)$
2) $(x-3)(x+2)(x+4)$
3) $(x+3)(x-5)(x-2)(x-4)$
4) $(x-3)(x+5)(x+2)(x+4)$



9. Vinny collects population data, $P(h)$, about a specific strain of bacteria over time in hours, h, as shown in the graph below.



Which equation represents the graph of $P(h)$?

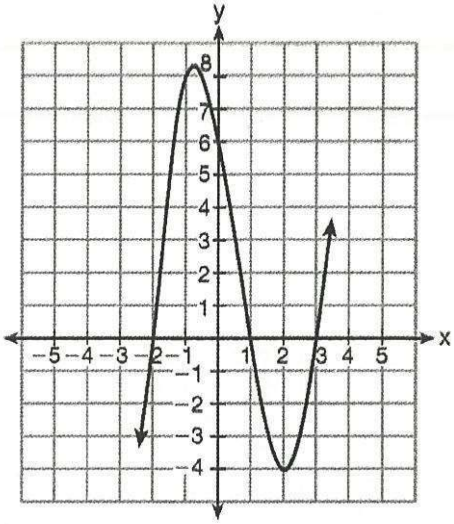
1) $P(h) = 4(2)^h$
2) $P(h) = \frac{46}{5}h + \frac{6}{5}$
3) $P(h) = 3h^2 + 0.2h + 4.2$
4) $P(h) = \frac{2}{3}h^3 - h^2 + 3h + 4$

10. Which equation(s) represent the graph below?

I $\quad y = (x+2)(x^2 - 4x - 12)$

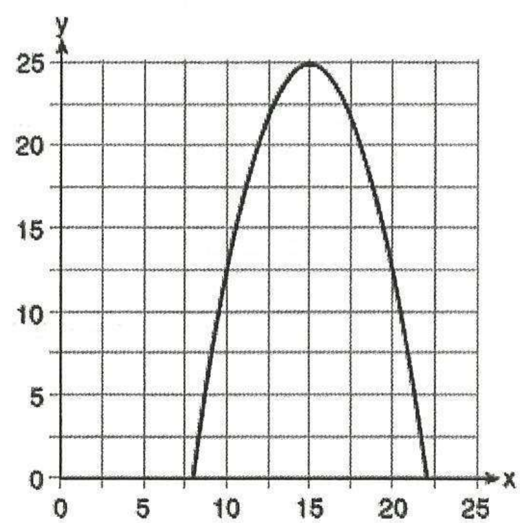
II $\quad y = (x-3)(x^2 + x - 2)$

III $\quad y = (x-1)(x^2 - 5x - 6)$



1) I, only
2) II, only
3) I and II
4) II and III

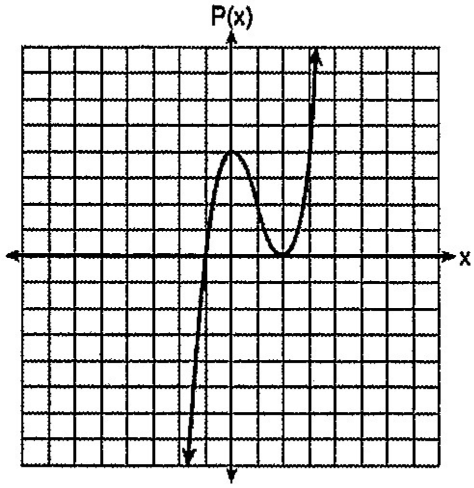
11. The graph of a quadratic function is shown below.



An equation that represents the function could be

1) $q(x) = \frac{1}{2}(x+15)^2 - 25$
2) $q(x) = -\frac{1}{2}(x+15)^2 - 25$
3) $q(x) = \frac{1}{2}(x-15)^2 + 25$
4) $q(x) = -\frac{1}{2}(x-15)^2 + 25$

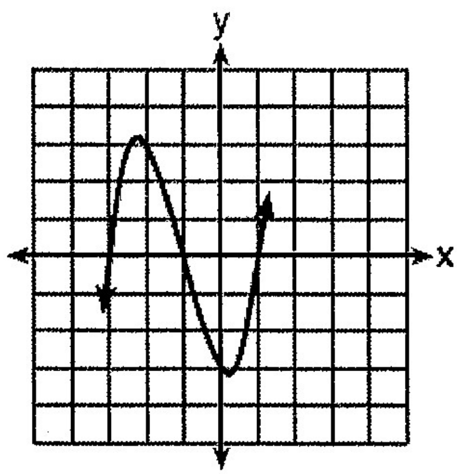
12. Wenona sketched the polynomial $P(x)$ as shown on the axes below.



Which equation could represent $P(x)$?

1) $P(x) = (x+1)(x-2)^2$
2) $P(x) = (x-1)(x+2)^2$
3) $P(x) = (x+1)(x-2)$
4) $P(x) = (x-1)(x+2)$

13. A cubic function is graphed on the set of axes below.



Which function could represent this graph?

1) $f(x) = (x-3)(x-1)(x+1)$
2) $g(x) = (x+3)(x+1)(x-1)$
3) $h(x) = (x-3)(x-1)(x+3)$
4) $k(x) = (x+3)(x+1)(x-3)$

Determining if a point is on the graph

Substitute x and y into the equation
If the two sides are equal, yes!
If the two sides are not equal, no!

or see if the point is in the table of values.

1. Which point is *not* on the graph represented by $y = x^2 + 3x - 6$?

1) $(-6, 12)$ $12 = (-6)^2 + 3(-6) - 6 = 12$ ✓
2) $(-4, -2)$ $-2 = (-4)^2 + 3(-4) - 6 = -2$ ✓
3) $(2, 4)$ $4 = (2)^2 + 3(2) - 6 = 4$ ✓
4) $(3, -6)$ $-6 = (3)^2 + 3(3) - 6 = 12$ ✗

2. Which ordered pair would *not* be a solution to $y = x^3 - x$?

1) $(-4, -60)$ $-60 = (-4)^3 + 4 = -60$ ✓
2) $(-3, -24)$ $-24 = (-3)^3 + 3 = -24$ ✓
3) $(-2, -6)$ $-6 = (-2)^3 + 2 = -6$ ✓
4) $(-1, -2)$ $-2 = (-1)^3 + 1 = 0$ ✗

3. Which ordered pair below is *not* a solution to $f(x) = x^2 - 3x + 4$?

1) $(0, 4)$ $4 = (0)^2 - 3(0) + 4 = 4$ ✓
2) $(1.5, 1.75)$ $1.75 = (1.5)^2 - 3(1.5) + 4 = 1.75$ ✓
3) $(5, 14)$ $14 = (5)^2 - 3(5) + 4 = 14$ ✓
4) $(-1, 6)$ $6 = (-1)^2 - 3(-1) + 4 = 8$ ✗

4. Which point is *not* in the solution set of the equation $3y + 2 = x^2 - 5x + 17$?

1) $(-2, 10)$ $3(10) + 2 = (-2)^2 - 5(-2) + 17$ $32 = 31$ ✗
2) $(-1, 7)$ $3(7) + 2 = (-1)^2 - 5(-1) + 17$ $23 = 23$ ✓
3) $(2, 3)$ $3(3) + 2 = (2)^2 - 5(2) + 17$ $11 = 11$ ✓
4) $(5, 5)$ $3(5) + 2 = (5)^2 - 5(5) + 17$ $17 = 17$ ✓

5. How many of the equations listed below represent the line passing through the points $(2, 3)$ and $(4, -7)$?

$x=2$ $y=3$ $x=4$ $y=-7$

$5(2) + 3 = 13$ ✓ $5x + y = 13$ $5(4) + 7 = 13$ ✗

$3 + 7 = -5(2-4)$ ✓ $y + 7 = -5(x - 4)$ $-7 + 7 = -5(4-4)$ ✓

✓ $3 = -5(2) + 13$ ✓ $y = -5x + 13$ $-7 = -5(4) + 13$ ✓

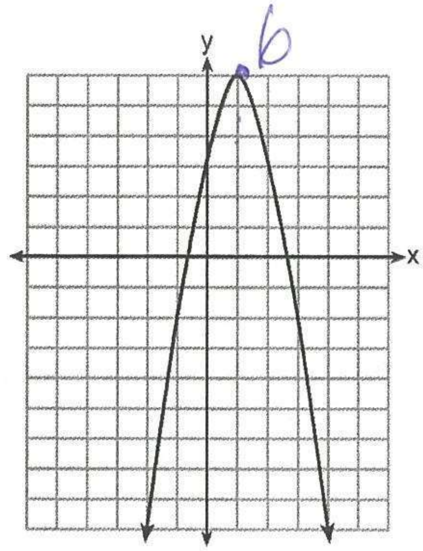
✗ $3 - 7 = 5(2-4)$ $y - 7 = 5(x - 4)$ $-7 - 7 = 5(4-4)$ ✗

1) 1
2) 2
3) 3
4) 4

Key Points

To compare key points, find the key point for each function. Use the graph, the table (2^{nd} graph), and the calculate menu (2^{nd} Trace).

1. Let f be the function represented by the graph below.



Let g be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$. Determine which function has the larger maximum value. Justify your answer. Type into calc

x	y
1	6.5
2	9
3	10.5
4	11
5	10.5
6	9
7	6.5

11

g(x)

11>6

2. Which quadratic function has the largest maximum?

1) $h(x) = (3 - x)(2 + x)$ 2nd Trace, max 6.25

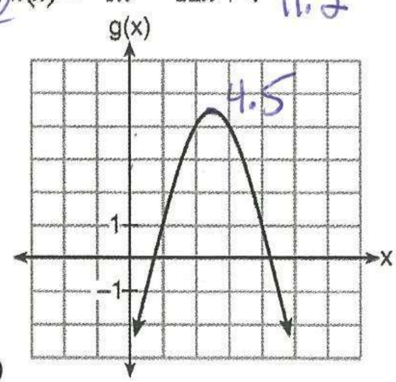
3) $k(x) = -5x^2 - 12x + 4$ 2nd Trace, max, adjust y max 11.2

2)

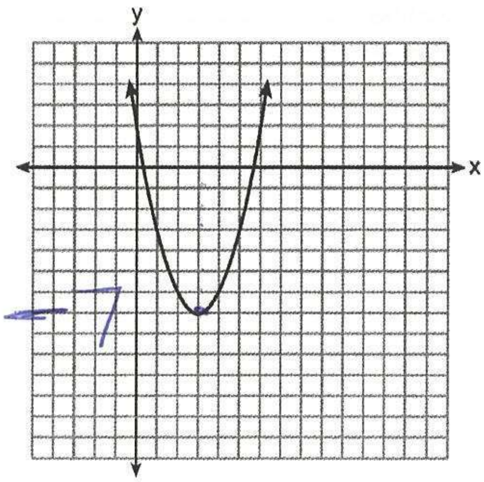
x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

≈9

4)



3. The graph representing a function is shown below.



Which function has a minimum that is *less* than the one shown in the graph?

2nd Trace min

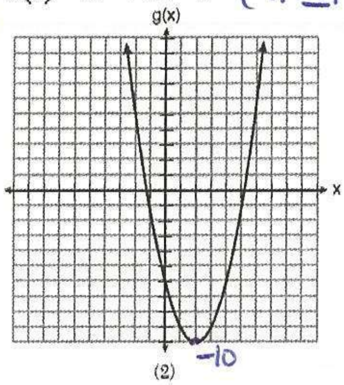
1) $y = x^2 - 6x + 7$ -2
2) $y = |x + 3| - 6$ -6
3) $y = x^2 - 2x - 10$ -11 adjust y min
4) $y = |x - 8| + 2$ 2

4. Which of the quadratic functions below has the *smallest* minimum value?

2nd Trace min

1) $h(x) = x^2 + 2x - 6$ (-1,-7)

2)



3) $k(x) = (x + 5)(x + 2)$ (-3.5,-2.25)

4)

x	f(x)
−1	−2
0	−5
1	−6
2	−5
3	−2

-6

5. Which statement is true about the quadratic functions g(*x*), shown in the table below, and $f(x) = (x - 3)^2 + 2$?

vertex: (3,2)
Zeros: None
AOS: X=3

X	y
0	11
1	6
2	3
3	2
4	3
5	6
6	11

x	g(x)
0	4
1	−1
2	−4
3	−5
4	−4
5	−1
6	4

Vertex: (3,-5)
Zeros: ≈0.5, ≈5.5
AOS: x=3

1) They have the same vertex. X
2) They have the same zeros. X
3) They have the same axis of symmetry. ✓
4) They intersect at two points. X

6. Which quadratic function has the largest maximum over the set of real numbers?

1) $f(x) = -x^2 + 2x + 4$ (1, 5)

3) $g(x) = -(x-5)^2 + 5$ (5, 5) 2nd Trace, maximum

2)

x	k(x)
−1	−1
0	3
1	5
2	5
3	3
4	−1

≈ 5.5

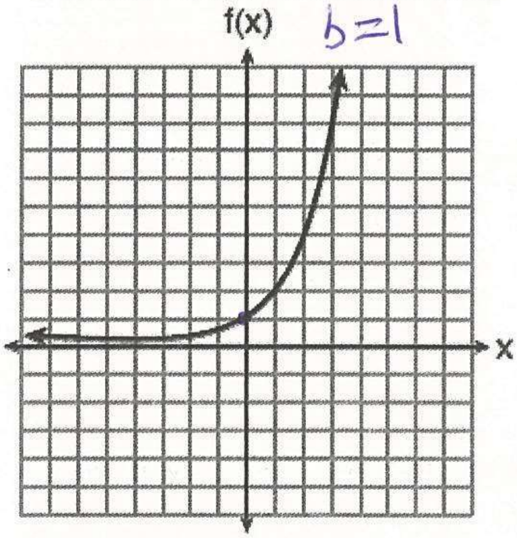
4)

x	h(x)
−2	−9
−1	−3
0	1
1	3
2	3
3	1

≈ 3.5

7. Three functions are shown below.

b = 1



b = −1

x	h(x)
−5	30
−4	14
−3	6
−2	2
−1	0
0	−1
1	−1.5
2	−1.75

b = 3

$g(x) = 3^x + 2$

Which statement is true?

1) The y-intercept for $h(x)$ is greater than the y-intercept for $f(x)$. −1 > 1 ✗

2) The y-intercept for $f(x)$ is greater than the y-intercept for $g(x)$. 1 > 3 ✗

3) The y-intercept for $h(x)$ is greater than the y-intercept for both $g(x)$ and $f(x)$. −1 > 1 ✗ −1 > 3 ✗

4) The y-intercept for $g(x)$ is greater than the y-intercept for both $f(x)$ and $h(x)$. 3 > 1 ✓ 3 > −1 ✓

8. The function $h(x)$, which is graphed below, and the function $g(x) = 2|x + 4| - 3$ are given. min = −3

Which statements about these functions are true?

I. $g(x)$ has a lower minimum value than $h(x)$. ✓

II. For all values of x, $h(x) < g(x)$. ✗

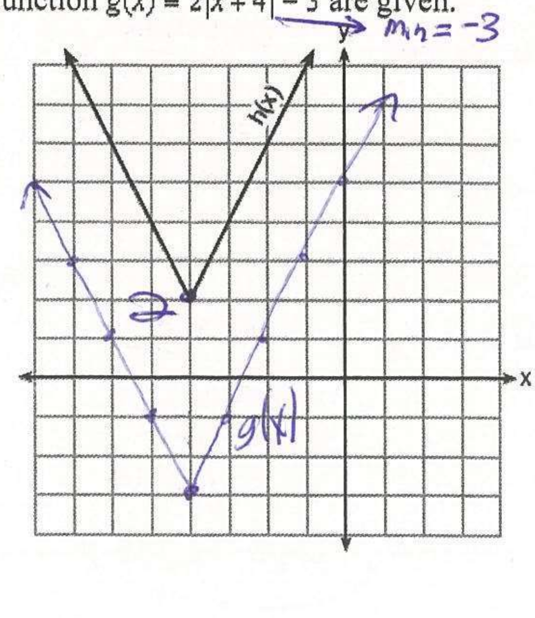
III. For any value of x, $g(x) \neq h(x)$. ✓

1) I and II, only

2) I and III, only

3) II and III, only

4) I, II, and III



Sequences:
Arithmetic: add a constant difference, Geometric: multiply by a common ratio
Explicit Formulas (From Reference Sheet)

Arithmetic: $a_n = a_1 + (n-1)d$ Geometric: $a_n = a_1(r)^{n-1}$

Recursive Formulas

Arithmetic: $a_1 =$; $a_n = a_{n-1} + d$ Geometric: $a_1 =$; $a_n = ra_{n-1}$

1. Write an explicit AND recursive equation for the following sequence and find the tenth term.
19, 16, 13, 10 … arithmetic
-3 -3 -3

$a_1=19$
$d=-3$

$a_n=a_1+(n-1)d$
$a_n = 19+(n-1)(-3)$

$a_n=19-3n+3$
$a_n = -3n+22$
$a_{10}=-3(10)+22$
$a_{10}=-8$

$a_1= 19$
$a_n=a_{n-1}-3$

2. Write an explicit AND recursive equation for the following sequence and find the ninth term.
2,8,32,128,… geometric
·4 ·4 ·4

$a_1=2$
$r=4$

$a_n=a_1(r)^{n-1}$
$a_n=2(4)^{n-1}$
$a_9=2(4)^{9-1}$
$a_9 = 131,072$

$a_1=2$
$a_n=4a_{n-1}$

3. Write an explicit AND recursive equation for the following sequence and find the eighth term.
2, 6, 18, 54, ... geometric
·3 ·3 ·3

$a_1=2$
$r=3$

$a_n=a_1(r)^{n-1}$
$a_n=2(3)^{n-1}$
$a_8=2(3)^{8-1}$
$a_8= 4374$

$a_1=2$
$a_n=3a_{n-1}$

4. Write an explicit AND recursive equation for the following sequence and find the 20th term.
63, 57, 51, 45, ... arithmetic
-6 -6 -6

$a_1=63$
$d=-6$

$a_n=a_1+(n-1)d$
$a_n=63+(n-1)(-6)$
$a_n=63-6n+6$
$a_n=-6n+69$

$a_{20}=-6(20)+69$
$a_{20}=-51$

$a_1 = 63$
$a_n= a_{n-1}-6$

5. Write an explicit AND recursive equation for the following sequence and find the 7th term.
3, -12, 48, -192, ... geometric
·-4 ·-4 ·-4

$a_1=3$
$r=-4$

$a_n=a_1(r)^{n-1}$
$a_n=3(-4)^{n-1}$
$a_7=3(-4)^{7-1}$
$a_7= 12,288$

$a_1=3$
$a_n=-4a_{n-1}$

6. The diagrams below represent the first three terms of a sequence.
Assuming the pattern continues, which formula determines a_n, the number of shaded squares in the nth term?

1) $a_n = 4n + 12$
2) $a_n = 4n + 8$
3) $a_n = 4n + 4$
4) $a_n = 4n + 2$

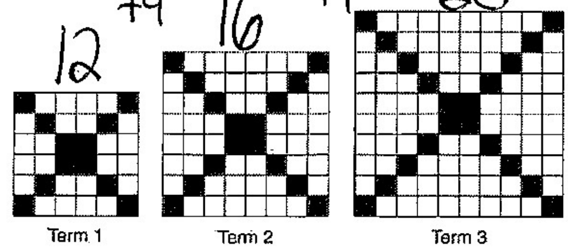
$a_n = a_1 + d(n-1)$ $\quad a_1 = 12$ $\quad d = 4$

$a_n = 12 + 4(n-1)$

$a_n = 12 + 4n - 4$

$a_n = 4n + 8$

12 $\quad$ +4 $\quad$ 16 $\quad$ +4 $\quad$ 20



7. Given the pattern below, which formula represents the number of triangles in this sequence?

$a_1 =$

3 $\quad$ +2 $\quad$ 5 $\quad$ +2 $\quad$ 7 $\quad$ +2 $\quad$ 9

1) $y = 2x + 3$
2) $y = 3x + 2$
3) $y = 2x + 1$
4) $y = 3x - 2$

$a_n = a_1 + d(n-1)$

$a_n = 3 + 2(n-1)$

$a_n = 3 + 2n - 2$

$a_n = 2n + 1$

-4 -4 -4

8. Which function defines the sequence $-6, -10, -14, -18, \ldots$, where $f(6) = -26$?

1) $f(x) = -4x - 2$
2) $f(x) = 4x - 2$
3) $f(x) = -x + 32$
4) $f(x) = x - 26$

$a_1 = -6$ $\quad d = -4$

$a_n = a_1 + d(n-1)$

$a_n = -6 - 4(n-1)$

$a_n = -6 - 4n + 4$

$a_n = -4n - 2$

9. In a sequence, the first term is 4 and the common difference is 3. The <u>fifth</u> term of this sequence is

1) -11
2) -8
3) 16
4) 19

$a_1 = 4$ $\quad d = 3$

$a_n = a_1 + d(n-1)$

$a_n = 4 + 3(n-1)$

$a_n = 4 + 3n - 3$

$a_n = 3n + 1$

$a_5 = 3(5) + 1$

$a_5 = 16$

10. In a geometric sequence, the first term is 4 and the common ratio is -3. The fifth term of this sequence is

1) 324
2) 108
3) -108
4) -324

$a_1 = 4$

$r = -3$

$a_n = a_1(r)^{n-1}$ $\quad a_5 = 4(-3)^{5-1}$

$a_n = 4(-3)^{n-1}$ $\quad a_5 = 324$

Sequences with Non-Consecutive Terms
Write out the terms and guess and check!

1. Determine the common difference of the arithmetic sequence in which $a_1 = 3$ and $a_4 = 15$.

$\underset{1}{3} \xrightarrow{+4} \underset{2}{7} \xrightarrow{+4} \underset{3}{11} \xrightarrow{+4} \underset{4}{15}$

$d=4$

2. The fifth term in an arithmetic sequence is 8 and the ninth term is 28. Find the common difference.

$\underset{5}{8} \xrightarrow{+5} \underset{6}{13} \xrightarrow{+5} \underset{7}{18} \xrightarrow{+5} \underset{8}{23} \xrightarrow{+5} \underset{9}{28}$

$d=5$

3. The first term in a sequence is 5 and the fifth term is 17. What is the common difference?

1) 2.4
2) 12
3) 3
4) 4

$\underset{1}{5} \xrightarrow{+3} \underset{2}{8} \xrightarrow{+3} \underset{3}{11} \xrightarrow{+3} \underset{4}{14} \xrightarrow{+3} \underset{5}{17}$

4. The third term in an arithmetic sequence is 10 and the fifth term is 26. If the first term is a_1, which is an equation for the nth term of this sequence?

1) $a_n = 8n + 10$
2) $a_n = 8n - 14$
3) $a_n = 16n + 10$
4) $a_n = 16n - 38$

$\underset{1}{-6} \xleftarrow{-8} \underset{2}{2} \xleftarrow{-8} \underset{3}{10} \xrightarrow{+8} \underset{4}{18} \xrightarrow{+8} \underset{5}{26}$

$d=8$ $\quad a_n = a_1 + d(n-1)$
$a_1 = -6$ $\quad a_n = -6 + 8(n-1)$
$a_n = -6 + 8n - 8$
$a_n = 8n - 14$

5. The 2nd term in a geometric sequence is 10 and the 6th term is ~~2560~~ 6250. Which is an equation for the nth term of this sequence?

1) $a_n = 10(5)^{n-1}$
2) $a_n = 10(2)^{n-1}$
3) $a_n = 2(5)^{n-1}$
4) $a_n = 2(2)^{n-1}$

$a_1 = 2$
$r = 5$
$a_n = 2(5)^{n-1}$

$\underset{1}{2} \xrightarrow{\cdot 5} \underset{2}{10} \xrightarrow{\cdot 5} \underset{3}{50} \xrightarrow{\cdot 5} \underset{4}{250} \xrightarrow{\cdot 5} \underset{5}{1250} \xrightarrow{\cdot 5} \underset{6}{\cancel{2560}\ 6250}$

6. What is a common ratio of the geometric sequence whose first term is 5 and third term is 245?

1) 7
2) 49
3) 120
4) 240

$\underset{1}{5} \xrightarrow{\cdot 7} \underset{2}{35} \xrightarrow{\cdot 7} \underset{3}{245}$

7. What is the fourth term of the geometric sequence whose third term is 20 and fifth term is 180?

1) 3
2) 60
3) 20
4) 70

$\underset{3}{20} \xrightarrow{\cdot 3} \underset{4}{60} \xrightarrow{\cdot 3} \underset{5}{180}$

Regression Equations
Turn Stat Diagnostics On (Mode, STATDIAGNOSTICS ON)
To write regression equations:

1) Stat, Edit
2) Stat, Calc, 4: LinReg or 0: ExpReg

r is the correlation coefficient. Negative slope has negative correlation coefficient, positive slope has positive correlation coefficient.
The closer the correlation coefficient is to 1 or -1, the stronger the correlation. The closer the correlation coefficient is to 0, the weaker the correlation is.
Read and round carefully! You may be asked to round to different values within different parts of the same question.

1. Which of the following correlation coefficients represents the strongest linear relationship?
(1) 0.79 (2) 0.36 (3) 0.12 (4) -0.87

2. Bella recorded data and used her graphing calculator to find the equation for the line of best fit. She then used the correlation coefficient to determine the strength of the linear fit. Which correlation coefficient represents the strongest linear relationship?
1) 0.9
2) 0.5
3) -0.3
4) -0.8

3. The results of a linear regression are shown below.
Which phrase best describes the relationship between x and y?
1) strong negative correlation
2) strong positive correlation
3) weak negative correlation
4) weak positive correlation

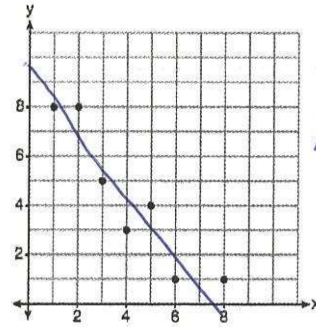
$y = ax + b$
$a = -1.15785$
$b = 139.3171772$
$r = -0.896557832$
$r^2 = 0.8038159461$

4. Which calculator output shows the strongest linear relationship between x and y?

(1)	(2)	(3)	(4)
Lin Reg	Lin Reg	Lin Reg	Lin Reg
$y = a + bx$	$y = a + bx$	$y = a + bx$	$y = a + bx$
$a = 59.026$	$a = .7$	$a = 2.45$	$a = -2.9$
$b = 6.767$	$b = 24.2$	$b = .95$	$b = 24.1$
$r = .8643$	$r = .8361$	$r = .6022$	$r = -.8924$

5. Analysis of data from a statistical study shows a linear relationship in the data with a correlation coefficient of -0.524. Which statement best summarizes this result?
1) There is a strong positive correlation between the variables.
2) There is a strong negative correlation between the variables.
3) There is a moderate positive correlation between the variables.
4) There is a moderate negative correlation between the variables.

6. What is the correlation coefficient of the linear fit of the data shown below, to the *nearest hundredth*?



Negative

1) 1.00
2) 0.93
3) −0.93
4) −1.00 → not a perfect fit

7. The percentage of students scoring 85 or better on a mathematics final exam and an English final exam during a recent school year for seven schools is shown in the table below. Write the linear regression equation for these data, rounding all values to the *nearest hundredth*. State the correlation coefficient of the linear regression equation, to the *nearest hundredth*. Explain the meaning of this value in the context of these data.

Percentage of Students Scoring 85 or Better	
Mathematics, x	**English, y**
27	46
12	28
13	45
10	34
30	56
45	67
20	42
L_1	L_2

$y = .96x + 23.95$

$r = .92$

There is a strong positive correlation between the percentage of students scoring 85 or better on Math and English.

8. Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below. Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*. The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Between the 2nd and 3rd hour.

Hours (x)	Average Number of Spores (y)
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380
L_1	L_2

Stat, Edit

Stat, Calc, 0: ExpReg

$y = 4.168(3.981)^x$

9. At Mountain Lakes High School, the mathematics and physics scores of nine students were compared as shown in the table below. State the correlation coefficient, to the *nearest hundredth*, for the line of best fit for these data. Explain what the correlation coefficient means with regard to the context of this situation.

LinReg

Mathematics	55	93	89	60	90	45	64	76	89
Physics	66	89	94	52	84	56	66	73	92

$y = .81x + 15.19$

$r = .92$

There is a strong, positive collelation between Mathematics and Physics scores

10. Erica, the manager at Stellarbeans, collected data on the daily high temperature and revenue from coffee sales. Data from nine days this past fall are shown in the table below. State the linear regression function, $f(t)$, that estimates the day's coffee sales with a high temperature of t. Round all values to the *nearest integer*. State the correlation coefficient, r, of the data to the *nearest hundredth*. Does r indicate a strong linear relationship between the variables? Explain your reasoning.

LinReg

$y = -58x + 6182$

$r = -.94$

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9
High Temperature, t	54	50	62	67	70	58	52	46	48
Coffee Sales, f(t)	\$2900	\$3080	\$2500	\$2380	\$2200	\$2700	\$3000	\$3620	\$3720

yes, because r is close to -1, there is a strong, negative collelation between temperature and coffee sales

11. The data given in the table below show some of the results of a study comparing the height of a certain breed of dog, based upon its mass. Write the linear regression equation for these data, where x is the mass and y is the height. Round all values to the *nearest tenth*. State the value of the correlation coefficient to the *nearest tenth*, and explain what it indicates.

LinReg

Mass (kg)	4.5	5	4	3.5	5.5	5	5	4	4	6	3.5	5.5
Height (cm)	41	40	35	38	43	44	37	39	42	44	31	30

$y = 1.9x + 29.8$

$r = .3$

There is a weak, positive relationship between mass and height of a certain breed of dog.

12. An application developer released a new app to be downloaded. The table below gives the number of downloads for the first four weeks after the launch of the app.
Write an exponential equation that models these data. Use this model to predict how many downloads the developer would expect in the 26th week if this trend continues. Round your answer to the nearest download.

ExpReg

Number of Weeks	1	2	3	4
Number of Downloads	120	180	270	405

$y = 80(1.5)^x$

$y = 80(1.5)^{26} = 3030140$

13. Emma recently purchased a new car. She decided to keep track of how many gallons of gas she used on five of her business trips. The results are shown in the table below.
Write the linear regression equation for these data where miles driven is the independent variable. (Round all values to the *nearest hundredth.*) State the linear correlation coefficient to the *nearest hundredth*. Explain its meaning in the context of the problem.

Miles Driven	Number of Gallons Used
150	7
200	10
400	19
600	29
1000	51

LinReg

$y = .05x - .92$

$r = 1.00$

There is a strong positive relationship between miles driven and number of gallons used.

14. The table below shows the number of math classes missed during a school year for nine students, and their final exam scores.

Number of Classes Missed (x)	2	10	3	22	15	2	20	18	9
Final Exam Score (y)	99	72	90	35	60	80	40	43	75

Write the linear regression equation for this data set. Round all values to the *nearest hundredth*. State the correlation coefficient for your linear regression. Round your answer to the *nearest hundredth*. State what the correlation coefficient indicates about the linear fit of the data.

LinReg $y = -2.81x + 97.55$

$r = -.97$

There is a strong negative collelation between number of classes missed a final exam score.

15. The data table below shows the median diameter of grains of sand and the slope of the beach for 9 naturally occurring ocean beaches. Write the linear regression equation for this set of data, rounding all values to the *nearest thousandth*. Using this equation, predict the slope of a beach, to the *nearest tenth of a degree*, on a beach with grains of sand having a median diameter of 0.65 mm.

Median Diameter of Grains of Sand, in Millimeters (x)	0.17	0.19	0.22	0.235	0.235	0.3	0.35	0.42	0.85
Slope of Beach, in Degrees (y)	0.63	0.7	0.82	0.88	1.15	1.5	4.4	7.3	11.3

Lin Reg

x

$y = 17.159x - 2.476$

$y = 17.159(.65) - 2.476$

$y = 8.7°$

Probability with Two Way Tables

Conditional Probabilities: Circle the row/column that contains the condition. Condition always comes after the phrase given that. You will not always see the phrase given that.
"And" is not conditional.

One-hundred employees of a company were asked their opinion on paying high salaries to the CEO. Their responses are summarized in the following contingency table. Express the following probabilities as fractions and rounded to the nearest percent.

	In Favor	Against	
Male	15	45	60
Female	4	36	40
	19	81	100

1. P(male and in favor)

$\frac{15}{100}$

2. P(female and against)

$\frac{36}{100}$

3. P(male)

$\frac{60}{100}$

4. P(in favor)

$\frac{19}{100}$

5. P(male given that in favor)

$\frac{15}{19}$

6. P(against given that male)

$\frac{45}{60}$

7. P(in favor given that male)

$\frac{15}{60}$

8. P(female given that against)

$\frac{36}{81}$

9. Probability a male is in favor

$\frac{15}{60}$

10. Probability a female is against

$\frac{36}{40}$

11. A statistics class surveyed some students during one lunch period to obtain opinions about television programming preferences. The results of the survey are summarized in the table below. What percentage of the school's male students would prefer comedy?

Programming Preferences

	Comedy	Drama	
Male	70	35	105
Female	48	42	90
	118	77	195

$\frac{70}{105} = 66.\overline{6}\%$

12. A public opinion poll was taken to explore the relationship between age and support for a candidate in an election. The results of the poll are summarized in the table below.

Age	For	Against	No Opinion	
21–40	30	12	8	50
41–60	20	40	15	75
Over 60	25	35	15	75
	75	87	38	200

What percent of the 21-40 age group was for the candidate?
1) 15
2) 25
3) 40
4) 60

$\frac{30}{50} = 60\%$

13. A radio station did a survey to determine what kind of music to play by taking a sample of middle school, high school, and college students. They were asked which of three different types of music they prefer on the radio: hip-hop, alternative, or classic rock. The results are summarized in the table below.

What percentage of college students prefer classic rock?

	Hip-Hop	Alternative	Classic Rock	
Middle School	28	18	4	50
High School	22	22	6	50
College	16	20	14	50
	66	60	24	150

$\frac{14}{50} = 28\%$

What percentage of the students that prefer classic rock are college students?

$\frac{14}{24} \approx 58\%$

14. A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

	Favorite Type of Program			
	Sports	**Reality Show**	**Comedy Series**	
Senior	83	110	67	260
Freshmen	119	103	54	276
	202	213	121	536

A student response is selected at random from the results. State the *exact* probability the student response is from a freshman, given the student prefers to watch reality shows on television.

$\frac{103}{213}$

15. At Berkeley Central High School, a survey was conducted to see if students preferred cheeseburgers, pizza, or hot dogs for lunch. The results of this survey are shown in the table below.

	Cheeseburgers	**Pizza**	**Hot Dogs**	
Females	32	44	24	100
Males	36	30	34	100
	68	74	58	200

Based on this survey, what percent of the students preferred pizza?

1) 30
2) 37
3) 44
4) 74

$\frac{74}{200} = 37\%$

16. A middle school conducted a survey of students to determine if they spent more of their time playing games or watching videos on their tablets. The results are shown in the table below.

	Playing Games	**Watching Videos**	**Total**
Boys	138	46	184
Girls	54	142	196
Total	192	188	380

Of the students who spent more time playing games on their tablets, approximately what percent were boys?

1) 41
2) 56
3) 72
4) 75

$\frac{138}{192} = 71.875\%$

17. A survey was given to 12th-grade students of West High School to determine the location for the senior class trip. The results are shown in the table below.

	Niagara Falls	Darien Lake	New York City	
Boys	56	74	103	233
Girls	71	92	88	251
	127	166	191	484

To the *nearest percent*, what percent of the boys chose Niagara Falls?

1) 12
2) 24
3) 44
4) 56

$\frac{56}{233} \approx 24\%$

18. Jenna took a survey of her senior class to see whether they preferred pizza or burgers. The results are summarized in the table below.

	Pizza	Burgers	
Male	23	42	65
Female	31	26	57
	54	68	122

Of the people who preferred burgers, approximately what percentage were female?

1) 21.3
2) 38.2
3) 45.6
4) 61.9

$\frac{26}{68} \simeq 38.2$

19. Students were asked to name their favorite sport from a list of basketball, soccer, or tennis. The results are shown in the table below.

	Basketball	Soccer	Tennis	
Girls	42	58	20	120
Boys	84	41	5	130
	126	99	25	250

What percentage of the students chose soccer as their favorite sport?

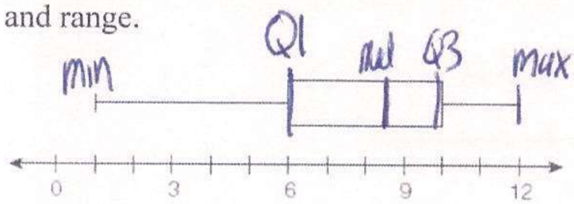
1) 39.6%
2) 41.4%
3) 50.4%
4) 58.6%

$\frac{99}{250} = 39.6\%$

Box Plots

The first dash is the minimum
The second dash in the first (lower) quartile
The third dash is the median (second quartile)
The fourth dash is the third (upper) quartile
The fifth dash is the maximum
Range = maximum – minimum
Interquartile Range = Q3 – Q1
Each section is 25% of the data

1. For the set of data below, find the lower quartile, median, upper quartile, interquartile range, and range.

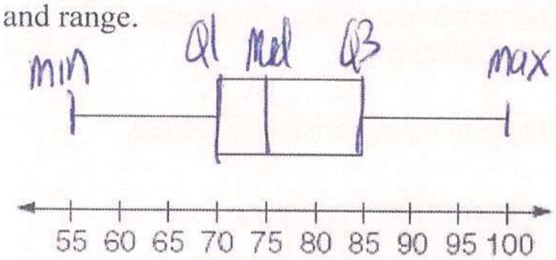


lower quartile = 6
median = 8.5
upper quartile = 10

IQR = Q3 - Q1
IQR = 10 - 6
IQR = 4

Range = max - min
Range = 12 - 1
Range = 11

2. For the set of data below, find the lower quartile, median, upper quartile, interquartile range, and range.



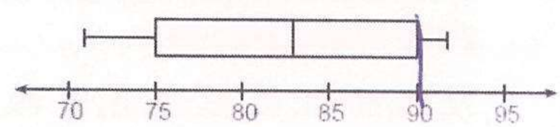
lower quartile = 70
median = 75
upper quartile = 85

IQR = Q3 - Q1
IQR = 85 - 70
IQR = 15

Range = max - min
Range = 100 - 55
Range = 45

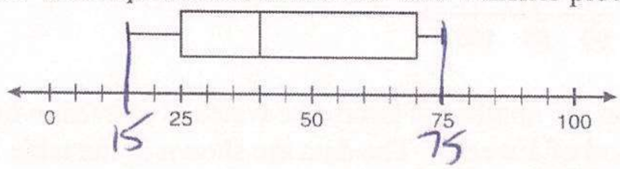
3. The box plot below summarizes the data for the average monthly high temperatures in degrees Fahrenheit for Orlando, Florida.
The third quartile is

1) 92
2) 90
3) 83
4) 71



4. What is the range of the data represented in the box-and-whisker plot shown below?

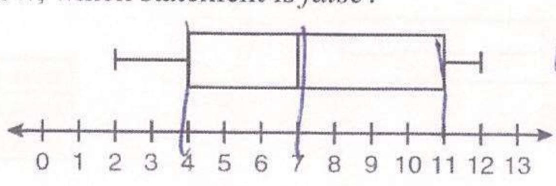
1) 40
2) 45
3) 60
4) 100



Range = max - min
Range = 75 - 15
Range = 60

5. Based on the box-and-whisker plot below, which statement is *false*?

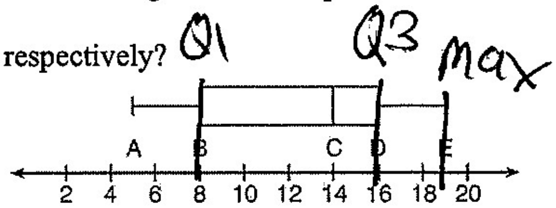
1) The median is 7. ✓
2) The range is 12. ✗
3) The first quartile is 4. ✓
4) The third quartile is 11. ✓



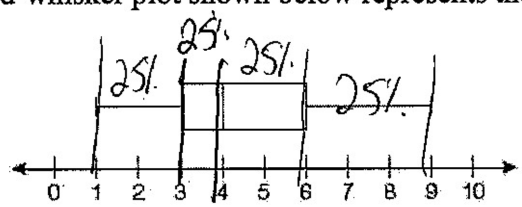
Range = 12 - 2 = 10

6. The box-and-whisker plot shown below represents the number of magazine subscriptions sold by members of a club.
Which statistical measures do points *B*, *D*, and *E* represent, respectively?

1) minimum, median, maximum
2) first quartile, median, third quartile
3) first quartile, third quartile, maximum
4) median, third quartile, maximum



7. A movie theater recorded the number of tickets sold daily for a popular movie during the month of June. The box-and-whisker plot shown below represents the data for the number of tickets sold, in hundreds.

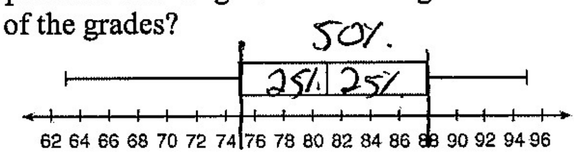


Which conclusion can be made using this plot?

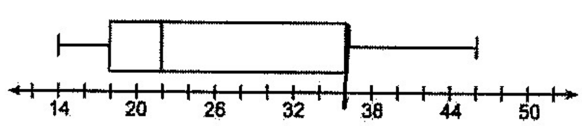
1) The second quartile is 600.
2) The mean of the attendance is 400.
3) The range of the attendance is 300 to 600.
4) Twenty-five percent of the attendance is between 300 and 400.

8. The box-and-whisker plot below represents a set of grades in a college statistics class. Which interval contains exactly 50% of the grades?

1) 63-88
2) 63-95
3) 75-81
4) 75-88



9. What is the value of the third quartile in the box plot shown below?



1) 18
2) 22
3) 36
4) 46

Measures of Central Tendency
Stat, Edit
Stat, Calc, 1-Var Stats
*If there is a frequency column/histogram, put L_2 into FreqList
$\bar{x} = mean$
$\sigma = standard\ deviation$
Med = Median
Range = maximum – minimum
Q1 = Lower Quartile
Q3 = Upper Quartile
Interquartile Range: Q3 – Q1
Mode = number that occurs the most (not given in calculator)
The greatest spread/variability has the greatest range.

1. Sara's test scores in mathematics were 64, 80, 88, 78, 60, 92, 84, 76, 86, 78, 72, and 90. Determine the mean, the median, lower quartile, upper quartile, range, interquartile range, population standard deviation, and the mode of Sara's test scores.

mean=79
median=79
lower quartile=74
upper quartile=87
range=max-min
range=92-60=32
IQR=Q3-Q1
IQR=87-74=13
population SD=~~6.14~~ 9.54
mode=78

2. Mickayla's test scores in social studies were 84, 77, 63, 72, 90, 71, 75, 76, 77, 81, 78, and 80. Determine the mean, the median, lower quartile, upper quartile, range, interquartile range, population standard deviation, and the mode of Mickayla's test scores.

Mean=77
Median=77
lower quartile=73.5
upper quartile=80.5
range=max-min
range=90-63=27
IQR=Q3-Q1
IQR=80.5-73.5=7
population SD=6.49
Mode=77

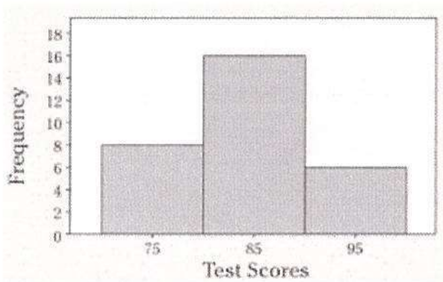
3. In the table, the data indicates the heights, in inches, of basketball players.
What is the mean, median, mode, population standard deviation, range, and interquartile range for this set of data? How many students are on this basketball team?

Height (Inches)	Frequency
77	2
76	0
75	5 mode
74	3
73	4
72	2
71	1

17 students

mean=74
median=74
Mode=75
population SD=1.6
range=max-min
range=77-71=6
IQR=Q3-Q1
IQR=75-73=2

4. The following histogram represents the test scores of students in a class.
What is the mean, median, mode, population standard deviation, range, and interquartile range for this set of data? How many students are in the class?



L1	L2
75	8
85	16
95	6

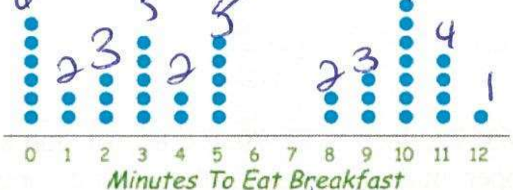
$\boxed{30}$

mean = 84.5
median = 85
pop SD = 6.8
range = 95−75 = 20
IQR = 85−75 = 10

5. The table below represents the time taken, in minutes, to eat breakfast. For this set of data, find the mean, median, mode, population standard deviation, range, and interquartile range. How many people were involved in this study?

mode

6 2 3 5 2 5 2 3 7 4 1



40 total ppl
(add up frequencies)

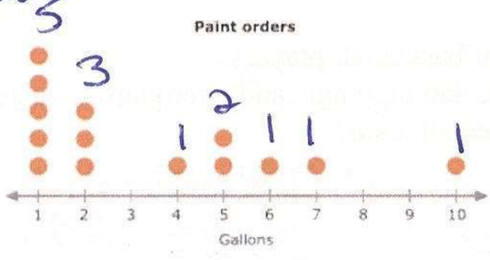
L1	L2
0	6
1	2
2	3
3	5
4	2
5	5
6	0
7	0
8	2
9	3
10	7
11	4
12	1

mean = 5.625
median = 5
mode = 7
pop SD = 4.0
range = max − min
range = 12 − 0 = 12
IQR = Q3 − Q1
IQR = 10 − 2 = 8

6. The following data represents the number of gallons of paint in a paint order in a given day. For this set of data, find the mean, median, mode, population standard deviation, range, and interquartile range. How many paint orders were placed on this day?

mode

5 3 1 2 1 1 1



L1	L2
1	5
2	3
3	0
4	1
5	2
6	1
7	1
8	0
9	0
10	1

mean = 3.4
median = 2
mode = 1
pop SD = 2.7
range = max − min
range = 10 − 1 = 9
IQR = Q3 − Q1
IQR = 5 − 1 = 4

14 orders

7. Which statement is true about the data set 3, 4, 5, 6, 7, 7, 10?

1) mean = mode
2) mean > mode
3) mean = median
4) mean < median

mean = 6
median = 6
mode = 7

8. The following are Regents scores in a math class.

59	56	64	69	55
67	55	57	55	68
64	69	65	71	45

Which of the following statement is true?

1) mean < mode
2) mode = lower quartile
3) standard deviation = range
4) interquartile range = standard deviation

mean = 61.26 IQR = 13
mode = 55 $\sigma x = 7..$
Q1 = 55
range = 71-45 = 26

9. The two sets of data below represent the number of runs scored by two different youth baseball teams over the course of a season.

Team *A*: 4, 8, 5, 12, 3, 9, 5, 2
Team *B*: 5, 9, 11, 4, 6, 11, 2, 7

Which set of statements about the mean and standard deviation is true?

1) mean *A* < mean *B*
   standard deviation *A* > standard deviation *B*
2) mean *A* > mean *B*
   standard deviation *A* < standard deviation *B*
3) mean *A* < mean *B*
   standard deviation *A* < standard deviation *B*
4) mean *A* > mean *B*
   standard deviation *A* > standard deviation *B*

A
mean = 6
$\sigma x = 3.3..$

B
mean = 6.875
$\sigma x = 3.2..$

10. Christopher looked at his quiz scores shown below for the first and second semester of his Algebra class.

Semester 1: 78, 91, 88, 83, 94
Semester 2: 91, 96, 80, 77, 88, 85, 92

Which statement about Christopher's performance is correct?

1) The interquartile range for semester 1 is greater than the interquartile range for semester 2.
2) The median score for semester 1 is greater than the median score for semester 2.
3) The mean score for semester 2 is greater than the mean score for semester 1.
4) The third quartile for semester 2 is greater than the third quartile for semester 1.

S1
mean = 86.8
Med = 88
Q3 = 92.5
IQR = 92.5 - 80.5
12

S2
mean = 87
Med = 88
Q3 = 92
IQR = 92 - 80
12

11. The 15 members of the French Club sold candy bars to help fund their trip to Quebec. The table below shows the number of candy bars each member sold.

Number of Candy Bars Sold				
0	35	38	41	43
45	50	53	53	55
68	68	68	72	120

Med = 53
range = max − min
range = 120 − 0 = 120

When referring to the data, which statement is *false*?

1) The mode is the best measure of central tendency for the data. ✗
2) The data have two outliers. ✓
3) The median is 53. ✓
4) The range is 120. ✓

12. The following table shows the heights, in inches, of the players on the opening-night roster of the 2015-2016 New York Knicks.

84	80	87	75	77	79	80	74	76	80	80	82	82

The population standard deviation of these data is approximately

1) 3.5
2) 13
3) 79.7
4) 80

$\sigma x = 3.5$

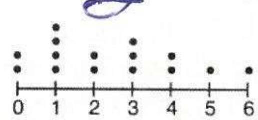
13. The ages of the last 16 United States presidents on their first inauguration day are shown in the table below. Determine the interquartile range for this set of data.

51	54	51	60
62	43	55	56
61	52	69	64
46	54	47	70

IQR = Q3 − Q1
IQR = 61.5 − 51 = 10.5

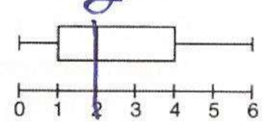
14. Different ways to represent data are shown below.

2

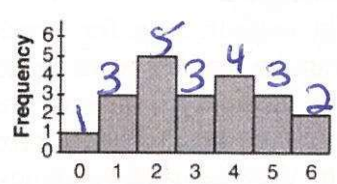


I

2



II



III

Which data representations have a median of 2?

1) I and II, only
2) I and III, only
3) II and III, only
4) I, II, and III

L1	L2
0	2
1	4
2	2
3	3
4	2
5	1
6	1

Med = 2

med = 3

L1	L2
0	1
1	3
2	5
3	3
4	4
5	3
6	2

15. Donna and Andrew compared their math final exam scores from grade 8 through grade 12. Their scores are shown below.

Donna	
8th	90
9th	92
10th	87
11th	94
12th	95

mean = 91.6
med = 92
Q3 = 94.5
IQR = 94.5 − 88.5 = 6

Andrew	
8th	78
9th	96
10th	87
11th	94
12th	93

mean = 89.6
med = 93
Q3 = 95
IQR = 95 − 82.5 = 12.5

Which statement about their final exam scores is correct?

1) Andrew has a higher mean than Donna. X
2) Donna and Andrew have the same median. X
3) Andrew has a larger interquartile range than Donna. ✓
4) The 3rd quartile for Donna is greater than the 3rd quartile for Andrew. X

16. Santina is considering a vacation and has obtained high-temperature data from the last two weeks for Miami and Los Angeles. Which location has the least variability in temperatures? Explain how you arrived at your answer.

range

Miami	76	75	83	73	60	66	76
	81	83	85	83	87	80	80

Range = 87 − 60
Range = 27

Los Angeles	74	63	65	67	65	65	65
	62	62	72	69	64	64	61

Range = 74 − 61
Range = 13

Los Angeles because it has a smaller range

17. The students in Mrs. Lankford's 4th and 6th period Algebra classes took the same test. The results of the scores are shown in the following table:

	$\bar{x}$	σ_x	n	min	Q_1	med	Q_3	max
4th Period	77.75	10.79	20	58	69	76.5	87.5	96
6th Period	78.4	9.83	20	59	71.5	78	88	96

$96-58=38$
$96-59=37$

Based on these data, which class has the larger spread of test scores? Explain how you arrived at your answer.

range

Period 4 has the greater range

18. The heights, in inches, of 12 students are listed below.

61,67,72,62,65,59,60,79,60,61,64,63

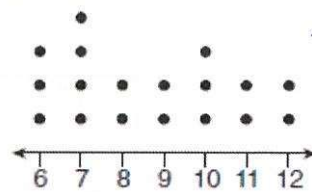
Which statement best describes the spread of these data?

1) The set of data is evenly spread.
there's an outlier
2) The median of the data is 59.5.
median=62.5
3) The set of data is skewed because 59 is the only value below 60.
1 value does not cause skew
4) 79 is an outlier, which would affect the standard deviation of these data. ✓

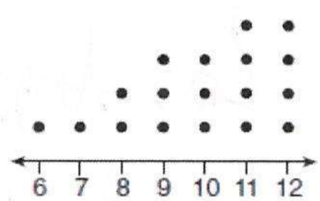
19. Noah conducted a survey on sports participation. He created the following two dot plots to represent the number of students participating, by age, in soccer and basketball.

mean $=8.\bar{6}$
med $=8.5$

L1	L2
6	3
7	4
8	2
9	2
10	3
11	2
12	2



Soccer Players' Ages



Basketball Players' Ages

L1	L2
6	1
7	1
8	2
9	3
10	3
11	4
12	4

mean $=9.8$
med $=10$

Which statement about the given data sets is correct?

1) The data for soccer players are skewed right. Uniform
2) The data for soccer players have less spread than the data for basketball players. they both have a range of 6
3) The data for basketball players have the same median as the data for soccer players. $8.5 \neq 10$
4) The data for basketball players have a greater mean than the data for soccer players. $9.8 > 8.\bar{6}$

Causal
Two events are causal if one causes the other to happen

1. Which situation describes a correlation that is *not* a causal relationship?
1) The rooster crows, and the Sun rises. A rooster crowing does not cause the Sun to rise
2) The more miles driven, the more gasoline needed
3) The more powerful the microwave, the faster the food cooks.
4) The faster the pace of a runner, the quicker the runner finishes.

2. Which situation describes a correlation that is a causal relationship?
1) The taller a person, the smarter they are.
2) The bigger a person's feet are, the bigger shoes they wear. Bigger feet causes them to need bigger shoes
3) The bigger a person's fingernails, the faster they can throw a baseball.
4) The longer a person's hair is, the more hats they wear.

3. Which relationship can best be described as causal?
1) height and intelligence
2) shoe size and running speed
3) number of correct answers on a test and test score the # of collect answers causes your score to be higher
4) number of students in a class and number of students with brown hair

4. Which situation describes a correlation that is *not* a causal relationship?
1) the length of the edge of a cube and the volume of the cube
2) the distance traveled and the time spent driving
3) the age of a child and the number of siblings the child has A child's age does not cause them to have more or less siblings.
4) the number of classes taught in a school and the number of teachers employed

5. Which situations described is not a causal relationship?
1) The number of baskets a team scores and their total amount of points
2) The number of points you score on a test and your test average
3) The amount of time you spend driving and the type of car you drive the amount of time you're driving does not cause you to need a certain car model
4) The faster you run a race the sooner you finish

Unit Conversions

CONVERSIONS

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilograms	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

To cancel out units, multiply by the conversion. The unit to cancel should have one on top and one on bottom. For a rate, the final units is the remaining top unit per the remaining bottom unit.

Convert the following units and round to the nearest tenth if necessary

1. 750 meter to kilometer

$750\cancel{m} \cdot \frac{1\ Km}{1000\cancel{m}}$

$\frac{750}{1000} = .75Km$

2. 220 centimeter to meter

$220\cancel{cm} \cdot \frac{1\ m}{100\cancel{cm}}$

$\frac{220}{100} = 2.2m$

3. 3.45 meter to centimeter

$3.45\cancel{m} \cdot \frac{100\ cm}{1\ \cancel{m}}$

$3.45(100) = 345cm$

4. 1.2 hours to minutes

$1.2\cancel{hr} \cdot \frac{60\ min}{1\ \cancel{hr}}$

$1.2(60) = 72min$

5. 6.2 miles to feet

$6.2\cancel{mi} \cdot \frac{5280\ ft}{1\ \cancel{mi}}$

$6.2(5280) = 32736\ ft$

6. 5000 feet to miles

$5000\cancel{ft} \cdot \frac{1\ mi}{5280\ \cancel{ft}}$ $\frac{5000}{5280} = .9mi$

7. 4 gallons to kiloliters

$4\ \cancel{gal} \cdot \frac{1\ \cancel{l}}{.264\ \cancel{gal}} \cdot \frac{1\ Kl}{1000\cancel{l}}$

$\frac{4}{.264(1000)} = .0\overline{15}Kl$

8. 2.4 feet to centimeters

$2.4\cancel{ft} \cdot \frac{12\ \cancel{in}}{1\ \cancel{ft}} \cdot \frac{2.54\ cm}{1\ \cancel{in}}$

$2.4(12)(2.54) = 73.152cm$

Convert the following rates and round to the nearest tenth if necessary

9. 100 km/hour to miles/minute

$\frac{100\ km}{1\ hr} \cdot \frac{1\ hr}{60\ min} \cdot \frac{.62\ mi}{1\ km}$

$\frac{100(.62)}{60} = 1.0\ mi/min$

10. 500 meters/second to kilometers/minute

$\frac{500\ m}{1\ sec} \cdot \frac{1\ Km}{1000\ m} \cdot \frac{60\ sec}{1\ min}$

$\frac{500(60)}{1000} = 30\ Km/min$

11. 12 gallons/second to liters/minute

$\frac{12\ gal}{1\ sec} \cdot \frac{1\ l}{.264\ gal} \cdot \frac{60\ sec}{1\ min}$

$\frac{12(60)}{.264} = 2727.3\ l/min$

12. 15 yards/hours to ft/minute

$\frac{15\ yds}{1\ hr} \cdot \frac{1\ hr}{60\ min} \cdot \frac{3\ ft}{1\ yds}$ $\frac{15(3)}{60} = .75\ ft/min$

13. 50 ft/sec to miles/hour

$\frac{50\ ft}{1\ sec} \cdot \frac{60\ sec}{1\ min} \cdot \frac{60\ min}{1\ hr} \cdot \frac{1\ mi}{5280\ ft}$

$\frac{50(60)(60)}{5280} = 34.1\ mi/hr$

14. 15 kilometers/sec to miles/hour

$\frac{15\ Km}{1\ sec} \cdot \frac{60\ sec}{1\ min} \cdot \frac{60\ min}{1\ hr} \cdot \frac{1\ mi}{1.609\ Km}$

$\frac{15(60)(60)}{1.609} = 33561.2\ mi/hr$

15. 20 cm/minute to feet/second

$\frac{20\ cm}{1\ min} \cdot \frac{1\ min}{60\ sec} \cdot \frac{1\ in}{2.54\ cm} \cdot \frac{1\ ft}{12\ in}$

$\frac{20}{60(2.54)(12)} = .01\ ft/sec$

16. 200 kilometers/hour to miles/second

$\frac{200\ Km}{1\ hr} \cdot \frac{1\ hr}{60\ min} \cdot \frac{1\ min}{60\ sec} \cdot \frac{1\ mi}{1.609\ Km}$

$\frac{200}{60(60)(1.609)} = .03\ mi/sec$

17. The Utica Boilermaker is a 15-kilometer road race. Sara is signed up to run this race and has done the following training runs:

I. 10 miles

II. 44,880 feet

III. 15,560 yards

Which run(s) are at least 15 kilometers?

1) I, only
2) II, only
3) I and III
4) II and III

$10\ mi \cdot \frac{1.609\ Km}{1\ mi} = 16.09\ Km$ ✓

$44{,}880\ ft \cdot \frac{1\ mi}{5280\ ft} \cdot \frac{1.609\ Km}{1\ mi}$ $\frac{44{,}880(1.609)}{5280} = 13.6765\ Km$ X

$15{,}560\ yds \cdot \frac{1\ mi}{1760\ yds} \cdot \frac{1.609\ Km}{1\ mi}$ $\frac{15{,}560(1.609)}{1760} = 14.225..$ X

18. Sarah travels on her bicycle at a speed of 22.7 miles per hour. What is Sarah's approximate speed, in kilometers per minute?

1) 0.2
2) 0.6
3) 36.5
4) 36.6

$\frac{22.7\ mi}{1\ hr} \cdot \frac{1.609\ Km}{1\ mi} \cdot \frac{1\ hr}{60\ min}$ $\frac{22.7(1.609)}{60} = .6\ Km/min$

19. A news report suggested that an adult should drink a minimum of 4 pints of water per day. Based on this report, determine the minimum amount of water an adult should drink, in fluid ounces, per week.

$\frac{4 \text{ pints}}{1 \text{ day}} \cdot \frac{2 \text{ cups}}{1 \text{ pints}} \cdot \frac{8 \text{ fl oz}}{1 \text{ cups}} \cdot \frac{7 \text{ day}}{1 \text{ week}}$ $4(2)(8)(7)$

448 fl oz/week

20. Bamboo plants can grow 91 centimeters per day. What is the approximate growth of the plant, in inches per hour?

1) 1.49
2) 3.79
3) 9.63
4) 35.83

$\frac{91 \text{ cm}}{1 \text{ day}} \cdot \frac{1 \text{ in}}{2.54 \text{ cm}} \cdot \frac{1 \text{ day}}{24 \text{ hr}}$

$\frac{91}{2.54(24)} = 1.49$ in/hr

21. A typical marathon is 26.2 miles. Allan averages 12 kilometers per hour when running in marathons. Determine how long it would take Allan to complete a marathon, to the *nearest tenth of an hour*. Justify your answer.

hours per marathon

$\cancel{\frac{12 \text{ km}}{1 \text{ hr}}} \cdot \frac{1 \text{ hr}}{12 \text{ km}} \cdot \frac{1.609 \text{ km}}{1 \text{ mi}} \cdot \frac{26.2 \text{ mi}}{1 \text{ marathon}}$

$\frac{1.609(26.2)}{12}$

3.5 hrs/marathon

22. A swimmer set a world record in the women's 1500-meter freestyle, finishing the race in 15.42 minutes. If 1 meter is approximately 3.281 feet, which set of calculations could be used to convert her speed to miles per hour?

1) $\frac{1500 \text{ meters}}{15.42 \text{ min}} \bullet \frac{60 \text{ min}}{1 \text{ hour}} \bullet \frac{1 \text{ meter}}{3.281 \text{ feet}} \bullet \frac{1 \text{ mile}}{5280 \text{ feet}}$

2) $\frac{1500 \text{ meters}}{15.42 \text{ min}} \bullet \frac{60 \text{ min}}{1 \text{ hour}} \bullet \frac{3.281 \text{ feet}}{1 \text{ meter}} \bullet \frac{1 \text{ mile}}{5280 \text{ feet}}$ mi/hr

3) $\frac{1500 \text{ meters}}{15.42 \text{ min}} \bullet \frac{3.281 \text{ feet}}{1 \text{ meter}} \bullet \frac{1 \text{ mile}}{5280 \text{ feet}}$ miles/min

4) $\frac{1500 \text{ meters}}{15.42 \text{ min}} \bullet \frac{60 \text{ min}}{1 \text{ hour}} \bullet \frac{1 \text{ mile}}{5280 \text{ feet}}$

23. It takes Tim 4.5 hours to run 50 kilometers. Which expression will allow him to change this rate to minutes per mile?

1) $\frac{4.5\text{ hr}}{50\text{ km}} \bullet \frac{1.609\text{ km}}{1\text{ mi}} \bullet \frac{60\text{ min}}{1\text{ hr}}$ min/mi

3) $\frac{50\text{ km}}{4.50\text{ hr}} \bullet \frac{1\text{ mi}}{1.609\text{ km}} \bullet \frac{1\text{ hr}}{60\text{ min}}$ mi/min

2) $\frac{50\text{ km}}{4.50\text{ hr}} \bullet \frac{1\text{ mi}}{1.609\text{ km}} \bullet \frac{60\text{ min}}{1\text{ hr}}$

4) $\frac{4.5\text{ hr}}{50\text{ km}} \bullet \frac{1\text{ mi}}{1.609\text{ km}} \bullet \frac{60\text{ min}}{1\text{ hr}}$

24. A construction worker needs to move 120 ft^3 of dirt by using a wheelbarrow. One wheelbarrow load holds 8 ft^3 of dirt and each load takes him 10 minutes to complete. One correct way to figure out the number of hours he would need to complete this job is

1) $\frac{120\text{ ft}^3}{1} \bullet \frac{10\text{ min}}{1\text{ load}} \bullet \frac{60\text{ min}}{1\text{ hr}} \bullet \frac{1\text{ load}}{8\text{ ft}^3}$

3) $\frac{120\text{ ft}^3}{1} \bullet \frac{1\text{ load}}{10\text{ min}} \bullet \frac{8\text{ ft}^3}{1\text{ load}} \bullet \frac{1\text{ hr}}{60\text{ min}}$

2) $\frac{120\text{ ft}^3}{1} \bullet \frac{60\text{ min}}{1\text{ hr}} \bullet \frac{8\text{ ft}^3}{10\text{ min}} \bullet \frac{1}{1\text{ load}}$

4) $\frac{120\text{ ft}^3}{1} \bullet \frac{1\text{ load}}{8\text{ ft}^3} \bullet \frac{10\text{ min}}{1\text{ load}} \bullet \frac{1\text{ hr}}{60\text{ min}}$

25. The following conversion was done correctly:

$$\frac{3\text{ miles}}{1\text{ hour}} \bullet \frac{1\text{ hour}}{60\text{ minutes}} \bullet \frac{5280\text{ feet}}{1\text{ mile}} \bullet \frac{12\text{ inches}}{1\text{ foot}}$$ inches/min

What were the final units for this conversion?

1) minutes per foot
2) minutes per inch
3) feet per minute
4) inches per minute

26. Olivia entered a baking contest. As part of the contest, she needs to demonstrate how to measure a gallon of milk if she only has a teaspoon measure. She converts the measurement using the ratios below:

$$\frac{4\text{ quarts}}{1\text{ gallon}} \bullet \frac{2\text{ pints}}{1\text{ quart}} \bullet \frac{2\text{ cups}}{1\text{ pint}} \bullet \frac{\frac{1}{4}\text{ cup}}{4\text{ tablespoons}} \bullet \frac{3\text{ teaspoons}}{1\text{ tablespoon}}$$

Which ratio is *incorrectly* written in Olivia's conversion?

1) $\frac{4\text{ quarts}}{1\text{ gallon}}$

3) $\frac{\frac{1}{4}\text{ cup}}{4\text{ tablespoons}}$

2) $\frac{2\text{ pints}}{1\text{ quart}}$

4) $\frac{3\text{ teaspoons}}{1\text{ tablespoon}}$

Reference Sheet for Algebra I (NGLS)

Conversions

1 mile = 5280 feet

1 mile = 1760 yards

1 pound = 16 ounces

1 ton = 2000 pounds

Conversions Across Measurement Systems

1 inch = 2.54 centimeters

1 meter = 39.37 inches

1 mile = 1.609 kilometers

1 kilometer = 0.6214 mile

1 pound = 0.454 kilogram

1 kilogram = 2.2 pounds

Quadratic Equation	$y = ax^2 + bx + c$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Equation of the Axis of Symmetry	$x = -\frac{b}{2a}$
Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Linear Equation Slope Intercept	$y = mx + b$
Linear Equation Point Slope	$y - y_1 = m(x - x_1)$

Exponential Equation	$y = ab^x$
Annual Compound Interest	$A = P(1 + r)^n$
Arithmetic Sequence	$a_n = a_1 + d(n - 1)$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Interquartile Range (IQR)	$IQR = Q_3 - Q_1$
Outlier	Lower Outlier Boundary $= Q_1 - 1.5(IQR)$
	Upper Outlier Boundary $= Q_3 + 1.5(IQR)$

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