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Algebra II

Newton's Law of Heating and Cooling

1. The Fahrenheit temperature of a heated object can be modeled by the function below.

$$F(t) = F_S + (F_0 - F_S)e^{-kt}$$

$F(t)$ = the temperature of the object after t minutes $=150$

t = time in minutes $=t$

F_S = the surrounding temperature $=68$

F_0 = the initial temperature of the object $=200$

k = a constant $=.05$

Hot chocolate at a temperature of 200°F (initial) is poured into a container. The room temperature is kept at a constant 68°F (surrounding) and k = 0.05.

After how much time, to the *nearest minute*, will the temperature of the hot chocolate be 150°F? (temperature after time passes)

$150 = 68 + (200-68)e^{-.05t}$

$-68 \quad -68$

$\frac{82}{132} = \frac{132e^{-.05t}}{132}$

$\ln\frac{41}{66} = \ln e^{-.05t}$

$\frac{\ln\frac{41}{66}}{-.05} = \frac{-.05t \ln e}{-.05}$

$10 = t$

After how much time, to the *nearest tenth of a minute*, will the temperature of the hot chocolate be 120°F?

$F(t) = 120$

$t = t$

$F_s = 68$

$F_0 = 200$

$k = .05$

$120 = 68 + (200-68)e^{-.05t}$

$-68 \quad -68$

$\frac{52}{132} = \frac{132e^{-.05t}}{132}$

$\ln\frac{13}{33} = \ln e^{-.05t}$

$\frac{\ln\frac{13}{33}}{-.05} = \frac{-.05t\ln e}{-.05}$

$18.6 = t$

2. The Fahrenheit temperature, $F(t)$, of a heated object at time t, in minutes, can be modeled by the function below. F_s is the surrounding temperature, F_0 is the initial temperature of the object, and k is a constant.

$$F(t) = F_S + (F_0 - F_S)e^{-kt}$$

Coffee at a temperature of 195°F (initial) is poured into a container. The room temperature is kept at a constant 68°F (surrounding temp) and k = 0.05. Coffee is safe to drink when its temperature is, at most, 120°F ($F(t)$). To the *nearest minute*, how long will it take until the coffee is safe to drink?

$F(t)$ = temperature after time passes $=120$

t = time in minutes $=t$

F_s = surrounding temp $=68$

F_0 = initial temp $=195$

k = constant $=.05$

$120 = 68 + (195-68)e^{-.05t}$

$-68 \quad -68$

$\frac{52}{127} = \frac{127e^{-.05t}}{127}$

$\ln\frac{52}{127} = \ln e^{-.05t}$

$\frac{\ln\frac{52}{127}}{-.05} = \frac{-.05t\ln e}{-.05}$

$18 = t$

surrounding initial

3. After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + (T_0 - T_a)e^{-kt}$$

$100 = 325 + (68 - 325)e^{-k(2)}$
$-325 \quad -325$
$-225 = -257e^{-2k}$
$\frac{-225}{-257} \quad \frac{}{-257}$
$\ln \frac{225}{257} = \ln e^{-2k}$
$\frac{\ln \frac{225}{257}}{-2} = \frac{-2k \ln e}{-2}$
$.066 = k$

T_a = the temperature surrounding the object = 325

T_0 = the initial temperature of the object = 68

t = the time in hours = 2

T = the temperature of the object after t hours = 100

k = decay constant = K

The turkey reaches the temperature of approximately 100° F after 2 hours. Find the value of k, to the *nearest thousandth*. Determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m.

$T_a = 325$
$T_0 = 68$
$t = 8am - 3PM = 7$
$T = T$
$K = .066$

$T = 325 + (68 - 325)e^{-.066(7)}$

$T = 163°$

surrounding initial

4. Empanadas are taken out of an oven when they reached a temperature of 168°F and put on the kitchen table at room temperature (68°F). After 8 minutes, the temperature of the empanadas is 125°F. The temperature of a cooled object can be given by the formula below:

$$T = T_a + (T_0 - T_a)e^{-kt}$$

T = the temperature of the object after t minutes = 125

t = time in minutes = 8

T_a = the surrounding temperature = 68

T_0 = the initial temperature of the object = 168

k = decay constant = K

$125 = 68 + (168 - 68)e^{-k(8)}$
$-68 \quad -68$
$\frac{57}{100} = \frac{100e^{-8k}}{100}$
$\ln \frac{57}{100} = \ln e^{-8k}$
$\frac{\ln(\frac{57}{100})}{-8} = \frac{-8k \ln e}{-8}$

Find the value of k, rounded to the *nearest thousandth*. Using your value of k, to the *nearest minute*, how long will it take for the empanadas to reach 100°F?

$.070 = k$

$T = 100$
$t = t$
$T_a = 68$
$T_0 = 168$
$K = .070$

$100 = 68 + (168 - 68)e^{-.070t}$
$-68 \quad -68$
$\frac{32}{100} = \frac{100e^{-.070t}}{100}$
$\ln \frac{8}{25} = \ln e^{-.070t}$

$\frac{\ln \frac{8}{25}}{-.070} = \frac{-.070t \ln e}{-.070}$

$16 = t$

$t = 1.5$ Surrounding air temp

5. Megan is performing an experiment in a lab where the air temperature is a constant 73°F and the liquid is 237°F. One and a half hours later, the temperature of the liquid is 112°F. Newton's law of cooling states $T(t) = T_a + (T_0 - T_a)e^{-kt}$ where:

initial T(t)

$T(t)$: temperature, °F, of the liquid at t hours = 112 $t = 1.5$

T_a: air temperature = 73

T_0: initial temperature of the liquid = 237

k: constant = K

$112 = 73 + (237-73)e^{-k(1.5)}$
$-73 \quad -73$
$\frac{39}{164} = \frac{164}{164}e^{-1.5k}$
$\ln \frac{39}{164} = \ln e^{-1.5k}$
$\frac{\ln \frac{39}{164}}{-1.5} = \frac{-1.5k \ln e}{-1.5}$
$.958 = k$

Determine the value of k, to the *nearest thousandth*, for this liquid. Determine the temperature of the liquid using your value for k, to the *nearest degree*, after two and a half hours. Megan needs the temperature of the liquid to be 80°F to perform the next step in her experiment. Use your value for k to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

$t=2.5$ T(t)

$T(t) = T$
$t = 2.5$
$T_a = 73$
$T_0 = 237$
$k = .958$

$T = 73 + (237-73)e^{-.958(2.5)}$
$T = 88°$

$T(t) = 80$
$t = t$
$T_a = 73$
$T_0 = 237$
$k = .958$

$80 = 73 + (237-73)e^{-.958t}$
$-73 \quad -73$
$\frac{7}{164} = \frac{164}{164}e^{-.958t}$
$\ln \frac{7}{164} = \ln e^{-.958t}$
$\frac{\ln \frac{7}{164}}{-.958} = \frac{-.958t \ln e}{-.958}$
$3.3 = t$

6. Objects cool at different rates based on the formula below.

$T = (T_0 - T_R)e^{-rt} + T_R$

T_0: initial temperature = 400

T_R: room temperature = 75

r: rate of cooling of the object = .0735

t: time in minutes that the object cools to a temperature, T = 5 $T = T$

Mark makes T-shirts using a hot press to transfer designs to the shirts. He removes a shirt from a press that heats the shirt to 400°F. The rate of cooling for the shirt is 0.0735 and the room temperature is 75°F. Tr Find the temperature of the shirt, to the *nearest degree*, after five minutes. At the same time, Mark's friend Jeanine removes a hoodie from a press that heats the hoodie to 450°F. After eight minutes, the hoodie measured 270°F. The room temperature is still 75°F. Determine the rate of cooling of the hoodie, to the *nearest ten thousandth*. The T-shirt and hoodie were removed at the same time. Determine when the temperature will be the same, to the *nearest minute*.

initial t T initial r equal → intersect

$T = (400-75)e^{-.0735(5)} + 75$
$T = 300°$

$T = 270$
$T_0 = 450$
$T_R = 75$
$r = r$
$t = 8$

$270 = (450-75)e^{-r(8)} + 75$
$-75 \quad -75$
$\frac{195}{375} = \frac{375}{375}e^{-8r}$
$\ln \frac{13}{25} = \ln e^{-8r}$
$\frac{\ln \frac{13}{25}}{-8} = \frac{-8r \ln e}{-8}$
$.0817 = r$

$75 + (400-75)e^{-.0735t} = (450-75)e^{-.0817t} + 75$
$Y1 = (400-75)e^{-.0735t} + 75$
$Y2 = (450-75)e^{-.0817t} + 75$
Intersect 2nd Trace
$t = 17$ minutes