

## Unit 5: Exponents and Logarithms Review Sheet

1. Justify why  $\frac{\sqrt[3]{x^2y^5}}{\sqrt[4]{x^3y^4}}$  is equivalent to  $x^{-\frac{1}{12}}y^{\frac{2}{3}}$  using properties of rational exponents, where  $x \neq 0$  and  $y \neq 0$ .

$$\frac{(x^2y^5)^{\frac{1}{3}}}{(x^3y^4)^{\frac{1}{4}}}$$

$$x^{\frac{2}{3}}y^{\frac{5}{3}}$$

$$x^{\frac{3}{4}}y^{\frac{1}{4}}$$

$$x^{-\frac{1}{12}}y^{\frac{2}{3}}$$

Radicals are fractional exponents

Get rid of parentheses  
Negative exponents are fractions

Clean it up  $\left\{ \begin{array}{l} \text{multiply} \\ \text{divide} \\ \text{reduce} \end{array} \right.$

2. For  $n$  and  $p > 0$ , is the expression  $\left(p^2n^{\frac{1}{2}}\right)^8 \sqrt{p^5n^4}$  equivalent to  $p^{18}n^6\sqrt{p}$ ? Justify your answer.

answer.  $(p^2n^{\frac{1}{2}})^8 (p^5n^4)^{\frac{1}{2}} = p^{18}n^6p^{\frac{1}{2}}$

$p^{16}n^4p^{\frac{8}{2}}n^2 = p^{18}n^6p^{\frac{1}{2}}$

$p^{\frac{37}{2}}n^6 = p^{\frac{37}{2}}n^6$  yes!

$(2) \frac{16}{2} + \frac{5}{2}$

$\frac{32}{2} + \frac{5}{2} = \frac{37}{2}$

$(2) \frac{18}{2} + \frac{1}{2}$

$\frac{36}{2} + \frac{1}{2} = \frac{37}{2}$

Solve the following equations for all values of  $x$  (reciprocal power)

3.  $3x^{\frac{2}{3}} - 11 = 289$   
 $+11 \quad +11$

$$3x^{\frac{2}{3}} = 300$$

$$\frac{3x^{\frac{2}{3}}}{3} = \frac{300}{3}$$

$$(x^{\frac{2}{3}})^{\frac{3}{2}} = (100)^{\frac{3}{2}}$$

$$x = 100000$$

4.  $2x^{\frac{1}{5}} - 2 = -8$   
 $+2 \quad +2$

$$\frac{2x^{\frac{1}{5}}}{2} = \frac{-6}{2}$$

$$(x^{\frac{1}{5}})^5 = (-3)^5$$

$$x = -243$$

Common exponential base

Solve the following equations for all values of x

5.  $27^x = 9^{x+2}$

$(3^3)^x = (3^2)^{x+2}$   $x=4$   
 $3^{3x} = 3^{2x+4}$   
 $3x = 2x+4$   
 $-2x -2x$

6.  $64^{x-2} = 256^{2x}$

$(2^6)^{x-2} = (2^8)^{2x}$   
 $6(x-2) = 8(2x)$   
 $6x-12 = 16x$   
 $-6x -6x$   
 $\frac{-12}{10} = \frac{10x}{10}$   
 $-\frac{6}{5} = x$

Expand the following logarithms

7.  $\ln \frac{\sqrt{x}}{y^3}$

$\ln x^{\frac{1}{2}} - \ln y^3$   
 $\frac{1}{2} \ln x - 3 \ln y$

8.  $\log \frac{m^3 \sqrt{n}}{k^2}$

$\log m^3 + \log n^{\frac{1}{2}} - \log k^2$   
 $3 \log m + \frac{1}{2} \log n - 2 \log k$

9. The expression  $\frac{1}{2} \log m - 3 \log n$  is equivalent to

- (1)  $\log \sqrt{m} + \log n^3$
- (2)  $\log \frac{1}{2} m - 3 \log n$

- (3)  $\log \frac{m^2}{3\sqrt{n}}$
- (4)  $\log \frac{\sqrt{m}}{n^3}$

$\log m^{\frac{1}{2}} / n^3$

10. If  $\log_b x = 3 \log_b p - \left[ 2 \log_b t + \frac{1}{2} \log_b r \right]$ , then the value of x is

- 1)  $\frac{p^3}{\sqrt{t^2 r}}$
- 2)  $p^3 t^2 r^{\frac{1}{2}}$
- 3)  $\frac{p^3 t^2}{\sqrt{r}}$
- (4)  $\frac{p^3}{t^2 \sqrt{r}}$

$\log_b \frac{p^3}{t^2 r^{\frac{1}{2}}}$

Solve the following equations  
 exponential form

11.  $\log_{16}(p^2 - p + 4) - \log_{16}(2p + 11) = \frac{3}{4}$

$\log_{16} \frac{p^2 - p + 4}{2p + 11} = \frac{3}{4}$

$16^{\frac{3}{4}} = \frac{p^2 - p + 4}{2p + 11}$

$\log_{16} 88 = p^2 - p + 4$   
 $-\log_{16} 88 \quad -\log_{16} 88$

$0 = p^2 - 17p - 84$   
 $0 = (p - 21)(p + 4)$   
 $p = 21 \quad p = -4$

cancel logs

13.  $\log(x - 6) - \log(3) = \log 6$

$\log \frac{x - 6}{3} = \log 6$

$\frac{x - 6}{3} = 6$

$x - 6 = 18$   
 $+6 \quad +6$

$x = 24$

log both sides

15.  $8 + 2(4)^{-5x} = 14$

$\frac{2(4)^{-5x}}{2} = \frac{6}{2}$

$\log 4^{-5x} = \log 3$

$\frac{-5x \log 4}{-5 \log 4} = \frac{\log 3}{-5 \log 4}$

$x = -.2$

12.  $\log_{(x+3)}(2x+3) + \log_{(x+3)}(x+5) = 2$

$\log_{(x+3)}(2x+3)(x+5) = 2$

$(x+3)^2 = (2x+3)(x+5)$   
 $(x+3)(x+3) = (2x+3)(x+5)$

$x^2 + 3x + 3x + 9 = 2x^2 + 10x + 15$   
 $x^2 + 6x + 9 = 2x^2 + 10x + 15$   
 $-x^2 - 4x - 6 = 0$

$0 = x^2 + 7x + 6$   
 $0 = (x+6)(x+1)$

$x = -6$   
 $x = -1$

14.  $\log_8(x - 40) - \log_8(x - 10) = \log_8(x + 2)$

$\log_8 \frac{x - 40}{x - 10} = \log_8(x + 2)$

$\frac{x - 40}{x - 10} = x + 2$

$x - 40 = x^2 - 10x + 2x - 20$   
 $x - 40 = x^2 - 8x - 20$   
 $-x + 40 = x^2 - 8x - 20$

$0 = x^2 - 9x + 20$   
 $0 = (x - 5)(x - 4)$   
 $x = 5 \quad x = 4$

No Solution

16.  $256 + 3(2)^{6x} = 2700$

$\frac{3(2)^{6x}}{3} = \frac{2444}{3}$

$\log 2^{6x} = \log 814.6$

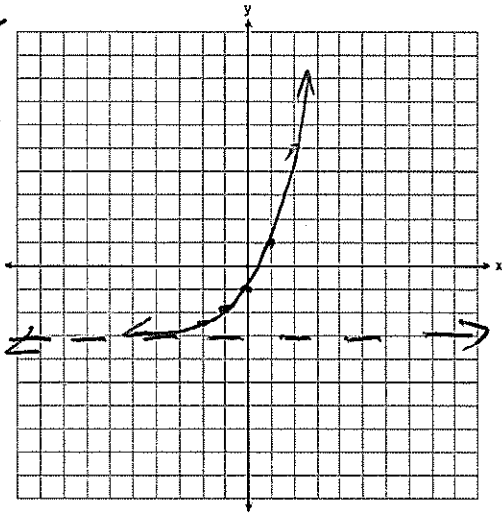
$\frac{6x \log 2}{6 \log 2} = \frac{\log 814.6}{6 \log 2}$

$x = 1.6$

Graph the following equations on the grid provided

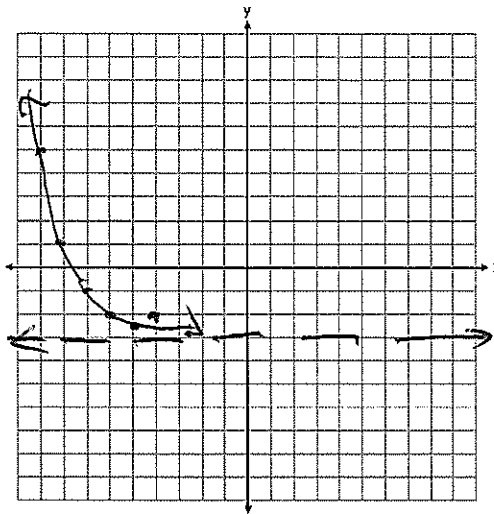
17.  $y = 2^{x+1} - 3$  asymptote:  $y = -3$

x	y
-2	-2.5
-1	-2
0	-1.5
1	-1
2	-0.5



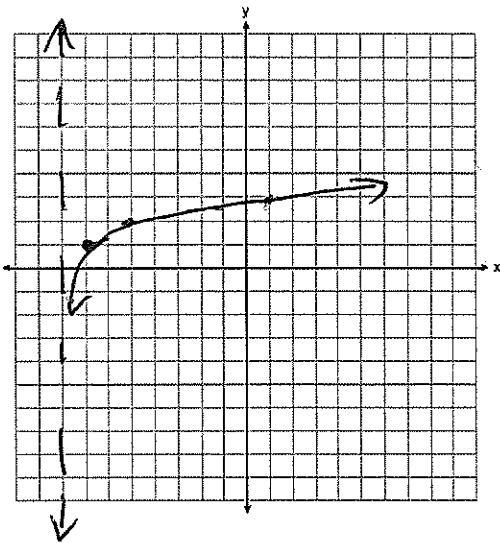
18.  $y = \left(\frac{1}{2}\right)^{x+6} - 3$  asymptote:  $y = -3$

x	y
-9	5
-8	1
-7	-1
-6	-2
-5	-2.5



19.  $y = \log_3(x+8) + 1$  asymptote:  $x = -8$

x	y
-8	1
-7	1.5
-5	2
-1	3



20.  $y = \log_2(x+9) - 3$  asymptote:  $x = -9$

x	y
-9	1
-8	-3
-7	-2
-5	-1
-1	0
7	1

