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Algebra II



Polynomial Equations with Irrational/Imaginary Solutions

Solve each of the following equations and express irrational answers in simplest radical form or simplest $a+bi$ form

1. $x^4 + 8x^2 = 9$
 $-9 -9$

$$x^4 + 8x^2 - 9 = 0$$

$$(x^2 + 9)(x^2 - 1) = 0$$

$x^2 + 9 = 0$	$x^2 - 1 = 0$
$\frac{-9}{+9} -9$	$\frac{-1}{+1} +1$
$\sqrt{x^2} = \sqrt{-9}$	$\sqrt{x^2} = \sqrt{1}$
$x = \pm 3i$	$x = \pm 1$

2. $x^4 + 4x^2 = 45$
 $-45 -45$

$$x^4 + 4x^2 - 45 = 0$$

$$(x^2 + 9)(x^2 - 5) = 0$$

$x^2 + 9 = 0$	$x^2 - 5 = 0$
$\frac{-9}{-9} -9$	$\frac{-5}{+5} +5$
$\sqrt{x^2} = \sqrt{-9}$	$\sqrt{x^2} = \sqrt{5}$
$x = \pm 3i$	$x = \pm \sqrt{5}$

3. $\left(\frac{x^3 + 3x^2}{x^2} + \frac{4x + 12}{4}\right) = 0$

$$x^2(x+3) + 4(x+3) = 0$$

$$(x^2 + 4)(x+3) = 0$$

$x^2 + 4 = 0$	$x + 3 = 0$
$\frac{-4}{-4} -4$	$\frac{-3}{-3} -3$
$\sqrt{x^2} = \sqrt{-4}$	$x = -3$
$x = \pm 2i$	

4. $x^3 - 3x^2 = -16x + 48$
 $+16x - 48 +16x - 48$

$$x^3 - 3x^2 + 16x - 48 = 0$$

$$\frac{x^3 - 3x^2}{x^2} + \frac{16x - 48}{16} = 0$$

$$x^2(x-3) + 16(x-3) = 0$$

$$(x^2 + 16)(x-3) = 0$$

$x^2 + 16 = 0$	$x - 3 = 0$
$\frac{-16}{-16} -16$	$\frac{-3}{+3} +3$
$\sqrt{x^2} = \sqrt{-16}$	$x = 3$
$x = \pm 4i$	

5. $x^4 - 6x^2 = -8$
 $+8 +8$

$$x^4 - 6x^2 + 8 = 0$$

$$(x^2 - 4)(x^2 - 2) = 0$$

$x^2 - 4 = 0$	$x^2 - 2 = 0$
$\frac{-4}{+4} +4$	$\frac{-2}{+2} +2$
$\sqrt{x^2} = \sqrt{4}$	$\sqrt{x^2} = \sqrt{2}$
$x = \pm 2$	$x = \pm \sqrt{2}$

6. $x^3 + 4x^2 - 2x = 8$
 $-8 -8$

$$x^3 + 4x^2 - 2x - 8 = 0$$

$$\frac{x^3 + 4x^2}{x^2} - \frac{2x - 8}{-2} = 0$$

$$x^2(x+4) - 2(x+4) = 0$$

$$(x^2 - 2)(x+4) = 0$$

$x^2 - 2 = 0$	$x + 4 = 0$
$\frac{-2}{+2} +2$	$\frac{-4}{-4} -4$
$\sqrt{x^2} = \sqrt{2}$	$x = -4$
$x = \pm \sqrt{2}$	

$$7. x^4 - 4x^2 - 32 = 0$$

$$(x^2 - 8)(x^2 + 4) = 0$$

$$\begin{array}{l|l} x^2 - 8 = 0 & x^2 + 4 = 0 \\ +8 & -4 & -4 \end{array}$$

$$\sqrt{x^2} = \sqrt{8} \quad \sqrt{x^2} = \sqrt{-4}$$

$$x = \pm\sqrt{8} \quad x = \pm 2i$$

$$\pm\sqrt{4 \cdot 2}$$

$$x = \pm 2\sqrt{2}$$

$$8. \frac{x^3 - 3x^2}{x^2 x^2} \frac{-5x + 15}{-5 - 5} = 0$$

$$x^2(x-3) - 5(x-3) = 0$$

$$(x^2 - 5)(x-3) = 0$$

$$\begin{array}{l|l} x^2 - 5 = 0 & x - 3 = 0 \\ +5 & +3 & +3 \end{array}$$

$$\sqrt{x^2} = \sqrt{5} \quad x = 3$$

$$x = \pm\sqrt{5}$$

$$9. 2x^3 - 3x^2 = -18x + 21$$

$$+18x - 21 \quad +18x - 21$$

$$\frac{(2x^3 - 3x^2 + 18x - 21)}{x^2} = 0$$

$$x^2(2x-3) + 9(2x-3) = 0$$

$$(x^2 + 9)(2x - 3) = 0$$

$$x^2 + 9 = 0 \quad 2x - 3 = 0$$

$$\begin{array}{l|l} x^2 + 9 = 0 & 2x - 3 = 0 \\ -9 & +3 & +3 \\ \sqrt{x^2} = \sqrt{-9} & \frac{2x}{2} = \frac{3}{2} \\ x = \pm 3i & x = \frac{3}{2} \end{array}$$

$$10. x^4 - 6x^2 - 27 = 0$$

$$(x^2 - 9)(x^2 + 3) = 0$$

$$\begin{array}{l|l} x^2 - 9 = 0 & x^2 + 3 = 0 \\ +9 & -3 & -3 \end{array}$$

$$\sqrt{x^2} = \sqrt{9} \quad \sqrt{x^2} = \sqrt{-3}$$

$$x = \pm 3 \quad x = \pm i\sqrt{3}$$

$$11. x^4 + 4x^3 + 4x^2 = -16x$$

$$+16x \quad +16x$$

$$\frac{x^4 + 4x^3 + 4x^2 + 16x}{x} = 0$$

$$x(x^3 + 4x^2 + 4x + 16) = 0$$

$$x[x^2(x+4) + 4(x+4)] = 0$$

$$x(x^2 + 4)(x + 4) = 0$$

$$x = 0 \quad x^2 + 4 = 0 \quad x + 4 = 0$$

$$\sqrt{x^2} = \sqrt{-4} \quad x = -4$$

$$x = \pm 2i$$

$$12. 3x^5 - 48x = 0$$

$$\frac{3x^5}{3x} - \frac{48x}{3x} = 0$$

$$3x(x^4 - 16) = 0$$

$$3x(x^2 + 4)(x^2 - 4) = 0$$

$$\begin{array}{l|l|l} \frac{3x}{3} = 0 & x^2 + 4 = 0 & x^2 - 4 = 0 \\ +4 & +4 & +4 & +4 \end{array}$$

$$x = 0 \quad \sqrt{x^2} = \sqrt{-4} \quad \sqrt{x^2} = \sqrt{4}$$

$$x = \pm 2i \quad x = \pm 2$$