

Polynomial Equations with Irrational/Imaginary Solutions

Solve each of the following equations and express irrational answers in simplest radical form or simplest $a+bi$ form

1. $x^3 + 2x^2 = 4x$
 $-4x \quad -4x$

$$x^3 + 2x^2 - 4x = 0$$

$$x(x^2 + 2x - 4) = 0$$

$$x = 0 \quad | \quad x^2 + 2x - 4 = 0$$

$$+4 \quad -4$$

$$x^2 + 2x + \square = 4 + \square$$

$$(x+1)(x+1) = 5$$

$$\sqrt{(x+1)^2} = \sqrt{5}$$

$$x+1 = \pm\sqrt{5}$$

$$x = -1 \pm \sqrt{5}$$

2. $x^3 + 3x = 2x^2$
 $-2x^2 \quad -2x^2$

$$x^3 - 2x^2 + 3x = 0$$

$$x(x^2 - 2x + 3) = 0$$

$$x = 0 \quad | \quad x^2 - 2x + 3 = 0$$

$$-3 \quad -3$$

$$x^2 - 2x + \square = -3 + \square$$

$$(x-1)(x-1) = -2$$

$$\sqrt{(x-1)^2} = \sqrt{-2}$$

$$x-1 = \pm\sqrt{-2}$$

$$x = 1 \pm i\sqrt{2}$$

3. $x^4 - 6x^2 = -8$
 $+8 \quad +8$

$$x^4 - 6x^2 + 8 = 0$$

$$(x^2 - 4)(x^2 - 2) = 0$$

$$(x+2)(x-2)(x^2-2) = 0$$

$$x+2=0 \quad | \quad x-2=0 \quad | \quad x^2-2=0$$

$$x=-2 \quad | \quad x=2 \quad | \quad \sqrt{x^2-2} = \sqrt{2}$$

$$x = -2, 2, \pm\sqrt{2}$$

4. $3x^4 + 6x^3 + 3x^2 = 0$

$$3x^2(x^2 + 2x + 1) = 0$$

$$3x^2(x+1)(x+1) = 0$$

$$3x^2 = 0 \quad | \quad x+1=0 \quad | \quad x+1=0$$

$$x=0 \quad | \quad x=-1 \quad | \quad x=-1$$

5. $2x^3 + 58x = 20x^2$
 $-20x^2 \quad -20x^2$

$$2x^3 - 20x^2 + 58x = 0$$

$$2x(x^2 - 10x + 29) = 0$$

$$2x = 0 \quad | \quad x^2 - 10x + 29 = 0$$

$$x = 0 \quad | \quad -29 \quad -29$$

$$x^2 - 10x + \square = -29 + \square$$

$$(x-5)(x-5) = -4$$

$$\sqrt{(x-5)^2} = \sqrt{-4}$$

$$x-5 = \pm 2i$$

$$x = 5 \pm 2i$$

6. $2x^3 - 3x^2 = -18x + 27$
 $+18x - 21 \quad +18x - 21$

$$\frac{2x^3 - 3x^2 + 18x - 27}{x^2} = 0$$

$$x^2(2x-3) + 9(2x-3) = 0$$

$$(x^2+9)(2x-3) = 0$$

$$x^2+9=0 \quad | \quad 2x-3=0$$

$$x^2 = -9 \quad | \quad 2x = 3$$

$$x = \pm 3i \quad | \quad x = \frac{3}{2}$$

7. $4x^3 - 10x^2 + 2x = 0$

$$2x(2x^2 - 5x + 1) = 0$$

$$2x = 0 \quad | \quad 2x^2 - 5x + 1 = 0$$

$$x = 0 \quad | \quad a=2 \quad | \quad x = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)}$$

$$b=-5 \quad | \quad c=1$$

$$x = \frac{5 \pm \sqrt{17}}{4}$$

8. $x^3 + 4x^2 - 2x = 8$
 $-8 \quad -8$

$$x^3 + 4x^2 - 2x - 8 = 0$$

$$x^2 \quad x^2 \quad -2 \quad -2$$

$$x^2(x+4) - 2(x+4) = 0$$

$$(x^2-2)(x+4) = 0$$

$$x^2-2=0 \quad | \quad x+4=0$$

$$+2 \quad +2 \quad -4 \quad -4$$

$$\sqrt{x^2-2} = \sqrt{2} \quad | \quad x = -4$$

$$x = \pm\sqrt{2}, -4$$

9. $x^4 - 4x^2 - 32 = 0$

$(x^2 - 8)(x^2 + 4) = 0$

$x^2 - 8 = 0$
 $+8 \quad +8$
 $\sqrt{x^2 = 8}$
 $x = \pm\sqrt{8}$

$x^2 + 4 = 0$
 $-4 \quad -4$
 $\sqrt{x^2 = -4}$
 $x = \pm 2i$

$x = \pm 2\sqrt{2}$

11. $3x^3 + 12x^2 = -6x$

$3x^3 + 12x^2 + 6x = 0$
 $3x(x^2 + 4x + 2) = 0$
 $3x = 0 \quad x^2 + 4x + 2 = 0$
 $x = 0 \quad x^2 + 4x + 4 = -2 + 4$

$(x+2)(x+2) = 2$
 $\sqrt{(x+2)^2 = 2}$
 $x+2 = \pm\sqrt{2}$
 $x = -2 \pm \sqrt{2}$

10. $\left(\frac{x^3 - 3x^2}{x^2} - \frac{5x + 15}{-5}\right) = 0$

$x(x-3) - 5(x-3) = 0$
 $(x^2 - 5)(x-3) = 0$
 $x^2 - 5 = 0 \quad x-3 = 0$
 $+5 \quad +5 \quad +3 \quad +3$
 $\sqrt{x^2 = 5}$
 $x = \pm\sqrt{5} \quad x = 3$

12. $x^4 + 4x^3 + 4x^2 = -16x$

$x^4 + 4x^3 + 4x^2 + 16x = 0$
 $x^3(x+4) + 4x(x+4) = 0$
 $(x^3 + 4x)(x+4) = 0$
 $x(x^2 + 4)(x+4) = 0$

$x = 0 \quad x^2 + 4 = 0 \quad x + 4 = 0$
 $-4 \quad -4 \quad -4 \quad -4$
 $\sqrt{x^2 = -4}$
 $x = \pm 2i \quad x = -4$

13. $2x^4 - 22x^2 + 36 = 0$

$x^4 - 11x^2 + 18 = 0$
 $(x^2 - 9)(x^2 - 2) = 0$
 $(x+3)(x-3)(x^2 - 2) = 0$
 $x+3 = 0 \quad x-3 = 0 \quad x^2 - 2 = 0$
 $-3 \quad -3 \quad +2 \quad +2$
 $\sqrt{x^2 = 2}$
 $x = \pm\sqrt{2}$
 $x = -3 \quad x = 3 \quad x = \pm\sqrt{2}$

15. $6x^3 + 18x^2 - 9x = 0$

$3x(2x^2 + 6x - 3) = 0$
 $3x = 0 \quad 2x^2 + 6x - 3 = 0$
 $x = 0$
 $a = 2 \quad x = \frac{-6 \pm \sqrt{6^2 - 4(2)(-3)}}{2(2)}$
 $b = 6$
 $c = -3 \quad x = \frac{-6 \pm \sqrt{60}}{4}$
 $x = \frac{-6 \pm 2\sqrt{15}}{4}$
 $x = \frac{-3 \pm \sqrt{15}}{2}$

14. $3x^5 - 48x = 0$

$3x(x^4 - 16) = 0$
 $3x(x^2 - 4)(x^2 + 4) = 0$
 $3x(x+2)(x-2)(x^2 + 4) = 0$
 $3x = 0 \quad x+2 = 0 \quad x-2 = 0 \quad x^2 + 4 = 0$
 $x = 0 \quad x = -2 \quad x = 2 \quad x^2 = -4$
 $x = \pm 2i$

16. $x^4 - 6x^2 - 27 = 0$

$(x^2 - 9)(x^2 + 3) = 0$
 $(x+3)(x-3)(x^2 + 3) = 0$
 $x+3 = 0 \quad x-3 = 0 \quad x^2 + 3 = 0$
 $-3 \quad -3 \quad +3 \quad +3$
 $\sqrt{x^2 = -3}$
 $x = \pm i\sqrt{3}$