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Date _____
Pre Calculus

Solving Polynomial Inequalities

Solve each of the following for x and graph on a number line.

1. $x^3 + 4x^2 - 9x - 36 < 0$
 $(x^2 + 4x)(x - 3) = 0$
 $\frac{x^2}{x^2} \frac{x^2}{x^2} \frac{-9}{-9} \frac{-36}{-4}$

$x^2(x+4) - 9(x+4) = 0$

$(x^2 - 9)(x + 4) = 0$ 

$(x+3)(x-3)(x+4) = 0$

$x = -3 \quad x = 3 \quad x = -4$ $x < -4$ or $-3 < x < 3$
 $(-\infty, -4) \cup (-3, 3)$

-5: $(-)(-)(-) < 0$ ✓

0: $(+)(-)(+) < 0$ ✓

2. $\frac{x^3 - 5x^2 - 4x - 20}{x^2 - 4} \geq 0$

$x^2(x-5) - 4(x-5) = 0$

$(x^2 - 4)(x - 5) = 0$

$(x+2)(x-2)(x-5) = 0$

$x = -2 \quad x = 2 \quad x = 5$



$-2 \leq x \leq 2$ or $x \geq 5$

$[-2, 2] \cup [5, \infty)$

-3: $(-)(-)(-) \geq 0$ X

0: $(+)(-)(-) \geq 0$ ✓

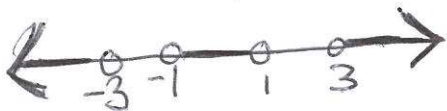
6: $(+)(+)(+) \geq 0$ ✓

3. $x^4 - 10x^2 + 9 > 0$

$(x^2 - 9)(x^2 - 1) = 0$

$(x+3)(x-3)(x+1)(x-1) = 0$

$x = -3 \quad x = 3 \quad x = -1 \quad x = 1$



-4: $(-)(-)(-)(-) > 0$ ✓

0: $(+)(-)(+)(-) > 0$ ✓

4: $(+)(+)(+)(+) > 0$ ✓

$x < -3$ or $-1 < x < 1$ or $x > 3$

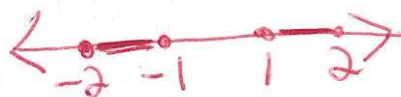
$(-\infty, -3) \cup (-1, 1) \cup (3, \infty)$

4. $x^4 - 5x^2 + 4 \leq 0$

$(x^2 - 4)(x^2 - 1) = 0$

$(x+2)(x-2)(x+1)(x-1) = 0$

$x = -2 \quad x = 2 \quad x = -1 \quad x = 1$



-3: $(-)(-)(-)(-) \leq 0$ X

-1.5: $(+)(-)(-)(-) \leq 0$ ✓

1.5: $(+)(-)(+)(+) \leq 0$ ✓

$-2 \leq x \leq -1$ or $1 \leq x \leq 2$

$[-2, -1] \cup [1, 2]$

$$5. x^3 + 3x^2 > 16x + 48$$

$$-16x - 48 - 16x - 48$$

$$\frac{(x^3 + 3x^2) - (16x + 48)}{x^2} = 0$$

$$\frac{x^2(x+3) - 16(x+3)}{x^2} = 0$$

$$(x^2 - 16)(x+3) = 0$$

$$(x+4)(x-4)(x+3) = 0$$

$$x = -4 \quad | \quad x = 4 \quad | \quad x = -3$$



$$-4 < x < -3 \text{ or } x > 4$$

$$(-4, -3) \cup (4, \infty)$$

$$-5: (-)(-)(-) > 0 \quad \times$$

$$-3.5: (+)(-)(-) > 0 \quad \checkmark$$

$$5: (+)(+)(+) > 0 \quad \checkmark$$

$$6. x^3 + 7x^2 \geq 30x$$

$$-30x - 30x$$

$$x^3 + 7x^2 - 30x = 0$$

$$x(x^2 + 7x - 30) = 0$$

$$x(x+10)(x-3) = 0$$

$$x = 0 \quad | \quad x = -10 \quad | \quad x = 3$$

$$-11: (-)(-)(-) \geq 0 \quad \times$$

$$-5: (-)(+)(-) \geq 0 \quad \checkmark$$



$$-10 \leq x \leq 0 \text{ or } x \geq 3$$

$$[-10, 0] \cup [3, \infty)$$

$$7. x^4 - 20x^2 < 64$$

$$+64 + 64$$

$$x^4 - 20x^2 + 64 = 0$$

$$(x^2 - 16)(x^2 - 4) = 0$$

$$(x+4)(x-4)(x+2)(x-2) = 0$$

$$x = -4 \quad | \quad x = 4 \quad | \quad x = -2 \quad | \quad x = 2$$



$$-4 < x < -2 \text{ or } 2 < x < 4$$

$$(-4, -2) \cup (2, 4)$$

$$-5: (-)(-)(-)(-) < 0 \quad \times$$

$$-3: (+)(-)(-)(-) < 0 \quad \checkmark$$

$$3: (+)(-)(+)(+) < 0 \quad \checkmark$$

$$8. x^3 + 6x^2 - 25x \leq 150$$

$$-150 - 150$$

$$\frac{(x^3 + 6x^2) - (25x + 150)}{x^2} = 0$$

$$\frac{x^2(x+6) - 25(x+6)}{x^2} = 0$$

$$x^2(x+6) - 25(x+6) = 0$$

$$(x^2 - 25)(x+6) = 0$$

$$(x+5)(x-5)(x+6) = 0$$

$$x = -5 \quad | \quad x = 5 \quad | \quad x = -6$$

$$-7: (-)(-)(-) \leq 0 \quad \checkmark$$

$$0: (+)(-)(+) \leq 0 \quad \checkmark$$



$$x \leq -6 \text{ or } -5 \leq x \leq 5$$

$$(-\infty, -6] \cup [-5, 5]$$

$$9. 2x^3 - 4x^2 > 16x$$

$$-16x - 16x$$

$$2x^3 - 4x^2 - 16x = 0$$

$$2x(x^2 - 2x - 8) = 0$$

$$2x(x-4)(x+2) = 0$$

$-2 < x < 0$ or $x > 4$
 $(-2, 0) \cup (4, \infty)$

$x=0$ | $x=4$ | $x=-2$

-3: $(-)(-)(-) > 0$ X
 -1: $(-)(-)(+) > 0$ ✓
 5: $(+)(+)(+) > 0$ ✓

$$10. x^4 \geq +125x^2 - 2500$$

$$-125x^2 + 2500$$

$$x^4 - 125x^2 + 2500 = 0$$

$$(x^2 - 100)(x^2 - 25) = 0$$

$$(x+10)(x-10)(x+5)(x-5) = 0$$

$$x = -10 \quad | \quad x = 10 \quad | \quad x = -5 \quad | \quad x = 5$$



$$x \leq -10 \text{ or } -5 \leq x \leq 5 \text{ or } x \geq 10$$

$$(-\infty, -10] \cup [-5, 5] \cup [10, \infty)$$

-11: $(-)(-)(-)(-) \geq 0$ ✓
 0: $(+)(-)(+)(-) \geq 0$ ✓
 11: $(+)(+)(+)(+) \geq 0$ ✓

-11: $(-)(-)(-)(-) \geq 0$ ✓

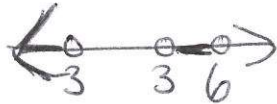
0: $(+)(-)(+)(-) \geq 0$ ✓

11: $(+)(+)(+)(+) \geq 0$ ✓

$$11. x^3 - 6x^2 < 9x - 54$$

$$-9x + 54 \quad -9x + 54$$

$$(x^3 - 6x^2 + 9x - 54) = 0$$



$$x^2(x-6) - 9(x-6) = 0$$

$$(x^2 - 9)(x-6) = 0$$

$$(x+3)(x-3)(x-6) = 0$$

$$x = -3 \quad | \quad x = 3 \quad | \quad x = 6$$

-4: $(-)(-)(-) < 0$ ✓

4: $(+)(+)(-) < 0$ ✓

$$12. x^4 + 2x^3 - 16x^2 \leq 32x$$

$$-32x - 32x$$

$$x^4 + 2x^3 - 16x^2 - 32x = 0$$

$$x^2(x^2 + 2x - 16) = 0$$

$$x[x^2(x+2) - 16(x+2)] = 0$$

$$x[(x^2 - 16)(x+2)] = 0$$

$$x(x+4)(x-4)(x+2) = 0$$

$$x = 0 \quad | \quad x = -4 \quad | \quad x = 4 \quad | \quad x = -2$$



-5: $(-)(-)(-)(-) \leq 0$ X

-3: $(-)(+)(-)(-) \leq 0$ ✓

1: $(+)(+)(-)(+) \leq 0$ ✓

$$-4 \leq x \leq -2 \text{ or } 0 \leq x \leq 4$$

$$[-4, -2] \cup [0, 4]$$